The particle zoo

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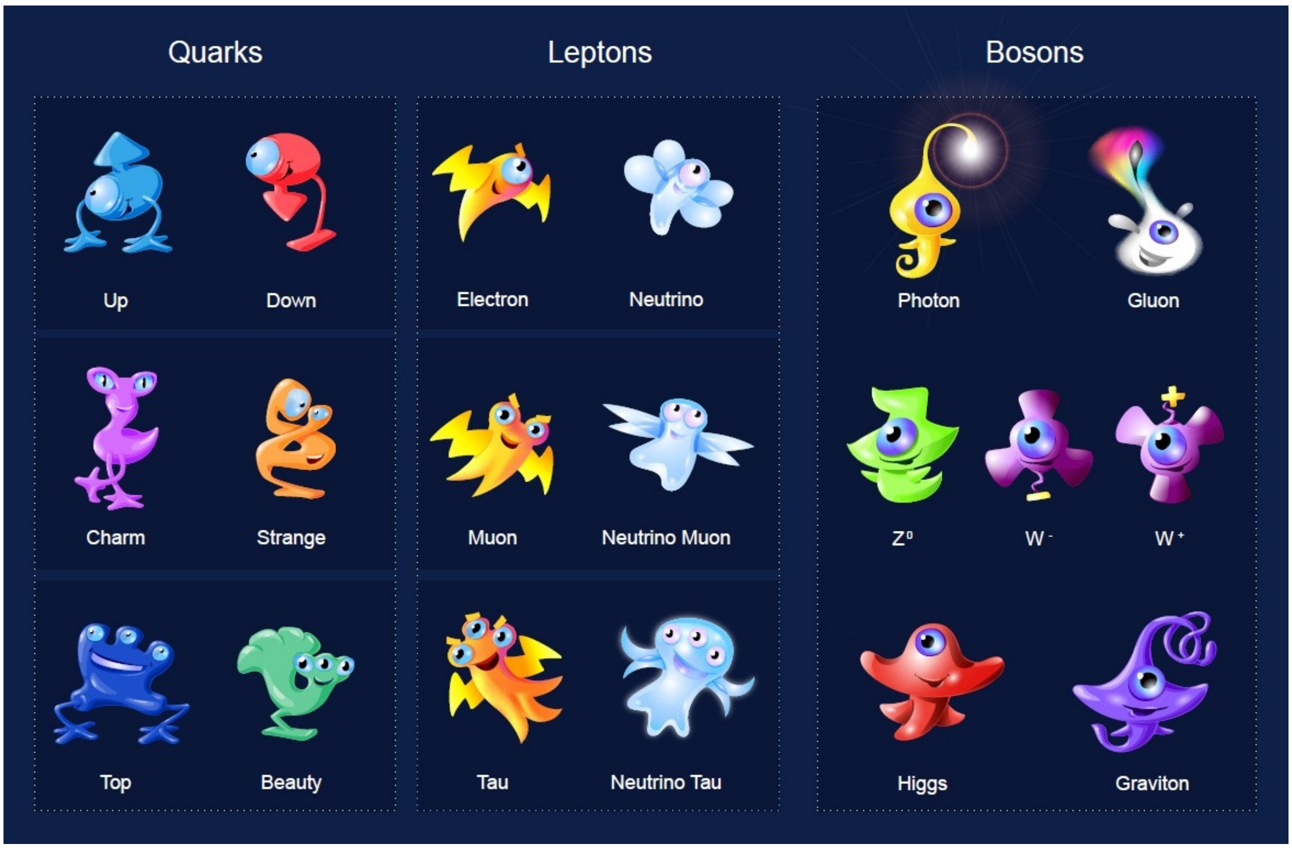

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Contents

Chapter 1

The particle zoo

In this chapter we are first going to get familiar with the basic properties of particles. We are going to discuss about mass and how one can calculate it, electric charge and magnetic dipole moment. We will also see how particles are characterised based on the value of their orbital angular momentum, spin and total angular momentum but also isospin. At the end of the chapter we are going to categorise particles and introduce the leptons, mesons, baryons and gauge bosons of the Standard Model. We will conclude by reviewing the basic conservation laws that different interactions of the Standard Model respect.

1.1 Elementary particles

In the Standard Model of particle physics, all matter and antimatter we know of is made of three categories of elementary particles:

- leptons, such as the electrons, muons, tau, the relevant neutrinos and their antiparticles,
- quarks, up, down, strange, charm, beauty, top and their antiquarks
- mediators, the massless photons and gluons and the massive $W^{\pm}$ and Z.

Lepton	Mass	Charge	L_e	L_μ	L_τ
e^-	0.511 MeV	-1(+1)	+1(-1)	0	0
ν_e	< 2 eV	0	+1(-1)	0	0
μ^-	105.7 MeV	-1(+1)	0	+1(-1)	0
ν_μ	< 0.17 MeV	0	0	+1(-1)	0
τ^-	1.777 GeV	-1(+1)	0	0	+1(-1)
ν_τ	< 15.5 MeV	0	0	0	+1(-1)

Table 1.1: The three generations of leptons, their masses, charges and their quantum numbers. In parenthesis, when different, the quantum numbers for antiparticles.

There are three generations of leptons: the one of electrons, positrons, the electron neutrino and antineutrino, the one of muons, the muon neutrino and antineutrino and the one of tau, the tau neutrino and the antineutrino. The basic properties and quantum numbers of each lepton are summarised in Table **??**. The table does not list antiparticles i.e. e^+, $\overline{\nu}_e$, μ^+, $\overline{\nu}_\mu$, τ^+ and $\overline{\nu}_\tau$, but note that their quantum numbers are reversed. That means that in total there are 12 leptons. Leptons fall in the category of spin-1/2 particles, as we will see towards the end of this chapter, and are thus fermions.

Similarly there are three generations of quarks, the building blocks that form all the composite particles in the Standard Model. Table **??** summarises their basic properties and quantum numbers. One distinct feature of quarks is that their charge is not integer multiple of the electron charge (q_e) but fractional: u, c and t all have $(2/3)\cdot q_e$, while d, s and b all have $-(1/3)\cdot q_e$. As in the case of leptons, the antiquarks all have reversed charge relative to their quark partners. Another distinct feature is the existence of additional quantum numbers, effective and applicable for each individual quark: upness

Quark	Mass	Charge	U	D	C	S	T	B
u	≈ 2.4 MeV	2/3(-2/3)	1(-1)	0	0	0	0	0
d	≈ 4.8 MeV	-1/3(1/3)	0	-1(1)	0	0	0	0
c	≈ 1.27 GeV	2/3(-2/3)	0	0	1(-1)	0	0	0
s	≈ 104 MeV	-1/3(1/3)	0	0	0	-1(1)	0	0
t	≈ 171 GeV	2/3(-2/3)	0	0	0	0	1(-1)	0
b	≈ 4.2 GeV	-1/3(1/3)	0	0	0	0	0	-1(1)

Table 1.2: The three generations of quarks, their masses, charges and their quantum numbers. In parenthesis, when different, the quantum numbers for antiquarks.

for u and $\overline{u}$, downness for d and $\overline{d}$, strangeness for s and $\overline{s}$, charm for c and $\overline{c}$, beauty for b and $\overline{b}$, top for t and $\overline{t}$. All signs for these quantum numbers are reversed between quarks and antiquarks. In the meantime, each quark or antiquark comes in three colours as we will see later in this chapter, so in total we have 36 particles. Quarks also fall in the category of spin-1/2 particles, as we will see towards the end of this chapter, and are thus fermions.

