CIEM5110-2: FEM, lecture 1.1

Introduction to the unit

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TU Delft

Agenda for today

1. Background and organization of this unit
2. Examples of applications of the finite element method (MSc thesis level)
3. Theory and quiz on notations
4. Basics of continuum mechanics

Positioning of the unit

You have had a first introduction in FEM

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  - Poisson equation
  - Derivation of the method

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- CIEM5000 Analysis of Slender Structures
  - Matrix-displacement method
  - Elimination of prescribed dofs
- **CIEM5110-2 Finite Elements for Structural Analysis**
  - Continuum mechanics
  - Beams and frames
  - Different analysis types: linear/nonlinear, static/dynamic, buckling/vibrations

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There will be more to learn about FEM:

- CIEM5210-1 Computational modeling of structures (also in CIEM5230 and somewhat in CIEM5240)
  - Work with commercial software (DIANA)
- CIEM5210-2 Advanced constitutive modeling
  - In-depth about material modeling
- CIEM1301 Advanced computational mechanics
  - Connect to recent literature
- CIEM1303 Upscaling techniques in construction materials design and engineering
  - Focus on multiscale modeling
- Data science and artificial intelligence for engineers (cross-over)
  - Possible application on FEM-AI interface
- CIEM0500 MSc thesis
  - Computational mechanics research
  - Computer-aided engineering

Learning objectives

After completing this course, you will be able to:

1. Explain what steps are taken to derive finite element formulations for different analysis types
   - linear elastic analysis
   - modal analysis
   - transient analysis
   - linear buckling analysis
   - geometrically nonlinear analysis
   - material nonlinear analysis
2. Select an appropriate analysis type for a given engineering problem
3. Perform finite element analysis to analyze the force distribution, stability and dynamic response of a frame structure
4. Compare results obtained from computer simulation and analytical calculations and assess the level of agreement between the results

Organization of the course

Two weekly slots for **contact hours**

- Tuesday 10:45-12:30.
- Thursday 13:45-15:30.

This Thursday: Interactive lecture including review of MUDE material

Assessment is by written exam

Assignments are exercise material and make the connection with units 1 and 3 of the module

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- We will first focus on derivations for continuum and frame problems (two different `models`)
- And then on the different analysis types (various `modules`)

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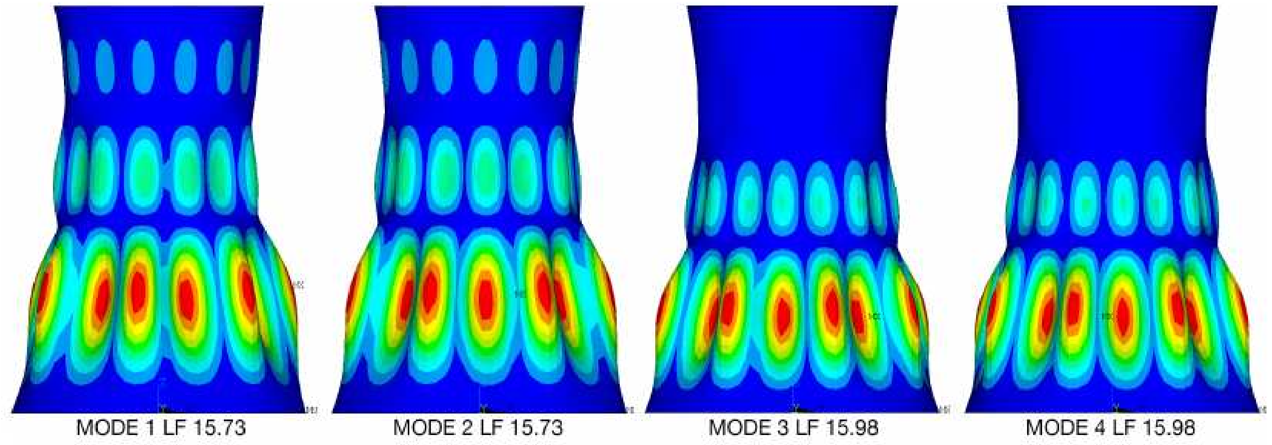
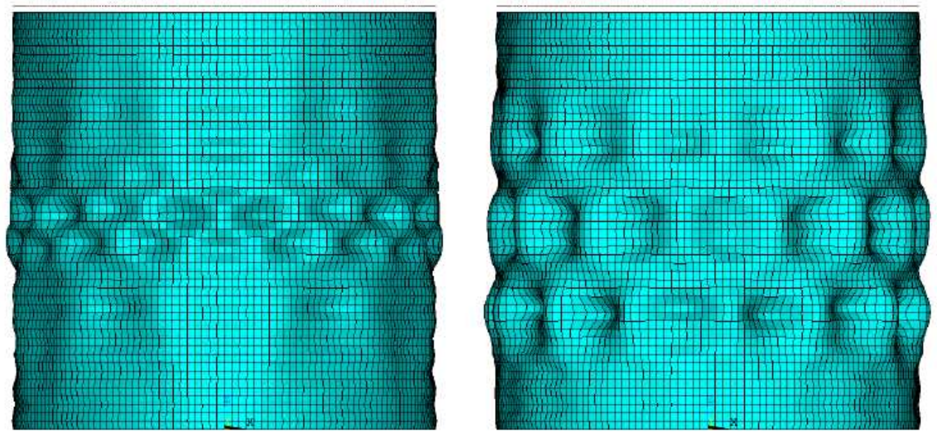
Now, let's look at some examples of these analysis types from past MSc theses

