(recent developments in)

Flavour Physics

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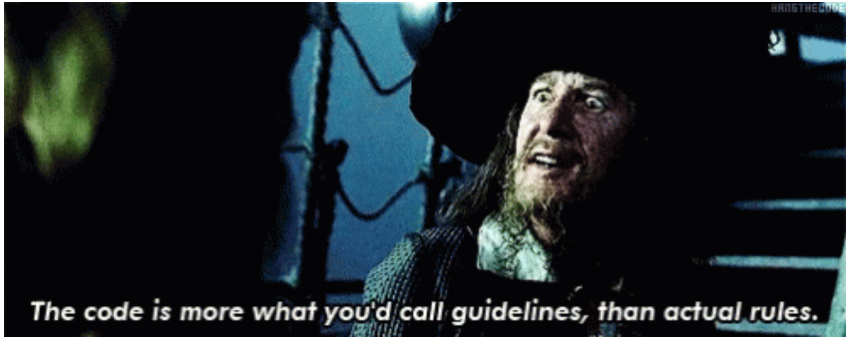
Slides prepared for the BND school 2023

Preliminary remarks

- lectures aimed at non-experts!

- “undemocratic” presentation
  - experimental part focused on B-hadron physics
  - missing a.o.: physics with kaons and D, electric-dipole moments, lepton-flavour violation, neutrinos, …

- please interrupt!
  - slides are more of ‘guideline’



Reference material

- books
  - Branco, Lavoura, Silva: “CP Violation”
  - Bigi and Sanda: “CP Violation”
  - …

- lecture notes (from some of the greatest, certainly not a complete list)
  - Y.Nir, “Flavour physics and CP Violation”, https://arxiv.org/abs/1605.00433
  - R. Fleischer, “B Physics and CP Violation”, http://arxiv.org/abs/hep-ph/0210323v3
  - A. Buras, “Flavour dynamics”, http://arxiv.org/abs/hep-ph/0101336v1
  - A. Lenz, https://www.tp.nt.uni-siegen.de/~lenz/Lecture_Flav_2021pdf
  - Y. Grossman and P. Tanedo: https://arxiv.org/abs/1711.03624
  - N. Tuning, “CP Violation”, http://www.nikhef.nl/~h71/Lectures/2020/ppII-cpviolation-14022020.pdf

Course organization

- 4 lectures of 90 minutes
  - ~60 minutes oral lecture 😅
  - ~30 minutes exercises
    https://github.com/wouterhuls/FlavourPhysicsBND2023/

- for the exercises need
  - laptop, pen and paper
  - access to Jupyter (your own installation, or Google Colab, SWAN, …)

- ack's: heavily borrowed from slides by Niels Tuning and Marcel Merk

Flavour physics lectures (4x45 minutes)

1. Flavour in the Standard Model
2. Neutral meson mixing
3. CP violation + experiments
4. Rare decays + recent developments

Let’s start with a few ‘Existential Questions” …

Existential questions

- universe's basic building blocks: electron, proton, neutron and neutrino
- consider their masses
  - neutrino: < 1 eV
  - electron: 0.5 MeV
  - proton: 938.27 MeV
  - neutron: 939.57 MeV

- why is the proton lighter than the neutron?
- what if it would be heavier?
- what if the electron were 4x heavier?

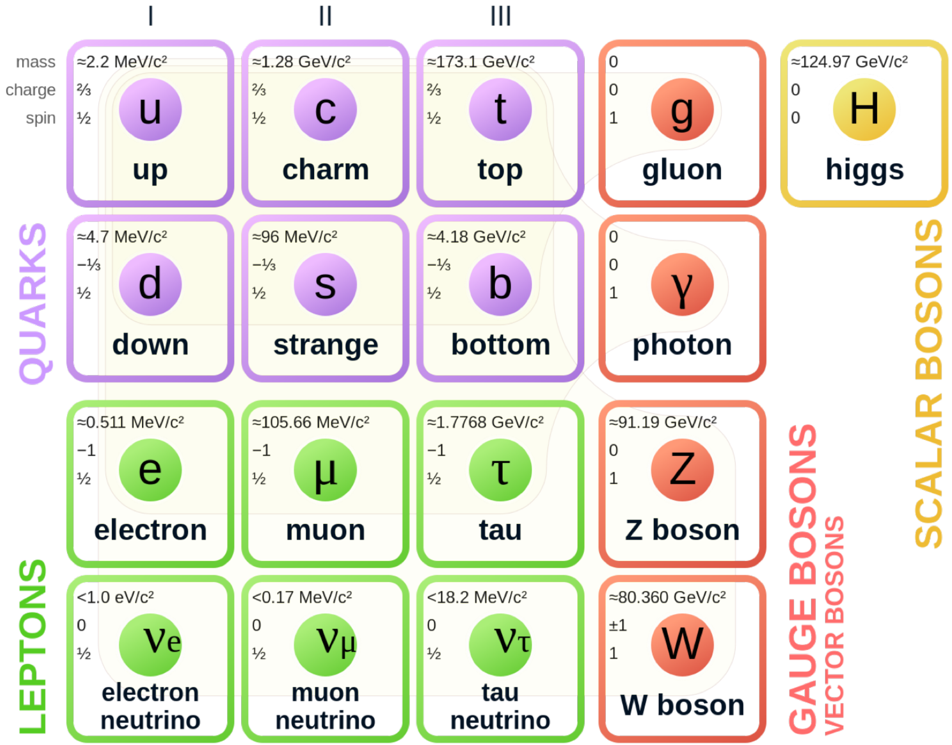
Existential questions

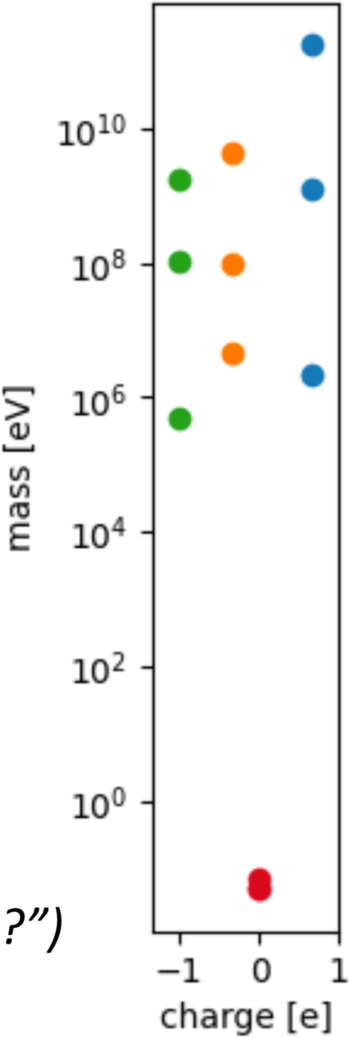
- universe’s basic building blocks: electron, up-quark, down-quark and neutrino
- consider their masses
  - neutrino: < 1 eV
  - electron: 0.5 MeV
  - up-quark: 2.2 MeV*
  - down-quark: 4.7 MeV*

- why is the up-quark lighter than the down-quark?
- what if it would be heavier?
- what if the electron were 4x heavier?

See R.Cahn, *The eighteen arbitrary parameters of the standard model in your everyday life* Rev. Mod. Phys., Vol. 68, No. 3, (1996)

Existential questions





- why are there three families? *(Rabi, 1936: “who ordered that?”)*
- why are the mass-scales so different?

Existential questions

weak interaction
quark mixing matrix

$$|V^{\mathrm{CKM}}| = \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.974 & 0.042 \\ 0.009 & 0.0413 & 0.999 \end{pmatrix}$$

weak interaction
lepton mixing matrix

$$|U^{\mathrm{PMNS}}| = \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.37 & 0.58 & 0.70 \\ 0.39 & 0.59 & 0.69 \end{pmatrix}$$

- why is the CKM matrix almost diagonal?
  - is there a relation between the mass hierarchy and the weak mixing?
- why is mixing in the lepton sector so different?
  - do neutrino masses have another source?

Existential questions

- why do we live in a matter dominated universe?

observation:

$$\frac{n_b - n_{\bar{b}}}{n_\gamma} \approx 6 \cdot 10^{-10}$$

SM prediction:

$$\frac{n_b - n_{\bar{b}}}{n_\gamma} \approx 10^{-18}$$

The Anthropic Principle?

- “What we observe is biased by our own existence.” (Brandon Carter, ‘73)

- for the science, see e.g. *(reference only, I didn’t read them yet!)*
    - “The Anthropic Landscape of String Theory”, L. Susskind (2003)
    - “The Emperor's Last Clothes?”, B. Schellekens (2008)

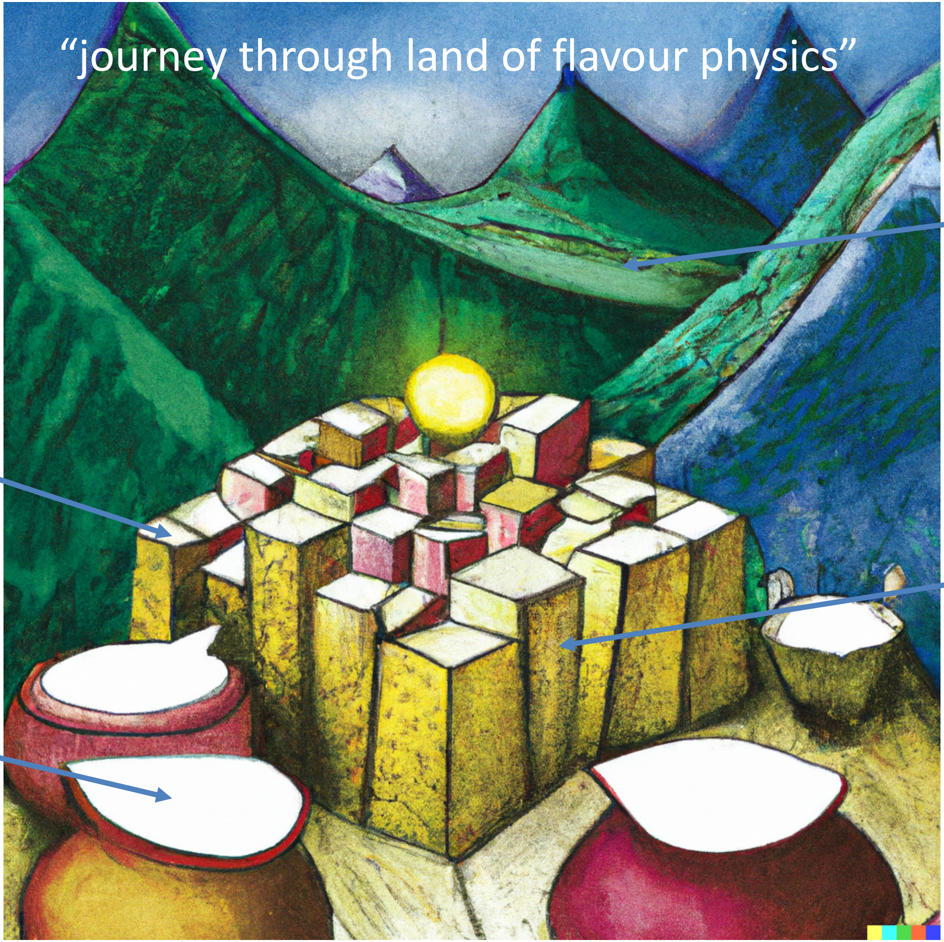
Explaining flavour?