Mediator	Mass	Charge
γ (photon)	0	0
W^-	≈ 80.4 GeV	-1
W^+	≈ 80.4 GeV	+1
Z^0	≈ 91.2 GeV	0
g (gluon)	0	0

Table 1.3: The mediators of interactions in the Standard Model with their masses and charges.

Finally, the Standard Model has mediators for the three interactions: the photon (γ) for the electromagnetic interactions, the $W^\pm$ and Z^0 for the weak interactions and the gluons (g) for the strong interactions. There are eight gluons that contain a combination of colour and anticolour. In total, the Standard Model has 12 mediators. These mediators fall in the category of spin-1 particles, as we will see towards the end of this chapter, and are thus bosons.

The last piece of the puzzle is the Higgs boson, discovered at the Large Hadron Collider (LHC) at CERN in Geneva in 2012. Its mass is around 125 GeV, with 0 charge and spin, hence a boson. We will briefly discuss about the Higgs boson in one of the last chapters.

Overall the Standard Model is built around 61 elementary particles.

1.2 Particle mass

One of the obvious ways that a particle can be identified is via its mass. From the law of Newton, if a particle of mass m is positioned in a force field that acts a force $\mathbf{F}$ on it, then the particle accelerates with $\mathbf{a}$ according to

$$\mathbf{F} = m\mathbf{A}$$

The equation above does not hold at the relativistic limit but in this case we talk about the rest mass of a particle. Masses of particles can vary significantly, from the massless photon (γ) or gluons (g) to the light neutrinos (e.g. $m_{\nu_e} \approx 1$ eV, the electron ($m_e \approx 0.511$ MeV) up to nuclei e.g. the proton (p, nucleus of the hydrogen atom) with $m_{\mathrm{p}} \approx 2000 m_e$. Note that the unit used to measure masses in particle physics is not anymore Kg or even g but multiples of "electronvolt"[1].

The mass of a particle can be deduced by calculating at the same time its momentum $\mathbf{p}$ and its energy E or its velocity $\mathbf{u}$. The mass then can be calculated using the relativistic equations

$$E^2 = p^2c^2 + m^2c^4$$

[1] The electronvolt (eV) is a typical unit of energy in particle physics which is equal to approximately 1.6×10^{-19} Joules. It is defined as the amount of energy gained or lost by a the charge of a single electron moving across an electric potential difference of one volt.

and

$$\mathbf{p} = m\gamma\mathbf{u},$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ with $\beta = \frac{u}{c}$.

Particle detectors are successful in helping measure the mass of charged particles. These detectors are usually embedded inside a magnetic field which is used to bend the trajectory of the particle from where on calculates its momentum. A schematic view of such a simple setup is given in fig. 1.1

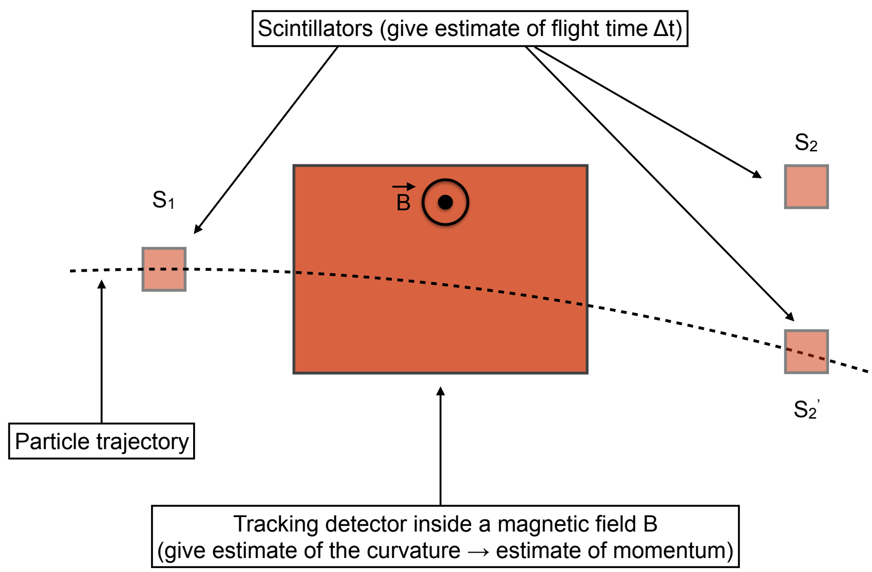

Fig. 1.1: A schematic view of a simple experimental setup that allows the measurement of particle masses.

This method obviously fails in the case of neutral particles or for particles whose lifetime is so sort that their momentum or energy can not be measured (i.e. resonances). In these cases the mass measurement is performed using the invariant mass. For this we will employ the knowledge the acquired in the Chapter **??**. Let us suppose that we have a $\mathrm{K}^0_{\mathrm{s}}$–meson with a mass of around 498 MeV decaying into a pair of pions i.e. a positive π^+ and a negative one π^- with masses close to 139 MeV(do not worry about what these strange particles are at this stage; we will discuss about them in detail in the following sections of this chapter). Let's denote with $P^\mu = (E, \mathbf{P})$ the 4–momentum of the incoming particle and with $P_1^\mu = (E_1, \mathbf{P_1})$ and $P_2^\mu = (E_2, \mathbf{P_2})$ the 4–momenta of the outgoing pions. This decay is also schematically given in fig. 1.2. The neutral particle leaves not trail in any particle tracking detector, even if the detector is embedded in an external magnetic field. Its trajectory is thus indicated with a dashed line in the figure. At some point it decays into a pair of charged pions that, in the presence of the magnetic field, have a curved trajectory from where information about their momenta can be deduced. There are also different detector techniques that can be employed that give us information about the identity of these particles and thus their masses. In addition, these two pions fly with an angle θ between them. Note that due to their decay topology that resembles a V–shape particles such as the $\mathrm{K}^0_{\mathrm{s}}$ of this example or the Λ–baryon (decaying into a proton and a negative pion) are called generally V0s. Let us now try to get an estimate of the mass of this mysterious neutral particle from the information of its "daughters". For this we are going to calculate the invariant quantities that we are very much aware of by now (working in natural units):

$$P_\mu P^\mu = (P_1 + P_2)_\mu (P_1 + P_2)^\mu \Rightarrow m^2_{\mathrm{K}^0_{\mathrm{s}}} = P_{1\mu} P_1^\mu + P_{2\mu} P_2^\mu + 2P_{1\mu} P_2^\mu \Rightarrow$$

$$m^2_{\mathrm{K^0_s}} = m^2_{\pi^-} + m^2_{\pi^+} + 2(E_1E_2 + P_1P_2\cos\theta) \Rightarrow$$

$$m^2_{\mathrm{K^0_s}} = 2m^2_{\pi} + 2(E_1E_2 + P_1P_2\cos\theta)$$

In particle physics experiments positioned in particle accelerators like the Large Hadron Collider (LHC), in a collision on let's say proton on proton many particles are produced. That means that researchers do not have just the "real daughters" at the final state to analyse. What they do is that they select a clean sample of pions with characteristic that fit the decay topology under discussion and they combine them in a statistical way to calculate the invariant mass making all possible combinations. As a result we get the right plot of fig. 1.2. This is a typical invariant mass plot, with the signal formed by combining pairs of pions that originate from decays of $\mathrm{K^0_s}$ lying on top of a combinatorial background (in this case really negligible) formed by pairs of pions not coming from a decay of a true $\mathrm{K^0_s}$.

