Structure of subatomic particles

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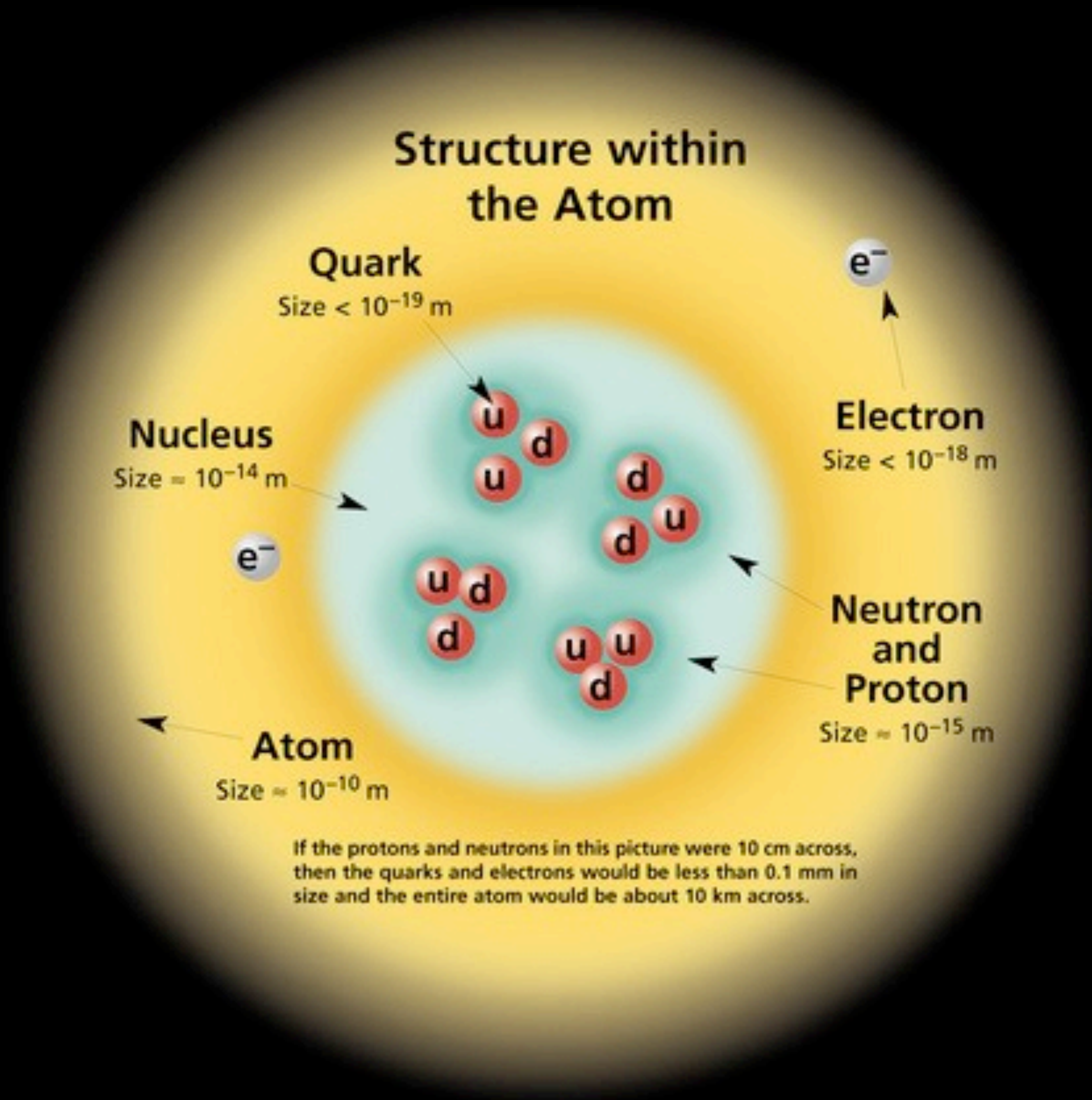
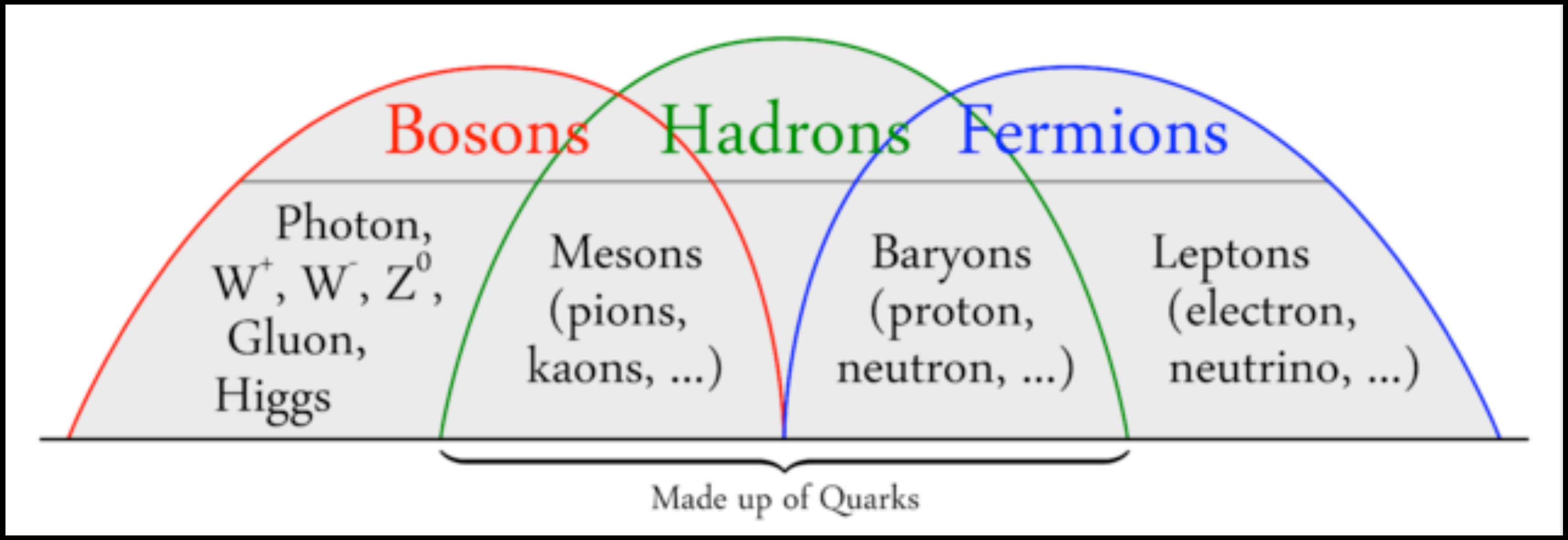
The world of subatomic particles



Categories of particles



Introduction

- For atoms, structure means the spatial distribution of electrons
  - It is described by the ground state wave function
  - For the hydrogen atom, neglecting spin, the probability density is given by

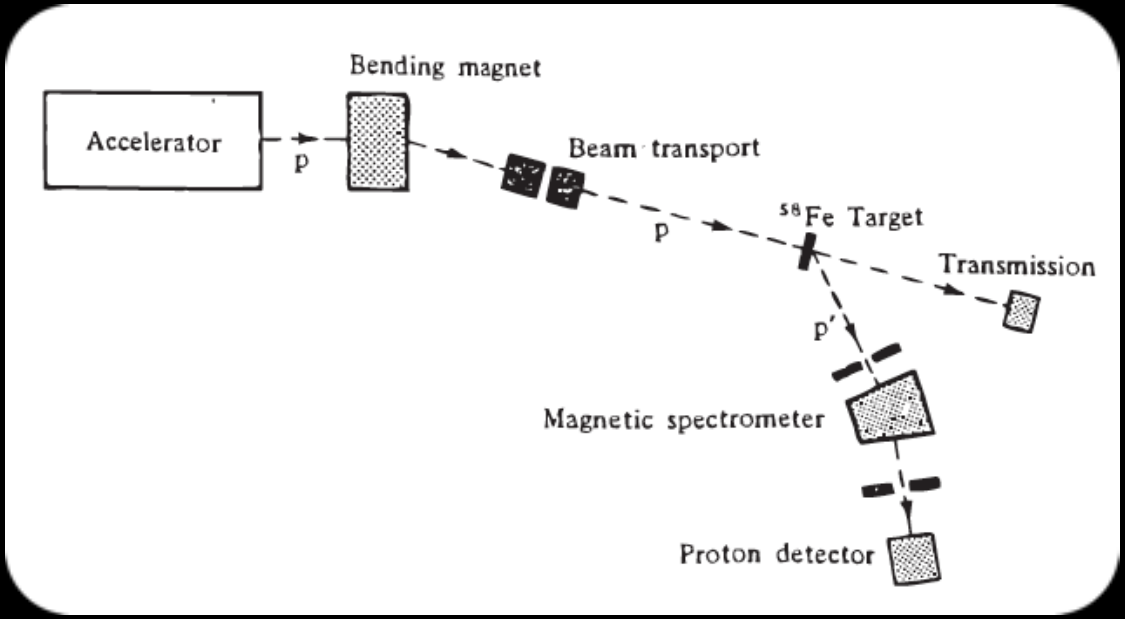
$$\rho(\vec{x}) = \psi^*(\vec{x})\psi(\vec{x})$$

