Wormholes from Evaporating Black Holes

Netta Engelhardt

MIT

work with A. Folkestad

Verlinde^2 Symposium

Happy Birthday, Erik and Herman!

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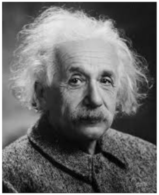

... Herman Verlinde

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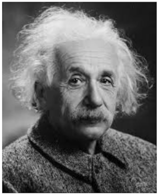

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Morgan Freeman

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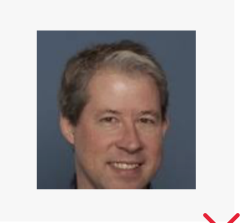

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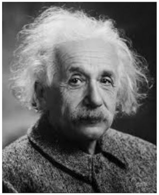

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Spacetime from Entanglement

- General expectation: entanglement builds spacetime van Raamsdonk, **2xVerlinde**, Jensen-Sonner, Maldacena, Maldacena-Susskind, etc...
- Realized in certain examples in AdS/CFT: TFD $\leftrightarrow$ Schwarzschild Maldacena, metric reconstruction from RT surfaces in a class of examples van Raamsdonk, Bao et al, Hubeny et al..., bulk equations of motion Lashkari et al, Faulkner et al, Swingle,......
- Heavily related to ER=EPR: $\mathcal{O}(G_N^{-1})$ entanglement builds spacetime; particularly relevant for the evaporating black hole van Raamsdonk, Maldacena Susskind, **2xVerlinde**.

Entanglement Builds Spacetime vs ER=EPR

Will distinguish between the two. First, the expectation that entanglement "builds" spacetime...

Expectation 1: Entanglement Builds Spacetime

If there's enough entanglement in some bipartite state $|\psi_{AB}\rangle$, and ρ_A, ρ_B each have a semiclassical gravitational bulk description, then the bulk dual to $|\psi_{AB}\rangle$ is connected.

Entanglement Builds Spacetime vs ER=EPR

Expectation 2: ER=EPR

If there's enough entanglement in some bipartite state $|\psi_{AB}\rangle$, then there exists a factorizing unitary $U_A \otimes U_B$ such that $U_A \otimes U_B|\psi_{AB}\rangle$ admits a semiclassical gravitational connected description.

Entanglement Builds Spacetime and ER=EPR

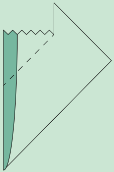
Well-motivated by the thermofield double $\leftrightarrow$ Schwarzschild black hole. Also recently illustrated by Balasubramanian et al in certain two-dimensional examples.

ER=EPR Prediction for BH evaporation: it is possible to act on the radiation of an old black hole with a unitary that maps it to a black hole connected to the old black hole via an ERB.

Realizing the ER=EPR Post-Page Prediction

Option 1: throw radiation into the other side

... but if there's no other side?



Could imagine gluing something at the event horizon, but in AdS/CFT that doesn't amount to a factorizing unitary.

Realizing ER=EPR Post-Page

Option 2: collapse the radiation into a black hole van Raamsdonk

Not a proof that the ERB forms in general and does not (in general) explain how it forms.

Another question: if there are multiple ways of collapsing the radiation, and only some of those ways result in an ERB, what is special about the connected states?

Preview of Talk

- Precise realization of the post-Page prediction of ER=EPR in general states satisfying the QES prescription.

arXiv:2201.08395v1 [hep-th] 20 Jan 2022

Canonical Purification of Evaporating Black Holes

Netta Engelhardt and Åsmund Folkestad

Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

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Abstract: We show that the canonical purification of an evaporating black hole after the Page time consists of a short, connected, Lorentzian wormhole between two asymptotic boundaries, one of which is unitarily related to the radiation. This provides a quantitative and general realization of the predictions of ER=EPR in an evaporating black hole after the Page time; this further gives a standard AdS/CFT calculation of the entropy of the radiation (without modifications of the homology constraint). Before the Page time, the canonical purification consists of two disconnected, semiclassical black holes. From this, we construct two bipartite entangled holographic CFT states, with equal (and large) amount of entanglement, where the semiclassical dual of one

