

# Are you old enough to buy this? 

Zero-Knowledge Age Restriction for GNU Taler

Özgür Kesim
31 May 2024

Code Blau GmbH, FU Berlin, TU Dresden

Prolog

## Sponsors

NGI Taler and NGI Pointer programs of the European Commission

Project
Concrete Contracts in the KMU-innovativ programm

Federal Ministry of Education and Research

## Who am I

Özgür Kesim,

- security consultant for 20+ years,
- PhD candidate at FU Berlin,
- member of GNU Taler dev-team.


## What to expect

## Deliverable

Present a solution to age restriction and its integration in GNU Taler.

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Show concepts from cryptography by example:
Zero-Knowledge protocol, Security Game and Security Proof

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Non-goals
Rigorous introduction into GNU Taler
Demos

## Chapters

Introduction

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The quest for a solution to age restriction

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Discussion \& Conclusion

## Introduction

Age Restriction in E-commerce

## Youth protection

Broad consensus in society about the necessity to protect minors from harmful content.

Also wanted from policy makers:
11. Member states should encourage the use of conditional access tools by content and service providers in relation to content harmful to minors, such as ageverification systems, ...

From the Recommendation Rec (2001) 8 of the Committee of Ministers to member states on self-regulation concerning cyber content of the Council of Europe.

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Verification of minimum age requirements in e-commerce.

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For age-restriction, the lowest level of authority is:

Parents, guardians and caretakers

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2. maintains the anonymity of buyers,
3. maintains unlinkability of transactions,
4. aligns with the principle of subsidiarity,
5. is practical and efficient.

## Digital cash withdrawal

```
Exchange
https://exchange-age.taler.ar/
Details
Withdraw
Transaction fees
5.0 ARS
-0.7 ARS
Total
4.3 ARS
```