$\psi(\vec{x})$ the electron wave function at the point $\vec{x}$

  - The structure includes all excited states
  - Only if all excited states are known is the structure of an atom fully known and determined
- For nuclei charge and mass distributions are not identical
- For nucleons, studying their structure implies shooting a beam of particles accelerated at high velocities on them
  - The nucleons, usually being at rest initially, recoil back from the interaction with very large velocities, close to the velocity of light
  - Their structures are described in terms of form factors
- For leptons, no internal structure is known so far

Elastic scattering vs spectroscopy

- In spectroscopy, one angle is selected and the spectrum of the scattered particles is studied at this angle
- In structure experiments, the detector configuration is very similar to the one used in spectroscopy experiments
  - the detectors look at the elastic peak
- Structure of particles is studied via either elastic or inelastic interactions between the particles of study and probes (beams) of incoming particles



- Elastic scattering experiments have provided a significant part of the information we now know about the structure of subatomic particles
- Elastic scattering experiments very similar to the spectroscopy ones
  - the intensity of the elastic peak is measured as a function of the scattering angle
    - the intensity changes with angle because of the different recoil of the target particles
  - the observed intensity is translated into a differential cross-section $d\sigma/d\Omega$
  - information about the structure of a particle is then deduced from the cross-section

Rutherford scattering

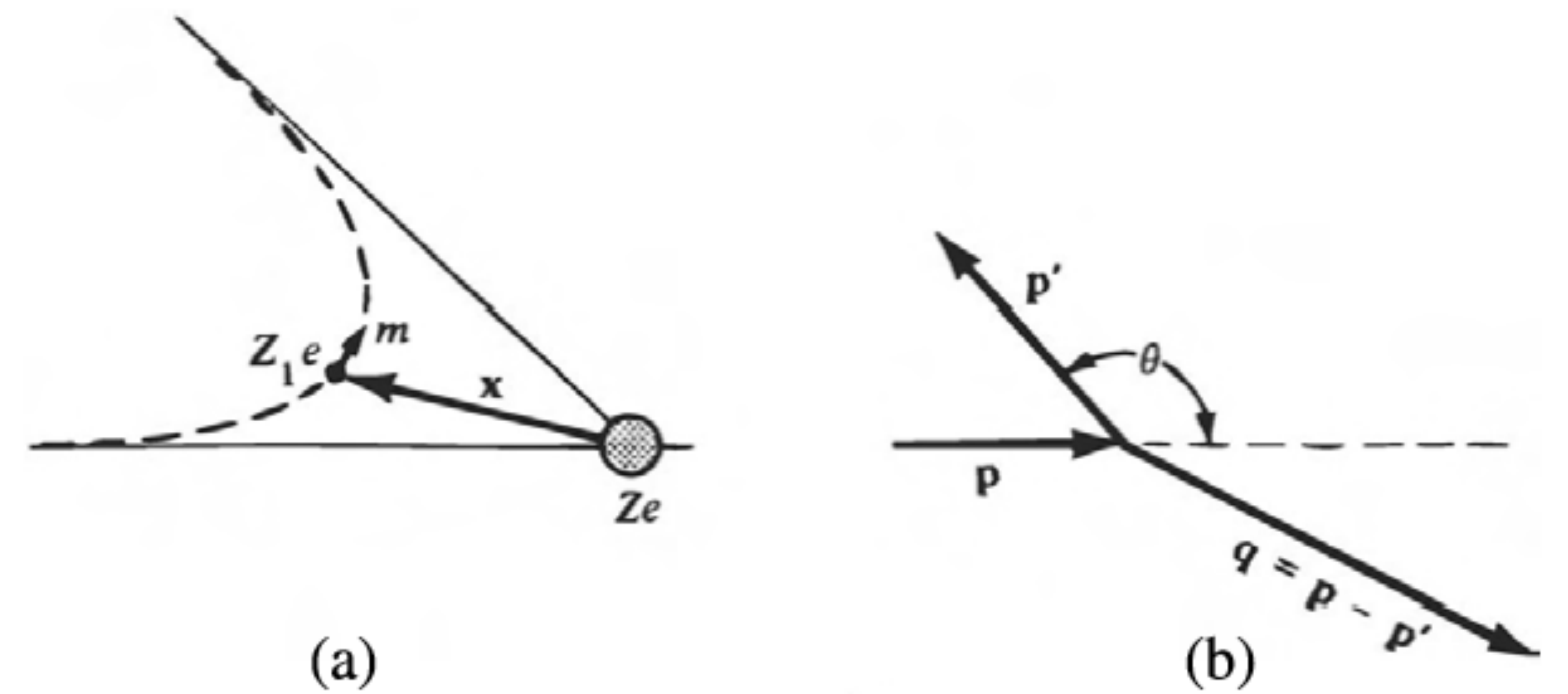


Figure 6.1: Rutherford scattering. (a) Classical trajectory of a particle with charge Z_1e in the field of a heavy nucleus with charge Ze. (b) Representation of the collision in momentum space.

- ✓ Elastic scattering of α-particles by the Coulomb field of the nucleus of charge Ze
- ✓ The cross section can be computed, with the same results, either classically or quantum-mechanically

electron-proton scattering vs energy: Rutherford scattering

- The Rutherford and Mott scattering are the low-energy limits of e-p scattering
- In both cases the electron energy is sufficiently low that the kinetic energy of the recoiling proton is negligible compared to its rest mass
- In this case the proton can be considered as a fixed, point-like source of 1/r electrostatic potential
- The cross-sections are calculated from scattering theory by using the first order terms in the perturbation expansion
- Rutherford scattering:
  - the proton recoil can be neglected and the electron is non-relativistic
  - The differential cross-section is given then by

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{4m^2(Z_1 Z e^2)^2}{q^4}.$$

electron-proton scattering vs energy: Mott scattering

- The Mott scattering is the limit where the electron is relativistic but the proton recoil can still be negligible
- These conditions apply when $m_e << E << m_p$
- The matrix element is given this time by

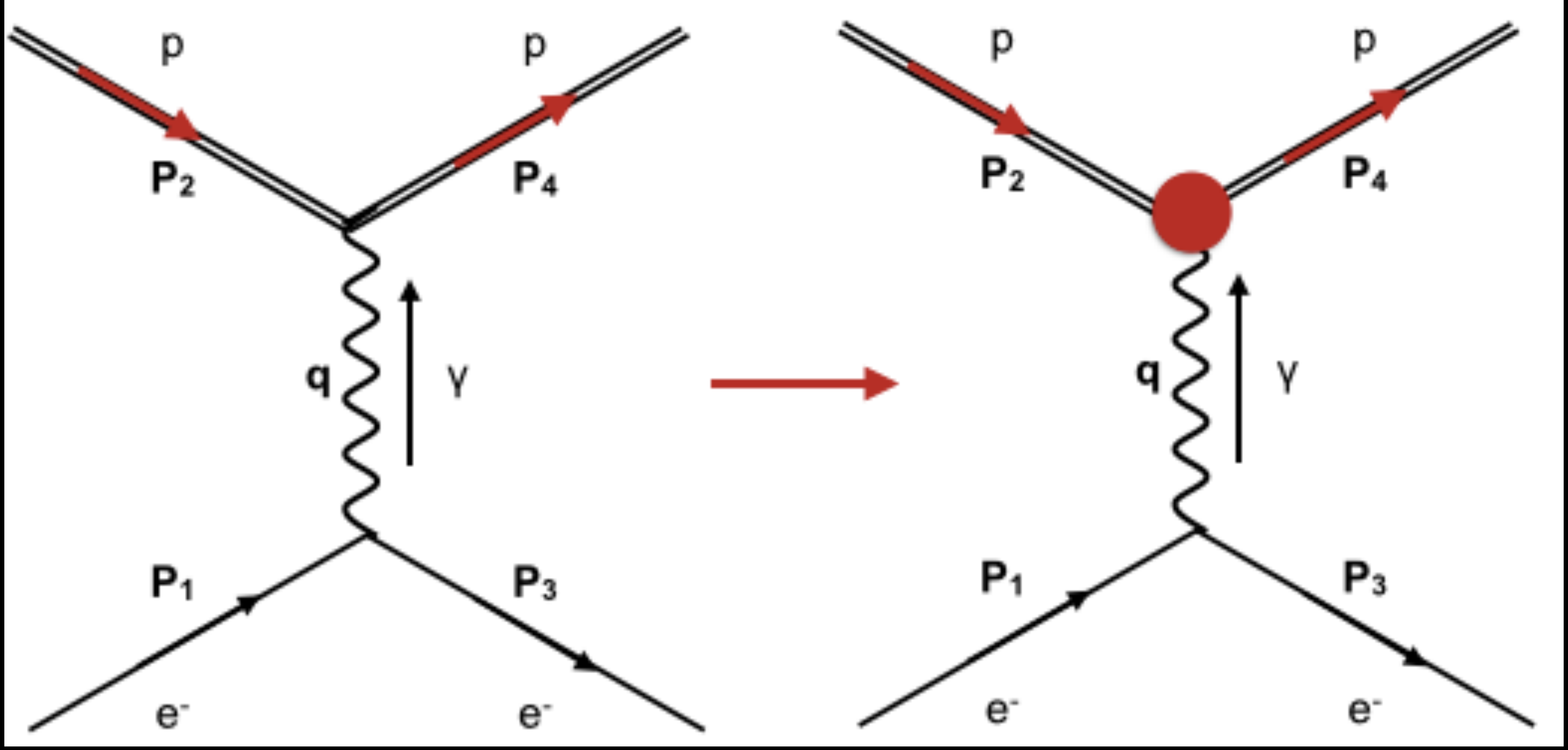
$$\langle |M_{if}|^2 \rangle \simeq \frac{m_p^2 e^4}{E^2 \sin^4\left(\frac{\theta}{2}\right)} \cos^2\left(\frac{\theta}{2}\right)$$