Example: Shell buckling, Tim Chen (2014)

Objective: investigate the influence of imperfections on shell buckling



Buckling modes of a cooling tower

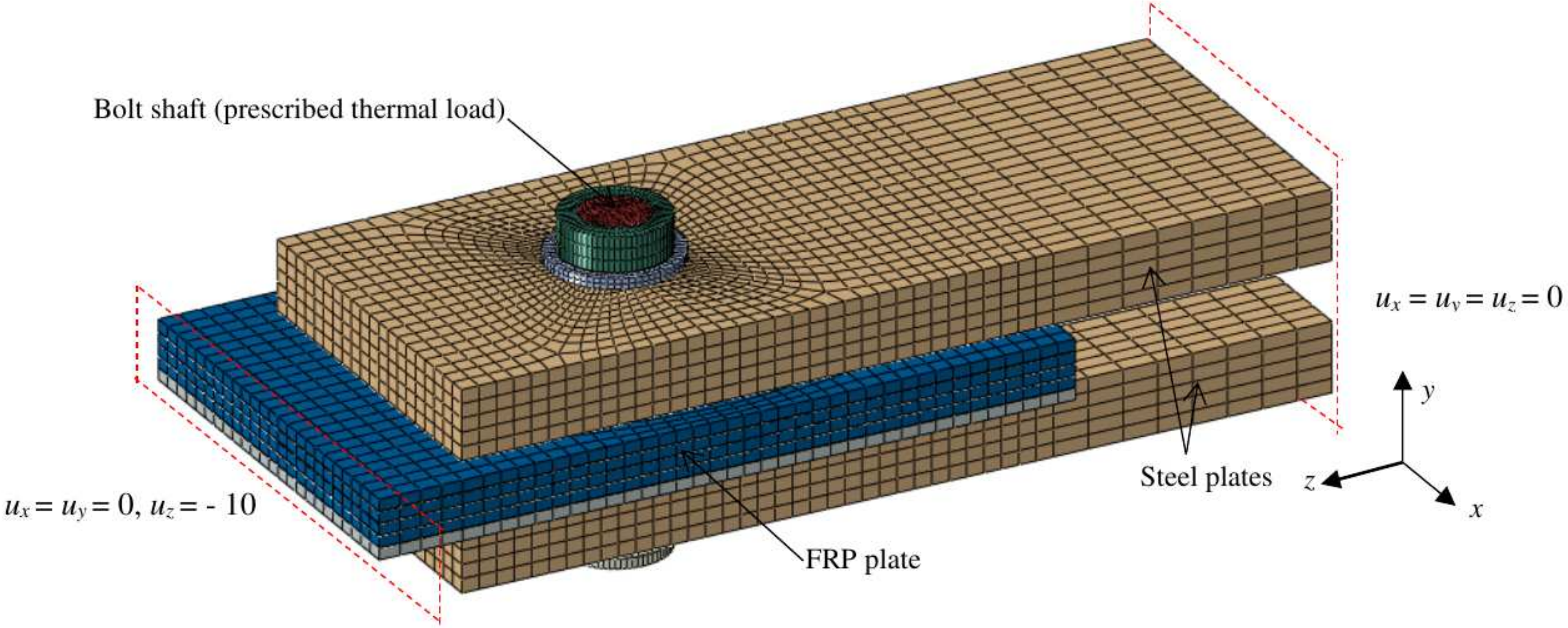


Post-buckling deformations of a cylinder

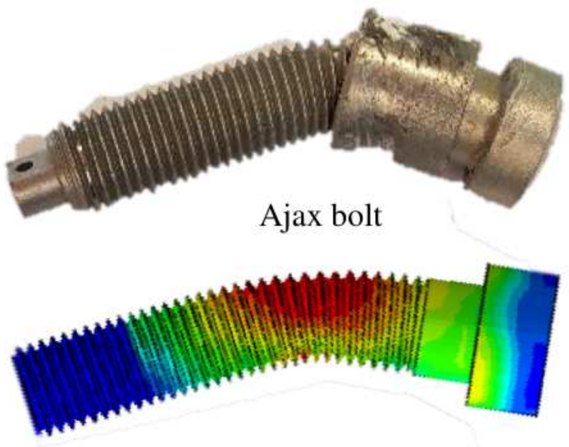
Analysis types: linear buckling analysis, geometrically nonlinear analysis

