Quantum Attacks on Lai-Massey Structure

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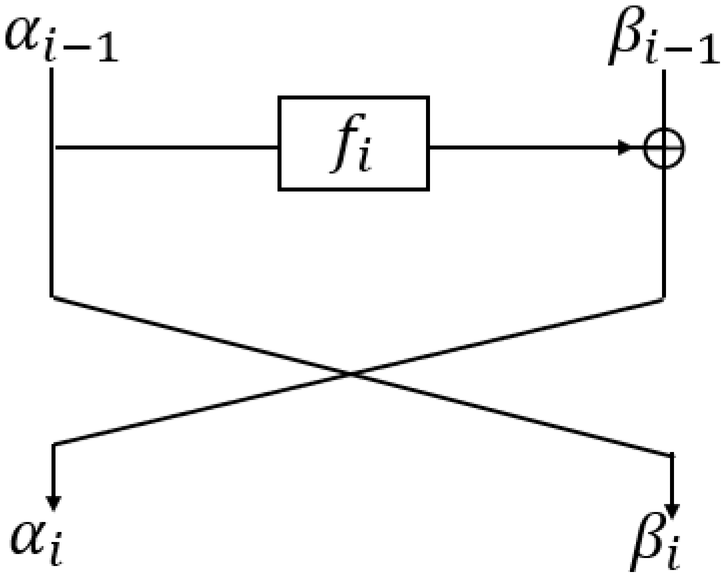
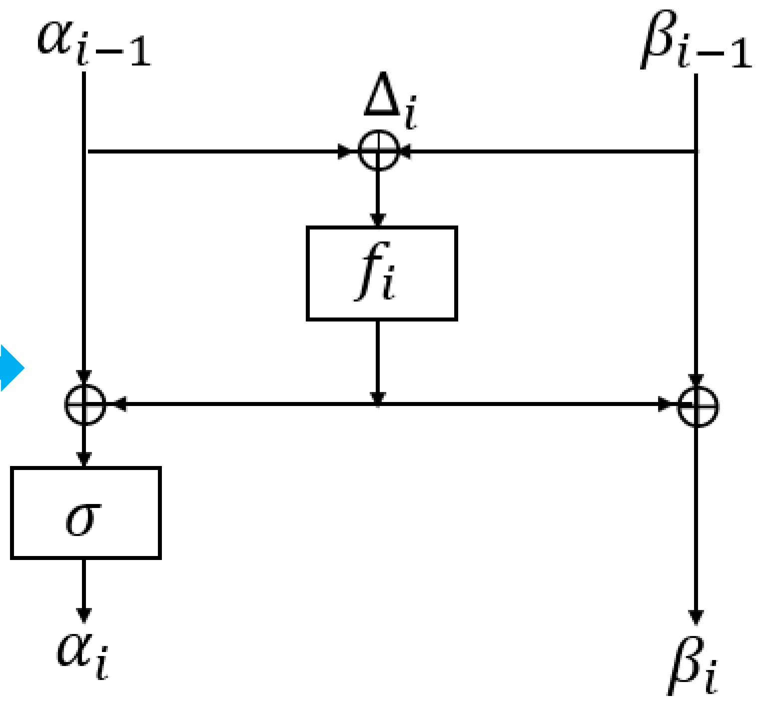


Feistel structure

the same security

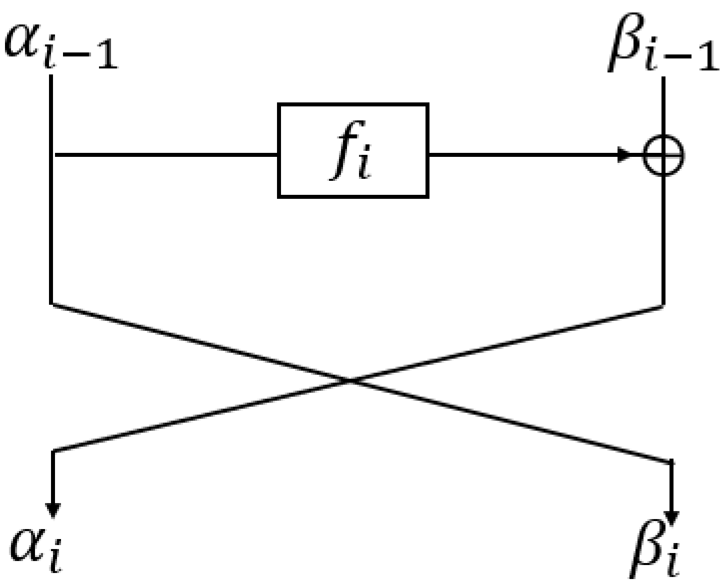
In classical



Lai-Massey structure

quasi-Feistel structure

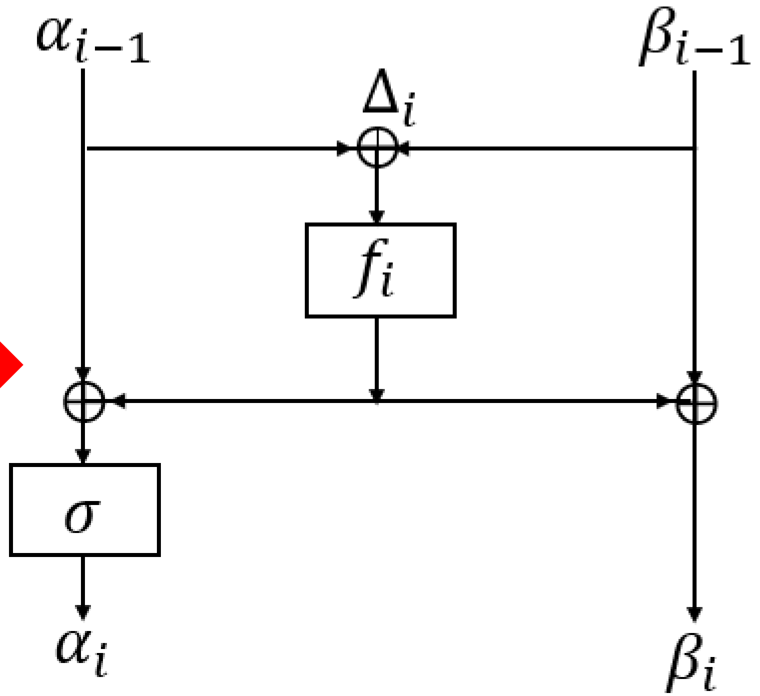
Aaram Yun et al.



Feistel structure

different security

In quantum?



