Extracting Interfaces from Software Components via Model Learning

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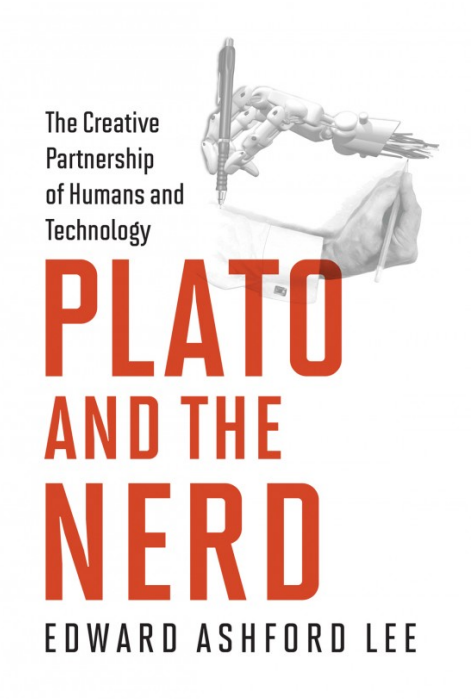
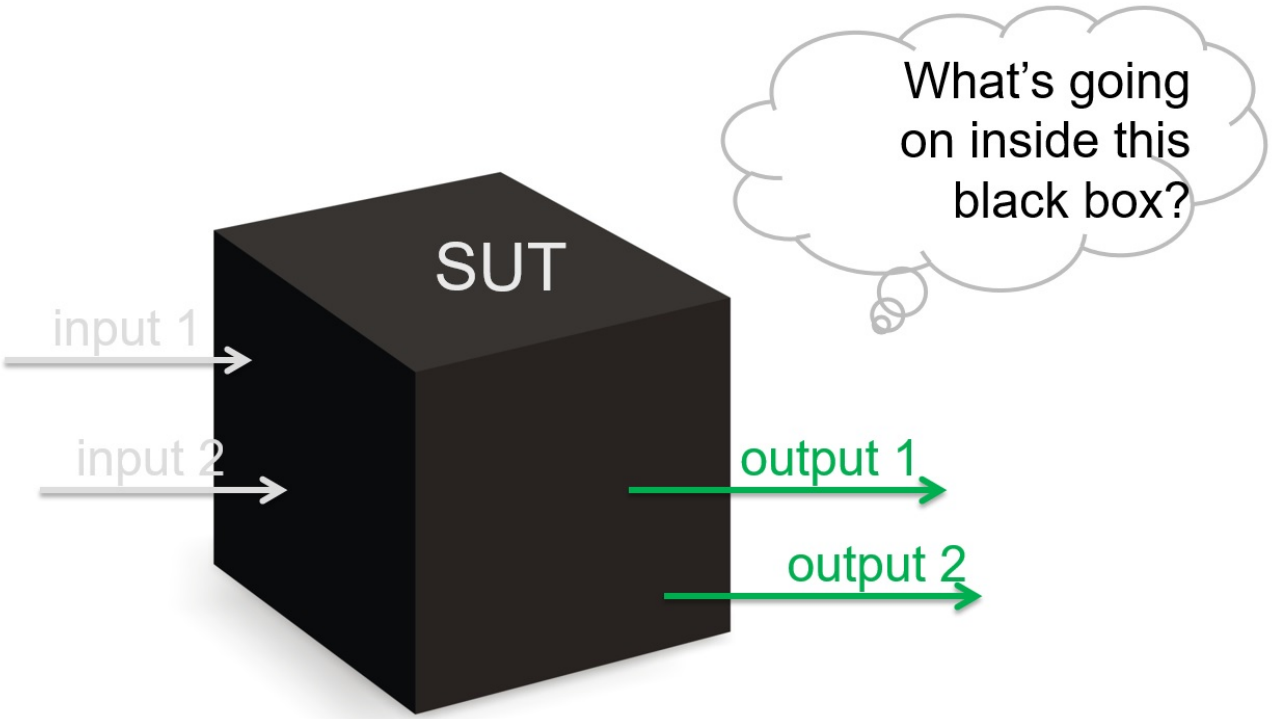
Outline

Plato and the Nerd

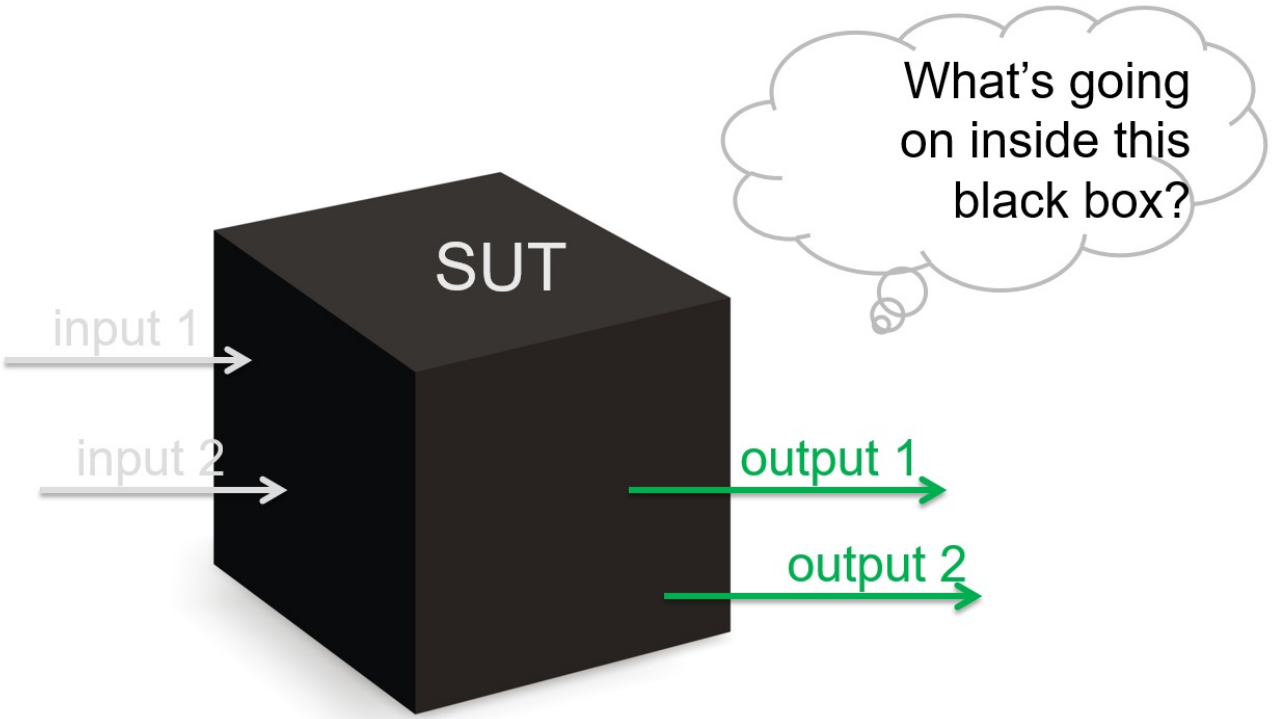


- Model: any description of a system that is not the thing-in-itself.
- Engineering perspective:
  "Can we build a system whose behavior matches that of a given model?"
- Science perspective:
  "Can we build a model whose behavior matches that of a given system?"
- This talk:
  By properly combining both perspectives (in an FSM setting) we can build better software.

Research Question



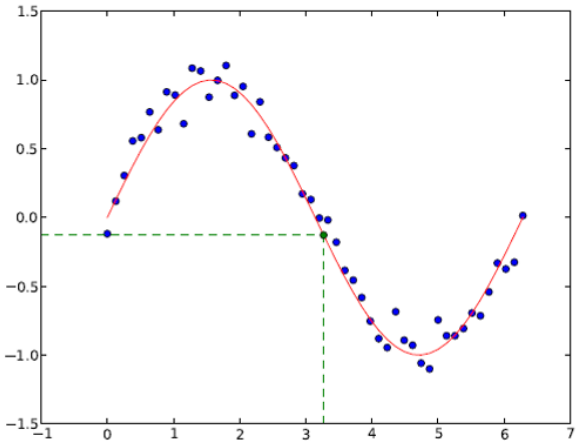
Research Question

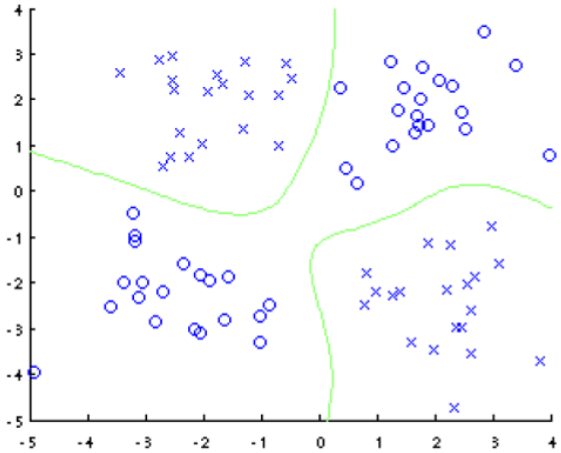


Here we assume SUT behaves deterministically and can be reset.

Machine Learning in General

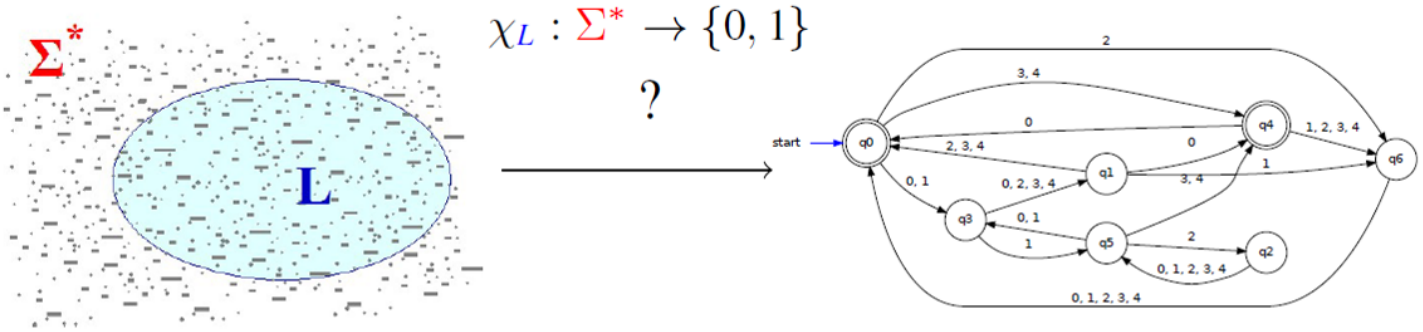
- Given a sample $M = \{(x, y) \mid x \in X, y \in Y\}$
- Find $f : X \to Y$ such that $f(x) = y, \forall (x, y) \in M$
- Predict $f(x)$ for all $x \in X$





Learning Regular Languages

Let Σ be an alphabet and let $L \subseteq \Sigma^*$ be a regular language (*the target language*)



- Edward F Moore, *Gedanken-experiments on sequential machines*, 1956
- E. Mark Gold, *System Identification via State Characterization*, 1972
- Dana Angluin, *Learning regular sets from queries and counterexamples*, 1987

Regular Languages and Congruences

Definition

The equivalence relation $\sim_L$ on Σ^* induced by a language $L \subseteq \Sigma^*$:

$$u \sim_L v \quad \textit{iff} \quad \forall w \in \Sigma^* : u \cdot w \in L \Leftrightarrow v \cdot w \in L$$

This relation is a right-congruence with respect to concatenation:

$$\forall u, v, w \in \Sigma^* : u \sim_L v \Rightarrow u \cdot w \sim_L v \cdot w$$

Theorem (Myhill-Nerode, 1958)

Language L is regular iff $\sim_L$ has finitely equivalence classes.

Visualisation

Consider the regular language $a(a \mid b)^* b$

	_	a	b	aa	ab	ba	bb	aaa	aab	aba
aa										
ba										

Visualisation

Consider the regular language $a(a \mid b)^* b$

	_	a	b	aa	ab	ba	bb	aaa	aab	aba
aa	0	0	1	0	1	0	1	0	1	0
ba	0	0	0	0	0	0	0	0	0	0

Hankel Matrix

	_	a	b	aa	ab	ba	bb	aaa	aab	aba
_	0	0	0	0	1	0	0	0	1	0
a	0	0	1	0	1	0	1	0	1	0
b	0	0	0	0	0	0	0	0	0	0
aa	0	0	1	0	1	0	1	0	1	0
ab	1	0	1	0	1	0	1	0	1	0
ba	0	0	0	0	0	0	0	0	0	0
bb	0	0	0	0	0	0	0	0	0	0
aaa	0	0	1	0	1	0	1	0	1	0
aab	1	0	1	0	1	0	1	0	1	0
aba	0	0	1	0	1	0	1	0	1	0

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aa	0	0	1	0	1	0	1	0	1	0
ab	1	0	1	0	1	0	1	0	1	0
ba	0	0	0	0	0	0	0	0	0	0
bb	0	0	0	0	0	0	0	0	0	0
aaa	0	0	1	0	1	0	1	0	1	0
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ab	1	0	1	0	1	0	1	0	1	0
ba	0	0	0	0	0	0	0	0	0	0
bb	0	0	0	0	0	0	0	0	0	0
aaa	0	0	1	0	1	0	1	0	1	0
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aa	0	0	1	0	1	0	1	0	1	0
ab	1	0	1	0	1	0	1	0	1	0
ba	0	0	0	0	0	0	0	0	0	0
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Hankel Matrix

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aa	0	0	1	0	1	0	1	0	1	0
ab	1	0	1	0	1	0	1	0	1	0
ba	0	0	0	0	0	0	0	0	0	0
bb	0	0	0	0	0	0	0	0	0	0
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b	0	0	0	0	0	0	0	0	0	0
aa	0	0	1	0	1	0	1	0	1	0
ab	1	0	1	0	1	0	1	0	1	0
ba	0	0	0	0	0	0	0	0	0	0
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Hankel Matrix

- $u \sim_L v$ iff rows of u and v in Hankel matrix for L have the same color

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- Language L is regular iff its Hankel matrix contains a finite number of distinct rows, i.e., colors

Hankel Matrix

- $u \sim_L v$ iff rows of u and v in Hankel matrix for L have the same color
- Language L is regular iff its Hankel matrix contains a finite number of distinct rows, i.e., colors
- The number of states in the smallest DFA for L equals the number of colors in the Hankel matrix

What is the FSM for this Hankel Matrix?