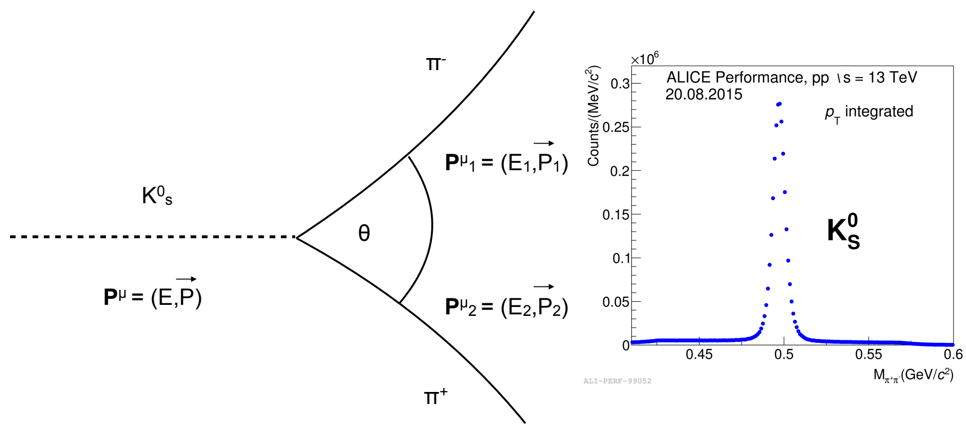

Fig. 1.2: A schematic view of the decay of a $\mathrm{K^0_s}$–meson into a pair of pions. The 4–momenta of the incoming neutral and outgoing charged particles are also indicated in the plot.

1.3 Orbital angular momentum and spin

Particles are also characterised by their angular momentum. The orbital angular momentum of a particle with momentum $\mathbf{p}$ is given by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}, \tag{1.3.1}$$

where $\mathbf{r}$ is the radius vector that connects the centre of mass of the particle to the point to which $\mathbf{L}$ refers to. Classically $\mathbf{L}$ can take any value. However in quantum mechanics $\hat{L}$, the quantum mechanical operator of the orbital angular momentum, has quantised eigen values. As we know, $\mathbf{p}$ is replaced in quantum mechanics by the operator:

$$\hat{p} = -i\hbar\nabla = -i\hbar(\partial/\partial x, \partial/\partial y, \partial/\partial z)$$

Consequently, the orbital angular momentum is also expressed in terms of its quantum mechanical operator $\hat{L}$ expressed as:

$$\hat{L} = \hat{r} \times \hat{p} = (x\hat{x} + y\hat{y} + z\hat{z}) \times \Big[-i\hbar\Big(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\Big)\Big] =$$

$$-i\hbar x\frac{\partial}{\partial x}(\hat{x}\times\hat{x}) - i\hbar y\frac{\partial}{\partial x}(\hat{y}\times\hat{x}) - i\hbar z\frac{\partial}{\partial x}(\hat{z}\times\hat{x})$$

$$-i\hbar x\frac{\partial}{\partial y}(\hat{x}\times\hat{y}) - i\hbar y\frac{\partial}{\partial y}(\hat{y}\times\hat{y}) - i\hbar z\frac{\partial}{\partial y}(\hat{z}\times\hat{y})$$

$$-i\hbar x\frac{\partial}{\partial z}(\hat{x}\times\hat{z}) - i\hbar y\frac{\partial}{\partial z}(\hat{y}\times\hat{z}) - i\hbar z\frac{\partial}{\partial z}(\hat{z}\times\hat{z}) =$$

$$i\hbar y\frac{\partial}{\partial x}\hat{z} - i\hbar z\frac{\partial}{\partial x}\hat{y} - i\hbar x\frac{\partial}{\partial y}\hat{z} + i\hbar z\frac{\partial}{\partial y}\hat{x} + i\hbar x\frac{\partial}{\partial z}\hat{y} - i\hbar y\frac{\partial}{\partial z}\hat{x} =$$

$$-i\hbar\Big(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\Big)\hat{x} - i\hbar\Big(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\Big)\hat{y} - i\hbar\Big(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\Big)\hat{z} \Rightarrow$$

$$\hat{L}_x = -i\hbar\Big(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\Big)$$

$$\hat{L}_y = -i\hbar\Big(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\Big)$$

$$\hat{L}_z = -i\hbar\Big(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\Big)$$

The different components of the orbital angular momentum satisfy the algebra of Lie:

$$[\hat{L}_i, \hat{L}_j] = i\hbar\varepsilon_{ij}^{k}\hat{L}_k,$$

where ε_{ij}^{k} is the Levi-Civita symbol that corresponds to 1 for different i, j, k indices that are properly ordered, -1 for different i, j, k indices that are not properly ordered and 0 in case any of the two indices are the same.

The wave function of a particle with a definite orbital angular momentum is an eigenfunction of L^2 and L_z such that

$$L^2|\psi_{lm}\rangle = l(l+1)\hbar^2|\psi_{lm}\rangle$$

$$L_z|\psi_{lm}\rangle = m_l\hbar|\psi_{lm}\rangle$$

The magnitude of L^2 can only take the values $l(l+1)\hbar^2$, where l is an integer number. The magnitude of L_z is $m_l\hbar$ and can take any integer value in the range: $-l, -l+1, ..., 0, ..., l-1, l$. That means that there can be $(2l+1)$ values.

Figure 1.3 presents the possible orientations of the angular momentum vector for $l = 2$.