Lai-Massey structure

Gembu Ito: 3 rounds Feistel structure can be attacked by using Simon's algorithm in quantum

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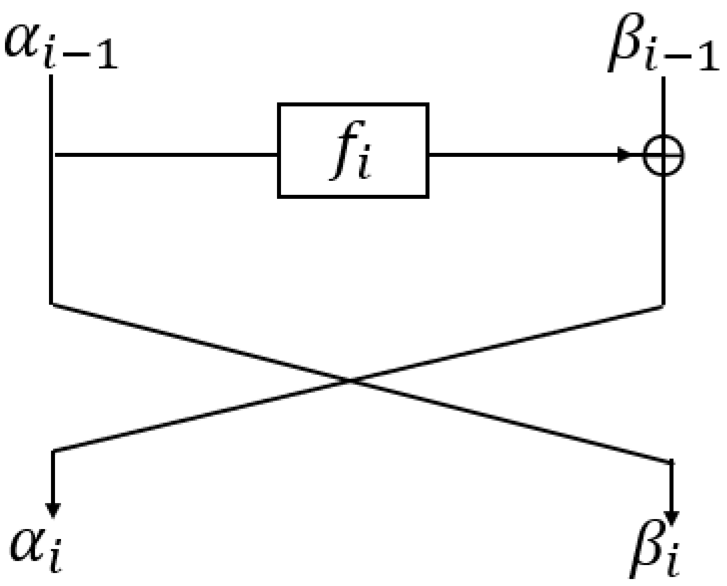
quasi-Feistel structure in quantum

Luo et al. : 3-round Lai-Massey structure can resist quantum attacks of Simon's algorithm

Question:

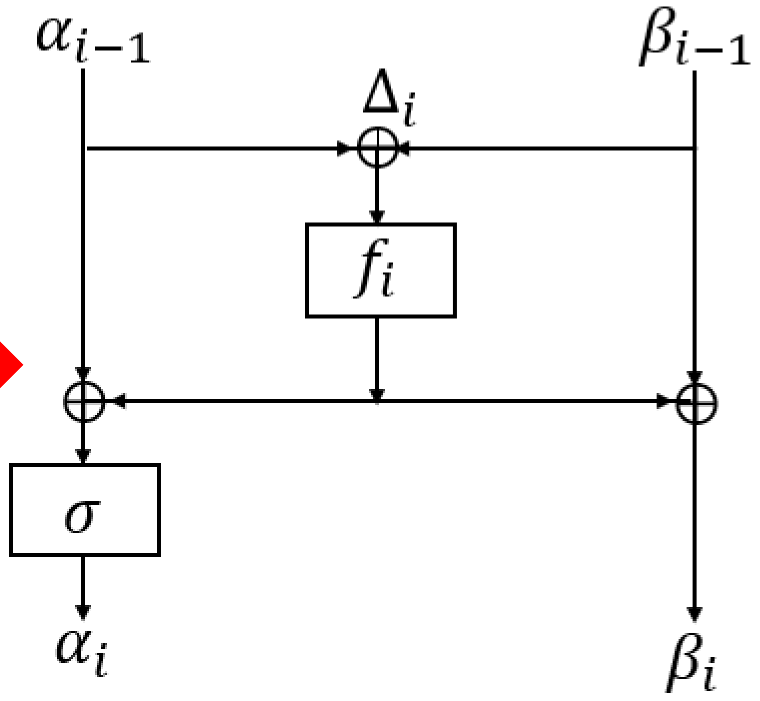
- Do Lai-Massey structure and Feistel structure have the same number of rounds that can be attacked in quantum?

- Can the attacks be extended to quasi-Feistel structures?



Feistel structure

different security ? In quantum



Lai-Massey structure

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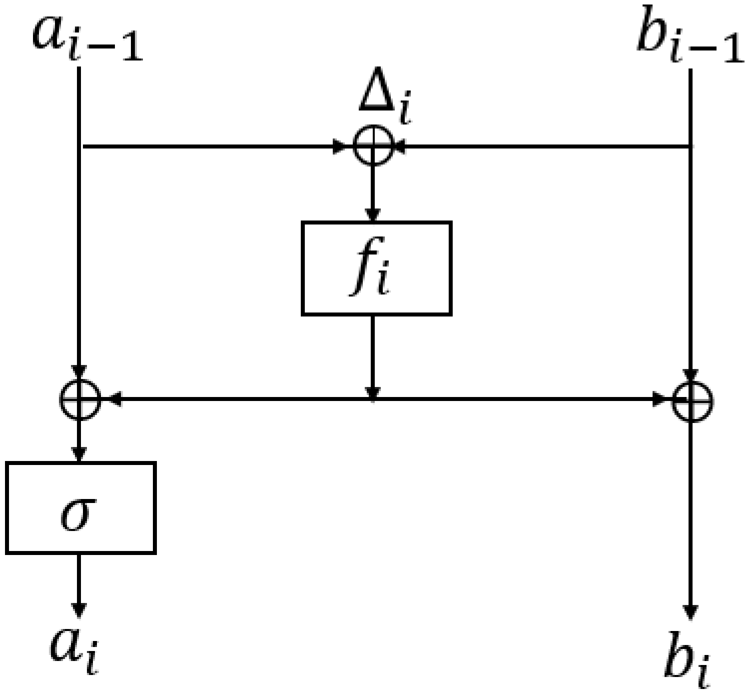
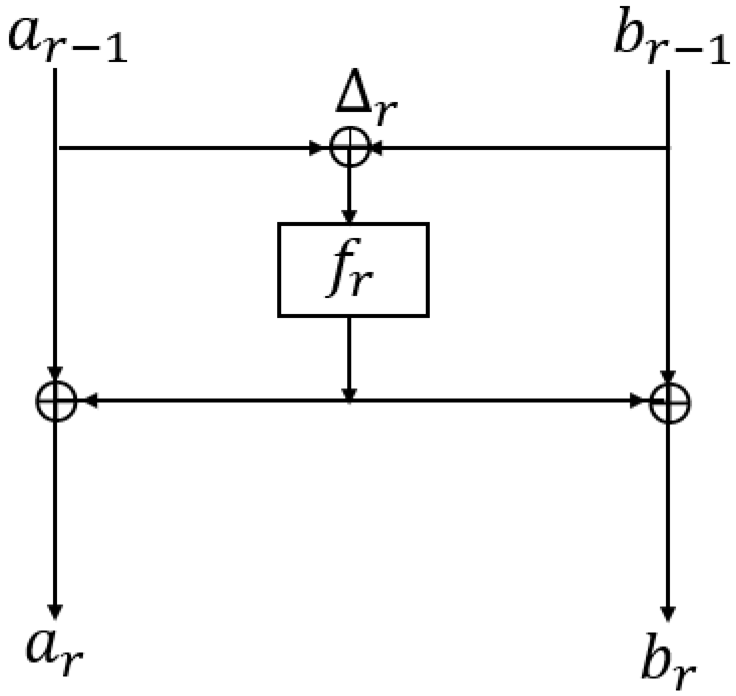
Quantum Attacks on Lai-Massey Structures

r-round Lai-Massey structure: $(a_r, b_r) = \mathrm{LM}'_r \circ \mathrm{LM}_{r-1} \circ \cdots \circ \mathrm{LM}_1$



The ith-round of Lai-Massey structure (LM_i)