- while the differential cross-section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = 4(Ze^2)^2 \frac{E^2}{(qc)^4}\left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$$

Form factors

- Form factors are introduced to account for the fact that some particles are not elementary but have internal structure
- The electron and in general leptons are elementary particles and thus are ideal tools to probe the internal structure of particles
  - A typical example of such kind of interaction is the electron-proton elastic scattering



Form factors (cont.)

- The connection between the form factor and the density distribution is given by a Fourier transform of the probability density

$$F(q^2)=\int d^3r\rho(\vec{r})e^{i\vec{q}\cdot\vec{r}/\hbar}$$

- Experiments measure $F(q^2)$ for various values of the square of momentum transfer q^2 and fit the data points to recover the continuum

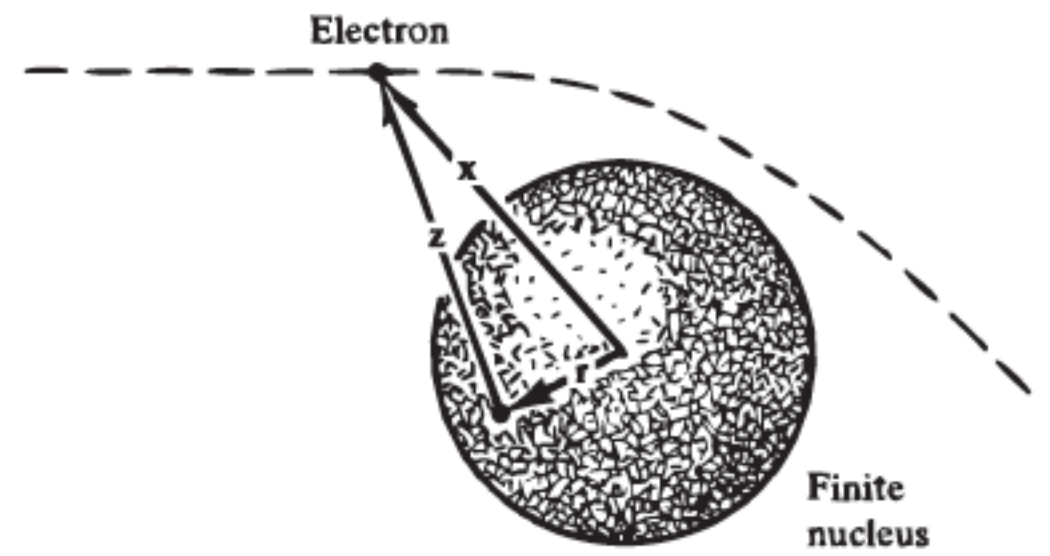


Figure 6.2: Scattering of a spinless electron by a spinless nucleus with extended charge distribution.

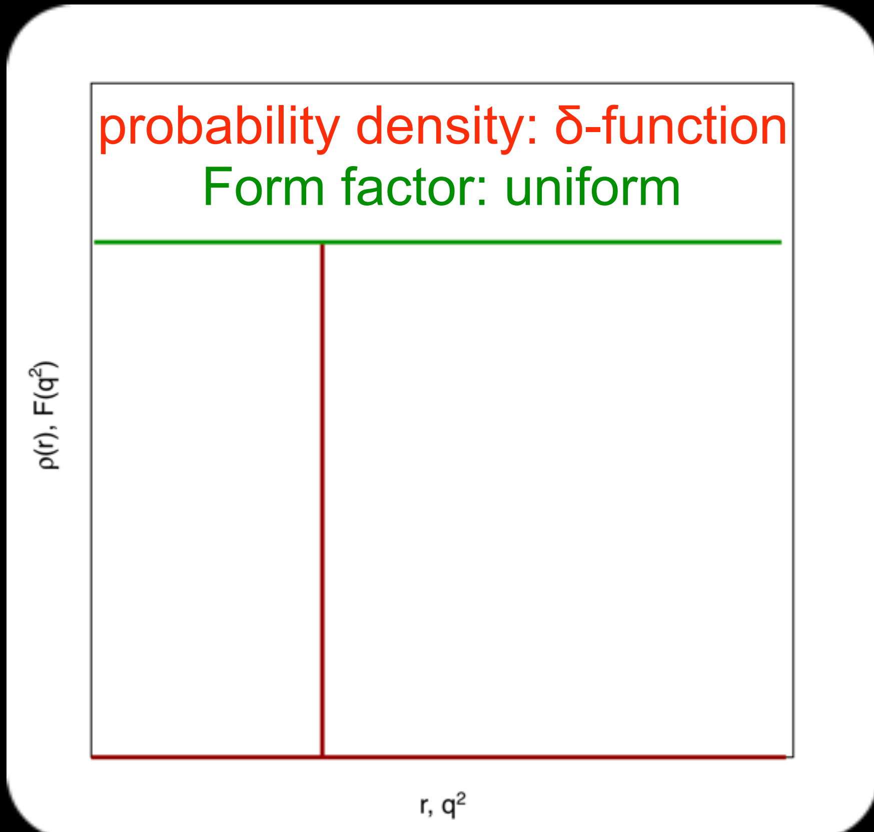
Form factors (cont.)

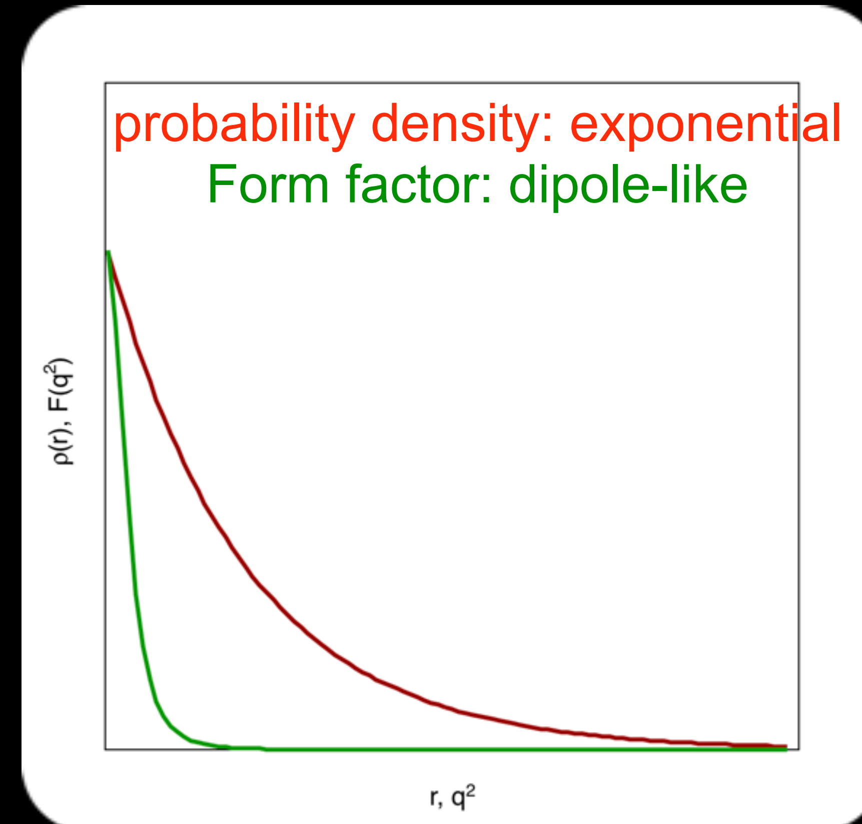
- The form factor depends on the square of the momentum transfer to the target and not on the energy of the incident particle
- $F(q^2)$ can thus be determined for a specific value of q^2 with projectiles of different energies
- However this q^2 dependence is only a first order approximation…higher order corrections are needed