## Age restriction

```
Not restricted \checkmark
```

Not restricted \checkmark
Not restricted
under }
under 10
under 12
under 14
under 16
under 18

```

\section*{The quest for a solution to age restriction}

A journey through cryptic territory

\section*{Basic assumption and ideas}

Assumption: Bank accounts are under control of adults/guardians.

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6. GOTO 2.

\section*{Helpful figure - Commit}


\section*{Helpful figure - Attest and Verify}


\section*{Helpful figure - Derive and Compare}


\section*{Helpful figure}

\section*{Commit:}

\section*{Attest and Verify: Derive and Compare:}



\title{
Specification of the Function Signatures
}

Searching for functions
Commit
Attest
Verify
Derive
Compare

\section*{Specification of the Function Signatures}

Searching for functions with the following signatures
Commit :
\[
(\mathrm{a}, \omega) \mapsto(\mathrm{Q}, \mathrm{P})
\]
\(\mathbb{N}_{M} \times \Omega \rightarrow \mathbb{O} \times \mathbb{P}\),

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Mnemonics:
\(\mathbb{O}=\) commitments, \(\mathrm{Q}=Q\)-mitment (commitment), \(\mathbb{P}=\mathbb{P r o o f s}\),

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Attest :
\((\mathrm{m}, \mathrm{P}) \mapsto \mathrm{T}\)
\(\mathbb{N}_{M} \times \mathbb{P} \rightarrow \mathbb{T} \cup\{\perp\}\),
Verify
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(\mathrm{~m}, \mathrm{Q}, \mathrm{~T}) & \mapsto b
\end{aligned}
\]
\[
\mathbb{N}_{\mathrm{M}} \times \mathbb{O} \times \mathbb{T} \rightarrow \mathbb{Z}_{2}
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Derive
Compare

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Derive :
\((\mathrm{Q}, \mathrm{P}, \omega) \mapsto\left(\mathrm{Q}^{\prime}, \mathrm{P}^{\prime}, \beta\right)\)
\(\mathbb{N}_{\mathrm{M}} \times \Omega \rightarrow \mathbb{O} \times \mathbb{P}\),
\(\mathbb{N}_{\mathrm{M}} \times \mathbb{P} \rightarrow \mathbb{T} \cup\{\perp\}\),
\(\mathbb{N}_{\mathrm{M}} \times \mathbb{O} \times \mathbb{T} \rightarrow \mathbb{Z}_{2}\),
\(\mathbb{O} \times \mathbb{P} \times \Omega \rightarrow \mathbb{O} \times \mathbb{P} \times \mathbb{B}\),

Compare

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Compare :
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Verify :
\(\mathbb{N}_{\mathrm{M}} \times \mathbb{O} \times \mathbb{T} \rightarrow \mathbb{Z}_{2}\),
\(\mathbb{O} \times \mathbb{P} \times \Omega \rightarrow \mathbb{O} \times \mathbb{P} \times \mathbb{B}\),
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\(\mathbb{O} \times \mathbb{P} \times \Omega \rightarrow \mathbb{O} \times \mathbb{P} \times \mathbb{B}\),
\(\mathbb{O} \times \mathbb{O} \times \mathbb{B} \rightarrow \mathbb{Z}_{2}\),
with \(\Omega, \mathbb{P}, \mathbb{O}, \mathbb{T}, \mathbb{B}\) sufficiently large sets.

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with \(\Omega, \mathbb{P}, \mathbb{O}, \mathbb{T}, \mathbb{B}\) sufficiently large sets.
We will define basic and security requirements later.
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Naïve scheme


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\(\Longrightarrow\) Unlinkability broken

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Given Derive() and Compare(), define the cut\&choose protocol DeriveCompare \({ }_{\kappa}\) as follows (sketch):

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& \text { Given Derive() and Compare(), define the cut\&choose protocol } \\
& \text { DeriveCompare }{ }_{\kappa} \text { as follows (sketch): } \\
& \qquad \begin{array}{r}
\text { Let } \kappa \in \mathbb{N}(\text { say: } \kappa=3) \\
\mathcal{C}: \quad \\
\text { 1. generates }\left(Q_{1}, \ldots, Q_{\kappa}\right) \text { and }\left(\beta_{1}, \ldots, \beta_{\kappa}\right) \text { from } \\
Q_{0} \text { by calling } \kappa \text { times Derive }\left(Q_{0}, P_{0}, \omega_{i}\right)
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\(\mathcal{C}: \quad\) 5. reveals \(h_{\gamma}:=H\left(\mathrm{Q}_{\gamma}, \beta_{\gamma}\right)\) and all \(\left(\mathrm{Q}_{i}, \beta_{i}\right)\), except \(\left(Q_{\gamma}, \beta_{\gamma}\right)\)

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7. evaluates Compare \(\left(Q_{0}, Q_{i}, \beta_{i}\right)\) for all \(i \neq \gamma\).

\section*{Achieving Unlinkability}

Given Derive() and Compare(), define the cut\&choose protocol DeriveCompare \({ }_{\kappa}\) as follows (sketch):
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\text { Let } \kappa \in \mathbb{N}(\text { say : } \kappa=3)
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\(\mathcal{C}\) : 1. generates \(\left(Q_{1}, \ldots, Q_{\kappa}\right)\) and \(\left(\beta_{1}, \ldots, \beta_{\kappa}\right)\) from \(Q_{0}\) by calling \(\kappa\) times Derive \(\left(Q_{0}, P_{0}, \omega_{i}\right)\)
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7. evaluates Compare \(\left(Q_{0}, Q_{i}, \beta_{i}\right)\) for all \(i \neq \gamma\).