Example: Bolted joints, Fruzsina Csillag (2018)

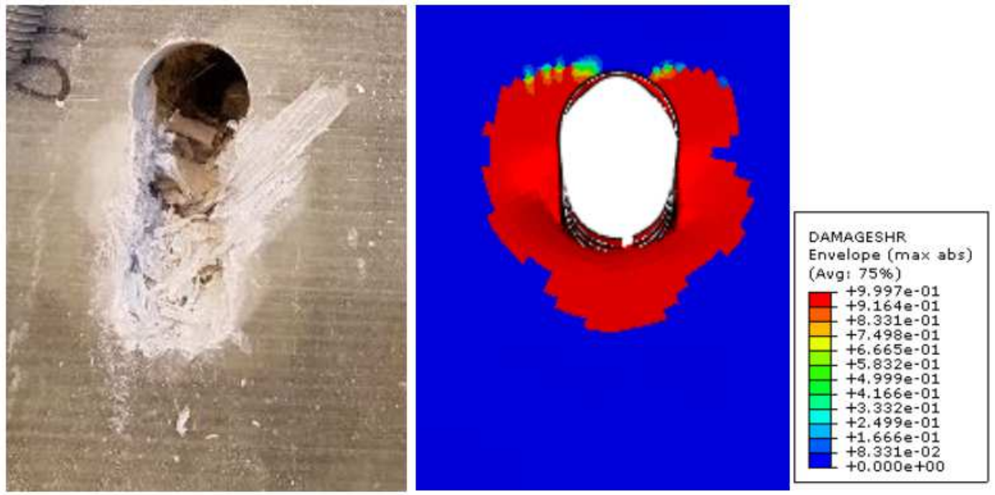
Objective: investigate the behavior of FRP-steel bolted connections



Setup



Deformation of the bolt

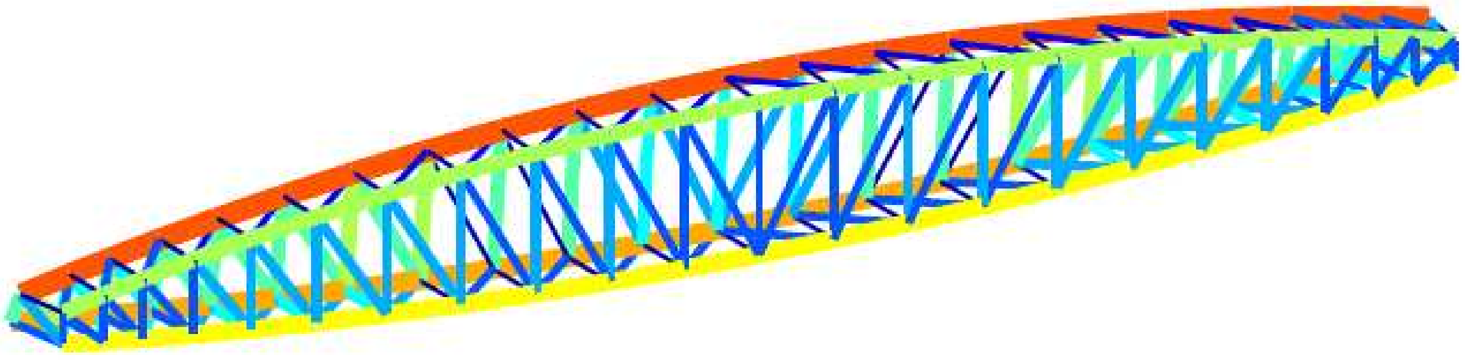


Damage of the FRP plate

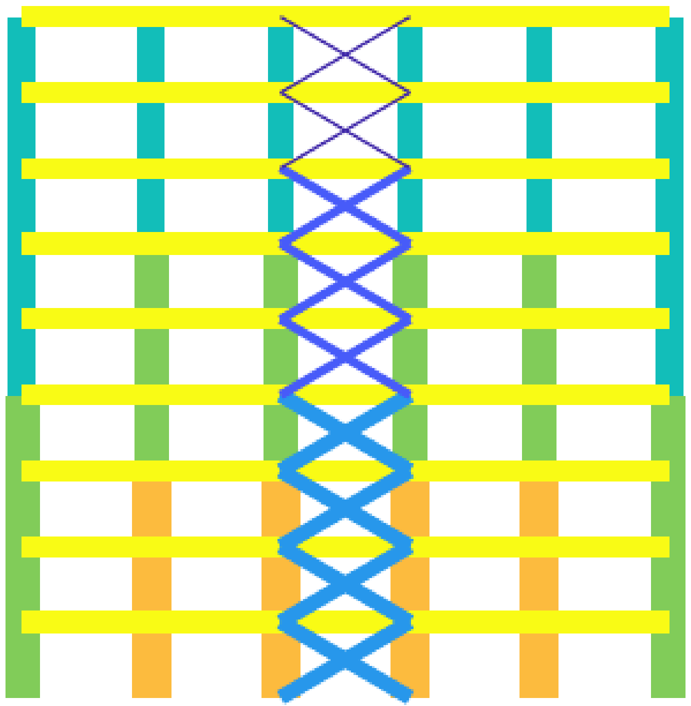
Analysis type: nonlinear analysis (plasticity, damage, contact)