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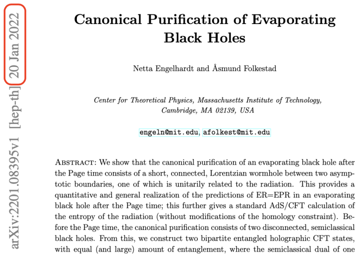

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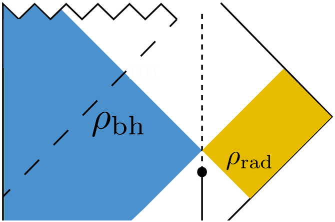
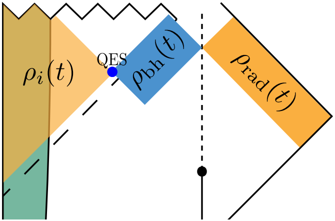
- Explicit holographic, semiclassical spacetime with well-behaved and understood CFT dual where bipartite entanglement doesn't build an ERB.

Overview

Evaporating AdS Black Holes

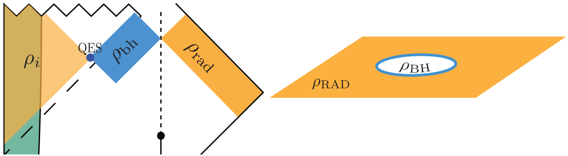




Black Hole vs Radiation

Three relevant bulk regions:

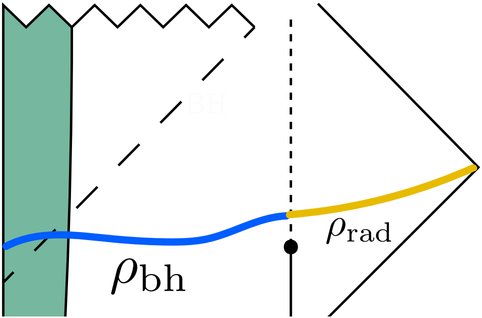
1. Region between subdominant (old) QES and dominant (new) QES: the island: i
2. Region between new QES and asymptotic AdS boundary: the "black hole" $\mathcal{W}_E[\rho_{\text{BH}}]$
3. Region between asymptotic AdS boundary and i^0: the "radiation": *rad*



ρ_{RAD} contains i in its entanglement wedge.

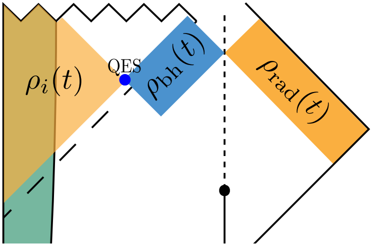
Universal Features: Pre- Page time

1. QESs for ρ_{BH}, $\rho_{\text{RAD}} \approx$ is empty (the classical extremal surface)
2. If black hole is single sided, this is the empty set.
3. Pre-Page time, the entanglement wedge of ρ_{BH} contains a Cauchy slice of the entire AdS bulk. The bulk state ρ_{BH} on this slice is not pure.



Universal Features: Post-Page time

1. QES for ρ_{BH}, ρ_{RAD} is far from the old classical HRT surface.
2. Entanglement wedge of ρ_{BH} no longer consists of a complete Cauchy slice of the entire bulk.
3. Entanglement wedge of ρ_{RAD} includes the now nontrivial island.



Expectation: entropy gradients after the Page time imply that *in broad generality*, old black holes have a new dominant QES. Argued in broad generality by Bousso&Shahbazi-Moghaddam

Overview

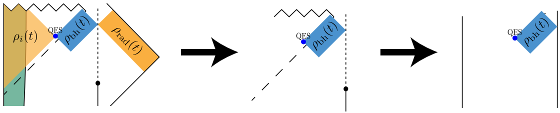
The Basic Idea

What we want: looking for a prescriptive algorithm that realizes ER=EPR after the Page time (or a counterexample).

Observation

There is a nontrivial QES ⇒ it is possible to purify ρ_{BH} by exchanging the island+radiation for a second asymptotic boundary in a single connected bulk.

We also know/expect that there is always nontrivial QES after the Page time.