Similarly for the spin of a particle, one can measure its magnitude S^2 and the third component S_z. The value of S^2 can be of the form:

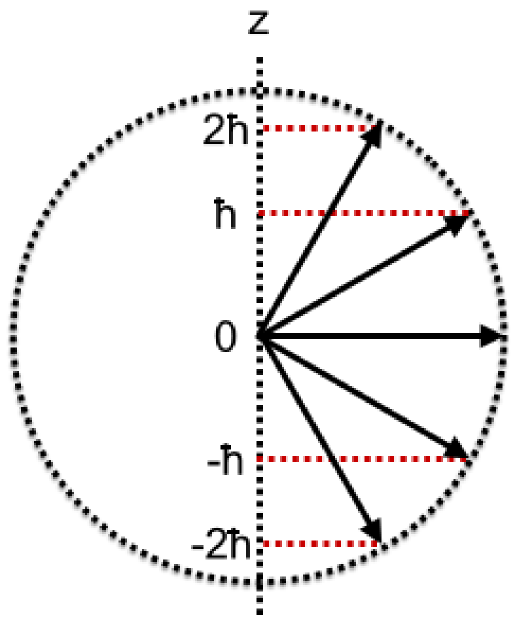

Fig. 1.3: Possible orientations of the angular momentum vector for $l = 2$.

$$S^2 \to s(s+1)\hbar$$

In the case of the spin though, the quantum number s can take half-integer values as well as integer ones i.e. s : $0, 1/2, 1, 3/2, 2, 5/2, 3, 7/2, \ldots$. For a given value of s, its third component S_z can have values of the form $m_s\hbar$, where m_s is an integer or half integer in the range of $[-s, s]$ i.e. $m_s : -s, -s+1, \ldots, 0, \ldots, s-1, s$. Both L_z and S_z can take $2k+1$ values, where k is either l or s, respectively.

Every particle can have any value of angular momentum but their spin is fixed. We will see that we call particles with half integer spin fermions (e.g. leptons, quarks, baryons) and the ones with integer spin bosons (e.g. mesons and force mediators).

Angular momentum and spin states are usually represented with a 'ket' i.e. $|l, m_l\rangle$ and $|s, m_s\rangle$. It can be that we are not interested in the value of the angular momentum or the spin separately but rather in the value of the total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$. It can also be that we are interested in the relevant values of the total momentum of a particle, composed by e.g. a quark and an anti-quark that, as we will see later, is called a meson. Combining angular momenta of two particles implies combining two states $|j_1, m_1\rangle$ and $|j_2, m_2\rangle$. Adding the third component is done naturally by

$$m = m_1 + m_2$$

But for the magnitude, it turns out that the quantum number j can take any value from $|j_1 - j_2|$ up to $|j_1 + j_2|$ in integer steps:

$$j = |j_1 - j_2|, |j_1 - j_2| + 1, \ldots, (j_1 + j_2) - 1, (j_1 + j_2)$$

1.3.1 Clebsch-Gordan coefficients

There are cases where instead of the total angular momentum, one requires the knowledge of the decomposition into the two different states $|j_1, m_1\rangle|j_2, m_2\rangle$ into states of total angular momentum $|j, m\rangle$:

$$|j_1,m_1\rangle|j_2,m_2\rangle = \sum_{j=|j_1-j_2|}^{(j_1+j_2)} C^{jj_1j_2}_{mm_1m_2}|j,m\rangle, \tag{1.3.2}$$

where $m = m_1 + m_2$. The numbers $C^{jj_1j_2}_{mm_1m_2}$ are called the Clebsch-Gordan coefficients. These numbers give the probability of getting a value of $j\cdot(j+1)\hbar$ for any allowed j if we measure J^2 on a system consisting of two angular momentum states $|j_1,m_1\rangle$ and $|j_2,m_2\rangle$. Figure 1.4 presents part of the Clebsch-Gordan coefficients extracted from the Particle Data Group (PDG). The square of the coefficients is given, which means that the reader should take the square root.

Example: The electron in the hydrogen atom occupies the orbital status $|2,-1\rangle$ and the spin state $|1/2,1/2\rangle$. What are the values of J^2 and what is the probability of each one of them?

The z-component of the angular momentum is $m = m_1 + m_2 = -1/2$. The possible states of the total angular momentum j are $j = l - s = 3/2$ and $j = l + s = 5/2$. Thus the two different states can be $|3/2,-1/2\rangle$ and $|5/2,-1/2\rangle$. We now need to extract the Clebsch-Gordan coefficients for the following final decomposition:

$$|2,-1\rangle|1/2,1/2\rangle = a\cdot|3/2,-1/2\rangle + b\cdot|5/2,-1/2\rangle$$

The coefficients can be found in fig. **??** by looking at the table labeled $2\times 1/2$ (i.e. combining a $j_1 = 2$ with a $j_2 = 1/2$ states) and finding the the horizontal row labeled $-1,1/2$ (i.e. combining a $m_1 = -1$ with a $m_2 = 1/2$ states). The relevant values are 2/5 and -3/5, which means that $a = -\sqrt{3/5}$ and $b = \sqrt{2/5}$. The final decomposition can thus be written as

$$|2,-1\rangle|1/2,1/2\rangle = -\sqrt{3/5}\cdot|3/2,-1/2\rangle + \sqrt{2/5}\cdot|5/2,-1/2\rangle$$

The probability of getting $j = 3/2$ is 3/5 and the relevant value for $j = 5/2$ is 2/5.

1.4 Isospin

In analogy to the quantum mechanical treatment of spin 1/2 particles, we define the quantum mechanical operator of isospin as

$$\hat{I}_i = \frac{1}{2}\hat{\sigma}_i$$

where σ_i are the Pauli matrices. The generators of isospin are the generators of SU(2) and form a non-Abelian i.e. non-commuting) Lie group that satisfies the following algebra:

$$[\hat{I}_1,\hat{I}_2] = i\hat{I}_3$$

$$[\hat{I}_2,\hat{I}_3] = i\hat{I}_1$$

$$[\hat{I}_3,\hat{I}_1] = i\hat{I}_2$$

The total isospin operator is given by

$$\hat{I}^2 = \hat{I}_1^2 + \hat{I}_2^2 + \hat{I}_3^2$$

and follow the commutations: $[\hat{I}^2,\hat{I}_1] = 0$, $[\hat{I}^2,\hat{I}_2] = 0$ and $[\hat{I}^2,\hat{I}_3] = 0$.

Finally, let's define the ladder operators that allow us to move between the $(2I+1)$ isospin states according to