Example: Steel structures, Tom van Woudenberg (2020)

Objective: design optimization limiting the number of different steel profiles



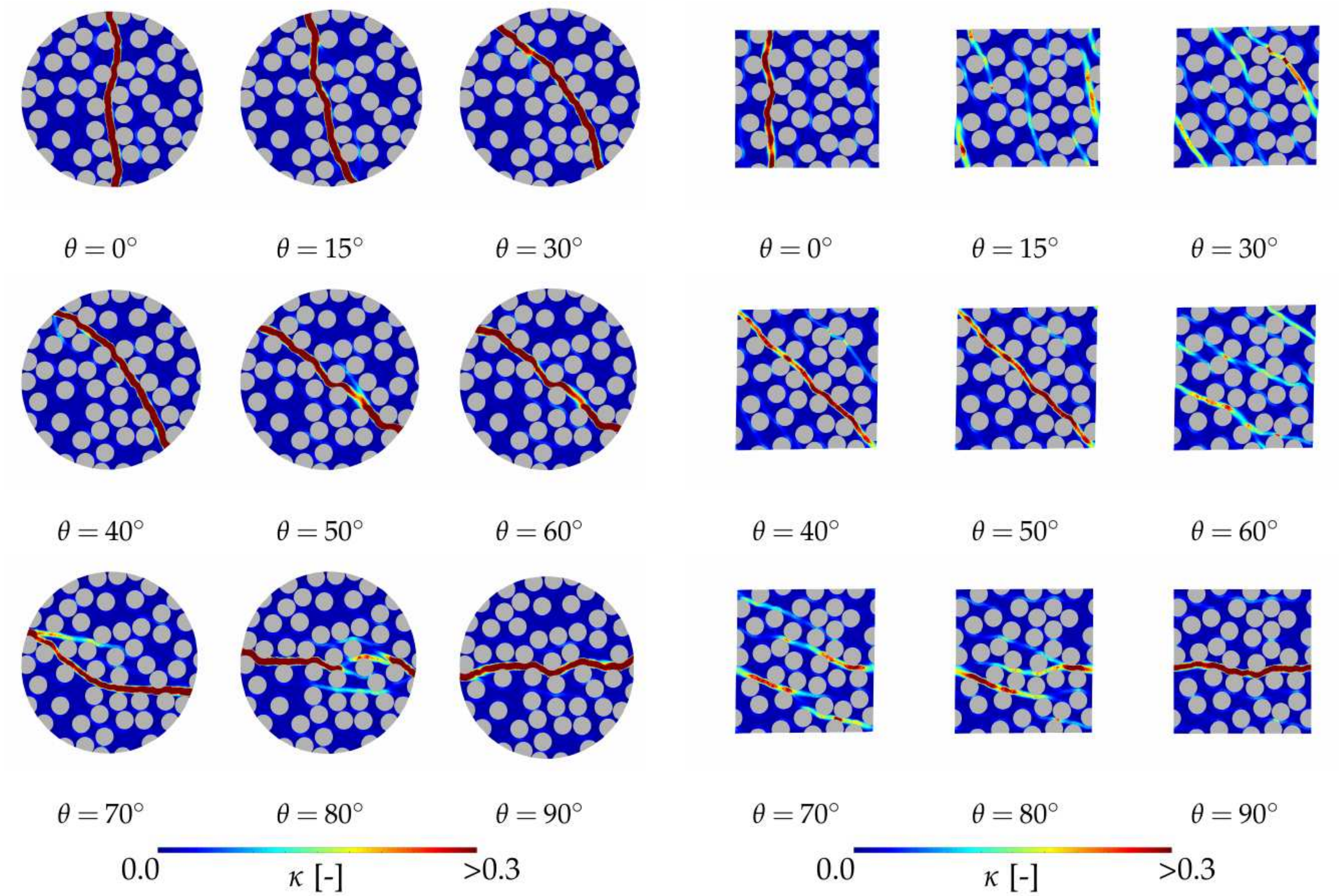
Truss structure



Frame structure

Analysis type: linear elastic analysis

Example: Circular micromodels, Pieter Hofman (2021)

