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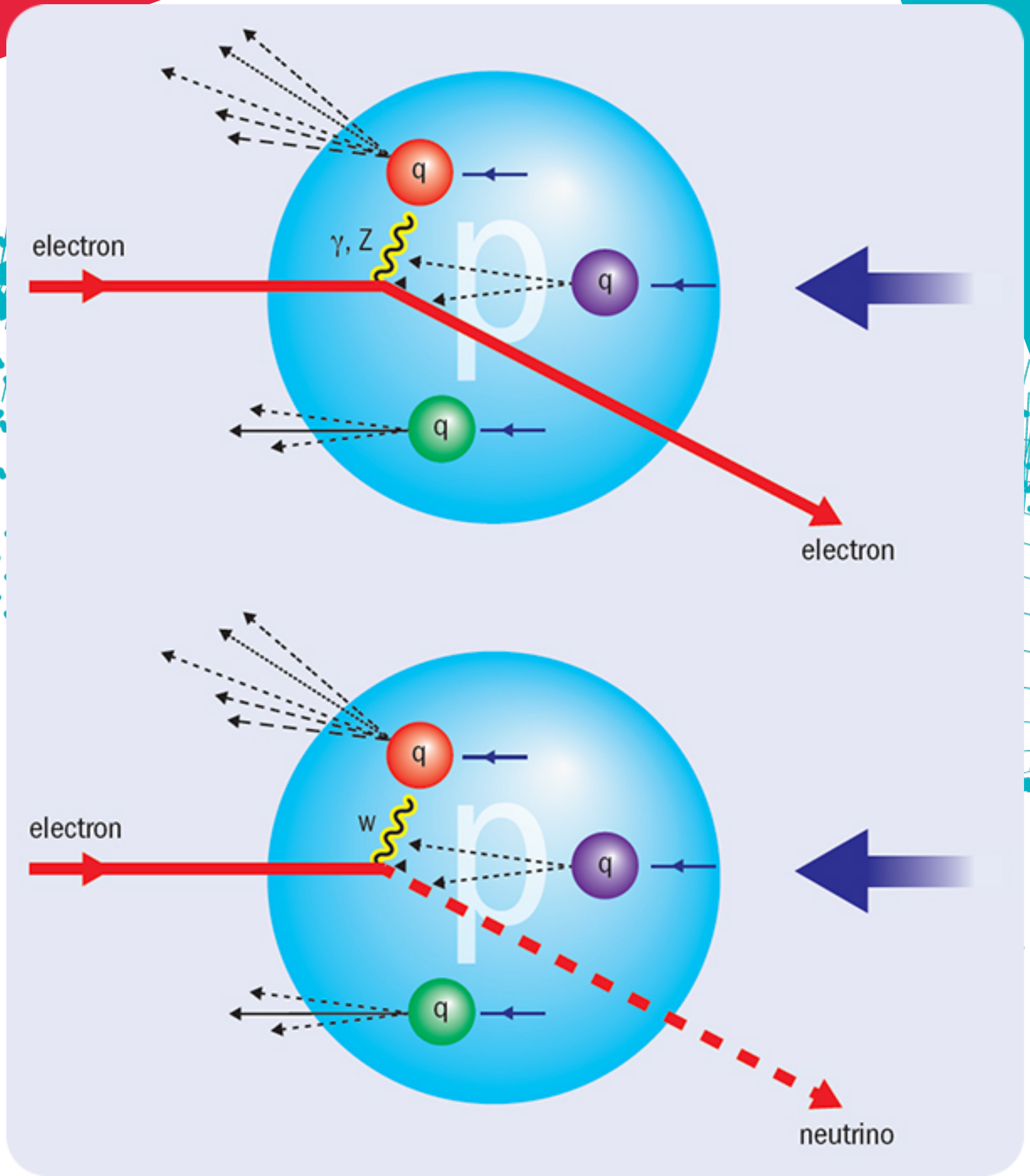
INTRODUCTION TO QUANTUM CHROMODYNAMICS



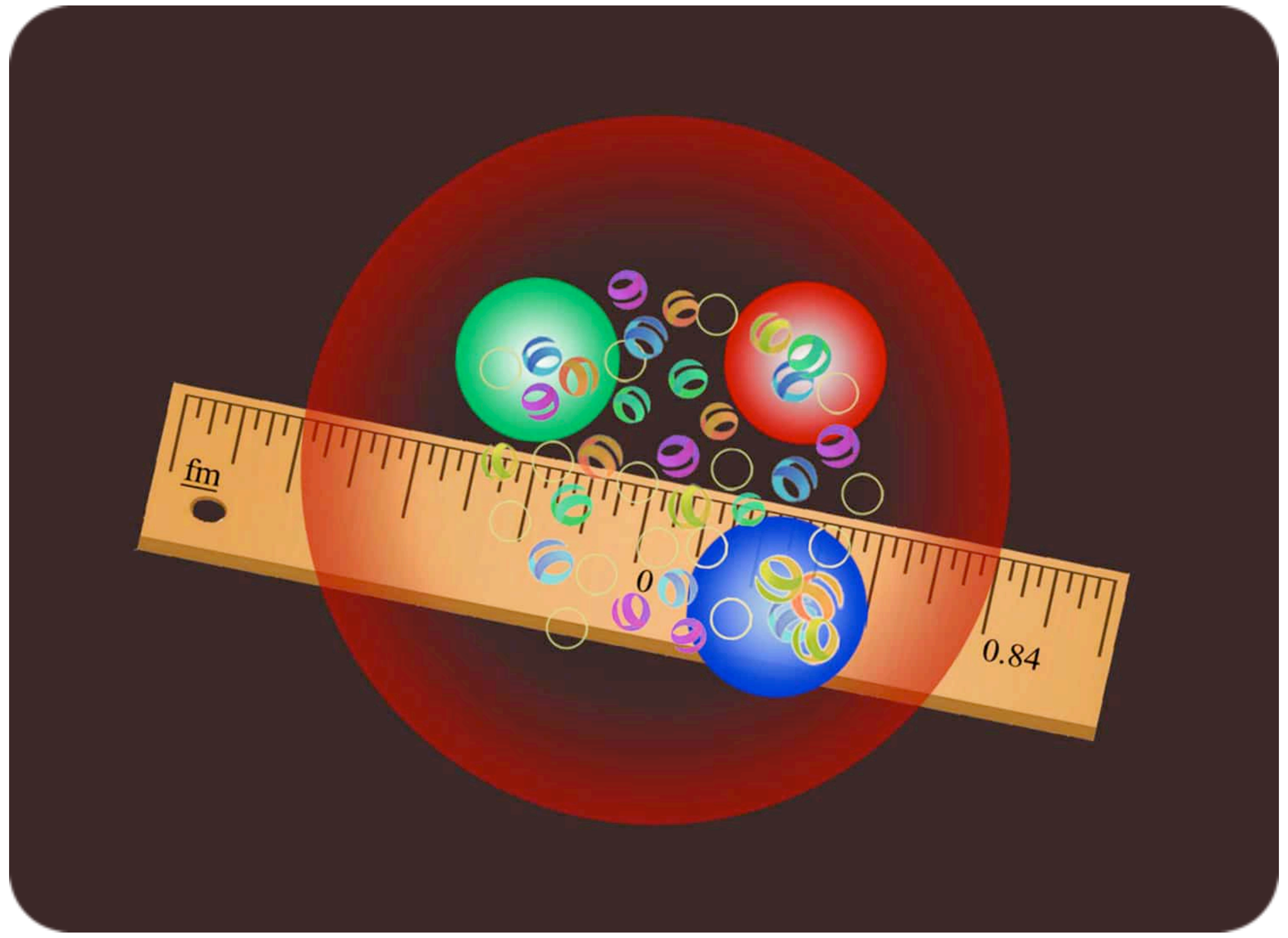
PARTICLE PHYSICS 2

Panos Christakoglou

SUMMARY

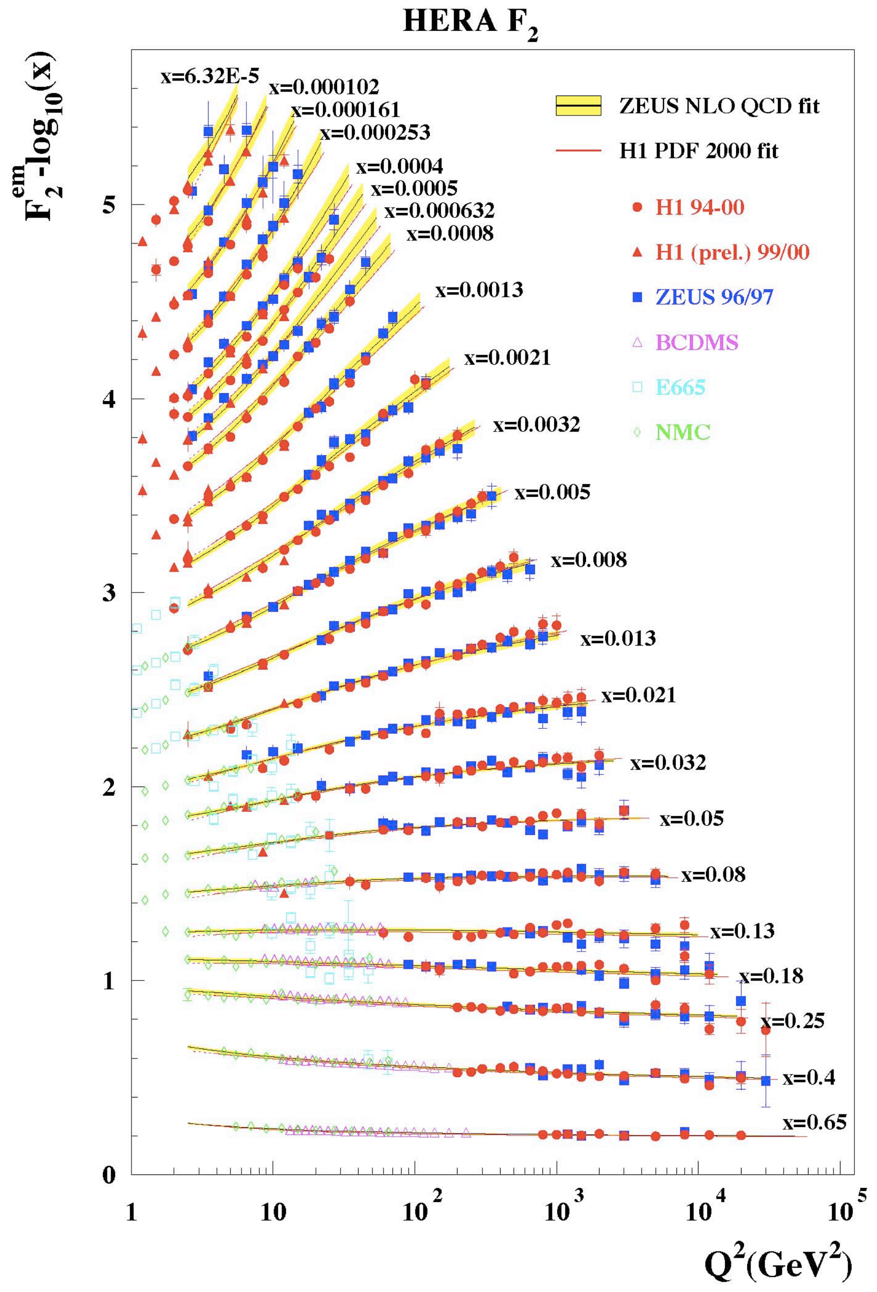
Last lecture

- Elastic electron-proton scattering
  - Form factors
  - Electric & magnetic charge distribution of a proton



Today's lecture

- Inelastic electron-proton scattering
  - Structure of protons
  - Deep inelastic scattering experiments
  - Parton model
  - Parton distribution functions



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EVIDENCE OF COLOUR

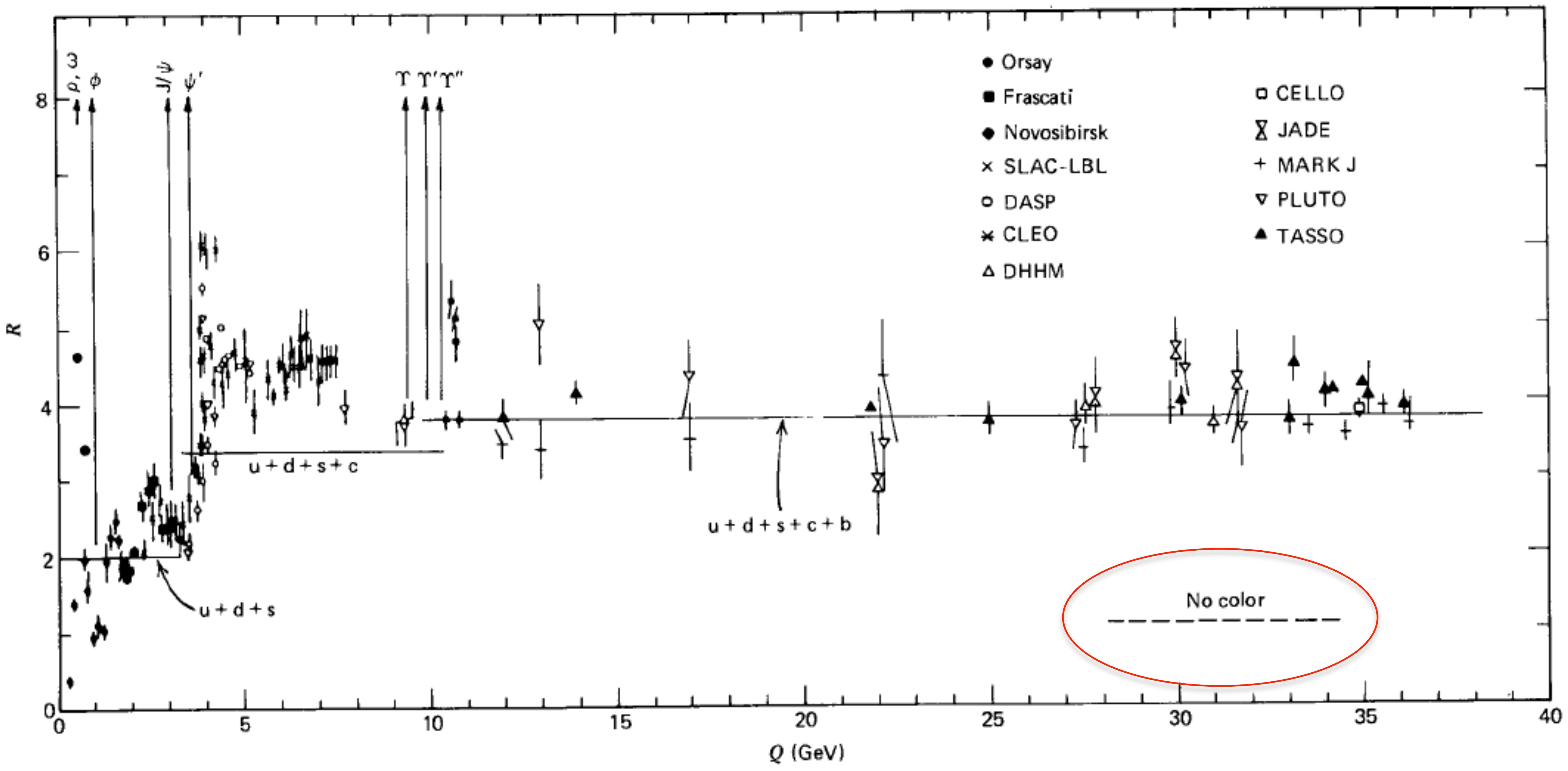

Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

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PROBING THE STRUCTURE OF PROTONS

The precise nature of $e^-p \rightarrow e^-p$ scattering process depends on the wavelength of the virtual photon in comparison with the proton radius

- at very low energies, where the electrons are non-relativistic and the wavelength of the virtual photon is large compared to the radius of the proton ($\lambda >> r_p$), the process is described by the elastic scattering of the electron in the static potential of an effective point-like proton

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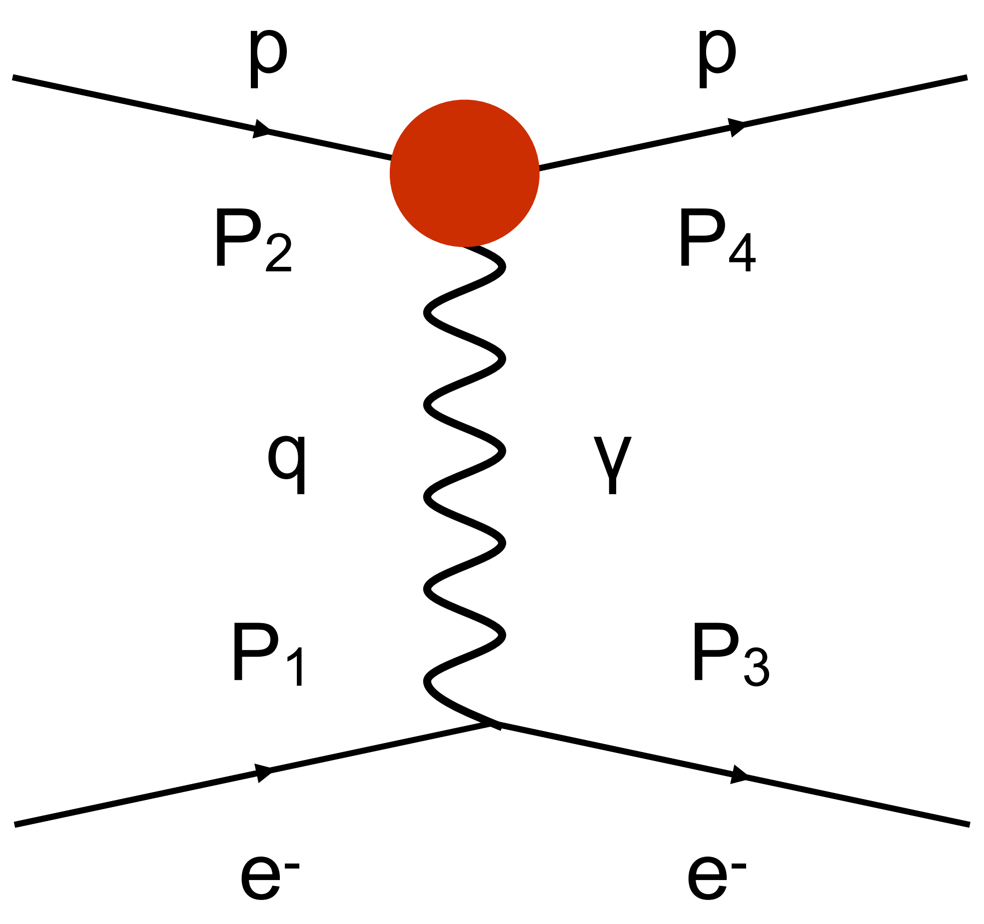
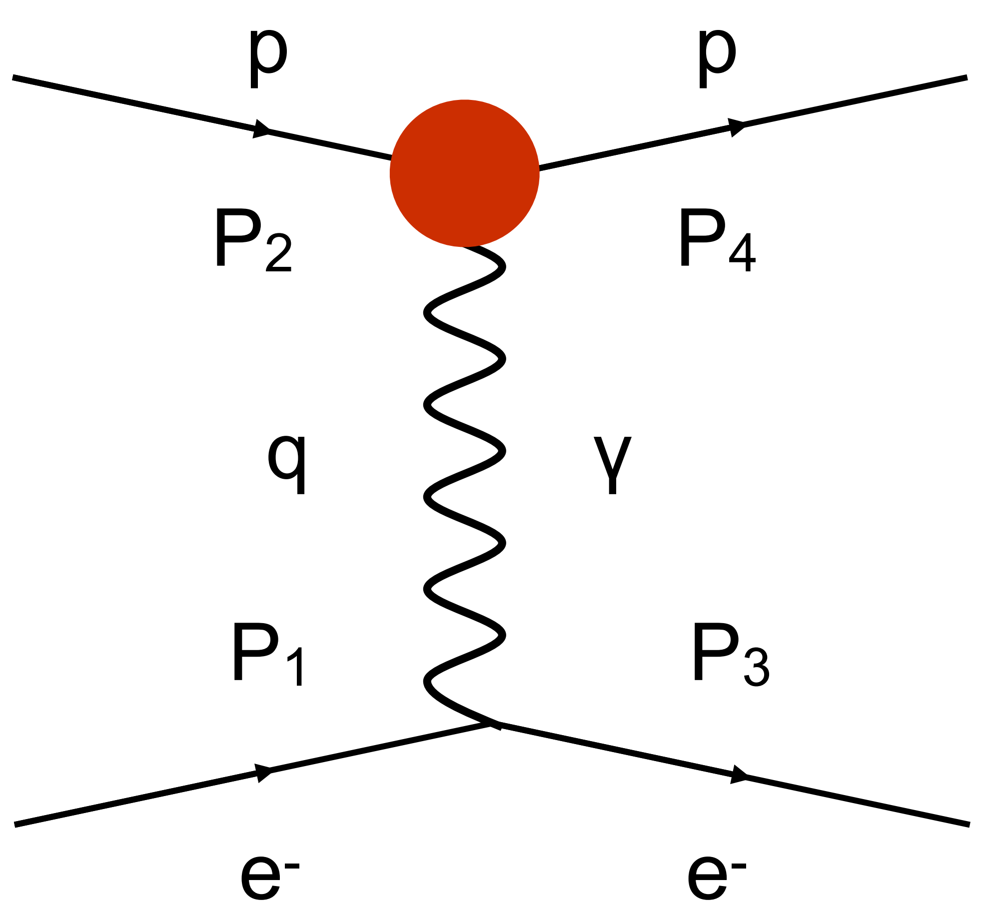


PROBING THE STRUCTURE OF PROTONS

The precise nature of $e^-p \rightarrow e^-p$ scattering process depends on the wavelength of the virtual photon in comparison with the proton radius

- At higher electron energies, where $\lambda \sim r_p$, the scattering process is no longer purely electrostatic and the cross-section needs to account for the extended charge and magnetic moment distribution of protons



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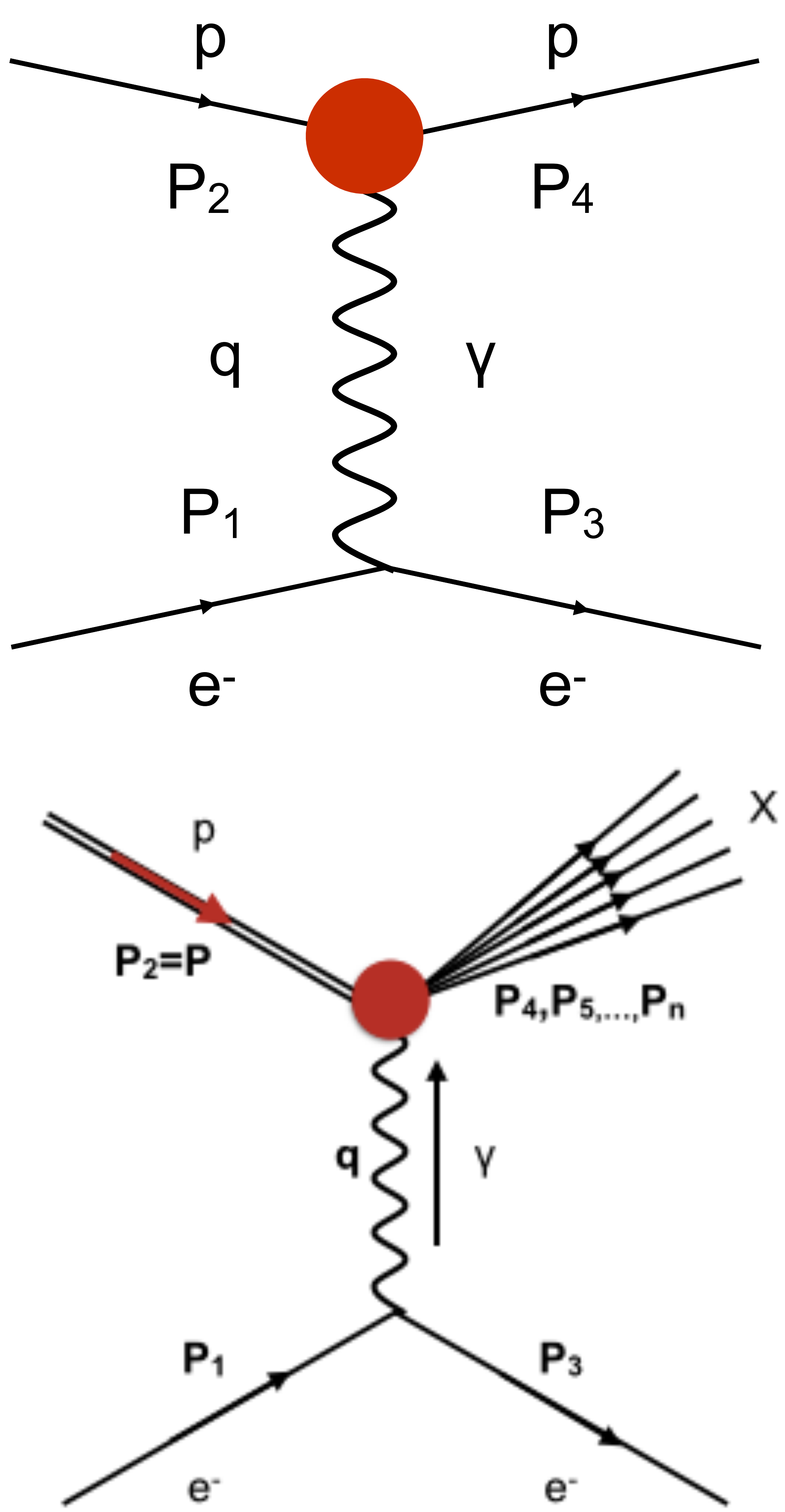
PROBING THE STRUCTURE OF PROTONS

The precise nature of $e^-p \rightarrow e^-p$ scattering process depends on the wavelength of the virtual photon in comparison with the proton radius

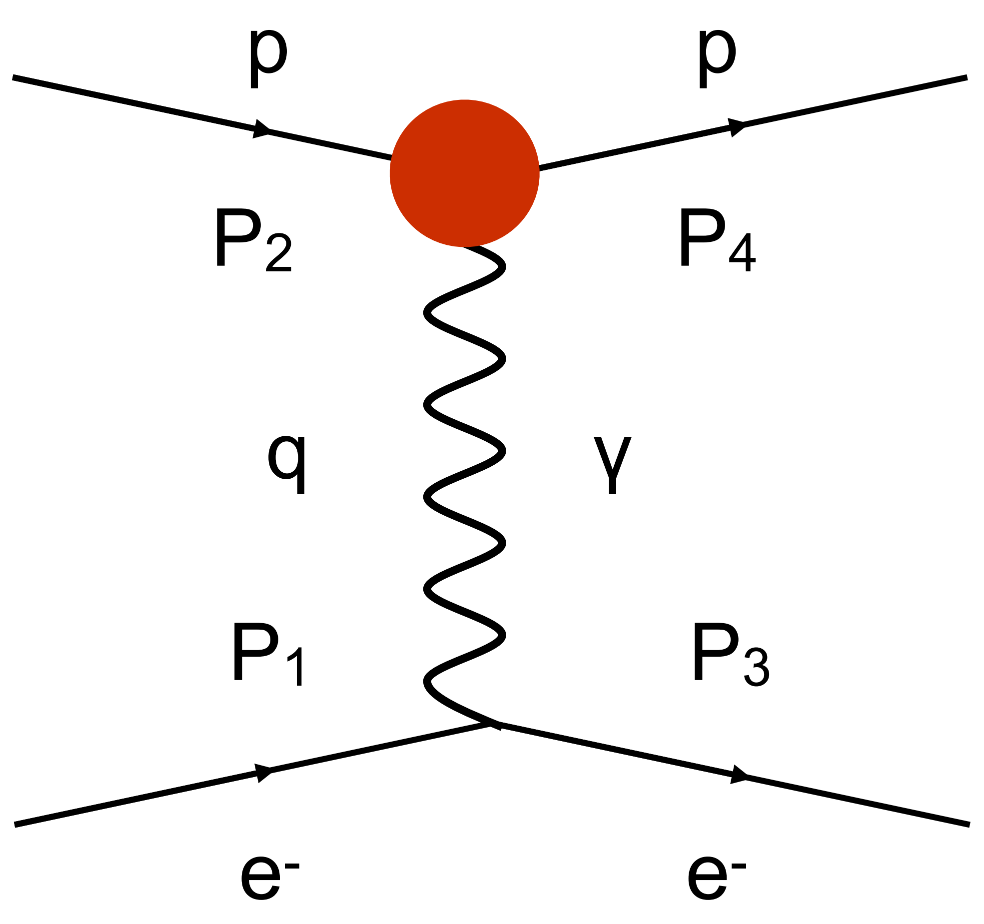
- when the wavelength of the virtual photon becomes relatively small ($\lambda < r_p$), the contribution from the elastic process becomes also small. The dominant process is of inelastic nature, where the virtual photon interacts with the constituent quark of the proton and the proton breaks up
    - the inelastic electron-proton scattering can be considered an elastic electron-quark scattering process