	_	a	b	aa	ab	ba	bb	aaa	aab	aba
_	1	0	1	1	1	1	0	0	1	0
a	0	1	1	0	1	0	1	1	1	1
b	1	1	0	0	1	1	1	1	1	1
aa	1	0	1	1	1	1	0	0	1	0
ab	1	0	1	1	1	1	0	0	1	0
ba	1	0	1	1	1	1	0	0	1	0
bb	0	1	1	0	1	0	1	0	1	1
aaa	0	1	1	0	1	0	1	1	1	1
aab	1	1	0	0	1	1	1	1	1	1
aba	0	1	1	0	1	0	1	1	1	1

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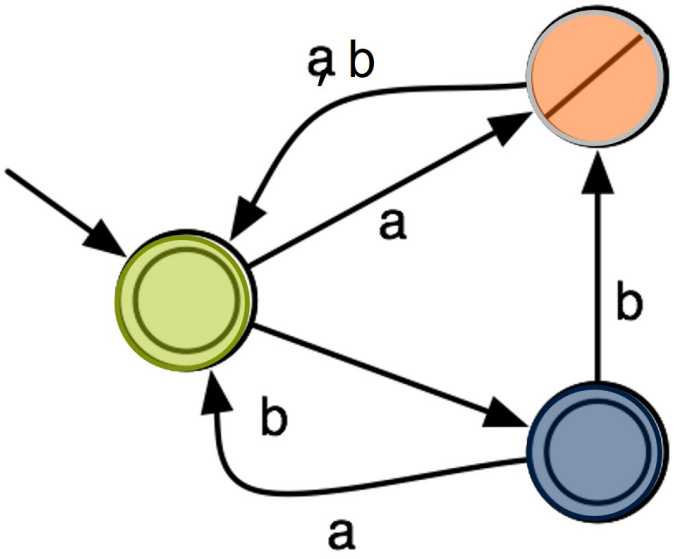
	_	a	b	aa	ab	ba	bb	aaa	aab	aba
_	1	0	1	1	1	1	0	0	1	0
a	0	1	1	0	1	0	1	1	1	1
b	1	1	0	0	1	1	1	1	1	1
aa	1	0	1	1	1	1	0	0	1	0
ab	1	0	1	1	1	1	0	0	1	0
ba	1	0	1	1	1	1	0	0	1	0
bb	0	1	1	0	1	0	1	0	1	1
aaa	0	1	1	0	1	0	1	1	1	1
aab	1	1	0	0	1	1	1	1	1	1
aba	0	1	1	0	1	0	1	1	1	1

Solution

	_
_	1
a	0
b	1
aa	1
ab	1
ba	1
bb	0



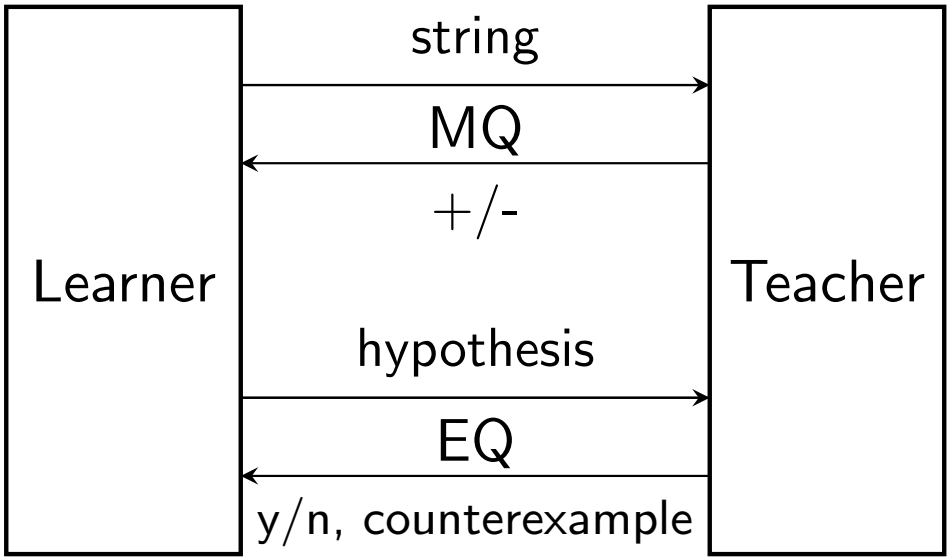
Colors of rows in Hankel matrix give us the states.
Access strings and one-letter extensions allow us to determine transitions.
Column for empty suffix gives us the accepting states.

What if Hankel Matrix is Incomplete?

	_	a	b	ba	aba	aa	baa	abaa	bb	bbb	bba
_		1	0		1			0		1	1
a	1			1			0				
b	0			1					1		
ab		1				0					
aba	1	0									
abaa	0										
bb		1	1								
bbb	1										
bba	1										

Problem to color such a matrix is NP-hard!

Minimally Adequate Teacher (Angluin)

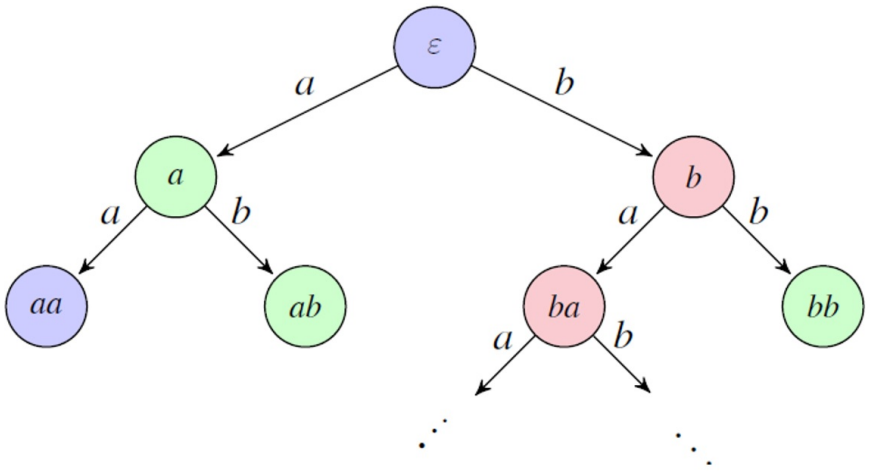


Learner asks membership queries and equivalence queries

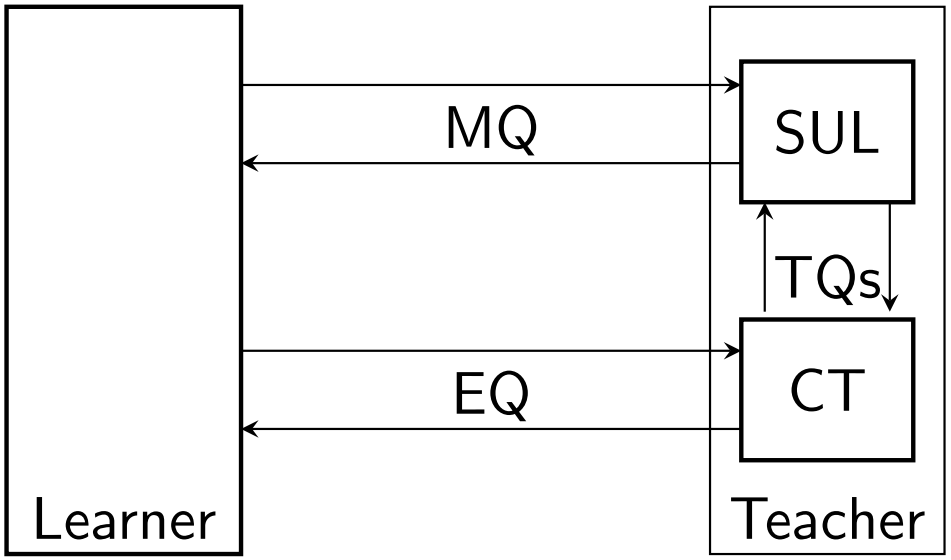
Angluin's L^* Algorithm

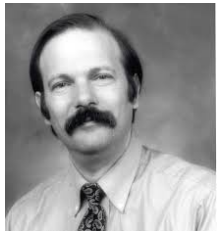
		E: ε	E: a
S	ε	$-$	$+$
	a	$+$	$-$
	b	$-$	$-$
	ba	$-$	$-$
R	aa	$-$	$+$
	ab	$+$	$-$
	bb	$+$	$-$
	baa	$-$	$-$
	bab	$+$	$-$

- S *states* of the canonical automaton
- The words/paths correspond to a spanning tree
- R cross- and back-edges/transitions



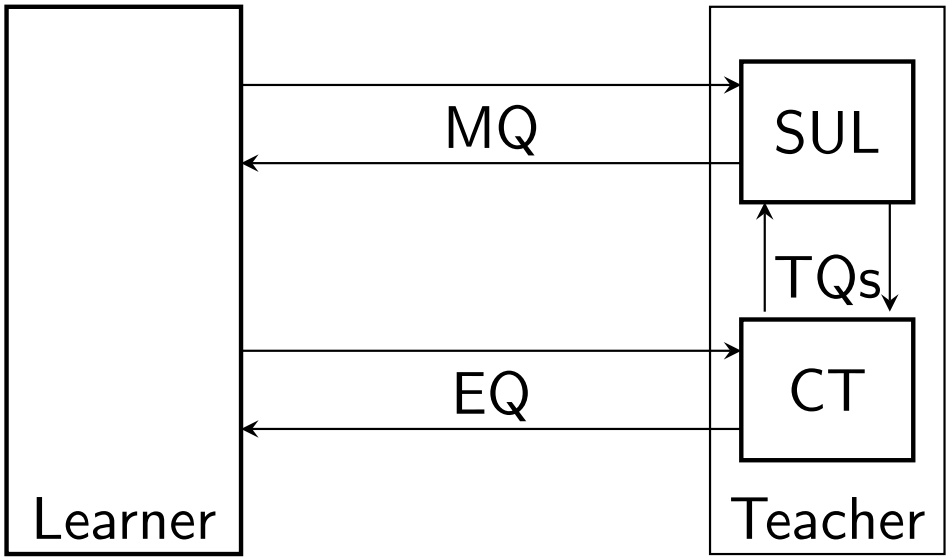
Black Box Checking (Peled, Vardi & Yannakakis)

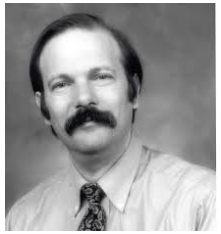



Learner: Formulate hypotheses
Conformance Tester (CT): Test correctness hypotheses

Black Box Checking (Peled, Vardi & Yannakakis)




Learner: Formulate hypotheses
Conformance Tester (CT): Test correctness hypotheses

Model learning and conformance testing two sides of same coin!

HOME NEWS DOWNLOADS FEATURES RESOURCES TEAM HELP

Welcome to the **LearnLib** home page! LearnLib is a *free, open-source* (Apache License 2.0) Java library for *active automata learning*. It is mainly being developed at the Chair for Programming Systems at TU Dortmund University, Germany; a complete list of contributors can be found on the team page.

Note: The open-source LearnLib is a from-scratch re-implementation of the former closed-source version. See the features page for a comparison of the feature sets of the two version.

