Declining search frictions, unemployment and self-employment

Declining frictions and self-employment

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Abstract: In most OECD countries, unemployment rates show no trend, which is puzzling if advancements in ICT decrease labour market frictions. We show, both analytically and quantitatively, that accounting for the secular decline in self-employment rates solves the puzzle. While declining labour market frictions can theoretically explain these trends, we provide contradictory causal evidence that the rollout of broadband Internet increased self-employment and decreased unemployment rates. We reconcile these observations with a

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new model featuring frictions in both labour *and* goods markets. We explain falling self-employment and non-trending unemployment quantitatively by labour market frictions declining relatively more than goods market frictions.

Keywords: Self-employment, unemployment, goods markets, labour markets, search frictions, Internet, matching efficiency

Classification: E24, J64, O33

1 Introduction

We revisit the effects of improvements in information and communication technologies (ICT) on labour market trends. As pointed out by Martellini and Menzio (2020), such improvements form a puzzle for our understanding of the unemployment rate. In the Diamond-Mortensen-Pissarides (DMP) model, the workhorse model of the labour market, improvements in the efficiency of match formation increase the rate at which vacancies and unemployed workers meet, decreasing unemployment and vacancies. However, as Martellini and Menzio show, unemployment, vacancy and job-finding rates in the United States exhibit no trend.

We argue that to understand labour market trends, trends in self-employment rates cannot be ignored. In most OECD countries, including the United States, self-employment rates are falling. While the DMP model completely abstracts from the distinction between payroll employees and self-employed workers, this distinction is crucial for two reasons. First, self-employed workers affect unemployment dynamics differently than payroll employees.

Self-employed workers do not lose their jobs at the same rate as payroll employees, and they do not congest the labour market to the same extent as unemployed workers do (Donovan *et al.*, 2023a). Second, it is implausible that the decision to become self-employed is fully independent from labour market conditions. Indeed, recent evidence shows that labour market frictions are important to explain cross-country variation in self-employment rates (e.g. Rud and Trapeznikova (2020)). Taking this finding as potential explanation for developments over time, falling self-employment rates can be expected to result from declining labour market frictions due to improvements in ICT.

Can the behaviour of self-employment in response to improvements in ICT provide an explanation for the absence of a trend in unemployment? Theoretically, declining labour market frictions may pull workers out of self-employment rather than decrease unemployment. However, we provide causal evidence that the rollout of broadband Internet, one of the most salient improvements in ICT, increased self-employment in a panel of OECD countries. If labour market frictions were the only driver of the self-employment rate, our finding would counterintuitively imply that broadband Internet increased frictions. However, we find no evidence that broadband increased the unemployment rate, and in some specifications, we find broadband to significantly decrease it.

We propose a model that can explain the joint behaviour of unemployment and self-employment rates. Our key innovation is to introduce a tractable formulation of goods market frictions. In our model, improvements in ICT thus decrease both labour *and* goods market frictions. We calibrate the model to explain the time series of self-employment and unemployment rates in the OECD, as well as to back out the effect of the rollout of broadband Internet on goods and labour market frictions. We find that falling self-employment and non-trending unemployment can be understood from a race between frictions: a decline in goods market frictions and a relatively larger decline in labour market frictions. Using a counterfactual analysis based on our empirical estimates, we also show that frictions would not have declined without the rollout of broadband Internet. Consequently, we provide quantitative evidence that broadband reduced labour and goods market frictions.

In the model, ex ante identical, risk-averse workers choose between self-employment and searching for a payroll job. The self-employed face the risk of not selling output on their own, while large firms insure payroll employees against goods market risk: employees obtain labour income even if they cannot sell in the goods market. However, searching for a payroll job comes with the risk of unemployment. Moreover, the self-employed are the sole claimants of the fruits of their labour while employees must share these with their employers.

We construct an equilibrium in which self-employment and payroll employment co-exist and in which workers are indifferent between self-employment and searching for a payroll job. In this equilibrium, payroll employees that manage to sell their output produce more than the self-employed. The resulting extra revenue pays for the insurance for those employees that cannot sell, and for the firms' recruiting costs. Now consider an off-equilibrium movement of workers from the labour market to self-employment. Because payroll employees produce more, moving workers from the labour market to self-employment acts as a negative supply shock, increasing prices. The self-employed benefit directly from higher prices through an increase in expected profits. However, job searchers benefit more; directly from an increase in wages *and* indirectly via a higher job-finding probability. These general equilibrium effects pull workers back to the labour market. Hence, changes in the self-employment rate trigger a supply externality on payroll job creation via changes in prices.

We show that reductions in goods market frictions unambiguously increase the self-employment rate. Even though large firms benefit equally from reductions in goods market frictions, lower goods market risk decreases the value of insurance that firms offer, increasing the incentives to become self-employed. At the same time, increased self-employment leads to higher prices. Higher prices foster job creation, so that when goods market frictions decline the unemployment rate falls. In contrast, reductions in labour market frictions make searching for a payroll job more attractive and decrease the equilibrium self-employment rate. Improvements in labour market matching efficiency act as a positive supply shock, exerting downward pressure on prices. Lower prices and higher matching efficiency in the labour market discourage workers from choosing self-employment and prompt them to search for jobs at firms. The fall in prices, however, also discourages firms from creating new jobs, to the point where job creation falls short from absorbing the additional workers abandoning self-employment. As a result, the unemployment rate increases when labour market frictions decline. Consequently, the supply externality of changes in the self-employment rate overturns the standard results on the effect of matching efficiency on the unemployment rate.

We calibrate our model to the time series for self-employment and unemployment and extract the values for the parameters governing the goods and labour market frictions. We provide evidence in favour of our model, rather than a model without flows from and to self-employment, by using data on aggregate price levels, which we take from the Penn World Tables. We show that the parameters governing the goods and labour market frictions move with prices as in our model: when labour market frictions fall, prices fall, and when goods market frictions fall, prices rise. While the latter correlation may be surprising given that declining goods market frictions would generally be considered a positive supply shock, rising prices make perfect sense in our model because of the supply externality.

We make three contributions. First, we provide new evidence on the effect of broadband Internet on self-employment and unemployment rate. Since our theory is concerned with general equilibrium effects, we use variation in broadband access across a panel of OECD countries between 1998 and 2017. To account for the possible endogeneity of broadband Internet, we instrument broadband adoption by a logistic diffusion model in which the availability of pre-existing technologies predicts broadband penetration, as in Czernich *et al.* (2011). We find that broadband Internet prompts more self-employment and that this effect is quantitatively important: the arrival of broadband Internet has halted three quarters of the average downward trend in self-employment rates. Our finding is robust to the inclusion of important institutional variables.[1] The effect on unemployment is negative but not consistently significant.

Second, we quantitatively show that the joint behaviour of self-employment and unemployment can be driven by declining search frictions if one considers not only labour, but also goods market frictions. We propose a novel, tractable model which we use to back out the underlying frictions in goods and labour markets. We find that, behind significant business cycle fluctuations, frictions in both markets on average decreased between 1998 and 2017. Falling self-employment rates thus help to explain the absence of a trend in unemployment in an environment of declining search frictions. Then, we combine our model and our

[1] Studies that seek to explain cross-country differences in self-employment rates and focus on institutional variation include Acs *et al.* (1994); Robson and Wren (1999); Blanchflower (2000); Parker and Robson (2004), and Torrini (2005).

empirical estimates to show that the rollout of broadband Internet, a canonical improvement in ICT, indeed decreased search frictions in both markets.

Third, we uncover a novel channel through which the goods market is intertwined with the labour market and eventually matters for the dynamics of unemployment. In earlier work (Petrosky-Nadeau and Wasmer, 2015; Kaplan and Menzio, 2016) changes in unemployment affect how consumers search in the goods market and this aggregate-demand effect feeds back to job creation. In our paper, changes in the split of employment into payroll and self-employment affect aggregate supply which, through changes in prices, affects job creation.

Related literature. Closely related to our paper is Martellini and Menzio (2020), who posed the question why advancements in ICT, likely reducing labour market frictions, did not result in falling unemployment. In their solution to this puzzle, worker-firm matches are inspection goods. The reservation match quality increases in response to declining search frictions. We argue that search frictions decline in both labour *and* goods markets and that interactions between self-employment and job-finding rates are important. Allowing for goods market frictions and selection into self-employment overturns the standard result on the effect of declining search frictions on unemployment. When search frictions decline, labour market tightness falls because of congestion caused by workers leaving self-employment, resulting in higher unemployment.

Birinci *et al.* (2021) study how an increase in applications prompted by improvements in search technology can result in fewer job separations but no rise in job-finding probabilities.

According to their answer, firms respond to an increase in applications by investing in identifying good matches, resulting in higher quality matches, and fewer separations. Martellini and Menzio (2021) study differential productivity growth for workers that benefit more or less from declining labour market frictions, while Menzio (2023) studies optimal product design under declining goods market frictions. None of these papers studies self-employment or the combination of declining labour and goods market frictions.

We are not the first to address the joint determination of payroll employment, unemployment and self-employment rates. Poschke (2019) and Feng *et al.* (2018) explain variations in employment status across a wide range of countries, while Rud and Trapeznikova (2020) focus on Sub-Saharan Africa. Bradley (2016) and Narita (2020) model and estimate the flows across payroll, self- and unemployment in a single country, and assess counterfactual policies. All of these papers stress the importance of labour market frictions, as we do, but neither considers goods market frictions. It is important to consider these, because in our model only declining goods market frictions decrease unemployment and increase self-employment.[2]

Our model of selection into self-employment puts technology and market frictions at the center stage. In contrast, a large body of earlier work summarized in Parker (2004) focuses on individual heterogeneity as the main determinant of selection into entrepreneurship, a subset of self-employment (Lucas, 1978; Jovanovic, 1982; Poschke, 2013; Kihlstrom and Laffont, 1979). Our model is complementary to this work as the notion of self-employment in our paper speaks predominantly to the self-employed who do not fit the definition of an entrepreneur.[3]

[2]Other papers that study the macroeconomic effects of goods market frictions (e.g. Michaillat and Saez (2015); Petrosky-Nadeau and Wasmer (2015); Kaplan and Menzio (2016)) do not consider self-employment.
[3]Levine and Rubinstein (2016) show that there is important heterogeneity within the pool of entrepreneurs regarding the exposure to income risk, the key distinction between payroll- and self-employment in our model. We further discuss the mapping between our theory and the measurement of self-employment in the data in Section 2.3.

Our paper is also related to the empirical literature on the effects of modern technologies on self-employment and unemployment. Using a time-series analysis, Blau (1987) finds that technological change helps to explain the rise in U.S. self-employment in the 1970's and 1980's. We find that broadband Internet had a similar effect in a panel of OECD countries even though aggregate self-employment rates fell. Also preceding significant broadband Internet adoption, Fairlie (2006) finds for the United States that ownership of a personal computer increases the probability that someone starts a business. We use a cross-country panel, because the effects of broadband likely depend on whether the self-employed's customers have access to broadband too. We are not aware of any papers that study the effect of broadband on self-employment.

Several papers exploit the geographical rollout of broadband within individual countries to study the effects on unemployment (Kolko, 2012; Hjort and Poulsen, 2019; Bhuller *et al.*, 2023; Briglauer *et al.*, 2019; Gürtzgen *et al.*, 2021; Denzer *et al.*, 2021; Zuo, 2021). We use cross-country data to capture the general equilibrium effects of broadband on unemployment and find negative point estimates, consistent with this literature.

2 Empirical Evidence

In this section we study the effect of improvements in ICT on self-employment and unemployment rates empirically. We focus on one of the most salient recent improvements in ICT: the rollout of broadband Internet. Because we are interested in the general equilibrium effects of improvements in ICT, we estimate a cross-country panel regression.

To allow the effects of the broadband Internet on labour market aggregates to be distributed over time, we follow the literature (Robson and Wren, 1999; Parker, 2004) and estimate a generalized error-correction model:

$$\Delta \ln Y_{it} = \beta \Delta X_{it} + \omega X_{it-1} - \gamma \ln Y_{it-1} + \alpha_i + \psi_t + \varepsilon_{it}. \tag{1}$$

In this equation, the dependent variable is the difference of (the logarithm of) the labour-market aggregate of interest (either the self-employment or the unemployment rate). On the right-hand side we have country and time fixed-effect variables α_i and ψ_t, respectively. These control for country-specific invariant characteristics, e.g. cultural attitudes to entrepreneurship or unemployment, and for common time variation, e.g. the global business cycle. X_{it} denotes a list of regressors that we describe below. Following the logic of the error-correction model, the β coefficients estimate the short-run effects of X_{it} on the dependent variable, while the ratios $\omega/|\gamma|$ measure the long-run effects.

2.1 Data

We obtain internationally comparable data on labour market variables from ILOSTAT (International Labour Organization, n.d.). We take data on the sizes of the populations of the unemployed, payroll employees, and self-employed in country i and year t.[4] Then, we calculate the unemployment rate U_{it} as:

$$U_{it} = \frac{\text{unemployed}_{it}}{\text{unemployed}_{it} + \text{employees}_{it} + \text{self-employed}_{it}} \times 100\%. \tag{2}$$

[4]We also collect information on own-account workers, constructing an alternative measure of self-employment, *Own-Account Work* rate. Further details on this are provided in Section 2.3.

and the self-employment rate SE_{it} as:

$$SE_{it} = \frac{\text{self-employed}_{it}}{\text{unemployed}_{it} + \text{employees}_{it} + \text{self-employed}_{it}} \times 100\%. \qquad (3)$$

We take data on broadband from the World Telecommunication/ICT Indicators Database (International Telecommunication Union, 2019), in which broadband Internet appears in 1998 (in seven countries only, with each less than 0.5% penetration). We study the broadband penetration rate, which is defined as the number of broadband subscriptions per 100 inhabitants.[5] Online Appendix C.1 contains further details on the data sources and definitions of the relevant variables.

To control for potential endogeneity, which could arise if an increase in self-employment or a reduction in unemployment would prompt further investment in broadband technology, we follow the instrumental-variable approach introduced in Czernich *et al.* (2011). The idea behind this instrument is that the most commonly used broadband standards use pre-existing infrastructure to connect homes and small- and medium-sized firms to the larger network. In particular, the copper wire of the voice telephony network and the coaxial cable of the cable TV network connect individual users to the Internet. Since the voice telephony and cable TV network have been built for other purposes than broadband Internet, they provide valid instruments for broadband penetration. We take the values of these instruments from the OECD (1999) Communications Outlook.

Instrument relevance has been shown by Czernich *et al.* (2011). They find that the adoption of broadband Internet is well-described by a logistic diffusion curve, where the

[5]Broadband Internet offers download speeds of at least 256 kbit/s. One subscription usually covers the whole household/establishment.

pre-existing voice telephony and cable TV infrastructure places a bound on the maximum reach of the broadband network in a country. Broadband penetration for each country and year can then be predicted with a nonlinear regression, featuring the maximum reach, the speed, and the inflection point of the diffusion process as parameters. Constant effects of the pre-existing voice telephony and cable TV infrastructure on changes in labour-market aggregates will be absorbed by our fixed country effects. Moreover, Czernich *et al.* (2011) show that the pre-existing infrastructure of the cable TV and voice telephony networks has no predictive value for the penetration of mobile telephony and computers, two other technologies that have been widely adopted around the same time as broadband Internet. Regarding the first-stage estimation, we merely extend their work to a longer time period. We report the details of the first-stage estimation, resulting in our regressor *Predicted Broadband*, in Online Appendix C.2.

We incorporate four additional regressors in our analysis. First, we control for the logarithm of GDP per capita, *GDP*. The earlier literature finds a negative relationship between the general level of economic development, proxied with GDP per capita, and the self-employment rate (Acs *et al.*, 1994; Poschke, 2019). By Okun's law a similar negative relationship exists between GDP and the unemployment rate.

Second, we account for the generosity of the official unemployment insurance system. In particular, following Parker and Robson (2004) and Torrini (2005), we control for the *Replacement Rate*, defined as the percentage share of the unemployment benefit in the median wage, averaged across types of households. The more generous the UI system, the more likely prospective workers are to engage in job search, if only to benefit from the system's generosity, rather than enter self-employment.

Table 1: *Sample descriptive statistics.*

Variable	Level			Change		
	Mean	Std. dev.	Obs.	Mean	Std. dev.	Obs.
Self-Employment	16.52	6.67	480	-0.18	0.52	456
Unemployment	5.48	3.49	480	-0.02	0.93	456
Own-Account Work	10.25	4.50	480	-0.07	0.43	456
Predicted Broadband	20.11	12.83	480	1.69	1.34	456
GDP Per Capita	37.44	9.09	480	0.50	0.99	456
Replacement Rate	56.25	17.10	408	-0.08	3.56	384
Tax Burden	33.13	9.99	432	-0.10	0.98	408
Public Sector	19.89	7.12	277	-0.09	1.38	249

Note: GDP per capita in $1000 (level and change). All other variables are reported in percentages (for levels) and in percentage points (for changes).

Third, following Robson and Wren (1999); Parker and Robson (2004) and Torrini (2005), the choice to become a self-employed can also be driven by tax evasion. Therefore, the higher taxes are, the larger the self-employment rate is likely to be. We thus define the *Tax Burden* as the average of the percentage-point tax wedge for a single-earner household, and married couples with two children and a single earner. This variable can also be relevant for the dynamics of unemployment (Hagedorn, Manovskii and Stetsenko, 2016).