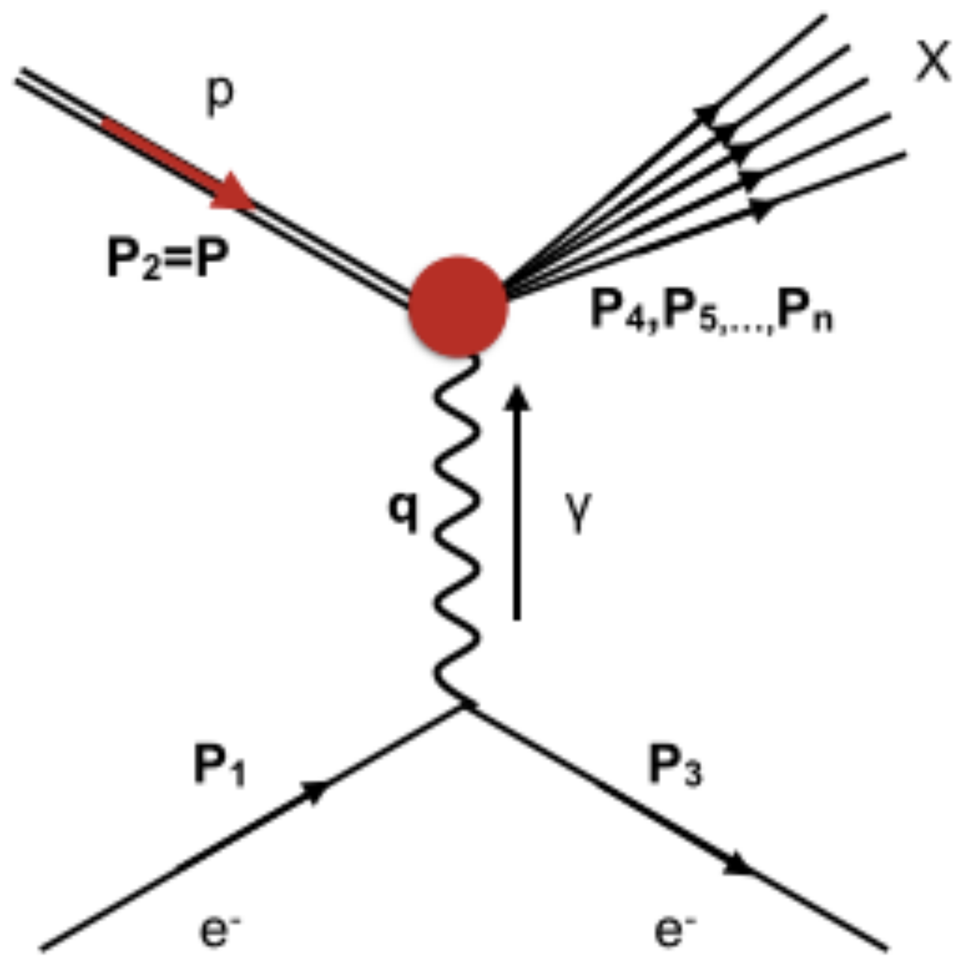


PROBING THE STRUCTURE OF PROTONS

The precise nature of $e^-p \rightarrow e^-p$ scattering process depends on the wavelength of the virtual photon in comparison with the proton radius

- at even higher energies, where the wavelength of the virtual photon is sufficiently short ($\lambda << r_p$) to resolve the detailed dynamic structure of the proton, the proton appears to consist of a sea of strongly interacting quarks and gluons





ELECTRON-PROTON SCATTERING

The Rutherford and Mott scattering are the low-energy limits of e^-p scattering

- In both cases the electron energy is sufficiently low that the kinetic energy of the recoiling proton is negligible compared to its rest mass
- In this case the proton can be considered as a fixed, point-like source of 1/r electrostatic potential
- The cross-sections are calculated from scattering theory by using the first order terms in the perturbation expansion
- Rutherford scattering:
  - the proton recoil can be neglected and the electron is non-relativistic
  - the spin-averaged matrix element is

$$\langle |M_{if}| \rangle^2 = \frac{m_p^2 m_e^2 e^4}{|\vec{P}|^4 \sin^4(\theta/2)}$$

  - The differential cross-section is given then by

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{1}{m_p + E_1 - E_1 \cos\theta}\right)^2 \langle |M_{if}| \rangle^2$$

SUBNUCLEAR STRUCTURE

In order not to see the proton as one compound object but rather to probe its internal structure, one has to bombard it with highly energetic particles

- Rutherford followed a similar trick by bombarding the thin gold foils with α-particles

Experiments later used higher energy projectiles and revealed that the (up to that moment known as) point-like core had some internal structure

- The scattering distributions were damped by the relevant form factors of the nucleus

In 1932 Chadwick discovered the neutron and it became clear that the nucleus consisted of protons and neutrons

James Chadwick (1891 - 1974)

MOTT SCATTERING

The Mott scattering is the limit where the electron is relativistic but the proton recoil can still be negligible

- These conditions apply when $m_e << E << m_p$
- The matrix element is given this time by

$$\langle |M_{if}| \rangle^2 \approx \frac{e^4}{E^2 \sin^4(\theta/2)} \cos^2(\theta/2)$$

- while the differential cross-section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} = \frac{a^2}{E^2 \sin^4(\theta/2)} \cos^2(\theta/2)$$

INTRODUCTION OF FORM FACTORS

To account for the finite extent of the charge distribution of the proton, the previous treatment needed to be modified by introducing a form factor

- The form factor accounts for the phase differences of the scattered wave from the different points of the charge distribution of the target
  - If the wavelength of the virtual photon is larger than the radius of the proton, then the contributions to the scattered wave are in phase and can be added constructively
  - When the wavelength of the virtual photon is smaller than the radius of the proton, the phases of the scattered wave will have a dependence on the position of the part of the charge distribution responsible for the scattering
    - in this case, when integrating over the entire charge distribution the negative interference between the different contributions reduces significantly the magnitude
  - The Mott scattering cross-section needs to be modified as $$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \to \frac{a^2}{4E^2\sin^4(\theta/2)}\cos^2(\theta/2)|F(q^2)|^2$$

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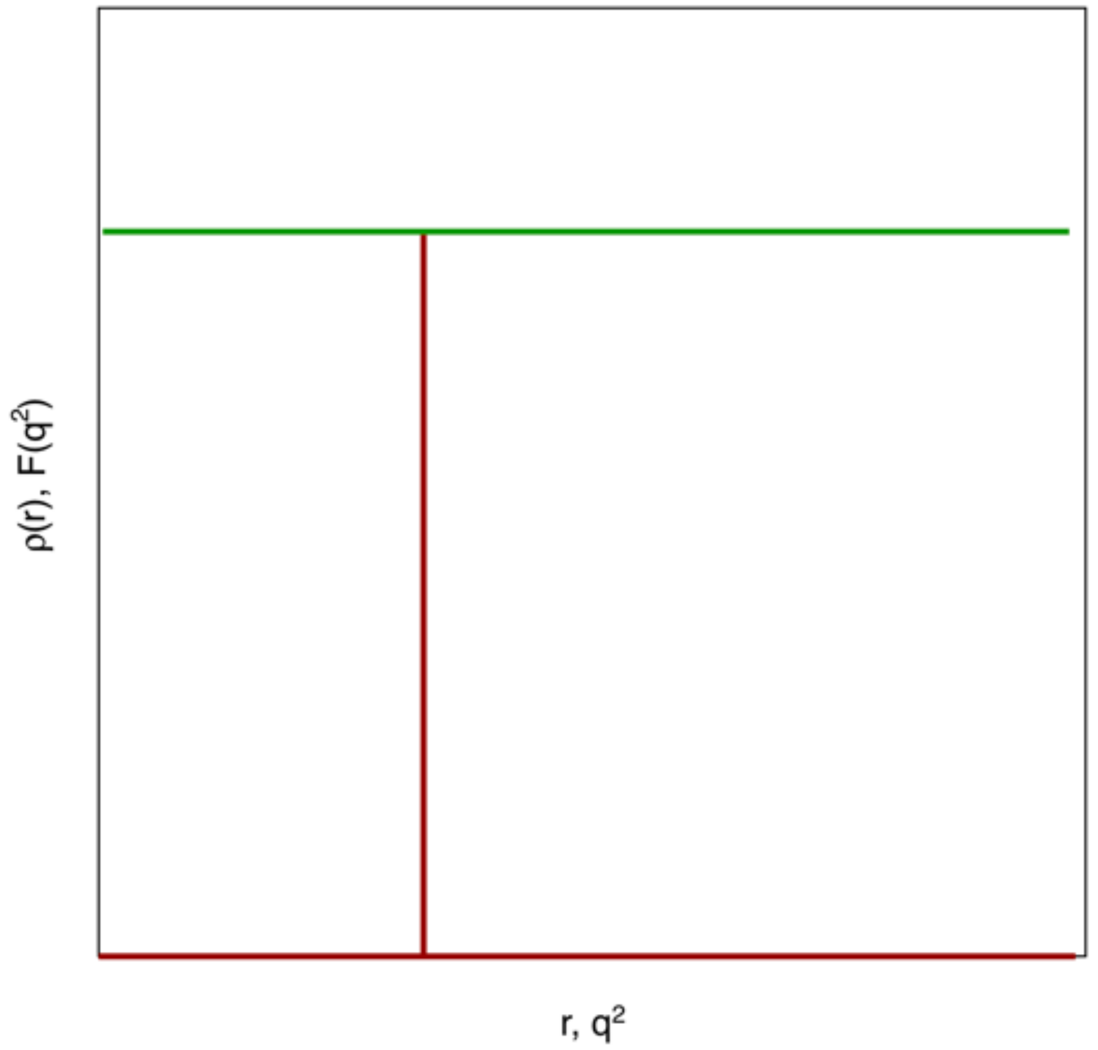
FORM FACTORS

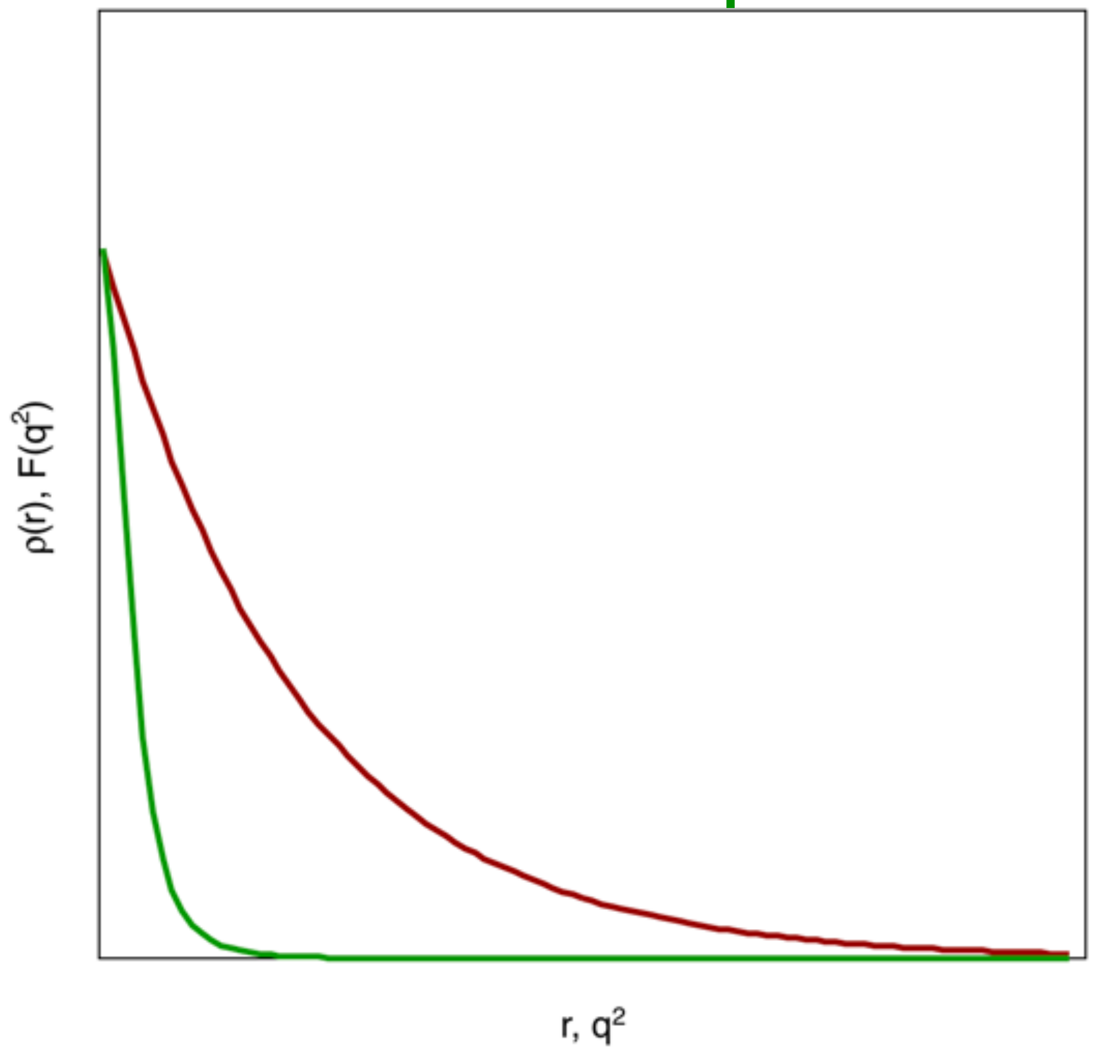
Table 6.1: PROBABILITY DENSITIES AND FORM FACTORS FOR SOME ONE-PARAMETER CHARGE DISTRIBUTIONS. [After R. Herman and R. Hofstadter, *High-Energy Electron Scattering Tables*, Stanford University Press, Stanford, CA, 1960.]

Probability Density, $\rho(r)$	Form Factor, $F(\boldsymbol{q}^2)$
$\delta(r)$	1
$\rho_0 \exp(-r/a)$	$(1+\boldsymbol{q}^2 a^2/\hbar^2)^{-2}$
$\rho_0 \exp[-(r/b)^2]$	$\exp(-\boldsymbol{q}^2 b^2/4\hbar^2)$
$\left.\begin{matrix}\rho_0, r \leq R \\ 0, r \geq R\end{matrix}\right\}$	$\frac{3[\sin(\lvert\boldsymbol{q}\rvert R/\hbar)-(\lvert\boldsymbol{q}\rvert R/\hbar)\cos(\lvert\boldsymbol{q}\rvert R/\hbar)]}{(\lvert\boldsymbol{q}\rvert R/\hbar)^3}$

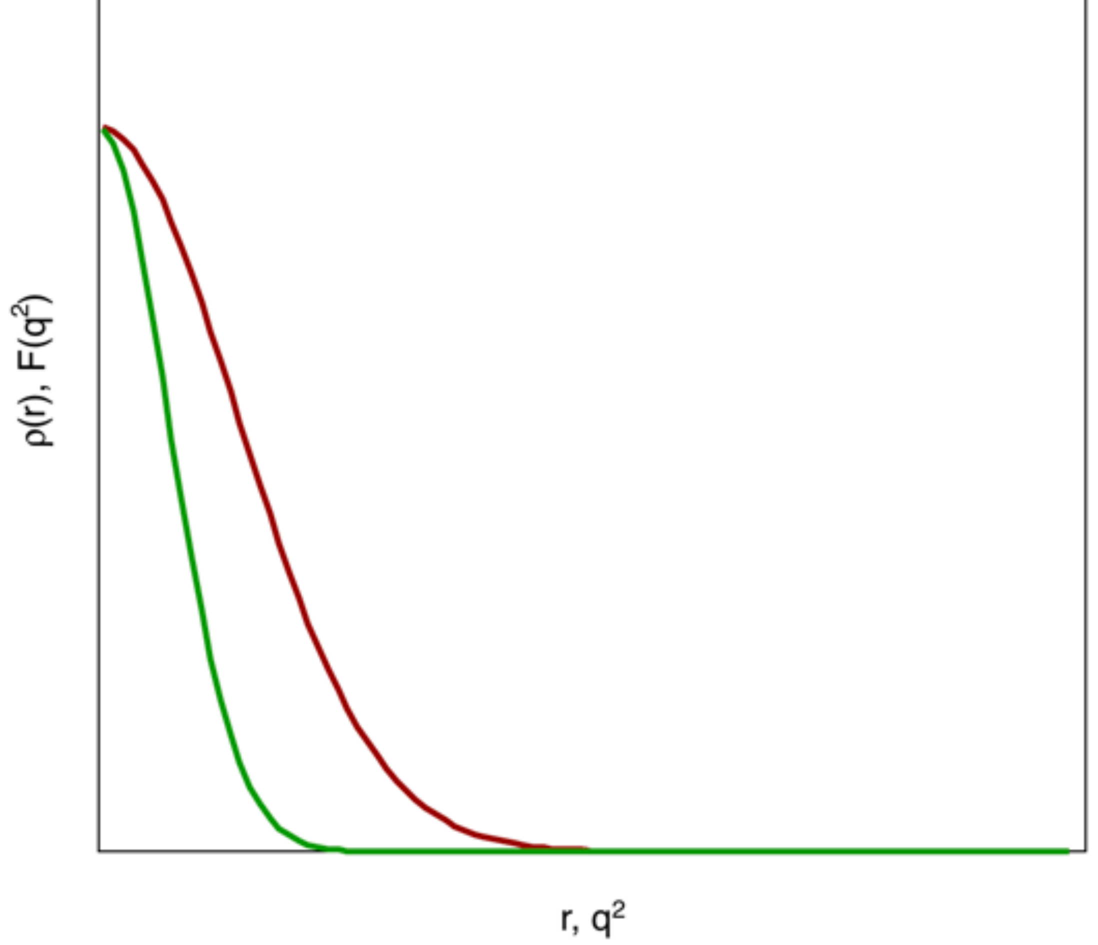
probability density: δ-function
Form factor: uniform



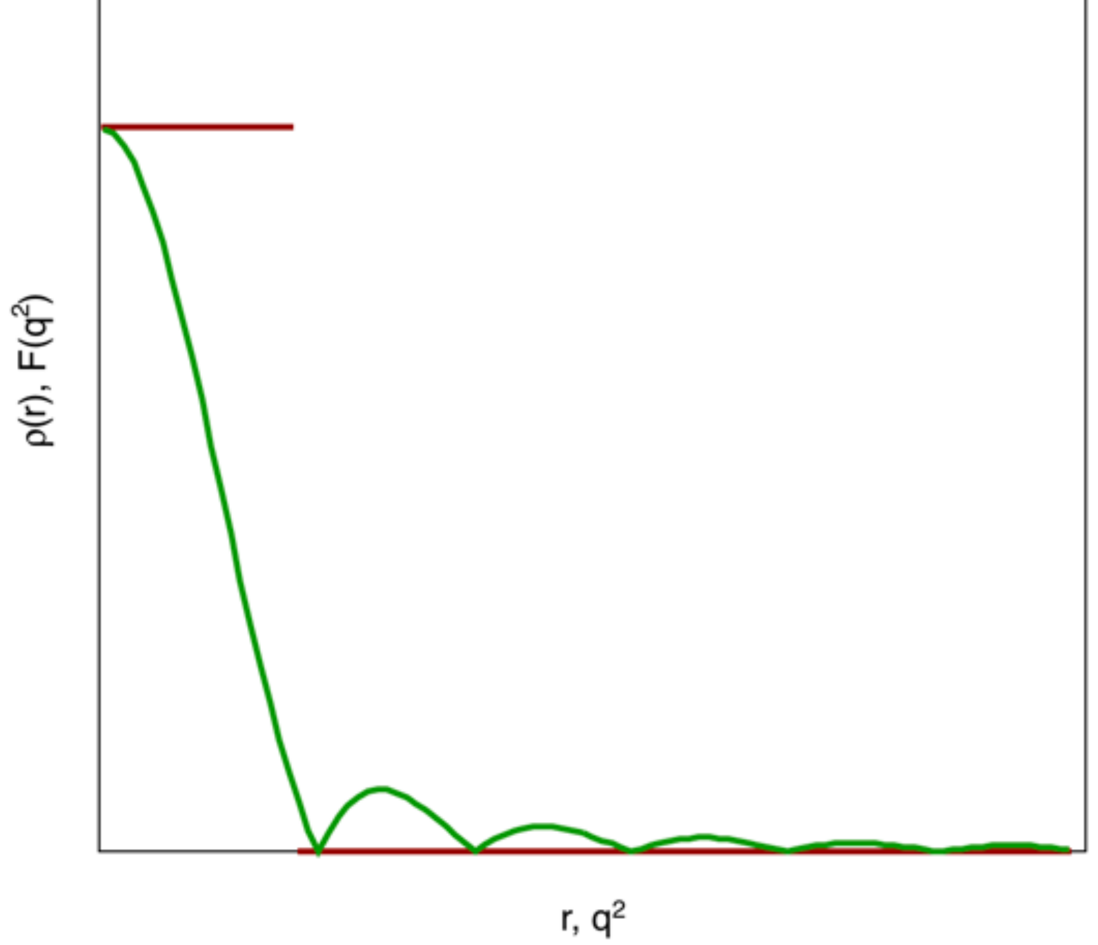
probability density: exponential
Form factor: dipole-like



probability density: gaussian
Form factor: gaussian



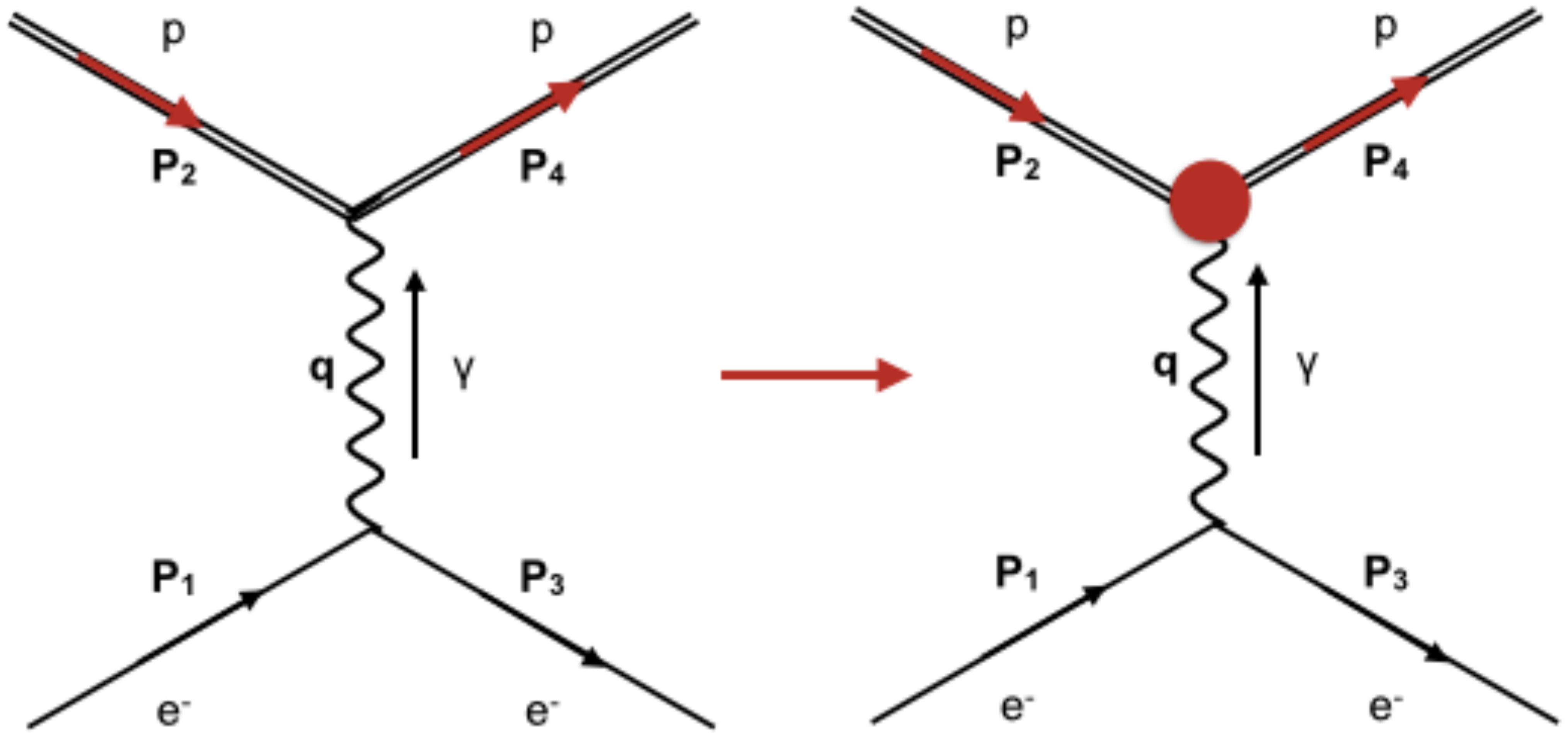
probability density: box
Form factor: sine-like



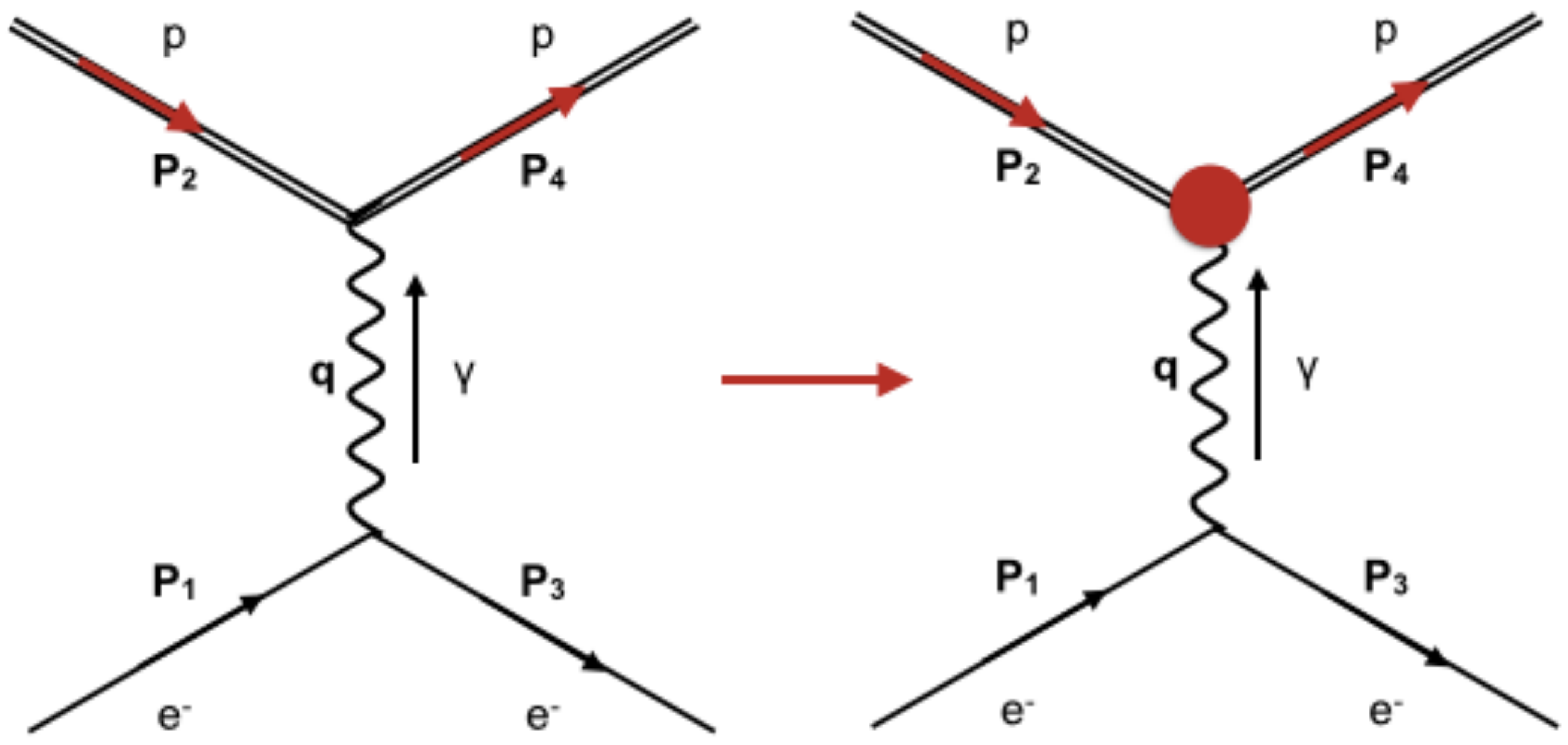
ELECTRON-PROTON ELASTIC SCATTERING

For electron-proton scattering at higher energies, with the wave-length of the virtual photon being still of the order of the proton radius, the recoil of the proton and the spin-spin magnetic interactions between the electron and the proton are not anymore negligible

The electron is relativistic and the matrix element is obtained profiting from the similarity of the interactions with the electron-muon scattering



ROSENBLUTH FORMULA



$$\frac{d\sigma}{d\Omega} = \left[\frac{\alpha}{4ME\sin^2(\theta/2)}\right]^2 \frac{E'}{E}\left(2K_1\sin^2(\theta/2) + K_2\cos^2(\theta/2)\right)$$

$$K_1 = -\mathbf{q}^2 G_M^2$$

$$K_2 = (2M)^2 \frac{G_E^2 - [\mathbf{q}^2/(2M)^2]G_M^2}{1 - \mathbf{q}^2/(2M)^2}$$