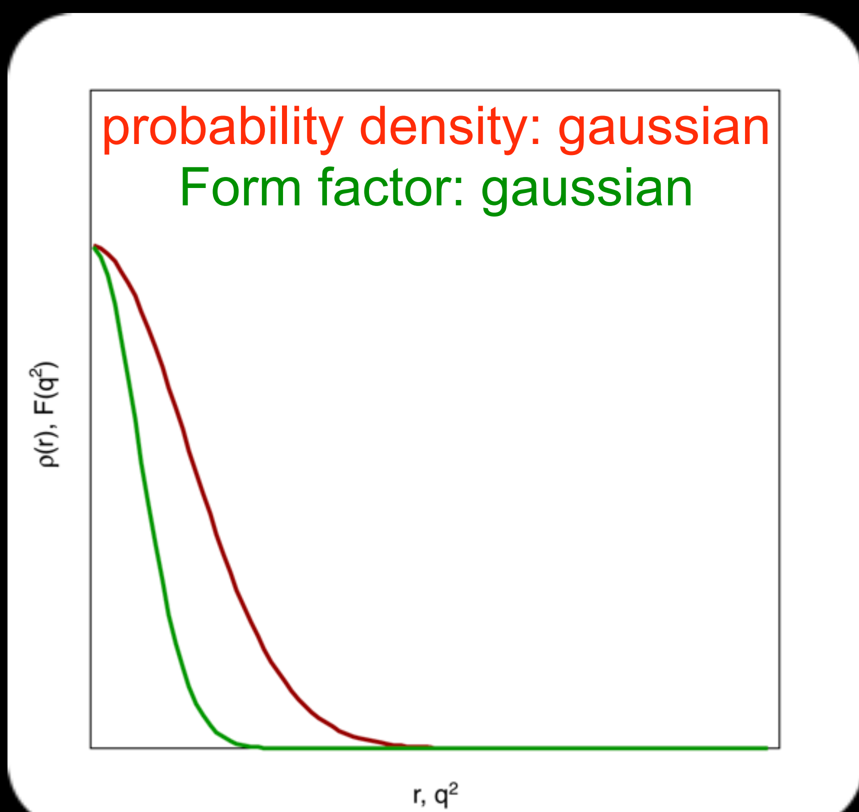
Table 6.1: PROBABILITY DENSITIES AND FORM FACTORS FOR SOME ONE-PARAMETER CHARGE DISTRIBUTIONS. [After R. Herman and R. Hofstadter, *High-Energy Electron Scattering Tables*, Stanford University Press, Stanford, CA, 1960.]

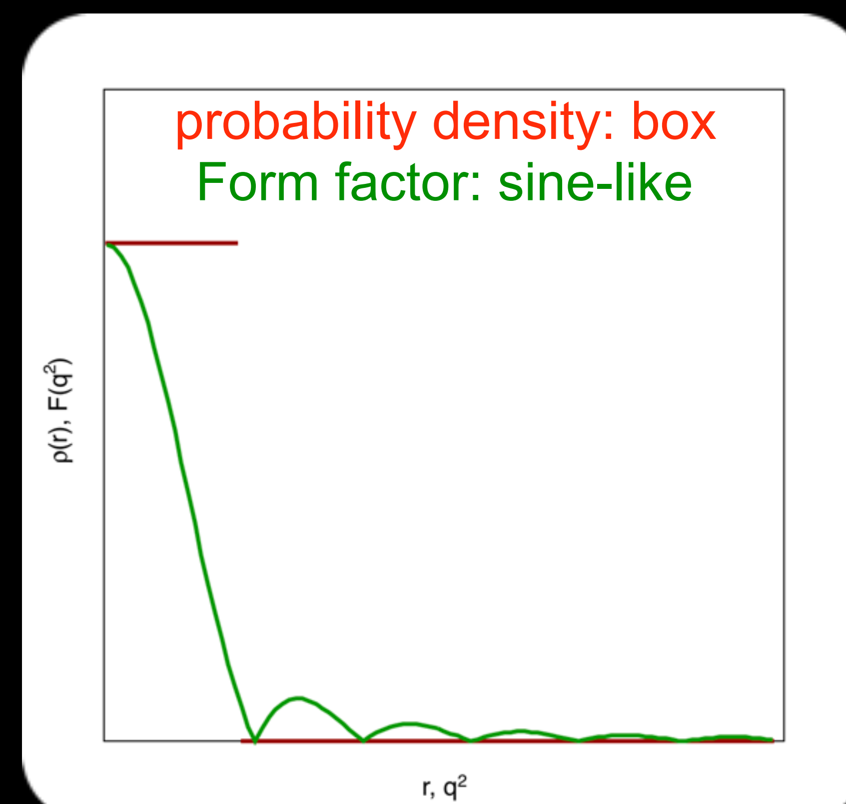
Probability Density, $\rho(r)$	Form Factor, $F(\boldsymbol{q}^2)$
$\delta(r)$	1
$\rho_0 \exp(-r/a)$	$(1+\boldsymbol{q}^2 a^2/\hbar^2)^{-2}$
$\rho_0 \exp[-(r/b)^2]$	$\exp(-\boldsymbol{q}^2 b^2/4\hbar^2)$
$\left.\begin{matrix}\rho_0, r \le R \\ 0, r \ge R\end{matrix}\right\}$	$\frac{3[\sin(\lvert\boldsymbol{q}\rvert R/\hbar)-(\lvert\boldsymbol{q}\rvert R/\hbar)\cos(\lvert\boldsymbol{q}\rvert R/\hbar)]}{(\lvert\boldsymbol{q}\rvert R/\hbar)^3}$

Form factors (cont.)

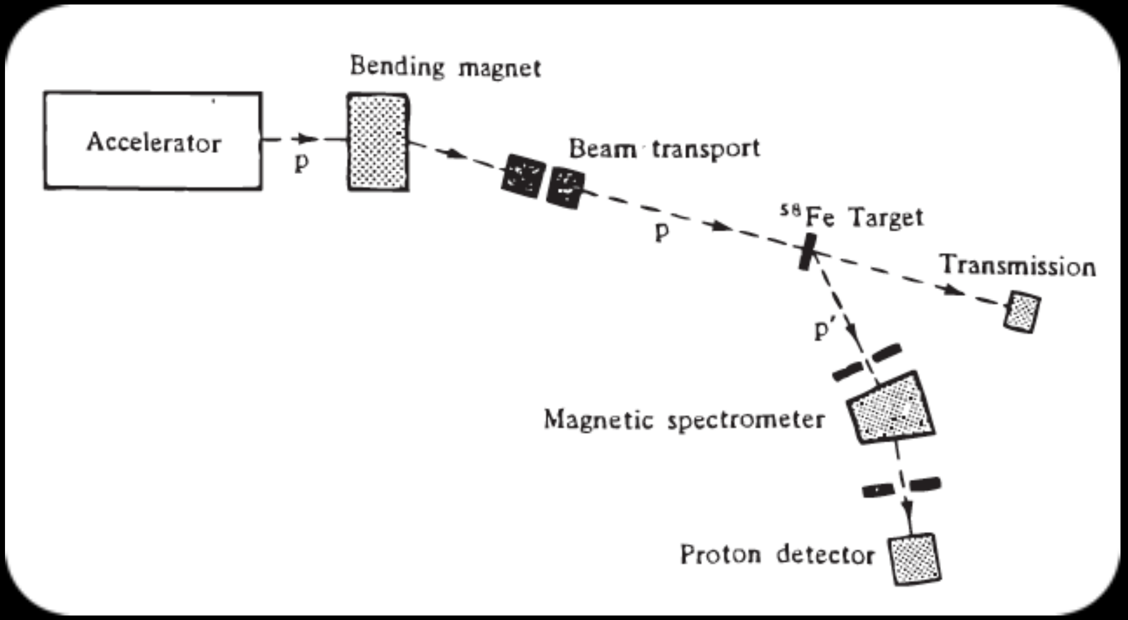








Charge distribution of spherical nuclei



- ✓ Investigation of nuclear structure by electron scattering performed using an electron accelerator that produces a beam with energies between 250 MeV and few GeV
- ✓ The electrons are transported into a scattering chamber where they hit the target
- ✓ The intensity of the scattering electrons is measured as a function of the scattering angle

Charge distribution of spherical nuclei (cont.)