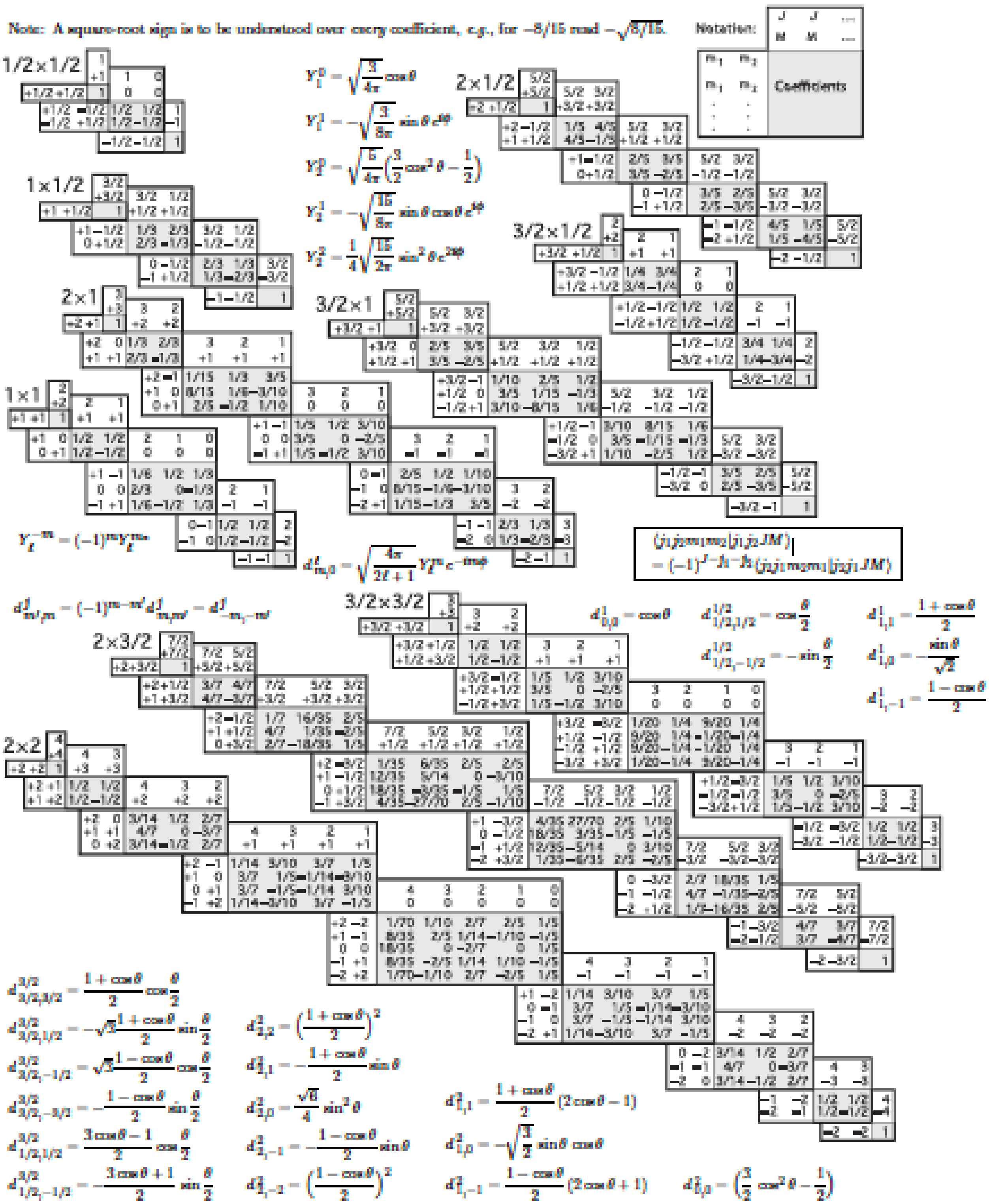

Fig. 1.4: The Clebsch-Gordan coefficients extracted from the Particle Data Group (PDG). The square of the coefficients is given, which means that the reader should take the square root.

$$\hat{I}_- = \hat{I}_1 - i\hat{I}_2$$

$$\hat{I}_+ = \hat{I}_1 + i\hat{I}_2$$

Applying $\hat{I}^2$ and $\hat{I}_3$ over a state $|I, I, 3\rangle$ gives

$$\hat{I}^2|I, I, 3\rangle = I(I+1)|I, I, 3\rangle$$

$$\hat{I}_3|I, I, 3\rangle = I_3|I, I, 3\rangle$$

with the eigenvalues being $I : 0, 1/2, 1, 3/2, ...$ and $I_3 : -I, -I+1, ..., 0, ..., I-1, I$.

It can be shown that

$$\hat{I}_+|I, I, 3\rangle = \sqrt{I(I+1) - I_3(I_3+1)}|I, I, 3\rangle$$

$$\hat{I}_-|I, I, 3\rangle = \sqrt{I(I+1) - I_3(I_3-1)}|I, I, 3\rangle$$

$$\hat{I}_+|I, I\rangle = 0$$

$$\hat{I}_-|I, -I\rangle = 0$$

1.5 Electric charge

A particle with charge q in the presence of an external electromagnetic field feels a force given by

$$\mathbf{F} = q(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B})$$

The total charge of a particle is not entirely indicative of the particle itself or its internal structure (if any) since in nature only particles with integer multiples of the electron–charge exist. As an example the proton that consists of a combination of $2u$ and one d quarks has a charge

$$p(uud) \to q(p) = 2q(u) + q(d) = 2(\frac{2}{3}) + (\frac{-1}{3}) = 1 = +q(e)$$

but also the positron, which belongs to a completely different category of particles has the same charge as the proton.

1.6 Magnetic dipole moment

A classical particle with charge and spin contains currents and exhibits a magnetic dipole moment. If electric charge is distributed throughout a particle, then if the particle has spin it produces a magnetic dipole moment μ with magnitude given by

$$|\mu| = \frac{1}{c}(\text{current}) \times (\text{area})$$

The direction of μ is perpendicular to the plane of the loop.

Let us now consider a particle of charge q, moving with velocity $\mathbf{v}$ in a circular orbit with radius r. The particle revolves with a period $T = 2\pi r/v$ and produces a current $I = qv/2\pi r$. The magnetic dipole moment will be then given by:

$$\mu = \frac{1}{c}\mathbf{I} \times \mathbf{a} = \frac{1}{c}\frac{2}{2\pi r}|\mathbf{v}|\hat{v} \times \pi r^2 \hat{r} = \frac{q}{2c} vr(\hat{v} \times \hat{r})$$

But $\mathbf{L} = \mathbf{r} \times \mathbf{p} = m(\mathbf{r} \times \mathbf{v})$ so that the previous equation takes the form:

$$\mu = \frac{q}{2mc}\mathbf{L}$$

This tells us that the direction of the magnetic dipole moment is the one of the orbital angular momentum and that the ratio of the magnitudes μ/L is characteristic of the particle since

$$\frac{|\mu|}{|\mathbf{L}|} = \frac{q}{2mc}$$

The operator of μ is related to the total angular momentum via $\mu = (\text{const.})\mathbf{J}$, where $(\text{const.}) = g(e/2mc)$:

$$\mu = g\frac{e}{2mc}\mathbf{J}$$

The variable g is a dimensionless constant that measures the deviation of the magnetic moment from the simple value of $(e/2mc)$:

$$\mu = g\frac{e\hbar}{2mc}\frac{\mathbf{J}}{\hbar} \Rightarrow \mu = g\mu_0\frac{\mathbf{J}}{\hbar}$$

where $\mu_0 = e\hbar/2mc$ the unit of magnetic moment called magneton.

In atomic physics and for electrons $m = m_e$ we define the Bohr magneton according to

$$\mu = \mu_{\mathrm{B}} = \frac{e\hbar}{2m_e c}$$

In subatomic physics we define the nuclear magneton according to

$$\mu = \mu_{\mathrm{N}} = \frac{e\hbar}{2m_p c}$$

which implies that $\mu_{\mathrm{N}} \approx 10^{-3}\mu_{\mathrm{B}}$.

1.7 The quark model

During the decade of 1960, a number of new states i.e. baryon and meson resonances were discovered. The pattern of the multiplets that these particles form was accounted for in terms of quark constituents. This was the basis of the so-called quark model that we are going to discuss in this paragraph.