Background

- Read some Papers on LearnLib
- Papers citing LearnLib at Google Scholar

Search

EXTERNAL LINKS

LearnLib @ GitHub
AutomataLib @ GitHub

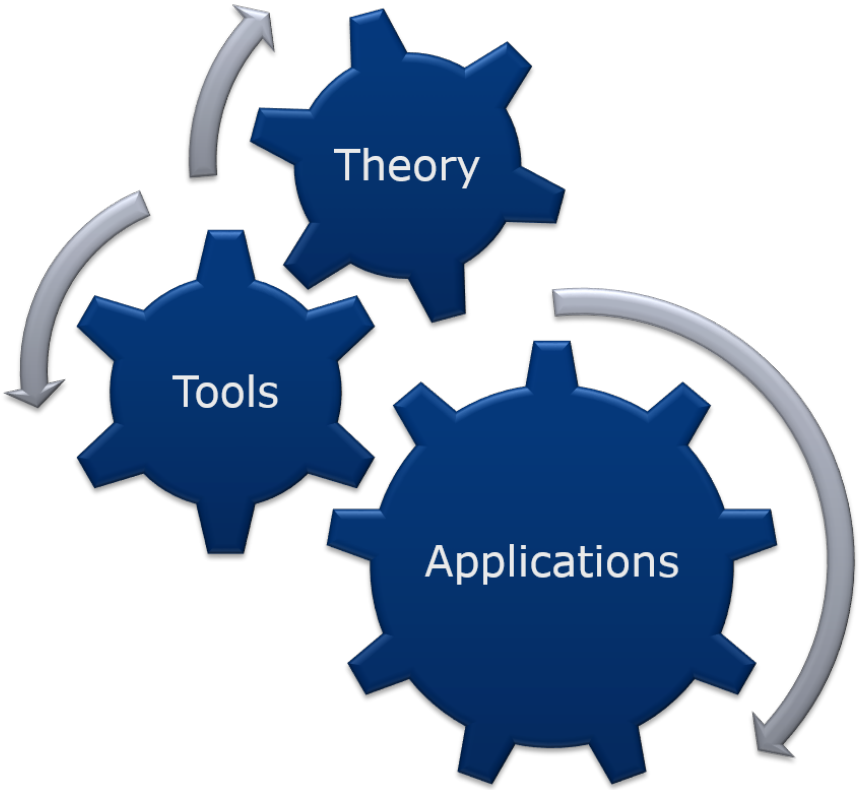
RECENT POSTS

Open Source release of LearnLib


Prof. Dr. Bernhard Steffen

Implements MAT framework for DFAs and Mealy machines

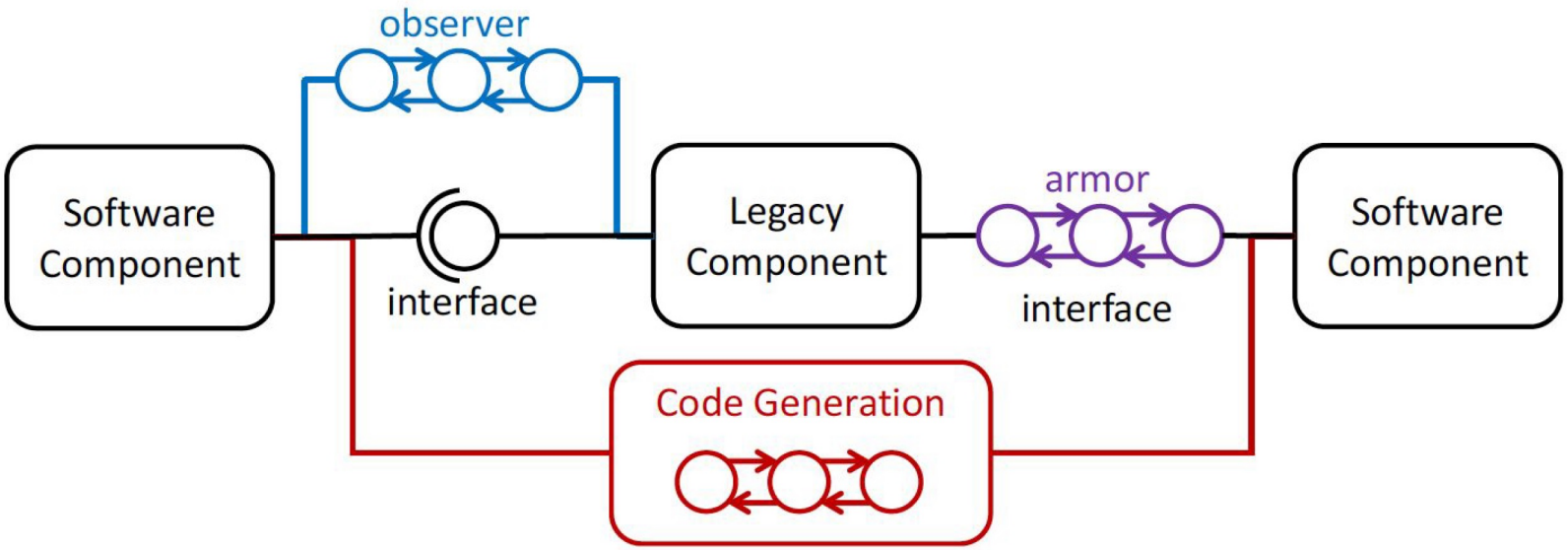
Our Research Method



Engine Status Manager in Océ Printer (ICFEM'15)

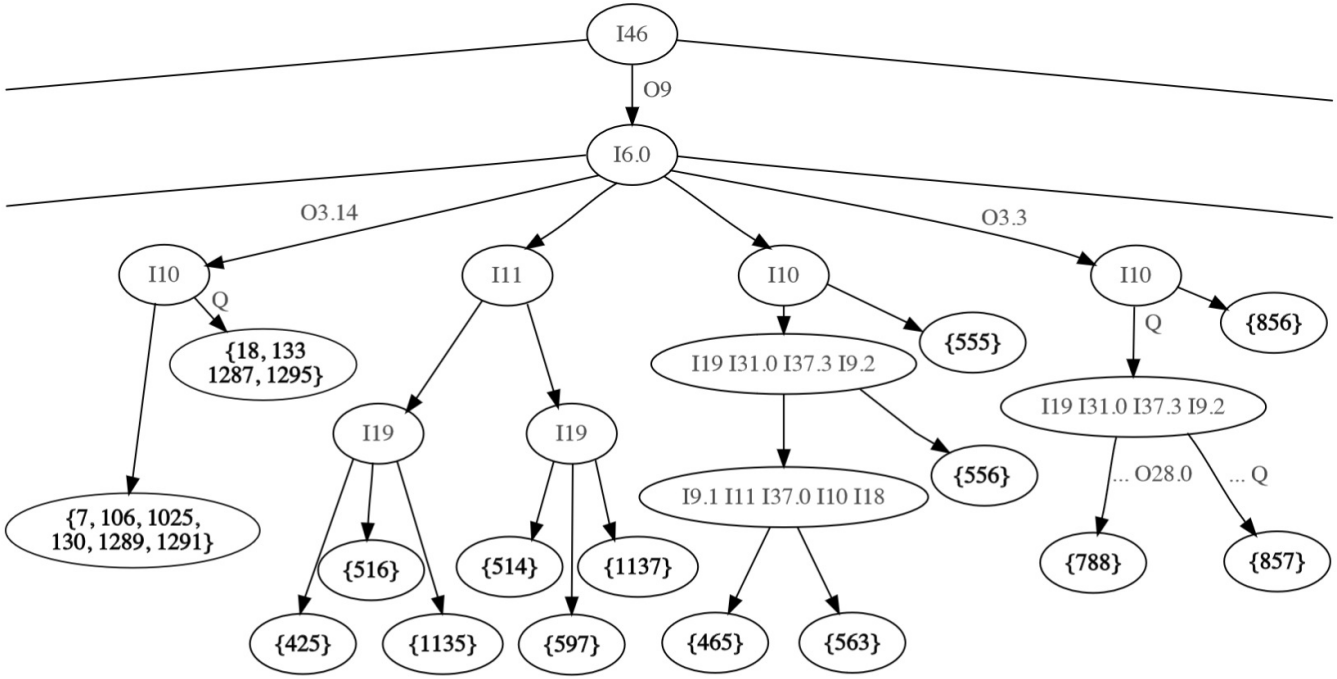
Can we learn interface models of realistic printer controllers?

Potential Applications Interface Models

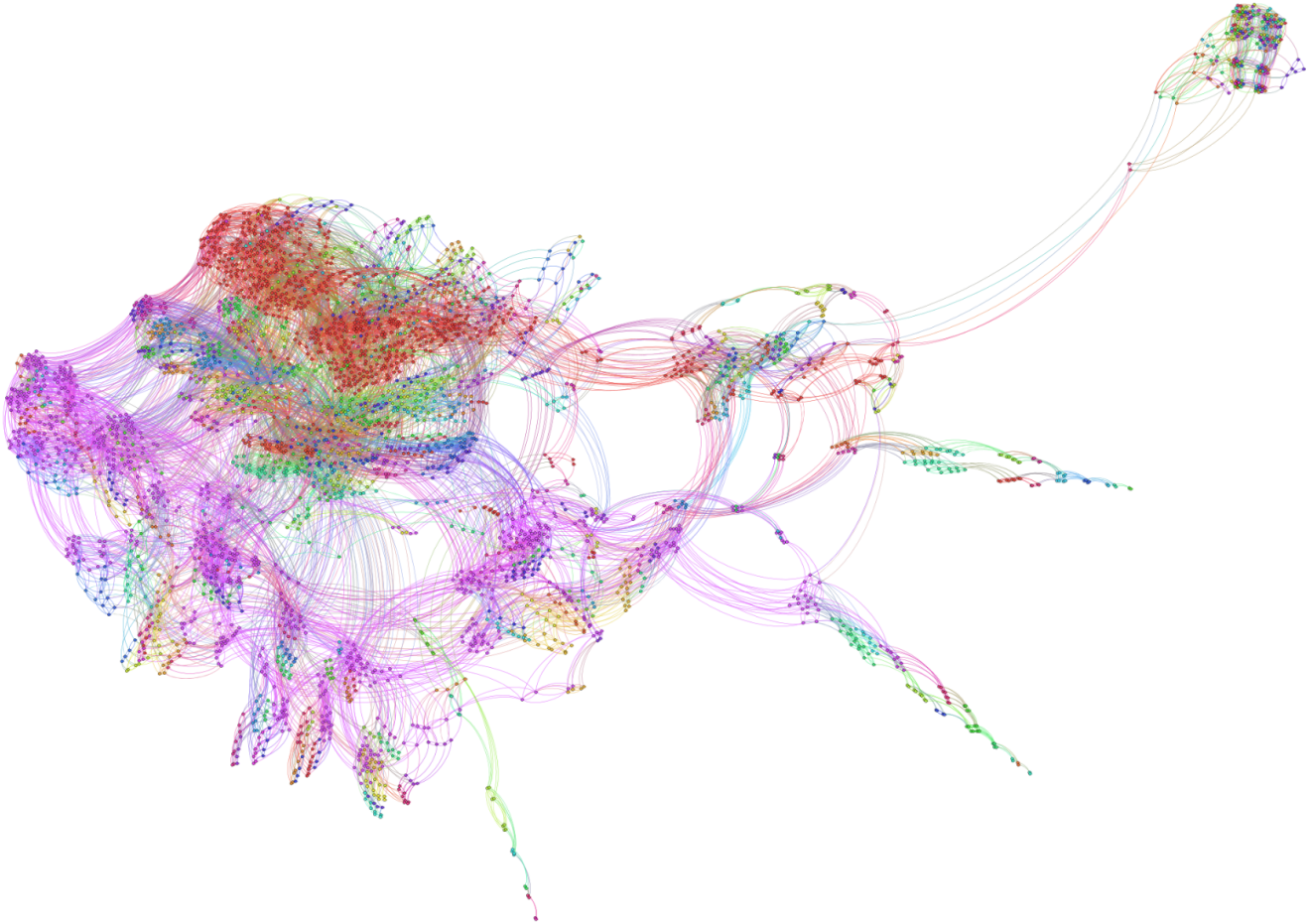


Conformance Testing Becomes Bottleneck!

No existing conformance testing methods (W, Wp, HSI, ADS, UIOv, P, H, SPY,..) was able to find counterexamples for some hypotheses models of the printer software. We had to develop a new hybrid ADS method, based on work of Lee & Yannakakis.



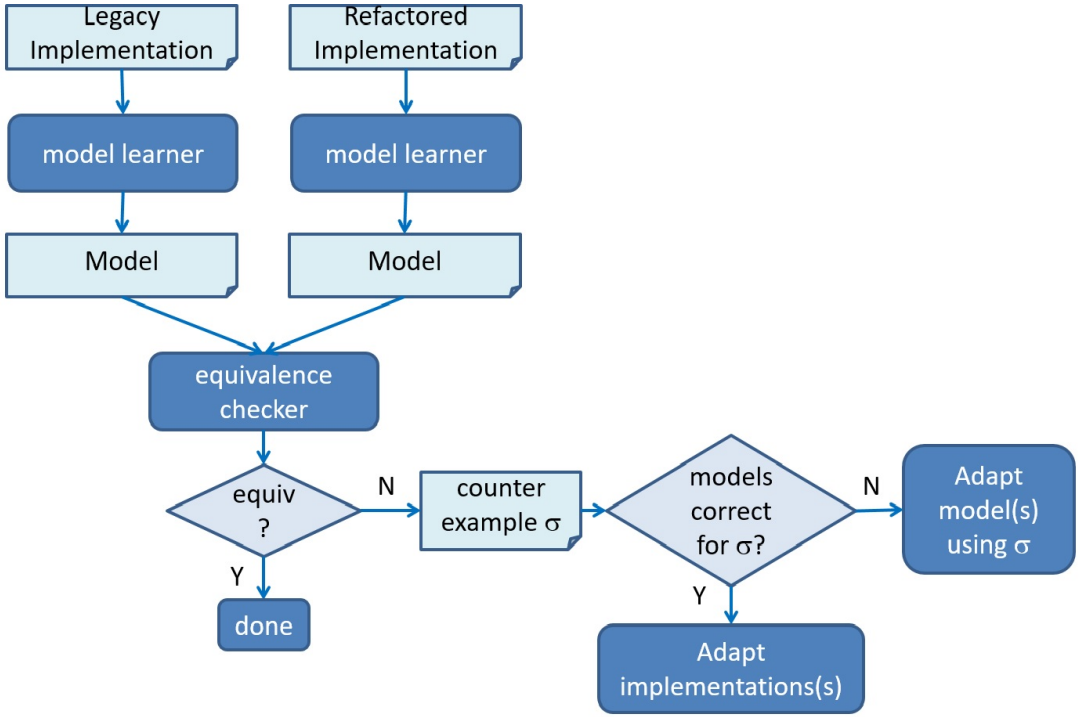
Mealy Machine for Engine Status Manager



Power Control Service from Philips Healthcare (iFM'16)

Are legacy component and refactored implementation equivalent?

Refactoring Legacy Implementations



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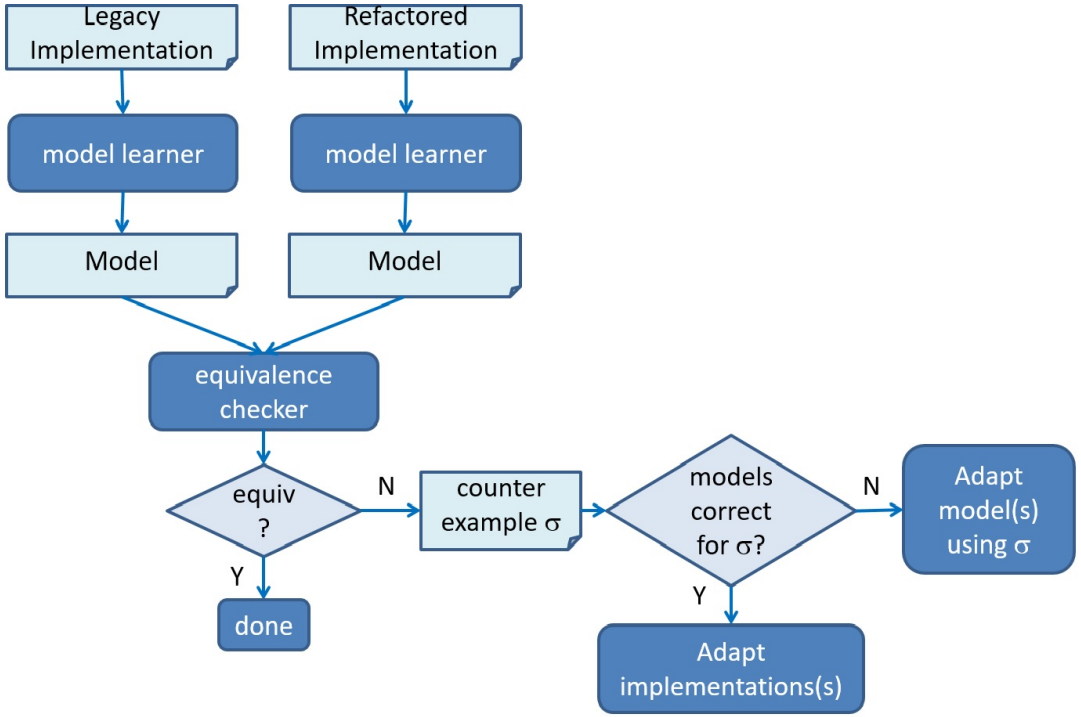

This approach allowed us to find several bugs in refactored implementations of power control service.