The rth-round of Lai-Massey structure (LM'_r)

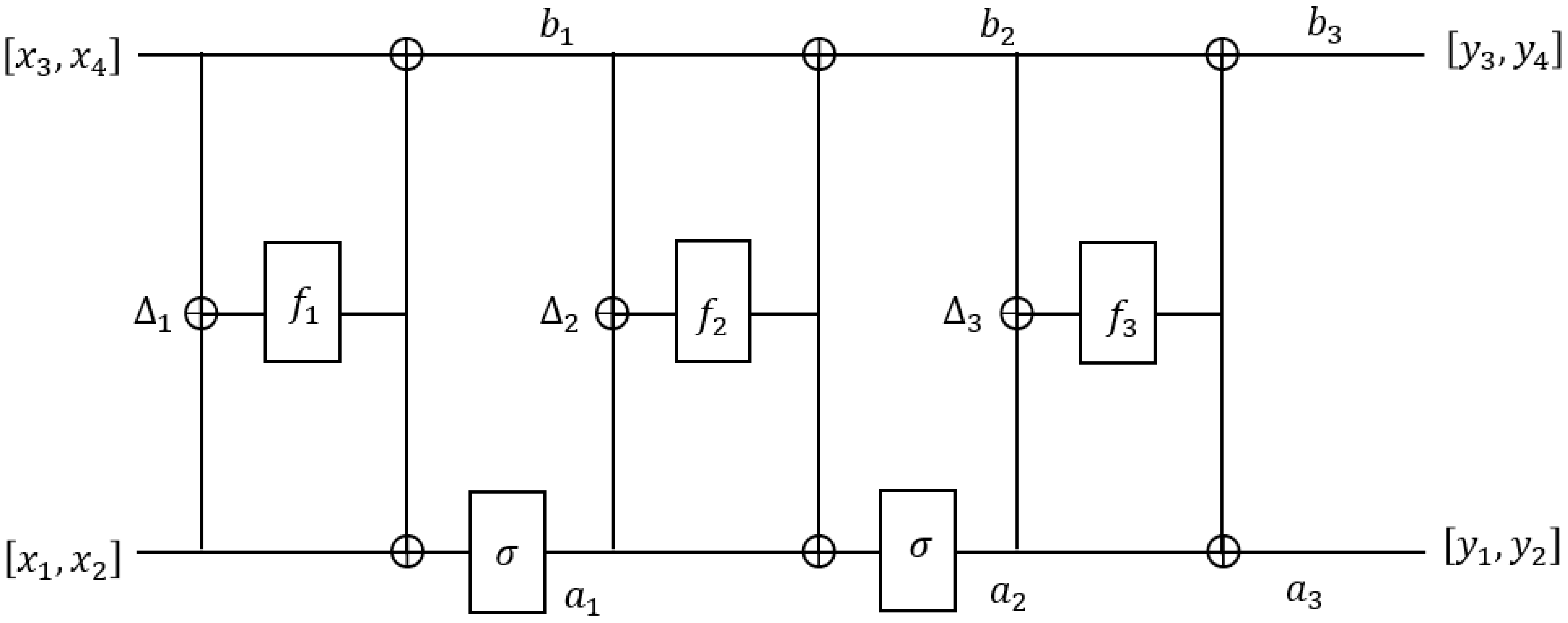
σ has the orthomorphism property: σ and $x \mapsto \sigma(x) - x$ are both permutations.

The instantiated Lai-Massey structure used in **FOX**:

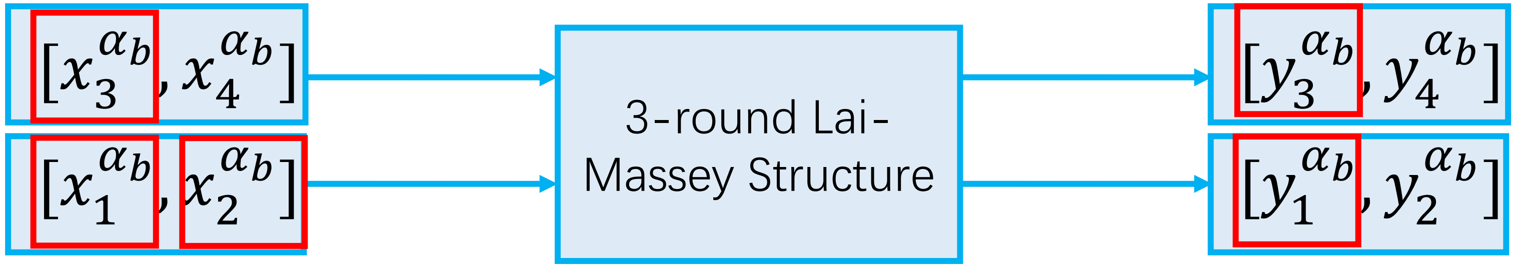
$$\sigma(x_L, x_R) = (x_R, x_L \oplus x_R)$$

$[a, b] \in \{0,1\}^n$: a, b represent the highest $n/2$ bits and the lowest $n/2$ bits respectively

Quantum Chosen-Plaintext Attack Against 3-round Lai-Massey Structure



- Let $x, x' \in \{0,1\}^{n/2}$. $([x_1^{\alpha_b}, x_2^{\alpha_b}], [x_3^{\alpha_b}, x_4^{\alpha_b}]) \overset{\text{def}}{=} ([x \oplus \alpha_b, x'], [x, x' \oplus \alpha_b])$.