The differential cross-section for the scattering of 500 MeV electrons from ^{40}Ca

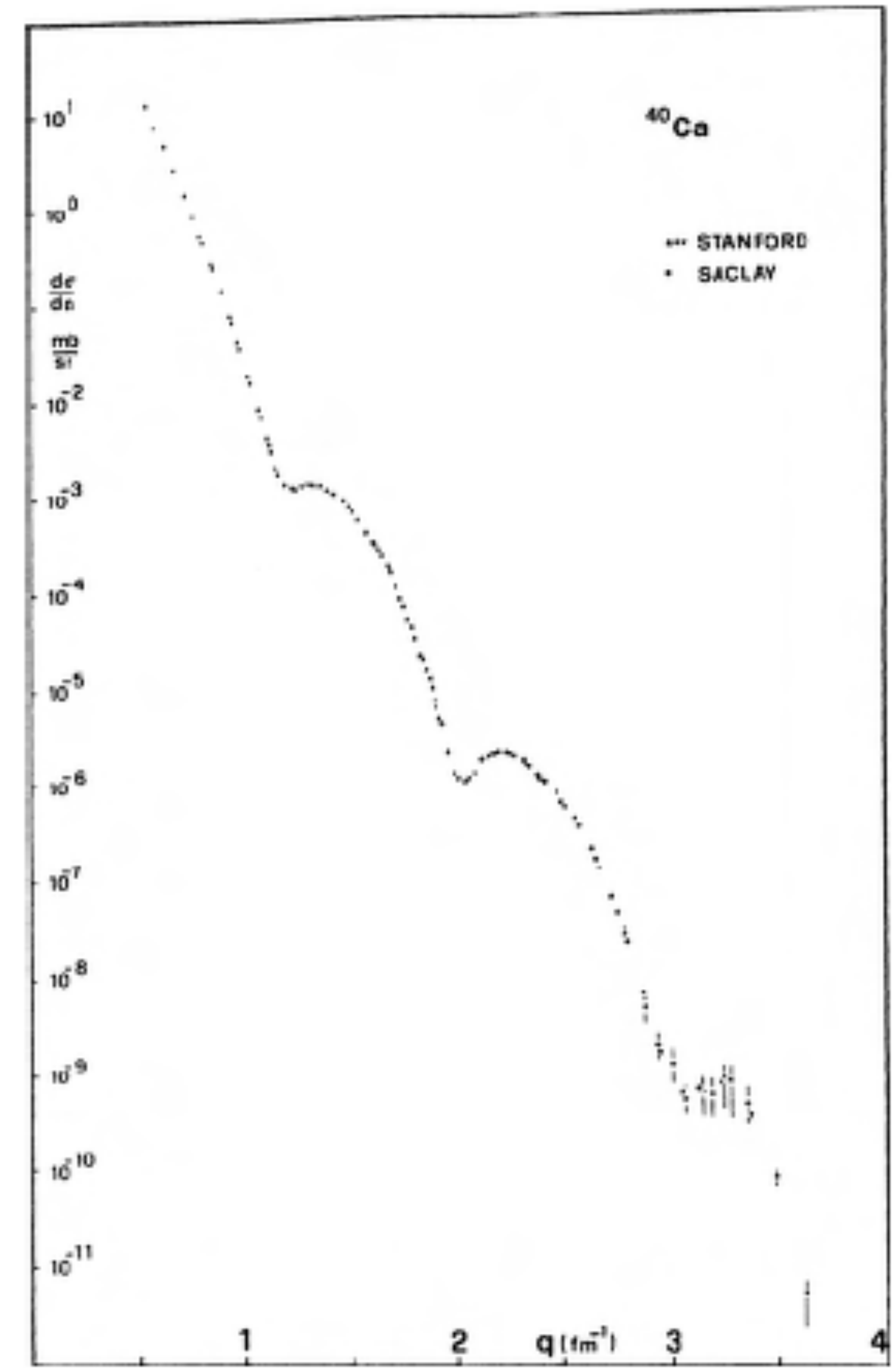


Figure 6.3: Elastic scattering cross section of electrons from ^{40}Ca from experiments performed at Stanford and Saclay, France. [Courtesy I. Sick, *Phys. Lett.* 88B, 245 (1979).]

Charge distribution of spherical nuclei (cont.)

- The crudest approximation of the nuclear charge distribution is via an one-parameter function e.g. a uniform or Gaussian distribution
  - One-parameter functions give poor fits
- Two-parameter functions are more suitable instead e.g. the Fermi distribution

$$\rho(r) = \frac{N}{1+e^{(r-c)/a}}$$

  - Where N is the normalisation constant, c and α are parameters describing the nucleus
  - c is called the half-density radius, t is the surface thickness
  - Both are connected with α via $t=(4\ln 3)a$

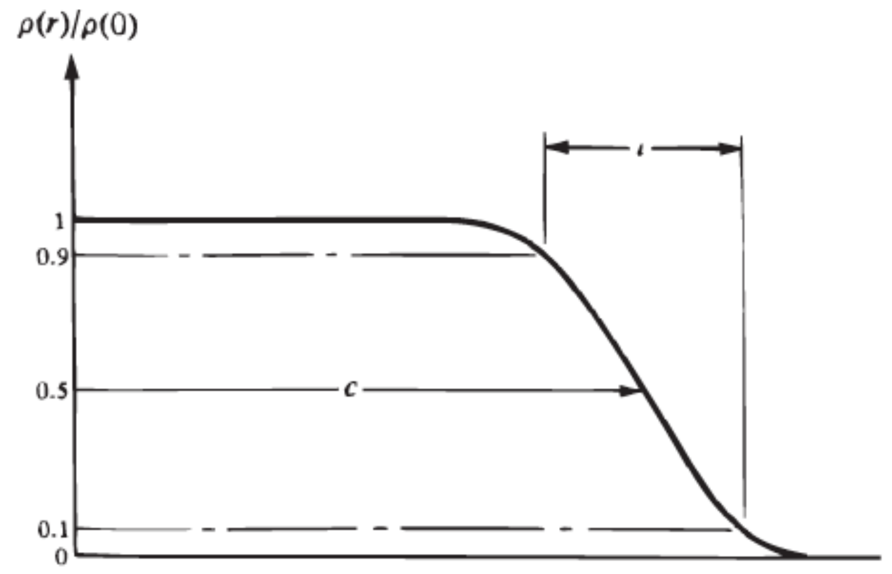


Figure 6.4: Fermi distribution for the nuclear charge density. c is the approximate half-density radius and t the surface thickness.

Charge distribution of spherical nuclei (cont.)

- For medium and heavyweight nuclei the root-mean-square charge radius is

$$\langle r^2 \rangle^{1/2} = r_0 A^{1/3} \qquad r_0 = 0.94 fm$$

  - where A is the mass number of the nucleus
  - the nuclear volume is thus proportional to the number of nucleons
  - Nuclear density is approximately constant

- The half-density radius and the skin thickness are approximately

$$c(fm) = 1.18 A^{1/3} - 0.48 \qquad t \approx 2.4 fm$$

  - the density of nucleons at the centre is $\rho_n \approx 0.17 nucleon/fm^3$ approaching the density of nuclear matter

Charge distribution of spherical nuclei (cont.)