ASML Twinscan

ASML Challenge

Can active automata learning be used to support refactoring of legacy software at ASML?

ASML machines run on legacy software. Recent components have been designed using model-based techniques. Can we learn those?

Can we learn the hundreds of design and interface models used for high level control of the wafer flow during lot operation?

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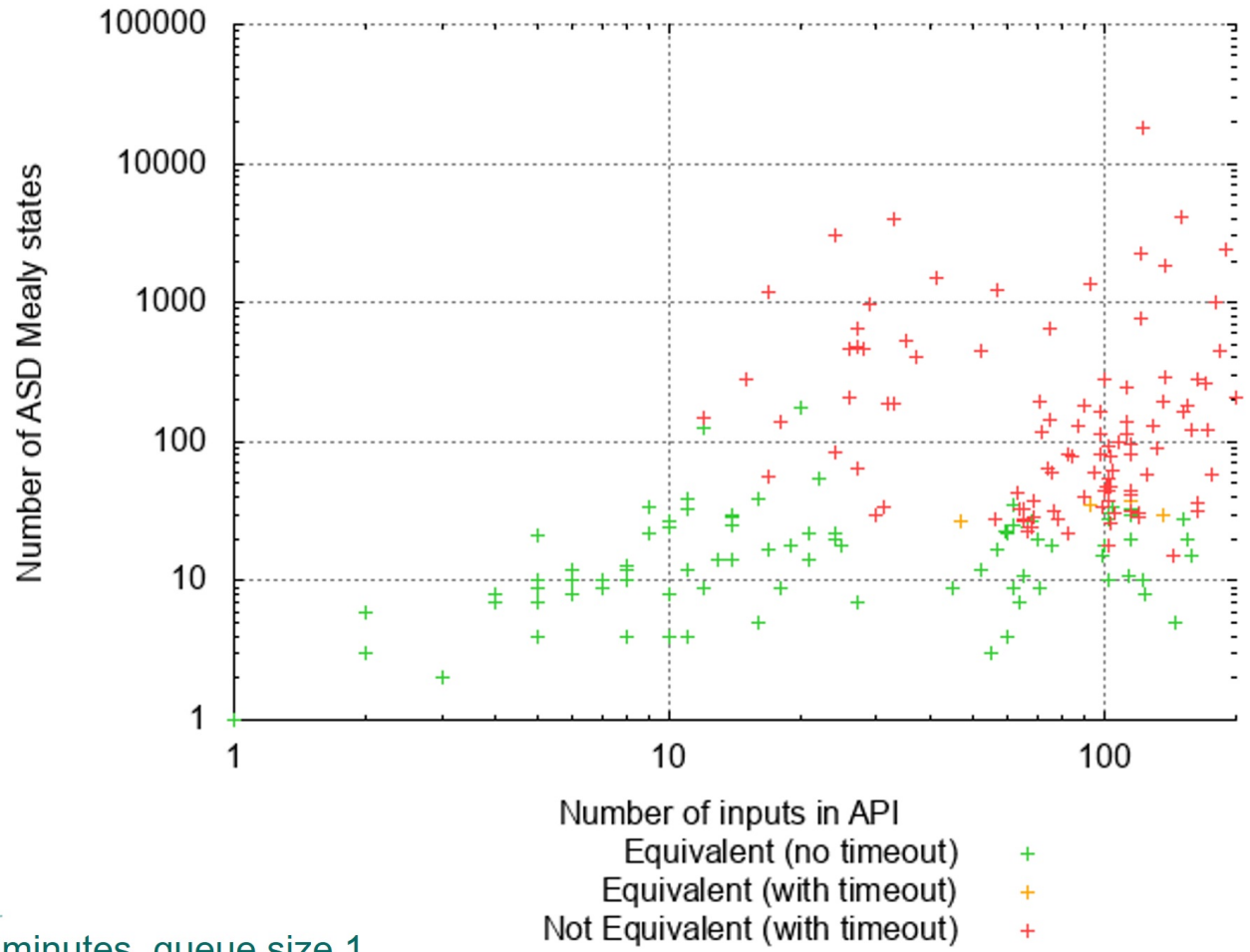
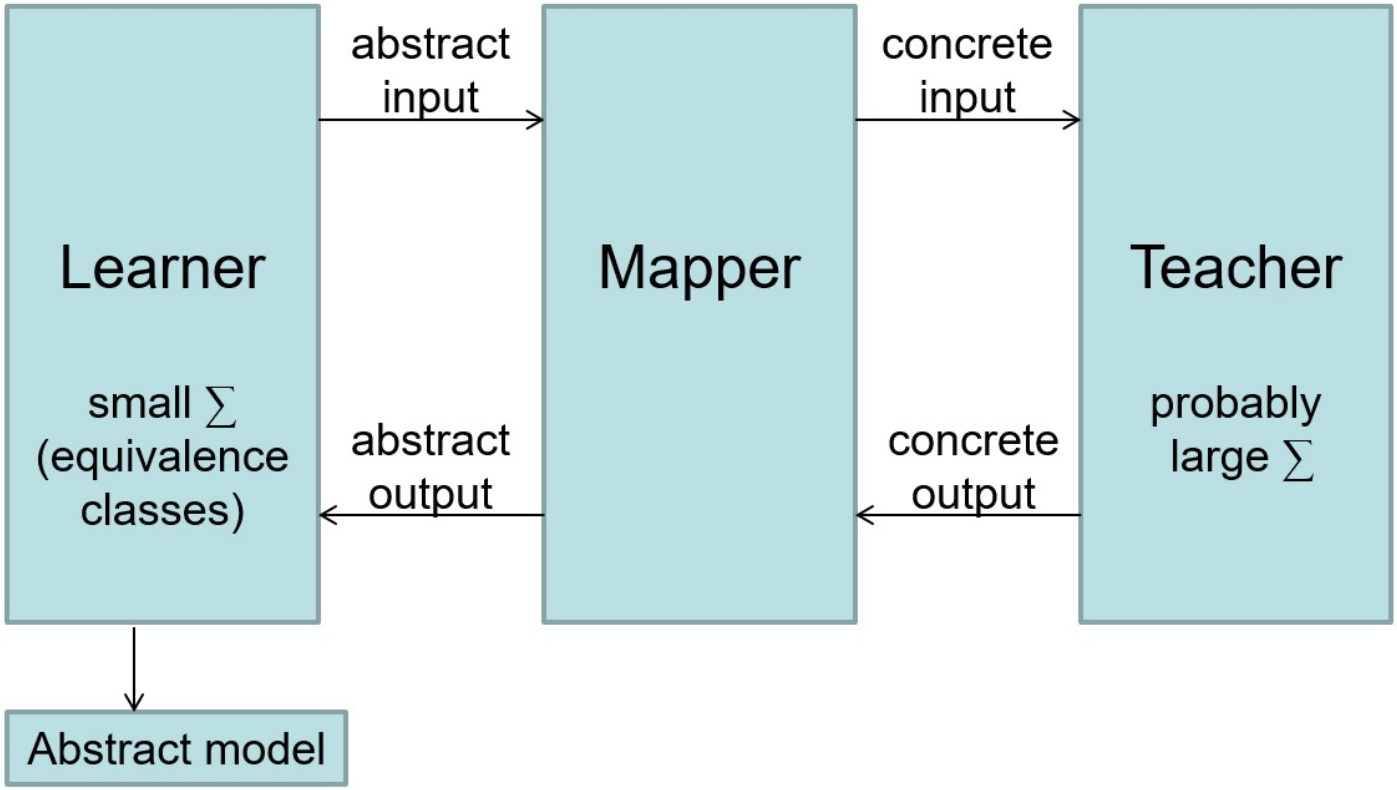
$\Rightarrow$ RERS @ TOOLympics'19

Results LearnLib on ASML Benchmarks

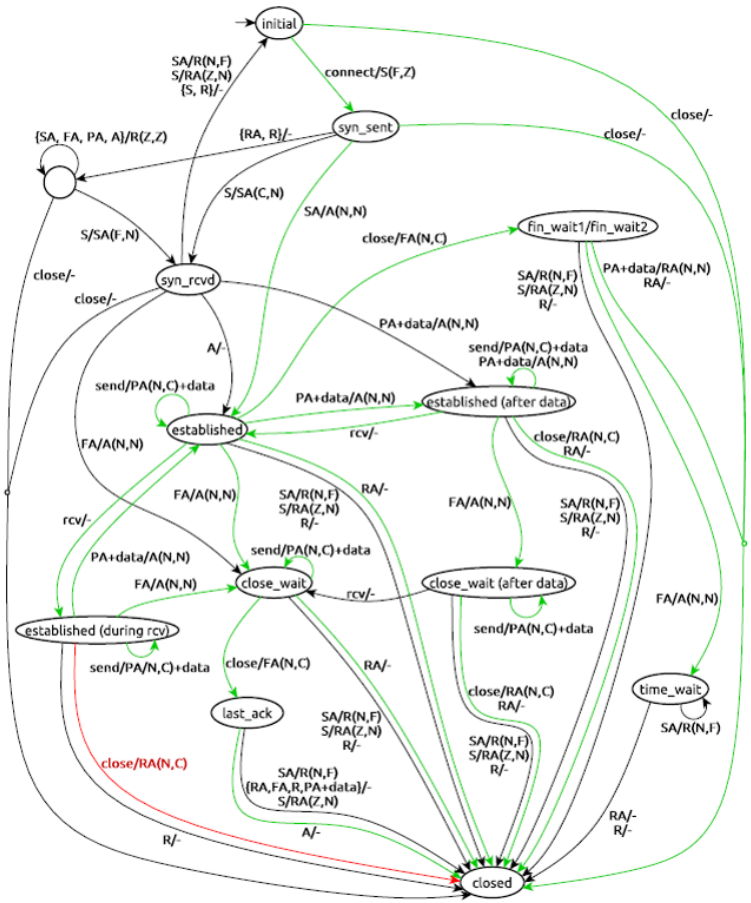


60 minutes, queue size 1

A Theory of Mappers (FMSD, 2015)



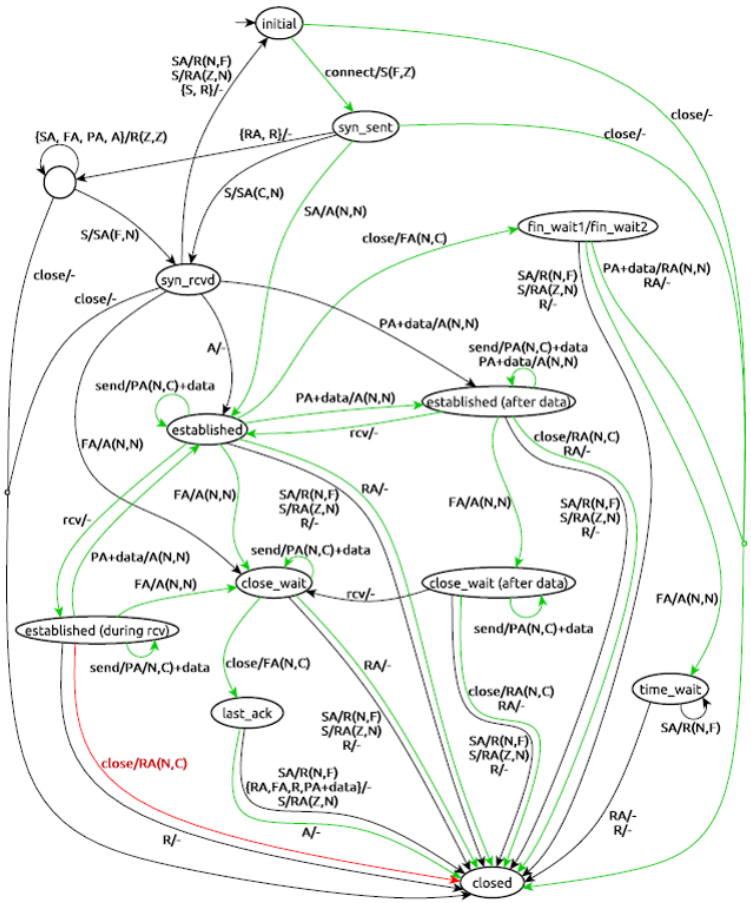
Bugs in Protocol Implementations



Standard violations found in implementations of major protocols:

- TLS (Usenix Security'15)
- TCP (CAV'16)
- SSH (Spin'17)

Bugs in Protocol Implementations



Standard violations found in implementations of major protocols:

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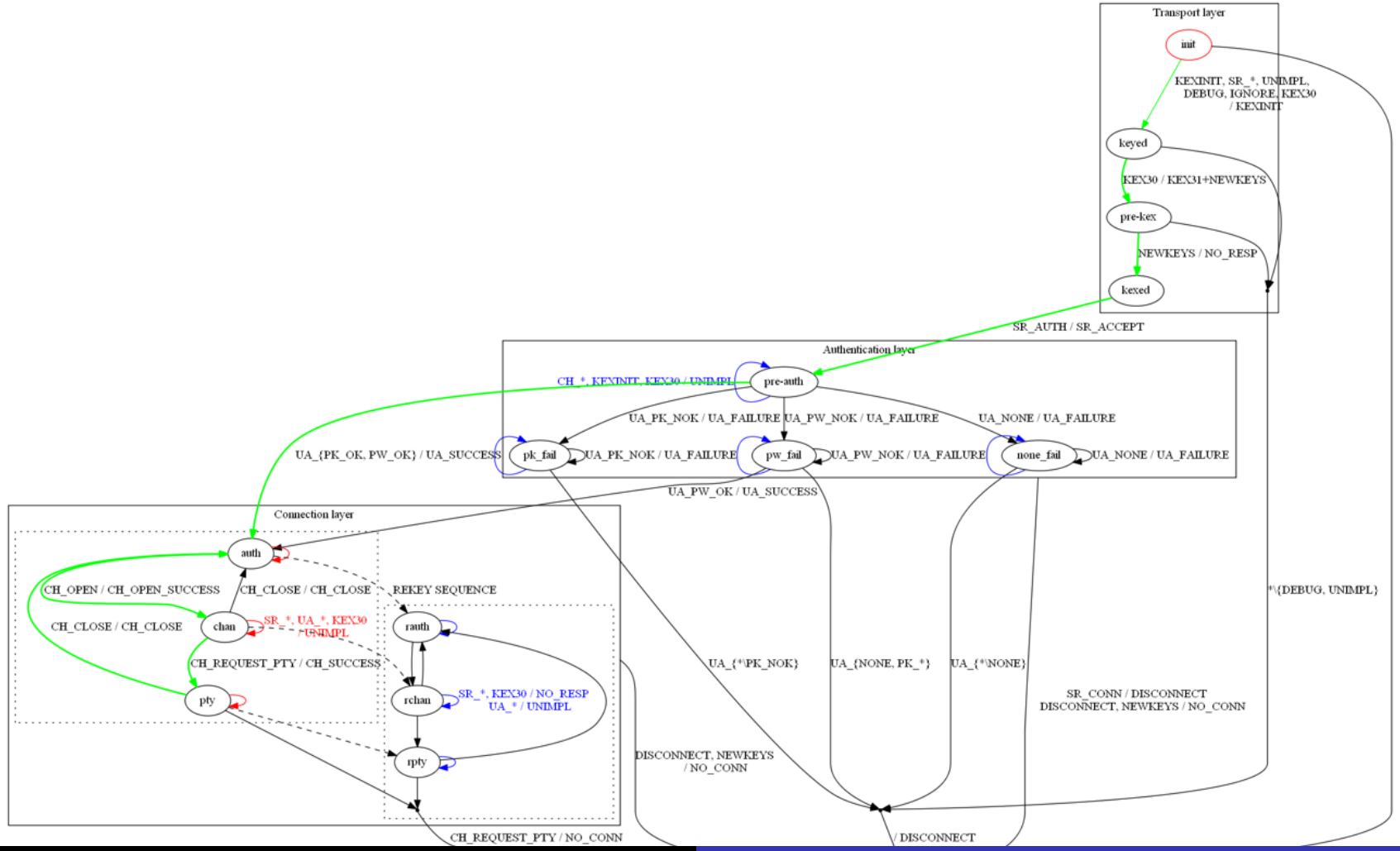
These findings led to bug fixes in implementations.

Learned Model for SSH Implementation



SSH Model Checking Results

	Property	Key word	OpenSSH	Bitvise	DropBear
Security	Trans.		✓	✓	✓
	Auth.		✓	✓	✓
Rekey	Pre-auth.		X	✓	✓
	Auth.		✓	X	✓
Funct.	Prop. 6	MUST	✓	✓	✓
	Prop. 7	MUST	✓	✓	✓
	Prop. 8	MUST	X*	X	✓
	Prop. 9	MUST	✓	✓	✓
	Prop. 10	MUST	✓	✓	✓
	Prop. 11	SHOULD	X*	X*	✓
	Prop. 12	MUST	✓	✓	X

Other Protocol Case Studies

- Biometric Passport
- EMV Protocol
- Session Initiation Protocol (SIP)
- Message Queuing Telemetry Transport (MQTT) protocol
- Quick UDP Internet Connections (QUIC) protocol
- WiFi
- IEC 60870-5-104 protocol
- ...

Fingerprinting TLS Implementations

There are many different TLS implementations. How can we figure out to which implementation we are talking?
Our approach:

- Learn state machine models of hundreds of TLS implementations
- Consider disjoint union of these state machines and add reset transitions
- Use algorithm of Lee & Yannakakis to compute adaptive distinguishing sequence

(joint work with Erwin Janssen and Joeri de Ruiter, SIDN)

Lorentz Workshop

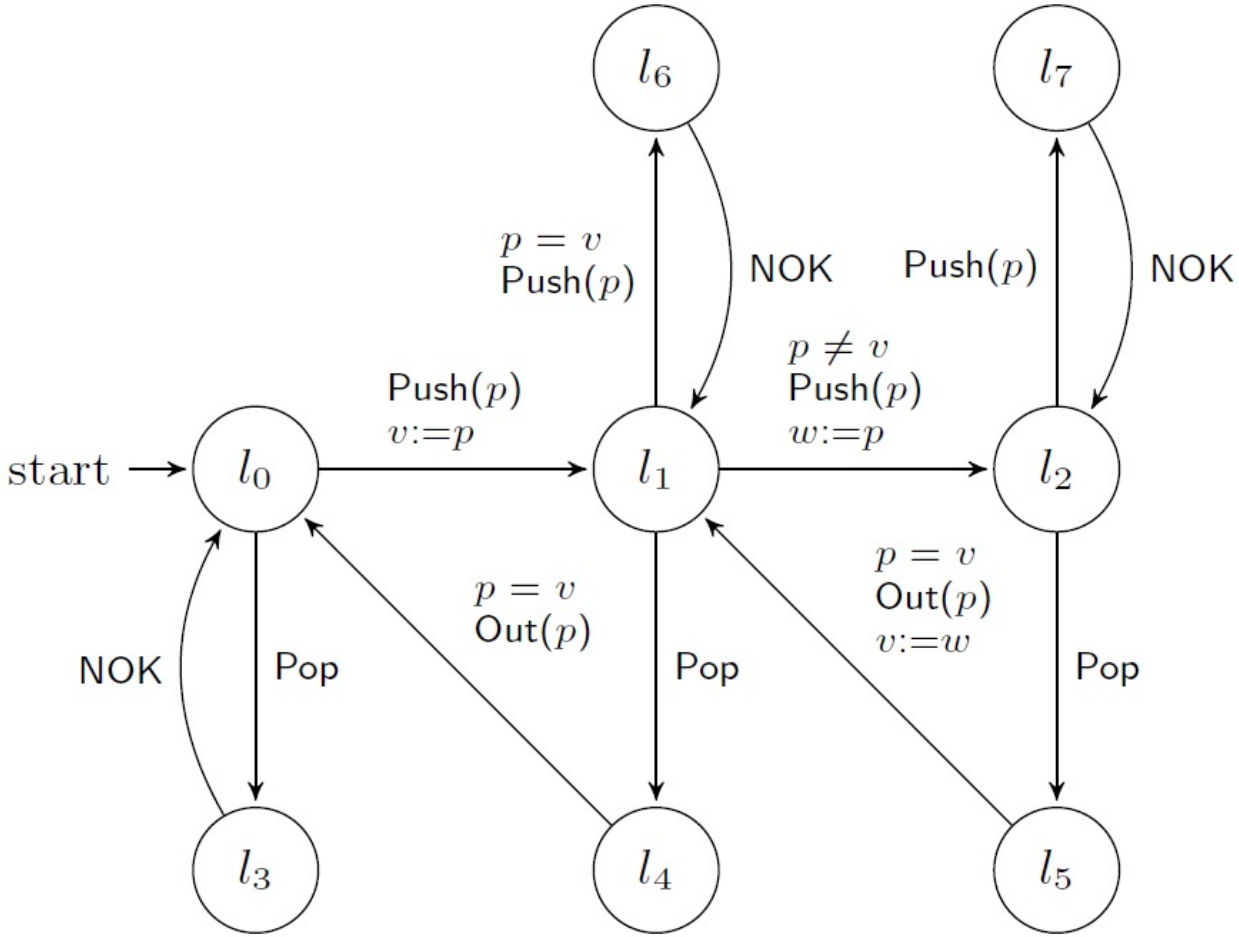
Participants from automata learning, model-based testing, cryptography, and security protocol implementation.

Working groups on e.g.,

- WiFi
- side channels in TLS
- LTE

Register Automata

Actions may carry data parameters that may be stored in registers:



Data Types

Register automata may be parametrized by a (relational) structure: a pair $\langle \mathcal{D}, \mathcal{R} \rangle$ where $\mathcal{D}$ is an unbounded domain of data values, and $\mathcal{R}$ is a collection of relations on $\mathcal{D}$.

Examples of simple structures include:

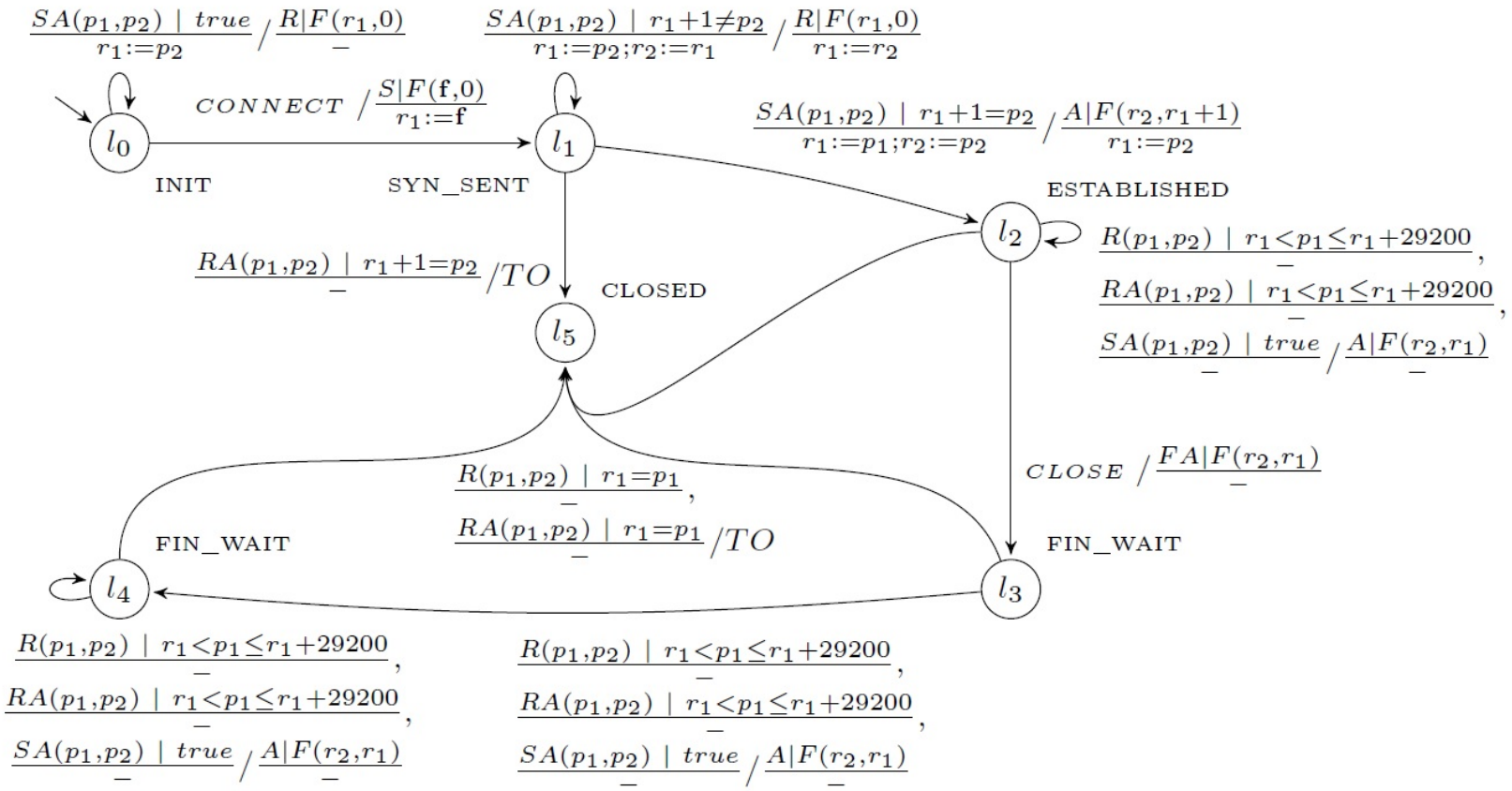
- $\langle \mathbb{N}, \{=\} \rangle$, the natural numbers with equality;
- $\langle \mathbb{R}, \{<\} \rangle$, the real numbers with inequality: this structure also allows one to express equality between elements.

Transition guards are conjunctions of negated and unnegated relations from $\mathcal{R}$.