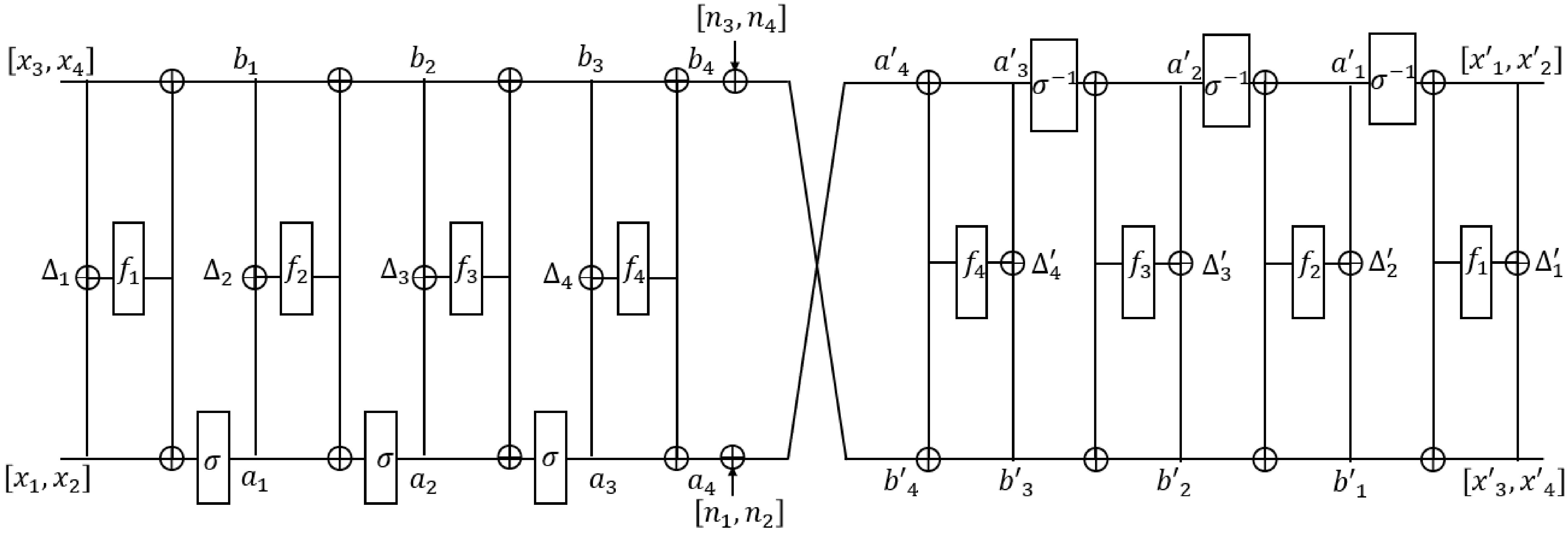


- We can construct a periodic function g_1 with period $s = f_1[\alpha_0, \alpha_0] \oplus f_1[\alpha_1, \alpha_1]$ by letting

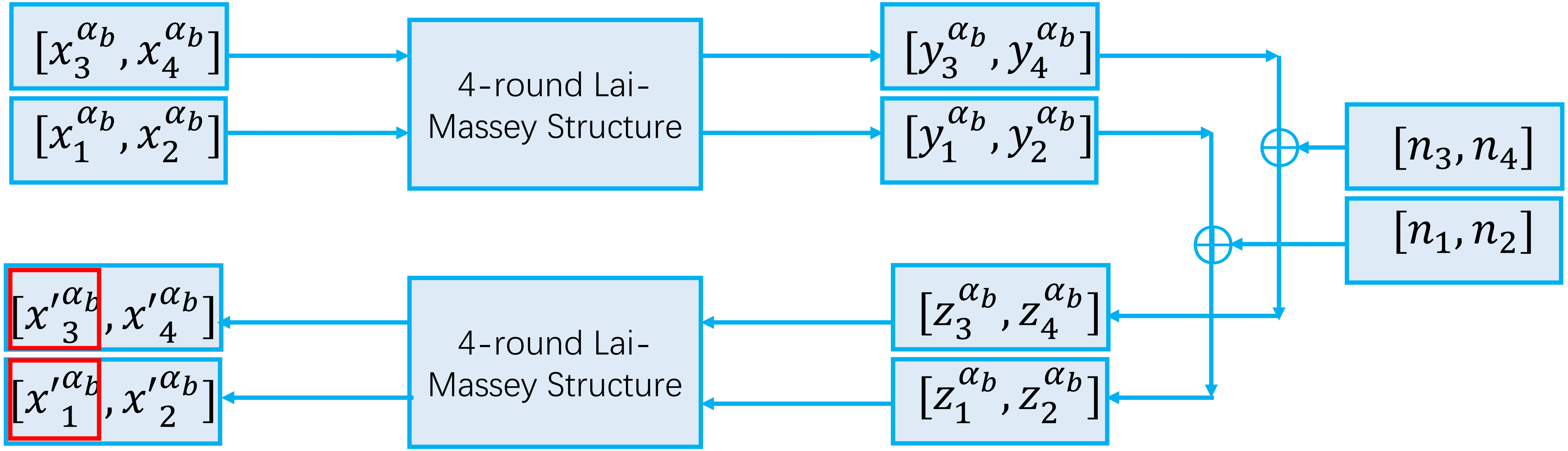
$$g_1 \mapsto x_1^{\alpha_0} \oplus x_2^{\alpha_0} \oplus x_3^{\alpha_0} \oplus y_1^{\alpha_0} \oplus y_3^{\alpha_0} \oplus \\ x_1^{\alpha_1} \oplus x_2^{\alpha_1} \oplus x_3^{\alpha_1} \oplus y_1^{\alpha_1} \oplus y_3^{\alpha_1}$$

- we can construct a quantum CPA distinguisher by using Simon' algorithm in $O(n)$ quantum queries.

Quantum Chosen-Ciphertext Attack Against 4-round Lai-Massey Structure



- Let $x, x' \in \{0,1\}^{n/2}$. $([x_1^{\alpha_b}, x_2^{\alpha_b}], [x_3^{\alpha_b}, x_4^{\alpha_b}]) \overset{\text{def}}{=} ([x \oplus \alpha_b, x'], [x, x' \oplus \alpha_b])$.



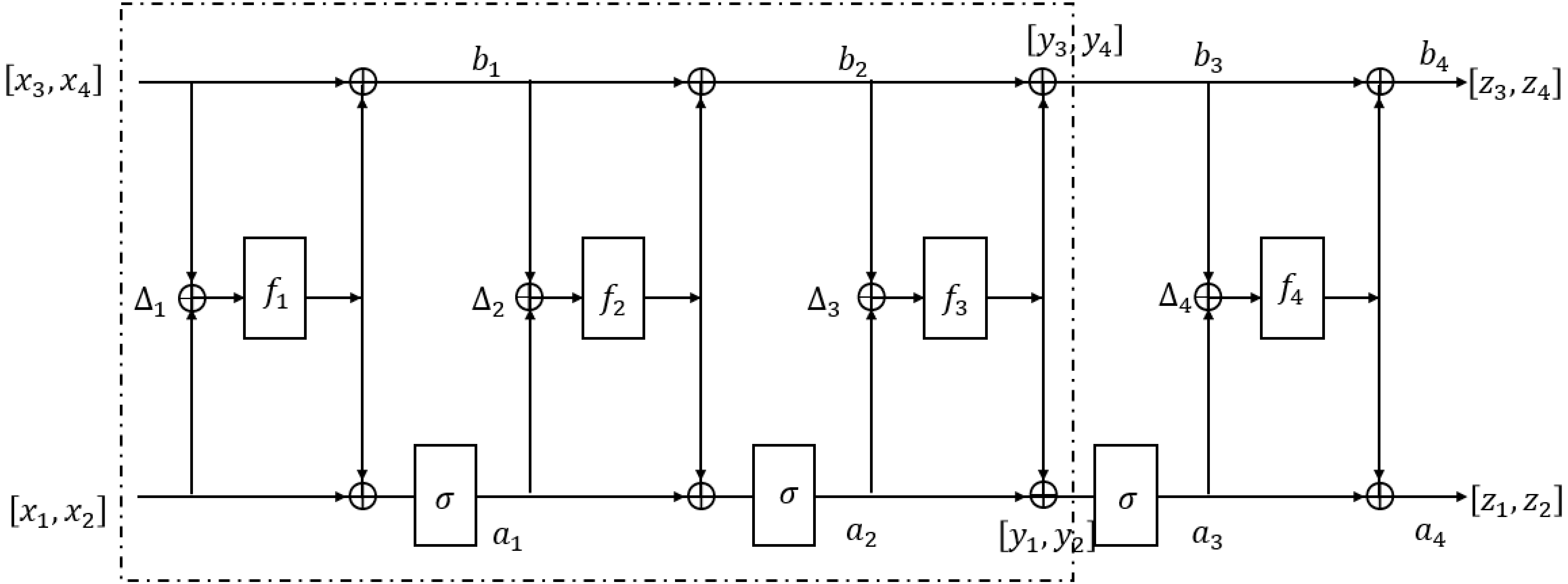
Let $n_1 = n_2 = n_3 = n_4 = \alpha_0 \oplus \alpha_1$