If all steps succeed, \(Q_{\gamma}\) is the new commitment.

\section*{Achieving Unlinkability}

With DeriveCompare \({ }_{\kappa}\)
- \(\mathcal{E}\) learns nothing about \(\mathrm{Q}_{\gamma}\) or \(H\left(\mathrm{Q}_{\gamma}\right)\),
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Notes:
- similar to the cut\&choose refresh protocol in GNU Taler
- still need to define Derive() and Compare().

Refined scheme

\section*{\(\mathcal{E}\)}
\(\mathcal{G}\)



Refined scheme
\(\mathcal{E}\)
\(\mathcal{G}\)

\(\mathcal{M}\)

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We need more requirements.

\section*{Requirements}

\section*{Basic Requirements}

Candidate functions
(Commit, Attest, Verify, Derive, Compare) must meet basic requirements:
- Existence of attestations
- Efficacy of attestations
- Derivability of commitments and attestations

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More details in the published paper and Appendix.

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Meeting the security requirements means that adversaries can win those games only with negligible advantage.

Adversaries are arbitrary polynomial-time algorithms, acting on all relevant input.

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\section*{Simplified Example}

Game \(G_{\mathcal{A}}^{\mathrm{FA}}\) : Forging an attest

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& \text { 2. }(Q, P) \leftarrow \operatorname{Commit}(a, \omega) \\
& \text { 3. }(\mathrm{m}, \mathrm{~T}) \leftarrow \mathcal{A}(\mathrm{a}, \mathrm{Q}, \mathrm{P}) \\
& \text { 4. Return } 0 \text { if } \mathrm{m} \leq \mathrm{a} \\
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Adversary \(\mathcal{A}\) wins the game, if \(G_{\mathcal{A}}^{\text {FA }}\) returns 1 .
Requirement: Unforgeability of minimum age
\[
\underset{\mathcal{A} \in \mathfrak{A}\left(\mathbb{N}_{\mathrm{M}} \times \mathbb{O} \times \mathbb{P} \rightarrow \mathbb{N}_{\mathrm{M}} \times \mathbb{T}\right)}{\forall}: \operatorname{Pr}\left[G_{\mathcal{A}}^{\mathrm{FA}}=1\right] \leq \epsilon
\]

\section*{Our task}

Finding functions

\section*{(Commit, Attest, Verify, Derive, Compare)}
that meet the basic and security requirements.

A solution to our quest

\section*{Instantiation with ECDSA}

We propose a solution based on ECDSA.
Think: One key-pair per age group.

\section*{Definition of Commit with ECDSA}

To Commit to age group \(a \in\{1, \ldots, \mathrm{M}\}\)
1. Guardian generates ECDSA-keypairs, one per age group:
\[
\left\langle\left(q_{1}, p_{1}\right), \ldots,\left(q_{\mathrm{M}}, p_{\mathrm{M}}\right)\right\rangle
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2. Guardian then drops all private keys \(p_{i}\) for \(i>a\) :
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\left\langle\left(q_{1}, p_{1}\right), \ldots,\left(q_{\mathrm{a}}, p_{\mathrm{a}}\right),\left(q_{\mathrm{a}+1}, \perp\right), \ldots,\left(q_{\mathrm{M}}, \perp\right)\right\rangle
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Proof: \(\overrightarrow{\mathrm{P}}_{\mathrm{a}}:=\left(p_{1}, \ldots, p_{\mathrm{a}}, \perp, \ldots, \perp\right)\)

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Proof: \(\overrightarrow{\mathrm{P}}_{\mathrm{a}}:=\left(p_{1}, \ldots, p_{\mathrm{a}}, \perp, \ldots, \perp\right)\)
3. Guardian gives child \(\left\langle\overrightarrow{\mathrm{Q}}, \overrightarrow{\mathrm{P}}_{\mathrm{a}}\right\rangle\)

\section*{Attest and Verify with ECDSA}

\section*{Child has}

\begin{tabular}{|l|l|}
\multicolumn{4}{c}{ key ID's } \\
age \\
groups & \begin{tabular}{ll}
\(1:\) & b5bb9d \\
\(2:\) & \(801 f a \theta\) \\
\(3:\) & \(19 d 8 d e\) \\
\(4:\) & \(52 f 23 c\) \\
\hline
\end{tabular}
\end{tabular}

- ordered public-keys \(\vec{Q}=\left(q_{1}, \ldots \ldots \ldots, q_{\mathrm{M}}\right)\),
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To Attest a minimum age (group) \(\mathrm{m} \leq \mathrm{a}\) :
Sign a message with ECDSA using private key \(p_{\mathrm{m}}\).
The signature \(\sigma_{\mathrm{m}}\) is the attestation.


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Contract

Age group: 3
Attestation:
\begin{tabular}{l} 
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\hline
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- Signature \(\sigma_{\mathrm{m}}\)

To Verify a minimum age (group) m:
Verify the ECDSA-Signature \(\sigma_{\mathrm{m}}\) with public key \(q_{\mathrm{m}}\).

\section*{Reminder: Derive and Compare}


\section*{Derive and Compare with ECDSA}

Child has \(\overrightarrow{\mathrm{Q}}=\left(q_{1}, \ldots, q_{\mathrm{M}}\right)\) and \(\overrightarrow{\mathrm{P}}=\left(p_{1}, \ldots, p_{\mathrm{a}}, \perp, \ldots, \perp\right)\).

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To Derive new \(\overrightarrow{\mathrm{Q}}^{\prime}\) and \(\overrightarrow{\mathrm{P}}^{\prime}\) : Choose random \(\beta \in \mathbb{Z}_{\mathrm{g}}\) and calculate
\[
\begin{aligned}
