Design of the communication structure of cooperative adaptive cruise controllers for vehicle platoons

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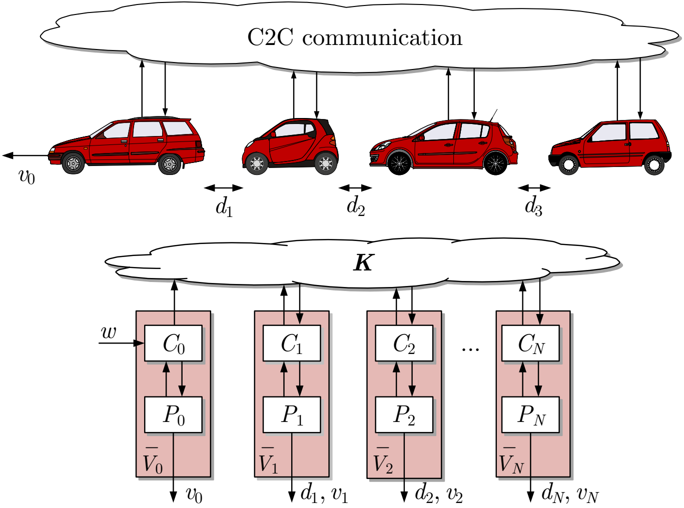
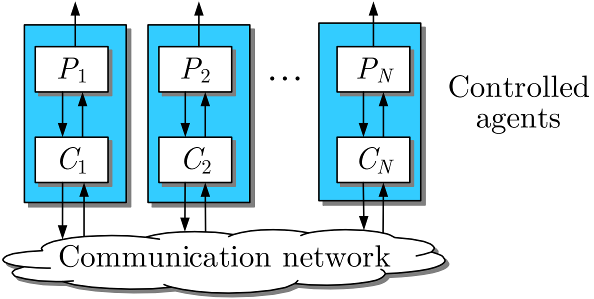
Control of vehicle platoons

CACC (*cooperative adaptive cruise control*)



Networked control of multi-agent system



Network thinking:

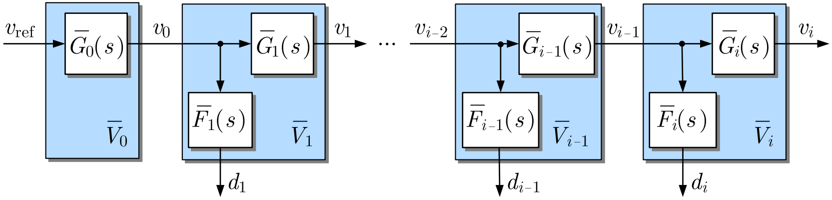
- *What are the appropriate properties of the controlled agents?*
- *What is a reasonable communication structure?*

Design of the communication structure of cooperative adaptive cruise controllers for vehicle platoons

1. Requirements on the platoon behaviour
2. Properties of the controlled vehicles
3. Communication structure design

Requirements on the platoon behaviour

Requirements on the platoon behaviour



Requirements on the platoon behaviour:

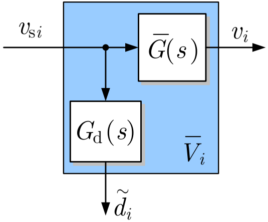
(R2) **Synchronisation:** $\lim_{t\to\infty} |v_i(t) - v_0(t)| = 0$

(R3) **Time-headway policy:** $d_{i\mathrm{ref}}(t) = \beta_i v_i(t) + \alpha_i$

(R4) **Collision avoidance:** $d_i(t) \geq \alpha_i, \quad t \geq 0$

(R5) **Continuous progression:** $v_i(t) \geq 0, \quad t \geq 0$

Model of the vehicle with ACC



$$\bar{V}_i : \begin{cases} V_i(s) = \bar{G}_i(s)\, V_{si}(s) \\ \tilde{D}_i(s) = \bar{F}_i(s)\, V_{si}(s), \end{cases} \qquad i = 1, 2, ..., N$$