Learning Tools for Register Automata

- Tomte, Radboud University, can only handle $\langle \mathbb{N}, \{=\} \rangle$
- LearnLib, TU Dortmund, can only handle $\langle \mathbb{N}, \{=\} \rangle$
- RALib, Uppsala/Dortmund, can handle some richer structures

TCP Protocol Case Study (FMICS-AVoCS'17)



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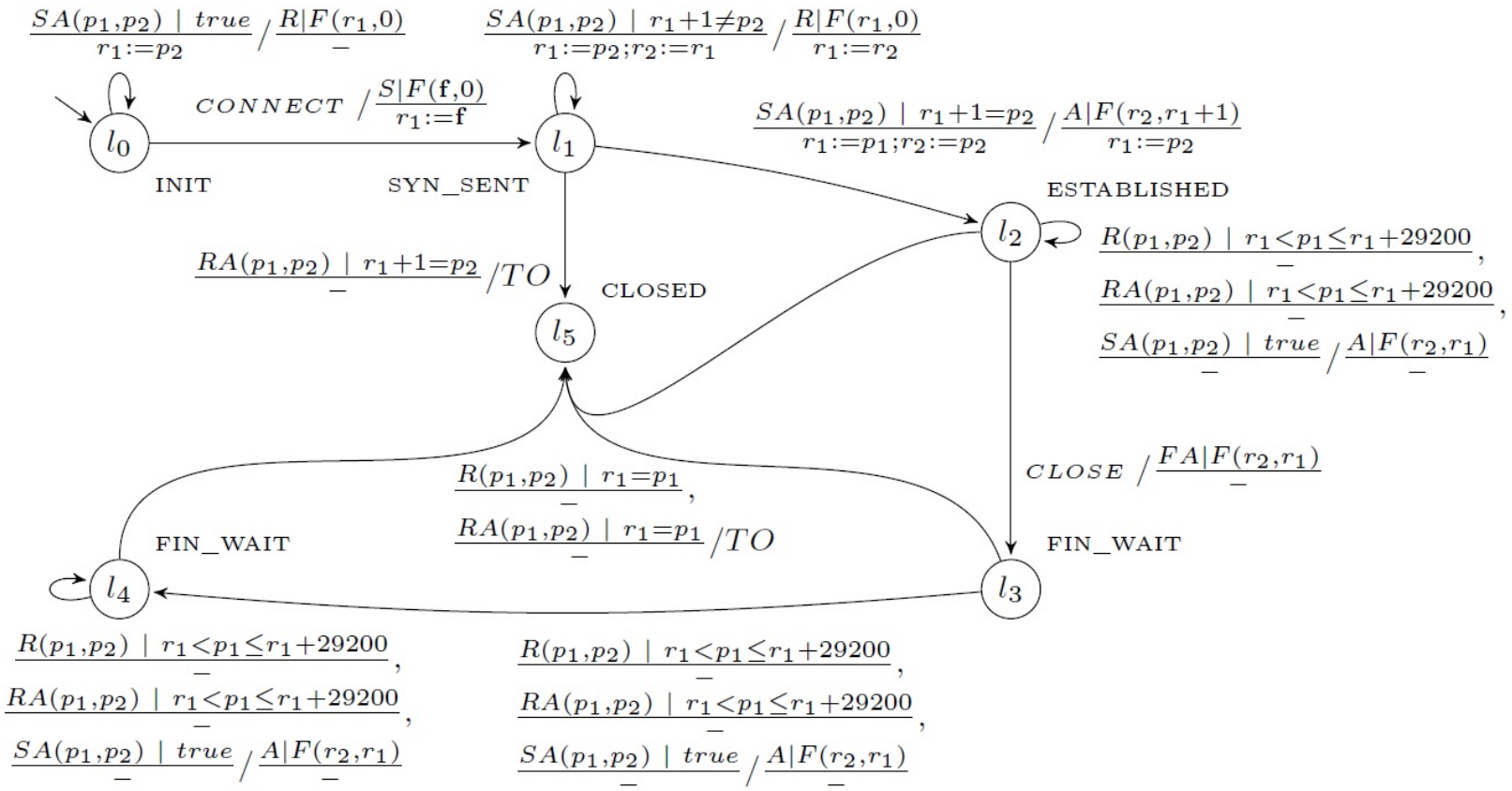

These findings led to bug fix in Linux TCP implementation!

Limits of Black-box Learning?

- Model learning is an highly effective bug finding technique

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Limits of Black-box Learning?

- Model learning is an highly effective bug finding technique
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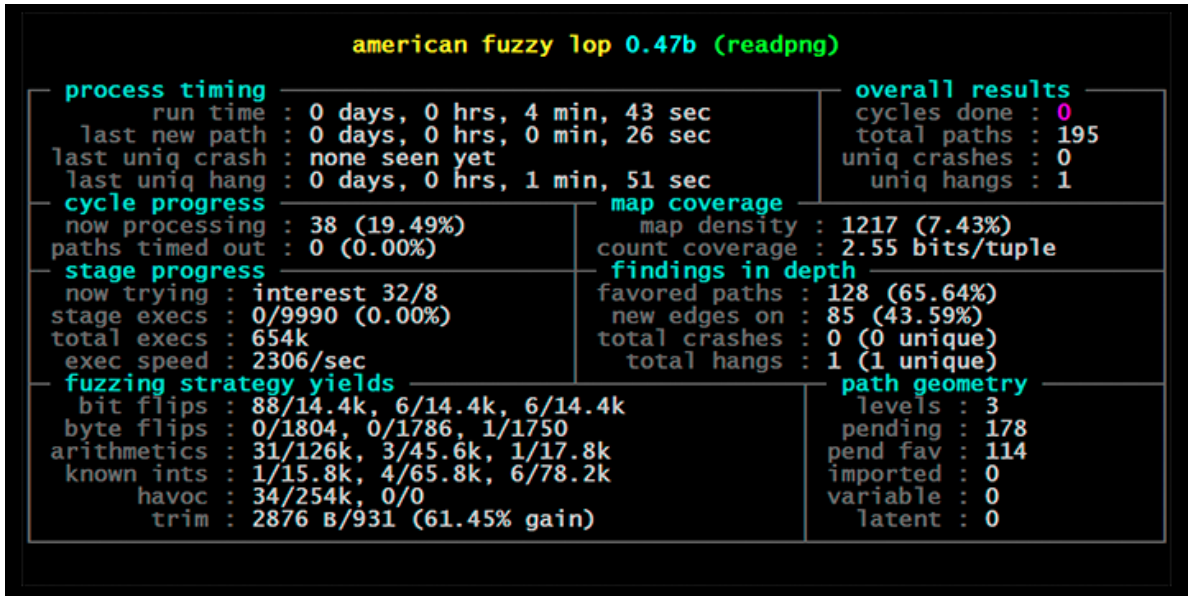
- Can we use white-box information while preserving the extensionality of black-box models?

Yes, we can!

Fuzzing



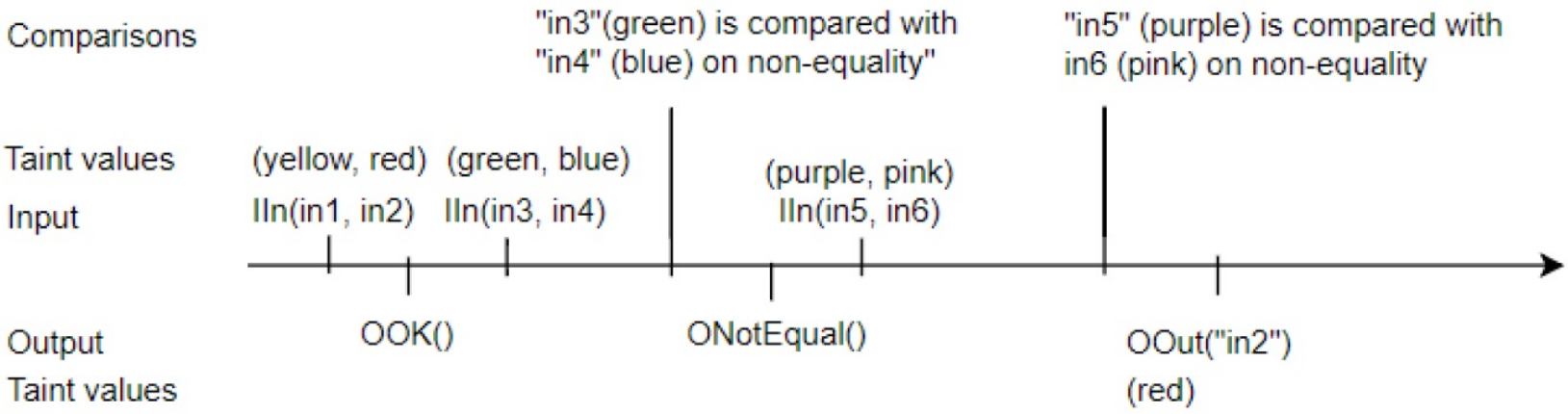
By combining LearnLib, hybrid ADS testing, and the American fuzzy lop fuzzer (AFL), my group, together with colleagues from Delft, won the RERS 2016 challenge.

Taint Analysis

Taint Analysis

- White-box technique for code analysis
- Instruments code to track input values
- Many tools focus on specific vulnerabilities, e.g. buffer overflows and sql injections
- Usually implemented using Dynamic Binary Analysis, e.g. Valgrind
- We use Python library from Pygmalion tool from Andreas Zeller et al.

What Does Pygmalion Do For Us?



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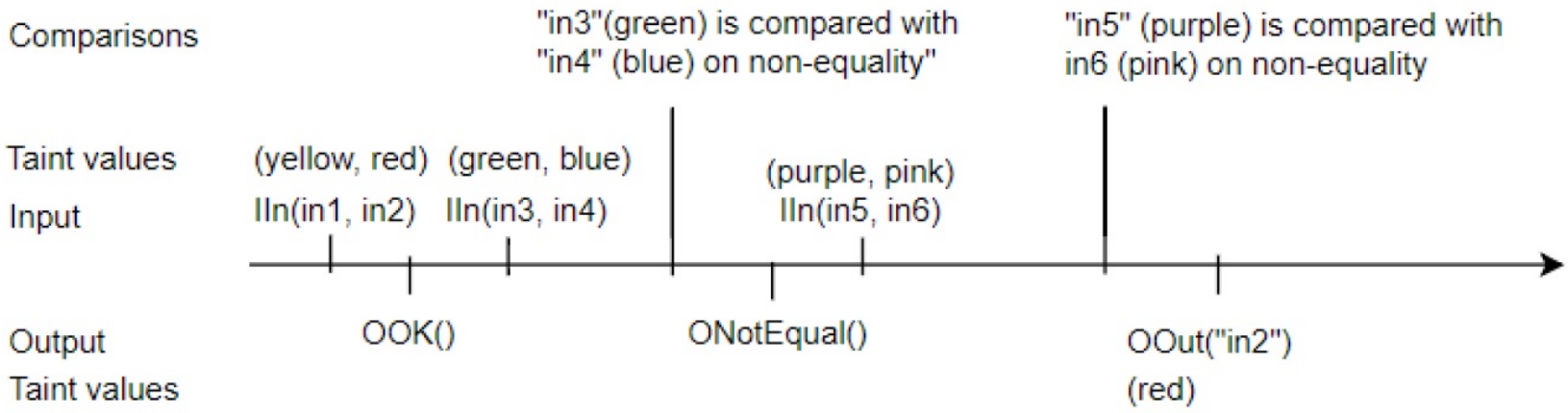
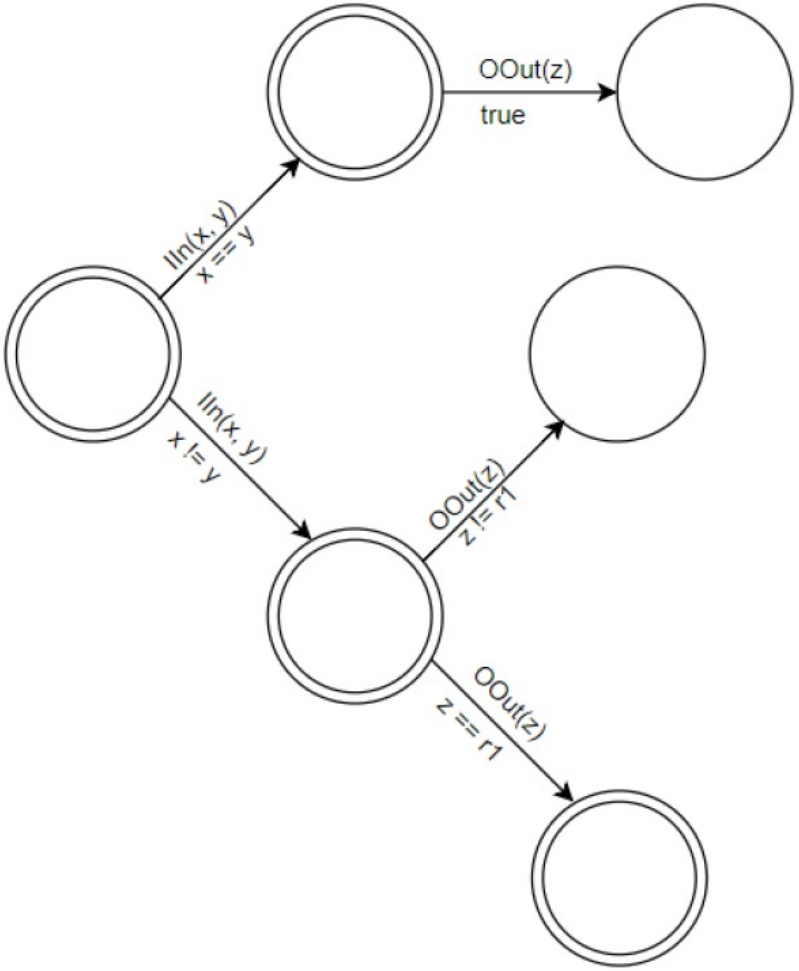


Potential of exponential gains during learning!

Architecture RAlib Tool for Learning Register Automata

Tree Oracle



Ongoing Work

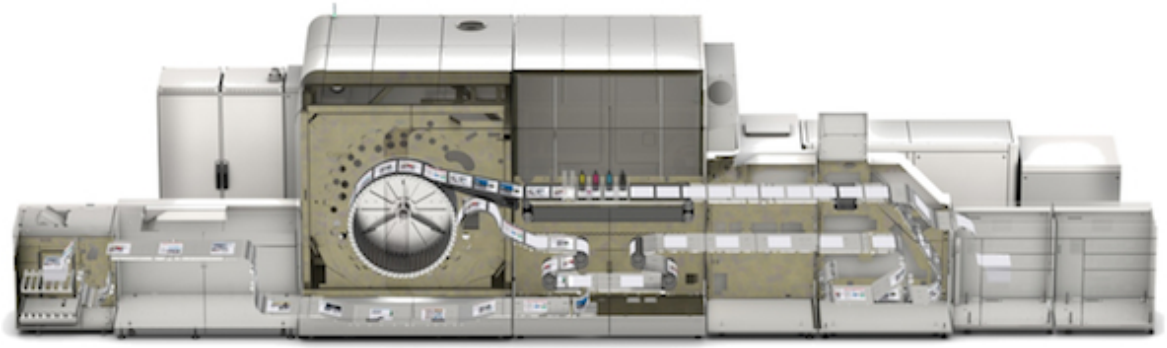
Replace tree oracle in RAlib by a version that uses taint analysis.