Finally, we introduce a *Public Sector* variable as the percentage share of public-sector employment in total employment, which is shown to be relevant for the aggregate self-employment rates in Torrini (2005). This variable proxies for the incidence of safe, stable jobs in the economy. Thus, we expect that countries with big public-sector employment have lower self-employment rates. Public-sector employment may also reduce unemployment (Bradley, Postel-Vinay and Turon, 2017).

Replicating the steps of Czernich *et al.* (2011) in obtaining data on broadband diffusion, we end up with a dataset on 24 OECD countries for the years 1998-2017. We report descriptive statistics of our sample in Table 1. Descriptive statistics on a country level are reported in Table C3 in Online Appendix C.3.

2.2 Results

Table 2: *Effects of broadband Internet on the self-employment rate.*

	(1) ΔSE rate	(2) ΔSE rate	(3) ΔSE rate	(4) ΔSE rate	(5) ΔSE rate	(6) ΔSE rate
Lagged SE rate	-0.074***	-0.074***	-0.100***	-0.185***	-0.090***	-0.176***
	(0.020)	(0.021)	(0.028)	(0.032)	(0.023)	(0.036)
ΔPredicted B-band	0.024***	0.025***	0.020**	0.042***	0.029***	0.038***
	(0.008)	(0.008)	(0.009)	(0.009)	(0.007)	(0.012)
Lagged Predicted B-band	0.003***	0.003***	0.002**	0.006***	0.004***	0.006***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
ΔGDP		0.008	0.012	0.010	0.010	0.034
		(0.073)	(0.080)	(0.076)	(0.075)	(0.080)
Lagged GDP		0.011	0.025	-0.009	0.037	-0.023
		(0.028)	(0.030)	(0.032)	(0.031)	(0.038)
ΔReplacement Rate			0.000			0.038
			(0.011)			(0.031)
Lagged Replacement Rate			-0.010			0.008
			(0.006)			(0.020)
ΔPublic Sector				-0.066*		-0.015
				(0.035)		(0.045)
Lagged Public Sector				-0.104***		-0.105***
				(0.027)		(0.030)
ΔTax Burden					0.036	0.058
					(0.038)	(0.039)
Lagged Tax Burden					-0.012	-0.003
					(0.020)	(0.022)
Observations	456	456	384	249	408	211
No. countries	24	24	24	22	24	22
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
$p(F=0)$	0.000	0.000	0.000	0.000	0.000	0.009
RMSE	0.027	0.027	0.026	0.022	0.027	0.022
R-squared	0.110	0.111	0.115	0.261	0.125	0.231

$^{*}\ p < 0.10$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

Note: The dependent variable is the change in the logarithm of the *Self-Employment* rate. The sample is 24 OECD countries in years 1998-2017. Cluster-robust standard errors in parentheses, except for columns (4) and (6) that report conventional standard errors because of insufficient degrees of freedom.

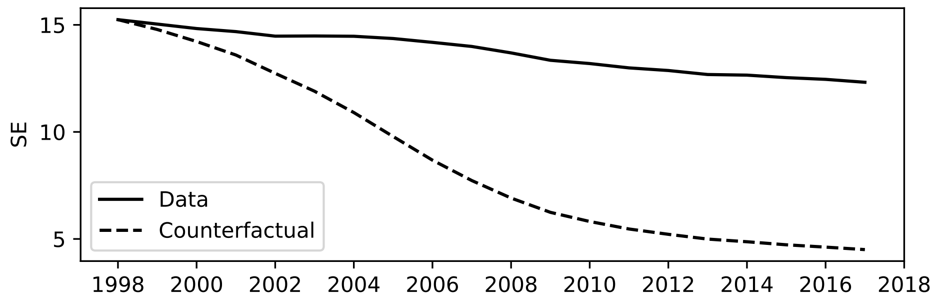
We estimate (1) in several steps, varying the contents of the X_{it} matrix. We always employ *Predicted Broadband* in our regressions. Then, we add *GDP*, subsequently incorporate the institutional variables one by one, and finally estimate the model with all regressors. All regressors apart from *Predicted Broadband* enter the estimation in logarithms and differences of logarithms, because *Predicted Broadband* consists of several zero and near-zero observations. Since the internationally comparable data on institutional variables provided by the OECD (n.d.) usually start two to three years later than our data on broadband, there is a trade-off between the length of the sample and the number of regressors we use.

Table 3: *Effects of broadband Internet on the unemployment rate.*

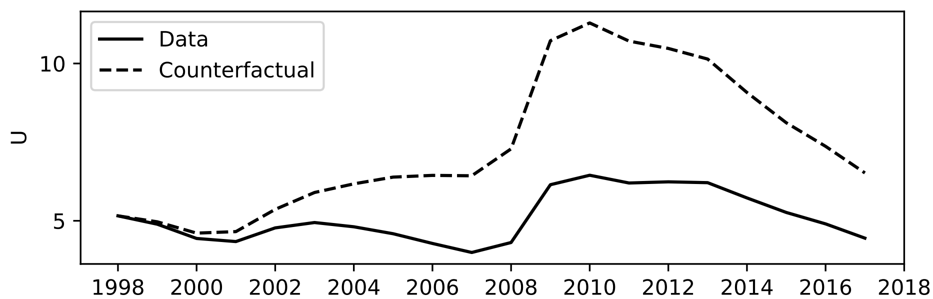
	(1)	(2)	(3)	(4)	(5)	(6)
	ΔU rate	ΔU rate	ΔU rate	ΔU rate	ΔU rate	ΔU rate
Lagged U rate	-0.090***	-0.092***	-0.135***	-0.051*	-0.100***	-0.088***
	(0.023)	(0.029)	(0.032)	(0.030)	(0.029)	(0.030)
ΔPredicted B-band	-0.034	-0.026	-0.129***	-0.053	-0.091**	-0.127**
	(0.049)	(0.042)	(0.037)	(0.041)	(0.044)	(0.049)
Lagged Predicted B-band	-0.001	-0.000	-0.018***	-0.006	-0.010**	-0.020***
	(0.006)	(0.004)	(0.004)	(0.005)	(0.004)	(0.006)
ΔGDP		-2.861***	-2.497***	-2.314***	-2.606***	-1.879***
		(0.509)	(0.560)	(0.354)	(0.568)	(0.327)
Lagged GDP		-0.319*	-0.407**	-0.236	-0.410**	-0.403**
		(0.172)	(0.174)	(0.165)	(0.181)	(0.172)
ΔReplacement Rate			0.129			0.094
			(0.116)			(0.126)
Lagged Replacement Rate			0.125***			0.048
			(0.042)			(0.085)
ΔPublic Sector				0.225		0.403**
				(0.147)		(0.181)
Lagged Public Sector				0.131		0.277***
				(0.083)		(0.087)
ΔTax Burden					-0.019	-0.093
					(0.173)	(0.163)
Lagged Tax Burden					-0.046	-0.118
					(0.134)	(0.092)
Observations	456	456	384	249	408	211
No. countries	24	24	24	22	24	22
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
$p(F=0)$	0.000	0.000	0.000	0.000	0.000	0.000
RMSE	0.112	0.101	0.090	0.105	0.096	0.091
R-squared	0.449	0.557	0.629	0.628	0.586	0.719

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The dependent variable is the change in the logarithm of the *Unemployment* rate. The sample is 24 OECD countries in years 1998-2017. Cluster-robust standard errors in parentheses, except for columns (4) and (6) that report conventional standard errors because of insufficient degrees of freedom.



(a) Self-Employment rate



(b) Unemployment rate

Figure 1: *Self-employment and unemployment rates with and without broadband Internet.*
Note: Labour-force-weighted sample average self-employment and unemployment rates as in the data and in a counterfactual scenario of no broadband roll-out implied by our estimates.

We report our results on the effects of broadband diffusion on the self-employment rate in Table 2. We find strong evidence of positive short-term and long-term effects of broadband diffusion. Point estimates of the relevant coefficients are positive in all specifications and significantly different from zero. Among the remaining regressors, only the size of the *Public Sector* is found to have significantly negative effects on the self-employment rate. Table 3 presents our findings on the unemployment rate. In this case, we estimate negative short-term and long-term effects of broadband diffusion. The coefficients are statistically significantly different from zero in fewer specifications relative to the results on the self-employment rate. On top of that, we find strong and robust evidence in favour of Okun's law, in the short- and long-run, as captured by negative and statistically significant coefficients on *GDP*. We also find that the generosity of the unemployment insurance system contributes to higher unemployment.

The dynamic and auto-regressive formulation of the estimation equation (1) complicates the interpretation of the quantitative significance of the estimated coefficients. For that reason, we consider a hypothetical stark scenario of no broadband Internet rollout, setting *Predicted Broadband* to zero in the entire sample. Then, we calculate predicted *Self-Employment* and *Unemployment* rates using the estimates from column (2) of Tables 2 and 3.[6] We consider our choice a fairly conservative one as it implies the weakest effects of the rollout of broadband on unemployment. Finally, we calculate counterfactual labour-force-weighted-averages of self-employment and unemployment rates in each year.

The results of this exercise is illustrated in Figure 1. We uncover a strong cumulative effect of broadband Internet diffusion on self-employment. If it was not for broadband Internet, the initial average of 15.2% would drop not to 12.3% but to 4.5% in 2017. The dynamic effects of the absence of broadband Internet are quite substantial for the unemployment rate as well. In 2017, the labour-force-weighted unemployment rate stood at 4.4%. Without broadband rollout, it would have been equal to 6.5%.[7]

[6]We chose this specification as the one with the largest number of regressors that still allows us to use our full sample. For example, the data on the *Replacement Rate* only start in 2001. Using a richer specification would thus not only shrink the size of the sample significantly, but it would also yield baseline observations of the dependent variables that already include the effects of at least 4 years of broadband Internet rollout.

[7]These results are not driven by the countries with the largest labour force in our sample. We find that countries with greater broadband diffusion tend to have higher self-employment and lower unemployment rates than their counterfactual trends would imply. For example, the cross-country correlation between the increase in *Predicted Broadband* from 1997 to 2018 and the corresponding changes in the *Self-Employment* and *Unemployment* rates, normalised by their initial values, are 0.54 and -0.37, respectively. See Appendix A.1 for further details.

2.3 Extensions and robustness

We further investigate the drivers and the robustness of our results along three lines. First, we study whether the effect of broadband Internet on the self-employment rate is as strong and significant for own-account workers, a subset of the broader population of the self-employed. Second, we find out which sectors drive the effect of broadband on the self-employment and own-account work rate. Third, we discuss the robustness of our estimation results to an alternative econometric specification of the main estimation equation.

2.3.1 Own-account workers

The theory of self-employment that we put forth in the next section relies on the income risk differentials between a payroll job and entering self-employment. In reality, the self-employed are a heterogeneous group in terms of their exposure to income risk. Arguably, an entrepreneur running a large incorporated business with many employees faces less risk than a freelancer working solely on their own. However, unincorporated entrepreneurs with very few employees have similar characteristics to own-account workers, as shown by Levine and Rubinstein (2016). Both the former and the latter group are included in our *Self-Employment* variable, potentially blurring the identification of the effects of the broadband Internet rollout. To sharpen the difference in income risk faced by the self-employed and those looking for a job at a firm, we alternatively proxy self-employment with the share of own-account workers in the labour force, defining the *Own-Account Work* rate OA_{it} as follows:

$$OA_{it} = \frac{\text{own-account workers}_{it}}{\text{unemployed}_{it} + \text{employees}_{it} + \text{self-employed}_{it}} \times 100\%. \tag{4}$$

As demonstrated in Table 1, own-account workers constitute approximately two-thirds of overall self-employment. We then estimate equation (1) with the *Own-Account Work* rate as the dependent variable. We report the results of this exercise in Appendix A.2, finding no substantial difference between the results reported in Table 2.

2.3.2 Sectoral Evidence

To dig deeper into what drives our findings on the effect of broadband Internet on the self-employment rate, we decompose it into six broad sectors. The details of this exercise are presented in Appendix A.3. We find that the rollout of broadband Internet significantly increased the self-employment rate in manufacturing, construction and, in the long run, market services. For the aggregate economy, these effects dominate a significantly negative effect on self-employment in the mining and utilities sector, in which the self-employment rate is negligible. The only significantly positive effect on the own-account work rate occurs in market services.

Our data allow us to further decompose the effect in market services in smaller sectors. We find that the positive long-run effect on the self-employment and own-account work rate in market services is primarily driven by effects on administrative and support service activities, and, to a lesser extent, professional, scientific and technical activities. Interestingly, the effect on the information and communication sector is negative, confirming anecdotal evidence that broadband Internet led to more concentration ('big tech') rather than self-employment or own-account work. However, while this may be true for the information and communication sector itself, the significant and positive effects for the aggregate economy suggest that broadband Internet has facilitated self-employment and own-account work in other sectors.

2.3.3 AR(1) residuals

To test the robustness of our results we also adapt a more flexible specification, estimating equation (1) together with:

$$\varepsilon_{it} = \rho \varepsilon_{it-1} + \epsilon_{it}, \tag{5}$$

allowing for autocorrelation of residuals. Estimating only equation (1) is equivalent to imposing the restriction $\rho = 0$ in (5). However, including (5) comes at the cost of shrinking the time dimension of our sample.[8]

The take-away message from this exercise, the details of which we report in Online Appendix C.4, is that we do not find evidence in favour of AR(1)-residuals either for *Self-Employment* or *Own-Account Work* rates. The estimated AR(1) coefficient is close to zero in both cases. However, for the *Unemployment* rate the AR(1) coefficient in equation (5) ranges from 0.4 to 0.6. We find much stronger and statistically significant negative effects of *Predicted Broadband* on unemployment in this more flexible specification. This finding confirms that our choice of point estimates used to construct the counterfactual no-broadband Internet series of unemployment was a conservative one.

[8]We chose our baseline specification, together with reporting robust standard errors, because it is the generally preferred approach when the number of units N is larger than the number of time periods T, which in our case stand at 24 countries and 20 years, respectively.

3 Model

In this section we propose a tractable model centered around a career choice between self-employment and payroll employment. According to our empirical evidence, the rollout of broadband Internet increased self-employment and, although not always significantly, decreased unemployment. We consider two potential interpretations of these findings. First, high-speed Internet has been accompanied by the development and use of platforms and mobile apps. We think of these innovations as increasing the likelihood that self-employed workers can find demand for their services. Inspired by our sectoral evidence, one can think of a consultant or architect that can work for a large firm, or that can work on their own and find customers on e.g. LinkedIn. Second, Internet also facilitates conventional job search and recruiting (Autor, 2001; Stevenson, 2009). Motivated by these explanations, our model features frictions in both labour and goods markets, and the rollout of broadband may have reduced frictions in both markets.

We consider an economy in discrete time inhabited by three types of agents - consumers, workers and firms - that interact in two markets: the goods market and the labour market. Time is infinite and all agents discount the future with a factor β. As in Rudanko (2009), workers are risk-averse and hand-to-mouth and can only be insured against income risk in payroll employment at large, risk-neutral firms.[9] We focus on the career choice and the risks associated with self-employment, in addition to the risks of entering the labour market.

A snapshot of the model is presented in Figure 2. At the beginning of a period, a fraction δ of existing payroll-employment relationships are destroyed. The workers that lose their job,

[9]Rudanko (2009) uses a slightly different nomenclature. In her model, risk neutral entrepreneurs can open a continuum of single-worker production units which she refers to as firms. We have large firms that hire a measure of workers, each staffing a single-worker production unit.

together with those already in unemployment and self-employment, face a choice between self-employment and searching for payroll employment. When workers choose to become self-employed, they try to sell their production directly to consumers, but they face the risk of not being able to enter the goods market. When workers choose to seek employment in a firm, they face the risk of unemployment. Once hired, however, the employee enters an employment relationship and is insured by the firm against the risk of not entering the goods market. Payroll employees can be offered insurance by firms because firms are large. Firms are large because each firm (from a fixed measure of firms) opens an endogenously determined measure of vacancies. In the goods market, self-employed workers and payroll employees supply a homogeneous and non-storable good to consumers. Because the market is competitive, the law of one price holds. We formalize this description and present the relevant details below.