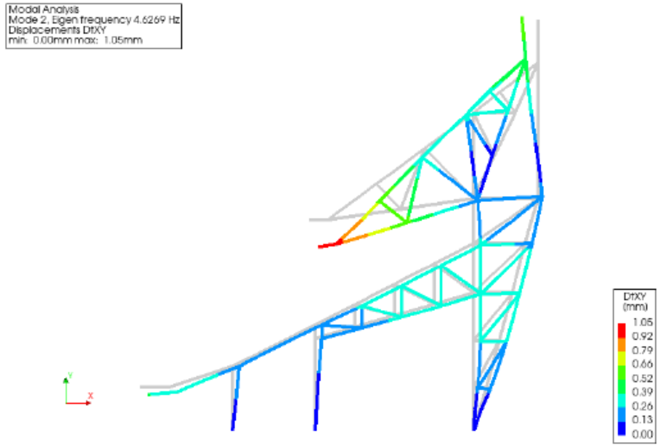


Objective:
make a micromodel that gives the same failure response in all directions

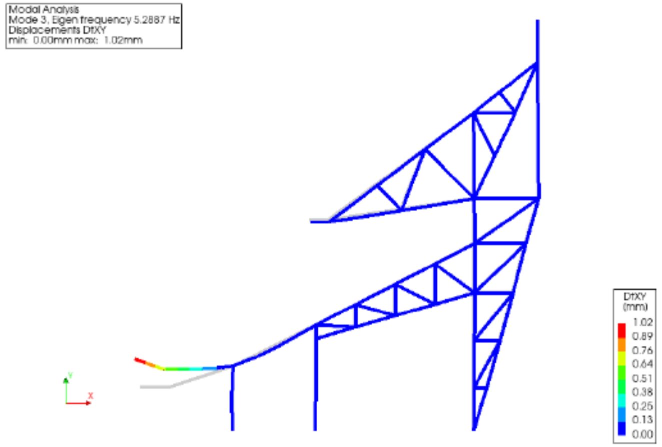
Analysis type:
material nonlinear analysis

Example: Stadium vibrations, Jelle Knibbe (2023)