Ongoing Work

Replace tree oracle in RAlib by a version that uses taint analysis.

First prototype finished (for integers with equality)

A Problem from Océ



Identify patterns in logs of printer behavior:[1]

$$S = \{abab, ababab, abababab, abba, abbba, abbbba, aaaa, aaaaaa, aaaaaaaa\}$$

[1] Based on work of Linard, Vaandrager & De La Higuera (LATA'19).

Tackling the Océ Problem

- To solve Océ problem we need to learn a union of regular languages from positive examples only
- But it is impossible to learn regular languages in the limit from positive examples! (Gold, 1967)
- Window languages (a.k.a. k-testable languages) (McNaughton & Papert, 1971) are learnable in the limit from positive examples.
- Can we learn unions of window languages? And if so, does this provide the patterns Océ is looking for?

Window Languages

Definition (k-test vector)

Let $k > 0$. A k-test vector is a tuple $Z = \langle I, F, T, C \rangle$ where:

- $I \subseteq \Sigma^{k-1}$ is a set of allowed prefixes
- $F \subseteq \Sigma^{k-1}$ is a set of allowed suffixes
- $T \subseteq \Sigma^{k}$ is a set of allowed segments
- $C \subseteq \Sigma^{<k}$ is a set of allowed short strings

We write $\mathcal{T}_k$ to denote the set of k-test vectors.

Window Languages

Window of size 3

Words matching *k-test vector* $Z = \langle I, F, T, C \rangle$ where:

- prefixes $I = \{ab\}$
- suffixes $F = \{ab, ba\}$
- segments $T = \{aba, abb, bab, bba\}$
- short strings $C = \{ab\}$

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ababababab

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$\boxed{\text{ab}}$ababab ab

ab $\in I$

Window Languages

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- short strings $C = \{ab\}$

$$\text{abababab}\boxed{\text{ab}}$$

$$\text{ab} \in F$$

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$\boxed{\text{aba}}$bababab

aba $\in T$

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a $\boxed{\text{bab}}$ ababab

bab $\in T$

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ab $\boxed{\text{aba}}$ babab

aba $\in T$

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aba $\boxed{\text{bab}}$ abab

bab $\in T$

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abab $\boxed{\text{aba}}$ bab

aba $\in T$

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ababa $\boxed{\text{bab}}$ ab

bab $\in T$

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ababab $\boxed{\text{aba}}$ b

aba $\in T$

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ababab$\boxed{\text{bab}}$

bab $\in T$

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abaaba

Window Languages

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$\boxed{\text{ab}}$aaba

ab $\in I$

Window Languages

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abaa $\boxed{\text{ba}}$

ba $\in F$

Window Languages

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$\boxed{\text{aba}}$aba

aba $\in T$

Window Languages

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- short strings $C = \{ab\}$

a $\boxed{\text{baa}}$ ba

baa $\notin T$

k-Testable Languages

Definition (from k-test vectors to languages)

Let $Z = \langle I, F, T, C \rangle$ be a k-test vector, for some $k > 0$. Then

$$\gamma_k(Z) \quad = \quad C \cup ((I\Sigma^* \cap \Sigma^* F) \setminus (\Sigma^*(\Sigma^k \setminus T)\Sigma^*)).$$

A language $L \subseteq \Sigma^*$ is k-testable in the strict sense (k-TSS) if there exists a k-test vector Z such that $L = \gamma_k(Z)$.

Note that k-TSS languages are regular.

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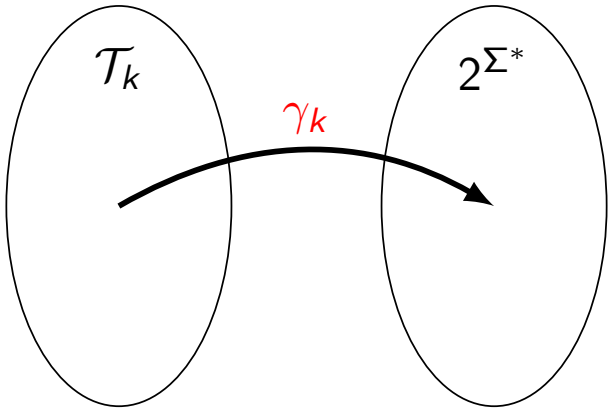

k-Testable Languages

Definition (from Languages to k-test vectors)

Let $L \subseteq \Sigma^*$ be a language and $k > 0$. Then $\alpha_k(L)$ is the k-test vector $\langle I_k(L), F_k(L), T_k(L), C_k(L)\rangle$ where

- $I_k(L) = \{u \in \Sigma^{k-1} \mid \exists v \in \Sigma^* : uv \in L\}$,
- $F_k(L) = \{w \in \Sigma^{k-1} \mid \exists v \in \Sigma^* : vw \in L\}$,
- $T_k(L) = \{v \in \Sigma^k \mid \exists u, w \in \Sigma^* : uvw \in L\}$, and
- $C_k(L) = (L \cap \Sigma^{<k-1}) \cup (I_k(L) \cap F_k(L))$.

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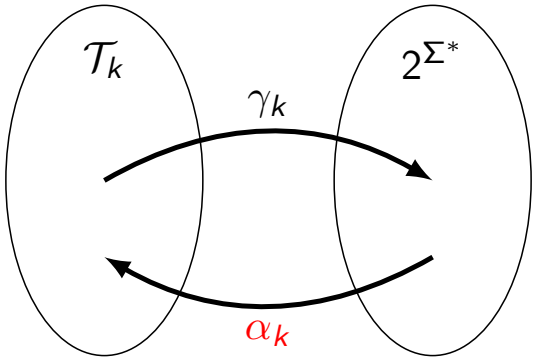

k-Test Vector Inclusion

Definition

Let $k > 0$. The relation $\sqsubseteq$ on $\mathcal{T}_k$ is given by

$$\langle I, F, T, C \rangle \sqsubseteq \langle I', F', T', C' \rangle \quad \Leftrightarrow \quad I \subseteq I' \wedge F \subseteq F' \wedge T \subseteq T' \wedge C \subseteq C'.$$

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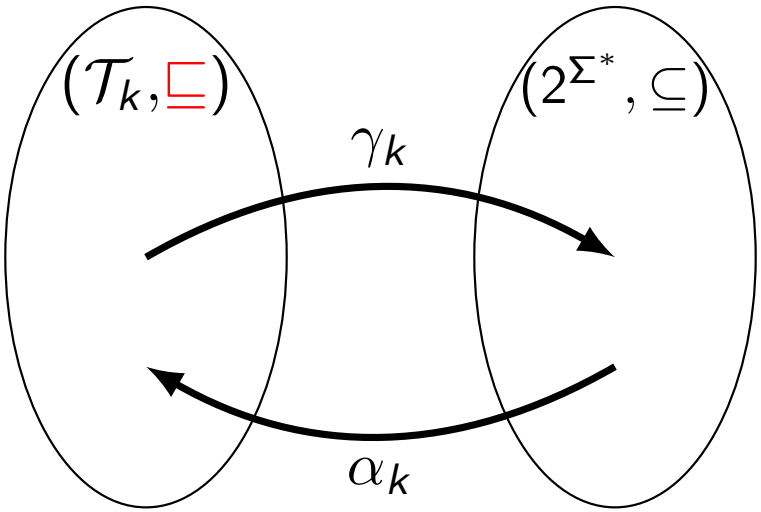


Order Preservation

Lemma

For $k > 0$ and for all languages L, L',

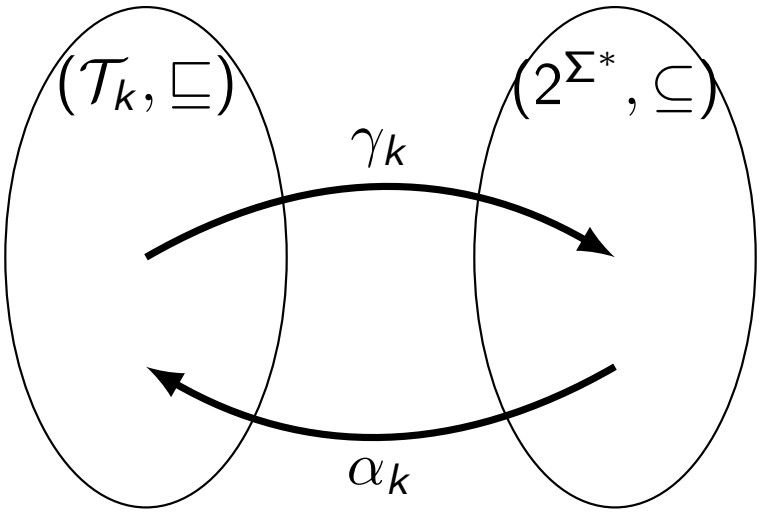
$$L \subseteq L' \quad \Rightarrow \quad \alpha_k(L) \sqsubseteq \alpha_k(L').$$

Lemma

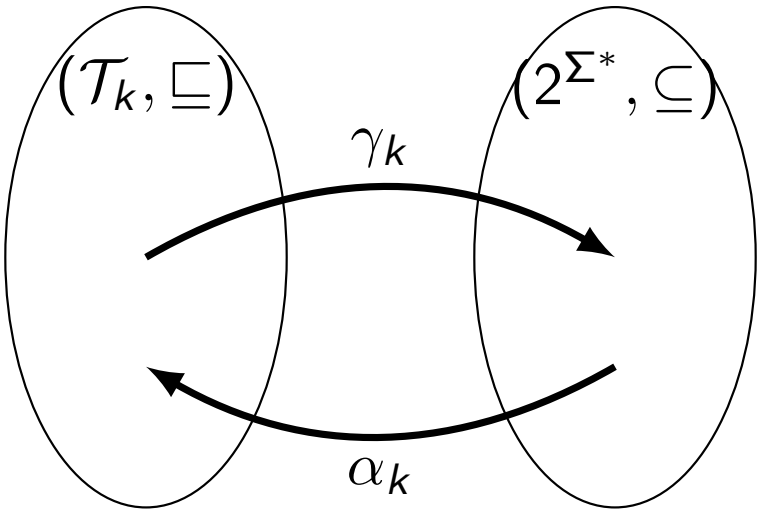
For all $k > 0$ and for all k-test vectors Z and Z',

$$Z \sqsubseteq Z' \quad \Rightarrow \quad \gamma_k(Z) \subseteq \gamma_k(Z').$$

Galois Connection



Galois Connection



Theorem (Galois Connection)

Let $k > 0$, $L \subseteq \Sigma^*$ a language, and Z a k-test vector. Then

$$\alpha_k(L) \sqsubseteq Z \quad \Leftrightarrow \quad L \subseteq \gamma_k(Z).$$

Galois Connections

- Particular correspondence between two partially ordered sets
- Many applications in mathematics
- Adjoint functors in category theory
- Describe many forms of abstraction in theory of abstract interpretation of programming languages

Galois Connection

Corollary

For all $k > 0$, $\gamma_k \circ \alpha_k$ and $\alpha_k \circ \gamma_k$ are monotone and idempotent.

Previously established as Theorem 3.2 in Garcia and Vidal (1990) and as Lemma 3.3 in in Yokomori and Kobayashi (1998).

Galois Connection

Corollary

For all $k > 0$, $L \subseteq \Sigma^*$ *and* $Z \in \mathcal{T}_k$,

$$\begin{aligned} \alpha_k \circ \gamma_k(Z) &\sqsubseteq Z \\ L &\subseteq \gamma_k \circ \alpha_k(L) \end{aligned}$$

Previously established as Lemma 3.1 in Garcia and Vidal (1990) and as Lemma 3.1 in Yokomori and Kobayashi (1998).

Galois Connection

Corollary

For all $k > 0$, $L \subseteq \Sigma^*$, and $Z \in \mathcal{T}_k$,

$$L \subseteq \gamma_k(Z) \quad \Rightarrow \quad \gamma_k \circ \alpha_k(L) \subseteq \gamma_k(Z).$$

Previously established as Theorem 3.1 in Garcia and Vidal (1990).

Galois Connection

Corollary

For all $k > 0$ and $Z \in \mathcal{T}_k$, $\gamma_k \circ \alpha_k \circ \gamma_k(Z) = \gamma_k(Z)$. Moreover, for any $Z' \in \mathcal{T}_k$,

$$\gamma_k(Z) = \gamma_k(Z') \quad \Rightarrow \quad \alpha_k \circ \gamma_k(Z) \sqsubseteq Z'.$$

Previously established as Lemma 1 in Yokomori and Kobayashi (1998).

Learning k-Testable Languages

Theorem (Garcia & Vidal (1990))

Any k-testable language can be identified in the limit from positive examples.

Union and Symmetric Difference

Definition

The union and symmetric difference of two k-test vectors $Z = \langle I, F, T, C \rangle$ and $Z' = \langle I', F', T', C' \rangle$ are given by:

$$\begin{aligned} Z \sqcup Z' &= \langle I \cup I', F \cup F', T \cup T', C \cup C' \cup (I \cap F') \cup (I' \cap F) \rangle \\ Z \triangle Z' &= \langle I \triangle I', F \triangle F', T \triangle T', C \triangle C' \triangle (I' \cap F) \triangle (I \cap F') \rangle \end{aligned}$$

Window Languages Not Closed Under Union

$$
\begin{aligned}
Z &= \langle \{aa\}, \{aa\}, \{aaa\}, \{aa\} \rangle \\
Z' &= \langle \{ba, bb\}, \{ab, bb\}, \{baa, bab, aaa, aab\}, \{bb\} \rangle
\end{aligned}
$$

Window Languages Not Closed Under Union

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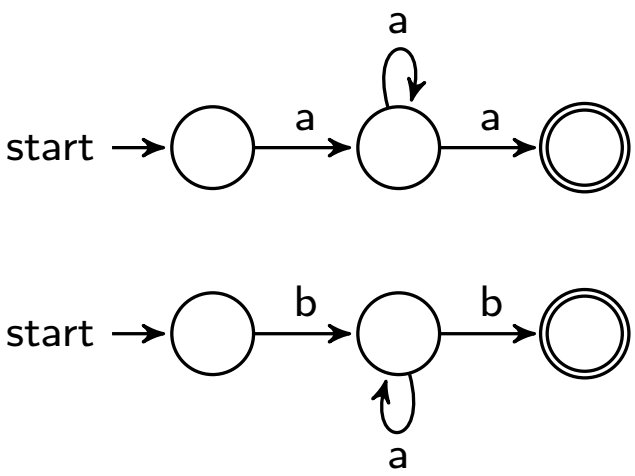
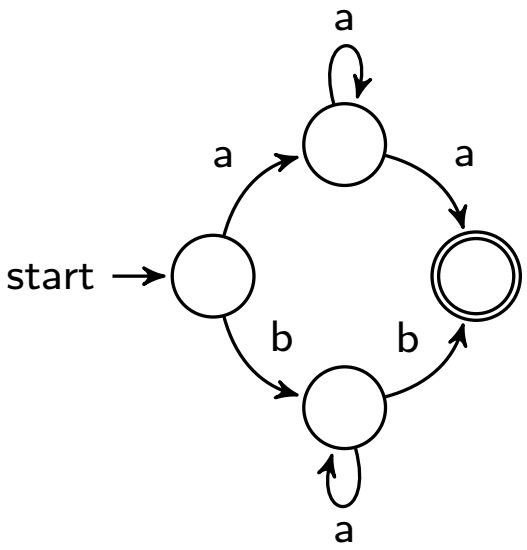


(a) $\gamma_3(Z)$ and $\gamma_3(Z')$.