3.1 Model setup

3.1.1 Consumers

Identical consumers live on a unit square $[0,1] \times [0,1]$ and freely enter a perfectly competitive goods market. Their preferences over the consumption good are captured by a linear utility function. To acquire q_t^c units of the consumption good at prevailing price p_t, they pay with a spot good – the numéraire – that they produce according to a strictly convex production function g. Thus, given the market price p_t, consumers essentially face a static problem and their utility of consuming q_t^c reads:

$$V^C(q_t^c; p_t) = \max_{q_t^c} q_t^c - g(p_t q_t^c) + \beta V^C.$$

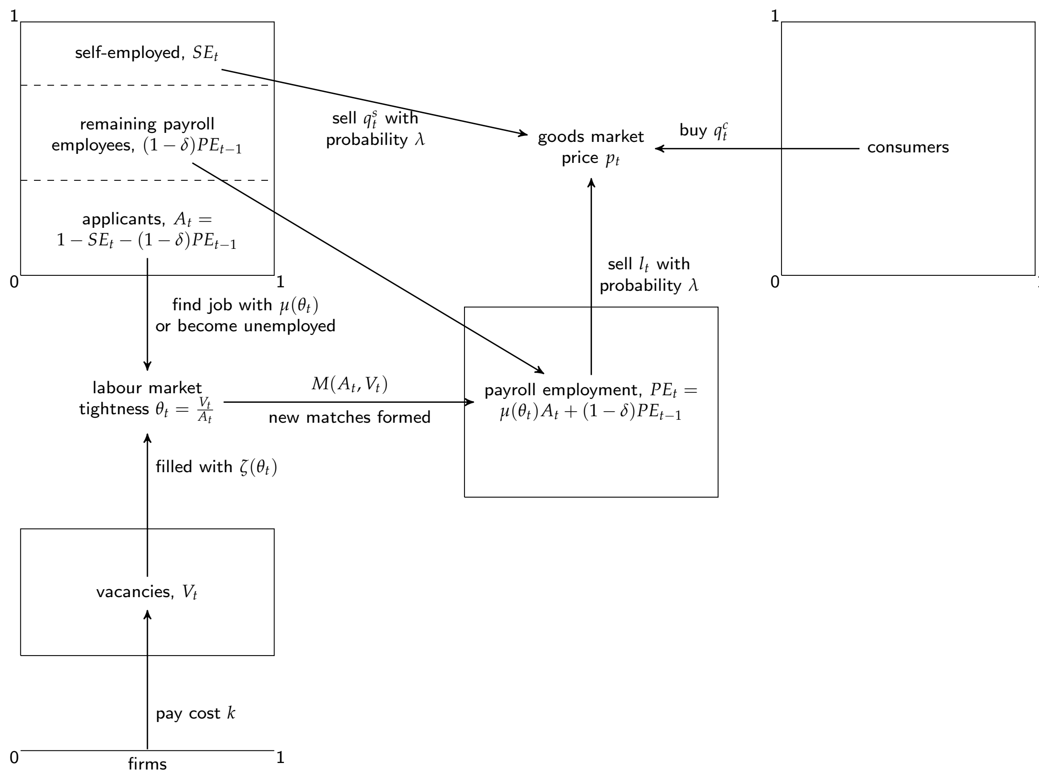


Figure 2: *Snapshot of the model.*

The optimal demand q_t^c is thus pinned down by the following equation:

$$g'(p_t q_t^c) = \frac{1}{p_t}, \tag{6}$$

which is also the aggregate demand equation.

3.1.2 Workers and Career Choice

Identical workers live on a unit square $[0,1] \times [0,1]$. They produce the consumption good desired by consumers at linear cost, and value the spot good supplied by consumers. In particular, they have a strictly increasing and strictly concave utility of consumption $u(c)$ so that $u'(c) > 0$ and $u''(c) > 0$. We also normalize $u(0) = 0$.

At the end of each period, workers can be self-employed SE_t, unemployed U_t, or employed at a firm in payroll employment PE_t, so that $SE_t + U_t + PE_t = 1$. At the beginning of a period, a fraction δ of the payroll employees loses their job, while the remainder remains employed. All workers except those that remain in payroll employment face a career choice: they can either become self-employed, which yields expected utility V^{SE}, or they can enter the labour market, which yields expected utility V^{LM}. [10] With the share of workers who choose to become self-employed equal to $0 \leq SE_t \leq (1-\delta)PE_{t-1}$, the measure of applicants searching for a job at a firm is given by $A_t = 1 - SE_t - (1-\delta)PE_{t-1}$. Thus, we rule out job search during self-employment. Assuming that the career choice is exclusive is simply the extreme version of the assumption that running a viable business (that actually results in earnings) takes time and reduces job search intensity.

3.1.3 Vacancies and Labour Market

Firms live on a unit segment $[0, 1]$. Each firm h perfectly elastically opens a measure of vacancies v_t^h at a cost of k units of the numéraire per vacancy. The overall stock of vacancies is $V_t = \int_0^1 v_t^h dh$. A filled vacancy becomes a one-worker production unit.

The labour market is characterized by search frictions. Search is random and the total number of matches between applicants A_t and vacancies V_t is given by a matching function

[10] In equilibrium, which we formally introduce in Section 3.2, it will never be optimal to quit payroll employment and become self-employed or search for another job. Furthermore, in equilibrium featuring positive SE_t and PE_t, workers will be indifferent between pursuing self- and payroll-employment, so that $V^{SE} = V^{LM}$. While workers have the option to switch careers next period, we can thus write discounted continuation values as βV^{SE} for self-employed (see equation (9)) and βV^{LM} for applicants (see equation (13)), instead of $\beta \max\left\{V^{SE}, V^{LM}\right\}$ in either case, for notational convenience.

$M_t = Em(A_t, V_t)$ that is homogeneous of degree one and increasing and concave in its arguments. The parameter $E > 0$ captures the exogenous matching efficiency. We model declining labour market frictions as an increase in E.

Let $\theta_t = V_t/A_t$ be labour market tightness, the ratio of vacancies to applicants. The probability that an individual vacancy is filled is then $\zeta(\theta_t) = M_t/V_t$. Correspondingly, each applicant finds a job with probability $\mu(\theta_t) = M_t/A_t$. Using the definitions of the job filling and job finding probabilities, we can also write $\zeta(\theta_t) = E\hat{\zeta}(\theta_t)$ and $\mu(\theta_t) = E\hat{\mu}(\theta_t)$ with $\hat{\zeta}(\theta_t) = m(A_t, V_t)/V_t$ and $\hat{\mu}(\theta_t) = m(A_t, V_t)/A_t$, so that $\hat{\zeta}'(\theta_t) < 0, \hat{\zeta}''(\theta_t) > 0$, $\hat{\mu}'(\theta_t) > 0$, and $\hat{\mu}''(\theta_t) < 0$.

Given last period's measure of payroll-employment PE_{t-1}, workers split between self-employed SE_t and applicants A_t. Given the resulting job-finding probability $\mu(\theta_t)$, end-of-period payroll employment PE_t is given by:

$$PE_t = \mu(\theta_t)A_t + (1-\delta)PE_{t-1},$$

and end-of-period unemployment U_t is given by:

$$U_t = (1 - \mu(\theta_t)) A_t.$$

Unemployed workers receive UI benefits b.

We will focus on steady state equilibria. In steady state, the measures of self-employed, unemployed, and payroll-employed workers do not change. Dropping the time-subscripts from now onwards, steady-state payroll employment is given by:

$$PE = (1 - SE)\frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))}, \tag{7}$$

and steady state unemployment is therefore:

$$U = (1 - SE)\frac{\delta(1 - \mu(\theta))}{\mu(\theta) + \delta(1 - \mu(\theta))}. \tag{8}$$

Finally, in steady state the measure of applicants is given by $A = U + \delta E$, so that:

$$A = (1 - SE)\frac{\delta}{\mu(\theta) + \delta(1 - \mu(\theta))}.$$

3.1.4 Frictional Entry in the Goods Market

Each self-employed worker and each one-worker production unit faces frictional entry into the competitive goods market: only with probability $\lambda \in (0, 1)$ can they enter and sell their production.[11] We will also say that when a worker manages to enter the goods market, this worker is visible in the goods market. The motivation for this formulation is that we think of search frictions in the goods market as consumers simply not being aware of sellers. Declining search frictions then allow more sellers to compete for the demand of consumers because they have become visible to them.[12] We model declining goods market frictions

[11]We consider the less tractable case of an endogenous probability to enter the goods market in Online Appendix D.3. We show that this extension does not affect the characterisation of equilibrium and the effects of declining search frictions on the self-employment rate.

[12]We do not only think of declining search frictions as increasing the likelihood that a seller is visible to a customer, but also as making it more likely that a previously unknown seller is considered sufficiently trustworthy by a customer to do business. Although we do not model such mechanisms explicitly, we do believe that ratings and reviews on platforms have decreased asymmetric information and facilitated trade.

as an increase in λ. Hence, we assume that large firms and self-employed workers benefit equally from reductions in search frictions in the goods market.[13]

3.1.5 Self-Employment

Because we assume that production cannot be stored, self-employed workers face a static problem. Given the price, they choose the quantity of the consumption good that they produce. The self-employed who do not become visible in the goods market do not produce and do not earn any income. Thus, the expected value of becoming self-employed and, upon making it to the goods market, supplying q^s, reads:

$$V^{SE}\left(q^s; p\right) = \max_{q^s} \lambda \left(u\left(pq^s\right) - q^s\right) + \beta V^{SE}, \tag{9}$$

with the optimal production choice q^s satisfying:

$$u'\left(pq^s\right) = \frac{1}{p}. \tag{10}$$

3.1.6 Payroll Employment

Because each firm opens a measure of vacancies, the law of large numbers applies and an individual firm faces uncertainty neither in the labour nor the goods market. Given $\zeta(\theta)$, the firm knows exactly how many of its vacancies will be filled, and given λ, how many of its employees will sell production for price p. As a result, the firm simply receives expected

[13]One can argue that before the arrival of broadband Internet large firms had smaller problems being visible to consumers than self-employed workers, so that the self-employed should benefit more from the arrival of broadband. However, throughout the paper we tie our hands and derive comparative statics for the case in which reductions in goods market frictions do not result in a higher self-employment rate essentially by assumption.

profits and can commit to pay salaries to employees who were hired but whose production units did not become visible in the goods market.

Employment contracts are characterised by greater complexity than spot trades in markets for goods. We assume that the employment contract follows from generalized Nash bargaining, with workers' bargaining power equal to ϕ and the value of unemployment as outside option for the worker. In particular, when a worker finds a vacancy, the firm and the worker bargain over the quantity l that the worker needs to produce when their production unit is visible in the goods market, the wage rate w that the worker will be paid for their production, and the salary d that the worker receives when their production unit is not visible in the goods market. Because vacancies can be opened perfectly elastically, unfilled vacancies have no value. Given a job-destruction rate δ, the steady-state value of a filled vacancy is thus:

$$V^J = \lambda\,(p - w)\,l - (1-\lambda)d + \beta(1-\delta)V^J, \tag{11}$$

Similarly, the value of payroll employment reads:

$$V^{PE} = \lambda\,(u\,(wl) - l) + (1-\lambda)u(d) + \beta\left(\delta V^{LM} + (1-\delta)V^{PE}\right), \tag{12}$$

in which V^{LM} is the value of entering the labour market as an applicant, which is given by

$$V^{LM} = \mu\,(\theta)\,V^{PE} + (1-\mu(\theta))\left(u(b) + \beta V^{LM}\right). \tag{13}$$

The value of unemployment is thus given by $u(b) + \beta V^{LM}$.

When the worker and the firm bargain over the employment contract, they take the value of unemployment and the (zero) value of an unfilled vacancy as outside options, so that the Nash product is

$$\max_{w,d,l}\left(V^{PE}(w,d,l)-u(b)-\beta V^{LM}\right)^{\phi}\left(V^{J}(w,d,l)\right)^{1-\phi}$$

We derive the properties of this contract in Appendix B.1. Notably, the risk-neutral firm and risk-averse worker agree to fully insure the worker against the goods market income risk: $wl=d$. The employees always receive income wl but produce only when their production unit is visible in the goods market. Hence, we can scrap d completely, and the contract on w and l satisfies the following conditions:

$$u'\left(wl\right)=\frac{1}{p}, \tag{14}$$

$$\phi\frac{(\lambda p-w)lu'(wl)}{1-\beta(1-\delta)}=(1-\phi)\frac{u\left(wl\right)-\lambda l-u(b)}{1-\left(1-\mu(\theta)\right)\beta(1-\delta)}. \tag{15}$$

With perfect insurance against the goods-market risk, the value of entering the labour market is:

$$V^{LM}=\frac{\mu\left(\theta\right)\left[u\left(wl\right)-\lambda l\right]+\left(1-\mu(\theta)\right)\left(1-\beta(1-\delta)\right)u(b)}{\left(1-\beta\right)\left[1-\left(1-\mu(\theta)\right)\beta(1-\delta)\right]}. \tag{16}$$

Furthermore, since unfilled vacancies have no value, expected firm profits per vacancy exactly cover vacancy posting costs, so that with insurance of employees (11) implies the following free-entry condition:

$$k=\zeta\left(\theta\right)\frac{\left(\lambda p-w\right)l}{1-\beta(1-\delta)}. \tag{17}$$

3.2 Steady state equilibrium

To close the model we require the goods market to clear. The market-clearing equilibrium price is defined implicitly as a price for which aggregate demand equals aggregate supply:

$$q^c = \lambda \left(SEq^s + (1 - SE) \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} l \right), \tag{18}$$

where supply follows from the measures of self-employed and payroll-employed workers in (7), and their respective expected production per worker.

The formal equilibrium concept we are after is a steady state mixed-strategy solution to the career choice, one in which payroll- and self-employment co-exist and are constant, pursuing each career yields the same expected utility, and workers outside of payroll employment randomize between them.

DEFINITION 1 (Mixed-strategy career-choice equilibrium). *A steady state mixed-strategy career-choice equilibrium (MSCC-equilibrium) is a tuple* $(SE, p, q^s, q^c, w, l, \theta)$ *such that:*

- $0 < SE < 1$, $0 < \theta$, *both types of employment are chosen in equilibrium and active in the goods market,*[14]

- *given* p, *each consumer demands* q^c *as prescribed by equation* (6), *each visible self-employed sells* q^s *given by equation* (10) *and* θ, w, l *satisfy equations* (14), (15), *and* (17),

[14] Note that with $\theta = 0$ there would be no payroll employment.

- *given θ, w, l, q^c and q^s; p and SE simultaneously clear the goods market in* (18)*, and make workers indifferent between self-employment and searching for a job at a firm:*

$$(1-\beta)V^{SE} = (1-\beta)V^{LM} \iff$$
$$\lambda\left(u(pq^s) - q^s\right) = \frac{\mu\left(\theta\right)\left[u\left(wl\right) - \lambda l\right] + \left(1-\mu(\theta)\right)\left(1-\beta(1-\delta)\right)u(b)}{1-\left(1-\mu(\theta)\right)\beta(1-\delta)}. \tag{19}$$

The equilibrium is a vector consisting of 7 variables that jointly solve equations (6), (10), (14), (15), (17), (18), and (19). In an MSCC-equilibrium, the self-employment rate SE is endogenously determined such that workers are ex ante indifferent between the two careers. Although workers are ex ante indifferent, they strictly prefer payroll employment over self-employment, and self-employment over unemployment.

4 Analytical results

We proceed by describing the MSCC-equilibrium in our model. We start with some analytical observations assuming an equilibrium exists. Then, we explain how to construct an MSCC-equilibrium. Next, we derive comparative statics with respect to reductions in frictions in the goods and in the labour market. Finally, we discuss the role of the main assumptions in our model.

4.1 MSCC equilibrium: characterisation, existence and comparative statics

LEMMA 1. *Let $(SE, p, q^s, q^c, w, l, \theta)$ be an MSCC-equilibrium. Then:*

1. *$wl = pq^s$, $w < p$, and $l > q^s$: an employee produces more than a self-employed.*

2. *There exists a differentiable and strictly increasing function $\psi(p)$ and a weight $\omega(\theta) \in (0, 1)$ given by*

$$\omega(\theta) \equiv \frac{\mu(\theta)\phi}{1 - (1 - \mu(\theta)\phi)\,\beta(1-\delta)}. \tag{20}$$

such that the workers' career-choice indifference reads

$$(1-\beta)V^{SE} = \lambda\psi(p) = \omega(\theta)\psi(p) + (1-\omega(\theta))\,u(b) = (1-\beta)V^{LM}, \tag{21}$$

the free-entry condition becomes

$$k = \zeta(\theta)\frac{(1-\phi)p\,[\psi(p) - u(b)]}{1 - \beta(1-\delta)\,(1-\mu(\theta)\phi)}, \tag{22}$$

so that equilibrium requires $\psi(p) - u(b) > 0$, and the sharing rule can be written as

$$\lambda l = (1-\phi)\frac{[1-\beta(1-\delta)]\,[\psi(p) - u(b)]}{1 - \beta(1-\delta)\,(1-\mu(\theta)\phi)} + q^s, \tag{23}$$

and hence in equilibrium $\lambda l > q^s$.

3. *The demand function is strictly monotone: $dq^c/dp < 0$.*

The proof is provided in Appendix B.2. For workers who secure it, labour income is equalized between the two careers in equilibrium, because the linear cost of effort makes utility transferable between firms and workers. However, employees who staff production units that are visible in the goods market produce more per capita than the visible self-employed, for two reasons. First, because they must generate profits to cover the vacancy creation costs, which explains the difference between q^s and λl. Indeed, the expected discounted markup of production by an employee over that of a visible self-employed measured in units of the numéraire exactly covers the expected recruiting costs $k/\zeta(\theta)$ per employee, as merging (22) and (23) reveals:

$$\frac{p\left(\lambda l - q^s\right)}{1-\beta(1-\delta)} = \frac{(1-\phi)p\left[\psi(p)-u(b)\right]}{1-\beta(1-\delta)\left(1-\mu(\theta)\phi\right)} = \frac{k}{\zeta(\theta)}.$$

Second, employees produce more than the self-employed because they pay for the insurance against the goods market risk that all employees obtain, by producing the additional $(1-\lambda)l$.[15]

Part 3 of Lemma 1 shows that aggregate demand is strictly decreasing in price p. To guarantee that aggregate supply is strictly increasing in price p, we make the following assumption.