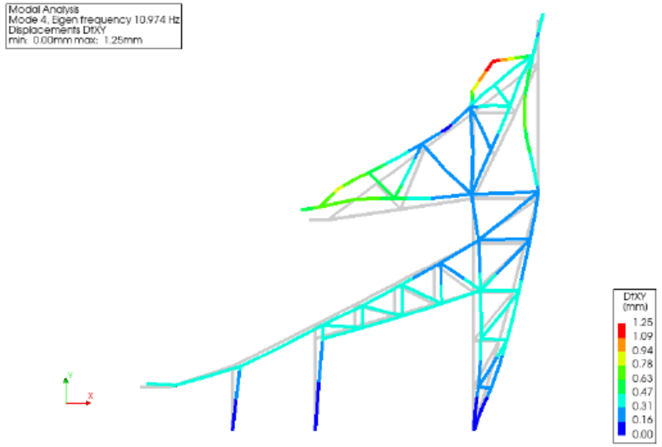
(a) Mode 1: 1.98 Hz



(b) Mode 2: 4.63 Hz



(c) Mode 3: 5.29 Hz



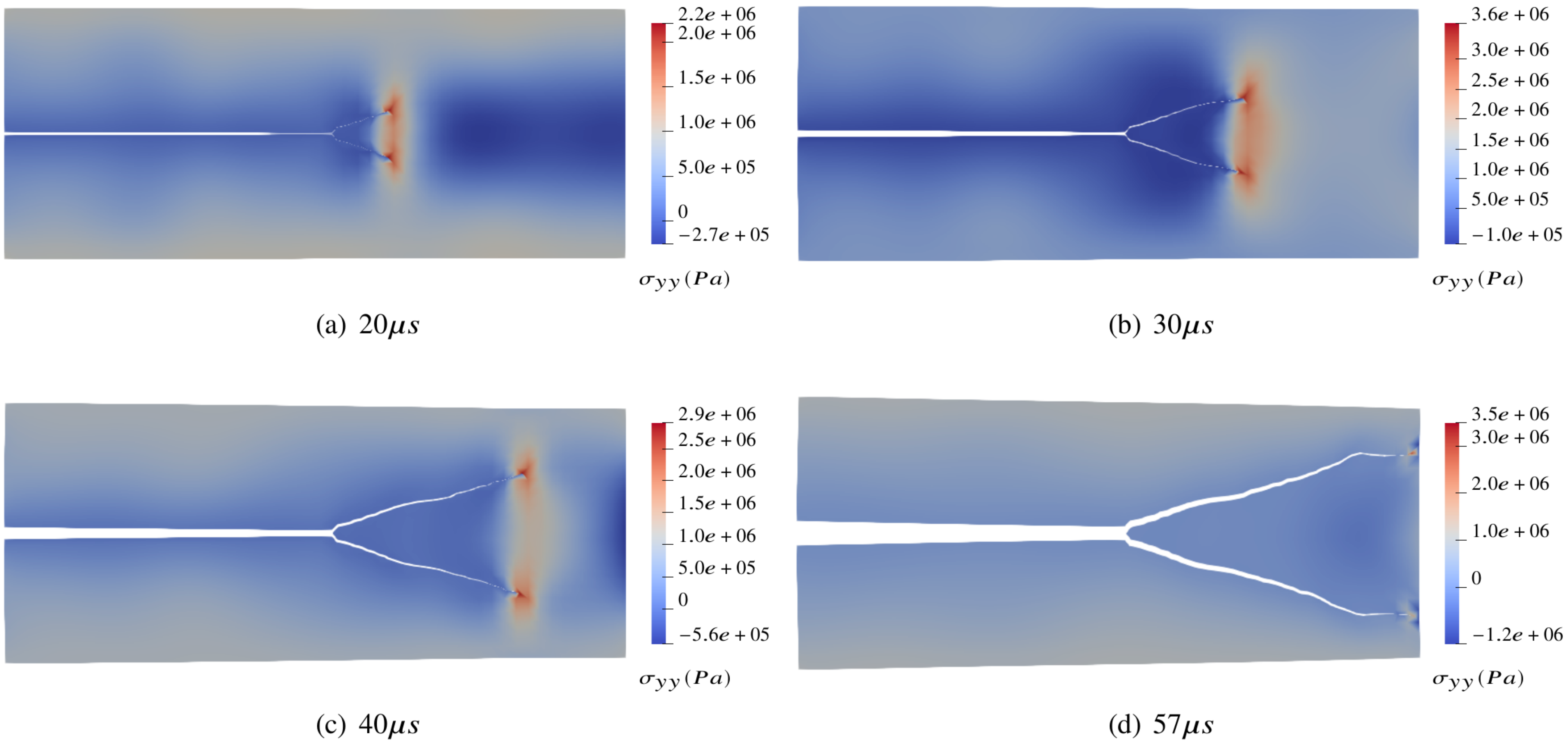
(d) Mode 4: 10.97 Hz

Objective: Analyze grandstand reliability under dynamic crowd loading

Analysis type: modal analysis

Example: Discontinuity-enriched finite element method, Jujian Zhang (2023)

Objective: extend an advanced discretization technique for cracks to dynamic applications



Analysis type: transient dynamic analysis

Tensors in different notations

	Tensor notation	Components	Index notation
Scalar (zero order tensor)	a	a	a
Vector (first order tensor)	$\mathbf{a}$	$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$	a_i
Matrix (second order tensor)	$\mathbf{A}$	$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$	A_{ij}
Fourth order tensor	$\boldsymbol{\mathcal{A}}$	-	$\mathcal{A}_{ijkl}$

Multiplication in different notations

	Matrix/vector notation	Tensor notation	Index notation
Matrix-vector product	$\mathbf{A}\mathbf{b}$	$\mathbf{A} \cdot \mathbf{b}$	$A_{ij}b_j$
Dot product	$\mathbf{a}^T\mathbf{b}$	$\mathbf{a} \cdot \mathbf{b}$	$a_i b_i$
Tensor product	$\mathbf{a}\mathbf{b}^T$	$\mathbf{a} \otimes \mathbf{b}$	$a_i b_j$
Double dot product	-	$\mathbf{A} : \mathbf{B}$	$A_{ij}B_{ij}$
Double dot product	-	$\boldsymbol{\mathcal{C}} : \mathbf{A}$	$\mathcal{C}_{ijkl}A_{kl}$
Matrix-matrix product	$\mathbf{A}\mathbf{B}$	$\mathbf{A} \cdot \mathbf{B}$	$A_{ik}B_{kj}$
Matrix-matrix with a transpose	$\mathbf{A}^T\mathbf{B}$	$\mathbf{A}^T \cdot \mathbf{B}$	$A_{ki}B_{kj}$
Cross product	$\mathbf{a} \times \mathbf{b}$	$\mathbf{a} \times \mathbf{b}$	$\epsilon_{ijk}a_j b_k$