- may never be able to ‘understand’ the 25+ parameters of the SM
  - understanding ‘why’ may be a matter of showing that from all the 10^500 string vacua ours is not an unlikely one

- still want to understand the dynamic principles of our universe
  - SM is not complete
    - what is dark matter, energy, quantized gravity?
    - what mechanism lead to a matter dominated universe?
  - it is believed that electroweak symmetry breaking and flavour physics plays central role in some of these questions

- so … let’s embark on a tour of “flavour physics”!

An AI view

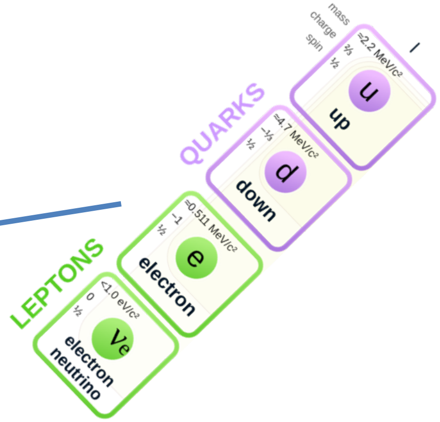


“journey through land of flavour physics”

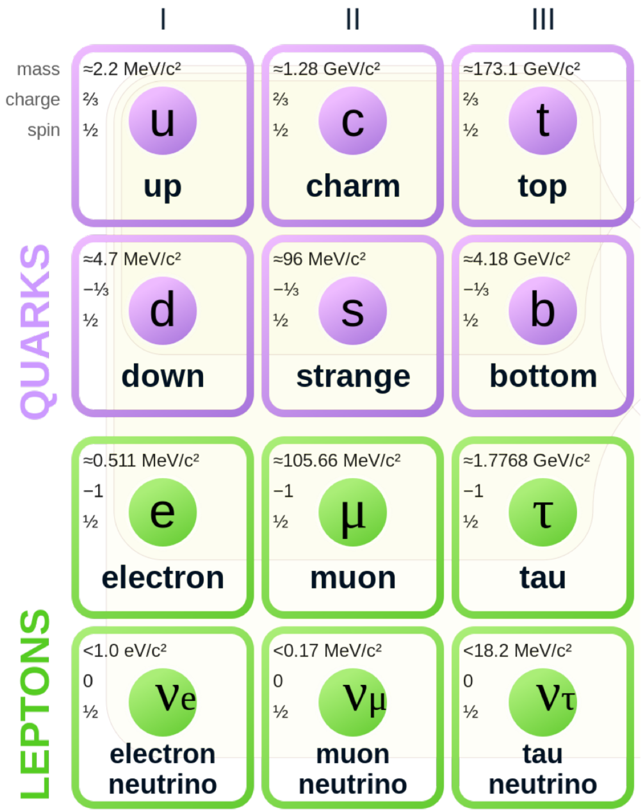
new gauge sectors with hidden valley?

4th and 5th family?

a can of ‘milk’?



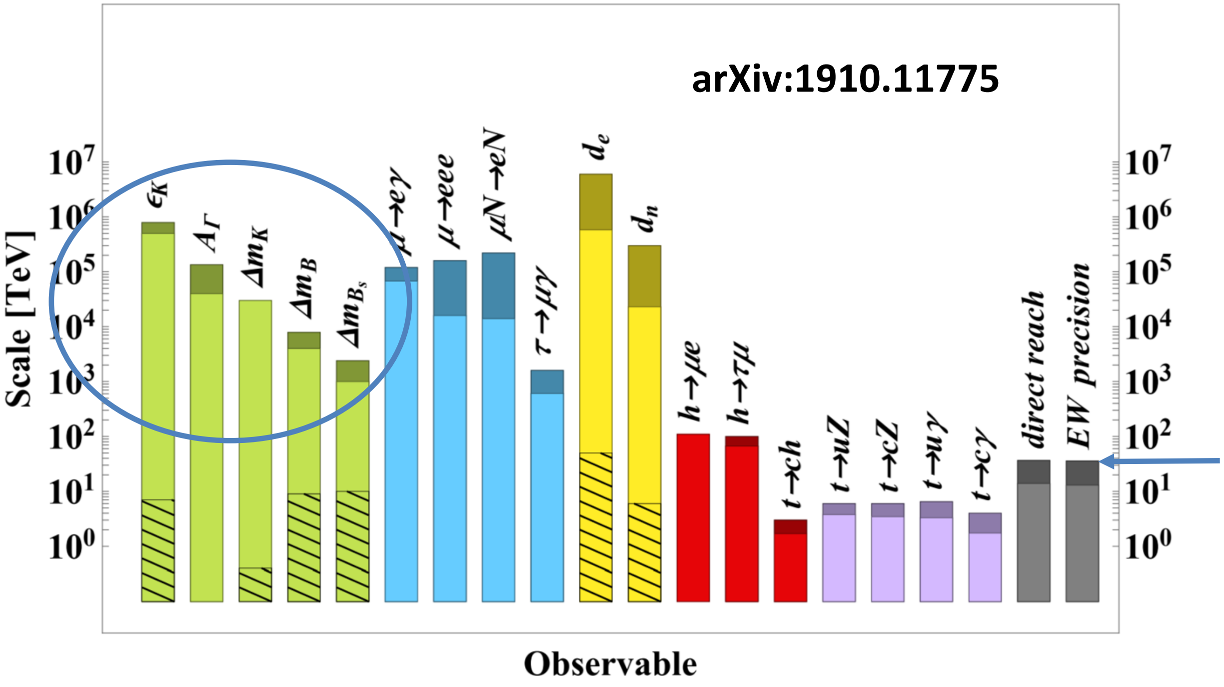
Flavour and the weak interaction



- EM and strong interaction ‘conserve flavour’
- only weak interaction allows for flavour-changing transitions
- ‘flavour physics’ is physics of the weak interaction and electro-weak symmetry breaking

Why flavour physics?

- flavour observables are very **sensitive to new physics at higher mass scales**
- this holds in particular for ‘**mixing**’, ‘**CP violation**’ and ‘**rare decays**’

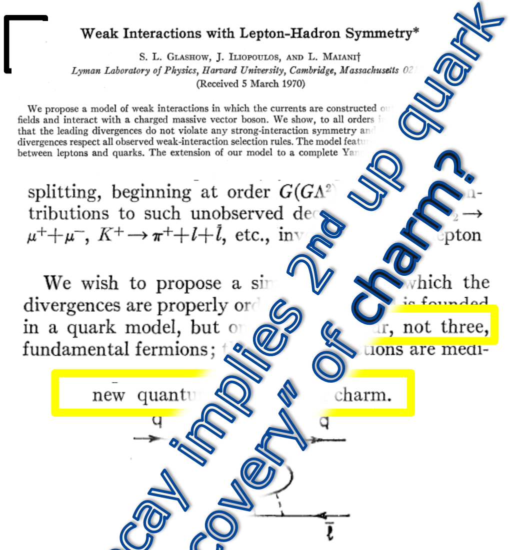


arXiv:1910.11775

LHC

Flavour physics: a tool for discovery

GIM mechanism in K⁰→μμ



Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02

(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed o fields and interact with a charged massive vector boson. We show, to all orders i that the leading divergences do not violate any strong-interaction symmetry an divergences respect all observed weak-interaction selection rules. The model featu between leptons and quarks. The extension of our model to a complete Yan

splitting, beginning at order $G(G\Lambda^2$ n- tributions to such unobserved de $_2 \to$ $\mu^+ + \mu^-$, $K^+ \to \pi^+ + l + \bar{l}$, etc., inv epton

We wish to propose a si which the divergences are properly or is founded in a quark model, but o r, not three, fundamental fermions; tions are medi-

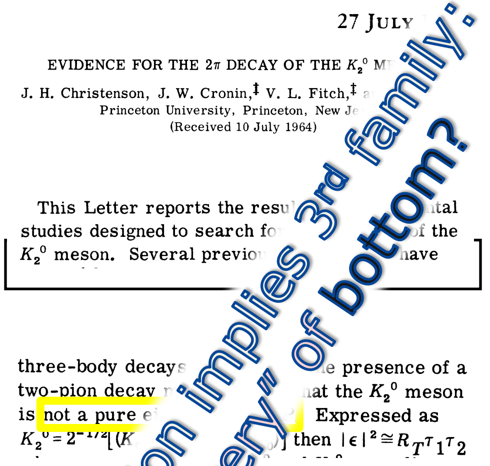
new quantu charm.

Rare decay implies 2nd up quark

“discovery” of charm?

Iliopoulos, Maiani,
v. D2 (1970) 1285

CP violation, $K_L^0 \to \pi\pi$



27 July

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 M

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ a

Princeton University, Princeton, New Je

(Received 10 July 1964)

This Letter reports the resu ntal studies designed to search fo of the K_2^0 meson. Several previo have

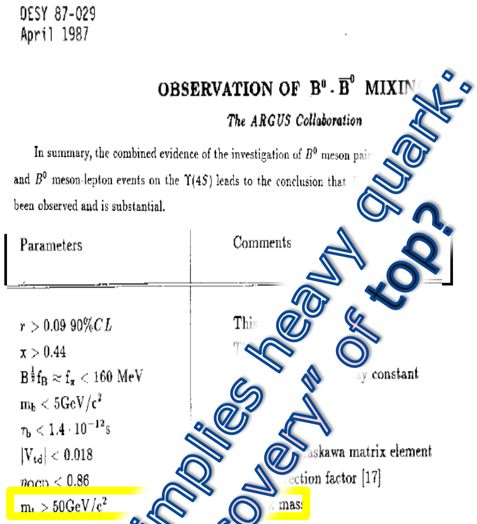
three-body decays e presence of a two-pion decay at the K_2^0 meson is not a pure e Expressed as $K_2^0 = 2^{-1/2}[(K$] then $|\epsilon|^2 \cong R_T \tau_1 \tau_2$

CP violation implies 3rd family:

“discovery” of bottom?

on, Cronin, Fitch, Turlay,
v.Lett. 13 (1964) 138-140

$B^0 \leftrightarrow \overline{B^0}$ mixing



DESY 87-029
April 1987

OBSERVATION OF $B^0 - \overline{B}^0$ MIXIN

The ARGUS Collaboration

In summary, the combined evidence of the investigation of B^0 meson pair and B^0 meson-lepton events on the $\Upsilon(4S)$ leads to the conclusion that been observed and is substantial.