Under assumption of unitarity, the full microscopic state is pure: $|\Psi\rangle$.

$$\rho_{\text{RAD}} = \text{tr}_{\text{BH}}(|\Psi\rangle\langle\Psi|)$$

The purification σ of ρ_{BH} is unitarily related to ρ_{RAD}:

$$\sigma = U_{\text{RAD}}\rho_{\text{RAD}}U^{\dagger}_{\text{RAD}}$$

Is there a general CFT procedure that yields a σ with a semiclassical dual such that

$$(U_{\text{RAD}} \otimes \mathbb{I}_{\text{BH}})|\Psi\rangle$$

has a connected two-boundary semiclassical bulk dual?

TFD Canonical Purification

Gibbs state:

$$\rho = \frac{1}{Z} \sum e^{-\beta E_n} |n\rangle\langle n|$$

Can purify by doubling the Hilbert space; we get TFD:

$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum e^{-\beta E_n/2} |n\rangle|n\rangle$$

Which gives us the complete (maximally extended) Schwarzschild-AdS black hole Maldacena '01.

Canonical Purification in General

The same idea works more generally for any mixed state:

1. Take some density matrix in the diagonal basis

   $$\rho = \sum p_i |\rho_i\rangle\langle\rho_i|$$

   and a Hilbert space $\mathcal{H}$.

2. Double the Hilbert space and define the pure state in the doubled Hilbert space:

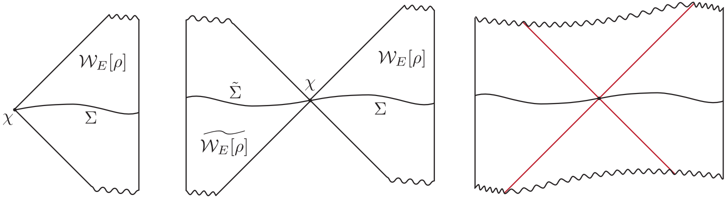
   $$|\sqrt{\rho}\rangle = \sum_i \sqrt{p_i}\, |\rho_i\rangle|\rho_i\rangle$$

3. Can think of it as "flipping bras to kets".

4. Clearly tracing out new d.o.f. returns ρ.

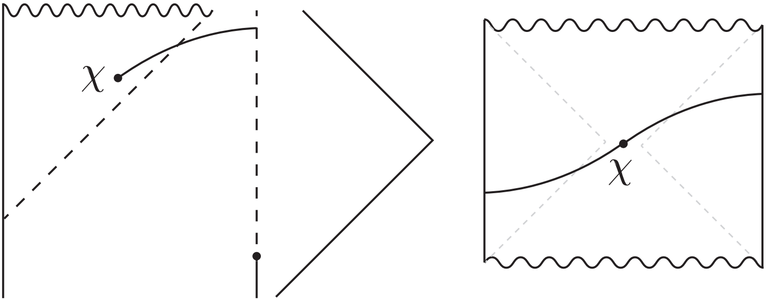
Holographic Dual of the Canonical Purification Engelhardt, Wall; Bousso, Chandrasekaran, Shahbazi-Moghaddam

Given a CFT in some mixed state ρ with a semiclassical dual entanglement wedge $\mathcal{W}_E[\rho]$, $|\sqrt{\rho}\rangle$ is given by a CPT conjugation of the spacetime around the QES χ.



Overview

Canonical Purification Realizes ER=EPR



Always possible to do this after the Page time.

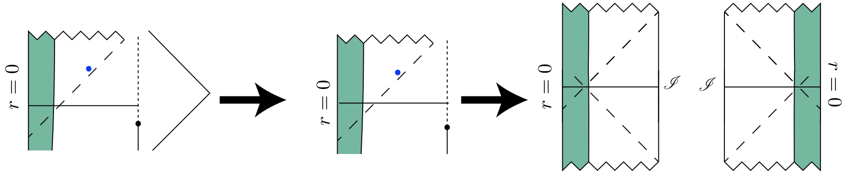
Canonical Purification for ER=EPR

ER=EPR after the Page time

Whenever the Page curve is correctly computed by QESs, there always exists a unitary that acts just on the radiation and yields a semiclassical ERB connected to the old black hole.

