Breaking Category Five SPHINCS$^+$ with SHA-256

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Summary of Result

- SPHINCS^+ is a stateless hash-based signature selected for standardization by NIST
- We present a forgery attack that reduces classical security by 40 bits
  - For submitted parameter sets:
    - That target Category 5
    - While using SHA-256
- Our attack builds on a previous attack by Antonov on the DM-SPR property of SHA-256 (a security assumption for SPHINCS^+)
- The SPHINCS^+ team has proposed a tweak which defeats our attack by using SHA-512 instead of SHA-256 (where necessary)

Outline

- Hash-Based Signatures: One-time, Multi-Use, Stateless
- SPHINCS$^+$ Basic Design, including WOTS$^+$
- DM-SPR Property and Antonov's Attack
- Using Antonov's Attack to Forge WOTS$^+$ (This Paper)
- Optimizations (This Paper)
- The SPHINCS$^+$ Tweak
- Conclusion

Hash-Based One-Time Signature (OTS)

- Most basic hash-based signature (Lamport 1979)
  - For bit b_i
    - Generate a secret S_{i0} for signing 0 and another S_{i1} for signing 1
  - Public key is

$$H(S_{00})|\ H(S_{00})|\ \ldots\ H(S_{(n-1)0})|\ H(S_{(n-1)1})$$

  - Can securely sign one n-bit digest
    - Signature is:

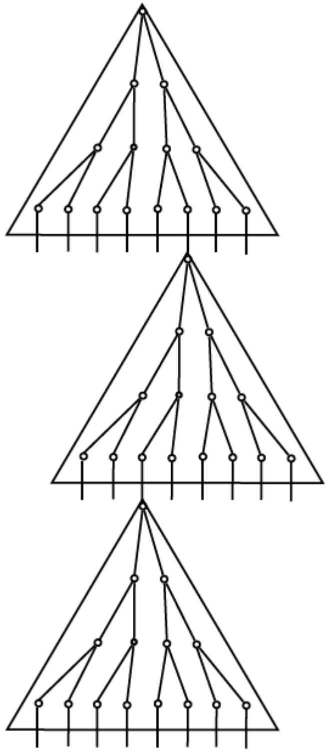
$$S_{0b_0}|\ \ldots\ |S_{(n-1)b_{(n-1)}}$$

- More advanced variants (e.g. WOTS+ – discussed later)
  - Reduce signature size using hash chains, etc.
  - Reduce PK size by using public key hash instead
    - We'll call the thing that gets hashed the *Public Key Preimage*

Multi-Use Hash-Based Signature

- Hash many OTS public keys together in a Merkle Tree
  - Only increases signature size logarithmically
  - But all OTS keys need to be precomputed
- Can get rid of precomputation by having OTS leaves of top tree sign roots of trees generated on the fly

- Hypertree:



Stateless Hash-Based Signature

- Create a hypertree by having multiple layers of Merkle trees with leaves from one signing the root of the next
- If the hypertree has enough leaves, the leaf can be chosen randomly with little risk of using the same leaf twice
  - Can make hypertree a lot smaller by using a few-time signature to sign the message
- Generate OTS keypairs pseudorandomly from seed and hypertree location so that each upper leaf always signs the same Merkle-Tree root

SPHINCS$^+$

Basic Design

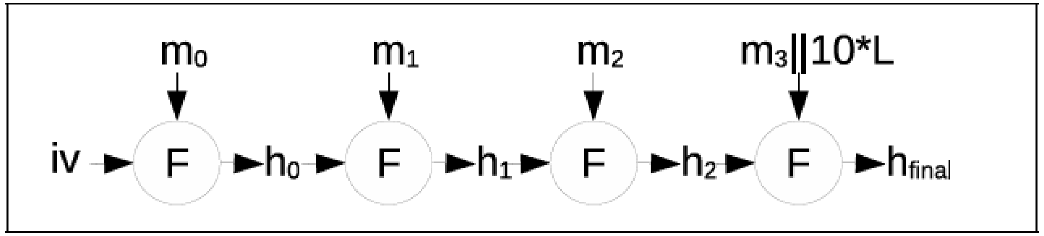
- Merkle roots are signed by WOTS$^+$
  - The Focus of our attack
- (Randomized) message digest is signed by FORS
  - FORS root is also signed by WOTS$^+$
- Hypertree path to FORS key is determined by extended message digest

SPHINCS+

Prefixes and *Distinct Function Multitarget Preimage Resistance* (DM-SPR)