Parameters	Comments
$r > 0.09$ 90%CL	This
$x > 0.44$	T
$B^{\frac{1}{2}} f_B \approx f_\pi < 160$ MeV	y constant
$m_b < 5\mathrm{GeV}/c^2$	
$\tau_b < 1.4 \cdot 10^{-12}$s	
$\lvert V_{td} \rvert < 0.018$	askawa matrix element
$\eta_{QCD} < 0.86$	ection factor [17]
$m_t > 50\mathrm{GeV}/c^2$	mass

Mixing implies heavy quark:

“discovery” of top?

S Coll.
s.Lett.B192:245,1987

(courtesy: N. Tuning)

Is flavour physics `complicated'?

- less-intuitive concepts: imaginary phases, different bases, oscillations

- difficult computations
  - lot's of Feynman diagrams
  - bound states, non-perturbative QCD, approximate symmetries

- very extensive phenomenology
  - e.g. PDG full of decay modes ("*Beetokaipaigamma…*")
  - need to develop some intuition for what is interesting

-> <u>aim</u>: make you understand a little more on your next HEP conference!

DISCRETE SYMMETRIES

Symmetries

Nobel Laureate T.D. Lee:

”The root of all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; these will be called ‘non-observables’.”

symmetry ⇔ unobservable ⇔ conserved quantity

Symmetries

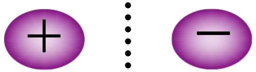
symmetry	⟺	example of unobservable
A. permutation symmetries		absolute identity of particle
B. continuous space time symmetries		absolute position, orientation, time
C. discrete symmetries (C,P,T)		handedness, direction of time, definition of sign of charge
D. internal (or ‘unitary’) symmetries		phase of a wave function

Emmy Noether: continuous symmetry (case B,D) → conservation law

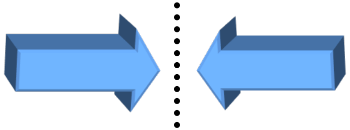
Discrete symmetries

- suppose we watch some physical process.
  can we determine unambiguously whether or not …
  - we are watching the process where all ***charges are reversed*** ?
  - we are watching the process ***through a mirror***?
  - we are watching the process in a ***film running backwards***?

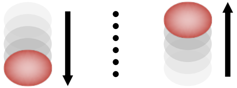
- *C*: charge conjugation



- *P*: parity transformation



- *T*: time reversal



Discrete symmetries

- classical theories invariant under C, P, T operations
  - Newton mechanics, Maxwell electrodynamics, QM
  - it is said these “conserve *C*, *P*, *T* symmetry”

- CPT theorem:
  “Lorentz invariant local quantum field theory
  with a Hermitian Hamiltonian must obey ***CPT*** symmetry”

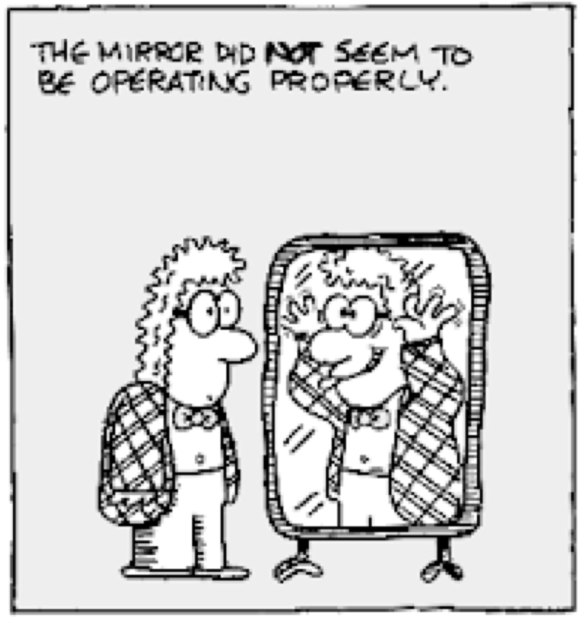
Parity transformation

parity transformation ***P***: inversion of *spatial* coordinates

$$\boxed{\vec{x} \rightarrow -\vec{x}}$$

equivalent to: mirror transformation in one axis followed by 180-degree rotation

→ often depicted by ‘mirror’



Time evolution in Heisenberg picture

consider process: $\phi_i \rightarrow \phi_f$

time evolution: $\boxed{\phi_f \;=\; \hat{U}_{fi}\,\phi_i}$

$\boxed{\hat{U}(t) \;=\; e^{-iHt/\hbar}}$ time evolution operator

Time evolution of mirror process

now consider the process in the mirror: $\boxed{\phi'_i = \hat{P}\phi_i}$

process is ‘symmetric under P’ if applying parity transformation *after* time-evolution is leads to same result as applying it *before*

$$\begin{tikzcd}
\phi_i \arrow[r] \arrow[d, "\boldsymbol{P}"'] & \phi_f \arrow[d] \\
\phi'_i \arrow[r, "\text{time}"'] & \phi'_f
\end{tikzcd}$$

parity conservation:

$$\boxed{\hat{P}\hat{U}\phi_i = \hat{U}\hat{P}\phi_i}$$

$$\Downarrow$$

$$\boxed{[\hat{P},\hat{U}] = 0} \Leftrightarrow \boxed{[\hat{P},H] = 0}$$

parity quantum number

operator commuted with H → conserved quantum number

$$\boxed{[\hat{P}, H] = 0}$$

⇒

P and H have common set of eigenvectors
with definite value for quantum number ‘parity’

applying parity twice brings us back where we were:

$$\boxed{\hat{P}^2 \, \phi_i \; = \; \phi_i}$$

⇒

eigenvalues are +1 and -1

caveat: could add arbitrary phase factor

parity even

parity odd

Is ‘parity’ a good quantum number?

general assumption until 1956: “laws of physics symmetric under parity”

in math: $\boxed{[\hat{P}, H] = 0}$

$$\boxed{H = H_{\text{free}} + H_{\text{EM}} + H_{\text{strong}} + H_{\text{weak}}}$$

well tested for electromagnetic and strong interaction (and gravity)

⇨ elementary particles must have ‘definite parity’

but do they?

The theta-tau puzzle

- around 1950, observation of two weakly decaying states with different parity:

$$\begin{aligned} \theta^+ \quad &\to \pi^+ + \pi^0 \\ \tau^+ \quad &\to \pi^+ + 2\pi^0 \quad \text{or} \quad 2\pi^+ + \pi^- \end{aligned}$$

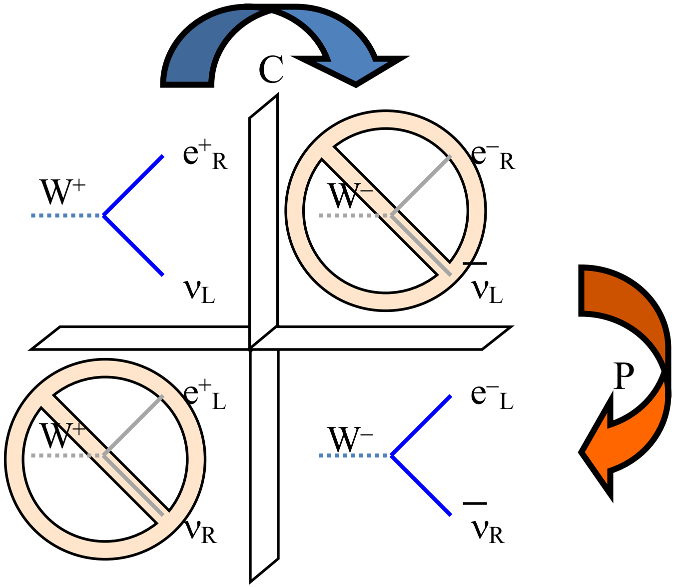
pion has odd parity →
- theta has even parity
- tau has odd parity

- big puzzle: why do tau and theta have **same mass and lifetime**?

- Lee & Yang in 1956:
  simplest explanation: this is one and the same particle,
  but **weak interaction violates parity symmetry**

- quick experimental confirmation (Wu, Ledermann, …)

C and *P* symmetry in the weak interaction



- weak interaction breaks ***C*** and ***P*** symmetry maximally
  - W couples to left-handed particles and right-handed anti-particles
- how about combined ***CP*** symmetry?

CP symmetry

- “CP symmetry” for fundamental processes:

$$\mathcal{P}(A \to B) \ = \ \mathcal{P}(\bar{A} \to \bar{B})$$

- In 1964, Christenson, Cronin, Fitch and Turlay observed ***CP*** *violation* (CPV) in decays of neutral kaons
  - can only properly explain their measurement tomorrow

- important for our story
  - CP violation is essential ingredient to understanding matter-anti-matter asymmetry in universe (“Sacharov Conditions”)
  - in the SM it originates from non-trivial phases in Higgs Yukawa couplings

C and P quantum numbers in the PDG

https://pdglive.lbl.gov

$\pi^{\pm} \qquad I^G(J^P) = 1^-(0^-)$

$\pi^{0} \qquad I^G(J^{PC}) = 1^-(0^{-+})$

$J:$ spin ('internal' angular momentum)

rotation symmetry

$P:$ parity
$C:$ charge conjugation

discrete symmetries
(without H_{weak})

$I:$ (strong) isospin
$G:$ G-parity ($G = Ce^{i\pi I_2}$)

SU(2) u <--> d symmetries
(without H_{EM} and H_{weak})

Discrete symmetry summary

- discrete symmetries: C, P, T
- CPT theorem: every reasonable theory obeys CPT symmetry
- strong and EW interaction are C, P and T symmetric
- weak interaction
  - maximally violated P and C symmetry
  - violates CP symmetry a little bit
- matter and anti-matter differ at the fundamental level

FLAVOUR IN THE STANDARD MODEL

Building the Standard Model

- ingredients to build renormalizable model
  1. choose gauge symmetries
  2. choose representation of matter fields under symmetries
  3. choose pattern of symmetry breaking
  4. add any other term that is renormalizable and does not break gauge invariance

- will introduce these concepts on next slides, though not exactly in this order

Step 1: massless fermion matter fields

- Dirac Lagrangian for set of **massless** fields

$$\boxed{\mathcal{L}_{\text{fermions}} = \sum_k i\overline{\psi}_k \gamma^\mu \partial_\mu \psi_k} \qquad \psi \in \{u_{i,\alpha}, d_{i,\alpha}, \ell_i, \nu_i\}$$

- sum includes
  - up quarks, down quarks, charged leptons, neutrinos
  - 3 generations (or ‘families’)
  - 3 versions of each quark (colour)
  - independent left and right components for each field (“chiral theory”)

