

Problem 1. Let S be a non-empty polyhedron: $S := \{x \in \mathbb{R}^n : Ax \leq b\}$ for some $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$.

1. Describe the recession cone R_S using A and b and prove the correctness of your description.
2. Describe $\text{int}(S)$ using A and b and prove the correctness of your description. Is $\text{int}(S) = \text{ri}(S)$ for any A, b ? If yes, provide a proof, if no, provide a counterexample.
3. Assume $x \in \text{ri}(S)$. Provide one example of a face $F \subset S$ (so $F \neq S$) such that $x \in F$ or prove that such a face does not exist.

Problem 2. Let K be a convex cone.

1. Show that if K ~~is pointed (i.e., it~~ contains no straight lines ~~)~~, then 0 is an extreme point of K .
2. Provide an example of a convex cone in $\mathbb{R}^2$ which ~~contains at least one straight line is not pointed~~ but for which 0 is its extreme point and explain why your example satisfies the required conditions.
3. Bonus question (not necessary but will give some bonus points): provide a counterexample showing that the set of cones required in 2. in the previous formulation of this exercise (i.e., non-pointed as defined in the slides, convex, having zero as an extreme point) is empty.