- ✓ The actual charge distribution is however by far more complex than the two component Fermi distribution due to shell structure effects

- ✓ It is possible to extract the charge distribution from electron scattering cross-section in a model-independent way by writing the charge distribution as a superposition of Gaussians

$$\rho \propto \sum_{i=1}^{N} A_i e^{-(\rho - R_i)^2/\delta^2}$$

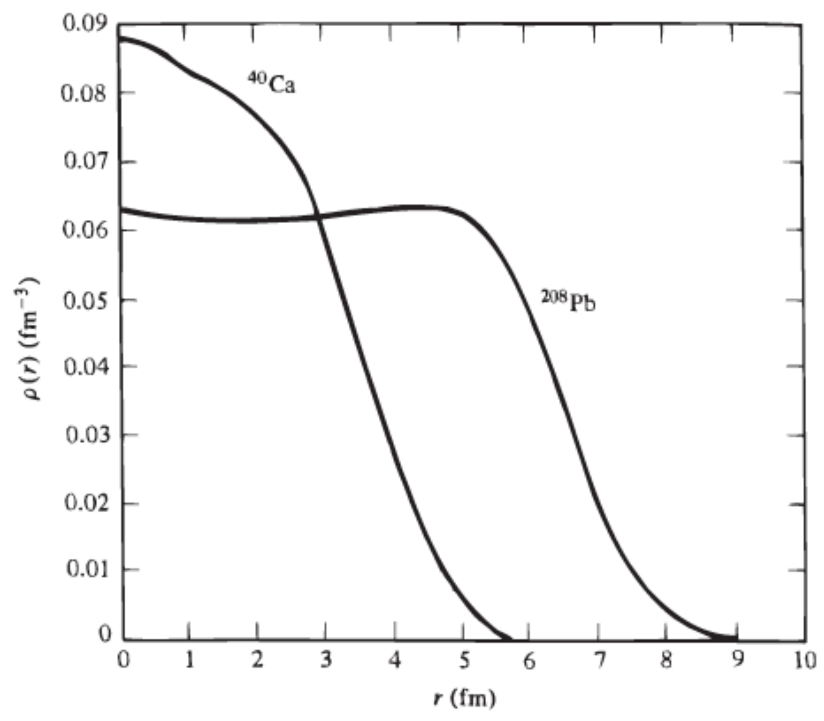


Figure 6.5: Probability distribution for ^{40}Ca and ^{208}Pb, as obtained from electron scattering. [From I. Sick. *Phys. Lett.* **88B**, 245 (1979).]

Elastic form factors of nucleons

- Nucleons are not point like particles
  - They have an internal structure, containing a combination of uud (protons) and udd (neutrons)
- The best way to explore the charge and current distributions of nucleons is again via the elastic scattering with electron beams
- For protons, one can use a liquid hydrogen target in the path of an electron beam and determine the differential cross-section of the scattered electrons
- For neutrons things become more complicated as there is no neutron target
  - One relies on deuteron targets and subtract the effect of protons
  - This usually leads to significantly large uncertainties in the measurements
- The form factors for spineless particles is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} |F(q^2)|^2$$

  - This formula needs to be generalised for spin-1/2 particles

Elastic form factors of nucleons (cont.)

- The form factor in the previous formula describes the electric charge distribution, and thus $F(q^2)$ is called the electric form factor
- A proton has also a magnetic moment with its "magnetisation" being distributed over the volume of the nucleon and is described by the magnetic form factor
- The generalisation of the previous formula is given by the Rosenbluth equation

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[\frac{G_E^2 + bG_M^2}{1+b} + 2bG_M^2 \tan^2\left(\frac{\theta}{2}\right)\right]$$

$$b = \frac{-q^2}{4M^2c^2} \qquad \left(\frac{d\sigma}{d\Omega}\right)_{Mott} = 4e^4 \frac{E'^2}{(qc)^4} \cos^2\left(\frac{\theta}{2}\right)\left(\frac{E'}{E}\right)$$

  - where G_E and G_M are the electric and magnetic form factors and they are both a function of q^2
  - M is the mass of the nucleon, θ is the scattering angle between the incoming and outgoing electrons with energies E and E', respectively
  - q is the momentum transfer to the nucleon

Elastic form factors of nucleons (cont.)

- For the static case we have

$$G_E(q^2=0) = \frac{Q}{e} \qquad G_M(q^2=0) = \frac{\mu}{\mu_N}$$

  - where Q and μ are the charge and magnetic moment of nucleons
  - More specifically, for protons and neutrons we have

$$G_E^p(q^2=0) = 1 \qquad G_E^n(q^2=0) = 0$$

$$G_M^p(q^2=0) = 2.79 \qquad G_M^n(q^2=0) = -1.91$$

Elastic form factors of nucleons (cont.)

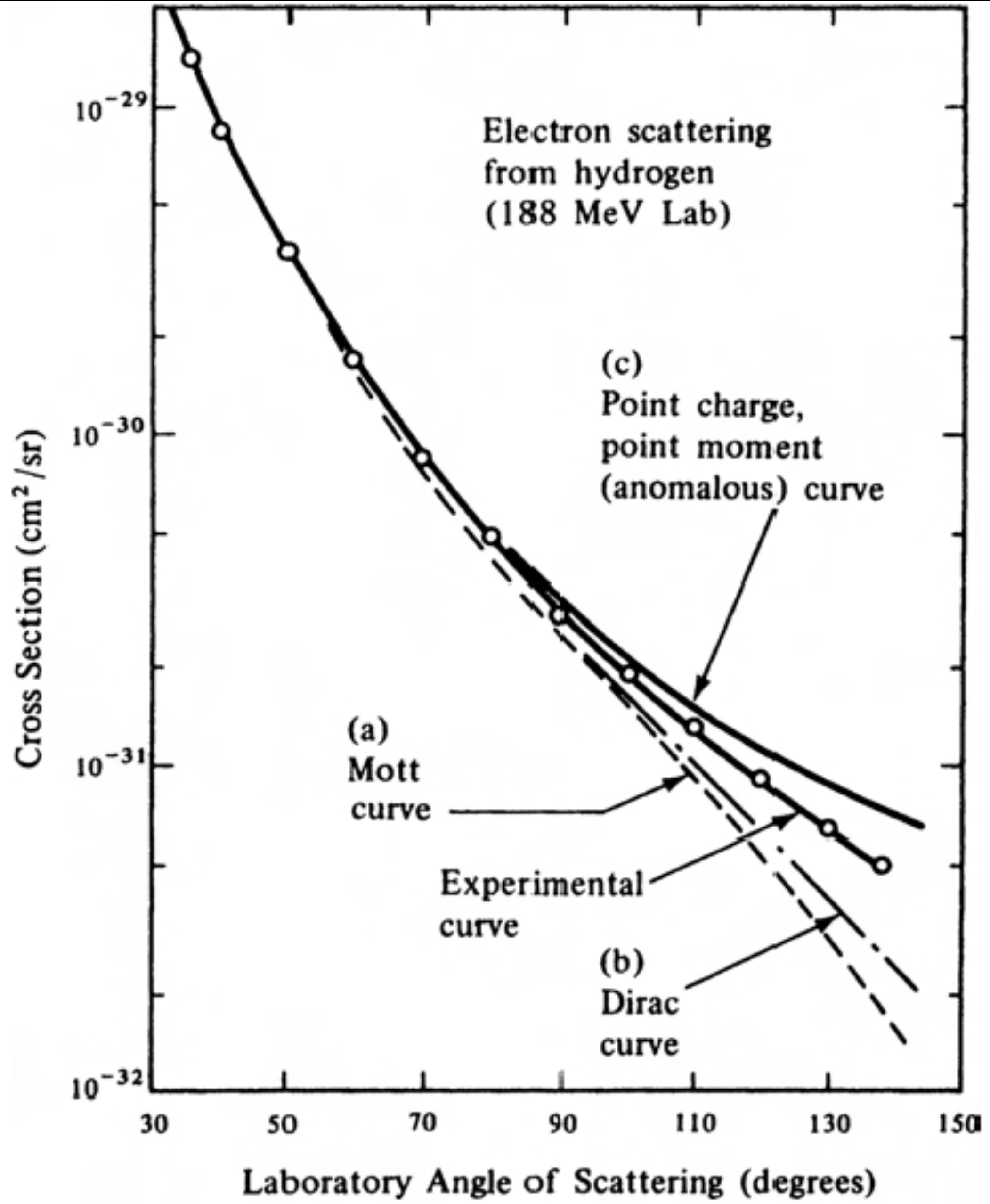


Figure 6.11: Electron–proton scattering with 188 MeV electrons. [R. W. McAllister and R. Hofstadter, *Phys. Rev.* **102**, 851 (1956).] The theoretical curves correspond to the following values of G_E and G_M: Mott (1;0), Dirac (1;1), anomalous (1;2.79).

Conclusion: Nucleons are not point like particles!