(b) $\gamma_3(Z) \cup \gamma_3(Z')$.

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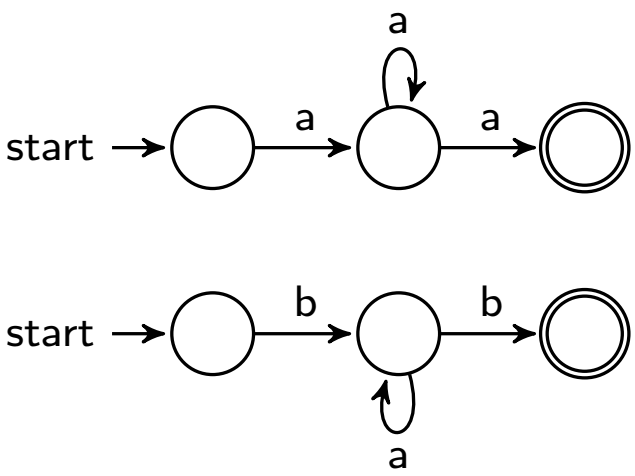

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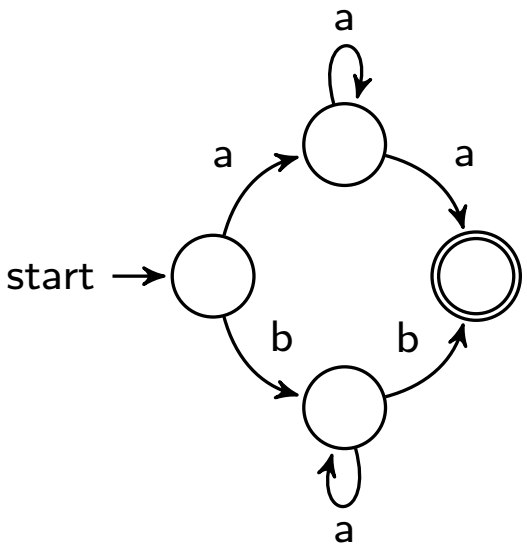

(b) $\gamma_3(Z) \cup \gamma_3(Z')$.

$$aab \in \gamma_3(Z \sqcup Z') \text{ but } aab \notin \gamma_3(Z) \cup \gamma_3(Z').$$

Learning Unions of k-Testable Languages

Theorem (Identification of unions in the limit)

Any language that is a union of k-testable languages can be identified in the limit from positive examples.

Distance

Definition (Size)

The size of a k-test vector $Z = \langle I, F, T, C \rangle$ is defined by:

$$|Z| \quad = |I| + |F| + |T| + |C \cap \Sigma^{<k-1}|.$$

Definition (Distance)

We define the distance between a pair of k-test vectors as:

$$d(Z, Z') \quad = |Z \triangle Z'|$$

Lemma (Metric)

Distance function is a metric on the set of k-test vectors.

Hierarchical Clustering Algorithm

Given a set $\mathcal{S}$ of words:

Hierarchical Clustering Algorithm

Given a set $\mathcal{S}$ of words:

1. compute k-test vectors $s = \{\alpha_k(\{x\}) \mid x \in \mathcal{S}\}$

Hierarchical Clustering Algorithm

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2. compute distance matrix D of vectors in s

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Given a set $\mathcal{S}$ of words:

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3. until no more merges are possible:

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   1. find closest pair of vectors Z and Z' s.t. $\gamma_k(Z \sqcup Z') = \gamma_k(Z) \cup \gamma_k(Z')$

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   2. replace Z and Z' by $Z \sqcup Z'$ in s

Hierarchical Clustering Algorithm

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3. until no more merges are possible:
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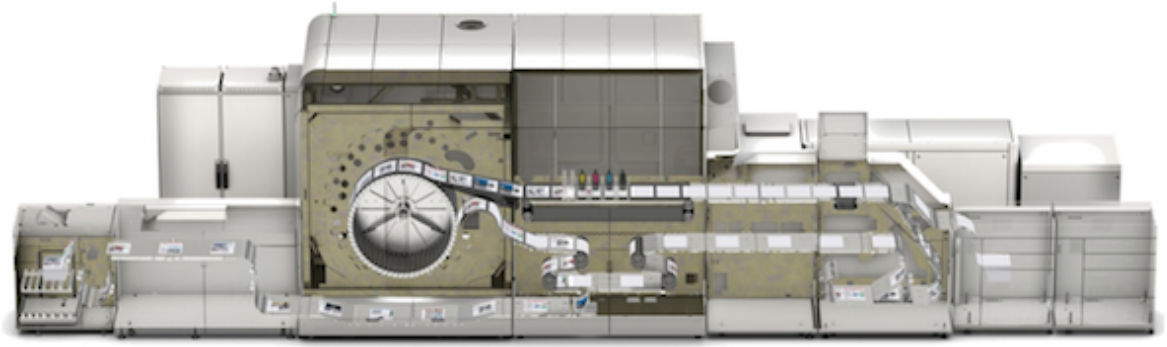
   2. replace Z and Z' by $Z \sqcup Z'$ in s
   3. update distance between $Z \sqcup Z'$ and remaining vectors in s

Case Study Océ

job	pattern	3-test vector	type of job
$aaaaa$	a^+	$Z = \langle \{aa\}, \{aa\}, \{aaa\}, \{a, aa\} \rangle$	homogeneous
$aaaaaaaaaa$			
$aaaaa \ldots aaa$			
$abababab$	$(ab)^+$	$Z = \langle \{ab\}, \{ab\}, \{aba, bab\}, \{ab\} \rangle$	heterogeneous
$abababababab$			
$abcabcabc$	$(abc)^+$	$Z = \langle \{ab\}, \{bc\}, \{abc, bca, cab\}, \{\} \rangle$	
$abcabcabcabcabc$			
$abcbcbcbca$	$a(bc)^+a$	$Z = \langle \{ab\}, \{ca\}, \{abc, bcb, cbc, cba\}, \{\} \rangle$	booklet



A Simple Galois Connection for Handling Subalphabets

Theorem

Let, for $i = 1, 2$, $\mathcal{M}_i = \langle I_i, O_i, Q_i, q_i^0, \rightarrow_i \rangle$ be (nondeterministic) Mealy machines with $I_1 \supseteq I_2$ and $O_1 = O_2$. Then

$$\mathcal{M}_1 \downarrow I_2 \leq \mathcal{M}_2 \quad \Leftrightarrow \quad \mathcal{M}_1 \leq \mathcal{M}_2 \uparrow I_1.$$

Here $\mathcal{M}_1 \downarrow I_2$ removes all transitions with input label not in I_2, and $\mathcal{M}_2 \uparrow I_1$ adds transitions to a chaos state for all inputs not in I_2.

A Galois Connection for Action Refinement

Assume we have sets X and Y of abstract inputs and outputs, and sets I and O of concrete inputs and outputs. An action refinement ρ is a pair of injective functions

$$\rho_i : X \to I^+ \qquad \rho_o : Y \to O^+$$

such that $\rho_i(x) \leq \rho_i(x') \Rightarrow x = x'$.

Galois Connection

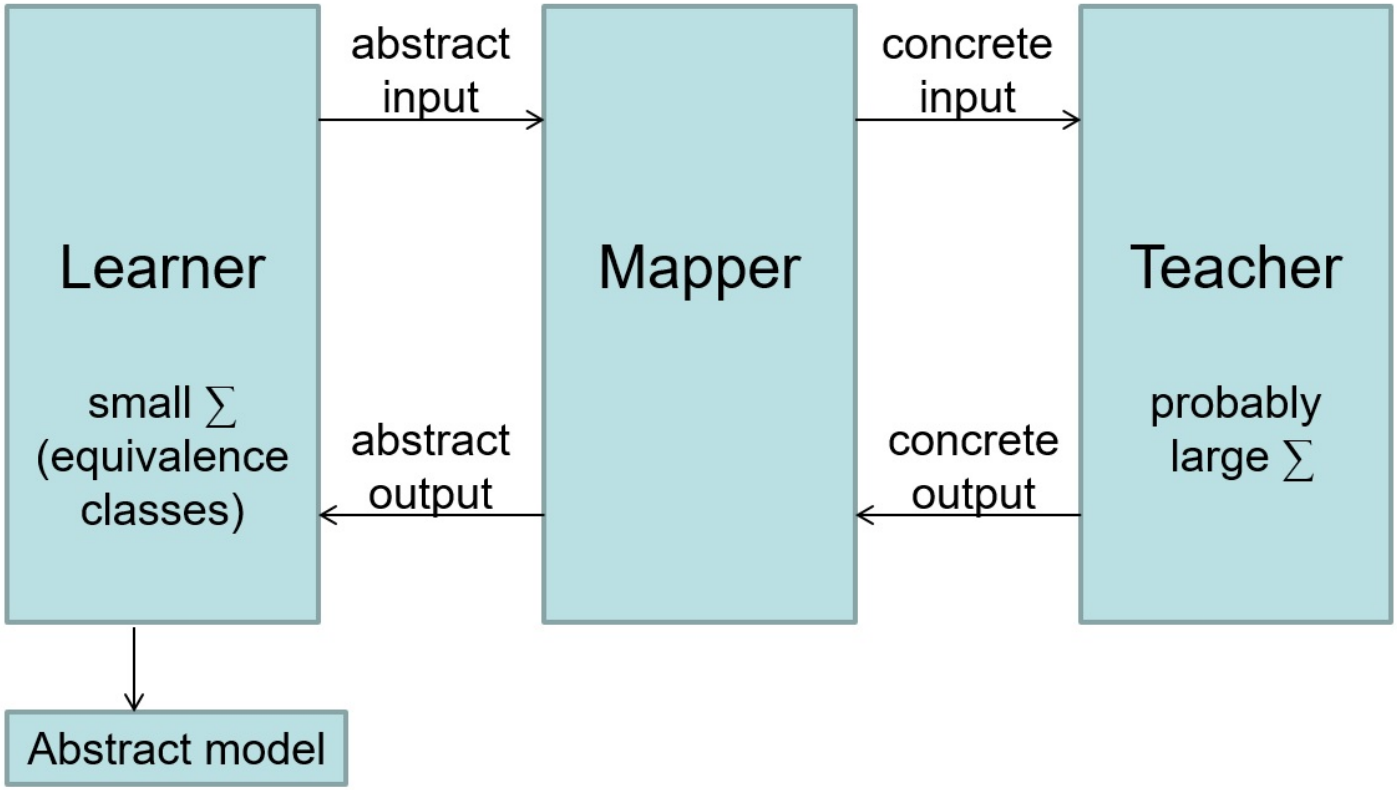
Then we can define monotone abstraction operators α_ρ and concretization operators γ_ρ such that:

Theorem

Let $\mathcal{M}$ be a Mealy machine over I and O, and let $\mathcal{N}$ be a Mealy machine over X and Y. If $\mathcal{M}$ and $\mathcal{N}$ "respect" refinement ρ then

$$\alpha_\rho(\mathcal{M}) \leq \mathcal{N} \quad \Leftrightarrow \quad \mathcal{M} \leq \gamma_\rho(\mathcal{N})$$

A Theory of Mappers (AJUV, 2015)



Transducers

Definition (Mapper)

A mapper for a set of inputs I and a set of outputs O is a deterministic Mealy machine $\mathcal{A} = \langle I \cup O, X \cup Y, R, r_0, \delta, \lambda \rangle$, where

- I and O are disjoint sets of concrete input/output symbols,
- X and Y are finite sets of abstract input/output symbols, and
- $\lambda : R \times (I \cup O) \to (X \cup Y)$, the abstraction function, respects inputs and outputs, that is, for all $a \in I \cup O$ and $r \in R$, $a \in I \Leftrightarrow \lambda(r, a) \in X$.

A Galois Connection that is Quite Useful

To every mapper $\mathcal{A}$ we may associate an abstraction operator $\alpha_{\mathcal{A}}$ and a concretization operator $\gamma_{\mathcal{A}}$. Then

Theorem

For a "surjective" mapper $\mathcal{A}$ and (nondeterministic) Mealy machines $\mathcal{M}$ and $\mathcal{H}$,

$$\alpha_{\mathcal{A}}(\mathcal{M}) \leq \mathcal{H} \quad \Leftrightarrow \quad \mathcal{M} \leq \gamma_{\mathcal{A}}(\mathcal{H})$$

Conclusions

Automata learning is emerging as a highly effective bug-finding technique, and slowly becoming a standard tool in the toolbox of the software engineer.

Future Work

1. Improved algorithms for black-box learning/testing FSMs
2. Better understanding of role Galois connections in learning; algorithms for finding Galois connections automatically
3. From Mealy machines to I/O automata
4. Learning EFSMs
5. Combinations of black-box and white-box learning
6. Algorithms for models with time and probabilities
7. Refactoring of legacy software excellent application domain