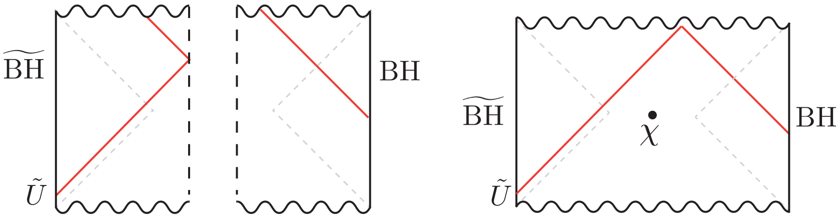
Overview

Canonical Purification Pre-Page

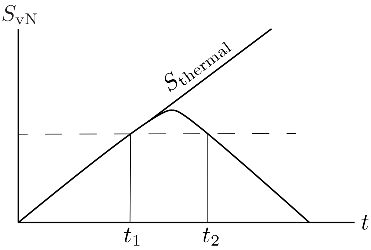


An Application: Affecting the Infalling observer

After the Page time, a signal sent from $\widetilde{\text{BH}}$ via some local unitary $\tilde{U}$ (i.e. some operation on the radiation) can effect an infalling observer from BH. Before the Page time, this can't happen. The canonical purification geometrizes this expectation precisely:



An Observation



Pre- vs Post-Page

Two semiclassical holographic spacetimes, with two boundaries and *the same von Neumann entropy*. One is connected and the other is not. Is this a counterexample to spacetime emergence from entanglement? What is the relevant difference betweent the two states that permits connectivity in one and not the other?

Maximally Mixed vs Slightly Mixed?

Obvious difference: post-Page black hole is maximally mixed. Pre-Page black hole is not.

$$S_{\text{th}}[t_1] > S_{\text{vN}}[\rho_{\text{BH}}(t_1)]$$

whereas

$$S_{\text{th}}[t_2] \approx S_{\text{vN}}[\rho_{\text{BH}}(t_2)]$$

Possibility: need both large $S_{\text{vN}}[\rho]$ *and* small $S_{\text{th}} - S_{\text{vN}}[\rho]$ for entanglement to “build” spacetime?

Adding a Pure State

But this is actually a red herring.

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And we can amplify the effect by working with a large number of matter fields.

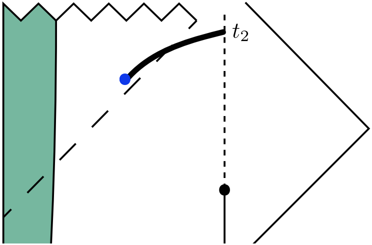
But maybe this changes the QES?

Adding a Pure State

Even if this moves the QES, it can't change S_{vN}. Since

$$S_{\text{vN}}[\rho(t_2)] = S_{\text{vN}}[\rho(t_2)'] < S_{\text{gen}}[\varnothing]|_{t=t_2}$$

there will always be a nontrivial dominant QES computing the entropy.



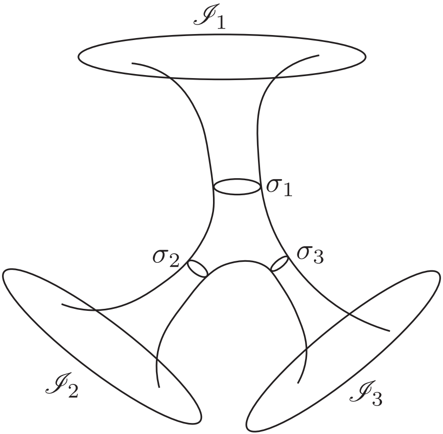
Thus the canonical purification will remain connected, S_{vN} remains constant, and we can increase S_{th}.

What about Complexity?

The pre-Page state is highly complex (has a nontrivial nonminimal QES), whereas the post-Page state is simple.

But we certainly expect to be able to construct highly complex connected wormholes. In particular, in the classical case, see Goel, Lam, Turiaci, Verlinde for partially entangled states with nonminimal compact (classical) extremal surfaces in a connected classical background.

Or a simpler example is a three-boundary wormhole.