Nobel prize in physics 1961

The Nobel Prize in Physics 1961
Robert Hofstadter, Rudolf Mössbauer

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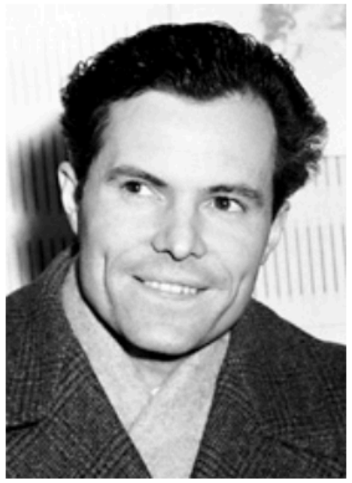
The Nobel Prize in Physics 1961

Robert Hofstadter
Prize share: 1/2



Rudolf Ludwig Mössbauer
Prize share: 1/2

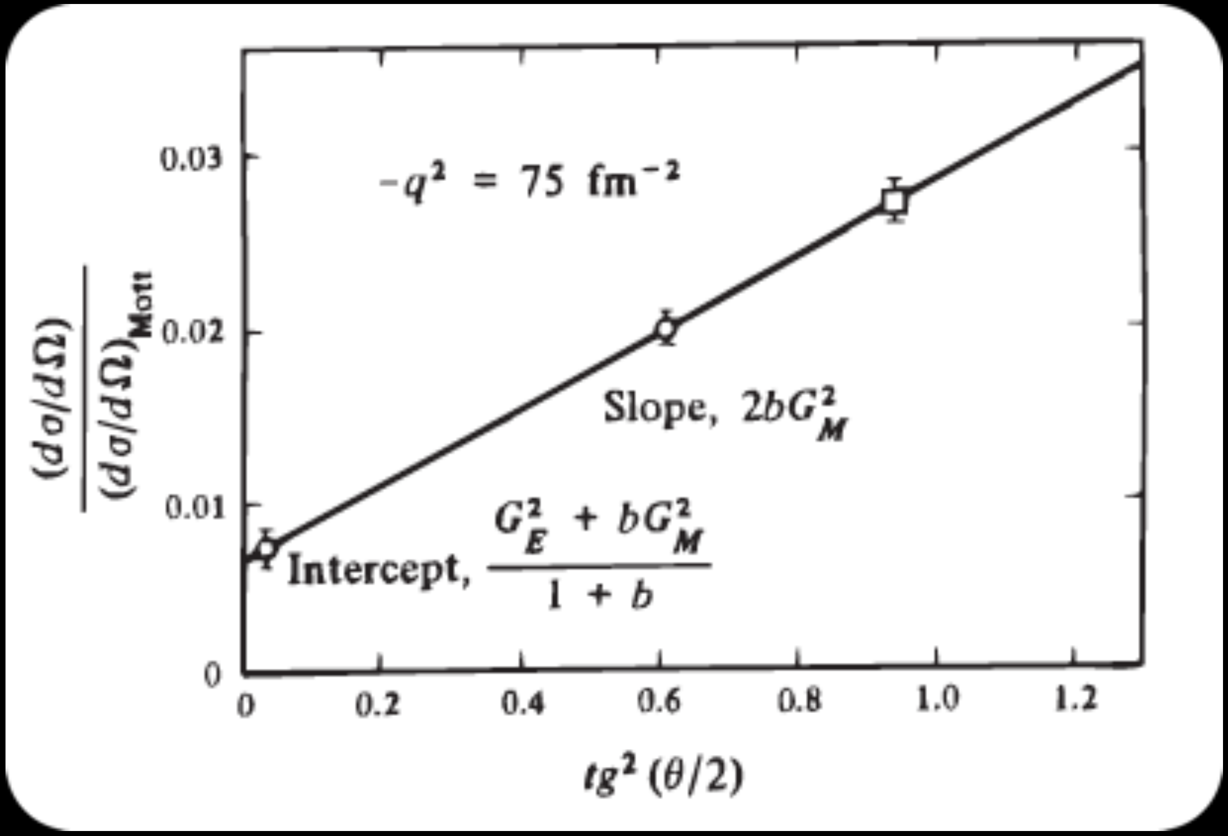
The Nobel Prize in Physics 1961 was divided equally between Robert Hofstadter *"for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons"* and Rudolf Ludwig Mössbauer *"for his researches concerning the resonance absorption of gamma radiation and his discovery in this connection of the effect which bears his name"*.

Elastic form factors of nucleons (cont.)

- To extract the form factors, we use this formula

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[\frac{G_E^2+bG_M^2}{1+b}+2bG_M^2\tan^2\left(\frac{\theta}{2}\right)\right]$$

  - and plot the ratio of the measured cross-section for a given q^2 value over the Mott cross-section as a function of $\tan^2(\theta/2)$
  - The slope gives the factor multiplying $\tan^2(\theta/2)$ from where we extract G_M^2
  - The intercept on the y-axis gives the other factor from where we extract G_E^2



Elastic form factors of nucleons (cont.)

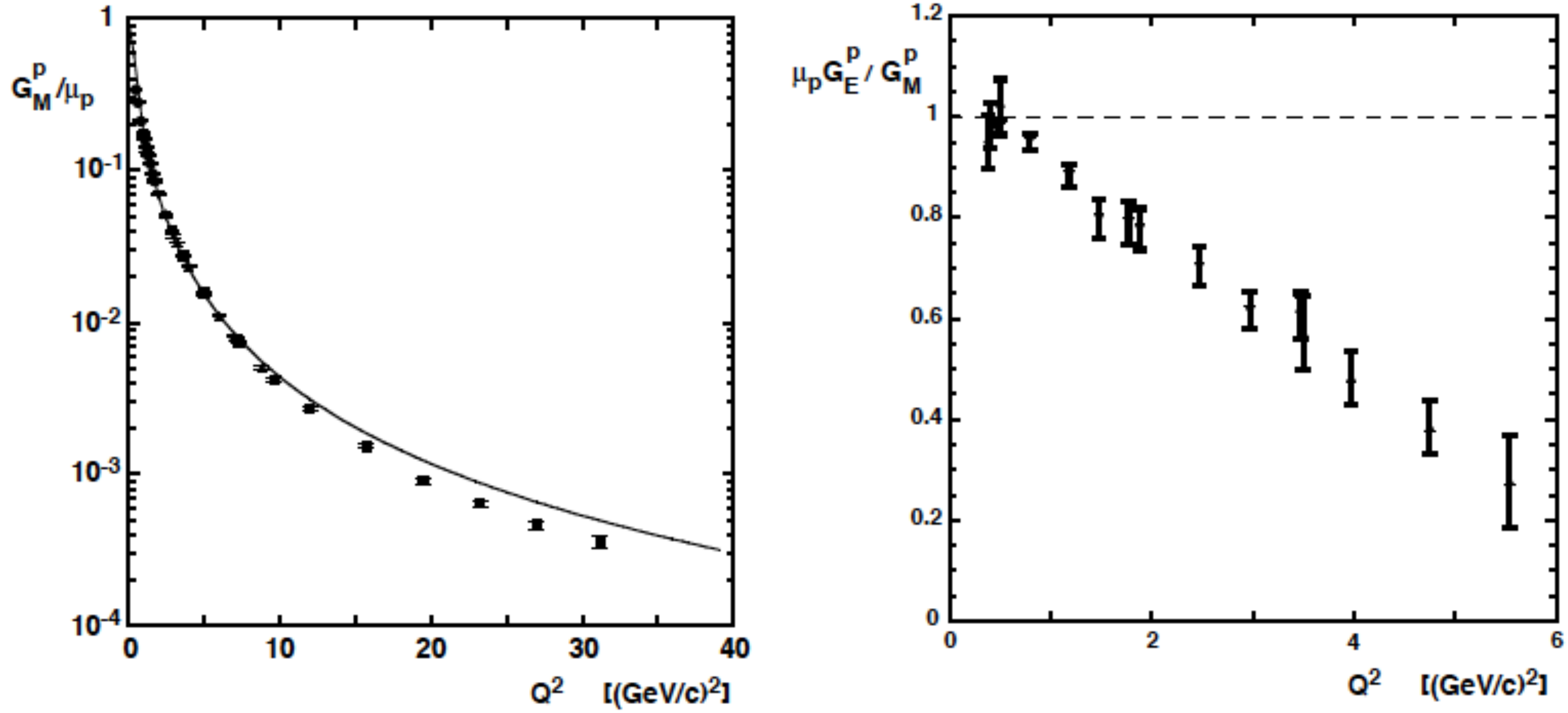


Figure 6.13: Left: Magnetic form factor for the proton plotted against the squared momentum transfer $|q|^2$. The different symbols correspond to different experiments. The 'dipole' function –described in the text and shown as a continuous line– describes the G_M data quite accurately below $|q|^2 \approx 10$ (GeV/c)^2. Right: G_E/G_M. The distributions of charge and magnetism in the proton are quite different. [See C.Hyde-Wright and K. de Jager, *Annu. Rev. Nucl. Part. Sci.* **54**, 217 (2004).]