According to the quark model all hadrons (i.e. all composite particles made of either three quarks, baryons, or a combination of a quark and an antiquark, mesons) are formed from a combination of quarks bound together by the strong force. The fundamental representation of SU(3) is a triplet which is the multiplet from which every other multiplet can be deduced. Figure 1.5 presents the quark (left) and antiquark (right) multiplets in SU(3). The two axis of the figure present the isospin I_3 (i.e. x-axis) and hyper charge $Y \equiv B + S$, where B and S are the baryon and strangeness numbers, respectively. Each quark is assigned spin-1/2 and baryon number $B = 1/3$. Baryons consist of three quarks and mesons of a combination of one quark and one anti-quark. The charge of each quark is given by

$$Q = I_3 + \frac{Y}{2}$$

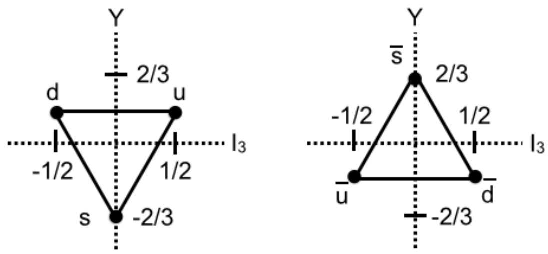

Fig. 1.5: The SU(3) quark (left) and antiquark (right) multiplets.

Mesons consist of a combination of one quark with one anti-quark. In terms of group theory, and for three quark flavours, the quarks are represented by the triplet, fundamental representation of SU(3) denoted as 3, whereas the anti-quarks form the conjugate representation $\overline{3}$. For these three quark flavours, we have nine combinations of $q\bar{q}$. These nine states divide into an SU(3) octet and singlet as can be seen in fig. 1.6:

$$3 \otimes \overline{3} = 8 \oplus 1$$

The combinations of quarks and anti-quarks that have $Y = 0$ and $I_3 = 0$ consist of combinations of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ states. One of them is taken to be the member of the isospin triplet π^-, X_1, π^+ and thus is identified as the neutral pion (π^0). The other is the singlet and contains all combinations on equal footing and is identified as the η' : $(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$. Finally the remaining particle is the η-meson which contains η : $(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$.

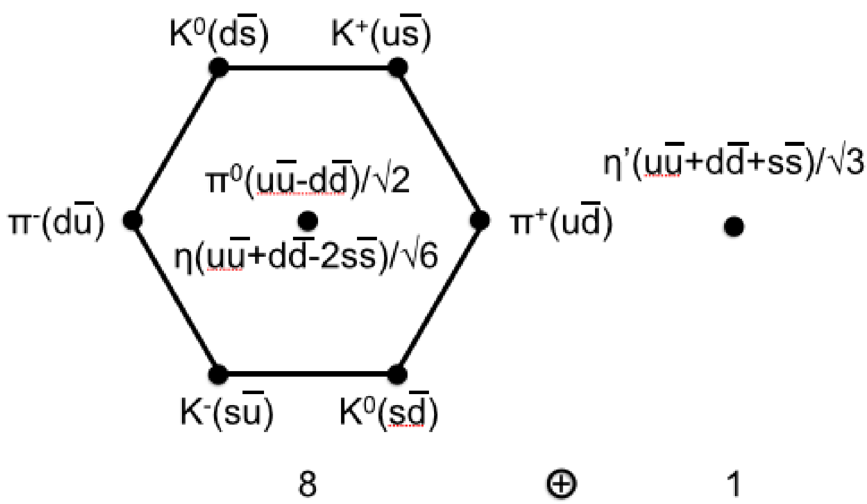

Fig. 1.6: The SU(3) octet (left) and singlet (right) that form the lightest pseudo-scalar meson multiplet.

For the same three quark flavours, we have 27 possible qqq combinations that form the lightest baryon multiplets. In order to discover these multiplets we first need to combine two quarks that form nine different states i.e. six symmetric (interchange of two quarks leaves the state untouched) and 3 antisymmetric (interchange of two quarks changes the sign of the wave function):

$$3 \otimes 3 = 6 \oplus \overline{3}$$

The resulting multiplets of the two-quark states are illustrated in fig.**??**

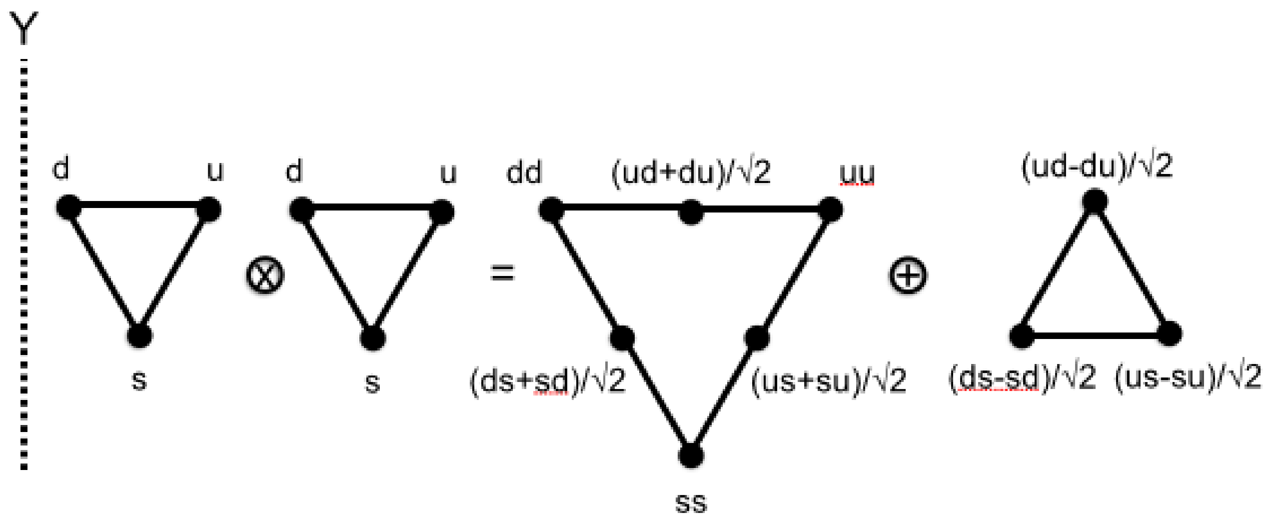

Fig. 1.7: The SU(3) lightest baryon multiplets.