ELASTIC FORM FACTORS OF NUCLEONS

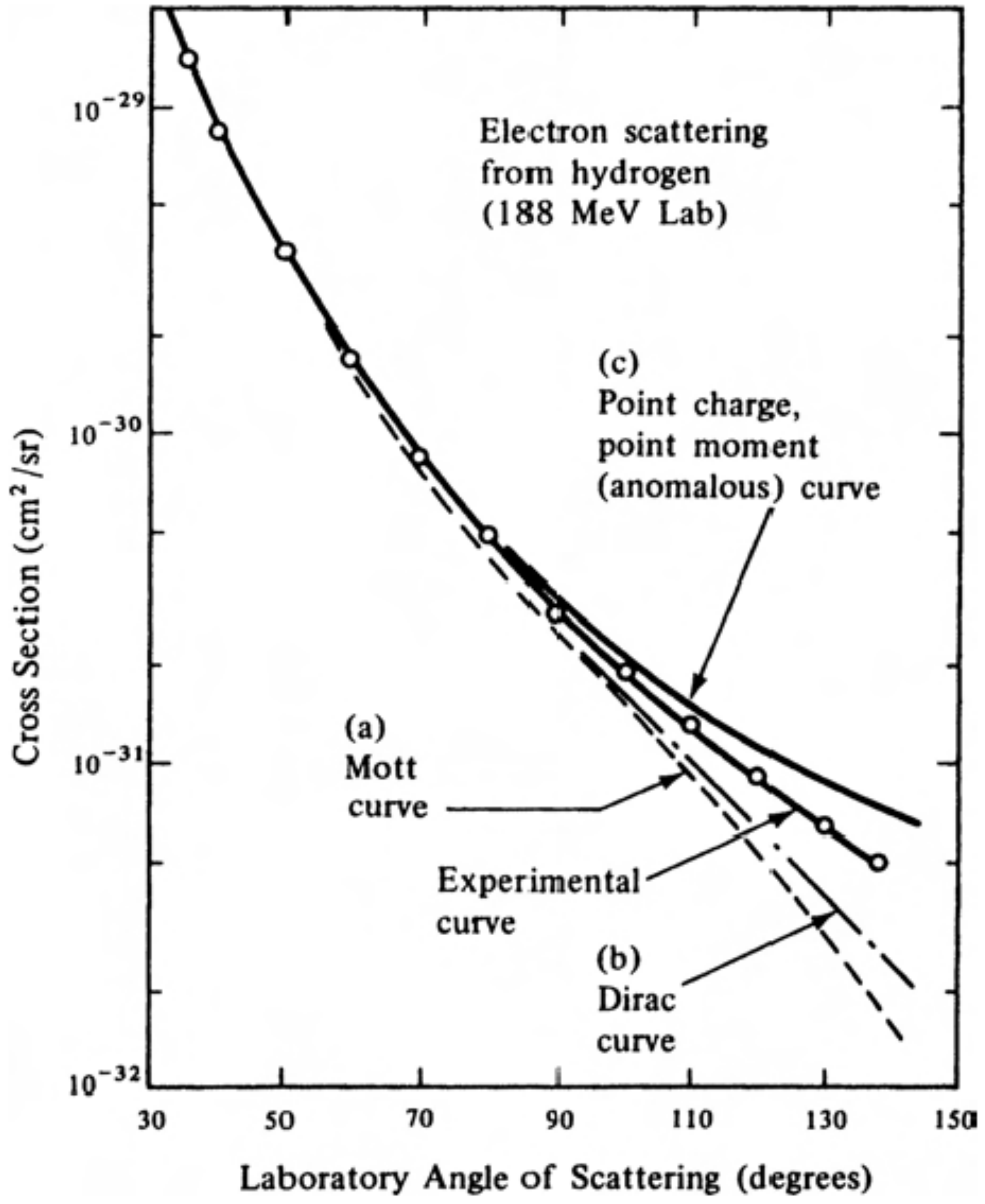
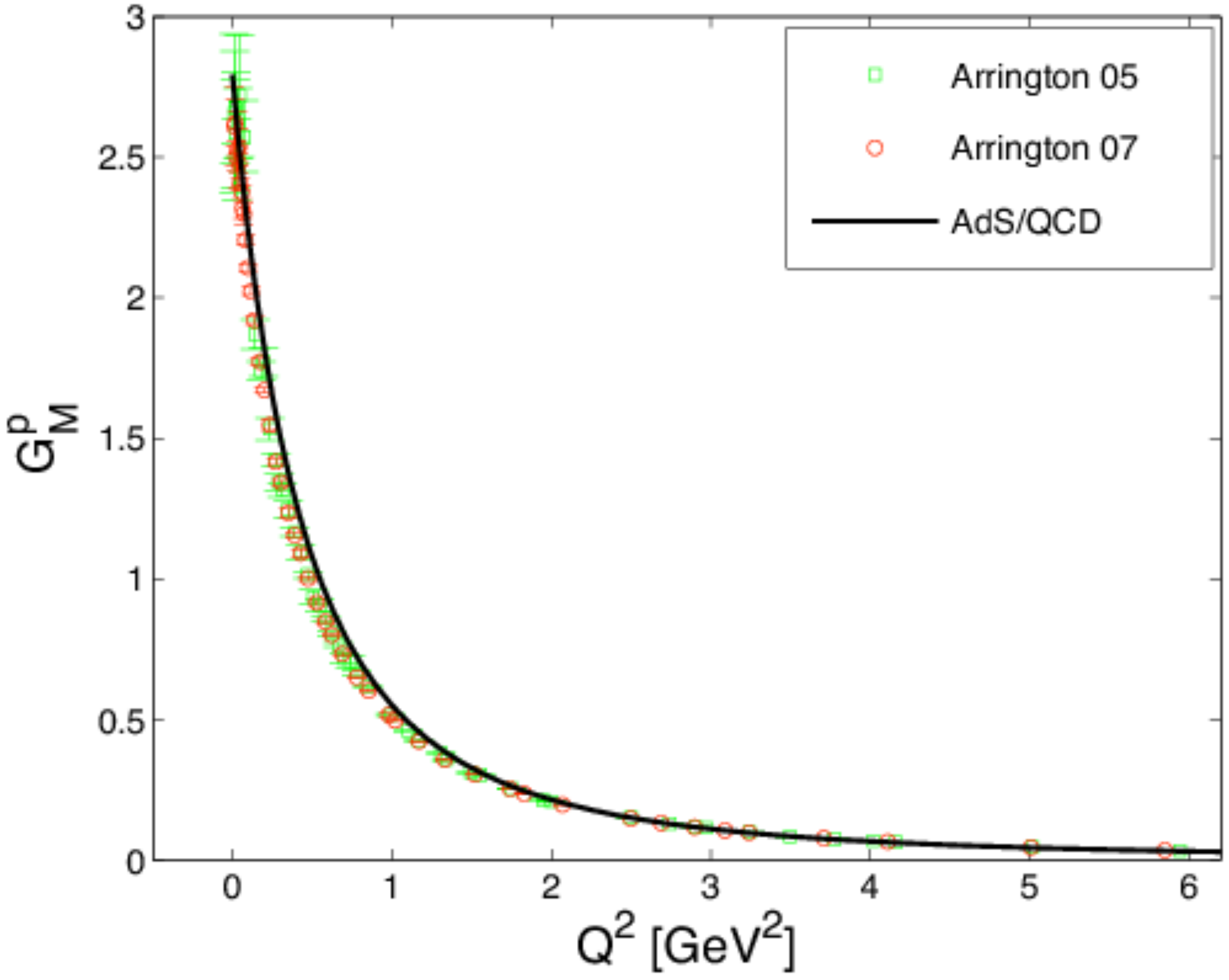


Figure 6.11: Electron–proton scattering with 188 MeV electrons. [R. W. McAllister and R. Hofstadter, *Phys. Rev.* **102**, 851 (1956).] The theoretical curves correspond to the following values of G_E and G_M: Mott (1;0), Dirac (1;1), anomalous (1;2.79).

Conclusion: Nucleons are not point like particles!

PROTON FORM FACTORS



$$G_E(q^2) = \left(1 - \frac{q^2}{0.71}\right)^{-2}$$

$$\langle r^2 \rangle = 6\left(\frac{dG_E(q^2)}{dq^2}\right)_{q^2=0} = (0.81 \cdot 10^{-13}\text{cm})^2$$

NOBEL PRIZE 1961

The Nobel Prize in Physics 1961
Robert Hofstadter, Rudolf Mössbauer

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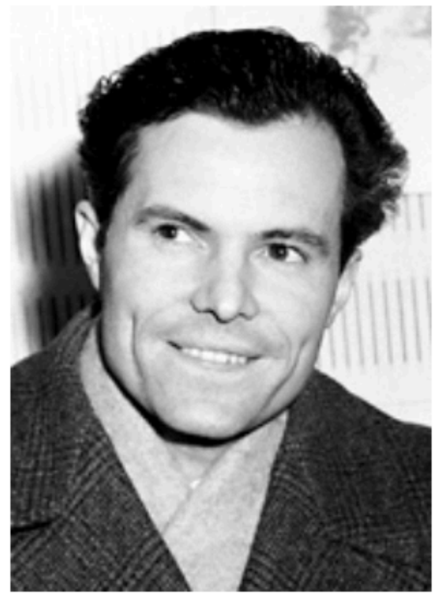
The Nobel Prize in Physics 1961

Robert Hofstadter
Prize share: 1/2



Rudolf Ludwig Mössbauer
Prize share: 1/2

The Nobel Prize in Physics 1961 was divided equally between Robert Hofstadter *"for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons"* and Rudolf Ludwig Mössbauer *"for his researches concerning the resonance absorption of gamma radiation and his discovery in this connection of the effect which bears his name"*.

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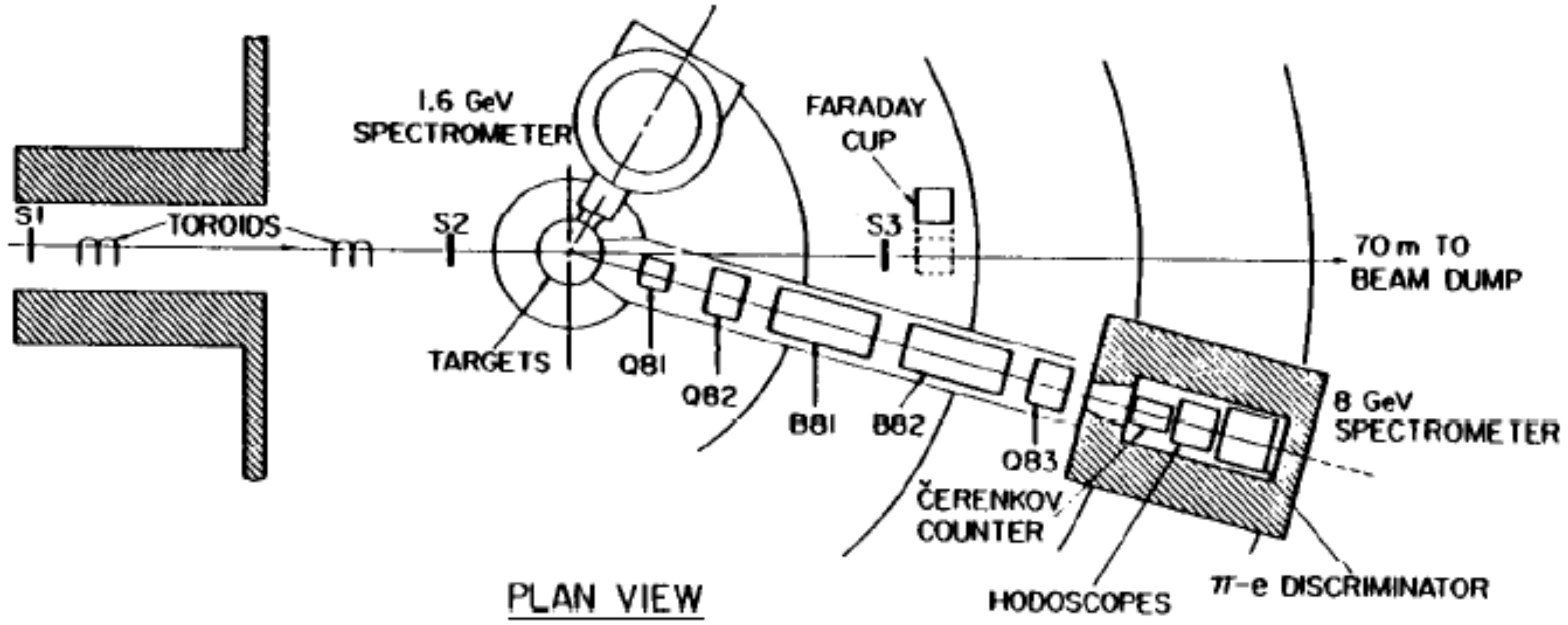
INTERNAL STRUCTURE OF PROTONS

Robert Hofstadter



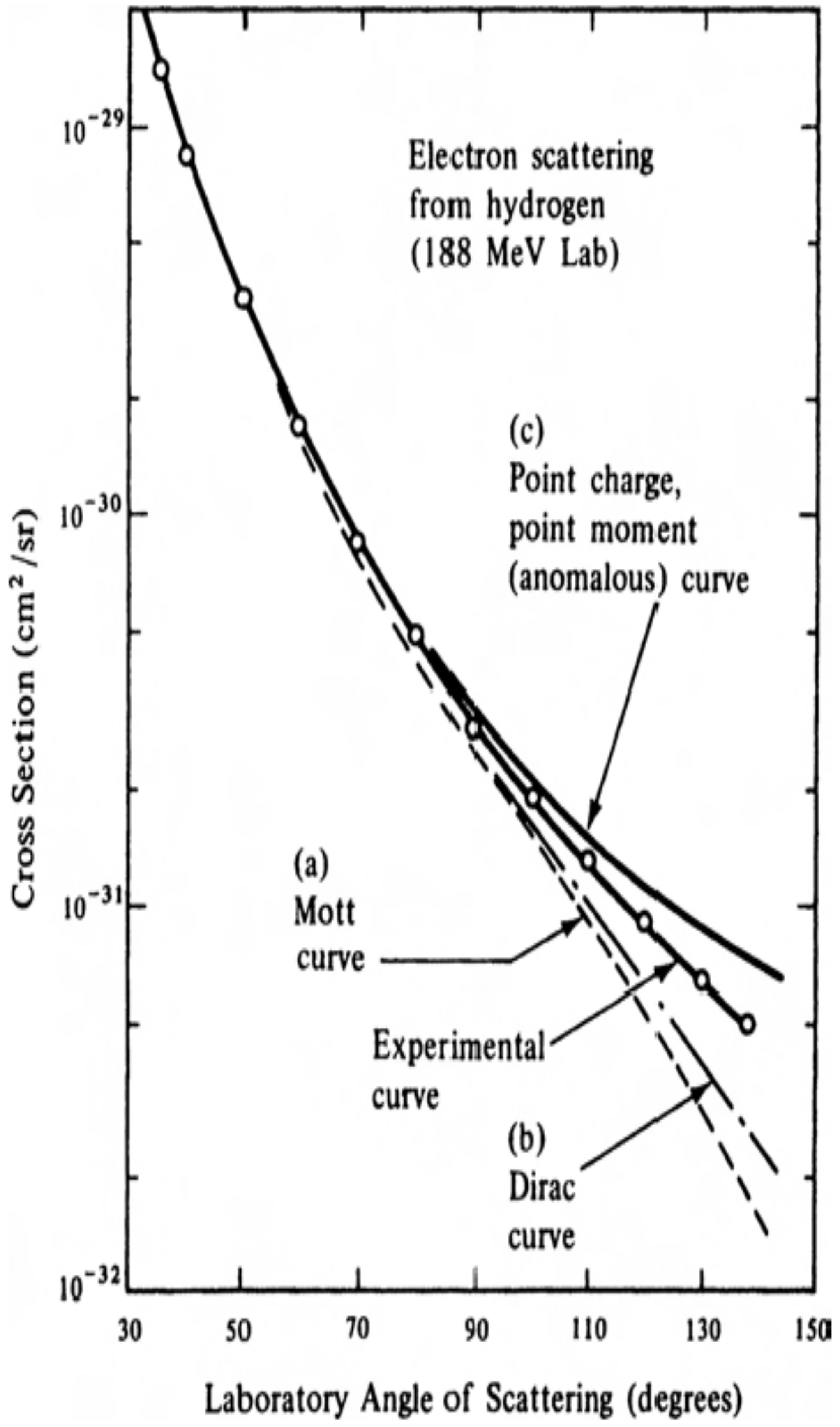


Figure 6.11: Electron–proton scattering with 188 MeV electrons. [R. W. McAllister and R. Hofstadter, *Phys. Rev.* **102**, 851 (1956).] The theoretical curves correspond to the following values of G_E and G_M: Mott (1;0), Dirac (1;1), anomalous (1;2.79).

WHAT WE KNOW SO FAR

We know

- from Rutherford about the atomic nucleus
- from Chadwick about neutrons
  - The nucleus has internal structure: the neutrons and protons
- from Hofstadter that protons and neutrons are composite particles
  - What is the structure of these particles?
  - What are they made of?

High energy electron-nucleon and neutrino-nucleon experiments to study proton and neutron structure

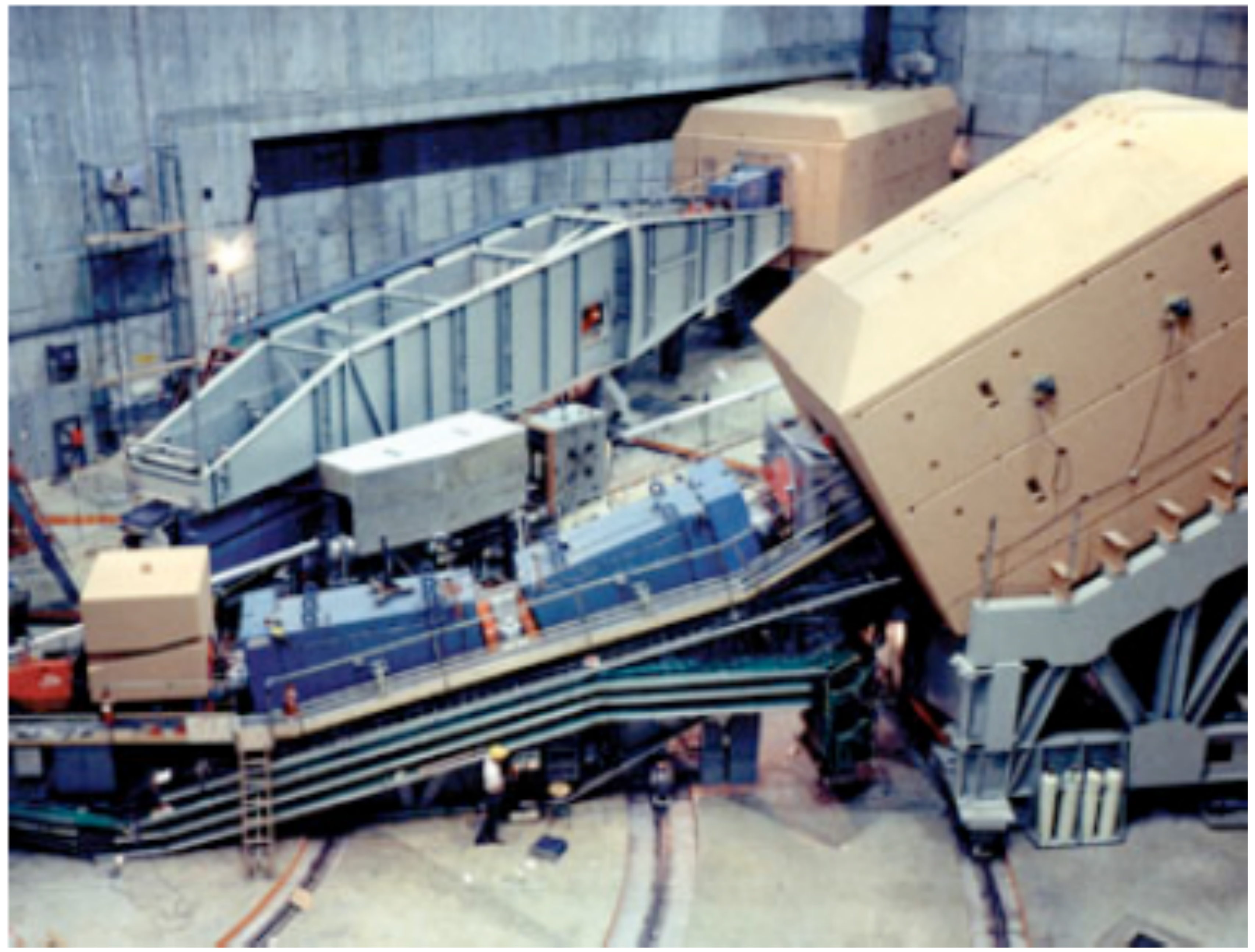
- Results turned out to be very important!!!

ARE NUCLEONS ELEMENTARY PARTICLES?

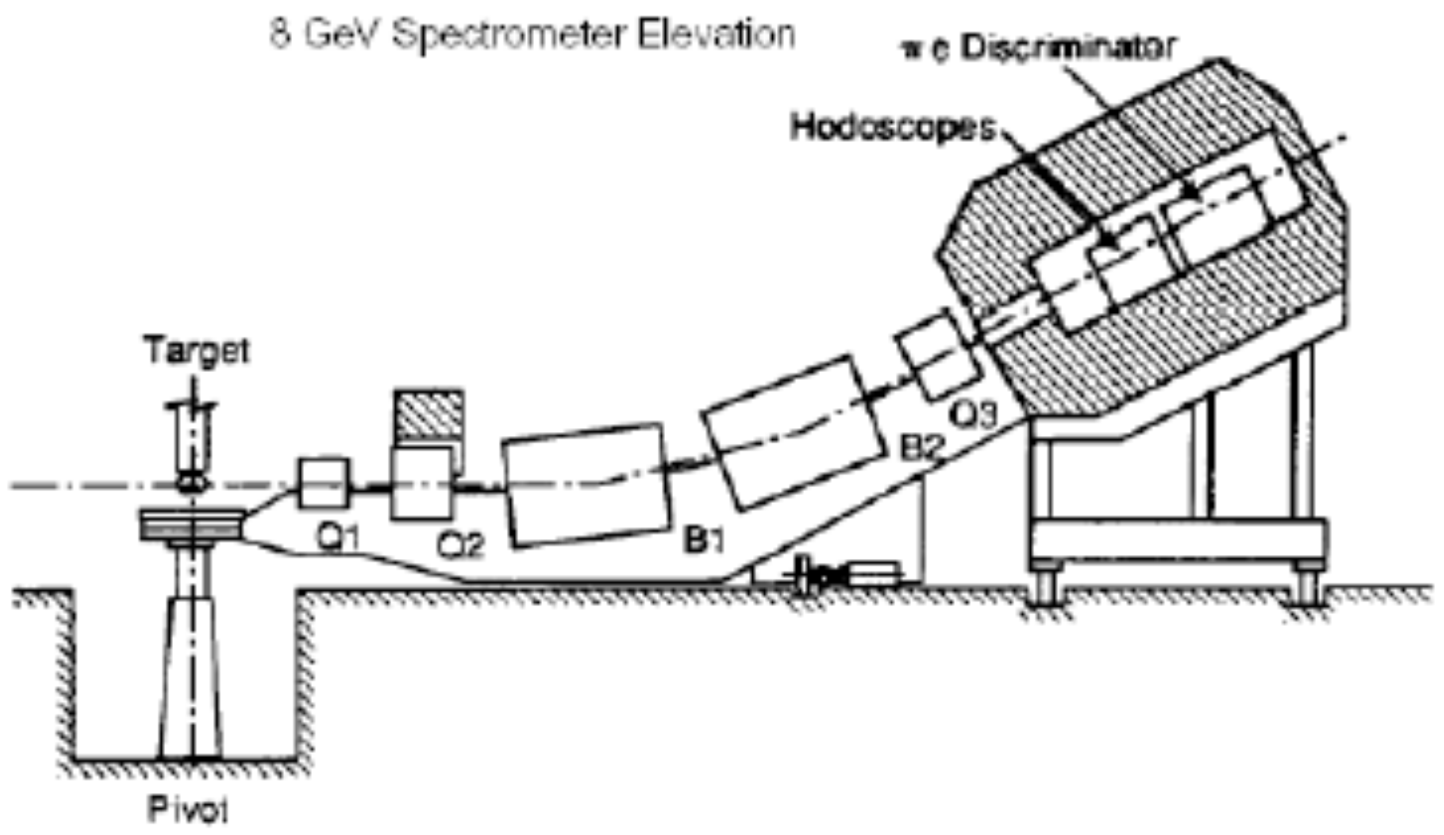
In 1956 Stanford staff met in Prof. W. Panofsky's home to discuss Hofstadter's suggestion to build a linear accelerator that was at least 10 times as powerful as the previous one (called Mark III) to study the structure of sub-nuclear matter. This idea was called "The M(onster)-project" because the accelerator would need to be 2 miles long!

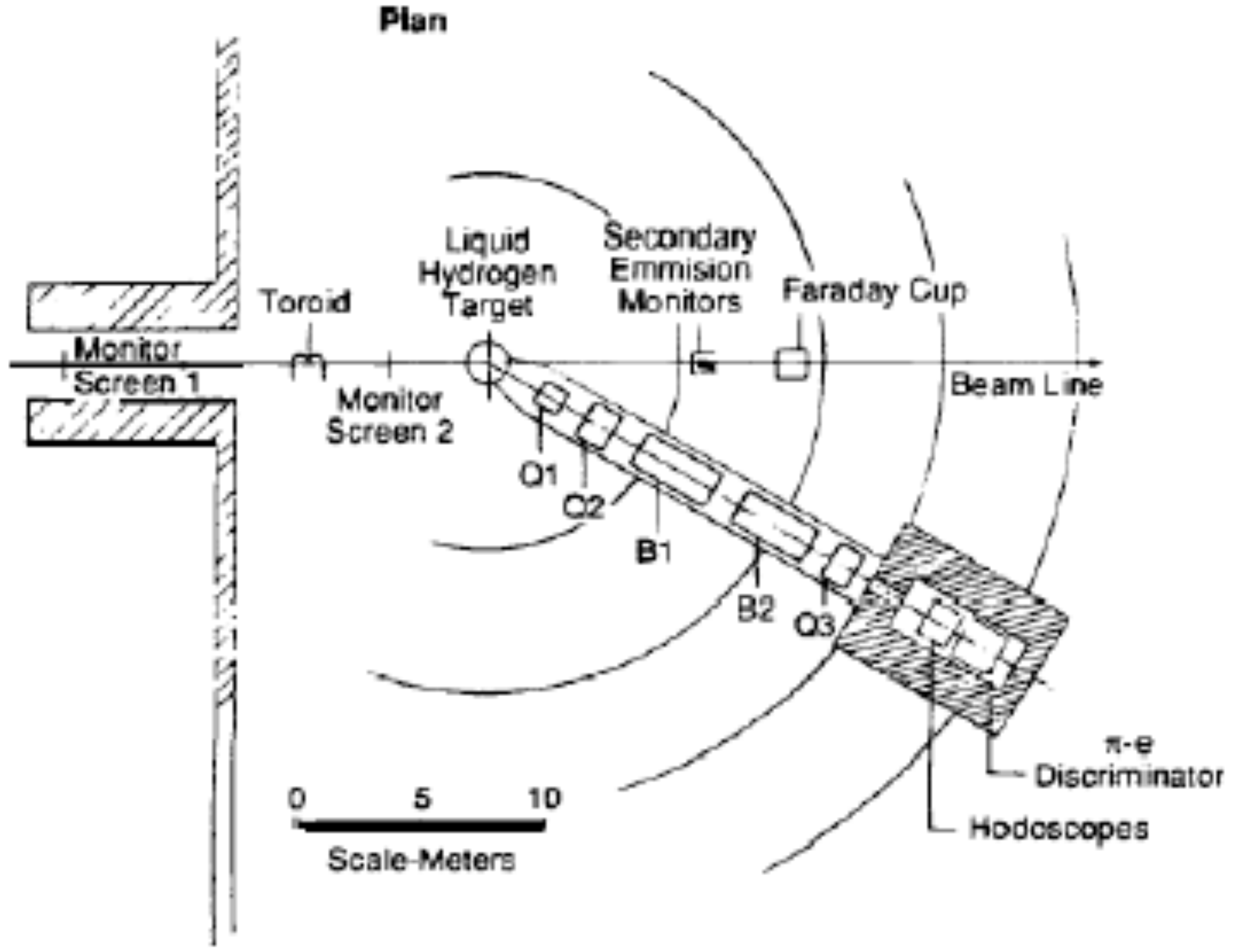
- 1957 A detailed proposal was presented
- 1959 Eisenhower said yes
- 1961 Congress approved the project ($114M)
- While excavating SLAC the workers discovered a nearly complete skeleton of a 10-foot mammal, Paleoparadoxia, which roamed earth 14 millions years ago…