- We can construct a periodic function g_2 with period $s = f_1[\alpha_0, \alpha_0] \oplus f_1[\alpha_1, \alpha_1]$ by letting

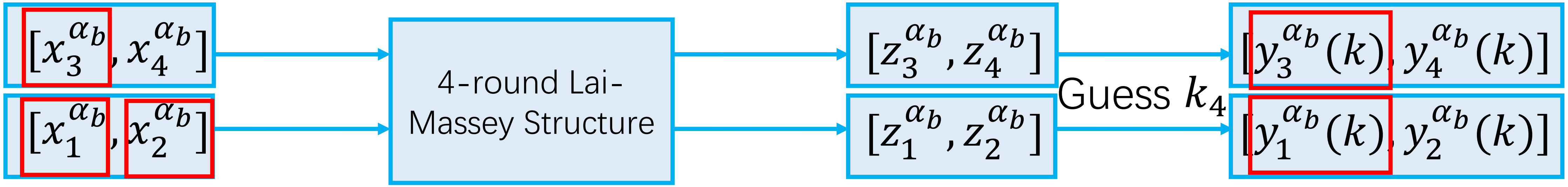
$$g_2 \mapsto x_1'^{\alpha_0} \oplus x_3'^{\alpha_0} \oplus x_1'^{\alpha_1} \oplus x_3'^{\alpha_1}$$

- we can construct a quantum CCA distinguisher by using Simon' algorithm in $O(n)$ quantum queries.

Quantum Key-recovery Attack on 4-round Lai-Massey Structure



- Let $x, x' \in \{0,1\}^{n/2}$. $([x_1^{\alpha_b}, x_2^{\alpha_b}], [x_3^{\alpha_b}, x_4^{\alpha_b}]) \stackrel{\text{def}}{=} ([x \oplus \alpha_b, x'], [x, x' \oplus \alpha_b])$.

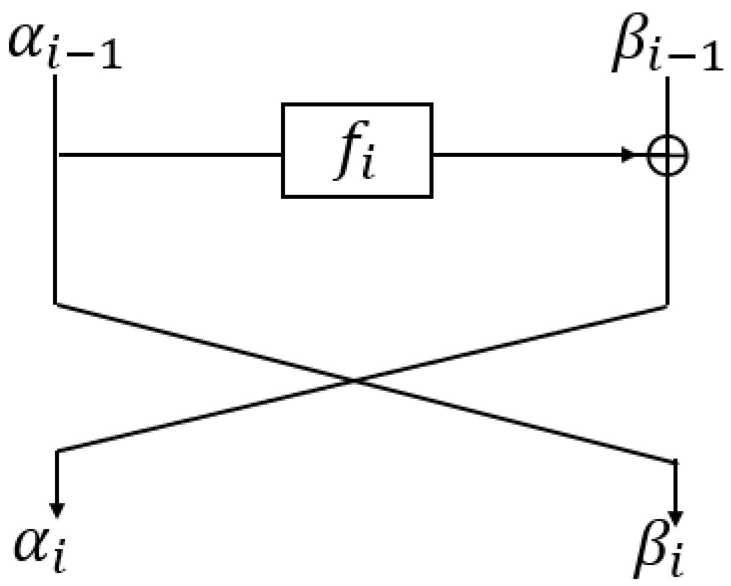


- We can construct a periodic function g_3 by letting

$$g_3 \mapsto x_1^{\alpha_0} \oplus x_2^{\alpha_0} \oplus x_3^{\alpha_0} \oplus y_1^{\alpha_0}(k) \oplus y_3^{\alpha_0}(k) \oplus \\ x_1^{\alpha_1} \oplus x_2^{\alpha_1} \oplus x_3^{\alpha_1} \oplus y_1^{\alpha_1}(k) \oplus y_3^{\alpha_1}(k)$$

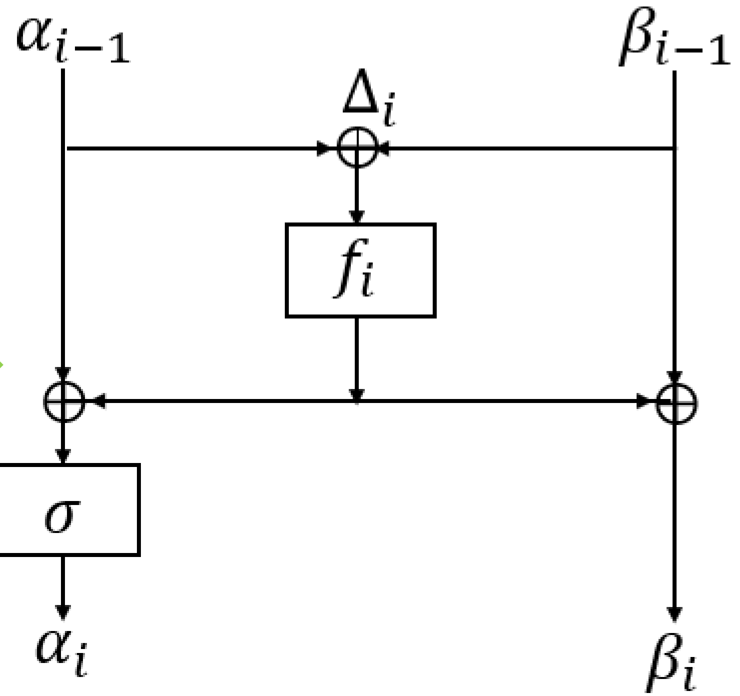
Then g_3 is a periodic function with period $s = f_1[\alpha_0, \alpha_0] \oplus f_1[\alpha_1, \alpha_1]$ if k guessed right.

- We can give a quantum Grover-meet-Simon attack with $O(n2^{m/2})$ quantum queries in quantum CPA.