Exercise

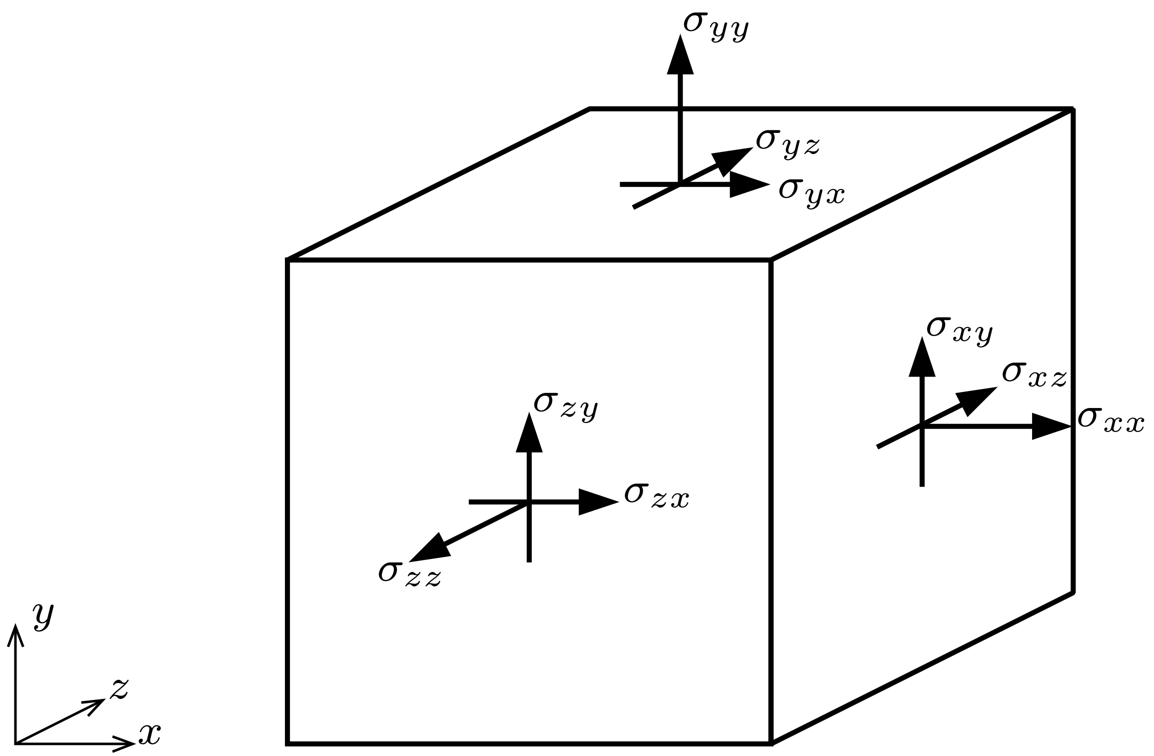
Test yourself with questions in the book

`https://interactivetextbooks.citg.tudelft.nl/computational-modelling`

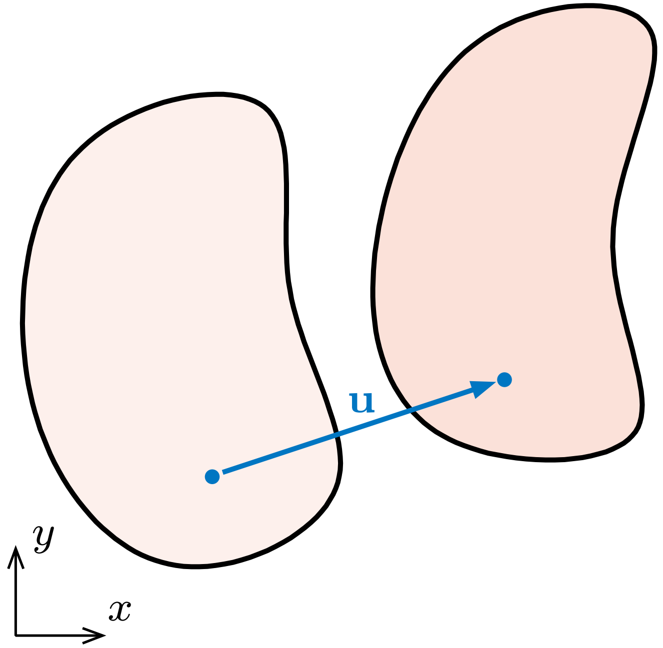
chapter **1.2 Linear algebra**

Continuum mechanics: stress and strain

Stress tensor on elementary cube



Displacement is a vector



And strain its symmetric gradient

$$\boldsymbol{\varepsilon} = \nabla_{\mathrm{s}}\mathbf{u} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$$