ASSUMPTION 1 (Monotone aggregate supply). *The utility function $u(c)$ is such that $dq^s/dp > 0$.*

This assumption is analogous to the absence of a wealth effect on labour supply, as in the case of GHH preferences. This condition is met by CRRA utility functions of the form $u(c) = Ac^{1-\sigma}/(1-\sigma)$ with $\sigma \in (0,1)\,, A > 0$, which is a common choice in quantitative and applied work using models similar to ours (see e.g. Berentsen *et al.* (2011)). Through (23), Assumption 1 also guarantees that $dl/dp > 0$. For preferences that do not satisfy

[15]As we abstract from leisure in the model, q^s and l should be interpreted in terms of production (and not as hours worked) which result from an identical production function for firms and the self-employed. Therefore, the model predicts employees to be more productive per capita than the self-employed at the cost of a higher disutility of effort in payroll employment.

Assumption 1, aggregate supply – the right-hand side of the goods market clearing condition (18) – may be non-monotone in price p, which may lead to a multiplicity of equilibria.[16]

The results in Lemma 1 reduce the dimensionality of the problem of finding an equilibrium. The linear cost of effort allows for writing the terms of the employment contract solely as a function of price, just as the decisions of consumers and the self-employed: q^c, q^s, l and w follow from p. As a result, the equilibrium is characterised by three nonlinear equations – workers' indifference (21), free entry of vacancies (22) and goods market clearing (18) – in three unknowns: p, θ and SE.

As we show below, in order to characterise the MSCC equilibrium further it is convenient to study the implications of the assumption of no unemployment benefits, $u(b) = 0$, first. This assumption permits us to reduce the dimensionality of the problem of finding the equilibrium even more. Unless $u(b) = 0$, unemployment insurance not only affects the division of the surplus between employees and firms, but also workers' indifference condition. The analytically more convenient case of $u(b) = 0$ brings additional insights, as encapsulated in the following Lemma.

LEMMA 2. *Assume $u(b) = 0$. Then moving a worker from self-employment to the labour market increases supply, because*

$$\lambda q^s < \lambda \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))} l. \tag{24}$$

The proof is provided in Appendix B.3. The left-hand side of equation (24) is the expected quantity produced by a self-employed worker. The right-hand side presents the expected production of an applicant: the quantity produced by a visible payroll employee, multiplied

[16] Even for such preferences, we could ensure uniqueness of an MSCC-equilibrium by choosing an appropriate buyers' production function g, resulting in an aggregate demand function that crosses supply only once.

by the probability of being visible, the probability of finding a job, and the expected duration of a worker-firm match.

Lemma 2 helps proving Theorem 1, our first main result. In the light of the discussion above, the proof of Theorem 1 has two parts. The first part is constructive and provides conditions for the equilibrium with $u(b) = 0$ to exist in a non-empty region of the parameter space. The second part of the proof ties equilibrium existence to the existence of a solution to an ordinary differential equation with respect to $u(b)$ with initial condition equal to $u(b) = 0$. We provide the constructive part of the proof in the main text and offer more details on the second part in Online Appendix D.1.[17]

THEOREM 1 (Existence and uniqueness of MSCC equilibrium). *Let Assumption 1 hold. For $u(b) = 0$, there exist numbers $0 < \underline{k}(\lambda, \phi, \beta, \delta) < \bar{k}(\lambda, \phi, \beta, \delta) < \infty$ such that an MSCC equilibrium exists and is unique, if $\lambda(1 - \beta(1 - \delta)) < \phi(1 - \lambda\beta(1 - \delta))$ and $k \in \left(\underline{k}(\lambda, \phi, \beta, \delta), \bar{k}(\lambda, \phi, \beta, \delta)\right)$. When these conditions are met, an MSCC equilibrium exists and is unique also for $u(b) > 0$ provided $u(b)$ is small enough.*

Proof. For $u(b) = 0$ the equilibrium conditions (21) and (22) simplify to:

$$\lambda = \omega(\theta) \equiv \frac{\mu(\theta)\phi}{1 - (1 - \mu(\theta)\phi)\,\beta(1 - \delta)}, \tag{25}$$

$$k = \frac{\zeta(\theta)(1 - \phi)p\psi(p)}{1 - \beta(1 - \delta)\,(1 - \mu(\theta)\phi)}. \tag{26}$$

For given λ, ϕ, and $\beta(1 - \delta)$, worker's indifference (25) pins down the equilibrium market tightness θ^*, unless λ and ϕ are such that this equation has no solution for $\mu(\theta) < 1$, so that

[17] It is possible that an MSCC equilibrium only exists when $u(b) > 0$ for a certain range of b. While such cases can easily be found and solved numerically, they are not as tractable as equilibria under $u(b) = 0$.

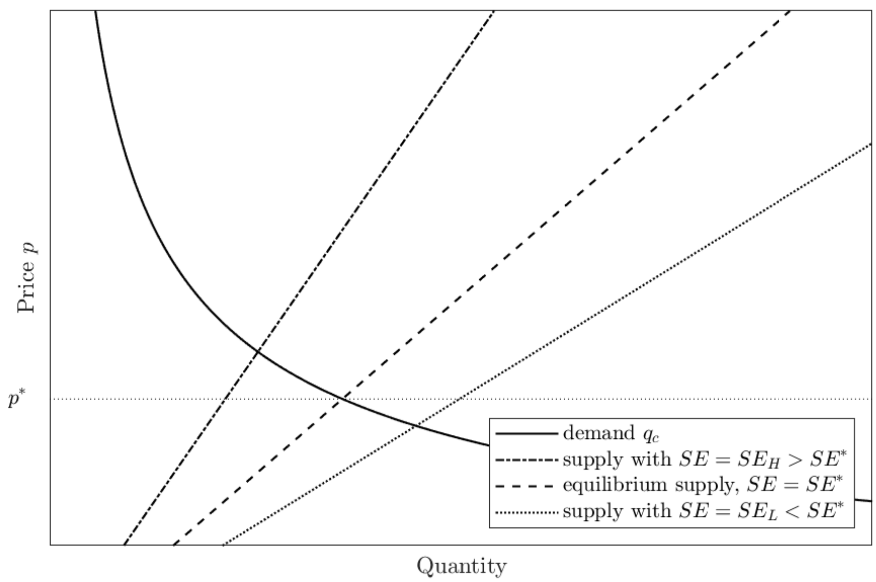


Figure 3: *Aggregate supply and demand curves.*
Note: Aggregate supply curves for three levels of the self-employment rate, SE_L, SE^* and SE_H with $SE_L < SE^* < SE_H$, crossing the aggregate demand curve. The goods market clears for SE^* at an equilibrium price p^* consistent with tightness θ^* pinned down by workers' indifference.

there is only one type of employment in equilibrium.[18] Given θ^*, the equilibrium price p^* is pinned down by free-entry condition (26) for a given k. An MSCC equilibrium then exists if, and only if, there exists a self-employment rate $SE^* \in (0, 1)$ such that the goods market clears given p^* and θ^*.

By virtue of Assumption 1 there is at most one self-employment rate that clears the goods market. For a given value of p, the quantities produced q^s and l are fixed. Moving the self-employment rate thus amounts to shifting the aggregate supply curve, as Lemma 2 shows that moving a worker from self-employment to the labour market increases supply.

[18]Note that the function $\psi(p)$ is well defined also outside of the region supporting an MSCC-equilibrium. One can think of it as the value function capturing the net utility of an optimal quantity choice made by a hypothetical self-employed confronted with price p. This object is well-defined even if $SE = 0$.

We illustrate this algebraic route to finding the equilibrium in Figure 3 with three values of the self-employment rate, SE_L, SE^* and SE_H, with $SE_L < SE^* < SE_H$. For each self-employment rate, there exists a corresponding aggregate supply curve, for a fixed value of labour market tightness θ^* (pinned down by workers' indifference in (25)) and fixed q^s and l pinned down by the equilibrium price p^* (implied by free entry in (26)).

The bounds on the vacancy posting cost compatible with an MSCC-equilibrium can then be found by considering the limiting cases of $SE \mapsto 0^+$ and $SE \mapsto 1^-$. The latter yields the highest price level, and the former yields the lowest price level which occurs in an MSCC-equilibrium for a given θ^*. These bounds map one-to-one to bounds on the vacancy-creation cost k, ensuring that the price level that clears the goods market is consistent with $0 < SE < 1$. Therefore, for sufficiently high values of k there will be no payroll employment, and for sufficiently low k there might be no self-employment. Furthermore, the larger λ, ϕ and δ are, and the smaller β is, the lower the values of the vacancy-creation cost k that are consistent with existence of the equilibrium.[19] □

The constructive proof of MSCC-equilibrium existence starts with the premise that workers are indifferent between careers. To understand the economic intuition behind the equilibrating forces in the model, consider taking some workers away from the pool of applicants and making them enter self-employment instead, while keeping the free-entry and goods-market clearing conditions satisfied. When more workers enter self-employment, they decrease congestion in the labour market, which benefits applicants. The free-entry condition requires that firms are compensated for a tighter labour market with a higher price, which increases the surplus per filled vacancy. The price rises because the self-employed

[19] In Online Appendix D.2 we solve for an equilibrium for one particular example of the utility functions.

produce less than other applicants. As a result, increasing the self-employment rate benefits job applicants via two channels. First, a higher self-employment rate increases the price and thus the surplus to be shared with employees. Second, the remaining applicants enjoy a greater likelihood of finding a job. The self-employed benefit only from the first effect. Formally, these effects of the self-employment rate on the incentives to enter self-employment rather than the labour market are:

$$\frac{d}{dSE}\left(V^{SE} - V^{LM}\right) = \underbrace{\lambda\psi'(p)\frac{dp}{dSE} - \omega(\theta)\psi'(p)\frac{dp}{dSE}}_{\text{net price effect}} - \underbrace{\omega'(\theta)\frac{d\theta}{dSE}\psi(p)}_{\text{tightness effect}} < 0. \tag{27}$$

However, as in equilibrium $\lambda = \omega(\theta)$, the self-employed and applicants benefit equally from the change in prices and the net price effect on workers' indifference is nil. Consequently, the effect of an increase in the job-finding rate is decisive and pulls workers back to the labour market. We illustrate these considerations in Figure 4. When $SE > SE^*$, the expected utility of becoming an applicant exceeds that of pursuing self-employment, $V^{SE} < V^{LM}$, pulling workers into the labour market. The converse is true for $SE < SE^*$.

Our second main result is on the comparative statics of the self-employment and unemployment rates with respect to matching efficiency in the labour market E and ease of entry in the goods market λ.

THEOREM 2 (Effects of reductions in frictions). *Let Assumption 1 hold. In any MSCC-equilibrium, a higher probability of selling in the goods market increases the equilibrium self-employment rate, $dSE/d\lambda > 0$, and decreases the unemployment rate, $dU/d\lambda < 0$. Improvements in matching efficiency in the labour market decrease the self-employment rate, $dSE/dE < 0$, and*

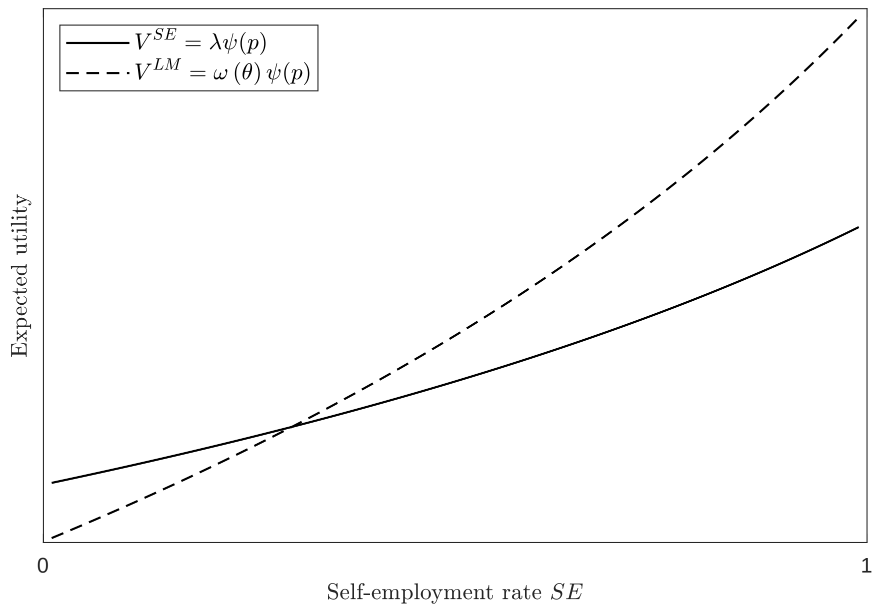


Figure 4: *Expected utility of entering self-employment or the labour market as a function of the self-employment rate.*
Note: The goods market clears and free entry of vacancies holds for all self-employment rates. Figure created with $u(c) = 2\sqrt{c}$ and $b = 0$. The lowest value on the vertical axis is greater than zero.

increase the unemployment rate, $dU/dE > 0$. Furthermore, reductions in goods market frictions increase, while reductions in the labour market frictions decrease the equilibrium price level, $dp/d\lambda > 0$ and $dp/dE < 0$.

The proof is provided in Appendix B.4. The direct effect of an increase in λ is to increase the likelihood of securing income in self-employment. For workers to remain indifferent, the job-finding probability must also increase, which requires labour market tightness to rise. A higher market tightness decreases the job-filling rate, and for the free-entry condition to remain satisfied, prices need to rise. Prices need to rise because due to the insurance in the equilibrium contract, firms do not directly benefit from increases in λ, as can be seen in (22). In equilibrium, higher prices and a higher labour market tightness result from a higher self-employment rate, since the self-employed produce less and do not congest the labour market. As a result, an increase in λ increases the equilibrium self-employment rate *and* the job-finding probability. By the virtue of (8), higher SE and higher $\mu(\theta)$ decrease the unemployment rate U. Not only do fewer workers expose themselves to unemployment risk, also the likelihood of finding a job increases. Even though a lower unemployment rate means fewer workers are idle, the reduction in aggregate supply from the shift towards self-employment dominates the increase in aggregate supply from a higher λ and lower unemployment rate. As a result, aggregate supply falls and prices go up.

When labour market matching efficiency E increases, it raises the job-filling probability. Firms open more vacancies, attracting workers from self-employment. However, when $u(b) = 0$, the equilibrium market tightness is fixed by worker's indifference in (25): the equilibrium job finding probability $\mu(\theta)$ cannot change when λ remains fixed. Hence, any increase in matching efficiency E must be exactly offset by adjustment in θ, which happens by the inflow of workers from self-employment. When $u(b) > 0$, the motive to abandon self-employment is even stronger. In this case, the congestion created by the inflow of workers from self-employment overturns the effects of the initial increase in E. Consequently, an increase in matching efficiency decreases the self-employment rate and cannot increase the job-finding probability. Consequently, the unemployment rate goes up. In equilibrium, prices fall because the effect of the reallocation of workers from self- to payroll employment dominates the adverse effect of higher unemployment on aggregate supply.

Hence, our model offers a new perspective on the puzzle identified by Martellini and Menzio (2020). They wonder why unemployment does not show a trend despite arguably plausible improvements in labour market efficiency. In our model, because of flows from and to self-employment, the unemployment rate can remain constant as long as search frictions decline *in both goods and labour markets*. To see this, consider a hypothetical joint decrease in frictions, $(\Delta\lambda, \Delta E)$ with $\Delta\lambda > 0$ and $\Delta E > 0$. The approximate response of the equilibrium unemployment rate, ΔU is:

$$\Delta U \approx \underbrace{\frac{dU}{d\lambda}}_{<0} \Delta\lambda + \underbrace{\frac{dU}{dE}}_{>0} \Delta E, \tag{28}$$

which can be of arbitrary sign by the virtue of Theorem 2. As can be seen in (8), the unemployment rate can remain constant either if the effects of $(\Delta\lambda, \Delta E)$ on SE and $\mu(\theta)$ perfectly offset each other so that the self-employment rate and the job-finding rate remain unchanged, or when, for example, an increase in the job-finding probability is balanced by a decrease in the self-employment rate. While our analytical results reveal how reductions in frictions in both markets *separately* impact on U and SE, identifying their joint effects is a quantitative question that we address in Section 5. In that section we also back out the contribution of broadband roll-out to the trends in frictions in both markets.

4.2 Discussion

To further explain the workings of the model, we turn to its two main assumptions. The first assumption is that large firms insure workers against goods market risk. The second assumption is that workers can freely choose their career.

4.2.1 Importance of income risk

Our model of career choice relies on differences in the exposure to risk in payroll- and self-employment. We show here that these differences are the *raison d'etre* of an MSCC-equilibrium. In the baseline model, firms enjoy the benefits of the law of large numbers. By the virtue of their size, they can guarantee income also to workers staffing production units that did not become visible in the goods market. Now suppose that firms were one-worker firms instead, so that they would only be able to pay a wage upon being visible in the goods market. The values of a filled vacancy and payroll employment would then read:

$$V^J = \lambda\,(p - w)\,l + \beta(1-\delta)V^J, \tag{29}$$

$$V^{PE} = \lambda\,(u\,(wl) - l) + \beta\left(\delta V^{LM} + (1-\delta)V^{PE}\right). \tag{30}$$

We show in Appendix B.5 that the employment contract that would result from Nash bargaining in this case, would satisfy:

$$u'(wl) = \frac{1}{p}, \tag{31}$$

$$\phi\frac{\lambda(p-w)lu'(wl)}{1-\beta(1-\delta)} = (1-\phi)\frac{\lambda\,[u\,(wl) - l] - u(b)}{1-(1-\mu(\theta))\,\beta(1-\delta)}, \tag{32}$$

Given these conditions, we can now provide the definition of a steady-state mixed-strategy career-choice equilibrium adjusted to one-worker firms.