with

$$\bar{F}_i(s) = \frac{1}{s}(1 - \bar{G}_i(s))$$

$$\text{ACC: } V_{si}(s) = V_{i-1}(s)$$

Properties of the controlled vehicles

Properties of the controlled vehicles

Permanent time-headway spacing:

$$d_i(t) = d_{i\mathrm{ref}}(t) = \alpha + \beta v_i(t), \quad t \geq 0, \quad i = 1, 2, ..., N$$

$$\begin{aligned} \Longleftrightarrow \quad d_i(0) &= \alpha + \beta v_i(0), \quad i = 1, 2, ..., N \\ \dot{d}_i(t) &= \beta \dot{v}_i(t), \quad t \geq 0 \end{aligned}$$

$$\Longleftrightarrow \quad \dot{d}_i(t) = v_{i-1}(t) - v_i(t)$$

Hence,

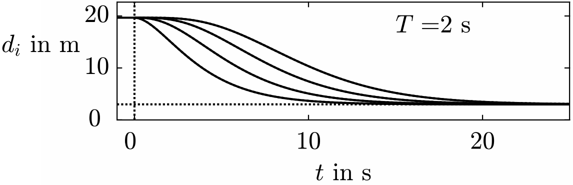
Ideal vehicle behaviour:

$$\bar{V}_i : \quad \dot{v}_i(t) = -\frac{1}{\beta} v_i(t) + \frac{1}{\beta} v_{i-1}(t), \quad i = 1, 2, ..., N$$

Ideal vehicle behaviour

Permanent time-headway spacing is ensured if and only if the vehicle dynamics is represented by

$$\bar{V}_i : \quad \dot{v}_i(t) = -\frac{1}{\beta} v_i(t) + \frac{1}{\beta} v_{i-1}(t), \quad i = 1, 2, ..., N.$$

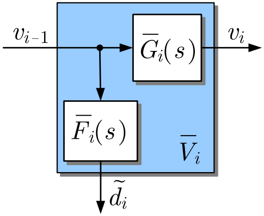


Consequence:

The platoon of ideal vehicles is guaranteed to avoid collisions.

Properties of the controlled vehicles

(R2) Synchronisation for constant velocity $v_{\text{ref}}(t) = \bar{v}$



$$\lim_{t\to\infty} v_i(t) = \bar{G}_i(0)\bar{v} \overset{!}{=} \bar{v}$$

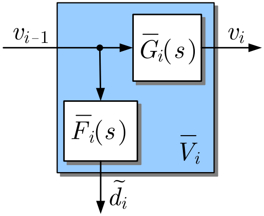
i.e.

Necessary condition for (R2) synchronisation:

$$\bar{G}_i(0) = 1$$

Properties of the controlled vehicles

(R3) Time headway for constant velocity $v_{\text{ref}}(t) = \bar{v}$



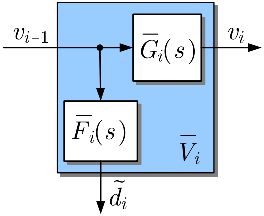
$$\lim_{t\to\infty} \tilde{d}_i(t) = \bar{F}_i(0)\bar{v} \stackrel{!}{=} \beta_i \bar{v}$$

i.e.

Necessary condition for observing (R3) time-headway policy:

$$\bar{F}_i(0) = \beta_i$$

(R3) Time headway for $v_{\text{ref}}(t) = \bar{v}$



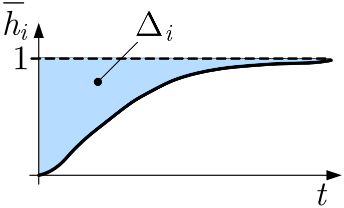
What does it mean?