THE SLAC-MIT EXPERIMENT



Friedman, Kendall and Taylor

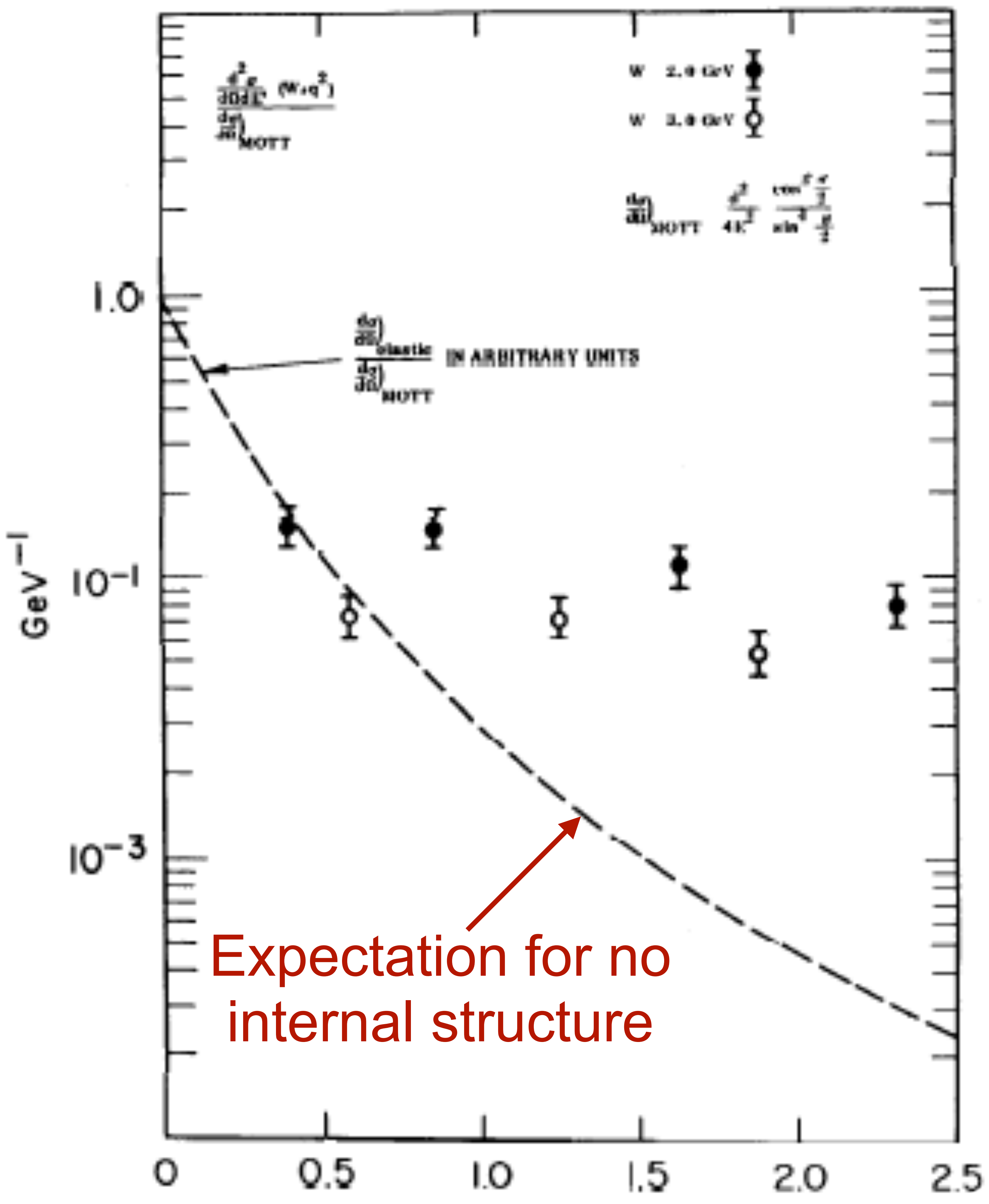




DEEP INELASTIC SCATTERING EXPERIMENTS

A real breakthrough in probing the internal structure of the proton came with a series of deep inelastic scattering experiments at SLAC - Stanford Linear Accelerator Centre

- Electrons were scattered off quasi-free point-like constituents inside the protons i.e. the quarks!
- Nobel prize in 1990 to Friedman, Kendall and Taylor
- These experiments were followed up by other experiments @ CERN and @ Fermilab using e, μ, ν and anti-ν beams as probes



NOBEL PRIZE IN 1991

The Nobel Prize in Physics 1990
Jerome I. Friedman, Henry W. Kendall, Richard E. Taylor

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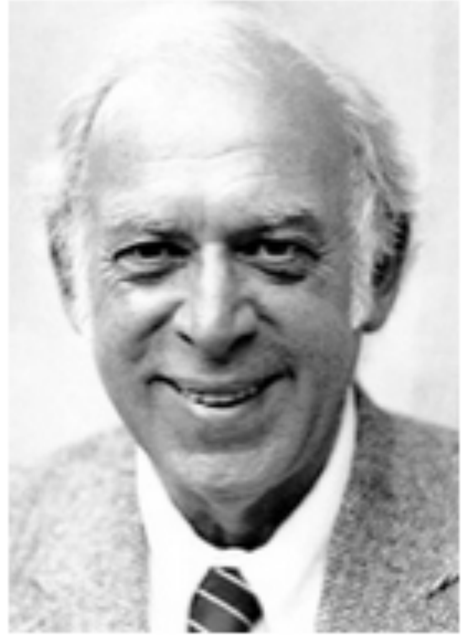
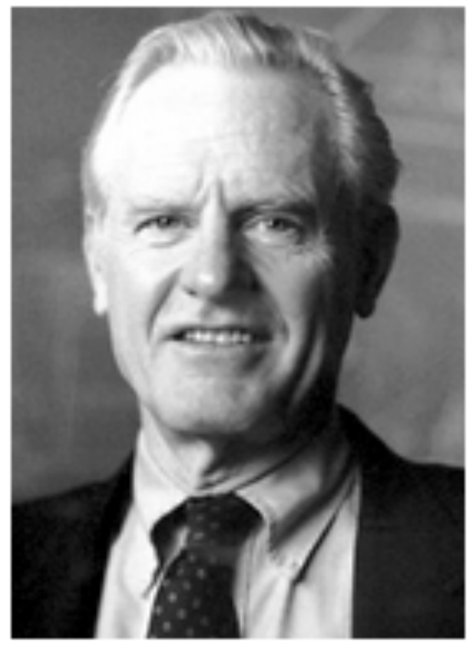
The Nobel Prize in Physics 1990



Jerome I. Friedman
Prize share: 1/3



Henry W. Kendall
Prize share: 1/3

Photo: T. Nakashima
Richard E. Taylor
Prize share: 1/3

The Nobel Prize in Physics 1990 was awarded jointly to Jerome I. Friedman, Henry W. Kendall and Richard E. Taylor *"for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"*.

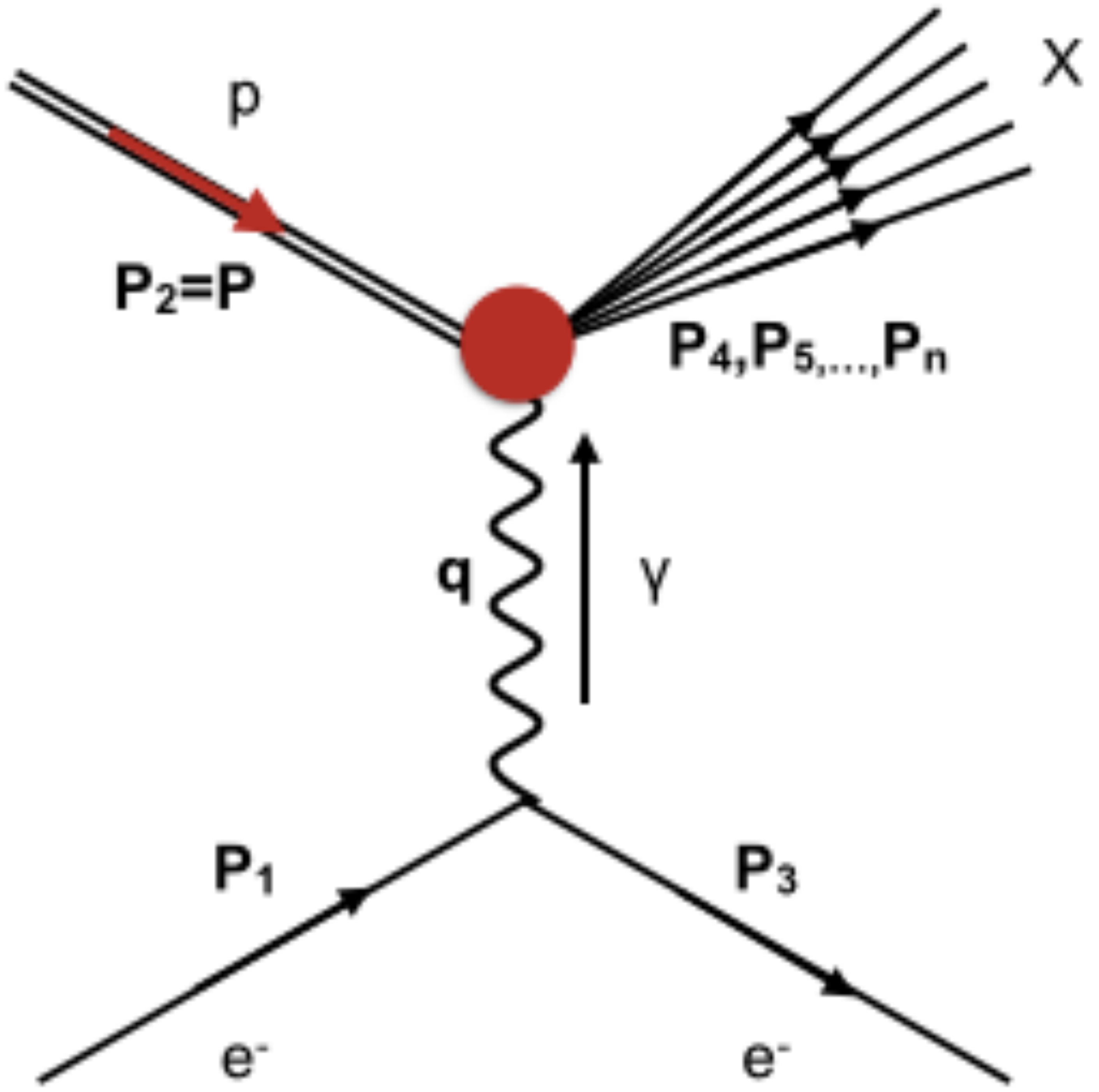
INELASTIC ELECTRON-PROTON SCATTERING

Elastic e-p scattering dominates at lower energies

- During this process an electron interacts with a proton which emerges intact
- The elastic x-section decreases with increasing energy
  - This is due to the finite size of the proton

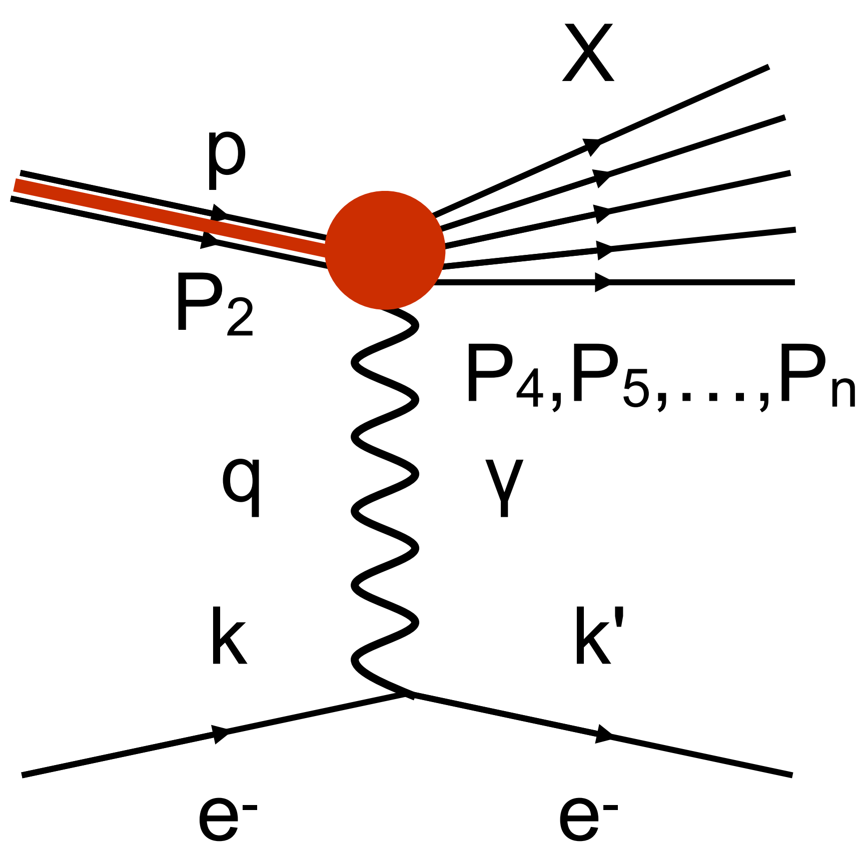
Cranking up the energy of this process, the interaction becomes inelastic

In experiments we record P_3 and what is usually measured is the inclusive cross-section in which all available final states are included



KINEMATICS OF INELASTIC SCATTERING

Build invariant quantities to describe the interaction between the virtual photon or the W and the proton

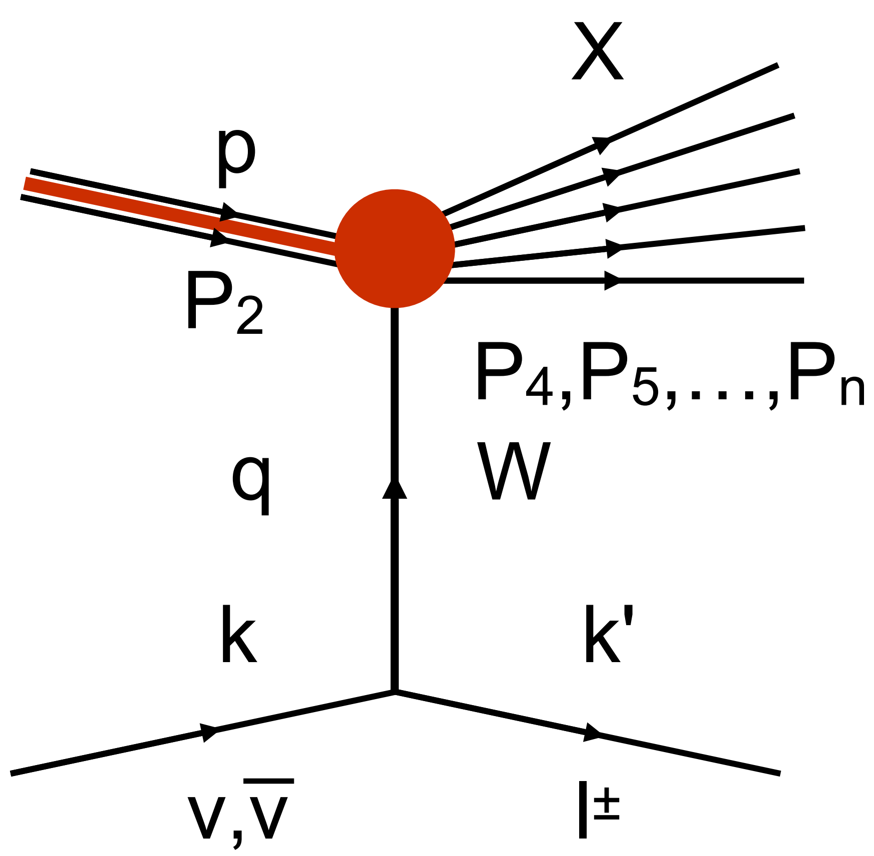


Momentum transfer squared: $Q^2 = -q^2$

Invariant mass of X squared: $W^2 = (P+q)^2$

Bjorken-x (the fraction of the proton's momentum carried by the struck quark): $x = \frac{Q^2}{2Pq}$

Fractional energy transfer in the lab: $y = \frac{Pq}{Pk}$

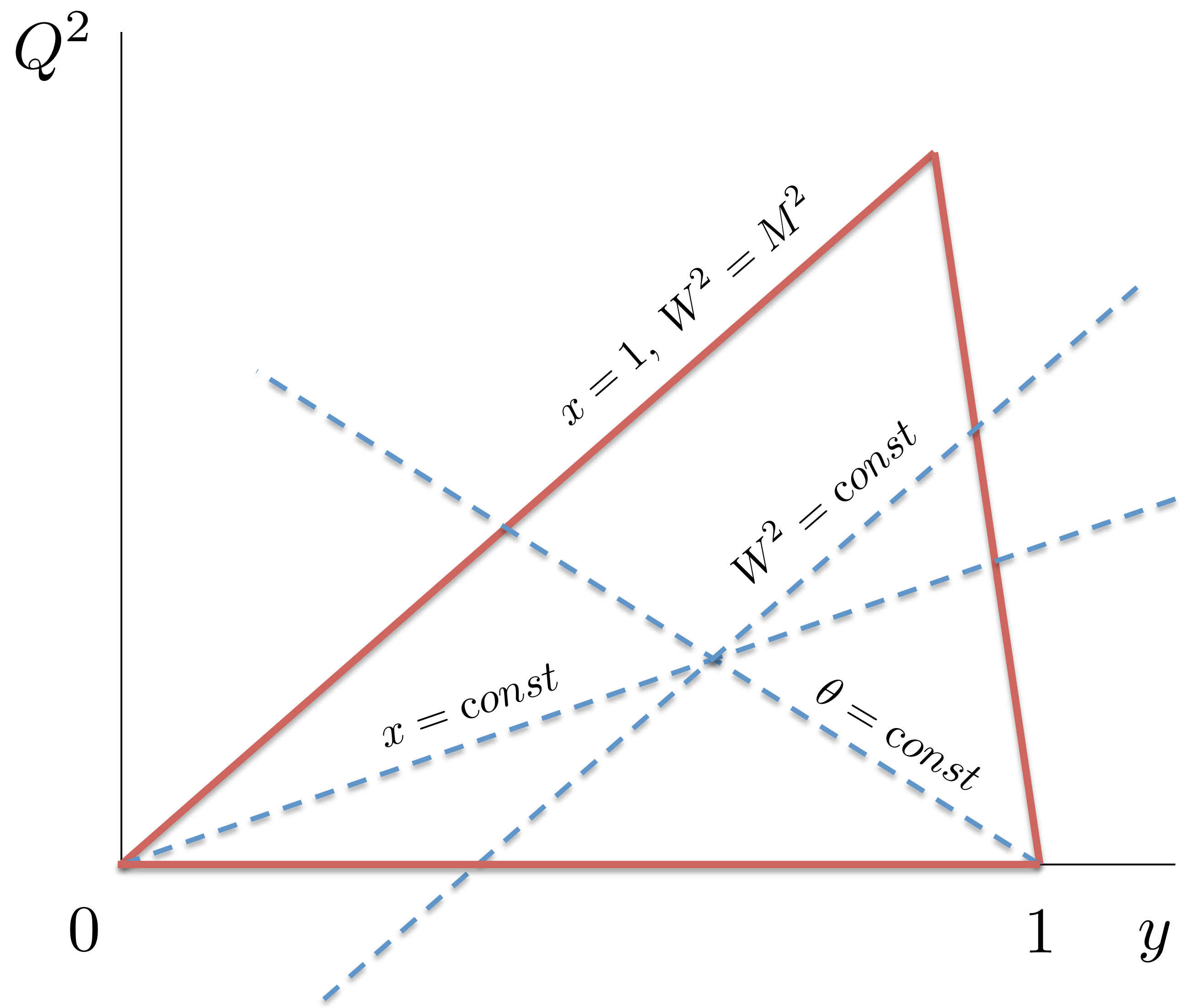


Square of the proton mass: $M^2 = P_\mu P^\mu = P^2$

Energy transfer in the lab: $\nu = \frac{Pq}{M}$

Centre of mass energy squared: $s = (P+k)^2$

KINEMATIC REGIONS



$$
\begin{aligned}
Q^2 &= 2MEy + M^2 - W^2 \\
&= 2MExy \\
&= 4(1-y)E^2 \sin^2\theta/2
\end{aligned}
$$

DEEP INELASTIC SCATTERING - DIS

The Rosenbluth formula gives the most general Lorentz invariant for of the elastic cross-section for the elastic electron-proton scattering via the exchange of a virtual photon

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{ep}} = \frac{a^2}{4E^2\sin^4(\theta/2)}\frac{E_3}{E_1}\left(\frac{G_E^2 + bG_E^2}{1+b}\cos^2(\theta/2) + 2bG_M^2\sin^2(\theta/2)\right)$$

Using the definitions of Q^2 and the elasticity y we can rewrite this as

$$\frac{d\sigma}{dQ^2} = \frac{4\pi a^2}{Q^4}\left[\frac{G_E^2 + bG_M^2}{1+b}\left(1 - y - \frac{M^2y^2}{Q^2}\right) + \frac{1}{2}y^2G_M^2\right]$$

We can now absorb the Q^2 dependence of the form factors and of b into two new functions $f_1(Q^2)$ and $f_2(Q^2)$ such that

$$\frac{d\sigma}{dQ^2} = \frac{4\pi a^2}{Q^4}\left[\left(1 - y - \frac{M^2y^2}{Q^2}\right)f_2(Q^2) + \frac{1}{2}y^2f_1(Q^2)\right]$$

In the case of elastic scattering (y=1) and the previous has a dependence only on Q^2

From the previous, $f_1(Q^2)$ is associated only with the magnetic interactions while $f_2(Q^2)$ has both magnetic and electric contributions

DIS: STRUCTURE FUNCTIONS

A more general form of the previous for inelastic scattering processes is obtained if one introduces the structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$, with the first having only magnetic contributions

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi a^2}{Q^4}\left[\left(1 - y - \frac{M^2y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

In the regime of DIS where $Q^2 >> M^2y^2$ the previous takes the form

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi a^2}{Q^4}\left[(1 - y)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

DIS: STRUCTURE FUNCTIONS

In DIS fixed target ep experiments Q^2, x and y can be obtained on an event-by-event basis from the observed energy E_3 and the scattering angle θ of the electron

$$Q^2 = 4E_1E_3\sin^2\left(\frac{\theta}{2}\right) \qquad x = \frac{Q^2}{2M(E_1 - E_3)} \qquad y = 1 - \frac{E_1}{E_3}$$

- The double differential cross-section $d^2\sigma/(dxdQ^2)$ is measured by counting the number of events in the range $x \rightarrow x + \Delta x$ and $Q^2 \rightarrow Q^2 + \Delta Q^2$
- At a given x and Q^2, $d^2\sigma/(dxdQ^2)$ can be determined for a range of y-values by e.g. varying E_1
- The y-dependence of the cross-section can be then used to disentangle the contributions of $F_1(x,Q^2)$ and $F_2(x,Q^2)$

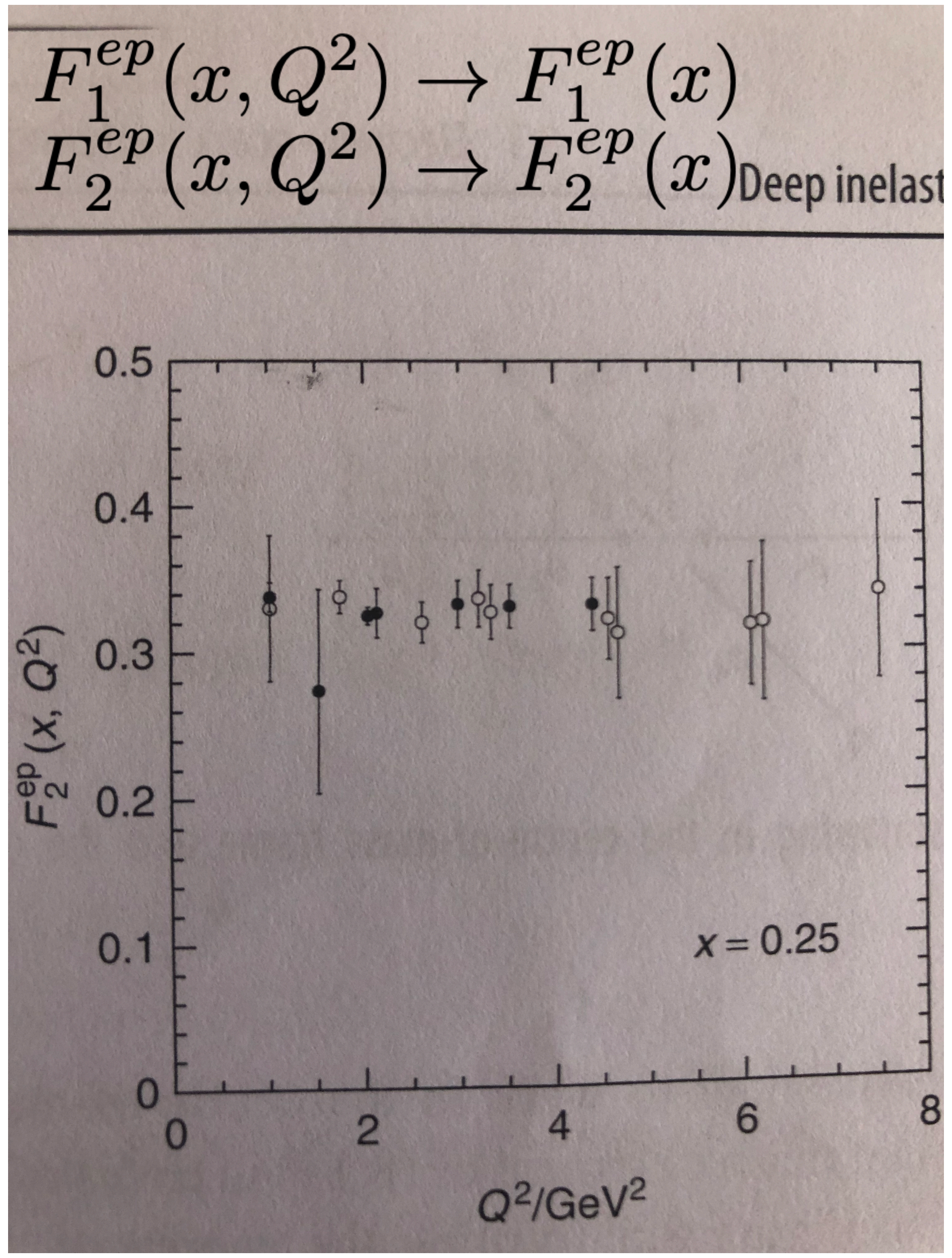
BJORKEN SCALING

The first systematic study of inelastic processed took place at SLAC

- An electron beam with energies between 5 to 20 GeV hit a liquid hydrogen target i.e. protons at rest
- The scattering angle of the electrons was measured with a movable spectrometer
- The differential cross-section was determined for a wide range of incident electron energies to determine the structure functions

BJORKEN SCALING

<u>Experimental observation</u>
at intermediate value of x (0.01 < x < 0.5)
the structure functions do not depend on Q^2



Finding: **The structure functions do not depend on Q^2**

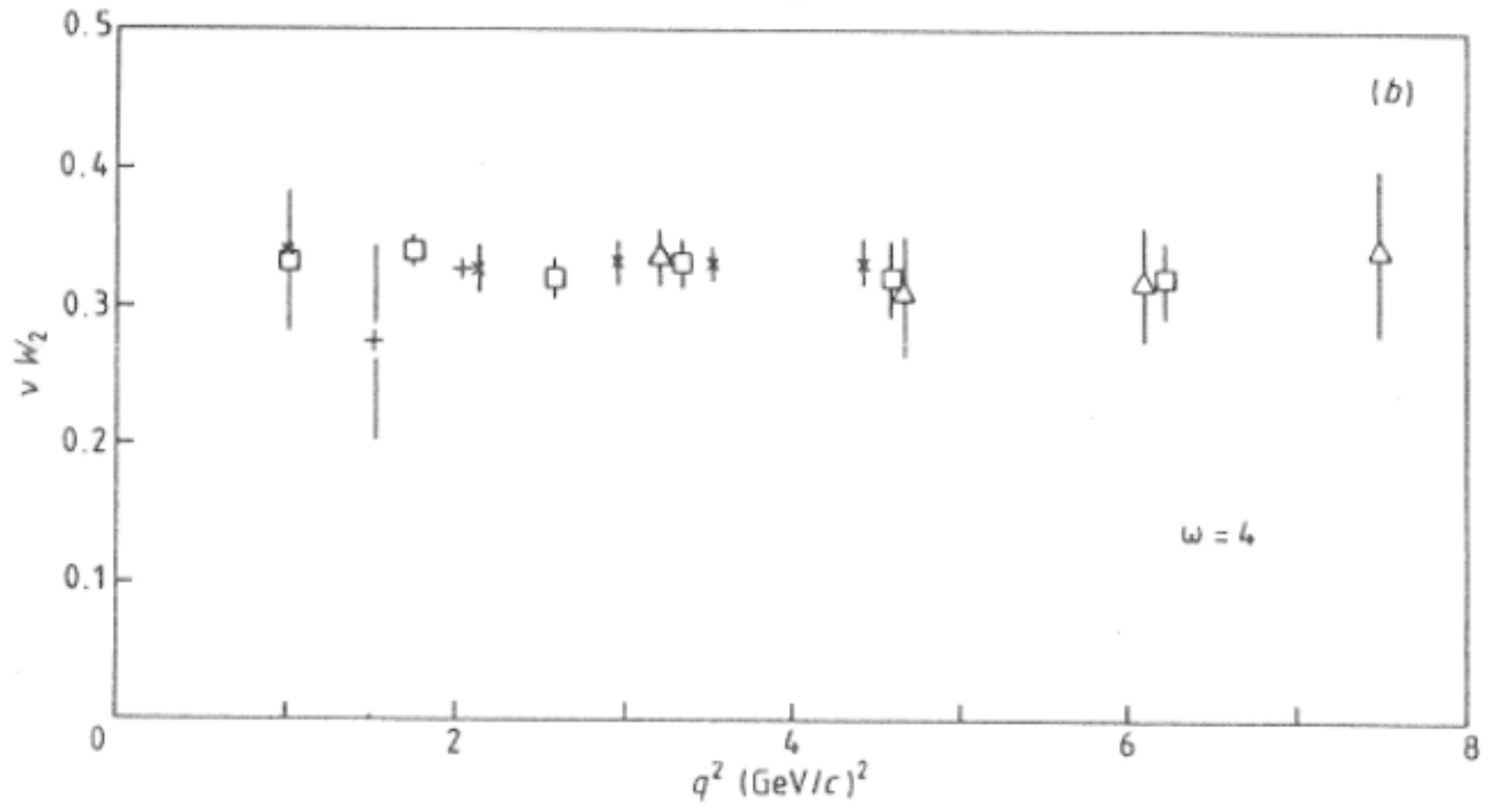
- This was predicted by Bjorken and can be explained as follows:
  - the wavelength of the virtual photon is inversely proportional to the momentum transfer Q and is connected to the resolution or better the resolving power of the internal structure
  - the higher the energy of the interaction between the electron and the proton, the higher the momentum transfer Q and the smaller the resolution