\vec{Q}^{\prime}=\left(q_{1}^{\prime}, \ldots \ldots \ldots, q_{\mathrm{M}}^{\prime}\right) & :=\left(\beta * q_{1}, \ldots \ldots, \beta * q_{\mathrm{M}}\right), \\
\overrightarrow{\mathrm{P}}^{\prime}=\left(p_{1}^{\prime}, \ldots, p_{\mathrm{a}}^{\prime}, \perp, \ldots, \perp\right) & :=\left(\beta p_{1}, \ldots, \beta p_{\mathrm{a}}, \perp, \ldots, \perp\right)
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Note:
- \(\beta * q_{i}\) is scalar multiplication on the elliptic curve.
- \(p_{i}^{\prime} * G=\left(\beta p_{i}\right) * G=\beta *\left(p_{i} * G\right)=\beta * q_{i}=q_{i}^{\prime}\)
\(\Longrightarrow \quad p_{i}^{\prime}\) actually is private key to \(q_{i}^{\prime}\)

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Exchange gets \(\overrightarrow{\mathrm{Q}}=\left(q_{1}, \ldots, q_{\mathrm{M}}\right), \overrightarrow{\mathrm{Q}}^{\prime}=\left(q_{1}^{\prime}, \ldots, q_{\mathrm{M}}^{\prime}\right)\) and \(\beta\)
To Compare, calculate: \(\left(\beta * q_{1}, \ldots, \beta * q_{\mathrm{M}}\right) \stackrel{?}{=}\left(q_{1}^{\prime}, \ldots, q_{\mathrm{M}}^{\prime}\right)\)

\section*{Instantiation with ECDSA}

Functions (Commit, Attest, Verify, Derive, Compare) as defined in the instantiation with ECDSA
- meet the basic requirements,
- also meet all security requirements.

Security proofs by reduction, details are in the paper.

\section*{Example: Proof of Unforgeability}

\section*{Proof by reduction:}

Game \(G_{\mathcal{A}}^{\text {FA }}\) : Forging an attest
1. \((\mathrm{a}, \omega) \stackrel{\$}{\stackrel{N}{M-1}} \times \Omega\)
2. \((\mathrm{Q}, \mathrm{P}) \leftarrow \operatorname{Commit}(\mathrm{a}, \omega)\)
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4. Return 0 if \(m \leq a\)
5. Return Verify (m, Q, T)

Requirement:
\(\underset{\mathcal{A}}{\forall}: \operatorname{Pr}\left[G_{\mathcal{A}}^{\mathrm{FA}}=1\right] \leq \epsilon\)

\section*{Example: Proof of Unforgeability}

\section*{Proof by reduction:}
1. Adversary wins if \(1=\operatorname{Verify}(m, Q, T)\).

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\section*{Example: Proof of Unforgeability}

\section*{Proof by reduction:}

Game \(G_{\mathcal{A}}^{\text {FA }}\) : Forging an attest
1. \((\mathrm{a}, \omega) \stackrel{\$}{\leftarrow} \mathbb{N}_{\mathrm{M}-1} \times \Omega\)
2. \((\mathrm{Q}, \mathrm{P}) \leftarrow \operatorname{Commit}(\mathrm{a}, \omega)\)
3. \((\mathrm{m}, \mathrm{T}) \leftarrow \mathcal{A}(\mathrm{a}, \mathrm{Q}, \mathrm{P})\)
4. Return 0 if \(m \leq a\)
5. Return Verify (m, Q, T)

\section*{Requirement:}
\(\forall_{\mathcal{A}}: \operatorname{Pr}\left[G_{\mathcal{A}}^{\mathrm{FA}}=1\right] \leq \epsilon\)
1. Adversary wins if \(1=\operatorname{Verify}(\mathrm{m}, \mathrm{Q}, \mathrm{T})\).
2. That means: \(\sigma\) was a valid ECDSA-signature, validated with \(q_{m}\).

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3. But adversary does not have the private key \(p_{m}\) to \(q_{m}\).
\(\Longrightarrow\) So winning this game would require to existentially forge the signature, which is negligible.

Integration with GNU Taler

\section*{GNU Taler}
https://www.taler.net

- Protocol suite for online payment services
- Based on Chaum's blind signatures
- Taxable, efficient, free software
- Allows for change and refund
- Privacy preserving: anonymous and unlinkable payments

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- Protocol suite for online payment services
- Based on Chaum's blind signatures
- Taxable, efficient, free software
- Allows for change and refund
- Privacy preserving: anonymous and unlinkable payments
- Coins are public-/private key-pairs \(\left(C_{p}, C_{s}\right)\).
- Exchange blindly signs \(H\left(C_{p}\right)\) with denomination key \(d_{p}\) :
\[
\beta\left(\sigma_{p}\right)=\operatorname{BlindSign}\left(\beta\left(H\left(C_{p}\right)\right), d_{p}\right)
\]
- Verification:
\[
\begin{gathered}
1 \stackrel{?}{=} \operatorname{Sig} \operatorname{Check}\left(H\left(C_{p}\right), D_{p}, \sigma_{p}\right) \\
\left(D_{p}=\text { public key of denomination and } \sigma_{p}=\text { signature }\right)
\end{gathered}
\]

\section*{Integration with GNU Taler}

\section*{Binding age restriction to coins}

To bind an age commitment Q to a coin \(C_{p}\), instead of blindly signing \(H\left(C_{p}\right)\)
\[
\beta\left(\sigma_{p}\right)=\operatorname{BlindSign}\left(\beta\left(H\left(C_{p}\right)\right), d_{p}\right)
\]
\(\mathcal{E}\) now blindly signs \(H\left(C_{p} \| H(Q)\right)\)
\[
\beta\left(\sigma_{p}\right)=\operatorname{BlindSign}\left(\beta\left(H\left(C_{p} \| H(Q)\right)\right), d_{p}\right)
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Therefore, verfication of a coin now requires \(H(Q)\), too:
\[
1 \stackrel{?}{=} \operatorname{Sig} \operatorname{Check}\left(H\left(C_{p} \| H(Q)\right), D_{p}, \sigma_{p}\right)
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\section*{Integration with GNU Taler}

Integrated schemes


\section*{Age restriction in the wallet}

\section*{Digital cash withdrawal}
```