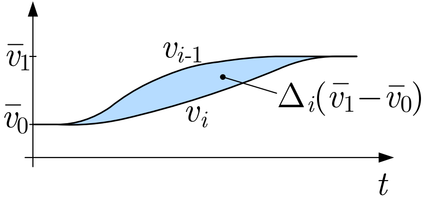
$$
\begin{aligned}
\bar{F}_i(0) &= \lim_{s\to 0} s\frac{1}{s}\frac{1}{s}(1-\bar{G}_i(s)) \\
&= \lim_{t\to\infty}\int_0^t (1-\bar{h}_i(\tau))\,\mathrm{d}\tau \\
&= \int_0^\infty (1-\bar{h}_i(\tau))\,\mathrm{d}\tau \qquad \overset{!}{=} \beta_i \qquad \text{(R3)}
\end{aligned}
$$

Properties of the controlled vehicles

Vehicle delay

$$\color{red}{\Delta_i} = \int_0^\infty (1 - \bar{h}_i(\tau))\, \mathrm{d}\tau$$



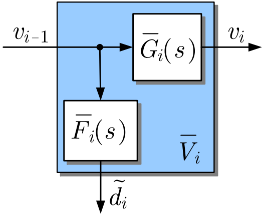


Necessary condition for observing the time-headway policy:

$$\Delta_i = \beta_i$$

Properties of the controlled vehicles

(R4) Collision avoidance
(R5) Continuous progression:



The controlled vehicle has to possess the following property:

$$v_{i-1}(t) \geq 0 \quad \Rightarrow \quad v_i(t) \geq 0,\ \tilde{d}_i(t) \geq 0.$$

That is, the vehicle has to be externally positive.

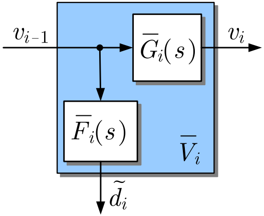
[Farina/Rinaldi]:
A system $y(t) = g(t) * u(t)$ is externally positive if and only if

$$g(t) \geq 0.$$

Properties of the controlled vehicles

(R4) Collision avoidance

(R5) Continuous progression:

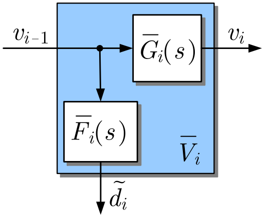


Necessary condition for (R4) collision avoidance and (R5) continuous progression:

$$\bar{f}_i(t) \geq 0, \quad \bar{g}_i(t) \geq 0, \; t \geq 0.$$

The velocity $v_i(t)$ is adapted to the reference $v_{\mathrm{s}i}(t) = v_{i-1}(t)$ without overshoot.

Properties of the controlled vehicles

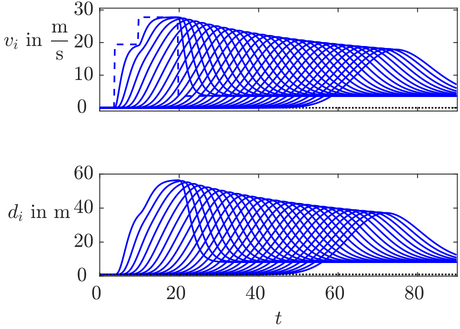


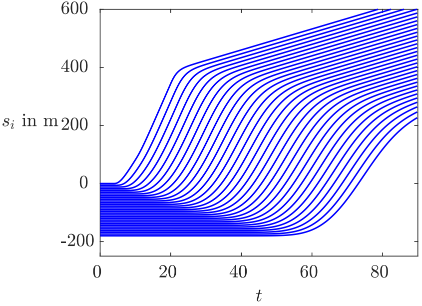
Summary of the design objectives:

Platoon behaviour		Vehicle behaviour
Synchronisation		$\bar{G}_i(0) = 1$
Time headway spacing	$\iff$	$\Delta_i = \beta_i$
Collision avoidance		$\bar{f}_i(t) \geq 0$
Continuous progression		$\bar{g}_i(t) \geq 0$

Properties of the controlled vehicles

Behaviour of a platoon



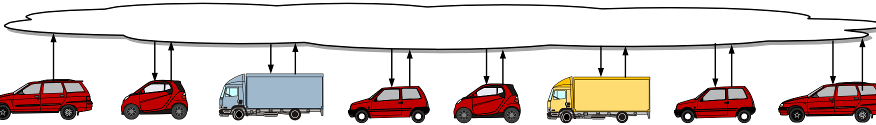


Communication structure design

Communication structure design

For vehicles with delay $\Delta_i = \beta_i$, ACC solves the control problem.