Physics message
Scaling suggestive of scattering from point-like constituents within the proton

BJORKEN SCALING

By increasing the energy we start probing the internal structure of the proton

- while at modest energies the structure functions have a dependence on both q^2 and x
- at higher energies the virtual photon interacts with a point-like particle i.e. the parton (or better the quark) which has no internal structure
  - at least not visible at the current energy regime!



$$MW_1(x, Q^2) \to F_1(x)$$

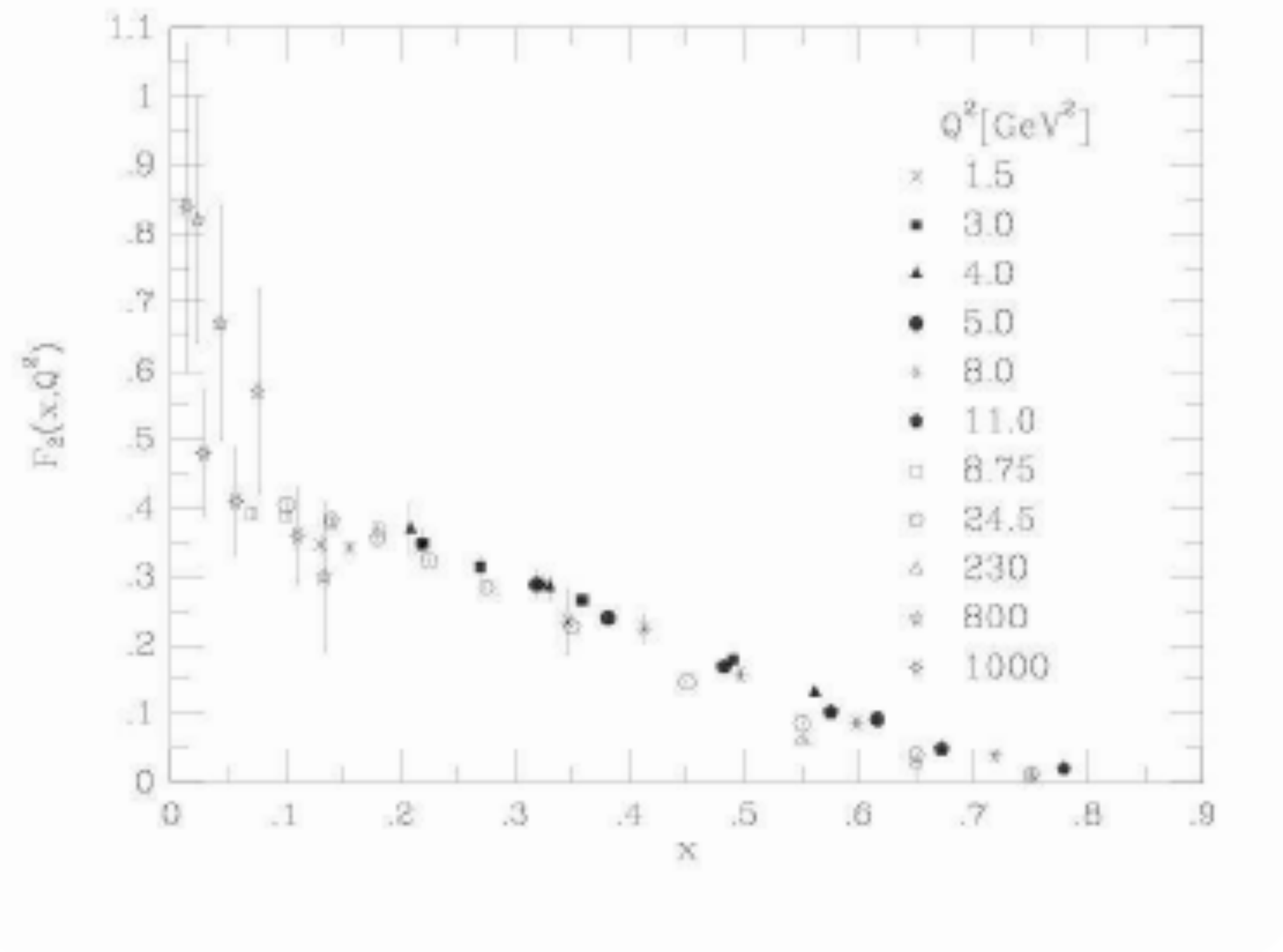
$$\frac{Q^2}{2Mx} W_2(x, Q^2) \to F_2(x)$$

Physics message
Scaling suggestive of scattering from point-like constituents within the proton → quarks???

BJORKEN SCALING

By increasing the energy we start probing the internal structure of the proton

- while at modest energies the structure functions have a dependence on both q^2 and x
- at higher energies the virtual photon interacts with a point-like particle i.e. the parton (or better the quark) which has no internal structure
  - at least not visible at the current energy regime!



$$MW_1(x,Q^2) \to F_1(x)$$

$$\frac{Q^2}{2Mx}W_2(x,Q^2) \to F_2(x)$$

Physics message
Scaling suggestive of scattering from point-like constituents within the proton → quarks???

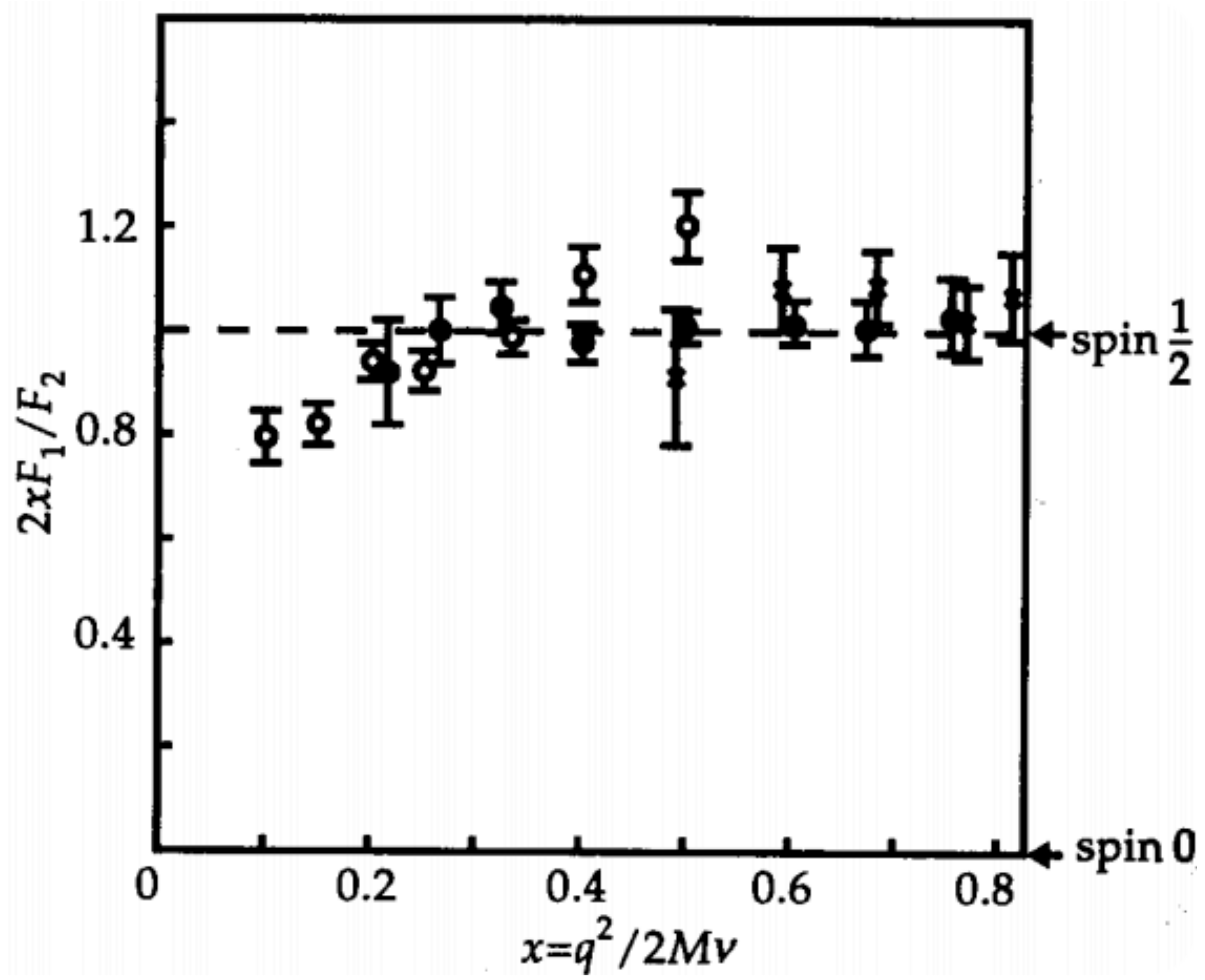
CALLAN-GROSS RELATION

The structure functions F_1 and F_2 are not independent but they follow the Callan-Gross relation within the range that the Bjorken scaling holds

- The electric and magnetic contributions to the scattering process are related by the fixed magnetic moment of **Dirac particles**

$$F_2(x) = 2xF_1(x)$$

Fig. 3.6. The ratio of the structure functions F_1 and F_2 within the scaling region provides a test of the so-called Callan–Gross relations (see (3.82))



Physics message
Underlying process is the elastic scattering from point-like spin-1/2 particles within the proton → quarks!!!

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THE PARTON MODEL

Bjorken's scaling hypothesis

- if scattering is caused by point-like constituents, then the structure functions should be independent of Q^2

Feynman's parton model

- a proton consists of constituents
- the term "parton" was used by Feynman at the early stages of his formulation and stands until our days
  - Physicists were reluctant to talk about quarks at that stage, let alone about gluons

THE PARTON MODEL

Main assumptions (proven also experimentally)

- Non-interacting point-like particles → Bjorken scaling i.e. $F_2(x,Q^2)=F_2(x)$
- Fractional charges (if partons=quarks)
- Spin-1/2 (i.e. Dirac) particles
- Valence and sea quark structure (sum rules)

The parton model made key predictions that could be tested experimentally!

THE PARTON MODEL

The matrix element for the (elastic) process $e^-q \rightarrow e^-q$ is given by

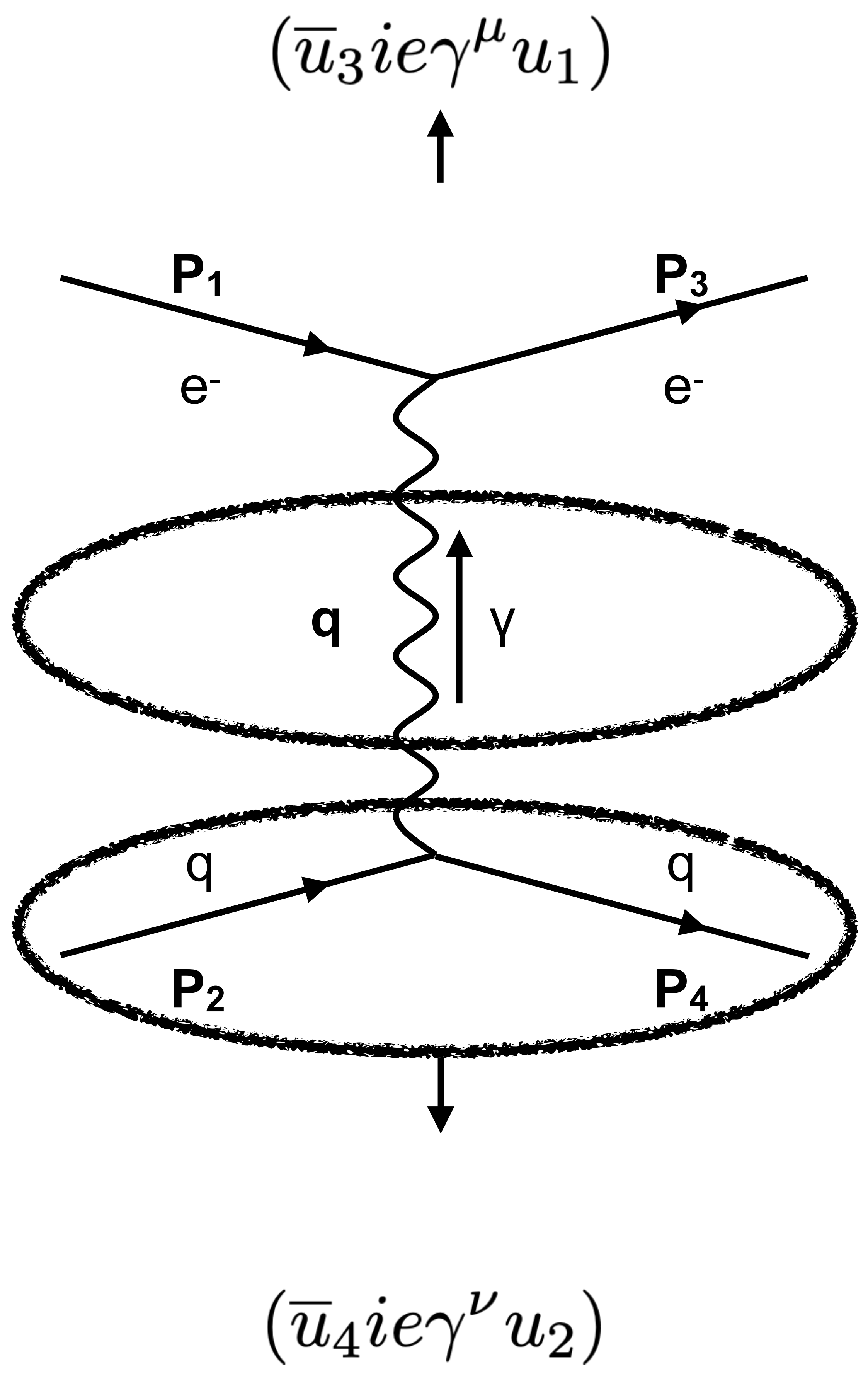
$$M_{if} = \frac{Qe^2}{q^2}(\overline{u}_3\gamma^\mu u_1)g_{\mu\nu}(\overline{u}_4\gamma^\mu u_2)$$

The spin average matrix element squared is given by

$$\langle|M_{if}|^2\rangle = 2Q^2e^4\left(\frac{s^2+u^2}{t^2}\right) = 2Q^2e^4\frac{(P_1P_2)^2+(P_1P_4)^2}{(P_1P_3)^2}$$

Madelstam variables

$$s = (P_1+P_2)^2 \qquad t = (P_1-P_3)^2 \qquad u = (P_1-P_4)^2$$

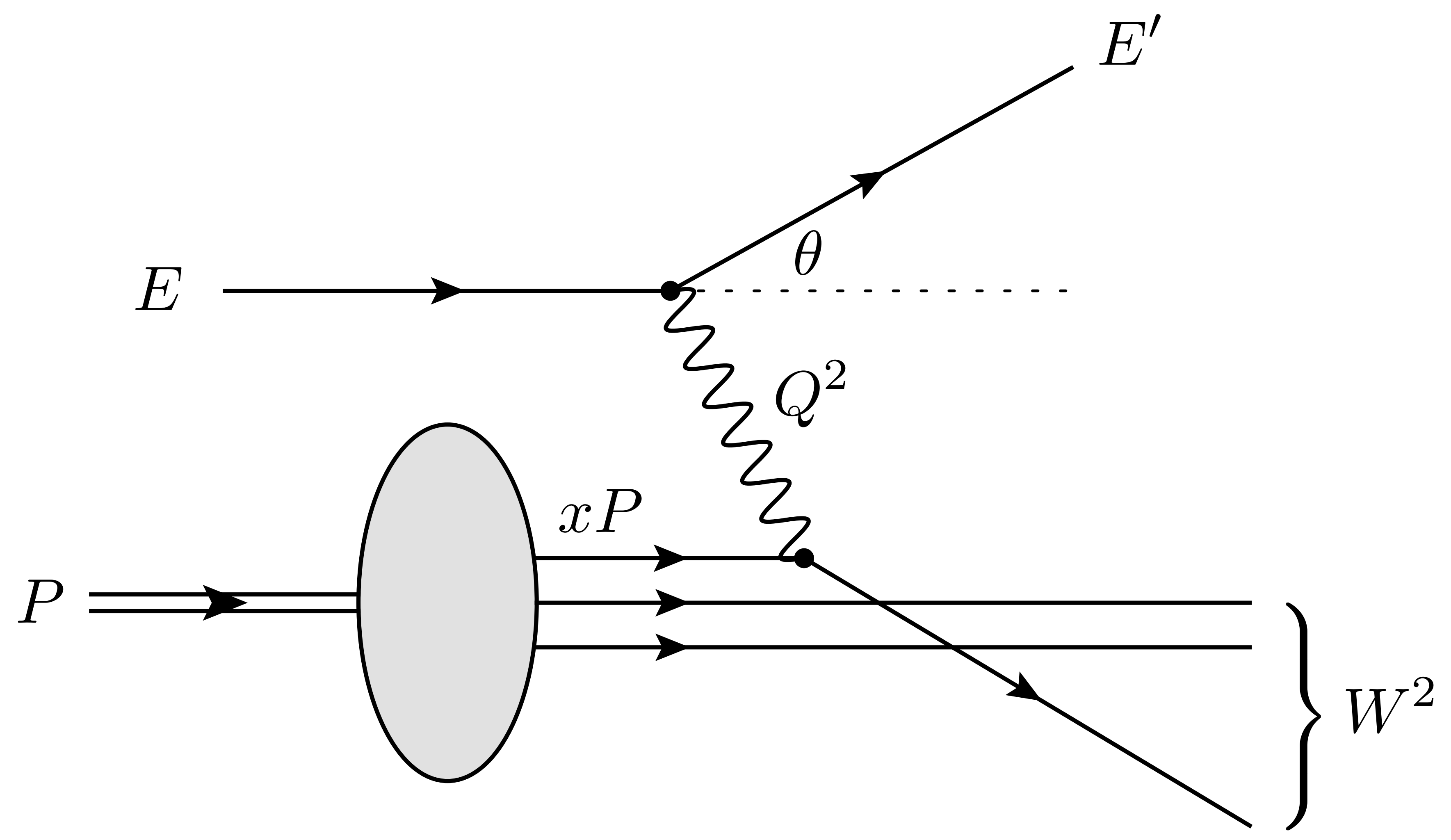


THE PARTON MODEL

A virtual photon can penetrate a proton and can interact with its constituents

We can thus probe the partons i.e. in this case the quarks

There are more than one type of such particles in the proton and each one carries a different fraction of the proton's momentum and energy



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THE PARTON MODEL

We introduce the parton distribution function f(x):

- the probability that the struck parton carries a fraction x of the proton's momentum
    - All fractions have to add up to unity such that

$$\sum_{i=1}^{N} \int x f_i(x) dx = 1$$

    - Where i is an index for all partons that do not interact with the photon

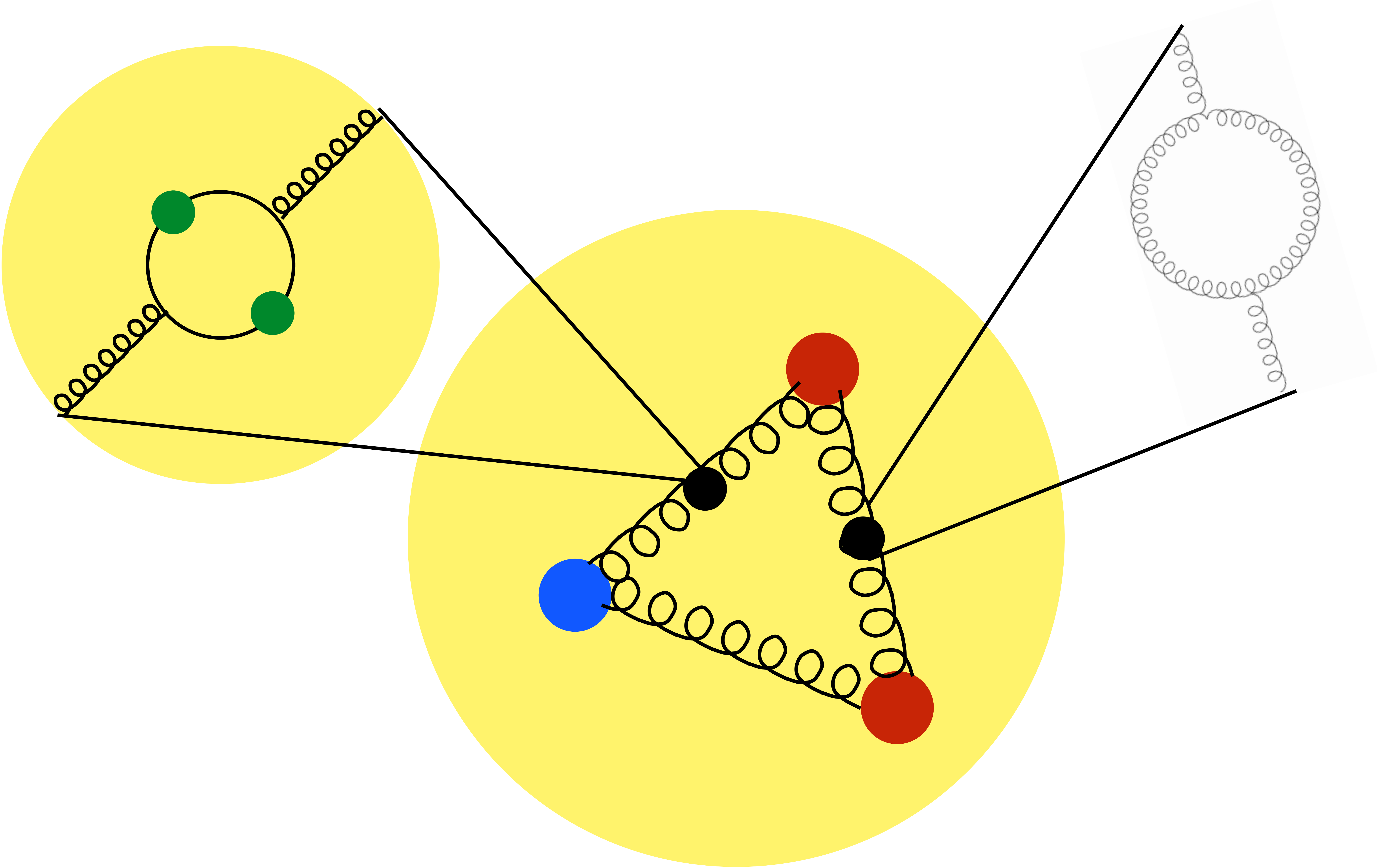
But inside the proton there are also other partons (e.g. sea quarks and gluons)

However there is a net excess of three quarks that carry the quantum numbers of the proton

- these are the valence quarks

These valence quarks are dressed with gluons and sea q-qbar pairs

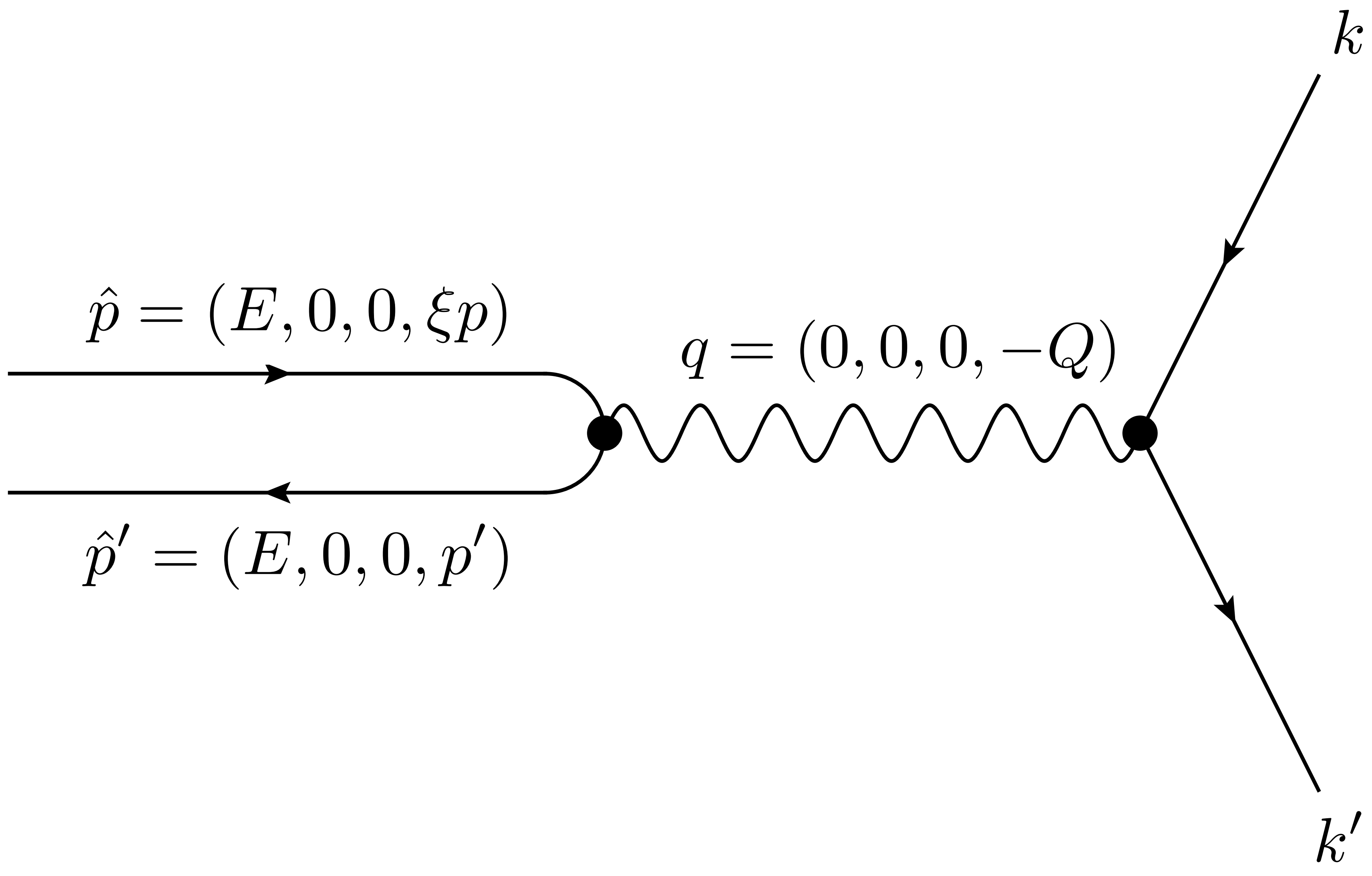
THE PARTON MODEL



DIS X-SECTION : THE BREIT FRAME

To calculate the DIS x-section we move to a frame where the masses can be neglected and we will use the Mandelstam variables, s, u, t

- We can boost the photon along its direction of propagation such that q_0 vanishes ($q^2 < 0$) $\Longrightarrow$ **the Breit frame** or infinite momentum frame
  - In this frame the proton moves with very large momentum towards the photon



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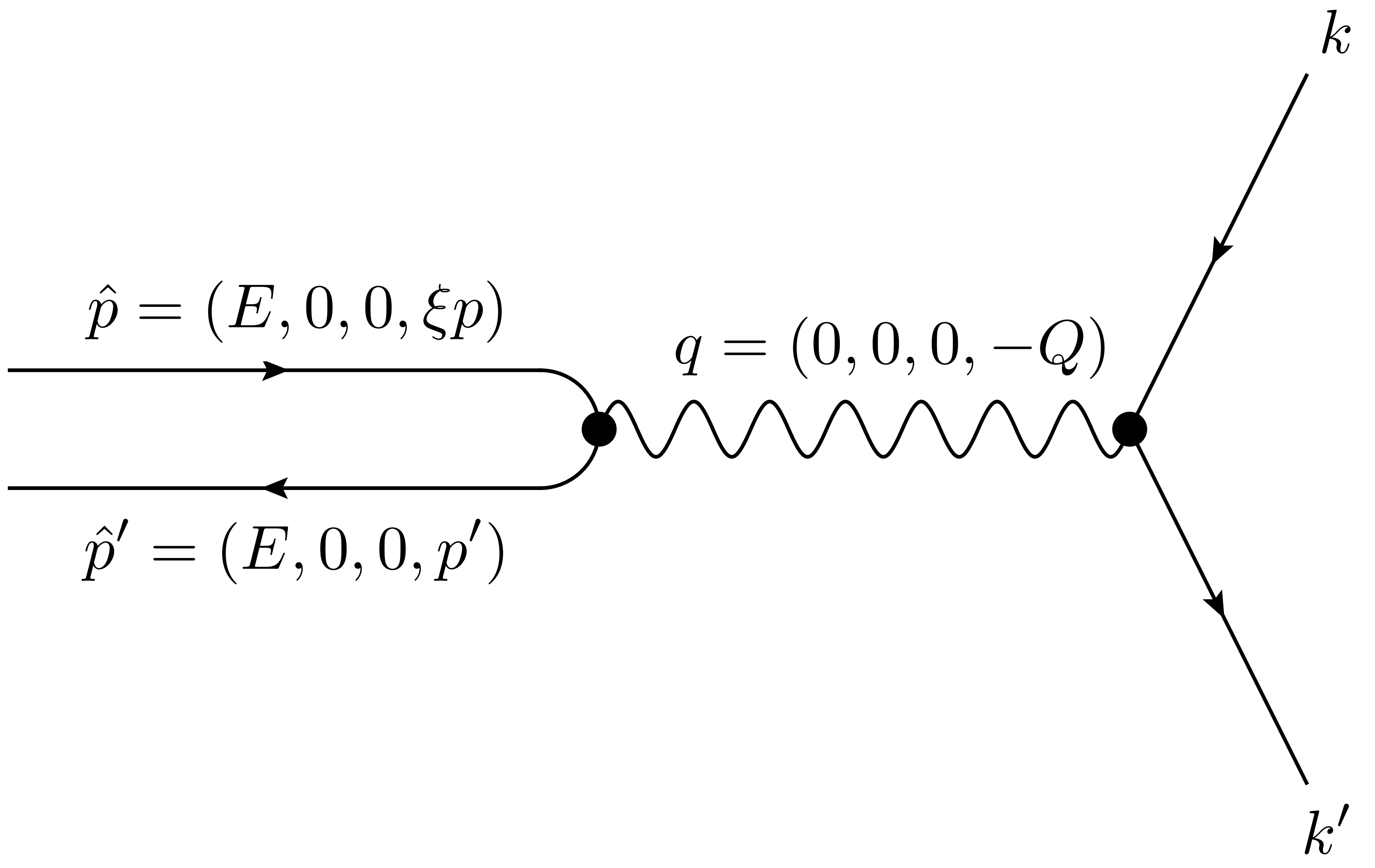
DIS X-SECTION : THE BREIT FRAME