Step 2: introduce gauge symmetry

- make doublets of the left-handed u/d fields

$$Q_i = \begin{pmatrix} u_{L,i} \\ d_{L,i} \end{pmatrix} \qquad D_i = d_{R,i} \qquad U_i = u_{R,i}$$

$$L_i = \begin{pmatrix} \nu_{L,i} \\ \ell_{L,i} \end{pmatrix} \qquad E_i = \ell_{R,i} \qquad N_i = \nu_{R,i}$$

- choose the gauge symmetry

$$U(1)_Y \otimes SU(2)_L \otimes SU(3)_{\mathrm{c}}$$

- choose the representation

field	$(\mathrm{c}, L)_Y$
Q^I	$(3, 2)_{1/3}$
L^I	$(1, 2)_{-1}$
u_R^I	$(3, 1)_{4/3}$
d_R^I	$(3, 1)_{2/3}$
ℓ_R^I	$(1, 1)_{-2}$
ν_R^I	$(1, 1)_0$

gauge transformations

- gauge transformation of Dirac fields

$$\boxed{\Psi' \;=\; \underbrace{e^{i\alpha(x)Y}}_{\text{“U(1) hypercharge”}} \; \underbrace{e^{i\beta(x)T}}_{\text{“SU(2) weak iso-spin”}} \; \underbrace{e^{i\gamma(x)L}}_{\text{“SU(3) color”}} \; \Psi}$$

- principle of local gauge invariance: $\boxed{\mathcal{L}' \;=\; \mathcal{L}}$

add covariant derivatives → gauge interactions

- introduce the covariant derivative (local gauge invariance)

$$\partial^\mu \to D^\mu \equiv \partial^\mu + ig_s \sum_a G_a^\mu L_a + ig \sum_b W_b^\mu T_b + ig' B^\mu Y$$

G,W,B: gauge (vector) fields

L,T,B: symmetry group generators

g_s, g, g’: **universal** coupling constants

- identical for quark/leptons
  (but some freedom in choosing Y)
- identical for all generations:
  flavour universality

add kinetic terms for gauge fields

- add kinetic terms for gauge bosons to complete Lagrangian

$$\boxed{\mathcal{L}_{\text{kinetic}} \;=\; \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{gauge bosons}} + \mathcal{L}_{\text{interactions}}}$$

$$\boxed{\overline{\psi}\left(i\gamma^{\mu}\partial_{\mu}\right)\psi}$$

free massless fermions

$$\boxed{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}$$

free massless gauge fields

$$\boxed{-\frac{g}{2}\overline{\psi}\gamma^{\mu}B_{\mu}Y\psi}$$

interaction terms

- up to this point fields are **massless**: mass terms break gauge invariance

mass terms?

- Dirac mass terms are

$$\boxed{\mathcal{L}_{\text{Dirac mass}} \;=\; -m\,\overline{\psi}\psi \;=\; -m\left(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R\right)}$$

- break gauge symmetry because left- and right-handed components transform differently (‘chiral theory’)

Step 3: introduce symmetry breaking

- add scalar complex doublet (4 real degrees-of-freedom)

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

representation under gauge group:

$$(c, L)_Y = (1, 2)_{+1}$$

- give it a Mexican-hat mass-term: this does not break symmetry

$$\boxed{\mathcal{L}_{\rm Higgs} \;=\; (D^\mu \phi^\dagger)(D_\mu \phi) - V(\phi)}$$

kinetic term

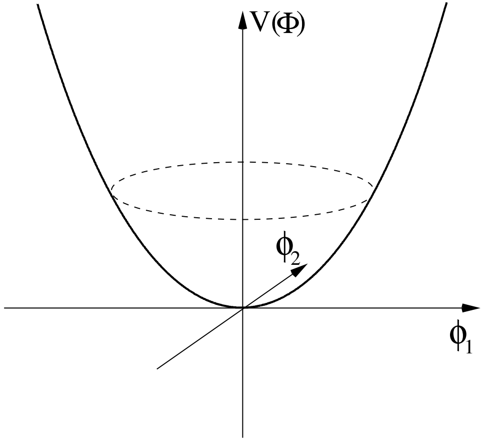
potential

$$\boxed{V(\phi) \;=\; \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2}$$

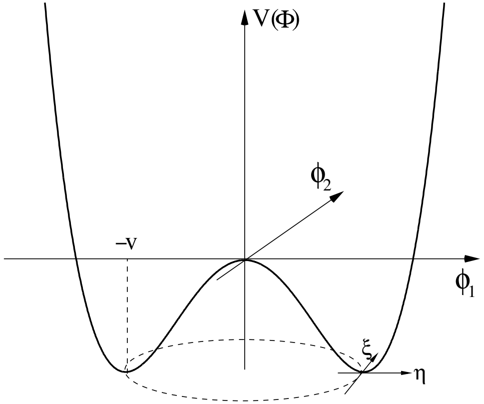
Step 3: introduce symmetry breaking

- choose parameters such that ground state has ‘broken symmetry’

$\mu^2 > 0$



$\mu^2 < 0$



- symmetry broken by *vacuum expectation value* (“vev”)

$$\boxed{v = \sqrt{\frac{-\mu^2}{\lambda}}}$$

Step 3: introduce symmetry breaking

- from all possible ground states, choose one where φ^0 has v.e.v.

$$\boxed{\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\xi_1 + i\,\xi_2\\ v + h + i\,\xi_3\end{pmatrix}}$$

$$\boxed{v = \sqrt{\frac{-\mu^2}{\lambda}}}$$

v: constant Higgs ‘vacuum expectation value’

h: dynamic real neutral scalar Higgs field

ξ: ‘eaten’ by SU(2) gauge bosons to give mass to W+, W- and Z

Step 4: add anything else allowed

- add terms that
  - do not break the gauge invariance
  - are renormalizable

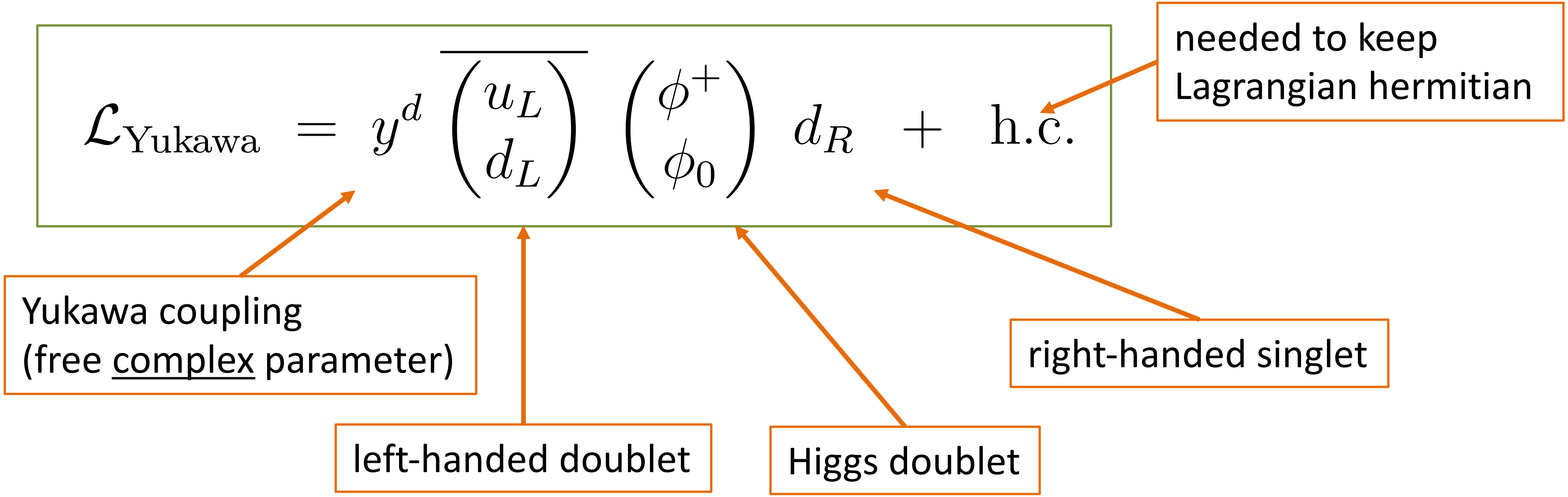
(this can be done before/after symmetry breaking: makes no difference)

- two kinds
  - “Higgs Yukawa interactions”
  - “Majorana neutrino mass” (Weinberg operator; will skip this)

Adding Yukawa interactions

$$\Psi' = e^{i\alpha(x)Y} \, e^{i\beta(x)T} \, e^{i\gamma(x)L} \, \Psi$$

- one example: for right-handed down quarks



$$\mathcal{L}_{\text{Yukawa}} = y^d \, \overline{\begin{pmatrix} u_L \\ d_L \end{pmatrix}} \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} d_R + \text{h.c.}$$

needed to keep Lagrangian hermitian

Yukawa coupling (free <u>complex</u> parameter)

left-handed doublet

Higgs doublet

right-handed singlet

- to make this work it is essential that Higgs doublet has $Y = Y_L - Y_R = +1$

Adding Yukawa interactions

- all Yukawa terms (in compact form)

$$\boxed{\mathcal{L}_{\text{Yukawa}} \;=\; y^d_{ij}\,\overline{Q}_i\,\phi\,D_j \;+\; y^u_{ij}\,\overline{Q}_i\,\tilde{\phi}^c\,U_j \;+\; (\text{leptons}) \;+\; \text{h.c.}}$$

- constraints from gauge symmetry:
  - terms that 'mix' leptons and quarks break $U(1)_Y$
  - **terms that 'mix' families are fine!**

- note: it is traditional to leave ν_R term away (but not well motivated anymore!)