The same security

In quantum



Feistel structure

Lai-Massey structure

Gembu Ito: 3/4 rounds Feistel can be attacked by using Simon's algorithm in quantum

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quasi-Feistel structure in quantum

Our: 3/4 rounds Lai-Massey structure can be attacked by using Simon's algorithm in quantum

Quantum Attacks against Quasi-Feistel structures

Quasi-Feistel structures

Combiner:

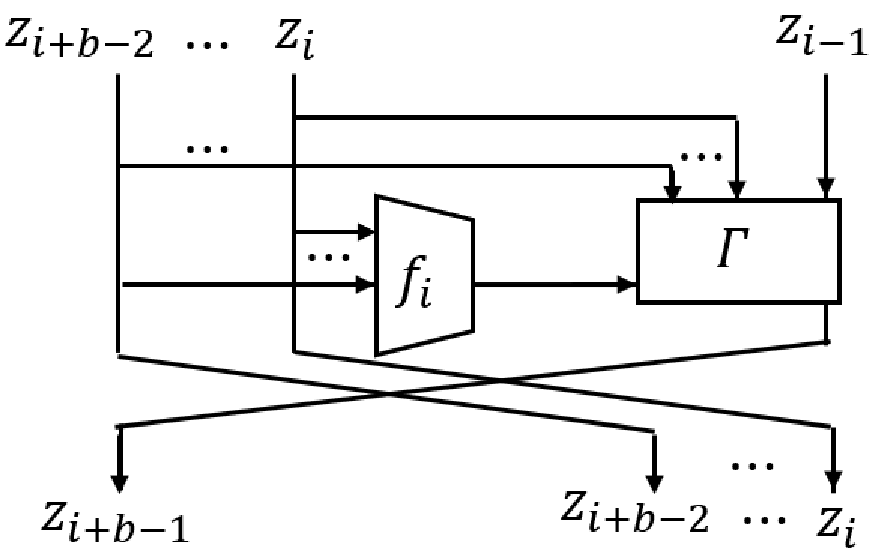
A function $\Gamma: \mathcal{X} \times \mathcal{X} \times \mathcal{Y} \to \mathcal{X}$ is a combiner over $(\mathcal{X}, \mathcal{Y})$, if

- for $y \in \mathcal{X}, z \in \mathcal{Y}$, $x \mapsto \Gamma(x, y, z)$ is a permutation, and
- for $x \in \mathcal{X}, z \in \mathcal{Y}$, $y \mapsto \Gamma(x, y, z)$ is a permutation.

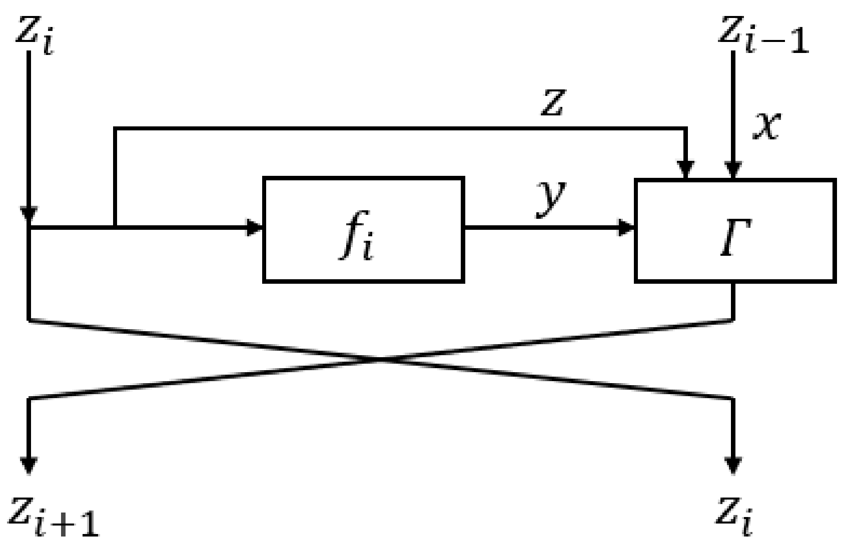
We denote $\Gamma[[x \star y \mid z]] \stackrel{\text{def}}{=} \Gamma(x, y, z)$.

- Feistel structure： $\Gamma\left[\left[x \star y \mid z\right]\right] = x \oplus y$
- Our Lai-Massey structure： $\Gamma\left[\left[x \star y \mid z\right]\right] = \sigma(x) \oplus \sigma^{-1}(y) \oplus \sigma^{-1}(\mathrm{z})$

b-branched, r-round quasi-Feistel structure:

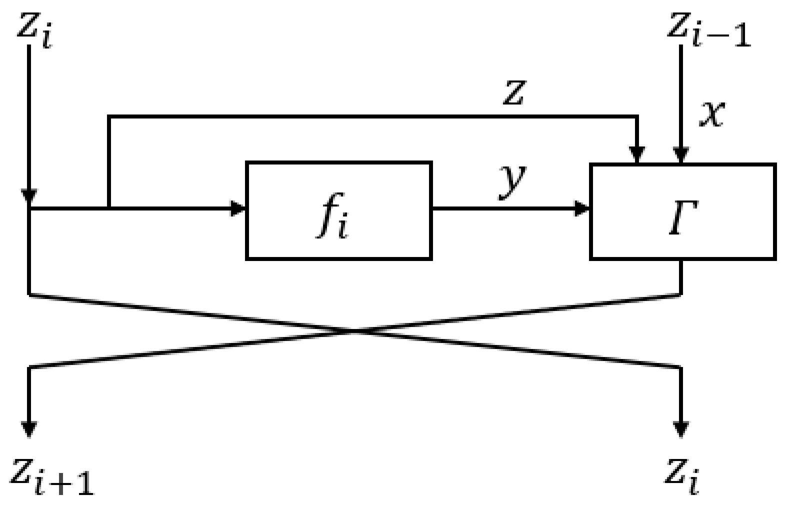


ith-round of quasi-Feistel structure

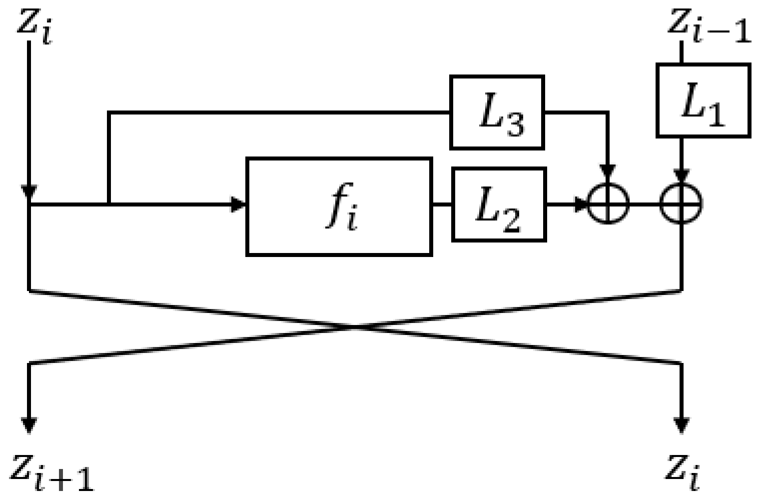


ith-round of balanced quasi-Feistel structure

1. $(z_0, z_1, \dots, z_{b-1}) \leftarrow P(x)$,
2. $z_{i+b-1} \leftarrow \Gamma\left[\left[z_{i-1} \star f_i(z_i \cdots z_{i+b-2}) \mid z_i \cdots z_{i+b-2}\right]\right]$ for $i = 1, \dots, r$.
3. $y \leftarrow Q^{-1}(z_r, z_{r+1}, \dots, z_{r+b-1})$.