The incoming quark moves with a momentum ξp along the z-axis, where ξ is the fraction of proton's momentum

The scattering is considered to be elastic (i.e. point-like q)



DIS X-SECTION

The total cross section is the sum of the partonic cross-sections

- Where we have introduced the partonic kinematic variables

$$d\sigma = \sum_{i=1}^{N} d\hat{\sigma}(\hat{s}, \hat{t}, \hat{u}) f_i(x) dx$$

$$\hat{s} = 2xpk = xs$$

$$\hat{t} = (k - k^{'})^2 = t$$

$$\hat{u} = -2xpk^{'} = xu$$

To calculate the total cross-section of the e-q process, we profit from the similarity with the e-μ scattering and we only change the muon part by the quark spinors and charges

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{a^2}{2s}\left(\frac{s^2 + u^2}{t^2}\right)$$

DIS X-SECTION

The partonic cross-section is given by

$$\frac{d\hat{\sigma}_i}{d\hat{t}} = \frac{2\pi a^2 e_i^2}{\hat{s}^2}\left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right)$$

We can now combine the partonic cross-section with the parton distribution function $f_i(x)$ to obtain the DIS cross section that is given by

$$\frac{d^2\sigma}{dx dQ^2} = \frac{2\pi a^2}{Q^4}\left[1 + (1-y)^2\right]\sum_{i=1}^{N} e_i^2 f_i(x)$$

- Where the variable y is the fractional energy transfer in the lab given by

$$y = \frac{Pq}{Pk} = \frac{Q^2}{(s - m_p^2)x}$$

We connect the parton distribution function to the structure function $F_2(x)$ by

$$F_2(x) = \sum_{i=1}^{N} e_i^2 x f_i(x)$$

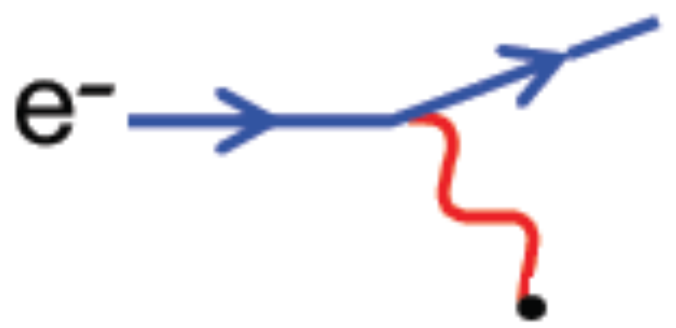
The final cross-section is

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi a^2}{Q^4}\frac{\left[1 + (1-y)^2\right]}{2x} F_2(x)$$

THE INTERNAL STRUCTURE OF THE PROTON

We start off with a "kid's microscope" with low resolution i.e. small momentum transfers $\Longrightarrow$ low energy

- able to detect the existence of a static electric potential



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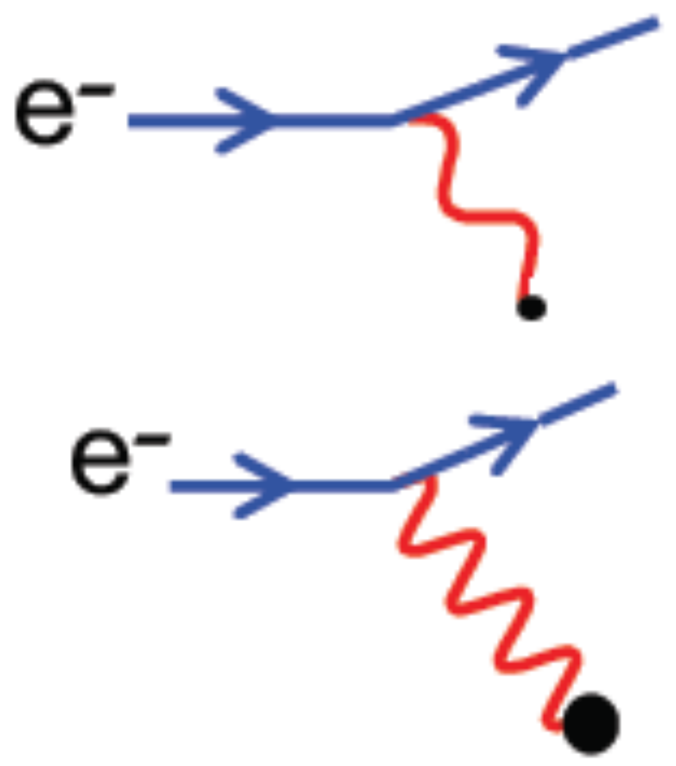
THE INTERNAL STRUCTURE OF THE PROTON

Increasing the energy leads to better resolution

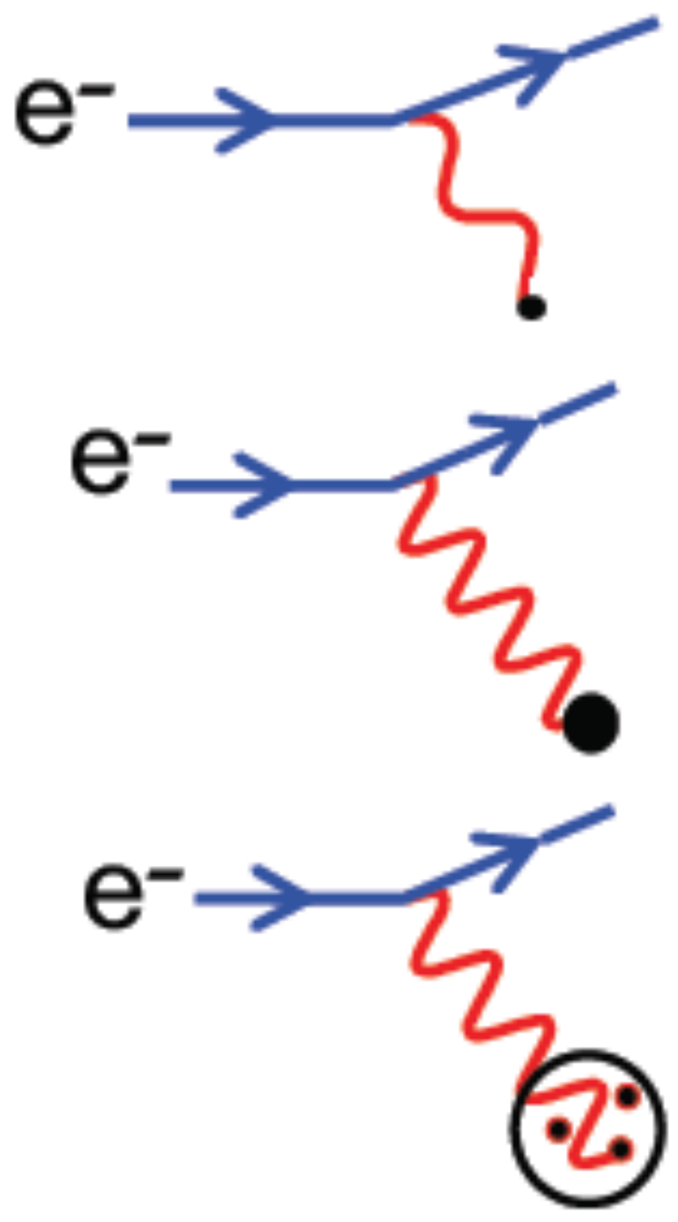
- our target has a sizeable charge distribution



THE INTERNAL STRUCTURE OF THE PROTON

For large momentum transfers

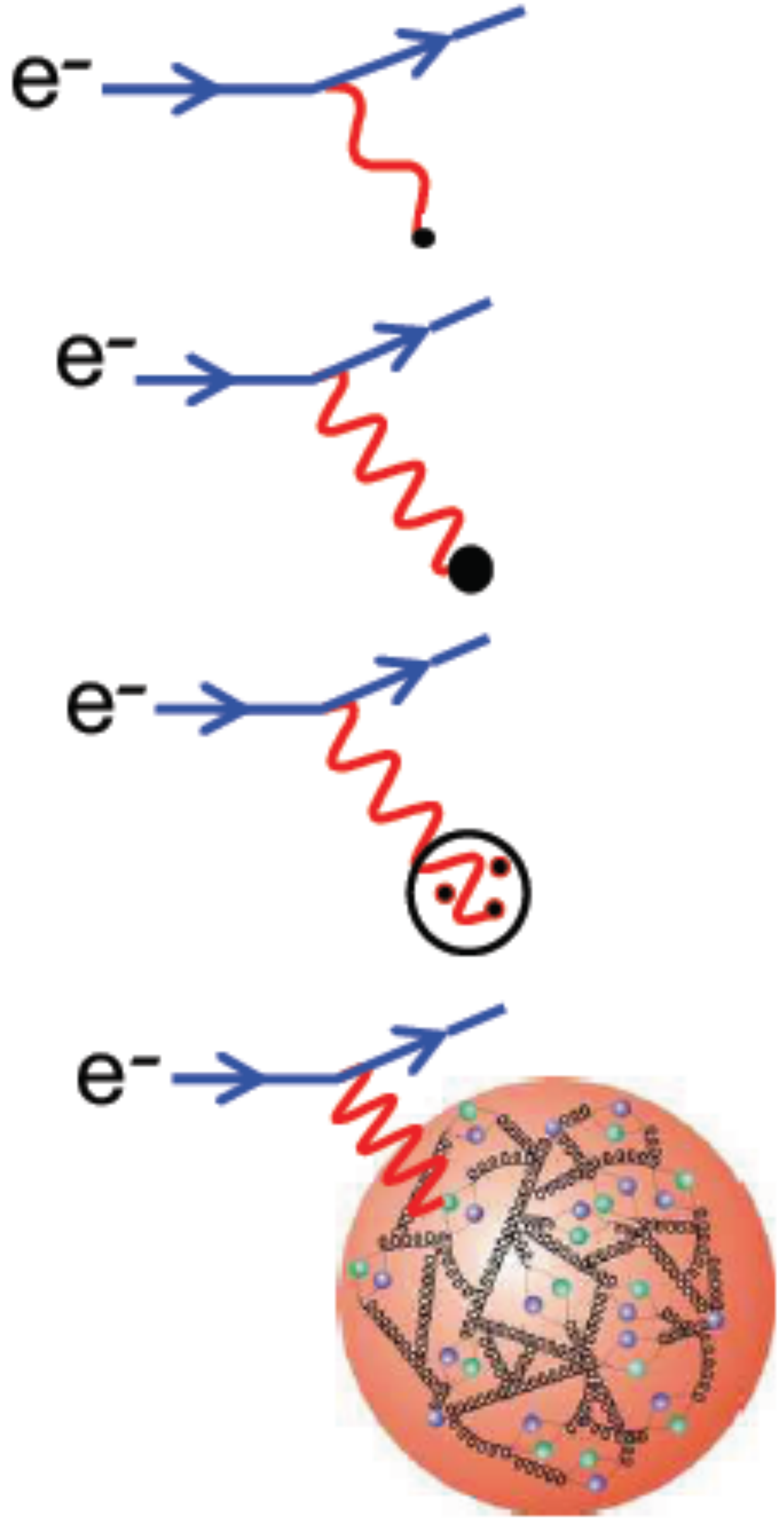
- our target has internal structure i.e. valence quarks
- Electron-proton inelastic scattering is seen as an electron-quark elastic scattering process



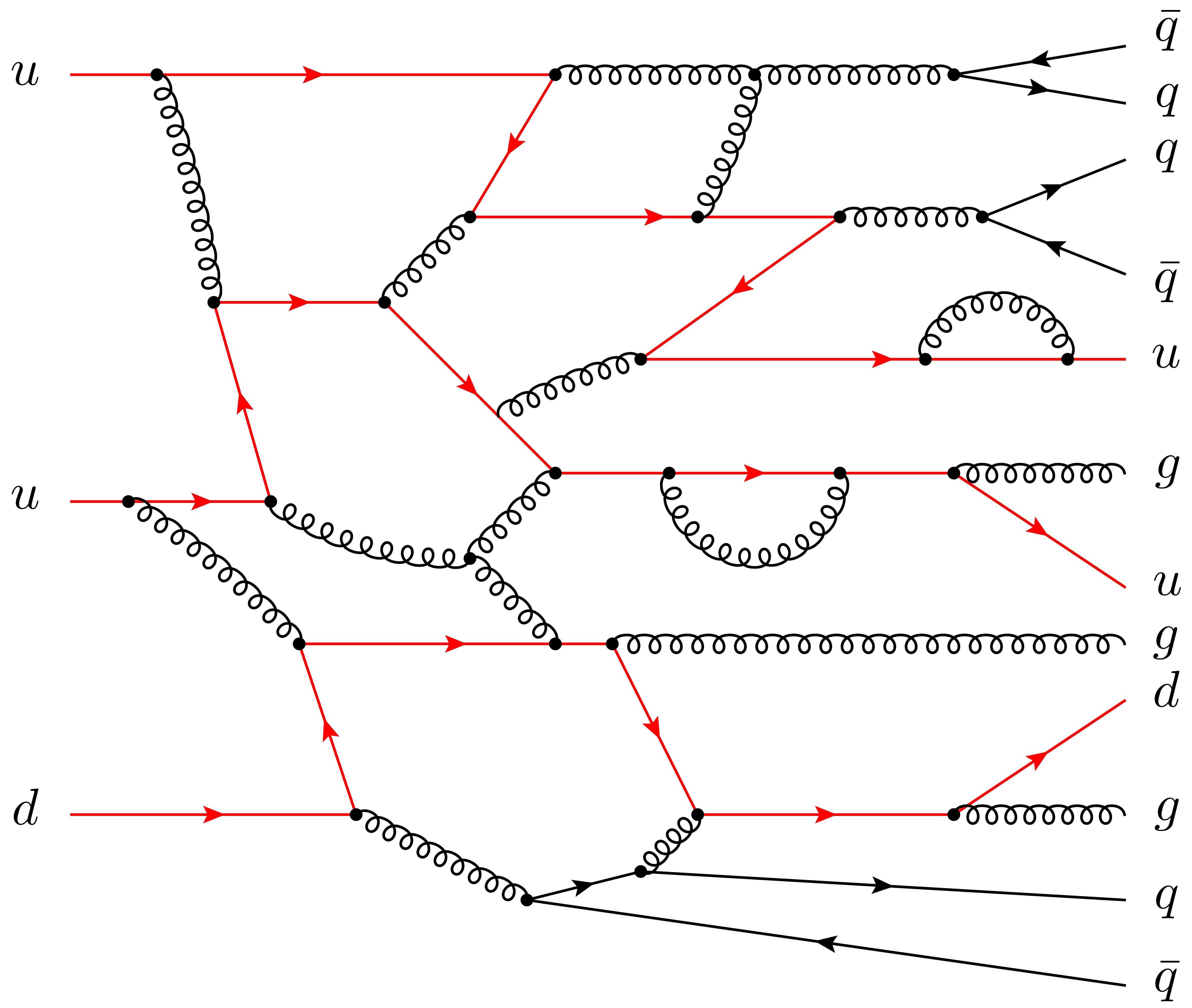
THE INTERNAL STRUCTURE OF THE PROTON

For even larger momentum transfers

- The internal structure of our target is even richer than we thought
- not only valence but also sea quarks and gluons!
- Introduce the parton distribution functions



HIGH RESOLUTION PROTON PICTURE



QUARK AND GLUON DISTRIBUTION FUNCTIONS

At high Q^2 we see a high resolution picture of the proton with not only the valence quarks but also the sea quarks and the gluons

We focus on the three lightest quarks i.e. u, d, s, as the heavier ones are subject to threshold effects

The structure function can be written as

$$\frac{1}{x}F_2^p(x) = \sum_{i=1}^{N} e_i^2 f_i^p(x)$$

$$\frac{1}{x}F_2^p(x) = \left(\frac{2}{3}\right)^2 [u^p(x) + \overline{u}^p(x)] + \left(\frac{1}{3}\right)^2 [d^p(x) + \overline{d}^p(x)] + \left(\frac{1}{3}\right)^2 [s^p(x) + \overline{s}^p(x)]$$

- where u(x), d(x) and s(x) are the probability distributions of u, d and s quarks within the proton (similarly for antiquarks)

$F_2(x)$ has six unknown quantities

- To overcome this one first relies on the fact that protons and neutrons are members of the isospin doublet and thus their quark content is related

$$\frac{1}{x}F_2^n(x) = \left(\frac{2}{3}\right)^2 [u^n(x) + \overline{u}^n(x)] + \left(\frac{1}{3}\right)^2 [d^n(x) + \overline{d}^n(x)] + \left(\frac{1}{3}\right)^2 [s^n(x) + \overline{s}^n(x)]$$

QUARK AND GLUON DISTRIBUTION FUNCTIONS

The quark content of the two nucleons are connected if one exchanges u with d and vice-versa

$$u^p(x) = d^n(x) = u(x)$$
$$d^p(x) = u^n(x) = d(x)$$
$$s^p(x) = s^n(x) = s(x)$$

Another constrain comes from the fact that the quantum numbers are carried by the valence quarks

One can also consider that the sea quarks occur to first order at the same rate and have similar momentum distributions

We define the valence and the sea quarks as

- So that

$$u_{\mathrm{v}}(x) = u(x) - \overline{u}(x) \qquad u_{\mathrm{s}}(x) = 2\overline{u}(x)$$
$$d_{\mathrm{v}}(x) = d(x) - \overline{d}(x) \qquad d_{\mathrm{s}}(x) = 2\overline{d}(x)$$
$$s_{\mathrm{v}}(x) = s(x) - \overline{s}(x) = 0 \quad s_{\mathrm{s}}(x) = 2\overline{s}(x)$$

$$u(x) + \overline{u}(x) = u_{\mathrm{v}}(x) + u_{\mathrm{s}}(x)$$

QUARK AND GLUON DISTRIBUTION FUNCTIONS

Summing over all the partons we should recover the quantum numbers of the proton

$$\int_0^1 u_\mathrm{v}(x)dx = 2 \qquad \int_0^1 d_\mathrm{v}(x)dx = 1$$

As a result the proton and neutron structure functions are written as

$$\frac{1}{x}F_2^p(x) = \frac{1}{9}[4u_\mathrm{v}(x) + d_\mathrm{v}(x)] + \frac{4}{3}S(x) \qquad \frac{1}{x}F_2^n(x) = \frac{1}{9}[u_\mathrm{v}(x) + 4d_\mathrm{v}(x)] + \frac{4}{3}S(x)$$

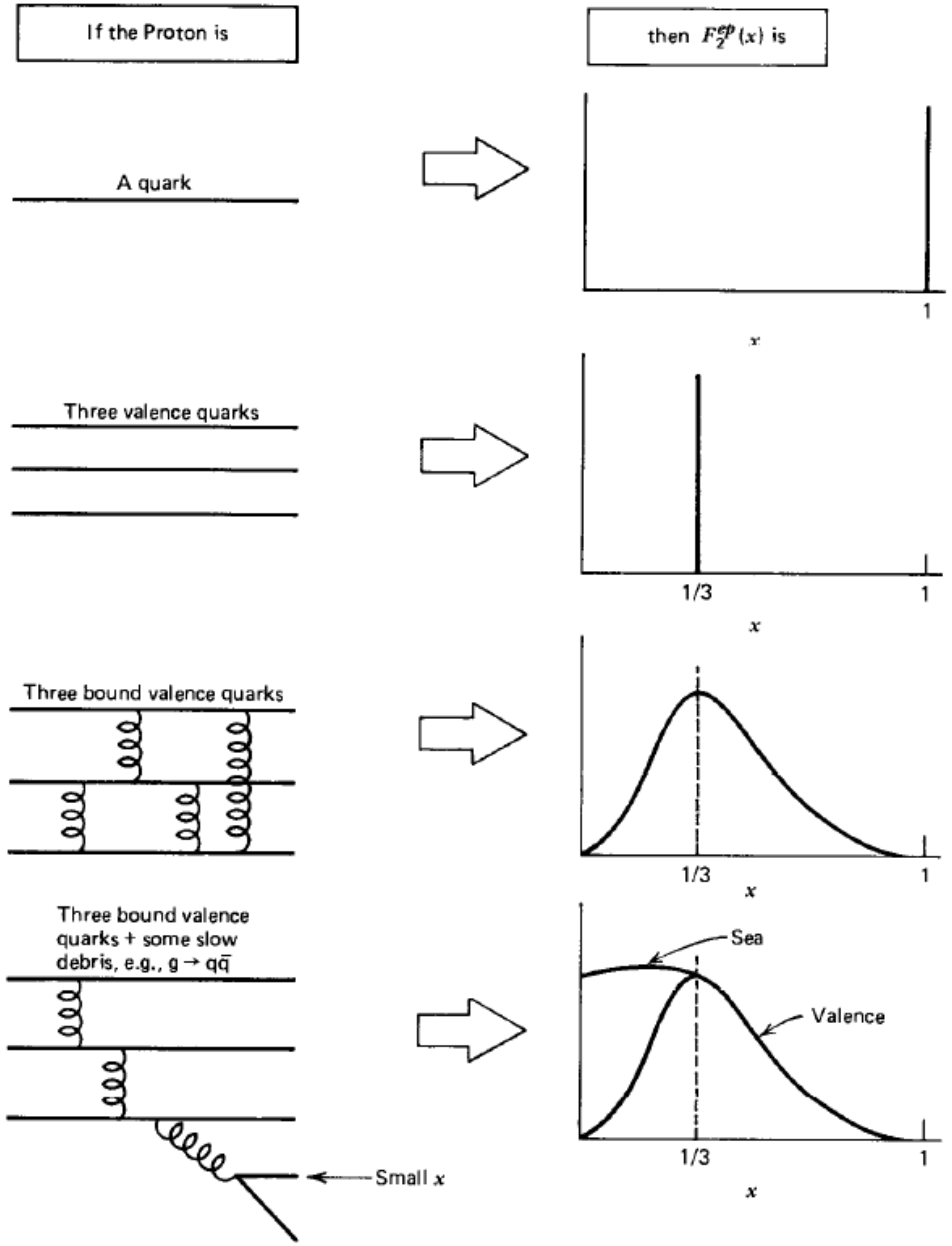
- where S(x) is the sea quark distribution

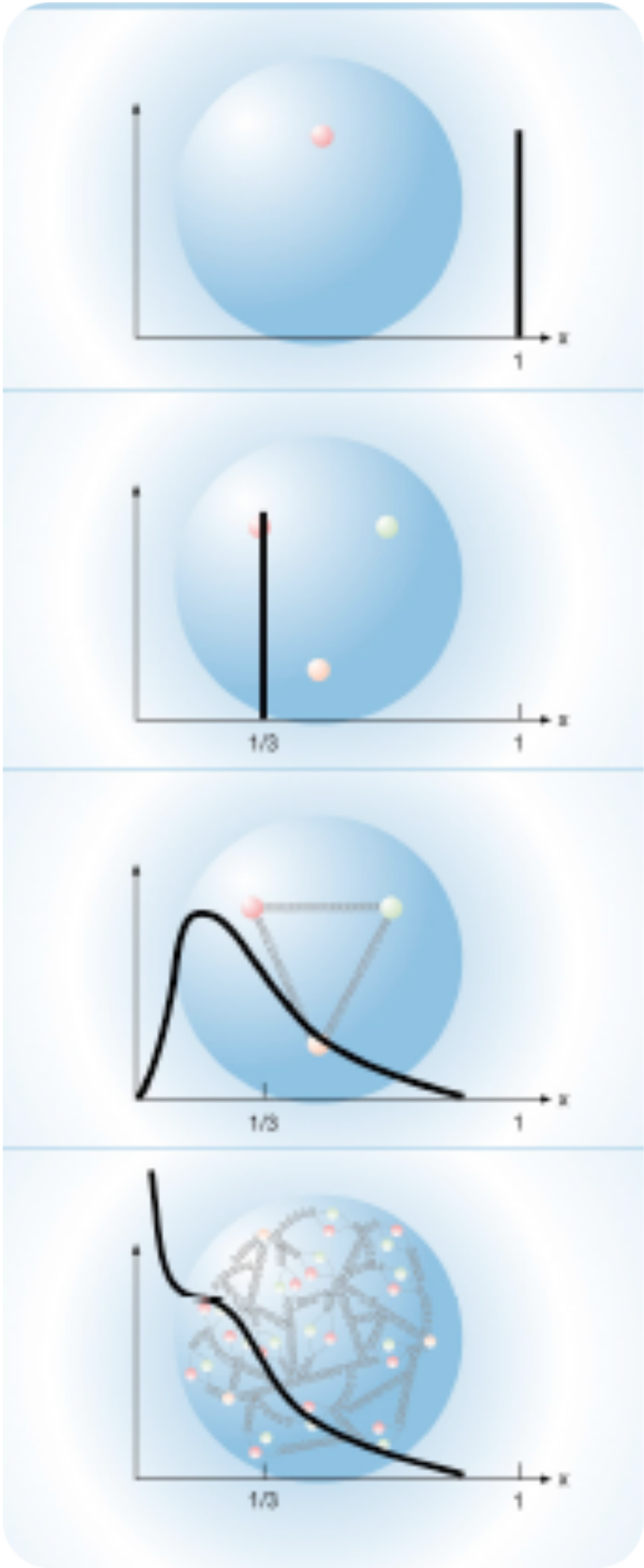
$$S(x) \equiv u_\mathrm{s}(x) = \overline{u}_\mathrm{s}(x) = d_\mathrm{s}(x) = \overline{d}_\mathrm{s}(x) = s_\mathrm{s}(x) = \overline{s}_\mathrm{s}(x)$$

When studying the small momentum part of the proton i.e. x→0, one probes the low momentum sea quarks

At high momenta i.e. x→1, the high momentum valence quarks leave little unoccupied room for the sea quarks and one probes mainly the valence quarks