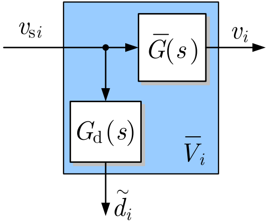
CACC is necessary, if a vehicle has delay $\Delta_i > \beta_i$.



Which communication links are necessary?

Communication structure design

Model of the platoon with CACC:



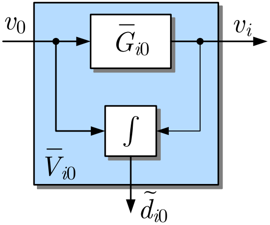
$$\bar{V}_i : \begin{cases} V_i(s) = \bar{G}_i(s)\, V_{si}(s) \\ \tilde{D}_i(s) = \bar{F}_i(s)\, V_{si}(s), \end{cases} \qquad i = 1, 2, ..., N$$

$$\text{CACC:} \quad V_{si}(s) = \sum_{j=0}^{i-1} \hat{a}_{ij} V_j(s)$$

How to choose $\hat{a}_{ij}$? $\quad \hat{a}_{ij} \geq 0, \quad \sum_{j=0}^{i-1} \hat{a}_{ij} = 1$

Communication structure design

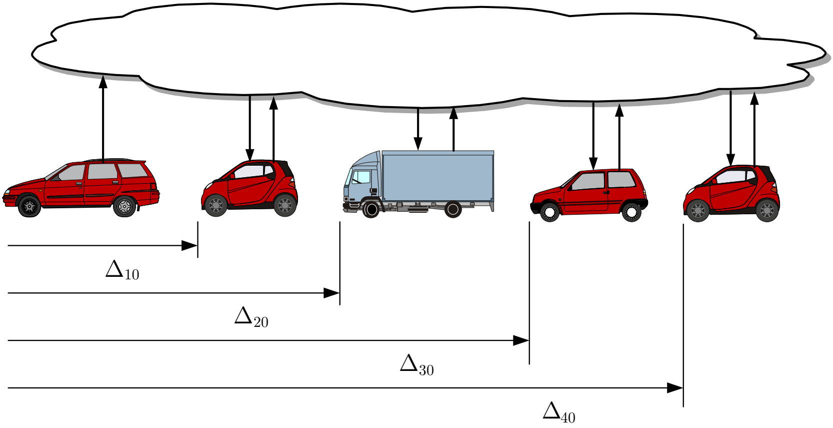
Cumulative delay of the truncated platoon



$$\Delta_{i0} = \int_0^\infty (1 - \bar{h}_{i0}(\tau))\,\mathrm{d}\tau$$

Communication structure design

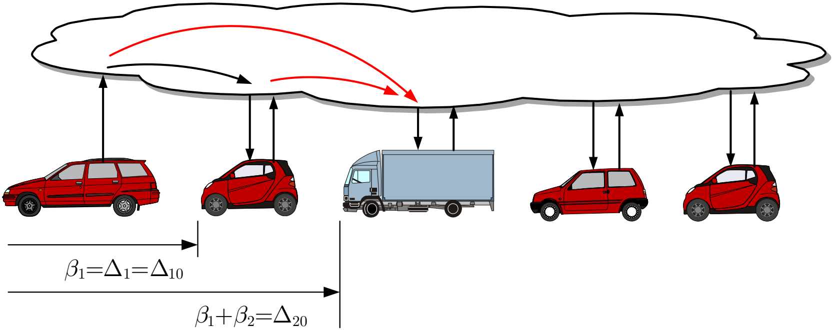
Cumulative delay Δ_{i0}:



$$
\begin{aligned}
\Delta_{00} &= 0 \\
\Delta_{i0} &= \Delta_i + \sum_{j \in \mathcal{N}_i} \hat{a}_{ij} \Delta_{j0}
\end{aligned}
$$