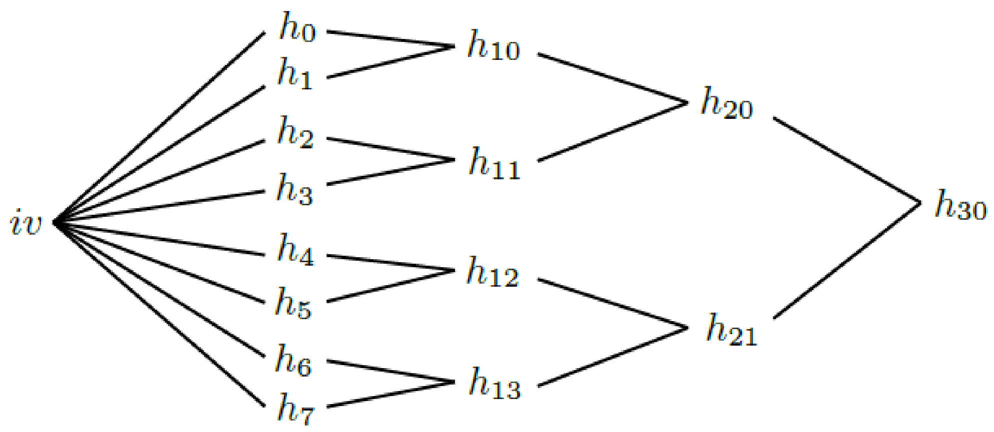
- Many places in hypertree where a preimage can create a forgery:
  - Hashes in Merkle Trees
  - Hash Chains in $WOTS^+$
  - Hash trees in FORS
  - FORS public key hash
  - $WOTS^+$ public key hash (Our attack here)
- New targets are revealed with every honest signature
- To avoid a 1 out of t multi-target preimage attack:
  - Make sure hash input at each hypertree location has a distinct prefix
  - Formalized as a tweakable hash function with DM-SPR property

Merkle-Damgård (SHA-256) Construction



- Hash an arbitrary-sized message using a fixed-input-length compression function, F
- Break message into B-bit blocks and repeatedly use F to produce an n-bit chaining value
- For SHA-256: B=512, n=256
- MD hash is proven collision resistant if F is
- But, MD doesn't always get more than $n/2$ bits of security for other properties:
  - Multicollisions for multi-block messages [Joux 2004]
  - Long message preimage attack [Dean 1999], [KS 2004]
  - Herding attack [KK 2005]
- What about DM-SPR?

Herding Attack



- Create many messages
  - With distinct fixed prefixes
  - That hash to the same value
- Build “Diamond Structure”
  - Distinct prefixes result in distinct internal states (h_1-h_7)
  - Use collision search on compression function to find message blocks that collide resulting in fewer distinct states (h_{10}-h_{13})
  - After adding logarithmically many (Above, 3) blocks to prefix, all messages hash to same chaining value (h_{30})

Antonov's Attack on SHA-256 DM-SPR [Antonov 2022]

- Collect t target hashes with different prefixes
- Find preimage with the same prefix for 1 of them
  - Use Herding to reach same state from all prefixes at the penultimate block
  - Use Multi-Target preimage search on compression function to find a block to append and reach a target
- Longest hash input in SPHINCS $^+$ is WOTS$^+$ public key hash
- That's still pretty short (34 blocks)
  - To balance cost of herding, multi-target preimage search, use some compression-function 3-collisions
    - Let t be $2^{10}3^{23} \approx 2^{46}$ instead of 2^{33}
    - 3-Collision search cost: $1.5 \cdot 3^{23} \cdot 2^{170.7} \approx 2^{208}$
    - Multi-Target preimage cost: $2^{256}/2^{46} \approx 2^{210}$

What's Left to Do?

- Antonov's attack lets us create a validly-signed $WOTS^+$ public key preimage
- But we need to know the corresponding private key to forge a $SPHINCS^+$ signature
  - This involves knowing preimages of parts of $WOTS^+$ public key
  - For validity, prefix must match hypertree location
  - But hypertree location depends which target we reached
  - No way to force correct prefix for all targets
- Or at least part of it…
  - As long as we can sign more than one possible digest with our $WOTS^+$ key
  - Can graft a forged Merkle-Tree root to the hypertree for less than 2^{256} work!

Our Attack: Outline

- Find a preimage of some $WOTS^+$ public key with enough private key info to sign **some** digests
- Brute-force search for a valid Merkle/FORS tree whose root has signable digest
- Sign the tree root with the attacked $WOTS^+$ key
- To forge a signature, try message randomization strings until the hypertree address is a descendent address of the tree root

WOTS+ Signature

- Write digest as base-w (16) number
- Append a base-w checksum
  - (960 – <sum of digits>)
- Sign each digit d_i of digest plus checksum by:
  - Hash $sk_{i,0}$ (with prefix) d_i times
  - Put the result in the signature
- Note: The signature of 0xF is just pk_i