THE INTERNAL STRUCTURE OF PROTONS





KEY PREDICTIONS: QUARK FRACTIONAL CHARGE

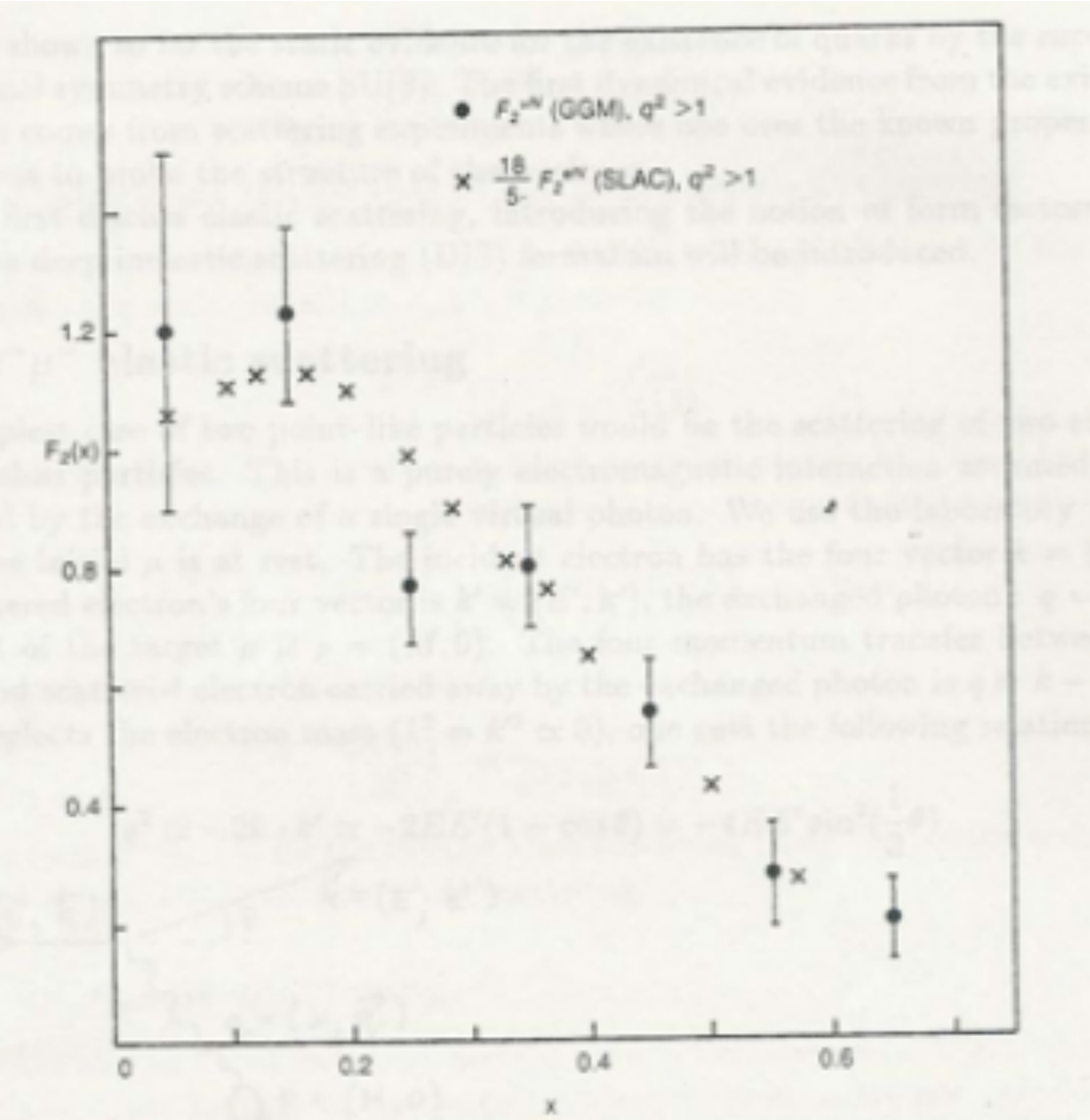
A good check of the quark fractional charge is the comparison between the structure functions in ep and vN scattering

$$F_2^{ep}(x) = \sum_q x e_q^2 [q(x) + \bar{q}(x)]$$

$$F_2^{ep}(x) = x\left[\frac{4}{9}(u+\overline{u}) + \frac{1}{9}(d+\overline{d})\right]$$

$$F_2^{en}(x) = x\left[\frac{1}{9}(u+\overline{u}) + \frac{4}{9}(d+\overline{d})\right]$$

$$F_2^{eN}(x) = \frac{5}{18} F_2^{\nu N}(x)$$



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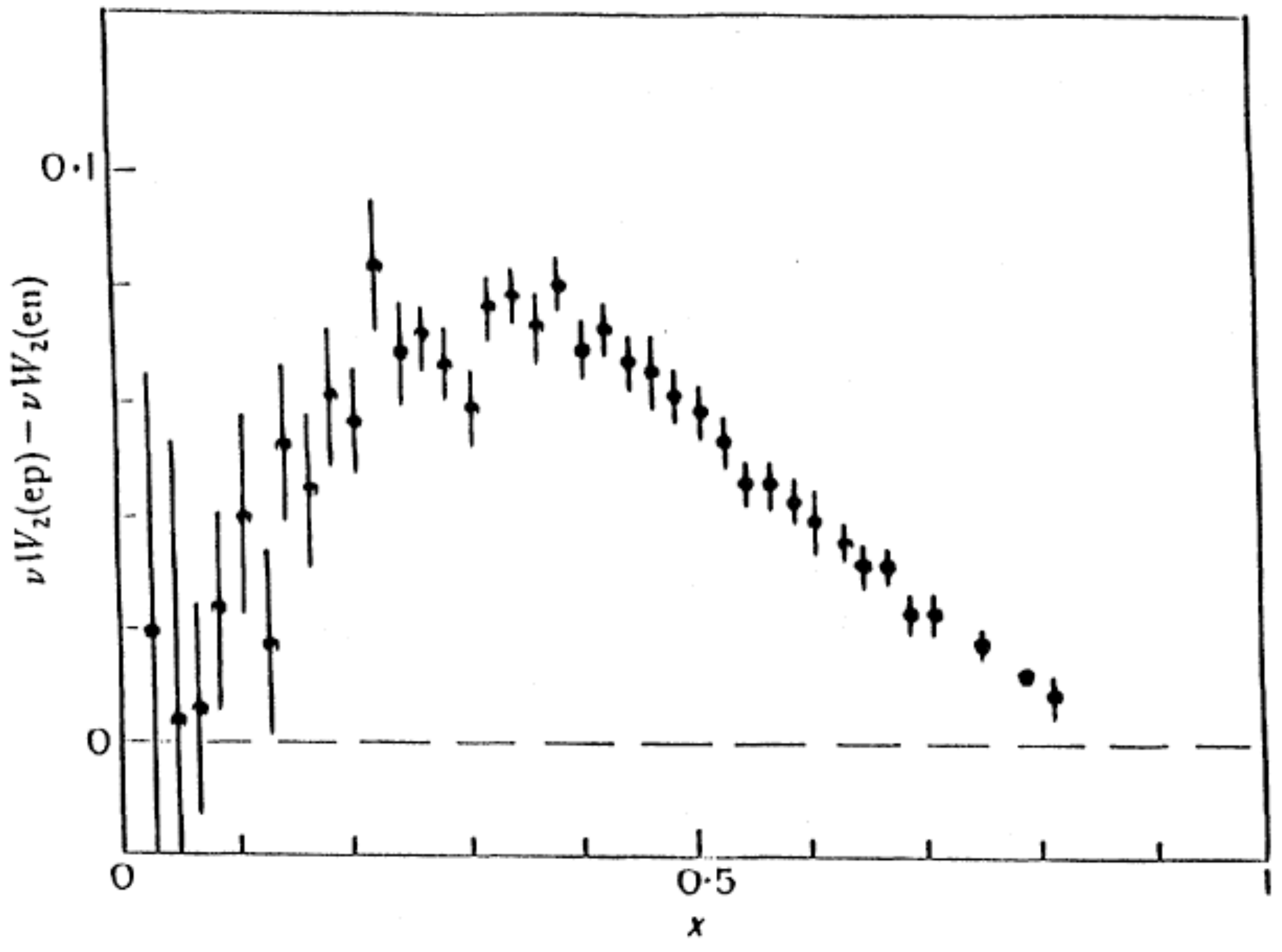
KEY PREDICTIONS: VALENCE QUARKS

A good check of the valence structure assigned to quarks is the comparison between the structure functions in ep and en scattering

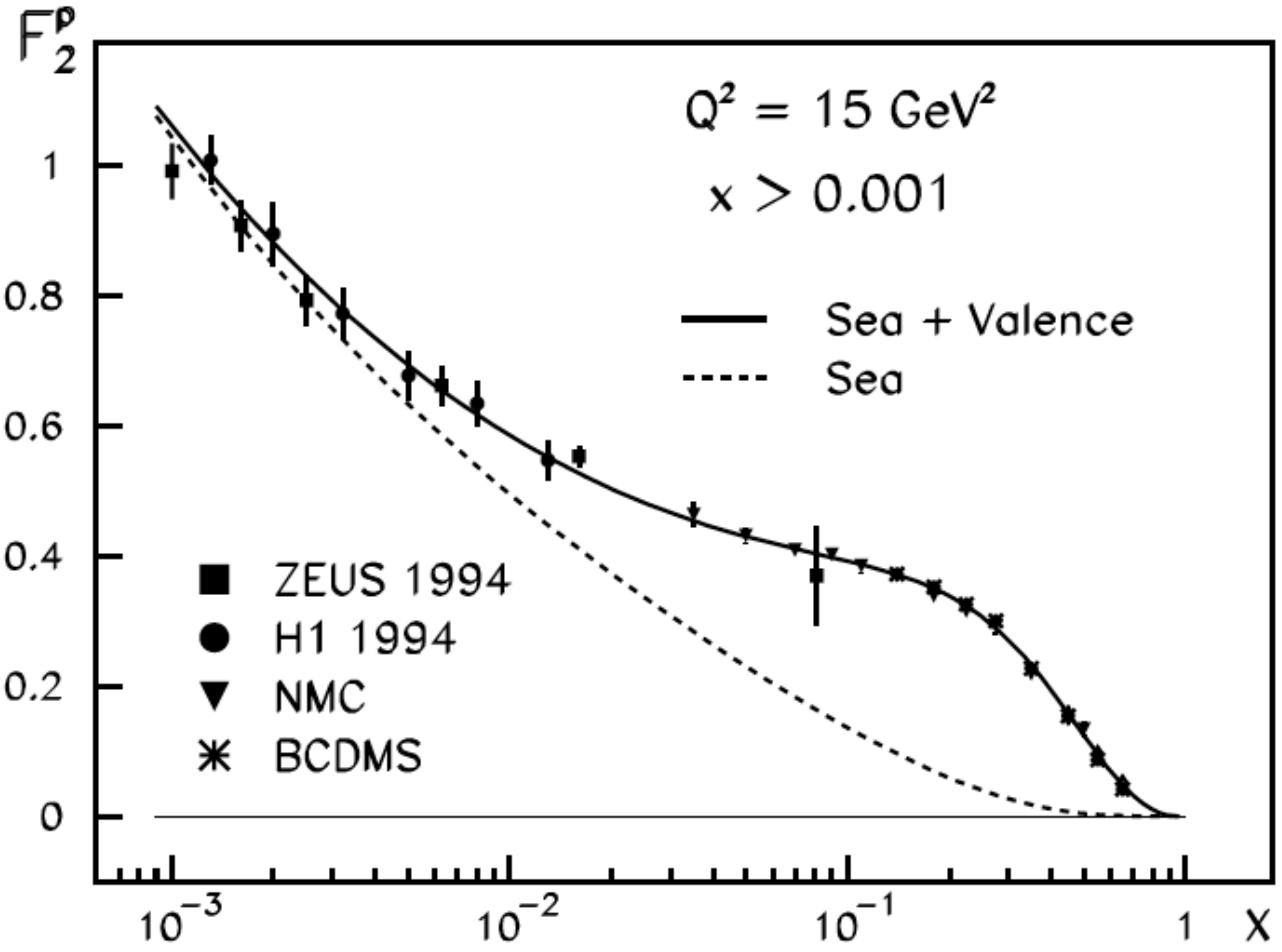
$$F_2^{ep}(x) - F_2^{en}(x) = \frac{1}{3}x[u_v(x) - d_v(x)]$$



$$F_2^{ep}(x) = x\left[\frac{4}{9}(u+\overline{u}) + \frac{1}{9}(d+\overline{d})\right]$$

$$F_2^{en}(x) = x\left[\frac{1}{9}(u+\overline{u}) + \frac{4}{9}(d+\overline{d})\right]$$

SEA AND VALENCE QUARK DISTRIBUTIONS



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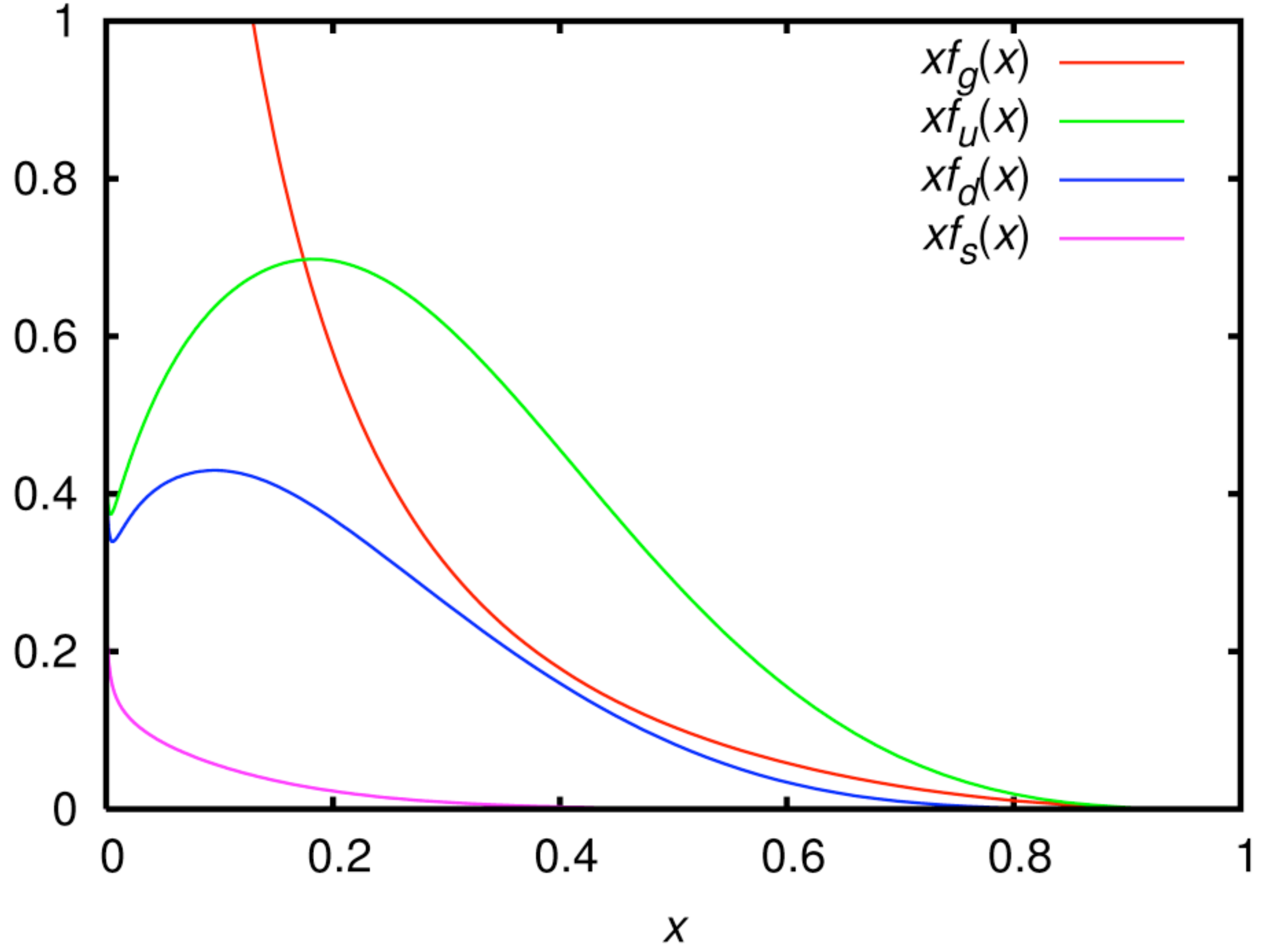
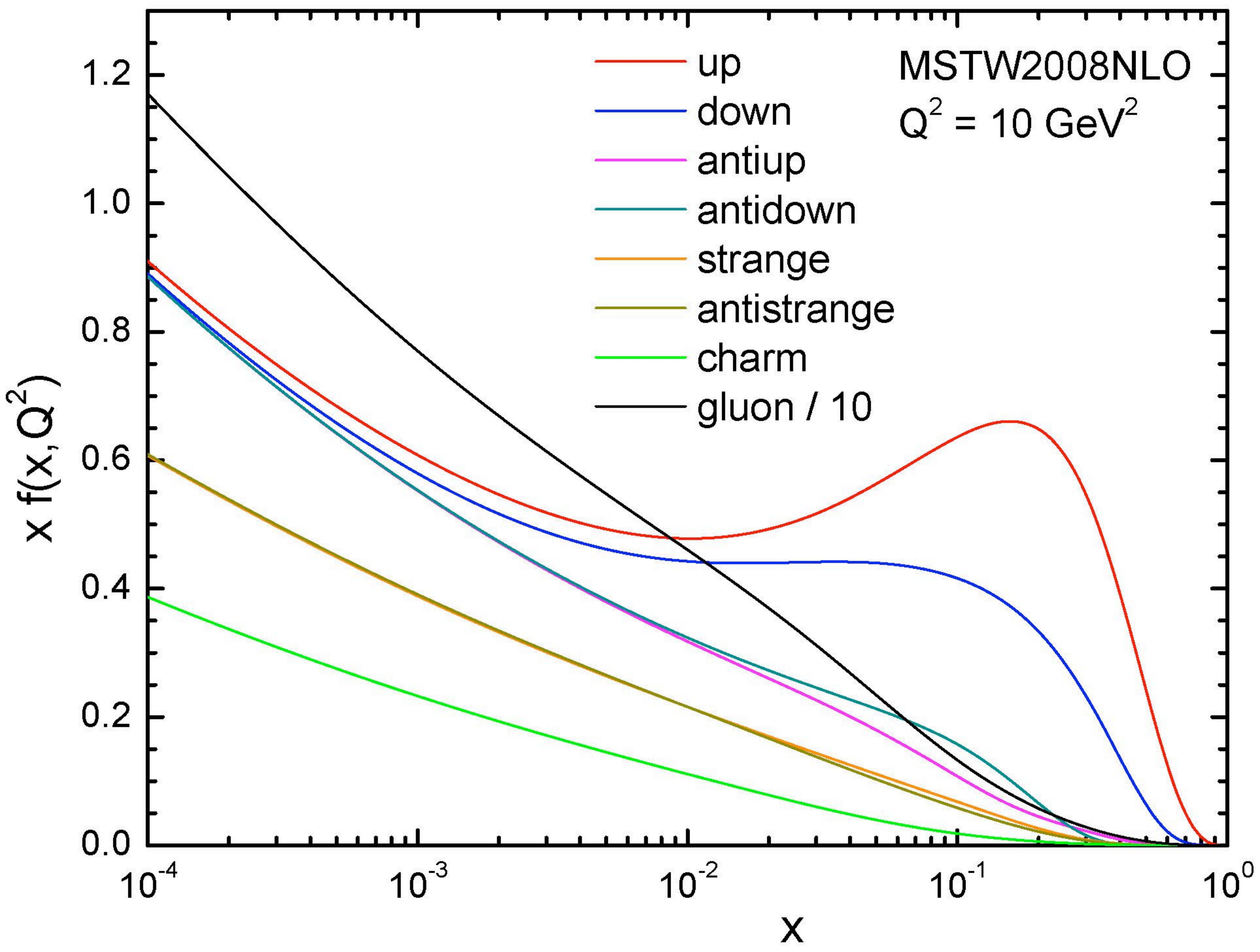
SEA AND VALENCE QUARK DISTRIBUTIONS

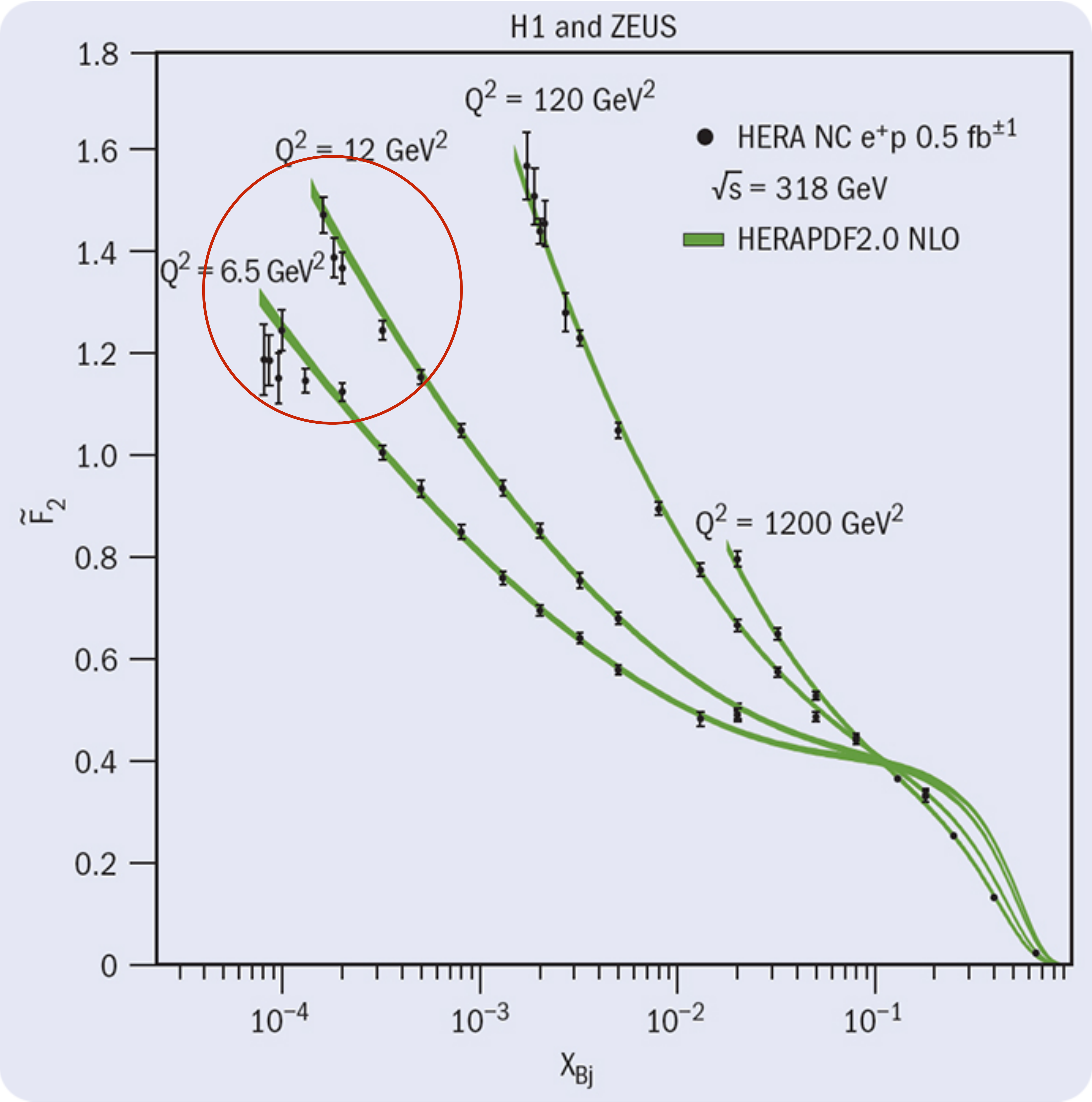
https://www.physics.smu.edu/scalise/cteq/





SEA AND VALENCE QUARK DISTRIBUTIONS

Note that the curves do not exhibit a turning point!



GLUON DISTRIBUTION

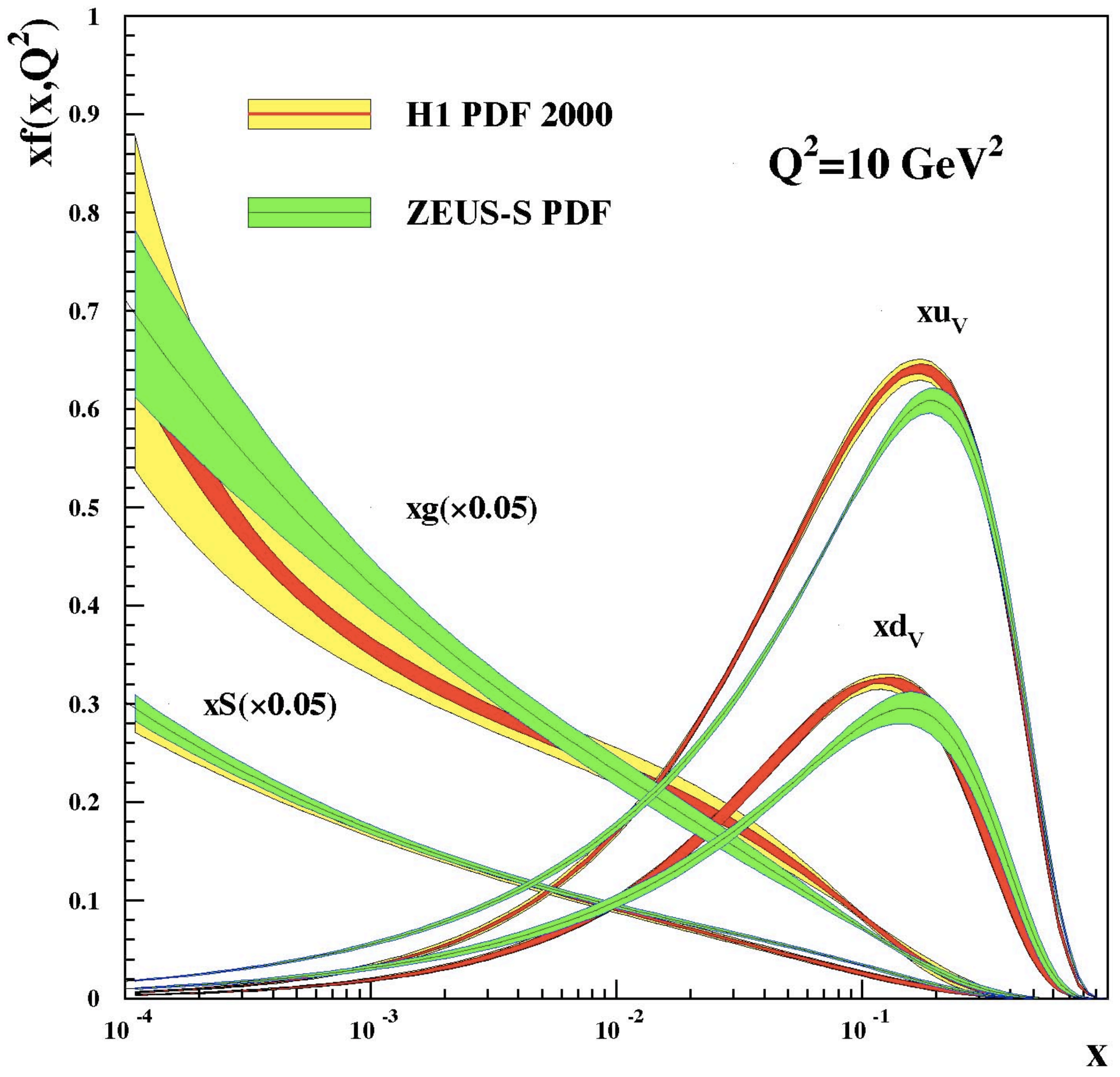
Integrating the quark distributions obtained from DIS and neutrino scattering experiments gives

$$\sum_{i=1}^{N} \int_0^1 x f_i(x) dx \approx 0.5$$

The missing momentum is carried by gluons!