Exchange
https://exchange-age.taler.ar/
Details
Withdraw
Transaction fees
5.0 ARS
-0.7 ARS
Total
4.3 ARS

```

\section*{Age restriction}
```

Not restricted \vee

```
Not restricted \vee
Not restricted
under }
under 10
under 12
under 14
under 16
under 18
```


## Discussion \& Conclusion

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Technical aspects and challenges

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- Know-Your-Customer (KYC) provides age information
- Parents can set age on a long-term wallet of a child
- cut\&choose protocol age-withdraw implemented


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Legal aspects and applicability

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- There will be limits where the scheme is considered acceptable.
- Our scheme offers an alternative to identity management systems (IMS), where applicable


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- GNU Taler defines the role of an Auditor:
- a seperate entity to supervise the operation of the exchange.


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Existing solutions are

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Our scheme offers an option that

- aligns with subsidiarity
- preserves privacy
- is efficient
- and an alternative to IMS


## Thank you!

 Questions?oec-taler@kesim.org
@oec@mathstodon.xyz

Interested in GNU Taler?
Intro: https://taler.net
Learn: https://docs.taler.net
Develop: https://git.taler.net, https://bugs.taler.net
Connect: https://ich.taler.net
NGI Taler: https://ngi.taler.net

## Taler Overview



## Basic Requirements - Details

back to Basic Requirements

## Existence of attestations

$$
\underset{\substack{a \in \mathbb{N} M \\
\omega \in \Omega}}{\forall}: \operatorname{Commit}(a, \omega)=:(Q, P) \Longrightarrow \operatorname{Attest}(m, Q, P)=\left\{\begin{array}{l}
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## Derivability of commitments and attestations ...

More details in the published paper.

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$\Longrightarrow \quad \sigma$ is a valid RSA signature to message $m$.

## Instantiation with Edx25519

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We also formally define another signature scheme, Edx25519:

- based on EdDSA (Bernstein et al.),
- generates compatible signatures,
- allows for key derivation from both, private and public keys, independently and
- is already in use in GNUnet.

Current implementation of age restriction in GNU Taler uses Edx25519.

## Related Work

- Current privacy-perserving systems all based on attribute-based credentials (Koning et al., Schanzenbach et al., Camenisch et al., Au et al.)
- Attribute-based approach lacks support:
- Complex for consumers and retailers
- Requires trusted additional authority
- Other approaches tie age-restriction to ability to pay ("debit cards for kids")
- Advantage: mandatory to payment process
- Not privacy friendly