Yukawa terms after symmetry breaking

$$\boxed{\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h\end{pmatrix}}$$

$$\boxed{\mathcal{L}^d_{\mathrm{Yukawa}} = y^d_{ij}\,\overline{\begin{pmatrix}u_{L,i}\\ d_{L,i}\end{pmatrix}}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} d_{R,j} \to \frac{1}{\sqrt{2}}\,y^d_{ij} v\,\overline{d}_{L,i} d_{R,j} + \frac{1}{\sqrt{2}}\,y^d_{ij}\,h\,\overline{d}_{L,i}\,d_{R,j}}$$

mass term
(but not diagonal)

Hqq coupling

- mass terms proportional to Yukawa couplings and vev:

$$\boxed{m^d_{ij} \equiv \frac{v}{\sqrt{2}}\,y^d_{ij}}$$

Mass eigenstates

- up to now Lagrangian written in terms of ‘**interaction eigen states**’

$$\boxed{\mathcal{L} = \overline{Q_i^I}\,(i\gamma^\mu\partial_\mu)\,Q_i^I \quad - \; g\,\overline{Q_i^I}\gamma^\mu(\mathbf{W}_\mu\cdot\mathbf{T})\overline{Q_i^I} \quad + \; y_{ij}^d\,\overline{Q}_i^I\,\phi\,D_j^I \quad + \quad \ldots}$$

(have not been very consistent with the ‘superscript I’)

- if we scatter particles, compute things in terms of ‘**mass eigenstates**’
  - natural basis in QFT perturbation theory

- this means for us: diagonalize mass terms in Lagrangian

Diagonalizing mass matrices

- mass matrices

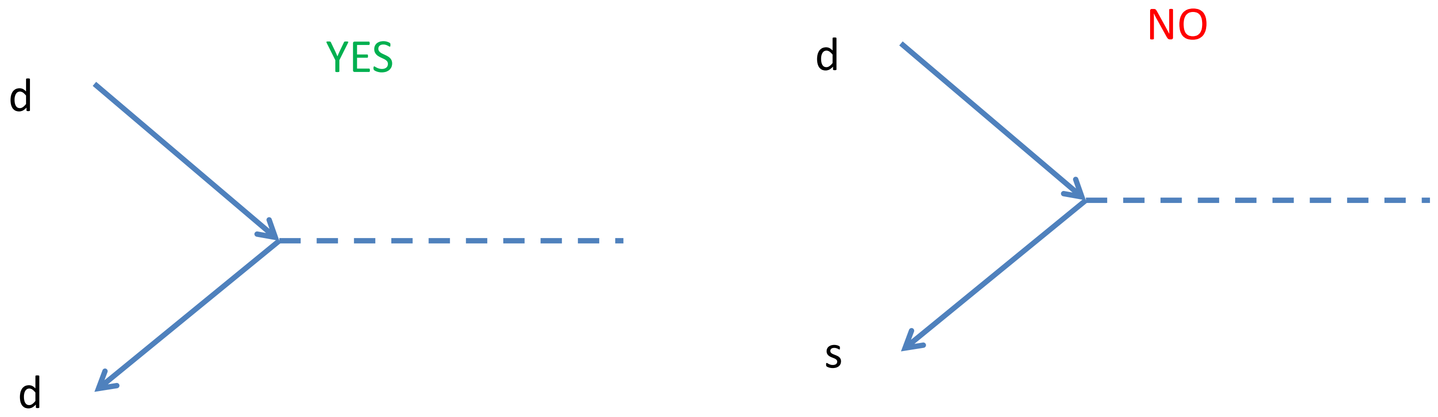
$$m_{ij}^{u} \equiv \frac{v}{\sqrt{2}} y_{ij}^{u} \qquad m_{ij}^{d} \equiv \frac{v}{\sqrt{2}} y_{ij}^{d} \qquad m_{ij}^{\ell} \equiv \frac{v}{\sqrt{2}} y_{ij}^{\ell} \qquad m_{ij}^{\nu} \equiv \frac{v}{\sqrt{2}} y_{ij}^{\nu}$$

- diagonalizing mass matrix is same as diagonalizing Yukawa matrices: Higgs-fermion interactions are diagonalized simultaneously

- two important **SM predictions**:
  - Higgs-fermion interaction strength is proportional to mass
  - no mixing of fermions from Higgs-fermion coupling: no “Higgs-induced flavour changing neutral coupling (“FCNC”)

Higgs FCNC

- “$\bar{\phi}\phi$h” terms couples mass eigenstates:



- no “Higgs induced flavour changing neutral couplings” (at tree level)

From your linear algebra course

- **complex matrix M** can be decomposed as

$$M = U_L^\dagger D U_R \qquad \Longleftrightarrow \qquad D = U_L M U_R^\dagger$$

U_L , U_R: **unitary matrices**
D: **diagonal matrix**

- decomposition is **not unique**
    - by changing phases column/row of U: can choose D **real and positive**
    - by re-arranging rows/columns of U: can choose **order** of diagonal elements

Diagonalizing mass matrix

- mass matrix:

$$m^q_{ij} = \left(U^{q\dagger}_L\right)_{ik} \underbrace{\tilde{m}_{kl}}_{\text{diagonal, real, positive}} (U^q_R)_{lj}$$

- mass term in Lagrangian:

$$\begin{aligned} \mathcal{L}^q_{\text{mass}} &= \bar{q}^I_{L,i}\, m^q_{ij}\; q^I_{R,j} \\ &= \bar{q}^I_{L,i}\; \left(U^{q\dagger}_L\right)_{ik} \tilde{m}^q_{kl}\, (U^q_R)_{lj}\;\; q^I_{R,j} \\ &\equiv \bar{q}_{L,i}\; \tilde{m}^q_{ij} q_{R,j} \end{aligned}$$

with "mass basis":

$$\begin{aligned} q_{L,i} &\equiv (U^q_L)_{ij}\, q^I_{L,j} \\ q_{R,i} &\equiv (U^q_R)_{ij}\, q^I_{R,j} \end{aligned}$$

How does this affect weak couplings?

$$\begin{aligned}\mathcal{L}_{\text{weak}} &= ig\sum_i \overline{Q_i^I}\gamma^\mu(\mathbf{W}_\mu\cdot\mathbf{T})\overline{Q_i^I} \\ &= \ldots \\ &= ig\sum_i\left[\overline{u}_{Li}^I\gamma^\mu\frac{W_\mu^+}{\sqrt{2}}d_{Li}^I + \overline{d}_{Li}^I\gamma^\mu\frac{W_\mu^-}{\sqrt{2}}u_{Li}^I + \overline{d}_{Li}^I\gamma^\mu W_\mu^0 d_{Li}^I + \overline{u}_{Li}^I\gamma^\mu W_\mu^0 u_{Li}^I\right]\end{aligned}$$

charged current

neutral current

- neutral weak current, and strong, and hypercharge:
  basis transformation has no effect!
- weak current: ‘u-d’ mix affected by basis transformation

The W^+ interaction term

$$\mathcal{L}_{W^+qq} = \sum_i \overline{u}^I_{L,i} \left(\frac{ig}{\sqrt{2}} \gamma^\mu W^+_\mu \right) d^I_{L,i}$$

insert mass basis

$$= \sum_{i,k,l} \overline{u}_{L,k} \, (U^u_L)_{ik} \left(\frac{ig}{\sqrt{2}} \gamma^\mu W^+_\mu \right) \left(U^{d\dagger}_L \right)_{il} d_{L,l}$$

combine U_R and U_L

$$\equiv \sum_{i,j} \overline{u}_{L,i} \left(\frac{ig}{\sqrt{2}} \gamma^\mu W^+_\mu \right) V^{CKM}_{ij} \, d_{L,j}$$

- in last step defined ”**CKM matrix**”

$$\boxed{V \equiv U^u_L \, U^{d\dagger}_L}$$

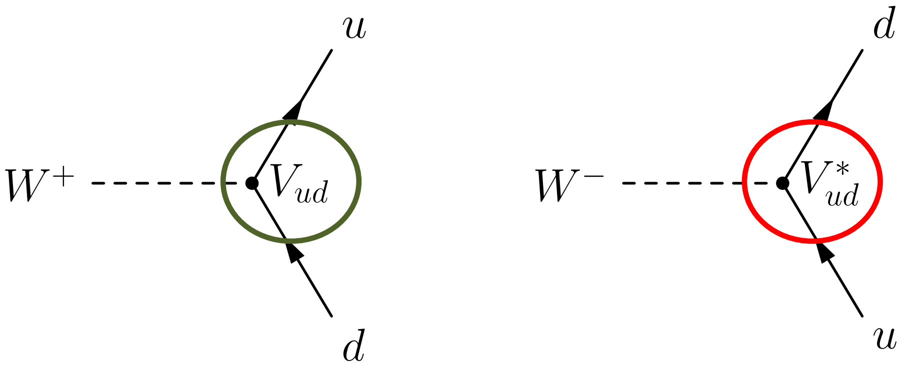
unitary!

The W^- interaction term

- playing the same game for W- vertex:

$$\mathcal{L}_{W^-qq} = \frac{ig}{\sqrt{2}} \sum_{i,j} \overline{d}_{L,i}\, \gamma^{\mu} W^{-}_{\mu}\, V^{\dagger}_{ij}\, u_{L,j}$$

- note **Hermitian conjugate** (important when we start to compute things)



Other effects of basis transformation?

- strong interaction, hypercharges, neutral W?
  - these are all of the form

$$\boxed{\mathcal{L}_{\text{int}} \propto \sum_i \overline{\psi}^I_{L,i} \,(\text{"flavour diagonal"})\, \psi^I_{L,i} + \overline{\psi}^I_{R,i} \,(\text{"flavour diagonal"})\, \psi^I_{R,i}}$$

  - no mix of up-down fields → not affected

- how about the U_R matrices?
  - do not appear in left-handed doublet interaction terms
  - not visible in any of the singlet terms
  - do not affect Lagrangian other than to diagonalize mass terms

Summary of flavour in the SM

- start from Lagrangian with flavour universal and diagonal interactions

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} u'_L \, \gamma_\mu \, W^\mu \, d'_L$$

- add Higgs interaction that are *not* flavour universal (because we can

$$\mathcal{L}_H = Y^d_{ij} \left(\overline{u'_i}, \overline{d'_i}\right)_L \binom{0}{v} d'_{jR} + Y^u_{ij} \left(\overline{u'_i}, \overline{d'_i}\right)_L \binom{v}{0} u'_{jR}$$

- diagonalize the mass matrix (because we measure mass eigenstates)

$$u_i = (V^u)_{ij} \, u'_j \qquad\qquad d_i = \left(V^d\right)_{ij} d'_j$$

- result: W interaction mixes families

Unitary?