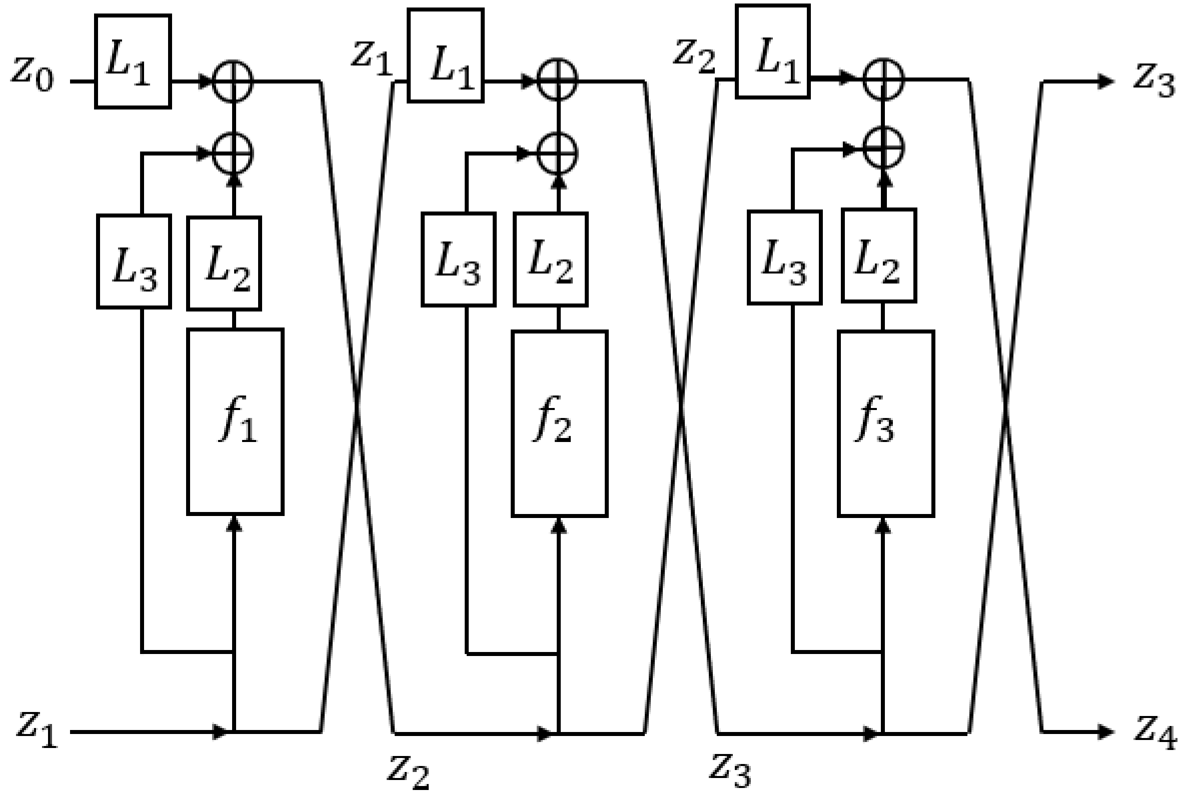


ith-round of balanced quasi-Feistel structure

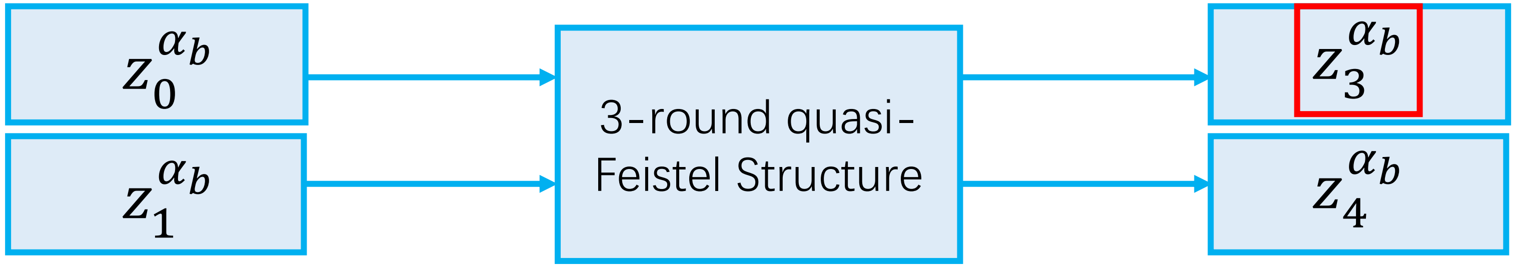


ith-round of balanced quasi-Feistel structure with linear combiner

Quantum Chosen-Plaintext Attack Against 3-round quasi-Feistel Structure



- Let $x \in \{0,1\}^n$. $(z_0^{\alpha_b}, z_1^{\alpha_b}) \overset{\text{def}}{=} (x, \alpha_b)$.

