Problem 1. Consider the function $f(x) = \sum_{i=1}^{n} |x_i|$.

1. Derive the conjugate function f^*, show the derivations.
2. Derive $\partial f(x)$ in an arbitrary point x, show the derivations.

Problem 2. Consider a convex, proper, closed function $f : \mathbb{R}^n \to \overline{\mathbb{R}}$, and let prox_f be the proximal operator of f defined as

$$\text{prox}_f(z) = \arg\min_{x \in \mathbb{R}^n} f(x) + 0.5\|x - z\|^2.$$

1. Show that for f with the given properties, the minimum in the definition of prox_f is attained and unique.
2. Show that prox_f is firmly non-expansive.
3. Does firm non-expansiveness imply non-expansiveness (Lipschitz-continuity with constant 1)? Does firm non-expansiveness imply continuity? Motivate your answers.
4. For a $z \in \mathbb{R}^n$, prove the following:

   $$z = \text{prox}_f(z) + \text{prox}_{f^*}(z),$$

   where f^* is the conjugate function of f.