Communication structure design

Cumulative delay Δ_{i0}:



$$\Delta_{20} = \Delta_2 + \hat{a}_{21}\beta_1 \overset{!}{=} \beta_1 + \beta_2$$

Hence,

$$\hat{a}_{21} = 1 - \frac{\Delta_2 - \beta_2}{\beta_1}, \quad \hat{a}_{20} = 1 - \hat{a}_{21}$$

Communication structure design

Theorem (Time-headway spacing with CACC)

For CACC with the following adjacency matrix

$$
\begin{aligned}
\hat{a}_{10} &= 1 \\
\hat{a}_{il_i} &= \frac{\sum_{j=l_i}^{i} \beta_j - \Delta_i}{\beta}, \quad i = 2, 3, ..., N \\
\hat{a}_{i,l_i-1} &= 1 - \hat{a}_{il_i}, \quad i = 2, 3, ..., N \\
\hat{a}_{ij} &= 0, \quad j \neq l_i,\ j \neq l_i - 1,
\end{aligned}
$$

where the indices l_i, $(i = 2, 3, ..., N)$ satisfy the inequalities

$$0 \leq \sum_{j=l_i}^{i} \beta_j - \Delta_i \leq \beta_{l_i}, \quad i = 2, 3, ..., N,$$

the platoon with CACC satisfies the requirement (R3).

Communication structure design

- Only 2 communication links are necessary for each vehicle:

$$\hat{a}_{i,l_i} = \frac{\sum_{j=l_i}^{i} \beta_j - \Delta_i}{\beta} \quad \text{and} \quad \hat{a}_{i,l_i-1} = 1 - \hat{a}_{il_i}.$$

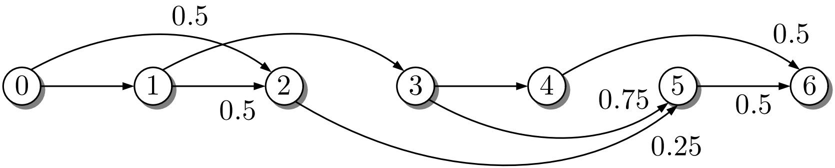
- The communication structure can be obtained in a systematic way:

$$0 \leq \sum_{j=l_i}^{i} \beta_j - \Delta_i \leq \beta_{l_i}, \quad i = 2, 3, ..., N.$$

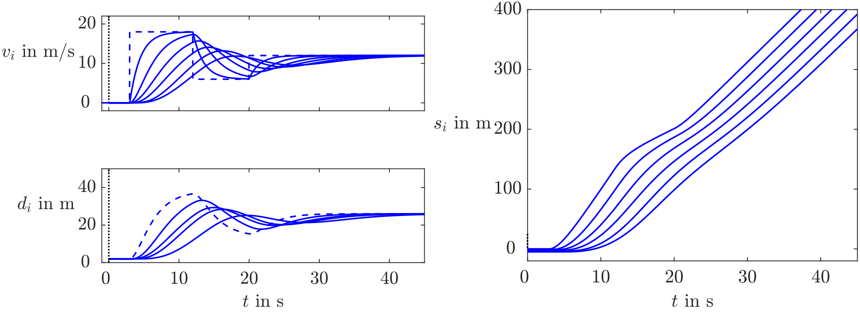
Communication structure design

Example: $\beta_i = 2$

$$\Delta_1 = 2, \quad \Delta_2 = 3, \quad \Delta_3 = 4, \quad \Delta_4 = 2, \quad \Delta_5 = 4.5, \quad \Delta_6 = 3$$