What remains to be done is to combine another quark which leads to

$$\begin{aligned} 3 \otimes 3 \otimes 3 &= (3 \otimes 3) \otimes 3 = (6 \oplus \overline{3}) \otimes 3 \\ &= (6 \otimes 3) \oplus (\overline{3} \otimes 3) = 10 \oplus 8 \oplus 8 \oplus \overline{1} \end{aligned}$$

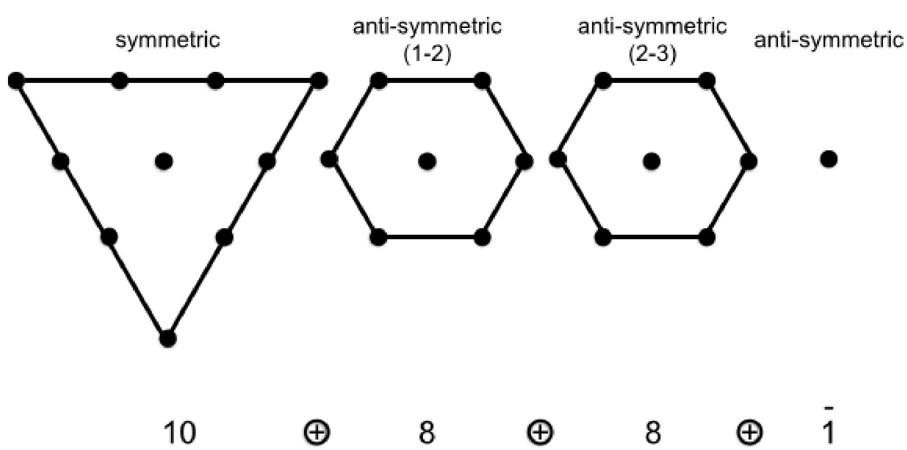

Fig. 1.8: The SU(3) lightest baryon multiplets.

The singlet state is completely anti-symmetric and consists of the linear combination:

$$(uds - usd + dsu - dus + sud - sdu)/\sqrt{6}$$

The second multiplet (i.e. octet) is partially anti-symmetric i.e. it is anti-symmetric under the interchange of the first and the second quark and can be seen in fig. 1.9.

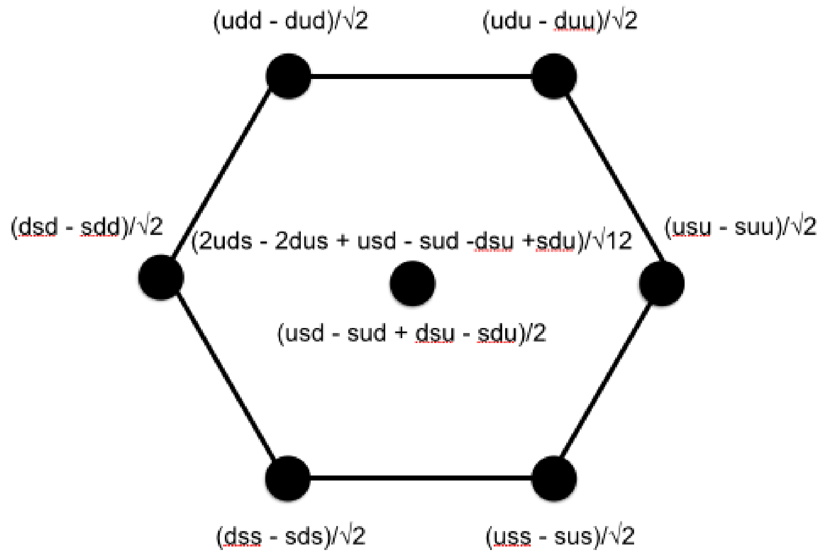

Fig. 1.9: The first anti-symmetric baryon octet.

The third multiplet (i.e. octet) is also partially anti-symmetric i.e. it is anti-symmetric under the interchange of the second and the third quark and can be seen in fig. 1.10. We can also construct one octet which is anti-symmetric under the inter-change of the first and the third quarks but this will not be independent.

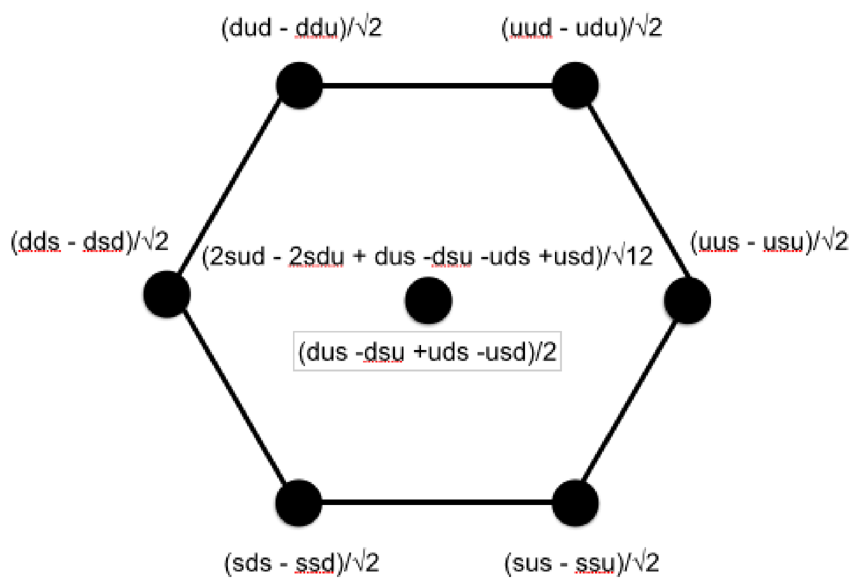

Fig. 1.10: The second anti-symmetric baryon octet.

Finally the first multiplet (i.e. 10) is completely symmetric under any interchange of quarks and can be seen in fig. 1.11.

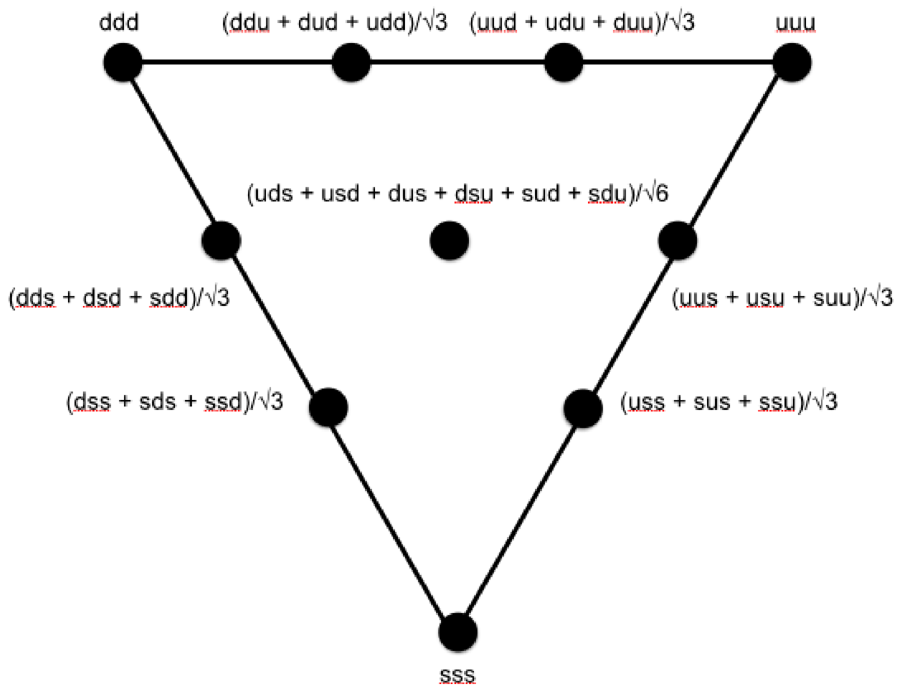

Fig. 1.11: The fully symmetric baryon multiplet.