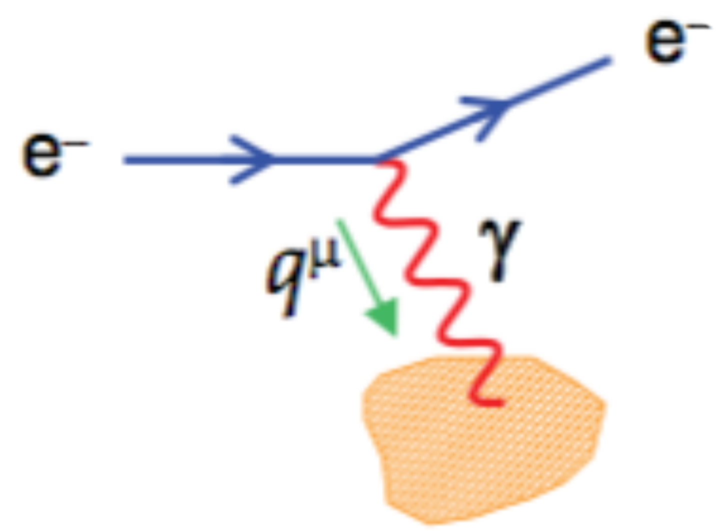
Introducing the gluon distribution function g(x), the correct sum rule reads

$$\sum_{i=1}^{N} \int_0^1 x f_i(x) dx + \int_0^1 x g(x) dx = 1$$

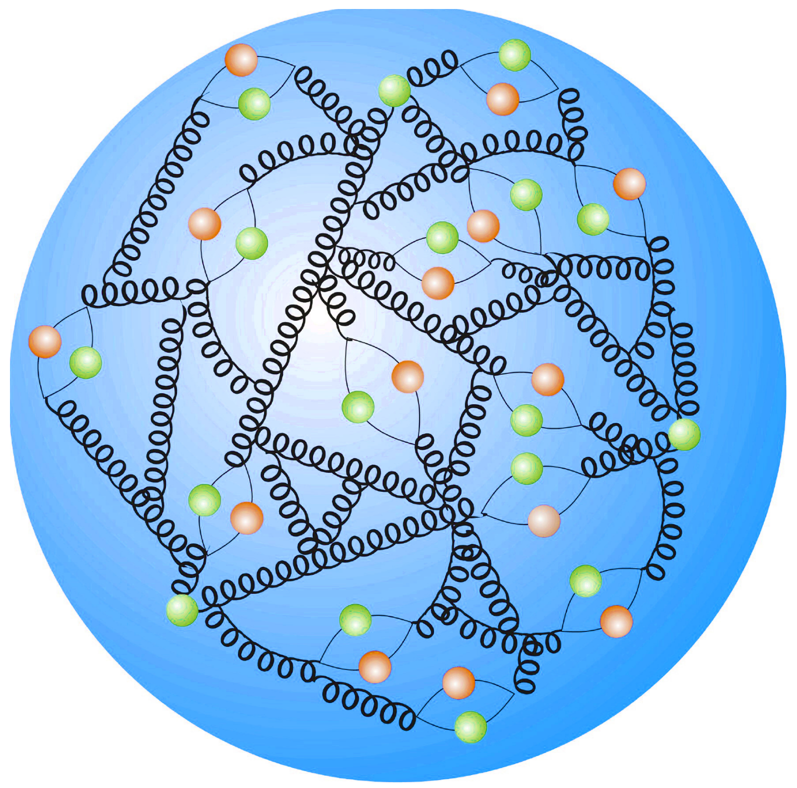


SCALING VIOLATIONS

In the last decades, experiments have probed the proton with virtual photons of ever increasing energy



Non-point like nature of the scattering becomes apparent when $\lambda_\gamma \sim$ size of scattering centre



Scattering from point-like quarks gives rise to **Bjorken scaling**: no q^2 cross section dependence

If quarks were not point-like, at high q^2 (when the wavelength of the virtual photon ~ size of quark) would observe rapid decrease in cross section with increasing q^2

To search for quark sub-structure want to go to highest q^2

HERA @ DESY

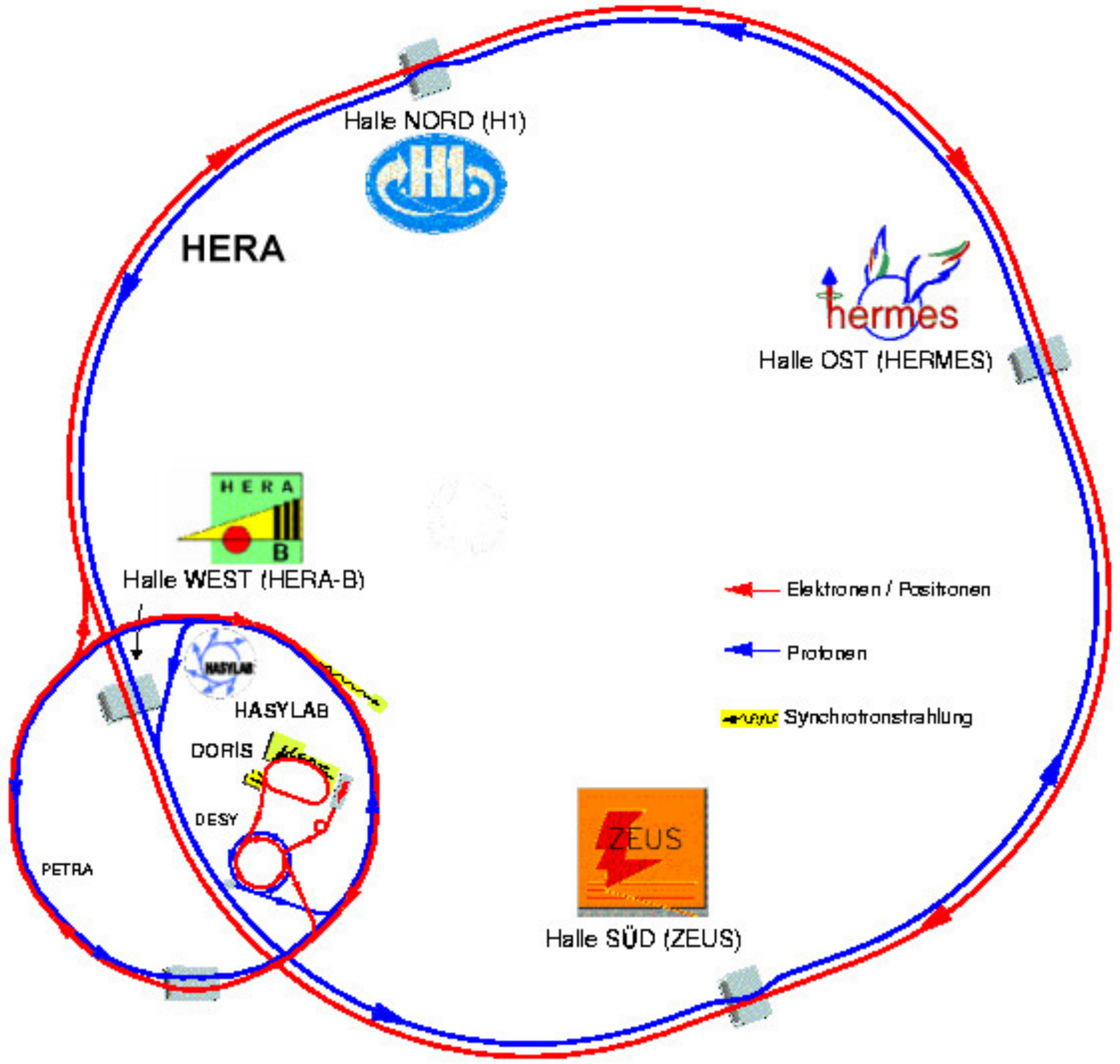
HERA COLLIDER @ DESY

Two major experiments: H1 and ZEUS

Probe proton at very large Q^2 and low x-Bjorken

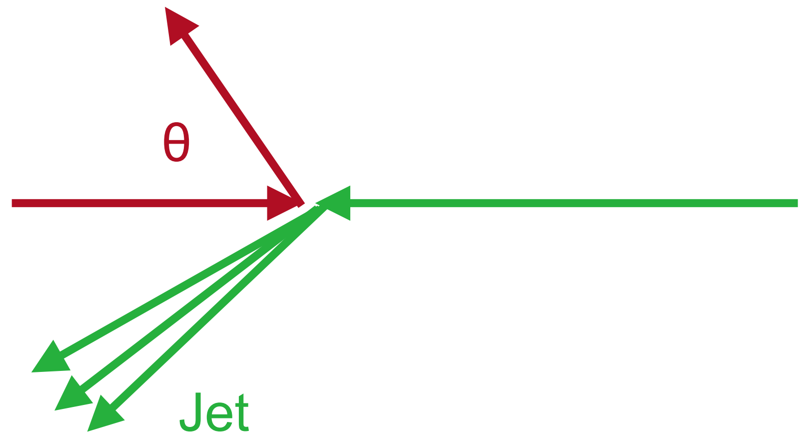
DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany

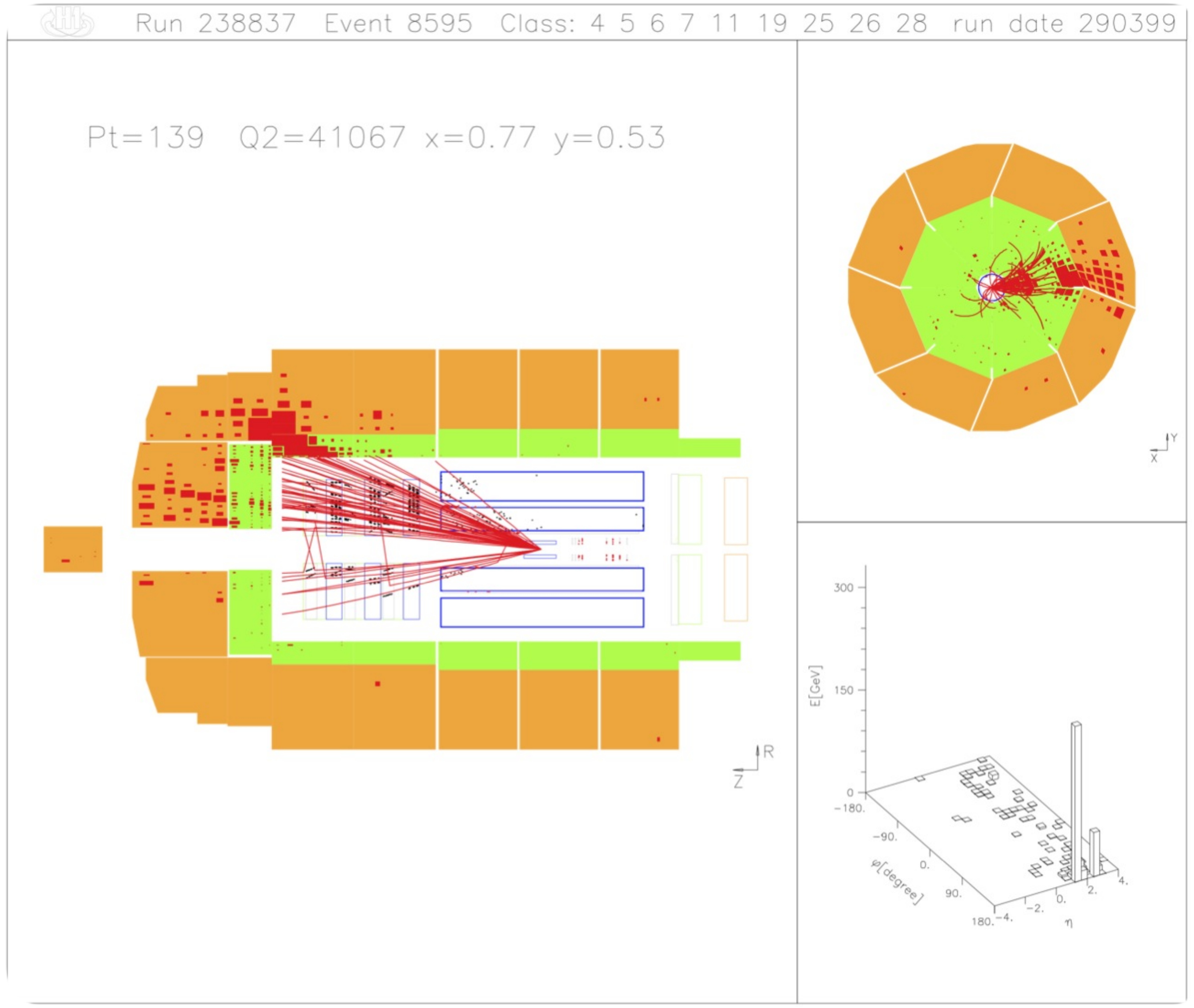
$e^{\pm}$ → 27.5 GeV   920 GeV ← p $\sqrt{s}$~318 GeV



EVENT DISPLAY FROM H1

Event kinematics determined from electron angle and energy





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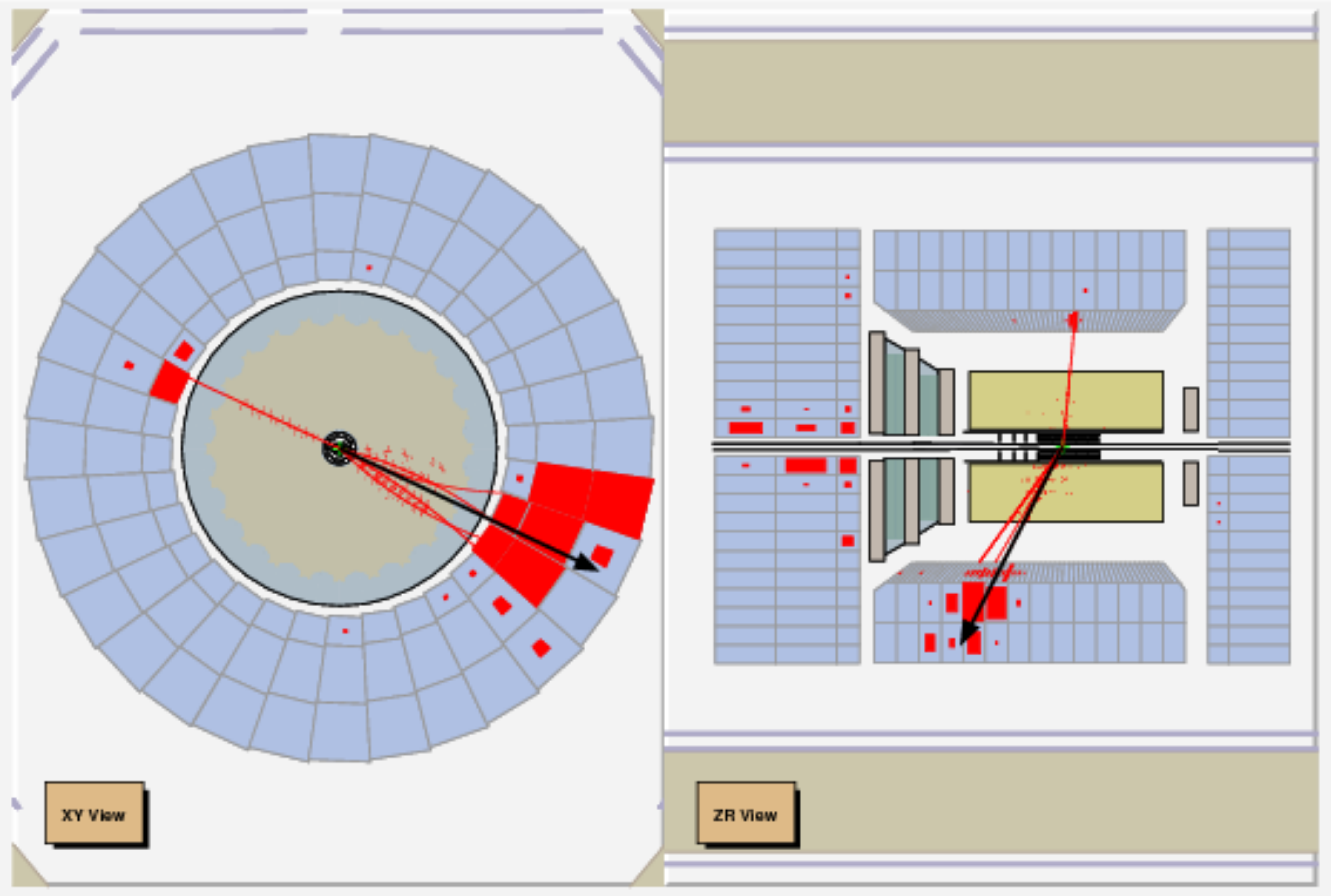
EVENT DISPLAY FROM ZEUS



VIOLATION OF BJORKEN SCALING

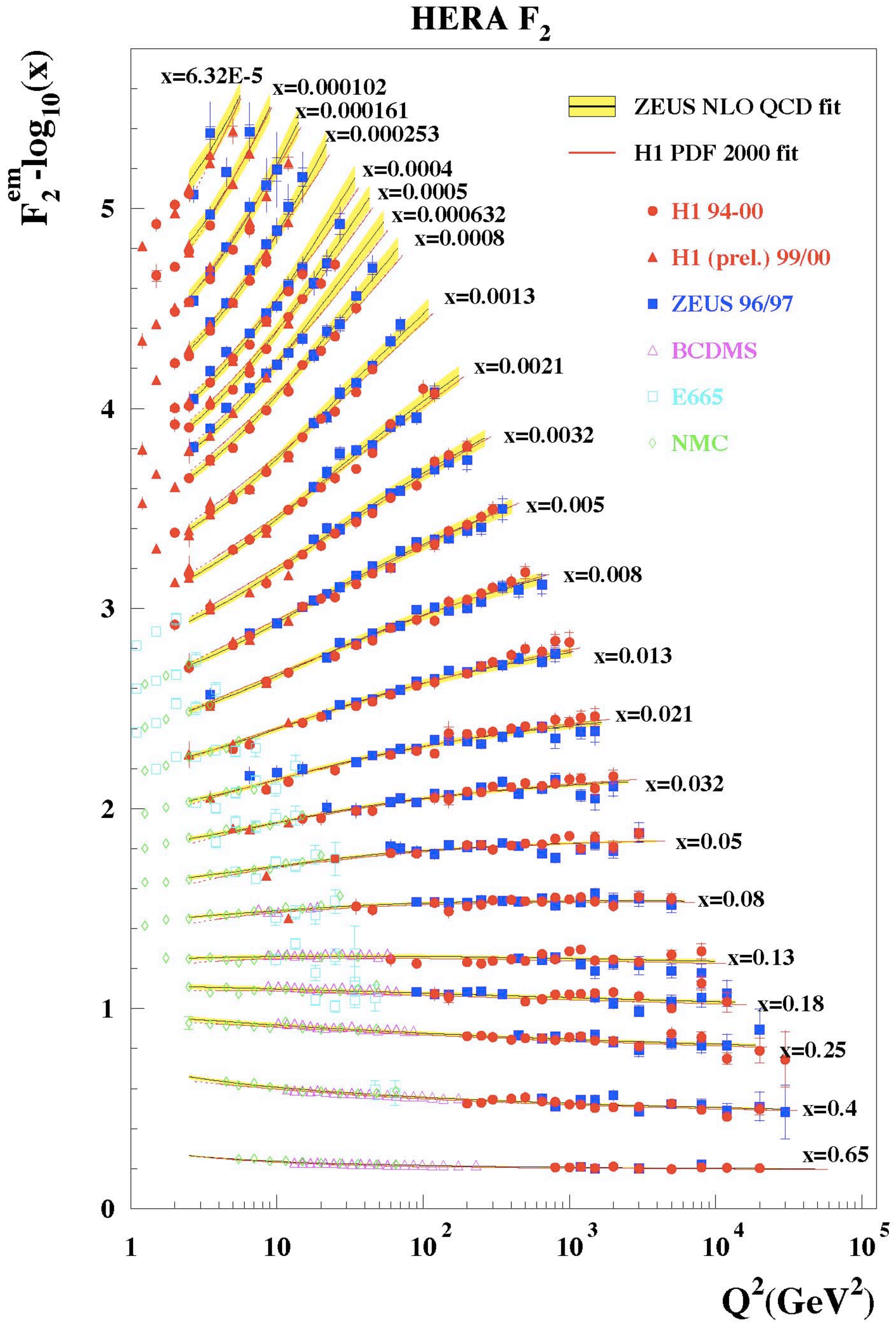
The plot shows the F_2 structure function of the proton as a function of Q^2 for different values of x, the Bjorken scaling variable.

The data include measurements from fixed target experiments as well as the HERA results.

The measurements are impressive as they span four orders of magnitude in both x and Q^2

At high x values the structure function does not vary with Q^2

- Bjorken scaling



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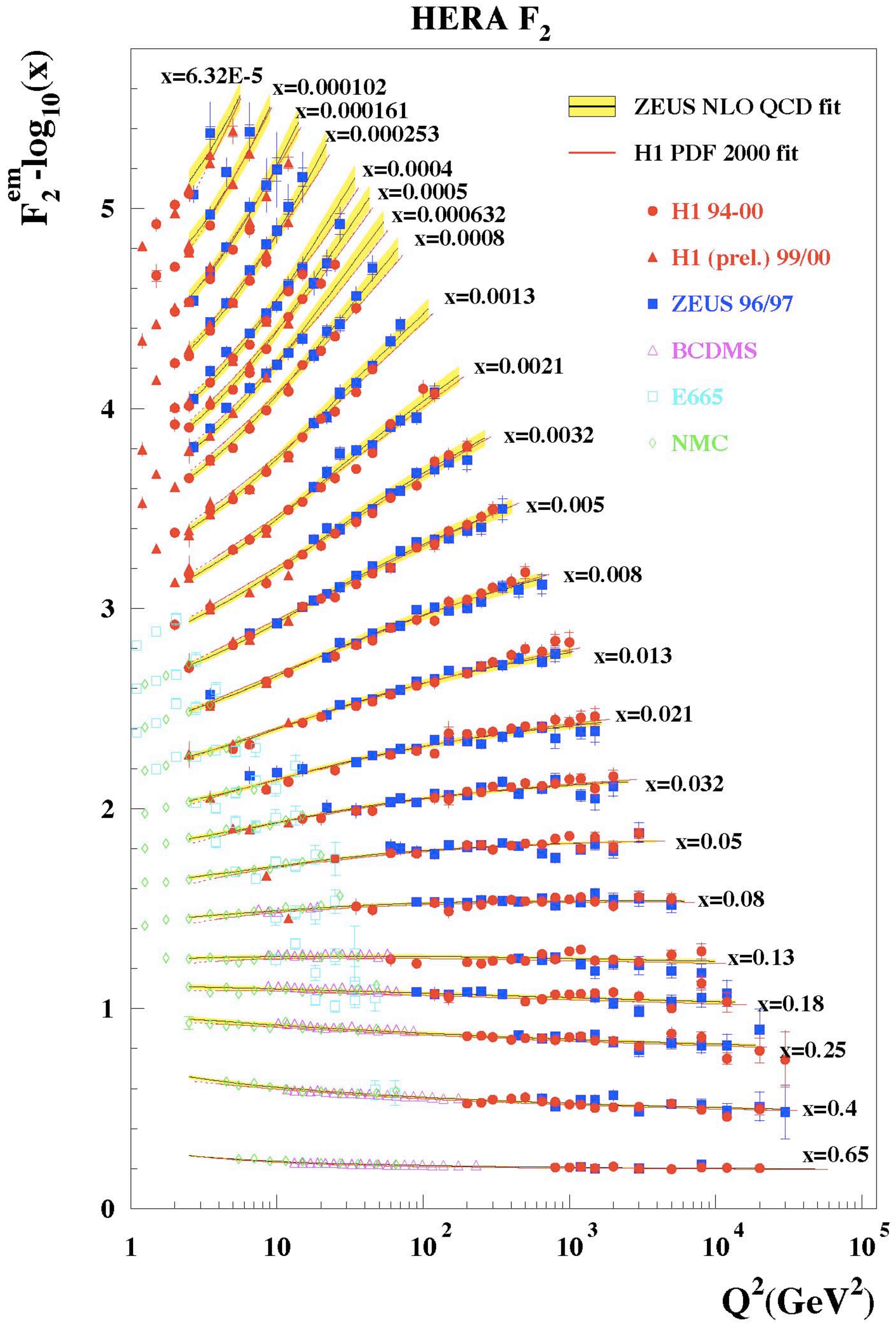
VIOLATION OF BJORKEN SCALING

As x decreases below ~0.1 this scaling fails, or is violated, and the structure function rises with Q^2.

The great success of QCD is that this behaviour is expected and can be calculated (using DGLAP evolution) given the structure function at some low Q^2 value usually around 4 GeV^2.

At high x (x>0.1) the scattering is from a valence quark and is independent of momentum transfer

As smaller x regions are studied the contribution from the gluons and sea quarks increase and these contributions are not constant but increase as you resolve smaller and smaller scales with increasing momentum transfer.



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Thank you for your attention!

LATEX

```
\Big(\frac{d\sigma}{d\Omega}\Big)_{\rm Mott} \rightarrow \frac{a^2}{4E^2\sin^4(\theta/2)}\cos^2(\theta/2)|F(q^2)|^2

F(q^2) = \int d^3 \rho(\vec{r}) e^{i\vec{q}\vec{r}/\hbar}

\frac{d\sigma}{d\Omega} = \Big(\frac{d\sigma}{d\Omega}\Big)_{\rm Mott} |F(q^2)|^2

\Big(\frac{d\sigma}{d\Omega}\Big)_{\rm ep} = \frac{a^2}{4E^2\sin^4(\theta/2)}\frac{E_3}{E_1}\Big(\frac{G_E^2 + bG_E^2}{1 + b}\cos^2(\theta/2) + 2bG_M^2\sin^2(\theta/2)\Big)

\Big(\frac{d\sigma}{d\Omega}\Big)_{\rm ep} = \frac{a^2}{4E^2\sin^4(\theta/2)}\frac{E_3}{E_1}\cos^2(\theta/2) \Big(\frac{G_E^2 + bG_E^2}{1 + b} + 2bG_M^2\tan^2(\theta/2)\Big)

\Big(\frac{d\sigma}{d\Omega}\Big)_{\rm ep} = \Big(\frac{d\sigma}{d\Omega}\Big)_{\rm Mott} \Big(\frac{G_E^2 + bG_E^2}{1 + b} + 2bG_M^2\tan^2(\theta/2)\Big)

\Big(\frac{d\sigma}{d\Omega}\Big)_{\rm ep} / \Big(\frac{d\sigma}{d\Omega}\Big)_{\rm Mott} \approx \Big(1 + 2b\tan^2(\theta/2)\Big)G_M^2(q^2)
```

```
Q^2 = -q^2

x = \frac{Q^2}{2Pq}

M^2 = P_{\mu}P^{\mu} = P^2

s = (P + k)^2

W^2 = (P + q)^2

y = \frac{Pq}{Pk}

\nu = \frac{Pq}{M}
```

```
\frac{d\sigma}{dQ^2} = \frac{4\pi a^2}{Q^4}\Big[\frac{G_E^2 + bG_M^2}{1 + b}\Big(1 - y - \frac{M^2y^2}{Q^2}\Big) + \frac{1}{2}y^2G_M^2\Big]

\frac{d\sigma}{dQ^2} = \frac{4\pi a^2}{Q^4}\Big[\Big(1 - y - \frac{M^2y^2}{Q^2}\Big)f_2(Q^2) + \frac{1}{2}y^2f_1(Q^2)\Big]

\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi a^2}{Q^4}\Big[\Big(1 - y - \frac{M^2y^2}{Q^2}\Big)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\Big]

\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi a^2}{Q^4}\Big[(1 - y)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\Big]
```

```
Q^2=4E_1E_3\sin^2\Big(\frac{\theta}{2}\Big)

x=\frac{Q^2}{2M(E_1 - E_3)}

y=1 - \frac{E_1}{E_3}

F_1^{ep}(x,Q^2) \rightarrow F_1^{ep}(x)

F_2^{ep}(x,Q^2) \rightarrow F_2^{ep}(x)

MW_1(x,Q^2) \rightarrow F_1(x)

\frac{Q^2}{2Mx}W_2(x,Q^2) \rightarrow F_2(x)
```

```
M_{if} = \frac{Qe^2}{q^2}(\overline{u}_3\gamma^{\mu}u_1)g_{\mu\nu}(\overline{u}_4\gamma^{\mu}u_2)

(\overline{u}_3ie\gamma^{\mu}u_1)

(\overline{u}_4ie\gamma^{\nu}u_2)

\langle |M_{if}|^2\rangle = 2Q^2e^4\Big(\frac{s^2 + u^2}{t^2}\Big) = 2Q^2e^4\frac{(P_1P_2)^2 + (P_1P_4)^2}{(P_1P_3)^2}

s = (P_1 + P_2)^2        t = (P_1 - P_3)^2        u = (P_1 - P_4)^2
```


Nikhef

LATEX

$$\sum_{i=1}^N\int xf_i(x)dx = 1$$

$$\frac{d^2\sigma}{d\hat{t}} = \frac{2\pi a^2e_i^2}{\hat{s}^2}\Big(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\Big)$$

$$\Big(\frac{d\sigma}{d\Omega}\Big)_{cm} = \frac{a^2}{2s}\Big(\frac{s^2 + u^2}{t^2}\Big)$$

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi a^2}{Q^4}\Big[1 + (1 - y)^2\Big]\sum_{i=1}^Ne_i^2f_i(x)$$

$$d\sigma = \sum_{i=1}^N d\hat{\sigma}(\hat{s},\hat{t},\hat{u})f_i(x)dx$$

$$y = \frac{Pq}{Pk} = \frac{Q^2}{(s - m_p^2)x}$$

$$\hat{s} = 2xpk = xs$$

$$F_2(x) = \sum_{i=1}^Ne_i^2xf_i(x)$$

$$\hat{t} = (k - k^{'})^2 = t$$

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi a^2}{Q^4}\frac{[1 + (1 - y)^2]}{2x}F_2(x)$$

$$\frac{1}{x}F_2^p(x) = \sum_{i=1}^Ne_i^2f_i^p(x)$$

$$\hat{u} = -2xpk^{'} = xu$$

$$\frac{1}{x}F_2^p(x) = \Big(\frac{2}{3}\Big)^2[u^p(x) + \overline{u}^p(x)] + \Big(\frac{1}{3}\Big)^2[d^p(x) + \overline{d}^p(x)] + \Big(\frac{1}{3}\Big)^2[s^p(x) + \overline{s}^p(x)]$$

$$\frac{1}{x}F_2^n(x) = \Big(\frac{2}{3}\Big)^2[u^n(x) + \overline{u}^n(x)] + \Big(\frac{1}{3}\Big)^2[d^n(x) + \overline{d}^n(x)] + \Big(\frac{1}{3}\Big)^2[s^n(x) + \overline{s}^n(x)]$$

$$u^p(x) = d^n(x) = u(x) \qquad u_{\rm v}(x) = u(x) - \overline{u}(x) \qquad u_{\rm s}(x) = 2\overline{u}(x) \qquad u(x) + \overline{u}(x) = u_{\rm v}(x) + u_{\rm s}(x)$$

$$d^p(x) = u^n(x) = d(x) \qquad d_{\rm v}(x) = d(x) - \overline{d}(x) \qquad d_{\rm s}(x) = 2\overline{d}(x)$$

$$s^p(x) = s^n(x) = s(x) \qquad s_{\rm v}(x) = s(x) - \overline{s}(x) = 0 \qquad s_{\rm s}(x) = 2\overline{s}(x)$$

$$\int_0^1u_{\rm v}(x) dx = 2$$

$$\frac{1}{x}F_2^p(x) = \frac{1}{9}[4u_{\rm v}(x) + d_{\rm v}(x)] + \frac{4}{3}S(x)$$

$$\frac{1}{x}F_2^n(x) = \frac{1}{9}[u_{\rm v}(x) + 4d_{\rm v}(x)] + \frac{4}{3}S(x)$$

$$\int_0^1d_{\rm v}(x) dx = 1$$

$$\sum_{i=1}^N\int_0^1xf_i(x)dx \approx 0.5$$

$$S(x) \equiv u_{\rm s}(x) = \overline{u}_{\rm s}(x) = d_{\rm s}(x) = \overline{d}_{\rm s}(x) = s_{\rm s}(x) = \overline{s}_{\rm s}(x)$$

$$\sum_{i=1}^N\int_0^1xf_i(x)dx + \int_0^1xg(x)dx = 1$$

$$F_2^{ep}(x) = \sum_qxe_q^2[q(x) + \overline{q}(x)]$$

$$F_2^{ep}(x) = x\Big[\frac{4}{9}(u + \overline{u}) + \frac{1}{9}(d + \overline{d})\Big]$$

$$F_2^{en}(x) = x\Big[\frac{1}{9}(u + \overline{u}) + \frac{4}{9}(d + \overline{d})\Big]$$

$$F_2^{eN}(x) = \frac{5}{18}F_2^{\nu N}(x)$$

$$F_2^{ep}(x) - F_2^{en}(x) = \frac{1}{3}x[u_{\rm v}(x) - d_{\rm v}(x)]$$