Communication structure design



Conclusions and outlook

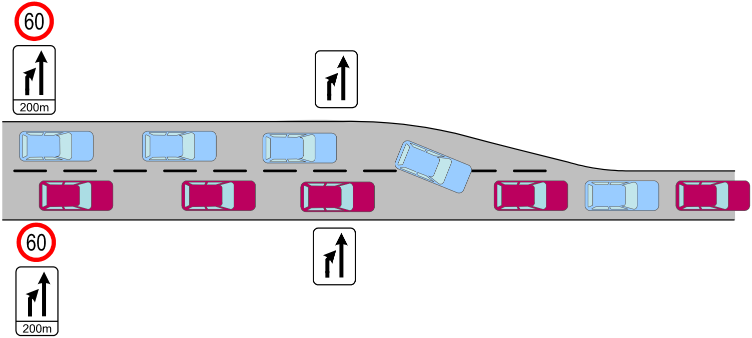
Conclusions and outlook

Summary of the design objectives:

$$\begin{array}{rcl} \textit{Synchronisation} & & \bar{G}_i(0) = 1 \\ \textit{Collision avoidance} & \Longleftrightarrow & \bar{f}_i(t) \geq 0 \\ \textit{Continuous progression} & & \bar{g}_i(t) \geq 0 \end{array}$$

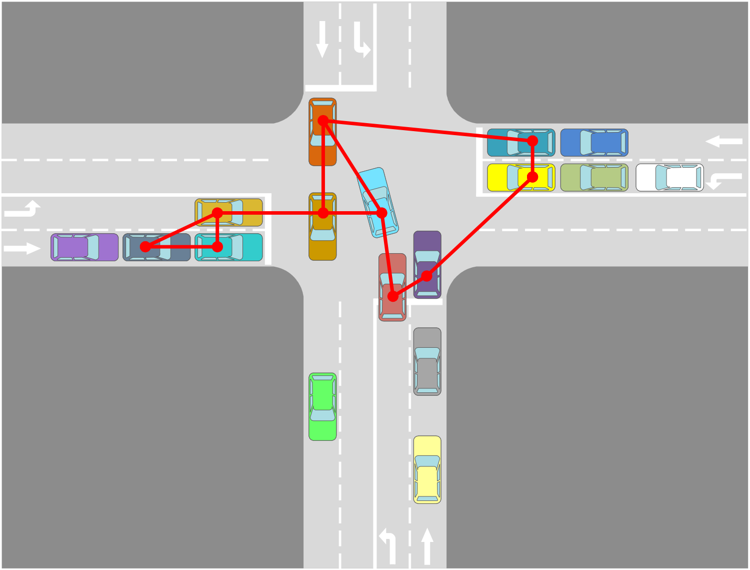
$$\textit{Time headway} \quad \Longleftrightarrow \quad \begin{array}{l} \Delta_i = \beta_i \ \textit{ with ACC} \\ \Delta_i > \beta_i \ \textit{ with CACC} \end{array}$$

Vehicle merging:

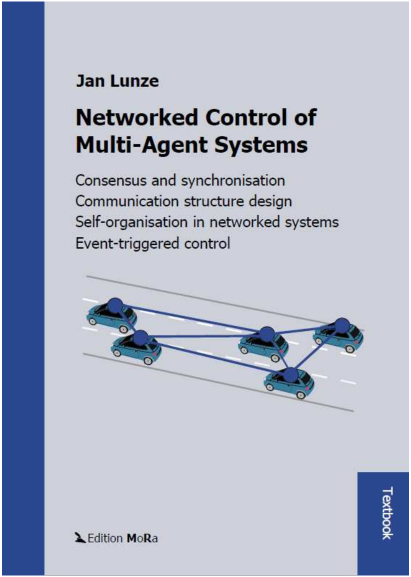


Conclusions and outlook

Automated crossroad management:



Conclusions and outlook



J. Lunze:
Networked Control of Multi-Agent Systems,
ISBN 9789463867139

Section 5.4:
Distance control of vehicle platoons