DEFINITION 2 (MSCC-equilibrium with one-worker firms). *A steady-state mixed-strategy career-choice equilibrium with one-worker firms is a tuple* $(SE, p, q^s, q^c, w, l, \theta)$ *such that:*

- $0 < SE < 1$, $0 < \theta$, *both types of employment are chosen in equilibrium and active in the goods market,*

- *given* p*, each consumer demands* q^c *as prescribed by equation* (6)*, each self-employed sells* q^s *given by equation* (10) *and* θ, w, l *satisfy equations* (31)*,* (32) *and the free entry condition*

$$k = \zeta(\theta) \frac{\lambda \left(p - w\right) l}{1 - \beta(1 - \delta)} \tag{33}$$

- *given* θ, w, l, q^c *and* q^s*,* p *and* SE *simultaneously clear the goods market as in* (18) *and make workers indifferent between self-employment and searching for a job at a firm:*

$$\begin{aligned} (1-\beta)V^{SE} &= (1-\beta)V^{LM} \\ \lambda\left(u(pq^s) - q^s\right) &= \frac{\mu\left(\theta\right)\lambda\left[u\left(wl\right) - l\right] + \left(1 - \mu(\theta)\right)\left(1 - \beta(1-\delta)\right)u(b)}{1 - \left(1 - \mu(\theta)\right)\beta(1-\delta)}. \end{aligned} \tag{34}$$

It turns out that such an equilibrium could only exist under knife-edge conditions that make the career choice ill-defined.

PROPOSITION 1. *If an MSCC-equilibrium with one-worker firms exists, then* $k = 0$*. Not only the incomes in the two types of employment are equalised,* $wl = pq^s$*, but so are the quantities produced,* $l = q^s$*, so that wages equal prices,* $w = p$*. Workers are indifferent not only between self-employment and entering the labour market, but also between self-employment and payroll employment.*

The proof is provided in Appendix B.6. With one-worker firms, workers' indifference condition reads:

$$\lambda\psi(p) = \omega(\theta)\lambda\psi(p) + (1 - \omega(\theta))u(b). \tag{35}$$

The only difference between this condition and its counterpart in the baseline model, equation (21), is that the surplus created by the worker-firm match is now $\lambda\psi(p)$ and not $\psi(p)$. This new condition can only hold under one of two knife-edge cases, which are both only consistent with free-entry in vacancy creation if opening a vacancy is costless. The first case requires $\omega(\theta) = 1$, which only happens when workers find jobs with certainty and, on top of that, capture the full surplus of a match. Then self-employment and entering the labour market are indistinguishable in every respect. The second case is that the surplus created by worker-firm matches is exactly equal to the utility from unemployment insurance, $\lambda\psi(p) = u(b)$. Then workers are indifferent between self-employment, payroll employment and unemployment.

A similar insight can be derived in the baseline model with large firms, under parameter values that render the insurance offered by firms irrelevant. To see this, consider the case with $u(b) = 0$ for simplicity, so that equilibrium is described by conditions (25) and (26). Now suppose that the frictions in the goods market completely vanish: $\lambda \mapsto 1$. Then, an MSCC equilibrium can only exist when employees capture the full surplus from the match, $\phi \mapsto 1$, *and* search frictions in the labour market vanish as well, $\mu(\theta) \mapsto 1$. However, a job-finding probability tending to one is only compatible with costless job creation by the virtue of (22), so that k must tend to zero. Furthermore, (23) implies that $l = q^s$ in the limit, and thus $w = p$. Consequently, when $\lambda \mapsto 1$, self-employment and entering the labour market become exactly identical so that the career choice margin is ill-defined again.

Therefore, payroll employment can be interpreted as a costly insurance mechanism that is delivered by firms against goods market risk. If such insurance either cannot be provided or is irrelevant, payroll- and self-employment become indistinguishable, and the creation of payroll employment must happen at no cost. Otherwise, the only equilibrium is a corner case of $SE = 1$.

4.2.2 Importance of career choice

Theorem 2 shows that our model overturns the standard result on the effect of matching efficiency on the unemployment rate. To highlight that allowing workers to choose their career is the reason for this implication of the model, we define an equilibrium without career choice, in which the self-employment rate is exogenously fixed. Such an equilibrium can be rationalised, for example, by assuming that selection into self-employment is purely based on individual-specific preferences or costs, regardless of the income differential between the two types of employment.

DEFINITION 3 (Fixed career-choice equilibrium). *A steady state fixed career-choice equilibrium (FCC-equilibrium) is a tuple* $(p, q^s, q^c, w, l, \theta)$ *such that, given* $SE \in (0, 1)$*:*

- *given* p*, each consumer demands* q^c *as prescribed by equation* (6)*, each self-employed sells* q^s *given by equation* (10) *and* θ, w, l *satisfy equations* (14) - (17)*,*
- *price* p *clears the goods market so that equation* (18) *holds.*

When we fix the split of workers between self-employment and pursuing a career at a firm, the predictions on how reductions in search frictions in the goods and the labour market affect the unemployment rate change their signs, as summarised in the following result.

PROPOSITION 2 (Comparative statics in FCC-equilibrium). *Let Assumption 1 hold. In any FCC-equilibrium, the unemployment rate increases in the ease of entry in the goods market and decreases* *in the matching efficiency in the labour market,* $dU/d\lambda > 0$ *and* $dU/dE < 0$*. Furthermore, prices fall whenever frictions decline,* $dp/d\lambda < 0$ *and* $dp/dE < 0$*.*

The proof is provided in Appendix B.7. Ignoring the career choice may thus result in a converse prediction on the effect of improvements in labour market matching efficiency on unemployment. This prediction changes sign because the inflow of workers from self-employment, which (more than) offsets the initial increase in the job-finding probability in an MSCC-equilibrium, is ruled out in an FCC-equilibrium. The effect of λ on the unemployment rate also switches sign, for two reasons. First, improvements in goods market frictions act as a positive supply shock that decreases the price level, depressing job creation by large firms. Second, the resulting decrease in labour market tightness is not mitigated by job applicants leaving the labour market for self-employment. In the baseline model, this shift towards self-employment led to a price increase, and this mechanism is absent in an FCC-equilibrium. To sum up, in an FCC-equilibrium reductions in frictions in both markets act as positive supply shocks. Their effects on unemployment are of opposite sign because firms are large, so that their profits are not directly impacted by λ.

5 Quantitative Analysis

In this section, we impose the model on the data to explain the joint dynamics of self-employment and unemployment rates in a panel of OECD countries. In particular, we use the model equilibrium conditions to back out the panel of matching efficiency parameters λ and E that fully rationalize the joint dynamics of self-employment and unemployment rates in the data. This exercise allows us to investigate whether falling self-employment rates and non-trending unemployment rates can stem from declining search frictions. Moreover, using our reduced-form estimation results to construct counterfactual series of self-employment and unemployment rates in case of no broadband Internet diffusion, we quantify the effect of broadband Internet rollout on the severity of frictions in the goods and in the labour market. Finally, we test the main mechanism of our model by regressing a panel of general price levels coming from an external data source that we do not target, on our model-implied panel of matching efficiency parameters.

5.1 Model Inputs, Calibration and Model Outputs

The inputs to our numerical exercise are time series on self-employment and unemployment $\{SE_{it}, U_{it}\}$ for countries i and years t. The outputs are the matching efficiency parameters $\{\lambda_{it}, E_{it}\}$. We take the model to the data assuming that each country-year data point is a separate steady state of our model. We make this simplifying assumption because we are primarily interested in comparing search frictions before and after, and with and without, the rollout of broadband Internet. This assumption is innocuous, both on quantitative and analytical grounds. Quantitatively, labour market flows in most countries are sufficiently large for steady states to be reasonable approximations of annual data. Analytically, remember that workers that are not in payroll employment are free to re-optimize their career choice every period. As long as wages are renegotiated every period in response to shocks, payroll employees never quit their jobs and the worker indifference condition shapes the equilibrium as in steady state.

Because we calibrate our model to separate time series for 24 different countries, we must take a stand on whether to make a parameter country-specific or not, accounting for objective data limitations. Because of the latter, we agnostically assume identical preference parameters, choosing $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma = 0.3$, a value within the range of parameters usually used in related work.[20] We also assume that the cost function of buyers is the inverse of the utility function, $g(\cdot) = u^{-1}(\cdot)$. Next, we set the discount factor β to 0.96, a standard value in the macro literature for calibration at an annual frequency.

As for the matching function, we assume $M(A, V) = E_{it} \times AV/(A+V)$ because it guarantees job-finding and job-filling probabilities in $[0, 1]$ for $E_{it} \in [0, 1]$. We also assume symmetric Nash bargaining. We leverage data on replacement rates in our sample and assume the UI parameter b to be a country-and-time specific function of a country-specific replacement rate. We use the country-specific mean of the *Replacement Rate* variable, because our data only start three years later than our data on broadband. We adjust this country-specific replacement rate for benefit expiration length, because in the model, unlike in reality, unemployment insurance payments are indefinite. This adjustment is time-and-country specific, because it takes into account that the duration of unemployment endogenously changes due to changes in the job-finding probability. Finally, we arrive at a country-and-time specific value of the unemployment benefit b_{it} because we multiply the adjusted replacement rate by country-and-time specific employee earnings $w_{it}l_{it}$.[21]

Next, we use the Labour Force Survey data from Donovan *et al.* (2023b) to calibrate δ. In these data, we observe all yearly transition probabilities between non-participation, unemployment, self-employment, and payroll employment for 14 of our OECD countries for on average 8.8 years within the time span of 1998-2017. Because data is missing for three

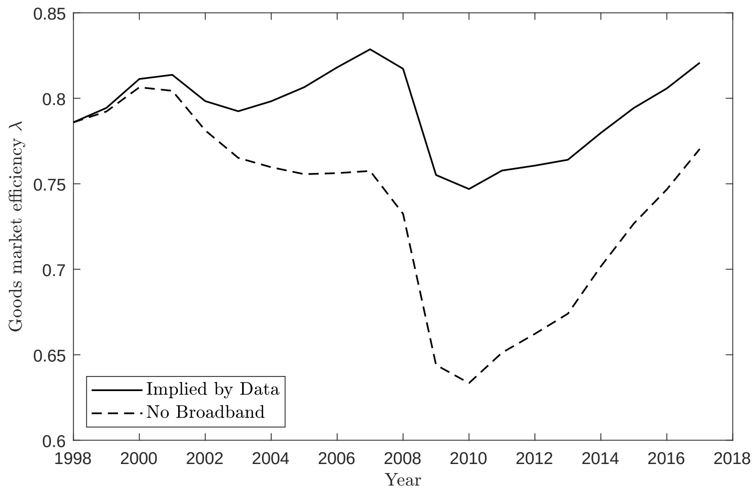
[20]For example, Lagos and Wright (2005) use values ranging from 0.16 to 0.27 in their baseline specification and report robustness checks which entail higher values of σ, going up to 0.5. In Online Appendix C.7 we perform a sensitivity analysis with respect to varying σ and find that our results are robust.

[21]Online Appendix C.5.1 contains further details on how we calibrate the value of unemployment benefits.

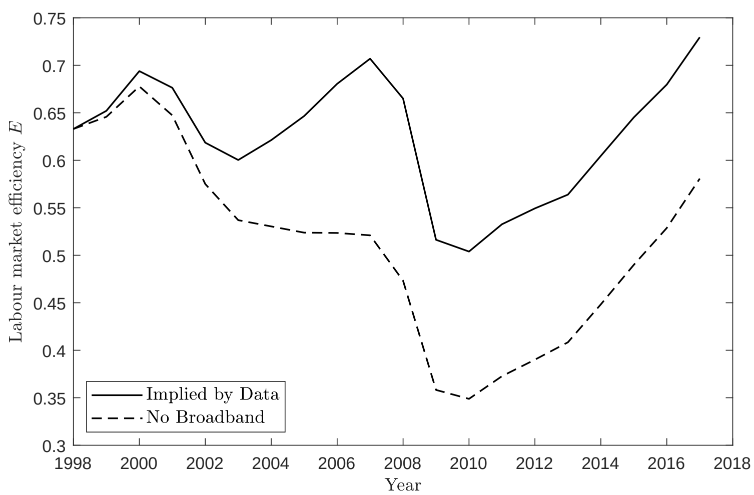
quarters of our sample, in our baseline calibration we set a single δ for all countries and years. Following a procedure that reconciles the timing assumptions of our model with the construction of the data, we arrive at a value of $\delta = 0.0246$.[22] We use the observations for the first year in our sample to identify the country-specific value of the vacancy posting cost k_i. Observe that for a given value of δ, self-employment and unemployment rates, Equation (8) uniquely pins down the job-finding probability in any given year as:

$$\mu(\theta)_{it} = \delta \frac{1 - U_{it} - SE_{it}}{(1-\delta)U_{it} + \delta(1 - SE_{it})}.$$

It is well-known that without information on vacancies, E and θ are not jointly identified in matching models. We follow Shimer (2005) and normalize $\theta = 1$ in the first year of our sample for each country. This normalization pins down the value of matching efficiency E_{i1} given $\mu(\theta)_{i1}$. Then, given the exogenously calibrated parameters, we solve the goods market clearing condition, and the worker's indifference condition for first-sample-year matching efficiency λ_{i1} and price level p_{i1}, and solve the free-entry condition for the vacancy posting cost k_i. We hold this vacancy posting cost constant throughout the sample on a country level. Finally, given the vector of parameters $\Theta = \{\sigma, \beta, \phi, \delta, b_{it}, k_i\}$ and time-series $\{SE_{it}, U_{it}\}$ we solve the three equilibrium conditions of our model – free-entry, goods market clearing, and worker's indifference – for λ_{it}, E_{it} and p_{it}.[23]



(a) Goods market



(b) Labour market

Figure 5: *Matching efficiency parameters in the goods and labour market*.
Note: These figures plot the labour-force-weighted averages of λ and E implied by the model imposed on time series of self-employment and unemployment rates as in the data and in the no-broadband counterfactual experiment.

5.2 Results

We present our main quantitative findings in Figure 5. The black lines in this figure show the labour-force-weighted average series of λ and E that rationalize the panel of self-employment and unemployment rates in the data.[24] These model-implied matching efficiency parameters both have two distinct features.

First, they capture the cyclical variation, decreasing around 2001-2003 and 2008-2010. Indeed, as shown by Sedláček (2014) in a standard matching model, a rise of unemployment shows up as a fall in labour matching efficiency if one allows this parameter to vary. Because business conditions deteriorate in recessions, it is comforting to find that the probability to sell decreases in recessions as well. Second, they nevertheless exhibit an increasing trend. The labour-force-weighted average of the goods market efficiency parameter λ increased from around 0.79 to slightly more than 0.82, a 4.4% increase. The increase of the labour-force-weighted average of E was more pronounced. It went from approximately 0.63 to 0.73, an increase of about 16%. As a result of a horse-race between reductions in the goods and in the labour market, the unemployment rate decreased slightly and the self-employment rate a bit more. Consequently, we conclude that non-trending unemployment is consistent with substantial improvements in labour matching efficiency if one considers that such improvements may pull workers out of self-employment rather than decrease unemployment.

[22]We provide full details of this procedure in Online Appendix C.5.2.

[23]As we have pinned down the value of k, we now have enough structure in the model to jointly identify θ and E and we no longer need to normalize $\theta = 1$ in subsequent years in the sample. λ_{it}, E_{it} and p_{it} in turn imply wages w_{it}, consumption of buyers q^c_{it}, a stock of vacancies V_{it}, and production in payroll- and self-employment, l_{it} and q^s_{it}, respectively.

[24]Figure C2 in Online Appendix C.6 presents the time series of λ and E for each country separately.

While we find that falling self-employment is consistent with declining search frictions, we know from our causal evidence that the rollout of broadband Internet increased the self-employment rate. To reconcile these findings, we exploit the panel of self-employment and unemployment rates implied by our counterfactual no-broadband scenario, as presented in Figure 1. Using this panel as alternative input, we repeat our numerical exercise and back out a counterfactual panel for λ and E. The dashed lines in Figure 5 present the resulting labour-force-weighted average series of λ and E. We find that the matching efficiency parameters, while still capturing cyclical fluctuations, are substantially below those implied by the data. The end-of-sample labour-force-weighted averages of λ and E in the absence of broadband Internet would have been equal to 0.77 and 0.58, respectively. This is equivalent to a 2% decrease in λ and an 8% drop in E relative to the beginning of the sample. To put it differently, we find that the roll-out of broadband Internet reduced search frictions by approximately 7% in the goods and roughly 26% in the labour market in 2017, relative to the no-broadband counterfactual scenario. These effects seem to have been largest at the Great Recession and show signs of fading away in the goods market but less so, if at all, in the labour market. Thus, we conclude that the rollout of broadband Internet has very significantly reduced search frictions in the labour market, and to a lesser, yet still profound degree, in the goods market. In the light of our theoretical results, this decline in goods market frictions is indispensable to understand the positive effect of the rollout of broadband Internet on the self-employment rate.