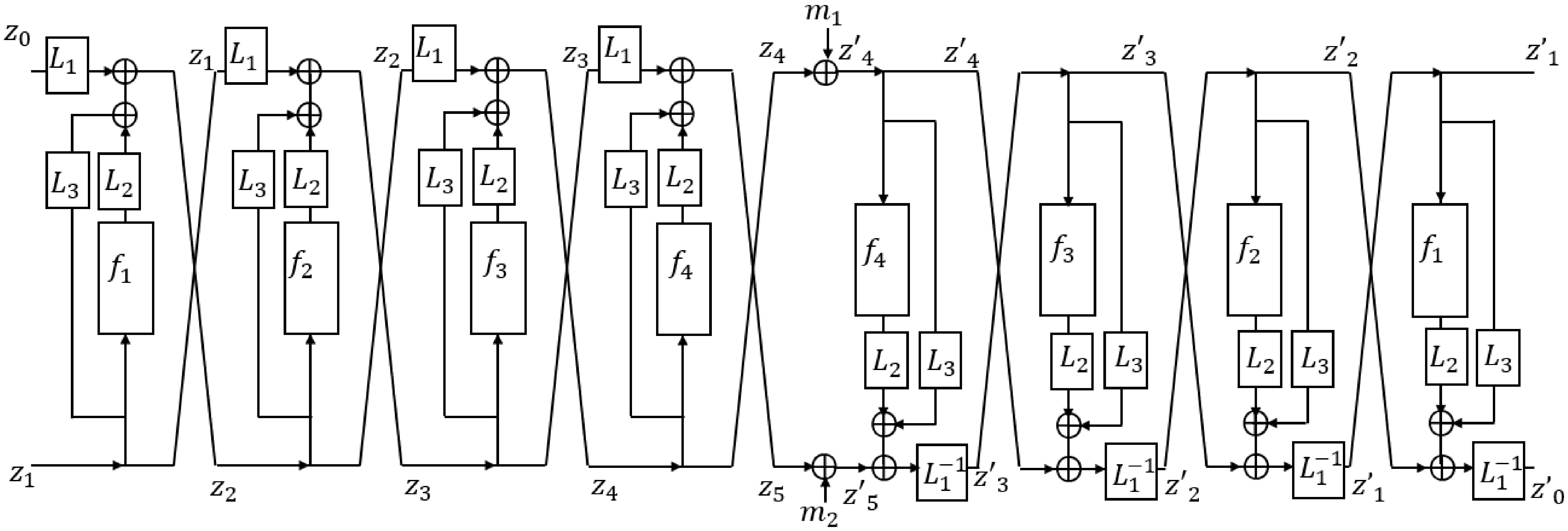


- We can construct a periodic function g_4 with period $s = L_1^{-1}L_2(f_1(\alpha_0)) \oplus L_1^{-1}L_2(f_1(\alpha_1)) \oplus L_1^{-1}L_3(\alpha_0) \oplus L_1^{-1}L_3(\alpha_1)$ by letting

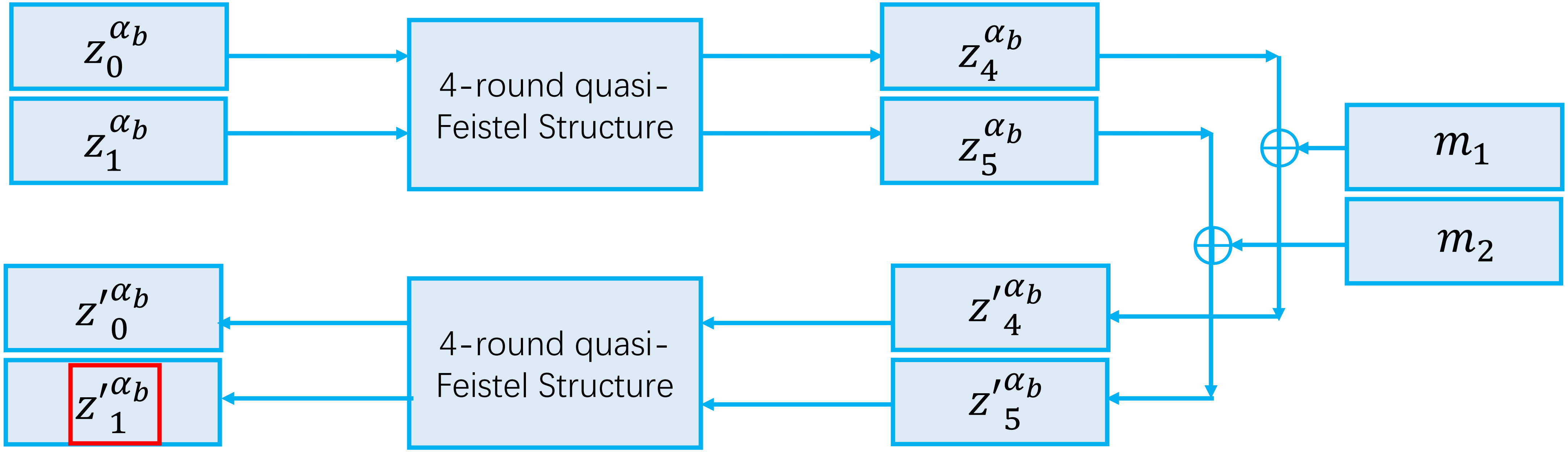
$$g_4 \mapsto z_3^{\alpha_0}(x) \oplus z_3^{\alpha_1}(x)$$

- we can construct a quantum CPA distinguisher by using Simon' algorithm in $O(n)$ quantum queries.

Quantum Chosen-Ciphertext Attack Against 4-round quasi-Feistel Structure



- Let $x \in \{0,1\}^n$. $(z_0^{\alpha_b}, z_1^{\alpha_b}) \stackrel{\text{def}}{=} (x, \alpha_b)$.



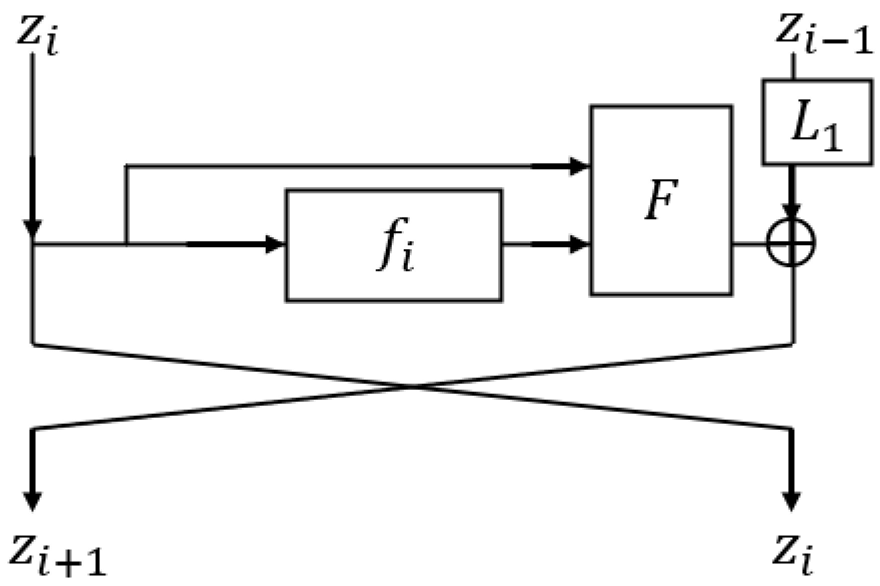
Let $m_1 = 0, m_2 = L_1 L_1(\alpha_0) \oplus L_1 L_1(\alpha_1)$

- We can construct a periodic function g_5 with period $s = L_1^{-1} L_2(f_1(\alpha_0)) \oplus L_1^{-1} L_2(f_1(\alpha_1)) \oplus L_1^{-1} L_3(\alpha_0) \oplus L_1^{-1} L_3(\alpha_1)$ by letting

$$g_5 \mapsto {z'}_1^{\alpha_0}(x) \oplus {z'}_1^{\alpha_1}(x) \oplus \alpha_0 \oplus \alpha_1$$

- we can construct a quantum CCA distinguisher by using Simon' algorithm in $O(n)$ quantum queries.

Acknowledgement



One of reviewers pointed out that the combiner Γ of balanced quasi-Feistel structure does not need to be all linear.
After our verification, only L_1 needs to be linear.

$$\Gamma[\![x \star y \mid z]\!] = L_1(x) \oplus F(y, z)$$

where L_1 is linear and F is a function.

Thanks