- important assumption in this step:

$$\mathcal{L}_{W^+ud} = \sum_{i,k,l} \overline{u}_{L,k} \, (U_L^u)_{ik} \, \boxed{\left(\frac{ig}{\sqrt{2}}\gamma^\mu W_\mu^+\right)} \left(U_L^{d\dagger}\right)_{il} d_{L,l}$$

$$= \sum_{i,k,l} \overline{u}_{L,k} \left(\frac{ig}{\sqrt{2}}\gamma^\mu W_\mu^+\right) (U_L^u)_{ik} \left(U_L^{d\dagger}\right)_{il} d_{L,l}$$

- flavour universality: all SU(2) quark multiplets must have same coupling g
- if not, then V is not a unitary matrix:

$$\boxed{\quad gV^{CKM} \;\rightarrow\; (U_L^u)_{ik} \; g_k \; \left(U_L^{d\dagger}\right)_{kj} \quad}$$

”Down-quark rotation”

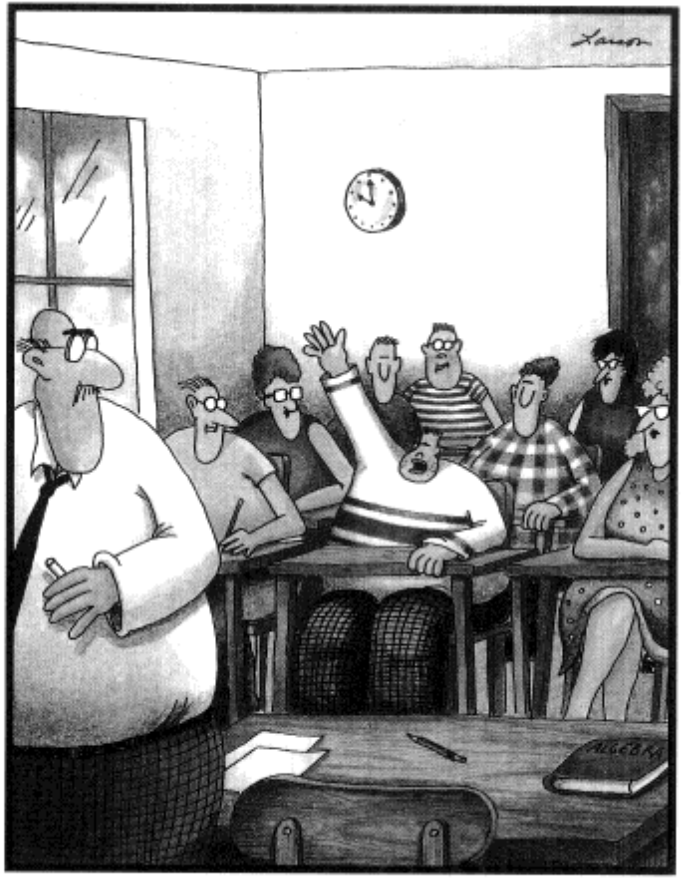
- it is customary to represent basis transformation as rotation of down quark states

$$\begin{bmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{bmatrix} .$$

down quark states
interacting with up quark
mass eigenstates

down quark mass
eigenstates

- contrary to what you find in some texts, states on left are NOT interaction eigenstates: they are states interacting with up-quark mass eigenstates



"Mr. Osborne, may I be excused?
My brain is full."

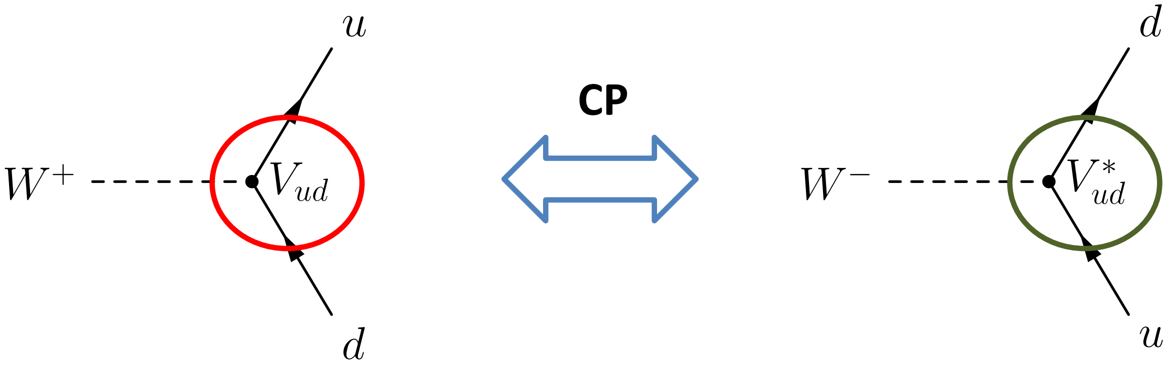
“Cabibbo-Kobayashi-Maskawa matrix”

$$V^{CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Glashow, Illiapolous and Maiani

CP violation

- W^+ and W^- terms represent CP-conjugate processes

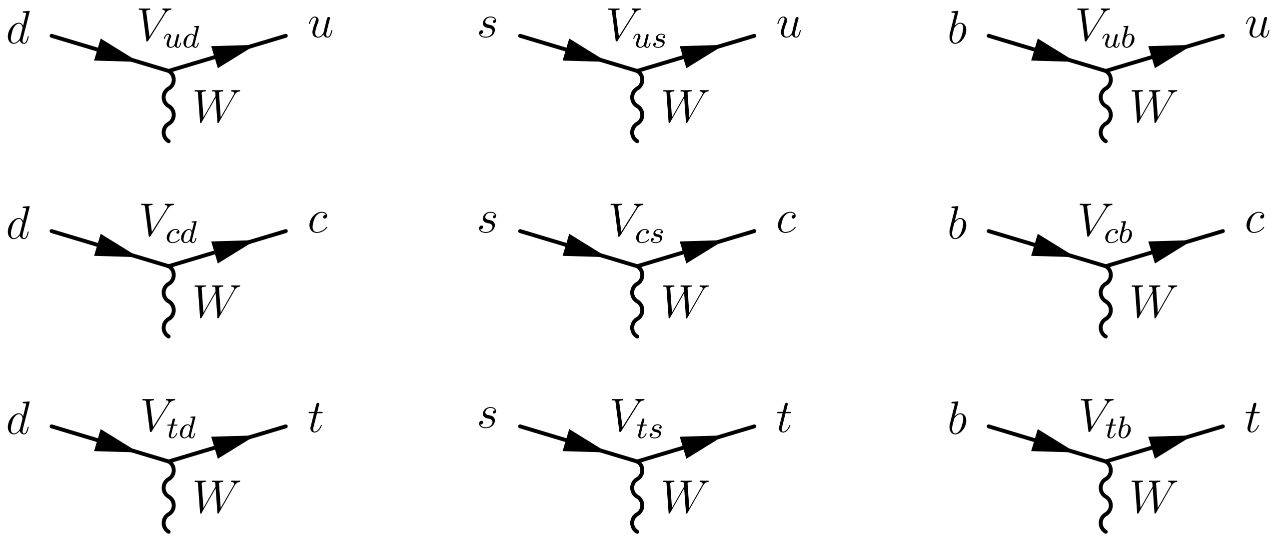


- if $\boldsymbol{V_{ud}}$ is not real, then corresponding amplitudes have different coupling
- Kobayashi and Maskawa (1970)
  - need at least 3 generation to have non-trivial complex element in $\boldsymbol{V}^{\mathrm{ckm}}$
  - by adding 3rd generation, can explain CP-violation in Kaon decays!

“physical non-zero phase” = “this theory is CP violating.”

Flavour changing interactions

- charged weak interactions leads to flavour transition through CKM matrix



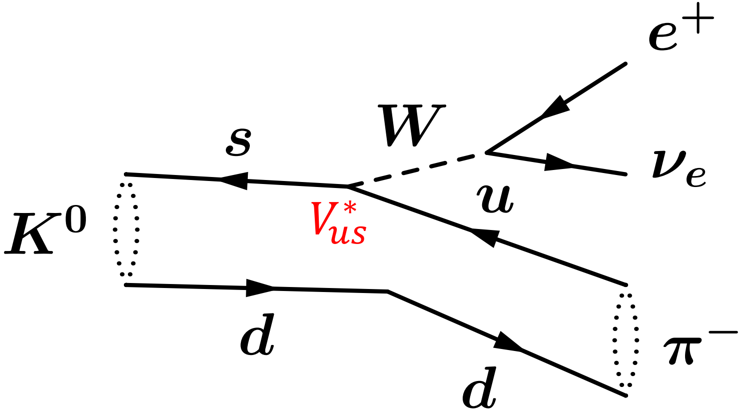
- we `measure’ CKM matrix by studying these interactions

Example: extracting Vus

$$K^0 \to \pi^- e^+ \nu_e$$

1. measure branching fraction
2. compare to prediction to extract V_{us}

Feynman diagram

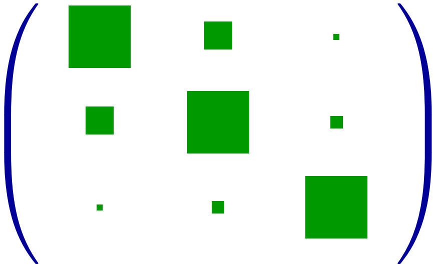


- main **difficulty: quarks only appear in bound states**!
- → theoretical developments in quark flavour physics are mostly about dealing with '**hadronic effects**'

The ‘Flavour Puzzle’

- *<u>unexplained structure</u>*: CKM matrix is almost diagonal

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim$$



- remember: quark order is choice → ordered quarks by mass

- are mass hierarchy and CKM hierarchy related?

CKM matrix parametrization

$$V \equiv U_L^u \, U_L^{d\dagger}$$

- CKM matrix is 3x3 unitary matrix
- how many physical parameters?
  - generic unitary 3x3 matrix: 9 real parameters
  - relative phase between quark fields unphysical: 4 parameters left
  - usually parametrized with **3 angles and 1 complex phase**

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

- Kobayashi-Maskawa phase **δ** is (the) source of CPV in quark sector of SM

CKM matrix parameters

- current values of parameters (actually 2015)

$$\begin{aligned}\sin\theta_{12} &= 0.22497 \pm 0.00069\\ \sin\theta_{23} &= 0.04229 \pm 0.00057\\ \sin\theta_{13} &= 0.00368 \pm 0.00010\\ \delta[^\circ] &= 65.9 \pm 2.0\ .\end{aligned}$$

- note:
  - mixing angles are small
  - complex phase is large

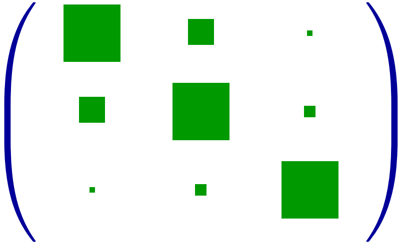
Wolfenstein parametrization

- observed structure exploited by “Wolfenstein parametrization”

$$\begin{aligned} \sin\theta_{12} &= 0.22497 \pm 0.00069 \\ \sin\theta_{23} &= 0.04229 \pm 0.00057 \\ \sin\theta_{13} &= 0.00368 \pm 0.00010 \\ \delta[^\circ] &= 65.9 \pm 2.0\ . \end{aligned}$$

$$\boxed{\begin{aligned} s_{12} &\equiv \lambda \\ s_{23} &\equiv A\lambda^2 \\ s_{13}e^{-i\delta} &\equiv A\lambda^3(\rho + i\eta) \end{aligned}}$$

$$\boxed{V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)}$$



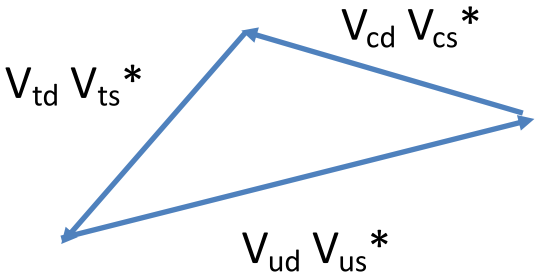
- amplitudes usually involve several CKM elements:
expansion in **powers of *λ*** is useful to see which combinations are large

Unitary triangles

- CKM matrix is unitary: leads to 6 ‘orthogonality relations’, e.g.

$$\boxed{\sum_{i=1}^{3} V_{id} V_{is}^{*} = 0}$$

- zero sum of three numbers represented by triangle in complex plane:

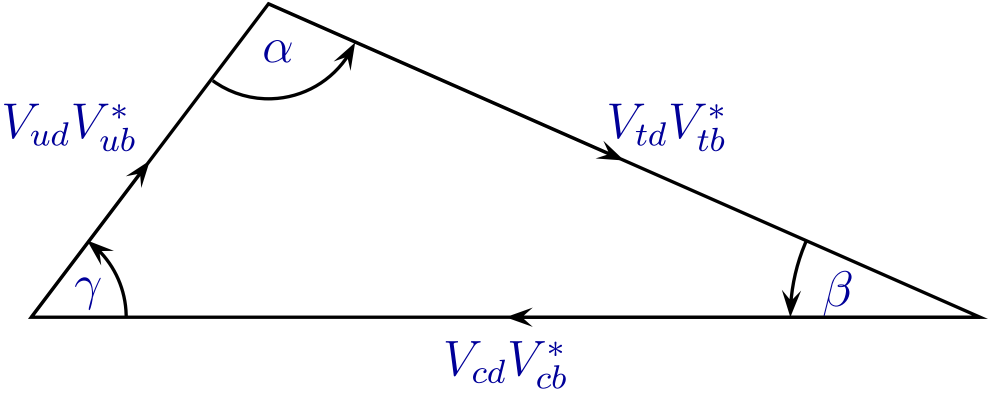


homework exercise: surface of all 6 unitary triangles is equal to Jarlskog invariant!