To highlight the importance of self-employment and goods market frictions for the understanding of trends in unemployment, we use the steady-state condition for the unemployment rate, Equation (8), and further decompose the effects of the broadband Internet

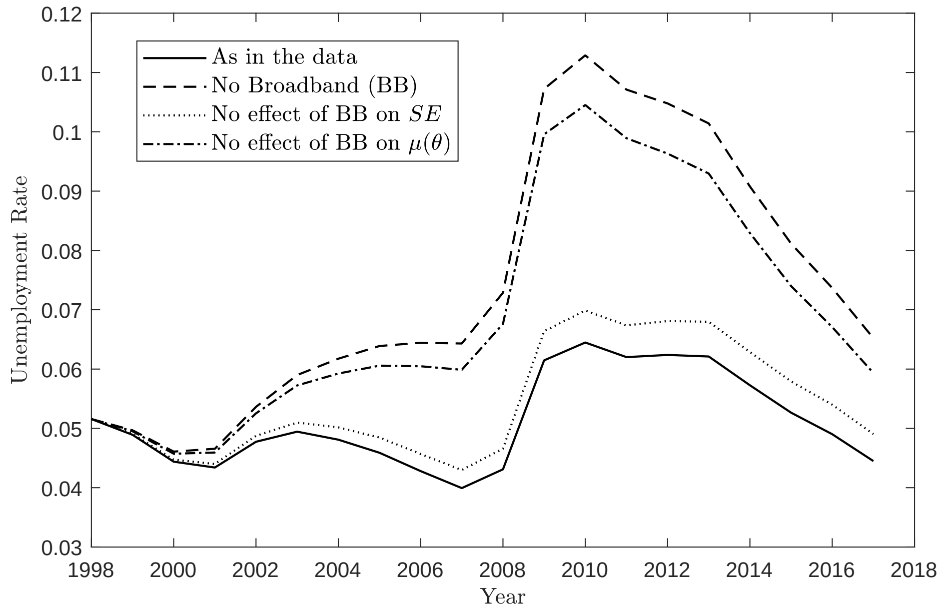


Figure 6: *Decomposition of the effect of broadband Internet on the unemployment rate.*
Note: Labour-force-weighted sample average unemployment rate as in the data and in three counterfactual scenarios: no broadband roll-out, broadband not affecting the self-employment rate, and broadband not affecting the job-finding probability.

rollout. Firstly, we calculate job-finding probabilities as implied by Equation (8) imposed on the data, but recalculate unemployment using the no-broadband series for the self-employment rate. Secondly, we take the self-employment rates as in the data, but recalculate unemployment using the no-broadband series for the job-finding probability. We present the results of this exercise in Figure 6, where we add these two new labour-force-weighted series of the unemployment rate to the data and no-broadband series that we already reported on the bottom panel of Figure 1. Two main insights emerge. First, changes in unemployment due to broadband do not only stem from mechanically accounting for variation in self-employment rates. In fact, assuming away the positive effect of broadband on SE

would only increase the unemployment rate from 4.4% to 4.9% in 2017. Second, the effect of broadband on the job-finding probability is quantitatively much more important. Without it, the unemployment rate would stand at 5.9% in that year, only 0.6 percentage point short of the full counterfactual-scenario value. Hence, the latter effect explains 71% of the effect of broadband on the unemployment rate in 2017 while the former accounts for 24%. This exercise further strengthens the claim that the goods market frictions are important for unemployment, as it is the increase in λ that leads to a higher $\mu(\theta)$ in our theoretical model.

5.3 Evidence on search frictions and the price level

The general price level plays an important role in the mechanism of our model. When goods market frictions decline, more workers choose to become self-employed. Because the self-employed produce less than the payroll employed, declining goods market frictions actually reduce supply and increase the price level. Conversely, declining labour market frictions pull more workers into payroll employment, increasing supply and decreasing prices.

To test these predictions, we use a panel of price levels as reported in the Penn World Tables (PWT) (Feenstra *et al.*, 2021) based on the International Comparisons Program (Feenstra *et al.*, 2015). Using the United States in 2005 as reference, the PWT report two price indices that allow for a comparison of consumption price levels across countries. The first index measures the price level of household consumption goods, whereas the second index also includes government consumption. We refer to these indices of the price levels for C and $C+G$, respectively. We present empirical results for both indices because, even though we abstract from a government in our model, we think it is useful to think of aggregate demand

Table 4: *Effects of λ and E on the price level.*

	(1) Price C	(2) Price C	(3) Price C	(4) Price C+G	(5) Price C+G	(6) Price C+G
λ	0.202***		0.356***	0.245***		0.446***
	(0.063)		(0.097)	(0.068)		(0.105)
E		0.034	-0.097**		0.037	-0.126**
		(0.030)	(0.047)		(0.033)	(0.051)
Observations	480	480	480	480	480	480
No. countries	24	24	24	24	24	24
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
F-test p-value	0.000	0.000	0.000	0.000	0.000	0.000
RMSE	0.073	0.074	0.073	0.080	0.081	0.079
R-squared	0.828	0.824	0.829	0.808	0.803	0.811

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The dependent variable is the price level of either household consumption (C), or household and government consumption ($C + G$). The sample consists of 24 OECD countries in years 1998-2017. Conventional standard errors in parentheses.

as potentially including the government. We present the time series for both price indices for all countries in our sample in Figure C3 in Online Appendix C.6.

To test whether PWT price levels move with declining goods and labour market frictions as in our model, we take the model-implied series of λ and E and use them in a regression explaining changes in the price level. Because the relationships between the matching efficiency parameters and prices are contemporaneous, we estimate a static panel model. We include country and year fixed-effects to explain price changes within the same country while controlling for the global business cycle. The results of our fixed-effects regressions are presented in Table 4, with the estimates for the price level of C in columns (1)-(3) and of $C + G$ in columns (4)-(6). Several insights emerge.

Firstly, the point estimates are qualitatively and quantitatively similar across the two price indices. Secondly, when we control only for either λ or E, as in columns (1)-(2) and (4)-(5), we find positive coefficients. These positive coefficients are difficult to explain without flows between self-employment and the labour market. For example, they are in stark contrast to the comparative statics in the Fixed career-choice equilibrium, in which reductions in goods and labour market frictions act as positive supply shocks. With higher λ and E, more workers should be able to offer their production, exerting a downward pressure on prices.

However, the finding that a higher λ comes with higher prices is perfectly consistent with the predictions of our model. In our model, reductions in goods market frictions pull workers away from applying for jobs, prompting them to choose self-employment instead. Self-employment workers produce less than payroll employees. The flows between self-employment and the labour market thus transform a reduction in goods matching frictions in a negative supply shock, consistent with the estimation results.

While the positive correlation between λ and prices is consistent with the predictions of our model, the same result for E is not. However, as the estimation results in columns (4) and (6) imply, controlling for the magnitude of frictions in only one of the two markets suffers from an omitted variable bias. Because the correlation coefficient of the λ and E time series is approximately 0.91, the effects of increases in E are confounded by its correlation with λ, resulting in spuriously positive estimates in columns (2) and (5). When we control for frictions in both markets jointly, we estimate a positive coefficient on λ and a negative one on E. This pattern is fully in line with Theorem 2. Reductions in labour market frictions pull workers from self-employment to the labour market. Even though unemployment increases, production increases and prices fall. In short, we find that the data support the predictions of the MSCC-equilibrium rather than the FCC-equilibrium.

6 Conclusions

We provide novel empirical evidence on the effects of advancements in ICT technologies on labour markets. Specifically, we find in a panel of OECD countries that the rollout of broadband Internet increased self-employment and decreased unemployment rates. We then propose a new model of career choice featuring frictional labour and goods markets. In our model, the career choice is driven by income risk and firms' ability to insure their employees against it. We show that reductions in goods market frictions increase the self-employment rate and reduce the unemployment rate while the converse is true, due to outflow of workers from self-employment, when labour market frictions decline. Modeling the goods market explicitly, we uncover a new, supply-side driven externality of changes in self-employment on unemployment.

We then use the model as a measurement device to back out the evolution of frictions in goods and labour markets as implied by actual data and a counterfactual experiment of no broadband Internet rollout. We find that, firstly, broadband Internet indeed reduced frictions in both markets, in particular, halting two-thirds of a decline in self-employment rates. Furthermore, without broadband Internet the declines in frictions would not materialize. The overall mild decline in self-employment rates and the lack of a trend in unemployment rates over our sample period are due to labour market frictions decreasing relatively more.

We think that the model we put forth in this paper offers grounds for further theoretical and quantitative work on the significance of the split of employment into self-employed workers and payroll employees for policy questions.

Appendix

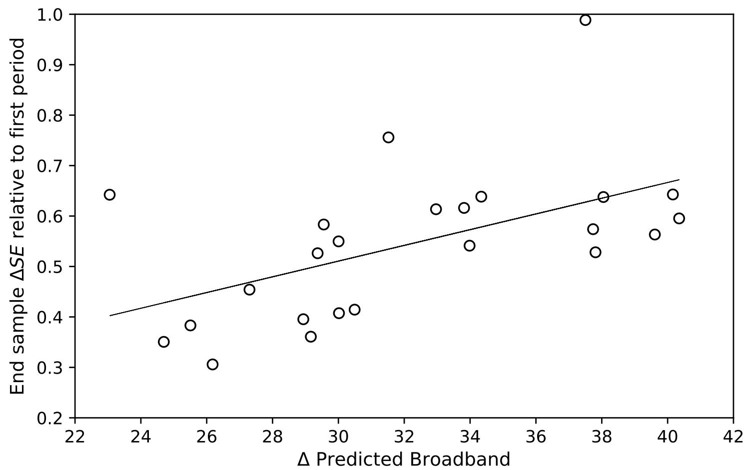
A Additional Empirical Evidence

A.1 Country-level evidence on the effects of broadband

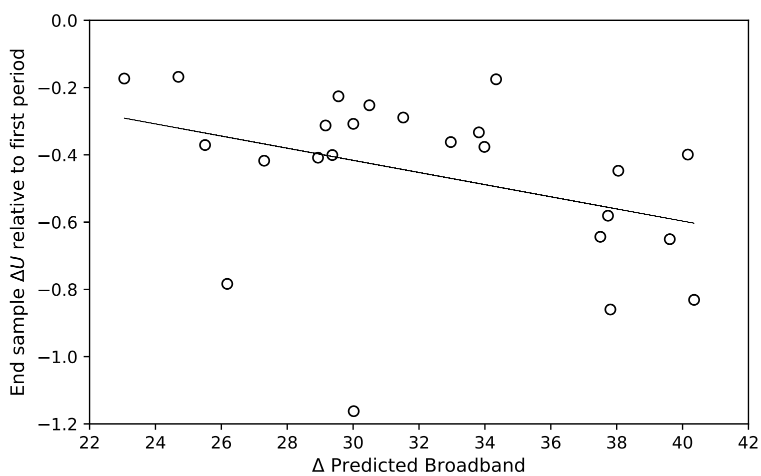
In the main body of the paper we present the evolution of the labour-force-weighted self-employment and unemployment rates as in the data and as implied by the counterfactual scenario of no broadband Internet rollout. The advantage of this approach is that it makes the results robust to potential small-country outliers. However, it necessarily removes information on between-country variation in the effects of broadband on SE and U rates. In Figure A1 we present scatter plots with linear regression lines of changes in *Predicted Broadband* from 1998 to 2017 on the horizontal axis, and the difference between the data and the counterfactual scenario of either SE or U rate in 2017 normalized by the 1998 observation of the labour market aggregate of interest. We find that there is quite strong and positive correlation between change in *Predicted Broadband* and increase in *Self-Employment* rate relative to its counterfactual trend and a slightly weaker, negative correlation between *Predicted Broadband* and change in *Unemployment* rate. The linear correlation coefficients are 0.54 and -0.37, respectively.

A.2 Effects of broadband diffusion on own-account work

In this section of the Appendix we present the results of estimating (1) with the own-account work rate as dependent variable. The results collected in Table A1 demonstrate that the quantitative and qualitative effects of the rollout of broadband Internet on this alternative



(a) Self-Employment rate



(b) Unemployment rate

Figure A1: *Broadband accumulation and its effects on SE and U.*
Note: These figures depict the correlation between changes in *Predicted Broadband* from 1998 to 2017 and the difference between actual and counterfactual values of self-employment (a) and unemployment (b) rates in 2017, normalised by the 1997 value of self-employment and unemployment rate, respectively.

Table A1: *Effects of Broadband Internet on the Own-Account Work Rate.*

	(1) ΔOA rate	(2) ΔOA rate	(3) ΔOA rate	(4) ΔOA rate	(5) ΔOA rate	(6) ΔOA rate
Lagged OA rate	-0.112***	-0.110***	-0.112***	-0.186***	-0.107***	-0.213***
	(0.027)	(0.026)	(0.033)	(0.032)	(0.029)	(0.041)
ΔPredicted B-band	0.017*	0.018*	0.020*	0.050***	0.029***	0.050***
	(0.009)	(0.008)	(0.011)	(0.012)	(0.007)	(0.016)
Lagged Predicted B-band	0.002*	0.002*	0.003*	0.007***	0.004***	0.008***
	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)
ΔGDP		-0.158	-0.180	-0.127	-0.179	-0.116
		(0.106)	(0.114)	(0.099)	(0.111)	(0.104)
Lagged GDP		-0.002	-0.000	-0.055	0.024	-0.066
		(0.048)	(0.039)	(0.043)	(0.045)	(0.050)
ΔReplacement Rate			-0.003			0.029
			(0.011)			(0.041)
Lagged Replacement Rate			-0.008			0.000
			(0.010)			(0.027)
ΔPublic Sector				-0.071		-0.030
				(0.046)		(0.059)
Lagged Public Sector				-0.102***		-0.127***
				(0.034)		(0.039)
ΔTax Burden					-0.020	-0.007
					(0.027)	(0.052)
Lagged Tax Burden					-0.013	-0.001
					(0.022)	(0.029)
Observations	456	456	384	249	408	211
No. countries	24	24	24	22	24	22
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
$p(F=0)$	0.000	0.000	0.000	0.000	0.000	0.026
RMSE	0.036	0.036	0.034	0.029	0.035	0.029
R-squared	0.123	0.128	0.104	0.239	0.112	0.213

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The dependent variable is the change in the logarithm of the *Own-Account Work* rate. The sample is 24 OECD countries in years 1998-2017. Cluster-robust standard errors in parentheses, except for columns (4) and (6) that report conventional standard errors because of insufficient degrees of freedom.

proxy for self-employment are similar to the ones reported in the main text. The estimated short-term effects of *Predicted Broadband* on the *Own-Account Work* rate range from being 30% larger to being 30% smaller than their counterparts on the *Self-Employment* rate. A similar picture emerges for the estimates of the long-term effects.

A.3 Effects of broadband per sector

In this section of the Appendix we present evidence on the effect of broadband on the self-employment rate and own-account work rate in different sectors of the economy.[25] We follow the classification into six broad sectors by the International Labour Organization (n.d.): Agriculture (AGR); Manufacturing (MAN); Construction (CON); Mining and quarrying, electricity, gas and water supply (MEL); market services (MKT); and non-market services (PUB). The ILO provides the total number of self-employed workers in each sector based on national labour force surveys, but not by age group. We transform these data into an internationally comparable sectoral self-employment rate using our earlier self-employment rate for workers older than 25. Our self-employment rate *se* in sector *s*, country *c* and year *t* is then given by

$$se_{s,c,t} = \underbrace{\frac{SE_{s,c,t}}{SE_{c,t}}}_{\text{LFS}} \times \underbrace{\frac{SE_{c,t}}{\text{labour force}_{c,t}}}_{\text{ILO modelled estimates.}} ,$$

and analogously for the own-account work rate. This transformation allows us to use the ILO modelled estimates that are comparable across countries, to focus on workers above 25 years old, and to include unemployment workers in the denominator of the self-employment rate, consistent with our model.[26] Nonetheless, we lose observations for New Zealand and for the early years for many other countries.

We estimate the effect of the rollout of broadband Internet on the self-employment rate in the six broad sectors. Table A2 reports the estimates of our benchmark regression that

[25]Because unemployed workers are not linked to a particular sector, we cannot do a similar exercise for the unemployment rate.
[26]The implicit assumption is that although sectoral self-employment numbers may not be comparable across countries, relative sectoral shares in self-employment are comparable across countries, and are proportional across age groups.

Table A2: *Effects of Broadband Internet on the Self-Employment Rate in Broad Sectors.*

	(1) AGR	(2) MAN	(3) CON	(4) MEL	(5) MKT	(6) PUB
Lag. Sec. SE Rate	-0.305***	-0.376***	-0.380***	-0.834***	-0.200***	-0.294***
	(0.056)	(0.055)	(0.042)	(0.067)	(0.040)	(0.045)
Δ Pred. B-band	0.056	0.079*	0.074**	-0.893***	0.037	0.037
	(0.048)	(0.045)	(0.034)	(0.221)	(0.023)	(0.037)
Lag. Pred. B-band	0.011	0.022**	0.025***	-0.143***	0.012***	0.005
	(0.009)	(0.009)	(0.006)	(0.040)	(0.004)	(0.007)
Observations	236	236	236	234	236	236
No. countries	23	23	23	23	23	23
Controls	GDP	GDP	GDP	GDP	GDP	GDP
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
F-test *p*-value	0.005	0.000	0.000	0.000	0.002	0.000
RMSE	0.070	0.065	0.050	0.305	0.033	0.054
R-squared	0.199	0.286	0.436	0.506	0.211	0.342

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The dependent variable is the change in the logarithm of the self-employment rate by broad sector. The sample consists of 23 OECD countries (excluding New Zealand) in years 1999-2017. Conventional standard errors in parentheses. The broad sectors are AGRiculture, MANufacturing, CONstruction, Mining and quarrying; Electricity, gas and water supply (MEL), Market services (MKT), and non-market services (PUB).

includes lagged and first-differenced GDP as regressor. In these regressions, we exclude institutional variables to minimize the loss of observations. We find that the rollout of broadband Internet significantly increased the self-employment rate in manufacturing, construction and, in the long run, market services. We also see that broadband Internet significantly reduced self-employment in the mining and utilities sector (MEL). Unsurprisingly, however, the self-employment rate is negligible in this sector.