Python's Lunch

Reconstruction is exponentially hard if and only if there exists a nonminimal QES. Brown et al; NE, Penington, Shahbazi-Moghaddam x2

A Puzzle

The Pre- vs Post-Page canonical purifications show that two states can

1. Have the same von Neumann entropy

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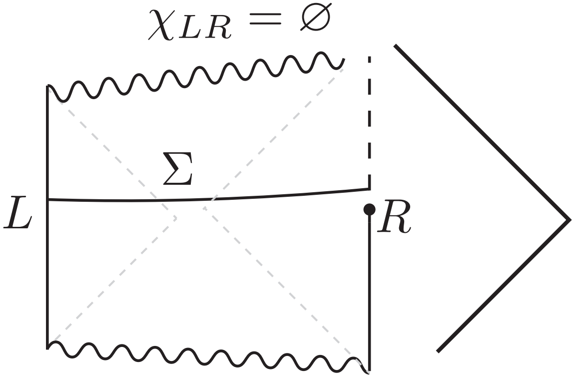
1. Have the same von Neumann entropy
2. Both be dual to well-behaved semiclassical bulk spacetimes
3. Can result from equally mixed ρ_{BH}'s.
4. Can both be high complexity

But one would be connected and the other not.

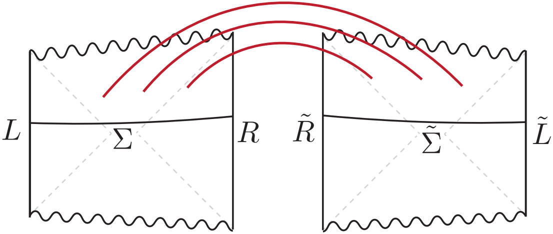
An Instructive Example

Let's take a *two*-sided black hole evaporating into a reservoir, and let's work with the entanglement wedge of $\rho_{\mathrm{BH,LR}}$ before the Page time:



An Instructive Example

We can canonically purify $\rho_{\mathrm{BH,LR}}$:

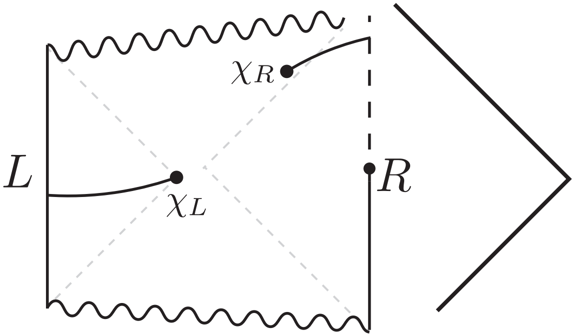


Here L and R are connected by an ERB but L and $\tilde{L}$, R and $\tilde{R}$ are not connected by an ERB.

An Instructive Example

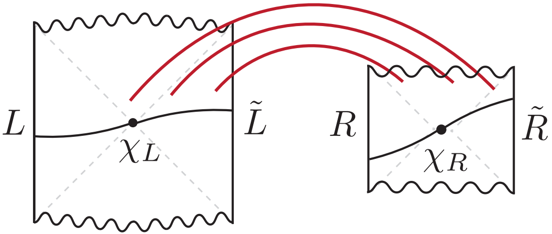
Now we consider the state $\rho_{\mathrm{BH,LR}}$ after the Page time.

$$\chi_{LR} = \chi_L \cup \chi_R$$



An Instructive Example

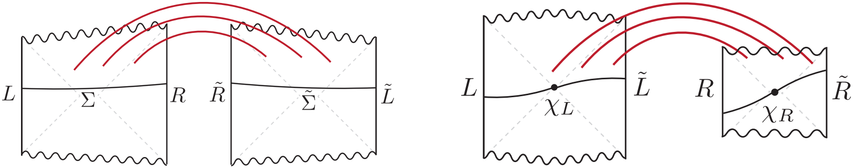
... and canonically purify:



Now L and $\tilde{L}$ are connected, but L and R are not.

Of course, we may pick ρ_{BH} to have the same von Neumann entropy in both cases.

Swapping Connectivity



What observable/quantity of $\rho_{\mathrm{LR\tilde{L}\tilde{R}}}$ determines if L and R are connected or L and $\tilde{L}$ are connected?

Unitaries Can't Distinguish

So what actually builds spacetime connectivity? We tried...

- Distillable entanglement
- Squashed entanglement
- Entanglement of purification/reflected entropy
- Capacity of entanglement
- Negativity
- ...