The unitary triangle

- only one of the 6 triangles has all sides of about equal sides:

$$V_{ub}^{*}V_{ud} \; + \; V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} \; = \; 0$$

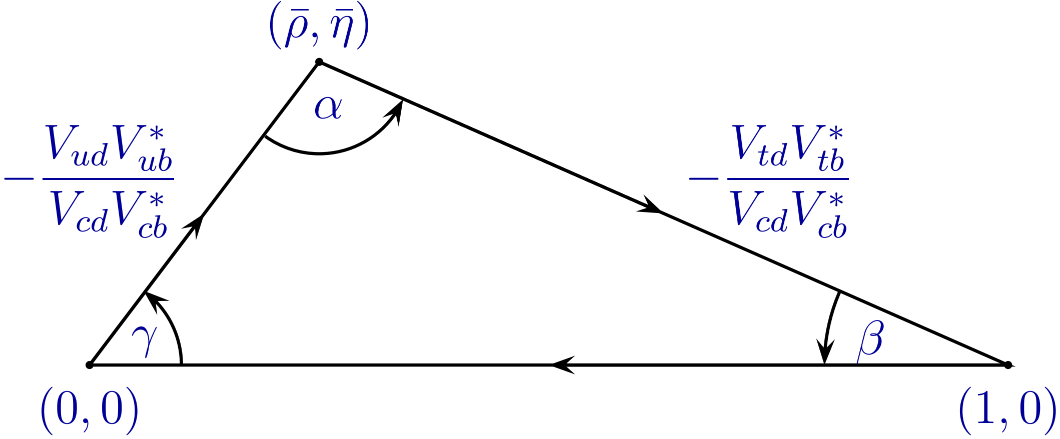


$$\alpha = \arg\left[-\frac{V_{td}V_{tb}^{*}}{V_{ud}V_{ub}^{*}}\right] \quad \beta = \arg\left[-\frac{V_{cd}V_{cb}^{*}}{V_{td}V_{tb}^{*}}\right] \quad \gamma = \arg\left[-\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}\right]$$

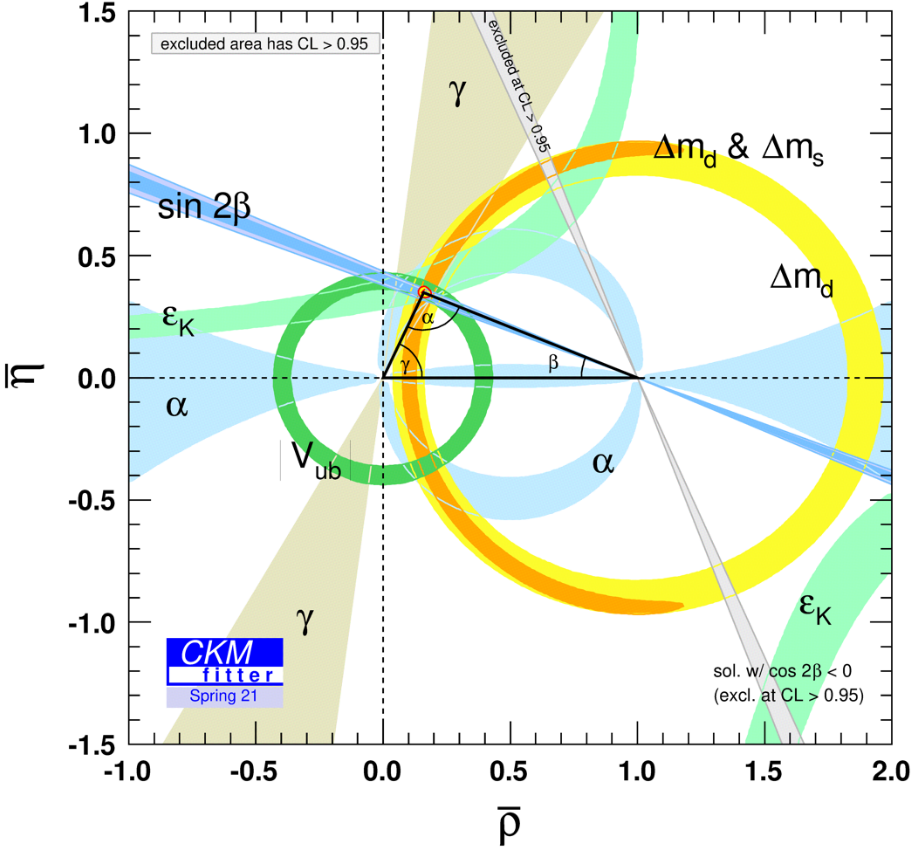
The unitary triangle

- it is customary to divide all sides by (-Vcd Vcb*):

apex of triangle corresponds (almost) to Wolfenstein parameters eta and rho

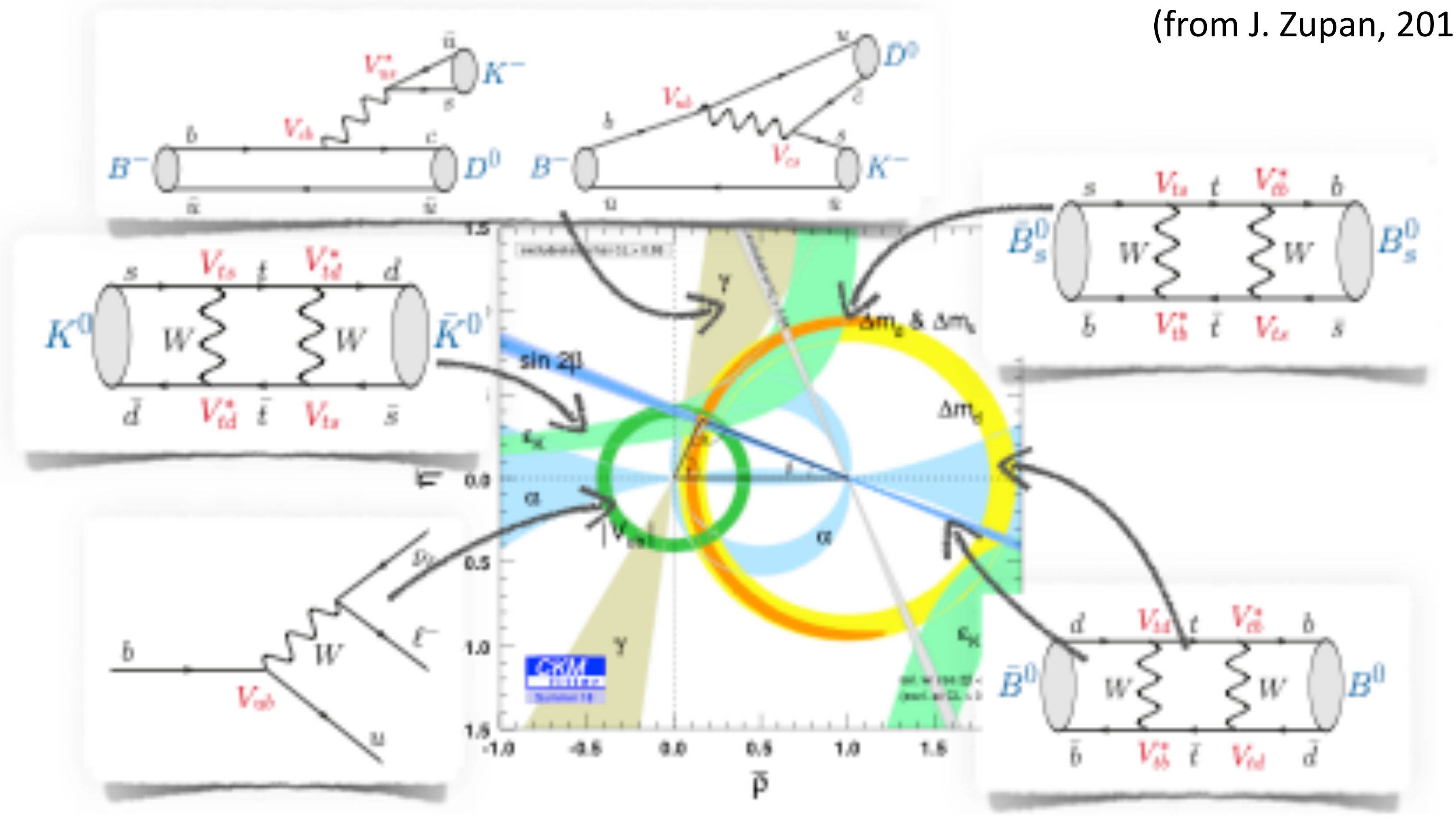


Testing the Standard Model



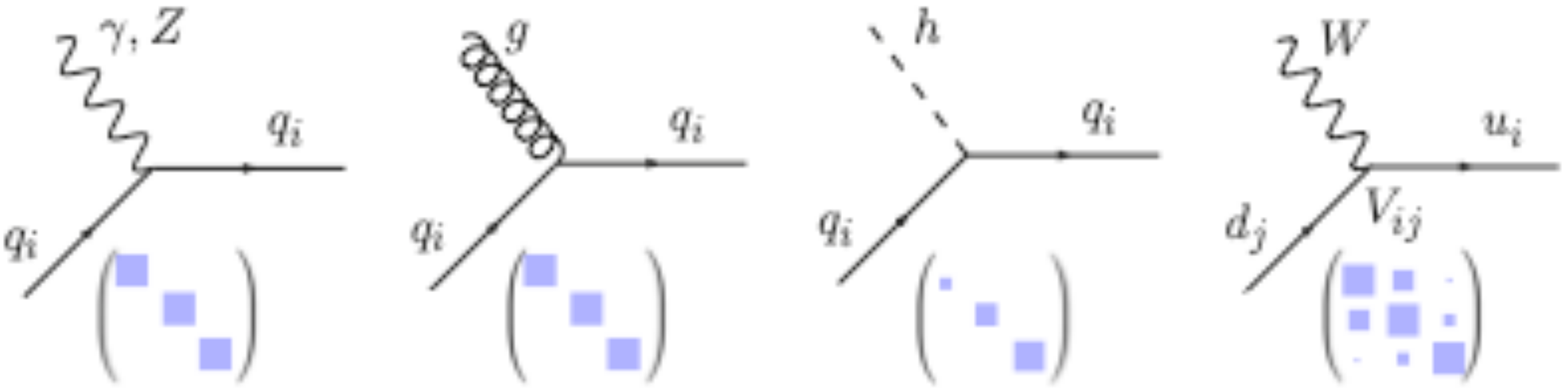
- unitary triangle: visualize consistency of SM

(from J. Zupan, 2019)



Summary of quark flavour in the SM

(J. Zupan, 2019)



- long-lived particles
- mixing
- CP violation
- …

How about leptons?