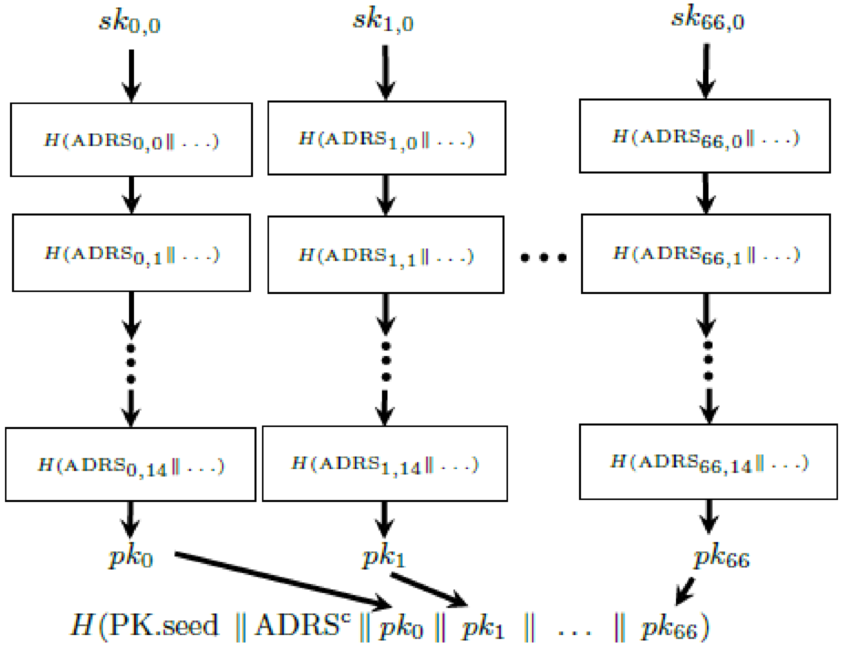


Fig. 2. A WOTS+ public key.

Finding a Merkle/FORS Root We Can Sign

- Aim to sign a digest like:

  xxxxxxxx xxxxxxxx xxxxxxxF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF

- Modify Antonov's multi-target preimage search to find a WOTS+ public key that can sign this
  - Treat the part that signs xxxx… as prefix – so we know $sk_{i,0}$ for this part
  - Use the last block of the prefix and the part that signs FFFF… for herding and multitarget preimage search
  - Target the SHA-256 state immediately before the first block that signs checksum
  - The part that signs the checksum will come from the target honest signature
- Can forge a signature on any Merkle/FORS root of the above form as long as checksum works out

Making Sure the Checksum Works Out

- For a digest like:

  xxxxxxxx xxxxxxxx xxxxxxxF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF

  - Checksum is $960 - 41 \cdot 15 - \sum x$
- We can increment, but not decrement, digits of honest checksum
  - Increment a digit by hashing (with prefix) sk_{i,d_i}
- Can choose targets with unusually small checksums
- Need $\sum x$ to be small enough with high enough probability

Batched Multicollision Search

- Best parameterization of our attack involves finding lots of 4-way collisions with distinct prefixes
- It is cheaper to search for lots of collisions at once
  - Finding a single 4-way collision costs $\sim 2^{192}$
  - Finding t 4-way collisions costs $\sim 2^{192} t^{1/4}$
  - (Ignoring prefixes and memory access costs)
- To get good memory access costs, use parallel collision search techniques
- To avoid wasting time colliding already-used prefixes
  - Compute collisions in smaller batches of size αt
- More detail in paper

Attack Complexity

Table 1. Summary of Our Results on SPHINCS$^+$ Category Five Parameters

Parameter Set	Cost				Reference
	Herd	Link	Signable	Total	
SPHINCS$^+$-256f	$2^{214.8}$	$2^{216.4}$	$2^{215.7}$	$\approx 2^{217.4}$	Section 4.3
SPHINCS$^+$-256s	$2^{214.8}$	$2^{216.4}$	$2^{215.7}$	$\approx 2^{217.4}$	Section 4.3

SPHINCS$^+$ Tweak [Hülsing 2022]

- In response to Antonov's attack on DM-SPR the SPHINCS$^+$ team issued a tweak to the SPHINCS$^+$ specification
  - Replaced SHA-256 with SHA-512, for hashing multi-block inputs in Category 3 and 5 parameters
  - Still some use of SHA-256, but doesn't seem exploitable

Conclusion

- Our attack shows that some submitted parameter sets of SPHINCS$^+$ are not as strong as claimed
- The problem is not the security proof for the SPHINCS$^+$ construction, but how its tweakable hash functions are instantiated
- Lesson: need to be very careful trying to get more than 128 bits of security from SHA-256

- On the upside:
  - SPHINCS$^+$'s proposed tweak seems to address these issues
  - SHA-256 on fixed-length inputs pretty reliably gets 128 bits of security, so it's unlikely this sort of oversight leads to a practical break

Thank You!

References

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