1.8 A new quantum number: colour

One of the major achievements of quantum field theory was the proof of the connection between spin and statistics:

- Bosons, described by the Bose-Einstein statistics, have integer spin and have symmetric wave functions i.e. $\psi(1,2) = \psi(2,1)$. In this category fall
  - mediators (elementary particles) such as the photon, the gluons and the carriers of the weak force,
  - composite particles such as mesons, consisting of a combination of a quark and an antiquark
- Fermions, (described by the Fermi-Dirac statistics, have half-integer spin and have anti-symmetric wave functions i.e. $\psi(1,2) = -\psi(2,1)$. In this category fall
  - elementary particles such as quarks and leptons,
  - composite particles such as baryons, consisting of three quarks.

Suppose that we have two particles and one is in state ψ_α and the other in ψ_β. If these two particles are different (e.g. an electron and a muon), then one can discuss about which of the two is in which state. Depending on the answer, we can have $\psi(1,2) = \psi_\alpha(1)\psi_\beta(2)$ or $\psi(1,2) = \psi_\alpha(2)\psi_\beta(1)$. However, if the particles are identical, then if they are bosons then the wave function is a symmetric combination of ψ_α and ψ_β:

$$\psi(1,2) = \frac{1}{\sqrt{2}}[\psi_\alpha(1)\psi_\beta(2) + \psi_\alpha(2)\psi_\beta(1)]$$

On the other hand, if the two particles are identical fermions then the wave function is anti-symmetric:

$$\psi(1,2) = \frac{1}{\sqrt{2}}[\psi_\alpha(1)\psi_\beta(2) - \psi_\alpha(2)\psi_\beta(1)]$$

That implies that if one tries to put two identical fermions in the same state then the final wave function collapses to 0 i.e. 0 probability for this state to exist. This is another manifestation of the Pauli exclusion principle.

The wave functions of particles up to this moment consist of the part that describe space and time ($\psi(\mathbf{r},t)$), the part that describe the spin ($\psi(s)$) and the part that describe the flavour ($\psi(flavour)$), such that ($\psi = \psi(\mathbf{r},t)\cdot\psi(s)\cdot\psi(flavour)$). This representation seems to be problematic if one considers e.g. the *uuu* baryon state (the so-called Δ^{++}). This baryon that consists of three identical fermions, should have an anti-symmetric wave function, however the function at this stage seems to be symmetric. To cure this, a new quantum number is introduced, the one of colour, with three possible values: red (R), green (G) and blue (B). The combination of quarks with different colours is done in such a way so that all hadrons are colourless particles. Thus, the wave function now reads

$$\psi = \psi(\mathbf{r},t)\cdot\psi(s)\cdot\psi(flavour)\cdot\psi(colour)$$

The colour quantum number can be represented by a single column matrix:

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \text{for R}$$

$$\begin{pmatrix}0\\1\\0\end{pmatrix} \text{for B}$$

$$\begin{pmatrix}0\\0\\1\end{pmatrix} \text{for G}$$

The quarks interact with each other, with the possibility to change the relevant colour. This is usually done at the quark-gluon vertex as we will see in the next chapters. This interaction is mediated by the gluons that carry the supplementary colour (and anti-colour). There are nine gluon species which are described by the SU(3) color symmetry:

$$3\otimes 3 = 8\oplus\overline{1}$$

The octet consists of the following states:

$$|1\rangle = \frac{1}{\sqrt{2}}(R\overline{B}+B\overline{R})$$

$$|2\rangle = \frac{1}{\sqrt{2}}[-i(R\overline{B}-B\overline{R})]$$

$$|3\rangle = \frac{1}{\sqrt{2}}(R\overline{R}-B\overline{B})$$

$$|4\rangle = \frac{1}{\sqrt{2}}(R\overline{G}+G\overline{R})$$

$$|5\rangle = \frac{1}{\sqrt{2}}[-i(R\overline{G}-G\overline{R})]$$

$$|6\rangle = \frac{1}{\sqrt{2}}(B\overline{G}+G\overline{B})$$

$$|7\rangle = \frac{1}{\sqrt{2}}[-i(B\overline{G} - G\overline{B})$$

$$|8\rangle = \frac{1}{\sqrt{6}}(R\overline{R} + B\overline{B} - 2G\overline{G})$$

The colour singlet is formed by $|9\rangle = \frac{1}{\sqrt{3}}(R\overline{R} + B\overline{B} + G\overline{G})$. This last configuration is not physical. As we will see in a later chapter, confinement requires that all naturally occurring particles are colour singlet states. This explains why the colour octet states can not be seen in nature. But $|9\rangle$ is also a colour singlet, and thus would have been easy to detect it in nature, as a free particle. In addition, being a state of a mediator it could be exchanged by e.g. a proton and a neutron (i.e. colour singlet states) which would imply that the strong force is long range, while we know that it is short range. All these indicates that the colour singlet $|9\rangle$ does not occur in our world.

1.9 Conservation laws

After reviewing all the previous properties of the various particles of the Standard Model (i.e. elementary and composite), it is time to briefly introduce the basic interactions and the corresponding conservation laws. The Standard Model describes three types of interactions:

- Weak interactions, mediated by the massive $W^{\pm}$ and Z^0 bosons. The weak force acts on leptons but also quarks (and thus hadrons) and can change the flavour of quarks. The decay of particles through the weak force takes anything between 10^{-13} sec and several minutes.
- Electromagnetic interactions, mediated by the massless photon (γ). The electromagnetic force acts on leptons and quarks (and thus hadrons). A typical decay lifetime is about 10^{-16} sec.
- Strong interactions, mediated by the eight gluons (g). It is the strongest interaction in the standard model. The strong force acts on gluons and quarks (and thus hadrons) and is responsible for binding composite particles made of quarks and gluons (e.g. protons). A typical decay lifetime is about 10^{-23} sec.

Deciding which interaction is responsible about a given decay of the form $A \to B + C$ among the three available is not straightforward unless we know the lifetime of the decay. However, there are some ”standard candles” that help us decide which force is involved:

- If an interaction involves neutrinos, then it's the weak force that is responsible.
- If an interaction involves photons, then the responsible force is the electromagnetic.
- If there is any (quark)flavour changing process, then it's the weak force that is responsible.

We now review the basic conservation laws in the list below.

- **Kinematic constrains**: Conservation of energy, momentum and angular momentum. As an example a particle cannot decay spontaneously into particles heavier than itself.
- **Conservation of electric charge**: All three interactions conserve electric charge.
- **Conservation of colour**: Both the weak and the electromagnetic forces do not feel the colour. It's only the strong interactions that can affect it and in these interactions colour is always conserved.
- **Conservation of baryon number**: In any interaction the baryon number (i.e. +1 for baryons and -1 for antibaryons) is conserved.
- **Conservation of lepton number**: Leptons do not feel the strong force so the lepton number is conserved by construction. In both the weak and the electromagnetic interactions, the individual lepton numbers i.e. electron, muon and tau lepton numbers are conserved.