- Yukawa term for leptons looks the same as for quarks:

$$\mathcal{L}_{\text{Yukawa, leptons}} = y^{\ell}_{ij}\,\overline{L}^{I}_{i}\,\phi\,\ell^{I}_{R,j} \;+\; y^{\nu}_{ij}\,\overline{L}^{I}_{i}\,\tilde{\phi}^{c}\,\nu^{I}_{R,j} \;+\; \text{h.c.}$$

- after symmetry breaking, perform similar basis transformation, but …

- customary: make different choice than for quarks
  - (most) **scattering experiments do not measure neutrino type**
  - choose charged-lepton mass matrix diagonal
  - choose charged weak interaction diagonal
  - → neutrino-mass matrix not diagonal

Choice of basis for lepton fields

quark basis *choice*

$$\mathcal{L}_{Wqq} \;=\; \frac{g}{\sqrt{2}}\,\bar{u}_L\, i\gamma_\mu \underset{\substack{\uparrow \\ \boxed{\text{not diagonal}}}}{\left(V_{uL}V_{dL}^\dagger\right)} d_L\, W^\mu$$

$$\mathcal{L}_{\text{mass},q} \;=\; \underset{\substack{\uparrow \\ \boxed{\text{diagonal}}}}{\tilde{m}^d_{ij}}\, \bar{d}^i_L\, d^j_R \;+\; \underset{\substack{\uparrow \\ \boxed{\text{diagonal}}}}{\tilde{m}^u_{ij}}\, \bar{u}^i_L\, u^j_R$$

lepton basis *choice*

$$\mathcal{L}_{Wqq} \;=\; \frac{g}{\sqrt{2}}\,\bar{u}_L\, i\gamma_\mu \underset{\substack{\uparrow \\ \boxed{\text{diagonal}}}}{\left(V_{uL}V_{dL}^\dagger\right)} d_L\, W^\mu$$

$$\mathcal{L}_{\text{mass},q} \;=\; \underset{\substack{\uparrow \\ \boxed{\text{diagonal}}}}{\tilde{m}^d_{ij}}\, \bar{d}^i_L\, d^j_R \;+\; \underset{\substack{\uparrow \\ \boxed{\text{not diagonal}}}}{\tilde{m}^u_{ij}}\, \bar{u}^i_L\, u^j_R$$

- note: there is ***no physics in the choice of basis***

The PNMS matrix

- Pontecorvo-Maki-Nakagawa-Sakata matrix

$$U^{PNMS} \equiv U_L^{\ell\dagger} U_L^{\nu\dagger} \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

states interacting
with charged lepton
mass eigenstates

mass eigenstates

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

- completely different hierarchy from quark mixing matrix

$$|U^{PMNS}| = \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.37 & 0.58 & 0.70 \\ 0.39 & 0.59 & 0.69 \end{pmatrix}$$

Sakharov conditions (1967)

- to create “net matter excess” need

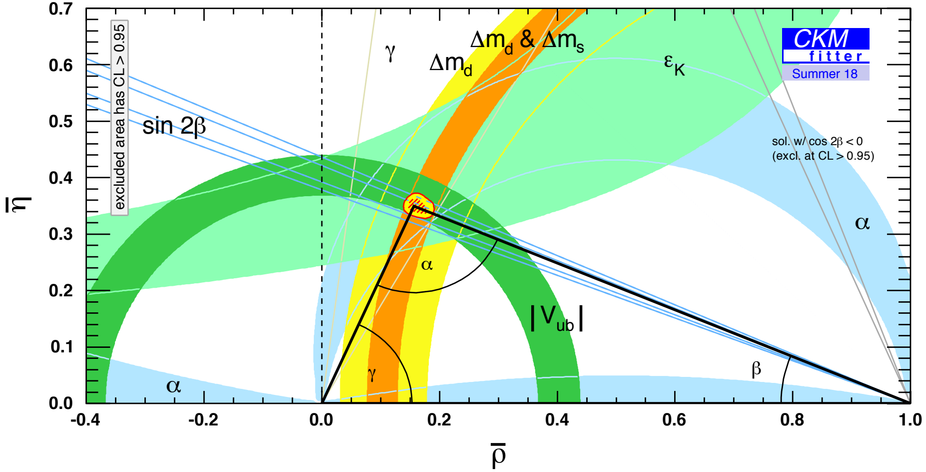
1. baryon number violating processes
   such that n(baryon) – n(anti-baryon) not constant

2. *C* and *CP* violating processes
   because of CP is conserved then for the process in 1 the CP-conjugated process has the same rate

3. non-thermal equilibrium
   because otherwise the reaction in 1 will be balanced by inverse reaction

How large is CP violation?

- Large CP violation requires *large mixing* and *large phases* in the CKM matrix.
  - Surface of unitarity triangle
  - Jarlskog invariant: $J = 3 \times 10^{-5}$



- CP violation also requires three generations with non-zero quark masses
  - In fact, ***different*** masses are required:
    - $m_u \neq m_c \;\; ; \;\; m_c \neq m_t \;\; ; \;\; m_t \neq m_u$
    - $m_d \neq m_s \;\; ; \;\; m_s \neq m_b \;\; ; \;\; m_b \neq m_d$

- Jarlskog criterion (1987) for amount of CP violation:

$$- \det\left[M_u M_u^\dagger , M_d M_d^\dagger\right] = 2\, i\, J\, (m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_u^2 - m_t^2) \\ \times \left(m_b^2 - m_s^2\right)\left(m_s^2 - m_d^2\right)\left(m_d^2 - m_b^2\right)$$

$$\boxed{M_{ij} = Y_{ij}\, v/\sqrt{2}}$$

Jarlskog invariant

- amount of CP violation can be represented by “Jarlskog Invariant”

$$\mathrm{Im}\left[V_{ij}V_{kl}V_{i\ell}^{*}V_{kj}^{*}\right] = \bigcirc\!\!\!\!\!J\sum_{mn}\epsilon_{ikm}\epsilon_{j\ell n}$$

no index summation

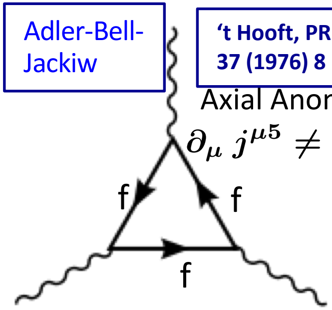
- in standard parametrization:

$$J = c_{12}c_{23}c_{13}^{2}s_{12}s_{23}s_{13}\sin\delta_{\mathrm{KM}} \approx \lambda^{6}A^{2}\eta$$ ~ 0.00003

Baryogenesis Puzzle – Electroweak Baryogenesis?

- Sacharov Conditions
  - ✓ All present in S.M.
  - Not Enough?

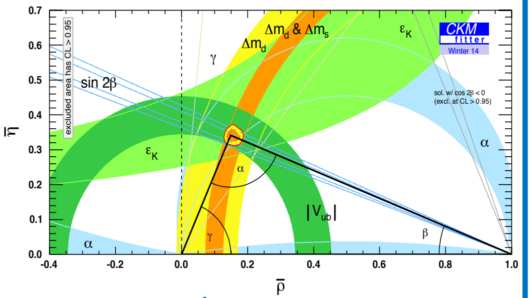
Baryon Number Violation



Axial Anomaly: $\partial_\mu j^{\mu 5} \neq 0$

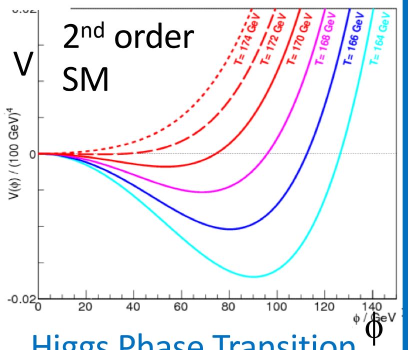
Quantum anomaly

C and CP Violation



Weak Interaction

ermal non-equilibrium



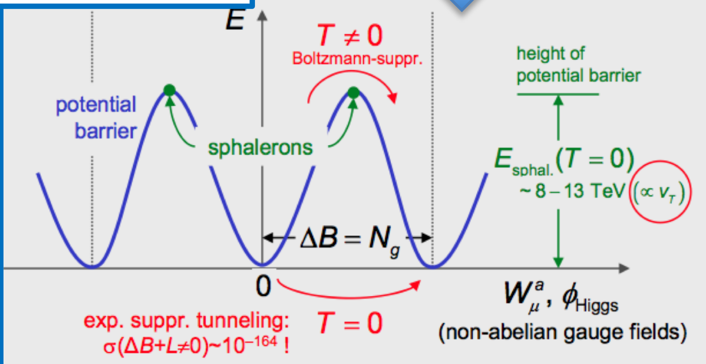
Higgs Phase Transition

1st order?

~70 GeV

- THDM: M_H ~ 125 OK

Sphalerons



CPV from CKM

- BAU: $\frac{\Delta n_B}{n_\gamma} \approx 10^{-10}$
- $A_{CP} = J_{inv} \times (m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_u^2 - m_t^2) \times (m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_d^2 - m_b^2)$
  - From CKM: $A_{CP}/T_c^{12} \approx 10^{-20}$ → Too small
  - Used $T_c \sim 100$ GeV

Exercises

- see README.md file at

  https://github.com/wouterhuls/FlavourPhysicsBND2023/

- **now: exercises 1-4**

- this is probably too much for 30 minutes. proposal:
  - make your pick
  - at least try this simple workbook exercise: particledatatable.ipynb
  - then I know if 'technically', we can run the more complicated workbooks as well