To understand slightly better which types of self-employed workers drive these findings, we run the same regression on the own-account work rate. As argued before, own-account workers may not include all workers that fit the conception of self-employment in our model, but includes few workers that do not fit our conception of self-employment. Table A3 shows that the significant negative effect of the mining and utilities sector persists, but that the significant positive effects for manufacturing and construction disappear. The positive long-run effect in market services also continues to exist for own-account workers. Consequently, the overall positive effect on the own-account work rate seems to be driven to a significant extent by market services. The decrease in the size of the coefficients in manufacturing and construction suggests that the increase in the self-employment rate in those sectors is to a larger extent driven by (small) employers rather than own-account workers. The absence of an effect in the non-market services sector (PUB), both for self-employment and own-account work, also suggests another interpretation to the negative coefficient on the size of the public sector in our regressions for the aggregate economy. While, as argued before, the public sector may provide relatively stable employment and thus dampen the incentives to become self-employed, our sectoral regressions suggest that a large public sector may also dampen a rise in self-employment because the rollout of broadband Internet does not affect self-employment much in this sector.

Interestingly, in the case of the market services sector, the data allow for a further sectoral decomposition according to the International Standard Industrial Classification (ISIC Rev.4). Table A4 shows that the only significantly positive effect is coming from administrative and support service activities (N). The coefficient on professional, scientific and technical activities (M) is also relatively large, but not significant, whereas the coefficient in the information and communication sector (J) is negative, but not significant either.

Zooming in on the own-account work rate, we see in Table A5 that the negative effect in the information and communication sector becomes significant. This finding may confirm anecdotal evidence that broadband Internet led to more concentration ('big tech') rather than self-employment. While this may be true in the information and communication sector

Table A3: *Effects of Broadband Internet on the Own Account Work Rate in Broad Sectors.*

	(1) AGR	(2) MAN	(3) CON	(4) MEL	(5) MKT	(6) PUB
Lag. Sec. OA Rate	-0.341***	-0.511***	-0.454***	-0.626***	-0.216***	-0.426***
	(0.055)	(0.066)	(0.051)	(0.096)	(0.043)	(0.042)
Δ Pred. B-band	0.030	0.078	0.026	-0.773***	0.037	0.054
	(0.046)	(0.063)	(0.045)	(0.258)	(0.029)	(0.041)
Lag. Pred. B-band	0.013	0.016	0.012	-0.132***	0.013**	0.002
	(0.008)	(0.012)	(0.008)	(0.043)	(0.005)	(0.007)
Observations	236	235	236	120	236	236
No. countries	23	23	23	12	23	23
Controls	GDP	GDP	GDP	GDP	GDP	GDP
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
F-test p-value	0.003	0.000	0.000	0.000	0.004	0.000
RMSE	0.066	0.092	0.066	0.253	0.043	0.060
R-squared	0.205	0.323	0.348	0.505	0.202	0.464

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The dependent variable is the change in the logarithm of the own account work rate by broad sector. The sample consists of 23 OECD countries (excluding New Zealand) in years 1999-2017. Conventional standard errors in parentheses. The broad sectors are Agriculture (AGR), Manufacturing (MAN), Construction (CON), Mining and quarrying; Electricity, gas and water supply (MEL), Market services (MKT), and non-market services (PUB).

itself, our findings do not provide support for this idea for the economy as a whole. Instead, we see that although there are no longer any significantly positive effects, the coefficients in the administrative and support service activities, and professional, scientific and technical activities, are substantial for own-account work too. These coefficients suggest that the overall positive long-run effect of broadband on the own-account work rate in the market services sector is primarily driven by these administrative and professional services, even though the coefficients are not significant. Although broadband Internet may thus have led to fewer own-account workers in the information and communication sector, the significant and positive effects for the aggregate economy suggest that broadband Internet has facilitated self-employment and own-account work in other sectors.

Table A4: *Effects of Broadband Internet on the Self-Employment Rate in Market Sectors (ISIC).*

	(1) G	(2) H	(3) I	(4) J	(5) K	(6) L	(7) M	(8) N
Lag. Sec. SE Rate	-0.275***	-0.526***	-0.581***	-0.397***	-0.323***	-0.444***	-0.520***	-0.382***
	(0.062)	(0.069)	(0.068)	(0.060)	(0.056)	(0.067)	(0.056)	(0.063)
Δ Pred. B-band	0.006	0.048	-0.045	-0.199	-0.143	0.004	0.092	0.159
	(0.054)	(0.103)	(0.091)	(0.175)	(0.257)	(0.177)	(0.095)	(0.216)
Lag. Pred. B-band	0.000	0.010	0.005	-0.064	-0.002	-0.015	0.035	0.101**
	(0.012)	(0.023)	(0.020)	(0.041)	(0.057)	(0.040)	(0.022)	(0.050)
Observations	210	210	210	208	210	208	207	206
No. countries	21	21	21	21	21	21	21	21
Controls	GDP	GDP	GDP	GDP	GDP	GDP	GDP	GDP
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test p-value	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000
RMSE	0.048	0.091	0.079	0.147	0.228	0.153	0.081	0.180
R-squared	0.181	0.295	0.360	0.286	0.285	0.248	0.468	0.632

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The dependent variable is the change in the logarithm of the self-employment rate by ISIC sector. The sample consists of 21 OECD countries (excluding Australia, Canada and New Zealand) in years 2001-2017. Conventional standard errors in parentheses. The ISIC (Rev. 4) sectors are Wholesale and retail trade; repair of motor vehicles and motorcycles (G), Transportation and storage (H), Accommodation and food service activities (I), Information and communication (J), Financial and insurance activities (K), Real estate activities (L), Professional, scientific and technical activities (M), and Administrative and support service activities (N).

B Proofs and Derivations

B.1 Nash bargaining

It follows from (11) and (12) that:

$$V^{J} = \frac{\lambda\,(p-w)\,l - (1-\lambda)d}{1-\beta(1-\delta)}, \tag{B.1}$$

$$V^{PE} = \frac{\lambda\,(u\,(wl) - l) + (1-\lambda)u(d) + \beta\delta V^{LM}}{1-\beta(1-\delta)}. \tag{B.2}$$

Table A5: *Effects of Broadband Internet on the Own Account Work Rate in Market Sectors (ISIC).*

	(1) G	(2) H	(3) I	(4) J	(5) K	(6) L	(7) M	(8) N
Lag. Sec. OA Rate	-0.250***	-0.531***	-0.689***	-0.525***	-0.429***	-0.435***	-0.382***	-0.274***
	(0.058)	(0.072)	(0.068)	(0.069)	(0.057)	(0.072)	(0.053)	(0.057)
Δ Pred. B-band	-0.014	-0.001	-0.045	-0.330*	-0.236	0.166	0.108	0.134
	(0.069)	(0.137)	(0.130)	(0.178)	(0.188)	(0.182)	(0.107)	(0.207)
Lag. Pred. B-band	-0.004	0.007	0.008	-0.099**	0.001	0.012	0.032	0.047
	(0.015)	(0.031)	(0.028)	(0.042)	(0.040)	(0.039)	(0.024)	(0.047)
Observations	210	210	203	205	181	173	207	201
No. countries	21	21	21	21	20	20	21	21
Controls	GDP	GDP	GDP	GDP	GDP	GDP	GDP	GDP
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
F-test p-value	0.012	0.000	0.000	0.000	0.000	0.000	0.000	0.000
RMSE	0.061	0.121	0.112	0.146	0.153	0.145	0.092	0.172
R-squared	0.194	0.280	0.436	0.403	0.489	0.299	0.299	0.717

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The dependent variable is the change in the logarithm of the own account work rate by ISIC sector. The sample consists of 21 OECD countries (excluding Australia, Canada and New Zealand) in years 2001-2017. Conventional standard errors in parentheses. The ISIC (Rev. 4) sectors are Wholesale and retail trade; repair of motor vehicles and motorcycles (G), Transportation and storage (H), Accommodation and food service activities (I), Information and communication (J), Financial and insurance activities (K), Real estate activities (L), Professional, scientific and technical activities (M), and Administrative and support service activities (N).

The first-order conditions of the Nash product maximisation are:

$$(1-\phi)\left(V^{PE}-u(b)-\beta V^{LM}\right)=\phi V^{J}u'(wl), \tag{B.3}$$

$$(1-\phi)\left(V^{PE}-u(b)-\beta V^{LM}\right)=\phi V^{J}u'(d), \tag{B.4}$$

$$\lambda(p-w)(1-\phi)\left(V^{PE}-u(b)-\beta V^{LM}\right)=\phi V^{J}\lambda\left(1-u'(wl)w\right). \tag{B.5}$$

Equations (B.3) and (B.4) immediately imply $u'(d) = u'(wl) \implies d = wl$. Substituting (B.3) into (B.5), we arrive at $u'(wl) = 1/p$. Moreover, with $d = wl$, (B.1) and (B.2) imply that:

$$V^J = \frac{(\lambda p - w)\, l}{1 - \beta(1-\delta)},$$
$$V^{PE} = \frac{u\,(wl) - \lambda l + \beta\delta V^{LM}}{1 - \beta(1-\delta)}.$$

Combining the latter with (13), we find (16):

$$V^{LM} = \frac{\mu\,(\theta)\,[u\,(wl) - \lambda l] + (1-\mu(\theta))\,(1-\beta(1-\delta))\,u(b)}{(1-\beta)\,[1-(1-\mu(\theta))\,\beta(1-\delta)]}$$

Substituting the value functions, (B.3) can be rewritten to the sharing rule of (15):

$$\phi\frac{(\lambda p - w) l u'(wl)}{1-\beta(1-\delta)} = (1-\phi)\frac{u\,(wl) - \lambda l - u(b)}{1-(1-\mu(\theta))\,\beta(1-\delta)}.$$

B.2 Proof of Lemma 1

1. Combining equations (10) and (14) we have $u'(pq^s) = 1/p = u'(wl) \implies wl = pq^s$. Next, for firms to make positive profits per filled vacancy it must hold that $w < \lambda p$ so that $w < p$, and thus $q^s < l$ follows trivially.

2. To obtain the function $\psi(p)$, we start with removing q^s in V^{SE}. From (10), we get $q^s = 1/p\,[u']^{-1}\,(1/p)$. Then, let:

$$\psi(p) \equiv u\left(\left[u'\right]^{-1}\left(\frac{1}{p}\right)\right) - \frac{1}{p}\left[u'\right]^{-1}\left(\frac{1}{p}\right) = u(pq^s) - q^s, \tag{B.6}$$

so that $(1-\beta)V^{SE} = \lambda\psi(p)$. We shall now demonstrate that $\psi(p)$ shows up in V^{LM} too. First, substitute (14) into (15) to arrive at:

$$\lambda l = (1-\phi)\frac{[1-\beta(1-\delta)]\,[u(wl) - wl/p - u(b)]}{1-\beta(1-\delta)\,(1-\mu(\theta)\phi)} + \frac{wl}{p}. \tag{B.7}$$

Substituting (B.7) into (16), the value of entering the labour market reads:

$$V^{LM} = \frac{(1-\beta(1-\delta))\,u(b) + \mu(\theta)\phi\,[u\,(wl) - wl/p - (1-\beta(1-\delta))\,u(b)]}{(1-\beta)\,[1-(1-\mu(\theta)\phi)\,\beta(1-\delta)]},$$

so that, because $wl = pq^s$ and thus $u(wl) - wl/p = u(pq^s) - q^s = \psi(p)$,

$$(1-\beta)V^{LM} = \frac{\mu(\theta)\phi\psi(p) + (1-\mu(\theta)\phi)\,(1-\beta(1-\delta))\,u(b)}{(1-\mu(\theta)\phi)\,(1-\beta(1-\delta)) + \mu(\theta)\phi}. \tag{B.8}$$

Note that $(1-\beta)V^{LM}$ is a weighted average of the time spent in employment and unemployment, with the weight given by

$$\omega(\theta) \equiv \frac{\mu(\theta)\phi}{1-(1-\mu(\theta)\phi)\,\beta(1-\delta)}.$$

Substituting this weight into (B.8), together with the definition of $(1-\beta)V^{SE}$, results in the worker indifference condition of (21). To derive the free-entry condition, substitute (B.7) into (17) with $wl = pq^s$, we obtain (22). To derive the sharing rule, substitute $\psi(p)$ into (B.7), resulting in (23). This sharing rule shows that the expected production λl is larger than q^s, and not just l. Next, we show that $\psi(p)$ is increasing in p. Since

$\psi(p) = u(pq^s) - q^s$, we have $\psi'(p) = u'(pq^s)q^s + dq^s/dp\,(pu'(pq^s) - 1)$. However, in the optimum the second term is equal to zero, so that $\psi'(p) = u'(pq^s)q^s > 0$.

3. Differentiation of (6) shows strict monotonicity of q^c in p.

B.3 Proof of Lemma 2

Lemma 1 shows that $q_s < \lambda l$, so that (24) holds if

$$\lambda < \frac{\mu(\theta)}{\mu(\theta) + \delta(1 - \mu(\theta))}. \tag{B.9}$$

When $u(b) = 0$, the workers' indifference condition in (21) reduces to

$$\lambda = \omega(\theta) \equiv \frac{\mu(\theta)\phi}{\mu(\theta)\phi + (1 - \mu(\theta)\phi)\,[1 - \beta(1 - \delta)]}$$

Consequently, the inequality in (B.9) holds if

$$\mu(\theta)\phi + (1 - \mu(\theta)\phi)\,[1 - \beta(1 - \delta)] > \phi\,[\mu(\theta) + \delta(1 - \mu(\theta))]\,, \quad \iff$$

$$1 - \mu(\theta)\phi - \beta + \beta\delta + \beta\mu(\theta)\phi - \beta\delta\mu(\theta)\phi > \phi\delta - \delta\mu(\theta)\phi, \quad \iff$$

$$1 - \beta - (1 - \beta)\mu(\theta)\phi > \delta\,[\phi - \beta - (1 - \beta)\mu(\theta)\phi]\,,$$

which is true because both $\delta \in (0, 1)$ and $\phi \in (0, 1)$.

B.4 Proof of Theorem 2

The proof relies on the three equilibrium conditions (the free entry condition, workers' indifference condition, and goods market clearing condition) and the relationships they imply between p, θ and SE. Furthermore, we first derive the results for the special case of $u(b) = 0$. The signs of the derivatives there will help us sign the effects of model parameters in the more general case of $u(b) > 0$. Observe that the MSCC-equilibrium is a level set of a mapping described by the three remaining equilibrium conditions involving the utility function $u(c)$, matching probabilities $\mu(\theta)$ and $\zeta(\theta)$, and $\psi(p)$ (which inherits smoothness from the utility function). Thus, any element of an MSCC is at least once continuously differentiable.

The case of $u(b) = 0$. Defining $\gamma(p) = p\psi(p)$ for brevity, the workers' indifference and free-entry conditions in (25) and (26) become:

$$\lambda = \omega(\theta) \equiv \frac{\mu(\theta)\phi}{1 - (1 - \mu(\theta)\phi)\, \beta(1-\delta)},$$
$$k = \frac{\zeta(\theta)(1-\phi)\gamma(p)}{1 - (1 - \mu(\theta)\phi)\, \beta(1-\delta)}.$$

For future reference, note that Lemma 1 implies that $\gamma'(p) > 0$. Similarly, from the definition of $\omega(\theta)$, we see that

$$\omega'(\theta) = \frac{[1 - \beta(1-\delta)]\, \mu'(\theta)\phi}{[1 - (1 - \mu(\theta)\phi)\, \beta(1-\delta)]^2} > 0,$$

so that $\omega'(\theta)$ inherits its sign from $\mu'(\theta)$.

Effects of λ on $\omega(\theta)$, $\mu(\theta)$ and θ. Implicitly differentiating the career-choice indifference condition we arrive at $1 = \omega'(\theta) d\theta/d\lambda$. Trivially, this amounts to $d\omega(\theta)/d\lambda = 1 > 0$ and $d\mu(\theta)/d\lambda > 0$, but it also reveals that $d\theta/d\lambda > 0$.

Effect of λ on p. Next, we differentiate the free-entry condition with respect to λ

$$\left[\underbrace{k\mu'(\theta)\phi\beta(1-\delta)}_{>0} - \underbrace{\zeta'(\theta)(1-\phi)\gamma(p)}_{<0} \right] \underbrace{\frac{d\theta}{d\lambda}}_{>0} = \underbrace{\zeta(\theta)(1-\phi)\gamma'(p)}_{>0} \frac{dp}{d\lambda},$$

which can only hold if $dp/d\lambda > 0$.