All of these are invariant under unitaries, and none could distinguish the difference.

Overview

Summary

- Constructed the canonical purification of an evaporating black hole
- Connected spacetime for the single-sided black hole after the Page time, disconnected before the Page time; connectivity swaps for a two-sided black hole.
- After the Page time, our protocol gives a prescription for realizing ER=EPR.
- Before the Page time, the geometry could potentially be unitarily mapped to an ERB.

Upshots

Unitarily-Invt Dictionary

The post-Page time canonical purification gives a standard AdS/CFT calculation of the entropy of the radiation (and unitary-invariants more generally).

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In favor of a Version of ER=EPR

After the Page time, there exists a factorizing unitary that maps that state to a semiclassical spacetime with an ERB.

Upshots

Unitarily-Invt Dictionary

The post-Page time canonical purification gives a standard AdS/CFT calculation of the entropy of the radiation (and unitary-invariants more generally).

In favor of a Version of ER=EPR

After the Page time, there exists a factorizing unitary that maps that state to a semiclassical spacetime with an ERB. But there may be a semiclassical description of the state that does not have an ERB.

So what builds spacetime connectivity?

Some questions:

1. What makes the connected spacetime special?

2. What determines if LR is connected or $L\tilde{L}$?

3. Why is connectivity so different at leading order vs subleading orders in $1/N$? Is this related to the story of Leuthesser, Liu; Witten where a type III von Neumann algebra emerges at strictly infinite N and a type II emerges when subleading corrections are included?

Thank you! And most importantly: happy (belated) birthday, Erik and Herman!

An Aside on Connectedness

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In this particular case, yes. We can do this using the fact that the post-Page black hole is simple (one way to see this is that the QES is practically on the event horizon: there is no Python's lunch in $\mathcal{W}_E[\rho_{\mathrm{BH}}]$.)

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Basic idea

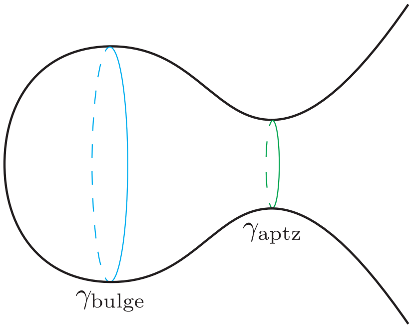
In such spacetimes, we can use a modified Gao-Jafferis-Wall type construction to try to send a causal signal from one boundary to the other. This will only succeed if the spacetime is connected.

Motivation: Strong Python's Lunch

A strengthening of the original Python's Lunch conjecture Brown, Gharibyan, Penington, Susskind

(Strong) Python's Lunch Brown, Gharibyan, Penington, Susskind; NE, Penington, Shahbazi-Moghaddam

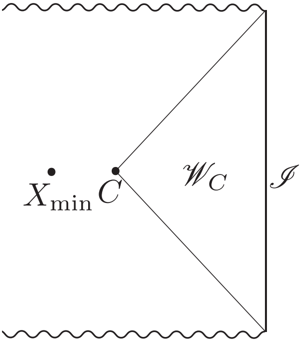
The only source of exponential complexity in the AdS/CFT dictionary can be identified with nonminimal QESs in the entanglement wedge.



Low Complexity

Low Reconstruction Complexity

If the entanglement wedge of each connected component of the boundary contains no QESs, then reconstruction is *low complexity*.



Simplifying the Spacetime

Using HKLL and timefolding, it is possible to push the event horizon back so it coincides with the outermost extremal surface.

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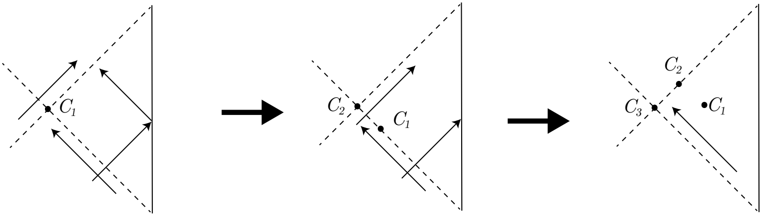
Example: JT + massless scalar:



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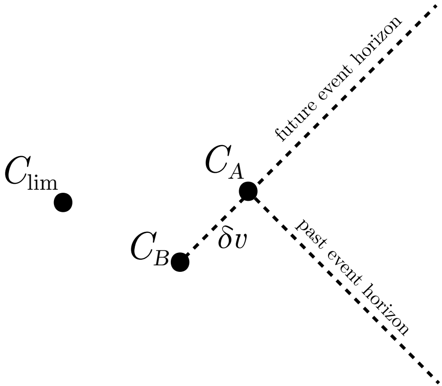

We can prove that C_{lim} limits to the outermost extremal surface. NE, Penington, Shahbazi-Moghaddam

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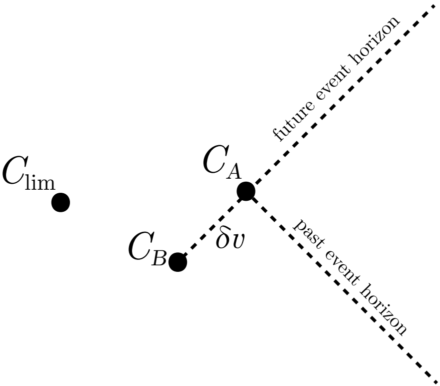

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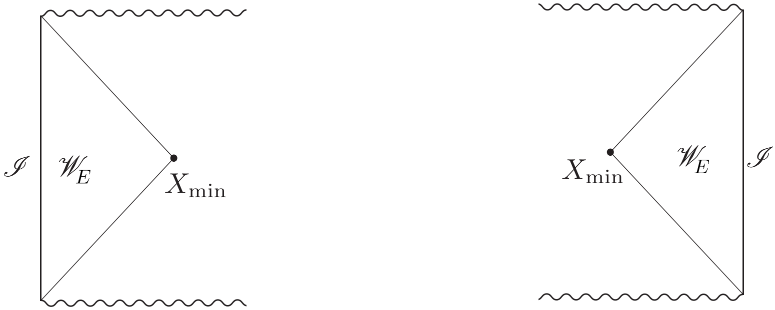
When the spacetime is low complexity, simple operations can reduce it to a TFD-like state in which the causal and entanglement wedges coincide*.

Connectedness for Low Complexity Spacetimes

Now consider a (possibly disconnected) low complexity spacetime with multiple boundaries.

Connectedness for Low Complexity Spacetimes

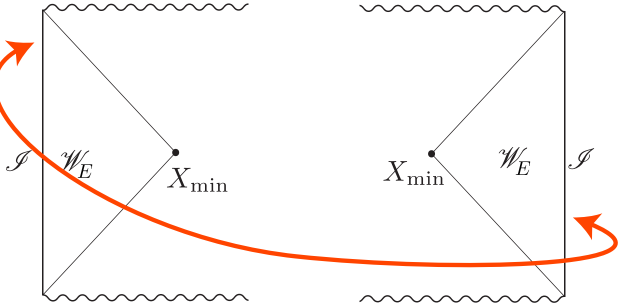
Apply the requisite simple operations to bring the causal and entanglement wedges of each boundary to coincide.



(Call the resulting spacetime "simple".)

Testing Connectedness

Couple the two dual CFT – e.g. double-trace deformation a la Gao-Jafferis-Wall.



Connectivity can then be diagnosed in the CFT via correlators, commutators, etc Maldacena, Stanford, Yang.

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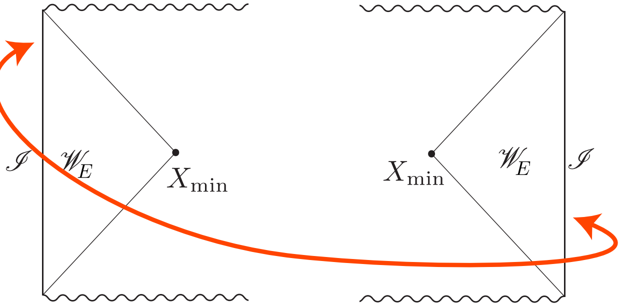

Connectivity can then be diagnosed in the CFT via correlators, commutators, etc Maldacena, Stanford, Yang. This gives one CFT way of diagnosing that our canonically purified post-Page black hole is connected.