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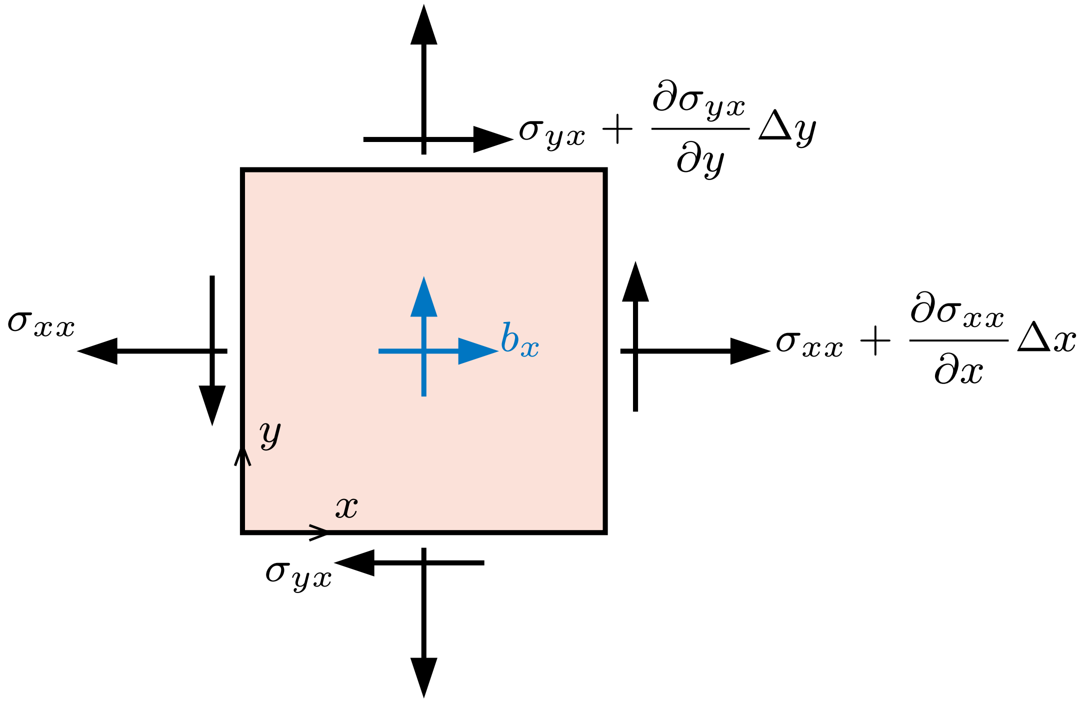
Continuum mechanics: equilibrium

Translational equilibrium (linear momentum):

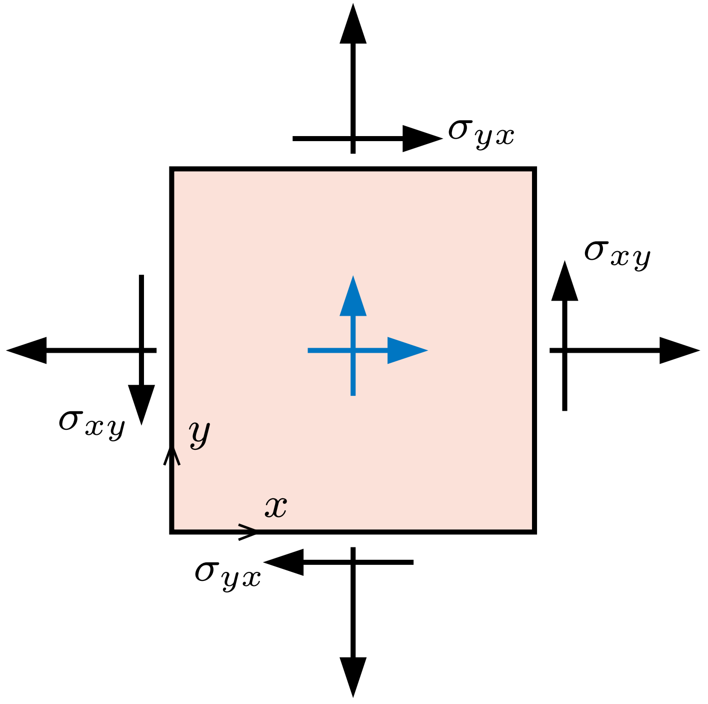


$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + b_x = 0 \quad \text{etc}$$

or

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = 0$$

Rotational equilibrium (angular momentum):



$$\sigma_{xy} = \sigma_{yx}, \quad \text{etc}$$

or

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$$

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Continuum mechanics: linear elasticity

Stress is related to strain through a fourth order tensor

$$\boldsymbol{\sigma} = \boldsymbol{\mathcal{D}} : \boldsymbol{\varepsilon} \quad \text{or} \quad \sigma_{ij} = D_{ijkl}\varepsilon_{kl}$$

In Voigt notation (with stress and strain as vectors) this can be written out, most conveniently in compliance format (relating strain to stress)

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2\nu \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{Bmatrix}$$

Continuum mechanics: 2D

There are three different ways to go from 3D to 2D:

- Plane stress ($\sigma_{zz} = 0$)
- Plane strain ($\epsilon_{zz} = 0$)
- Axisymmetry (starting from (r, z, θ) instead of (x, y, z), $\epsilon_{\theta\theta} = \dfrac{u_r}{r}$)

Outlook for this week

Find your way to the **online book**:

- `https://interactivetextbooks.citg.tudelft.nl/computational-modelling`
- `Course schedules - CIEM5110`: tailored reading guide
- `Codes - Pyjive`: link for python code
- Chapter `Introduction to finite elements`: MUDE material (incl. optional parts)

Study *Introduction to finite elements* before Thursday

There will be space for your questions

We will show how to derive the FE formulation for the PDE $\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = 0$