Effect of λ on SE. The total differential of l with respect to λ is

$$\frac{dl}{d\lambda} = \frac{dl}{dp}\frac{dp}{d\lambda} + \frac{dl}{d\theta}\frac{d\theta}{d\lambda}$$

Implicitly differentiating expected production λl in (23) with respect to p

$$\lambda \frac{dl}{dp} = \underbrace{\frac{(1-\phi)\left[1-\beta(1-\delta)\right]\psi'(p)}{1-\beta(1-\delta)\left(1-\mu(\theta)\phi\right)}}_{>0} + \underbrace{\frac{dq_s}{dp}}_{>0} > 0, \tag{B.10}$$

since supply by the self-employed responds positively to price increases, so that $dl/dp > 0$. Implicitly differentiating expected production λl in (23) with respect to θ

$$\lambda \frac{dl}{d\theta} = \frac{(1-\phi)\left[1-\beta(1-\delta)\right]\left[\psi(p)-u(b)\right]\beta(1-\delta)\phi\mu'(\theta)}{\left[1-\beta(1-\delta)\left(1-\mu(\theta)\phi\right)\right]^2} > 0, \tag{B.11}$$

which can be understood from the increase in recruiting costs when θ increases. We can thus conclude that for $b = 0$

$$\frac{dl}{d\lambda} = \underbrace{\frac{dl}{dp}}_{>0}\underbrace{\frac{dp}{d\lambda}}_{>0} + \underbrace{\frac{dl}{d\theta}}_{>0}\underbrace{\frac{d\theta}{d\lambda}}_{>0} > 0.$$

Finally, consider the effect of λ on SE. Implicitly differentiating (18)

$$\underbrace{\frac{dq_c}{dp}\frac{dp}{d\lambda}}_{<0} = \underbrace{\lambda SE\frac{dq_s}{dp}\frac{dp}{d\lambda}}_{>0} + \underbrace{\lambda(1-SE)\left[\frac{\delta\mu'(\theta)}{[\mu(\theta)+\delta(1-\mu(\theta))]^2}\frac{d\theta}{d\lambda}l + \frac{\mu(\theta)}{\mu(\theta)+\delta(1-\mu(\theta))}\frac{dl}{d\lambda}\right]}_{>0}$$
$$+ \underbrace{SEq_s + (1-SE)\frac{\mu(\theta)}{\mu(\theta)+\delta(1-\mu(\theta))}l}_{>0} + \lambda\frac{dSE}{d\lambda}\left[q_s - \frac{\mu(\theta)}{\mu(\theta)+\delta(1-\mu(\theta))}l\right],$$

by the virtue of Assumption 1 and Lemma 1, so that

$$\frac{dSE}{d\lambda}\left[q_s - \frac{\mu(\theta)}{\mu(\theta)+\delta(1-\mu(\theta))}l\right] < 0. \tag{B.12}$$

From (24) we know that $q_s < l\mu(\theta)/(\mu(\theta)+\delta(1-\mu(\theta)))$ for $b = 0$, so that we can conclude that $dSE/d\lambda > 0$.

Effects of λ on U. Remember that a higher λ increases self-employment and $\mu(\theta)$. From the steady-state definition of U in (8), we immediately infer that both of these effects decrease U, as:

$$\frac{dU}{dSE} = -\frac{\delta(1-\mu(\theta))}{\mu(\theta)+\delta(1-\mu(\theta))} < 0 \tag{B.13}$$
$$\frac{dU}{d\mu(\theta)} = -(1-SE)\frac{\delta}{[\mu(\theta)+\delta(1-\mu(\theta))]^2} < 0. \tag{B.14}$$

Consequently,

$$\frac{dU}{d\lambda} = \frac{dU}{dSE}\frac{dSE}{d\lambda} + \frac{dU}{d\mu(\theta)}\frac{d\mu(\theta)}{d\lambda} < 0.$$

Effects of E on $\mu(\theta)$, $\omega(\theta)$, θ, and $\zeta(\theta)$. We use that $\mu(\theta) \equiv E\hat{\mu}(\theta)$, so that worker indifference is now given by

$$\lambda = \omega(\theta) = \frac{\phi E\hat{\mu}(\theta)}{1 - (1 - E\hat{\mu}(\theta)\phi)\,\beta(1-\delta)}$$

Since θ is the only variable that can possibly adjust, it is immediate that $d\omega(\theta)/dE = d\mu(\theta)/dE = 0$. Implicitly differentiating worker indifference with respect to E yields

$$0 = \phi\,[1 + \beta(1-\delta)]\left(\hat{\mu}(\theta) + E\hat{\mu}'(\theta)\frac{d\theta}{dE}\right) \quad \Longrightarrow \quad -\underbrace{\hat{\mu}(\theta)}_{>0} = \underbrace{E\hat{\mu}'(\theta)}_{>0}\frac{d\theta}{dE},$$

so that $d\theta/dE < 0$. As a result the job-filling rate $\zeta(\theta) = E\hat{\zeta}(\theta)$ goes up.

Effects of E on p and SE. Differentiating the free-entry condition in (26) with respect to E:

$$0 = \underbrace{\hat{\zeta}(\theta)\gamma(p)}_{>0} + \underbrace{\zeta'(\theta)\gamma(p)}_{<0}\underbrace{\frac{d\theta}{dE}}_{<0} + \underbrace{\zeta(\theta)\gamma'(p)}_{>0}\frac{dp}{dE},$$

because $d\mu(\theta)/dE = 0$, so that we conclude that $dp/dE < 0$. Analogous to above but with signs switched, the total differential of l is given by

$$\frac{dl}{dE} = \underbrace{\frac{dl}{dp}}_{>0}\underbrace{\frac{dp}{dE}}_{<0} + \underbrace{\frac{dl}{d\theta}}_{>0}\underbrace{\frac{d\theta}{dE}}_{<0} < 0,$$

since dl/dp and $dl/d\theta$ in (B.10) and (B.11) are unchanged. Differentiating the market-clearing condition in (18) with respect to E thus results in

$$\frac{1}{\lambda}\underbrace{\frac{dq_c}{dp}\frac{dp}{dE}}_{>0} = \frac{dSE}{dE}\underbrace{\left(q_s - \frac{\mu(\theta)}{\mu(\theta)+\delta(1-\mu(\theta))}l\right)}_{<0} + \underbrace{SE\frac{dq_s}{dp}\frac{dp}{dE}}_{<0}$$
$$+ (1-SE)\left(\underbrace{\frac{\delta l}{[\mu(\theta)+\delta(1-\mu(\theta))]^2}\frac{d\mu(\theta)}{dE}}_{=0} + \underbrace{\frac{\mu(\theta)}{\mu(\theta)+\delta(1-\mu(\theta))}\frac{dl}{dE}}_{<0}\right),$$

so that $dSE/dE < 0$. Because of free entry of vacancies while $\zeta(\theta)$ increases, the price p must fall. Prices fall because SE goes down, resulting in a labour market inflow keeping $\mu(\theta)$ unchanged. Finally, we characterize the comparative statics for $u(b) > 0$.

Effect of E on U. Given that a higher E decreases SE but keeps $\mu(\theta)$ unchanged, it immediately follows from (B.13) that $dU/dE < 0$.

The case of $u(b) > 0$. Effects of λ on p and θ. Differentiating the workers' indifference condition in (25) yields:

$$\psi(p) + \underbrace{[\lambda - \omega(\theta)]}_{>0}\psi'(p)\frac{dp}{d\lambda} = \underbrace{\omega'(\theta)\left[\psi(p) - u(b)\right]}_{>0}\frac{d\theta}{d\lambda}, \tag{B.15}$$

because we know from (21) that for the mixed-strategy equilibrium to exist for $u(b) >$ 0, we need $\lambda > \omega(\theta)$, and because in any MSCC-equilibrium $\psi(p) > u(b)$. Implicitly differentiating the free-entry condition in (22):

$$\underbrace{\left(k\mu'(\theta)\phi\beta(1-\delta)-\zeta'(\theta)(1-\phi)p\left[\psi(p)-u(b)\right]\right)}_{>0}\frac{d\theta}{d\lambda}$$
$$=\underbrace{\zeta(\theta)(1-\phi)\left[\psi(p)-u(b)+p\psi'(p)\right]}_{>0}\frac{dp}{d\lambda} \tag{B.16}$$

Suppose that $dp/d\lambda = 0$. Then, (B.16) requires $d\theta/d\lambda = 0$ while (B.15) can only hold when $d\theta/d\lambda > 0$. Hence, by an argument of continuous differentiability, and using that $dp/d\lambda > 0$ at $u(b) = 0$, we can rule out $dp/d\lambda \leq 0$. Thus, $dp/d\lambda$ will not change its sign when we increase b as long as we remain in an MSCC-equilibrium, so that $dp/d\lambda > 0$. Then it is immediate from (B.16) that also $d\theta/d\lambda > 0$.

Effect of λ on SE. Note in (B.10) and (B.11) that $dl/dp > 0$ and $dl/d\theta > 0$ do not depend on b being zero, so that the sign restriction in (B.12) still holds. We know that at $b = 0$ we have $dSE/d\lambda > 0$ and $q_s - l\mu(\theta) / (\mu(\theta) + \delta(1-\mu(\theta))) < 0$. Since their product must be negative, neither of those can be equal to 0 for $b > 0$. From continuity we thus get that the same comparative statics hold for $b > 0$: the self-employment rate SE increases in λ.

Effect of λ on U. Allowing for $u(b) > 0$ does not change the results on the response of SE and $\mu(\theta)$ to λ, so that the predictions for $dU/d\lambda$ are robust to $u(b) > 0$ as well.

Effects of E. The workers' indifference condition in (21) yields

$$\underbrace{\left[\lambda-\omega(\theta)\right]\psi'(p)}_{>0}\frac{dp}{dE}=\underbrace{\phi\frac{(1-\omega(\theta)\beta(1-\delta))\left[\psi(p)-u(b)\right]}{1-(1-\mu(\theta)\phi)\,\beta(1-\delta)}}_{>0}\frac{d\mu(\theta)}{dE}, \tag{B.17}$$

while the free-entry condition in (22) yields

$$\frac{d\mu(\theta)}{dE}\underbrace{\frac{\phi\beta(1-\delta)k}{1-\phi}}_{>0}=\frac{d\zeta(\theta)}{dE}\underbrace{p\left[\psi(p)-u(b)\right]}_{>0}+\underbrace{\zeta(\theta)\left[\psi(p)-u(b)+p\psi'(p)\right]}_{>0}\frac{dp}{dE}. \tag{B.18}$$

Suppose that $dp/dE=0$. Then, (B.17) requires $d\mu(\theta)/dE=0$, which can only be the case for $d\theta/dE<0$ and thus $d\zeta(\theta)/dE>0$. However, when $dp/dE=0$ and $d\mu(\theta)/dE=0$, (B.18) can only hold for $d\zeta(\theta)/dE=0$. Consequently, we can rule out $dp/dE=0$, and $dp/dE<0$ at $u(b)=0$ implies $dp/dE<0$ also for $u(b)>0$ whenever an MSCC equilibrium exists. Hence, we conclude from (B.17) that also $d\mu(\theta)/dE<0$, and thus $d\theta/dE<0$ and $d\zeta(\theta)/dE>0$. Finally, $d\mu(\theta)/dE<0$ and $dp/dE<0$ are only consistent with more applicants and thus $dSE/dE<0$. Regarding the effect of E on U, observe that now not only self-employment decreases, but also the job-finding probability drops. Hence, it is the mirror opposite situation of how λ affects U in steady state. We conclude that increases in E decrease U also when $u(b)>0$.

B.5 One-worker firms

It follows from (29), (30) and (13) that

$$\begin{aligned}
V^{J}&=\frac{\lambda\left(p-w\right)l-(1-\lambda)d}{1-\beta(1-\delta)},\\
V^{PE}&=\frac{\lambda\left(u\left(wl\right)-l\right)+(1-\lambda)u(d)+\beta\delta V^{LM}}{1-\beta(1-\delta)},\\
V^{LM}&=\frac{\mu\left(\theta\right)\lambda\left[u\left(wl\right)-l\right]+\left(1-\mu(\theta)\right)\left(1-\beta(1-\delta)\right)u(b)}{\left(1-\beta\right)\left[1-\left(1-\mu(\theta)\right)\beta(1-\delta)\right]}.
\end{aligned}$$

The first-order conditions of the Nash product thus result in

$$(1-\phi)\left(V^{PE}-u(b)-\beta V^{LM}\right)=\phi V^{J}u'(wl), \tag{B.19}$$

$$\lambda(p-w)(1-\phi)\left(V^{PE}-u(b)-\beta V^{LM}\right)=\phi V^{J}\lambda\left(1-u'(wl)w\right). \tag{B.20}$$

Substituting the value functions into (B.20) yields the sharing rule in (32), while combining (B.19) and (B.20) yields the familiar result of $u'(wl)=1/p$.

B.6 Proof of Proposition 1

As in the baseline model, $u'(wl)=1/p$ results in $wl=pq^{s}$ and thus $wl/p=q^{s}$. The sharing rule in (32) can then be rewritten to yield

$$l=(1-\phi)\frac{[1-\beta(1-\delta)]\left[\psi(p)-u(b)/\lambda\right]}{1-\beta(1-\delta)\left(1-\mu(\theta)\phi\right)}+q^{s}, \tag{B.21}$$

which, substituted in (33), yields the free-entry condition

$$k=\zeta(\theta)\frac{(1-\phi)p\left[\lambda\psi(p)-u(b)\right]}{1-\beta(1-\delta)\left(1-\mu(\theta)\phi\right)}, \tag{B.22}$$

whereas the workers' indifference condition becomes (35). Then either $\omega(\theta)=1$, which requires that $\mu(\theta)=1$ and $\phi=1$, or $\omega(\theta)<1$, so that $\lambda\psi(p)=u(b)$. In either case, the sharing rule in (B.21) then shows that $l=q^{s}$ and thus $w=p$, and (B.22) shows that equilibrium requires $k=0$.

B.7 Proof of Proposition 2

Here we fix SE and abstract from the workers' indifference condition.

Effects of E**.** Differentiating the goods-market clearing condition in (18) we obtain:

$$\underbrace{\lambda(1-SE)\frac{\delta}{[\mu(\theta)+\delta(1-\mu(\theta))]^2}l}_{A}\frac{d\mu(\theta)}{dE} = \underbrace{\left(\frac{dq^c}{dp}-\lambda SE\frac{dq^s}{dp}-(1-SE)\frac{\mu(\theta)}{\mu(\theta)+\delta(1-\mu(\theta))}\frac{dl}{dp}\right)}_{B}\frac{dp}{dE}. \tag{B.23}$$

It is the case that $A > 0$ and $B < 0$. Next, we differentiate the free-entry condition.

$$\underbrace{k\beta(1-\delta)\phi}_{X}\frac{d\mu(\theta)}{dE} - \frac{d\zeta(\theta)}{dE}\underbrace{(1-\phi)p\left[\psi(p)-u(b)\right]}_{Y} = \underbrace{\zeta(\theta)(1-\phi)\left[\psi(p)-u(b)+p\psi'(p)\right]}_{Z}\frac{dp}{dE} \tag{B.24}$$

Here, $X > 0$, $Y > 0$ and $Z > 0$. Combining the two equations by substituting out dp/dE, we get:

$$\underbrace{-\frac{BY}{Z}}_{>0}\frac{d\zeta(\theta)}{dE} = \underbrace{\left[A-\frac{B}{XZ}\right]}_{>0}\frac{d\mu(\theta)}{dE}. \tag{B.25}$$

Consequently, the responses of the job-finding probability $\mu(\theta)$ and job-filling probability $\zeta(\theta)$ to increases in matching efficiency have identical signs. Note that it is impossible to have $d\zeta(\theta)/dE \leq 0$, because that would imply $d\theta/dE > 0$ and therefore $d\mu(\theta)/dE > 0$, contradicting identical signs. Increases in matching efficiency thus increase firm profits, so that firms open more vacancies. Consequently, labour market tightness increases, as the measure of workers searching for jobs is constant. The two effects jointly increase the job-finding probability: $\mu(\theta)$ increases. The job-filling probability thus increases as well. Aggregate supply rises, so that $dp/dE < 0$, which mitigates the increase in firm profits and market tightness.

Effects of λ. We can keep the notation from the analysis of the effects of E and write:

$$X\frac{d\mu(\theta)}{d\lambda} - Y\frac{d\zeta(\theta)}{d\lambda} = Z\frac{dp}{d\lambda} \tag{B.26}$$

$$A\frac{d\mu(\theta)}{d\lambda} = B\frac{dp}{d\lambda} - \underbrace{\left(SEq^s + (1-SE)\frac{\mu(\theta)}{\mu(\theta)+\delta(1-\mu(\theta))}l\right)}_{C}, \tag{B.27}$$

with the new element $C > 0$. These two conditions together rule out $dp/d\lambda \geq 0$ as that would imply $d\mu(\theta)/d\lambda < 0$ and $d\zeta(\theta)/d\lambda \leq 0$, which cannot hold simultaneously. Therefore, it must be that $dp/d\lambda < 0$, which implies $d\zeta(\theta)/d\lambda > 0$ and hence $d\mu(\theta)/d\lambda < 0$.

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