- ✓ The magnetic form factor of a proton is described fairly well by a dipole function for low values of q^2
- ✓ The proton charge and magnetic distributions are quite different

Elastic form factors of nucleons (cont.)

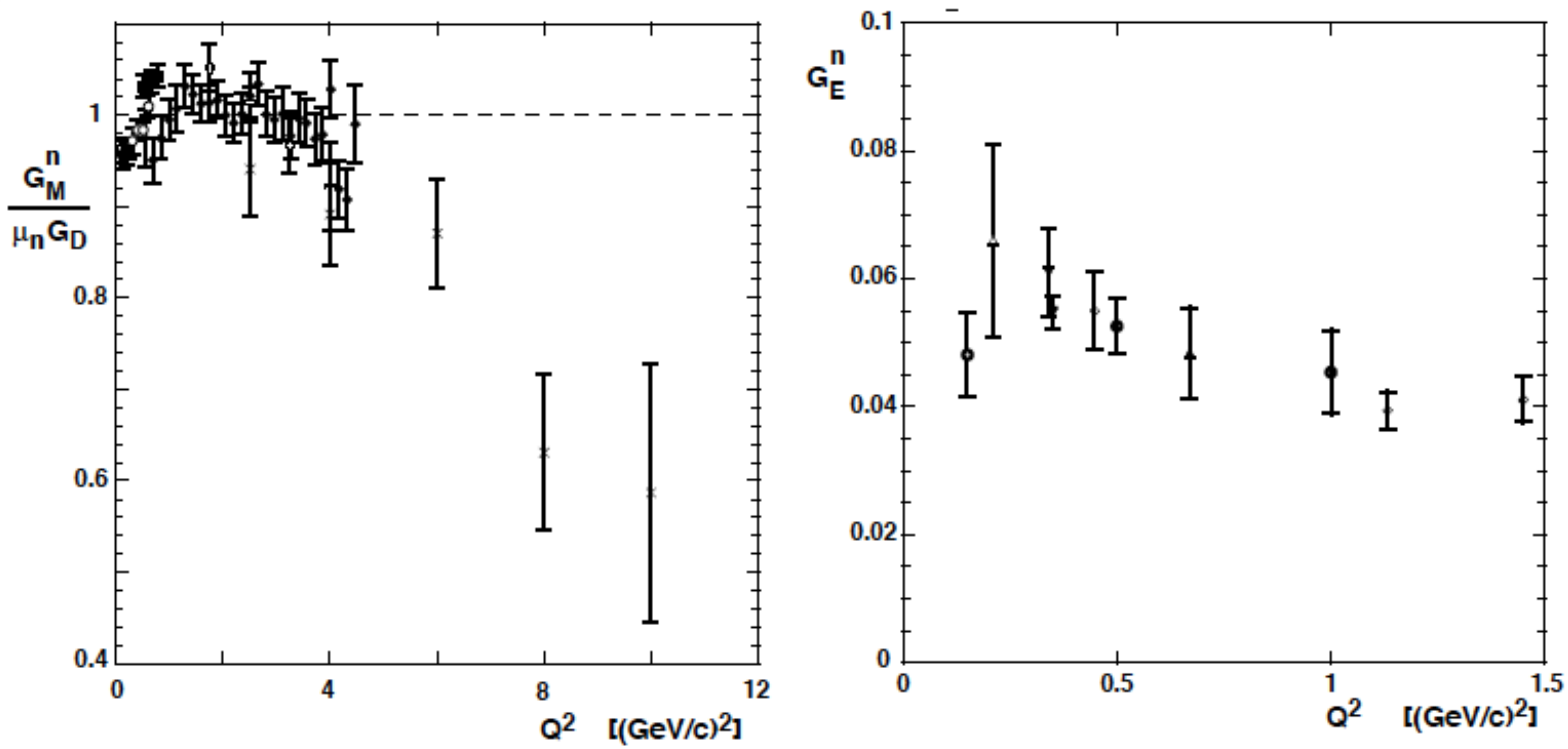
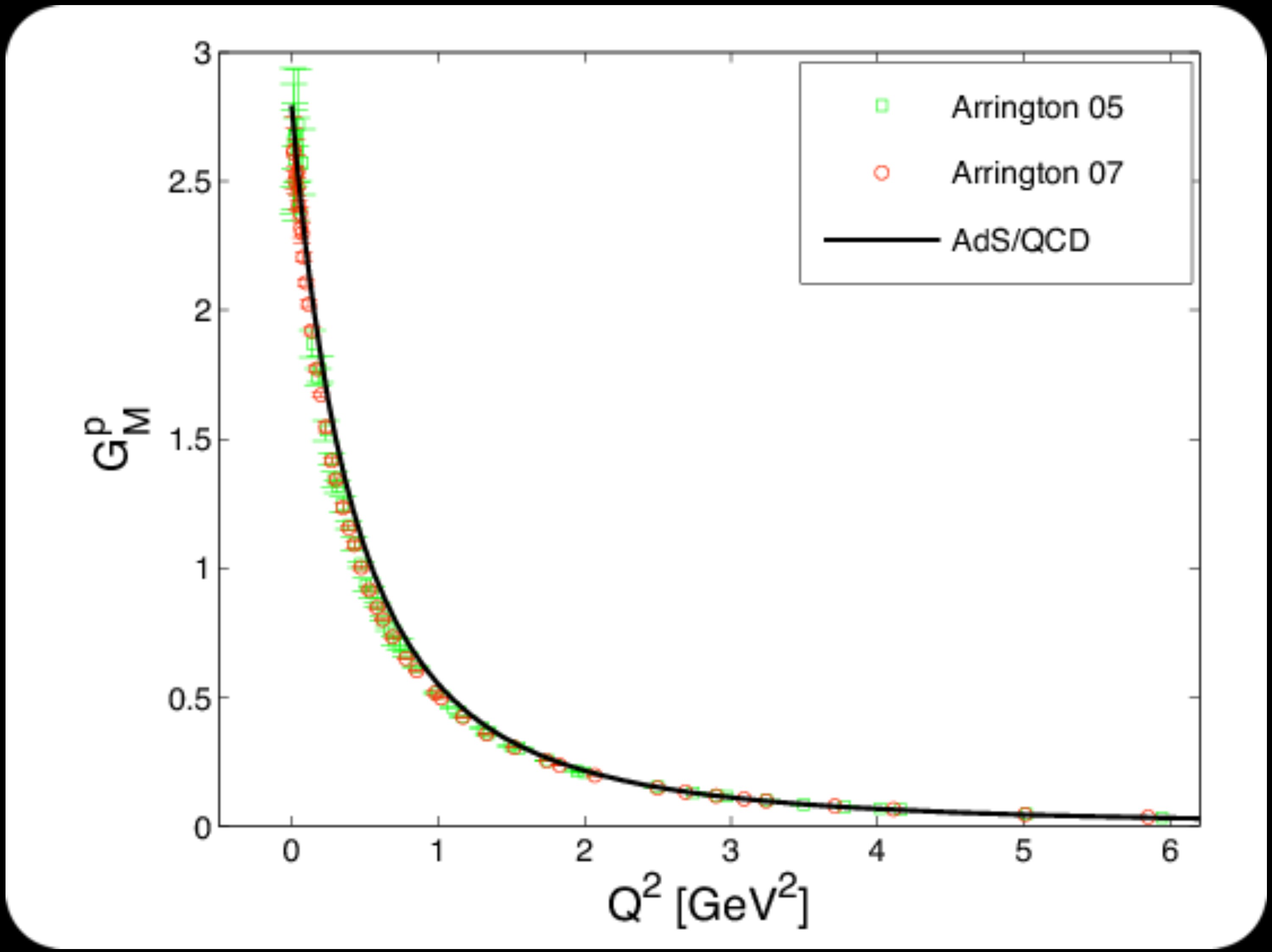


Figure 6.14: Magnetic (left) and electric (right) form factors for the neutron. Here we show the magnetic form factor divided by the dipole formula. The magnetic form factor shows rough agreement with the dipole formula for $|q|^2 < 5$ (GeV/c)^2. [See C.Hyde-Wright and K. de Jager, *Annu. Rev. Nucl. Part. Sci.* **54**, 217 (2004).]

- ✓ The magnetic form factor of a neutron is described fairly well by a dipole function for low values of q^2
- ✓ The neutron charge distribution is quite small

Size of proton



$$G_E(q^2) = \left(1 - \frac{q^2}{0.71}\right)^{-2}$$

$$\langle r^2 \rangle = 6\left(\frac{dG_E(q^2)}{dq^2}\right)_{q^2=0} = (0.81 \cdot 10^{-13}\mathrm{cm})^2$$