

# LSPM

# Question-Based Lecture

Week 2

# Week 2: Local (point) models

Questions?

Exercise from eLecture

# Exercise

Solve:

$$\frac{dy}{dt} = ky,$$

$$y(t_0) = y_i$$

$$y(t_0) = 0.02$$

$$k = -0.002$$

Calculate  $y(t_0 + 60)$

Use a time step of 60

# Analytical

$$y(t + \Delta t) = y(t_0) \cdot e^{kt}$$

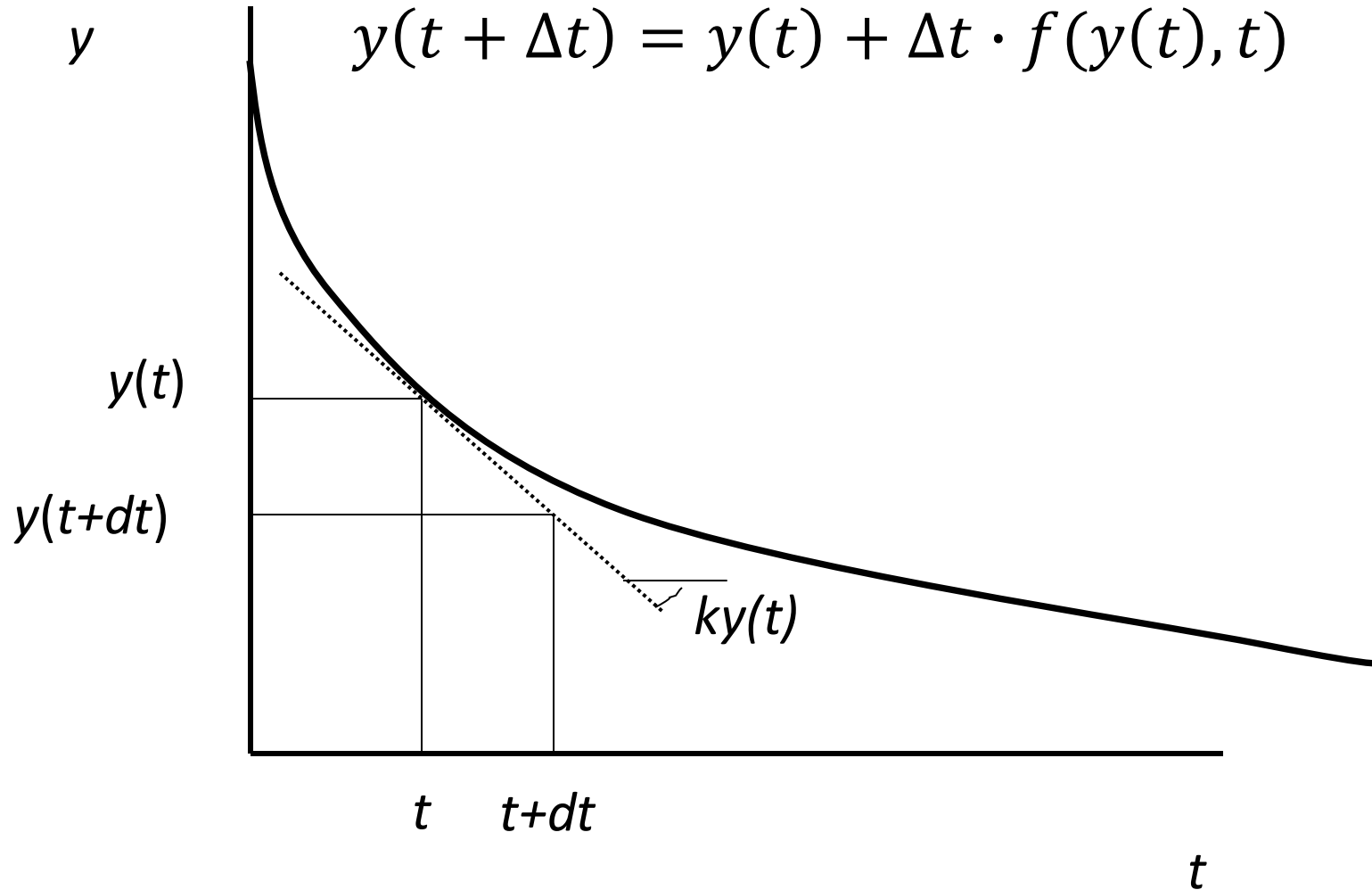
$$y(t + \Delta t) = 0.02 \cdot e^{-0.002 \cdot 60}$$

$$y(t + \Delta t) = 0.0177384087343$$

# Euler or Euler Cauchy

$$y(t + \Delta t) = y(t) + \Delta t \cdot f(y(t), t)$$

# Euler or Euler Cauchy



# Euler or Euler Cauchy

$$y(t + \Delta t) = y(t) + \Delta t \cdot f(y(t), t)$$

$$y(t + \Delta t) = y(t) + \Delta t \cdot ky$$

$$y(t + 60) = 0.02 + 60 \cdot -0.002 \cdot 0.02$$

$$y(t + 60) = 0.017600$$

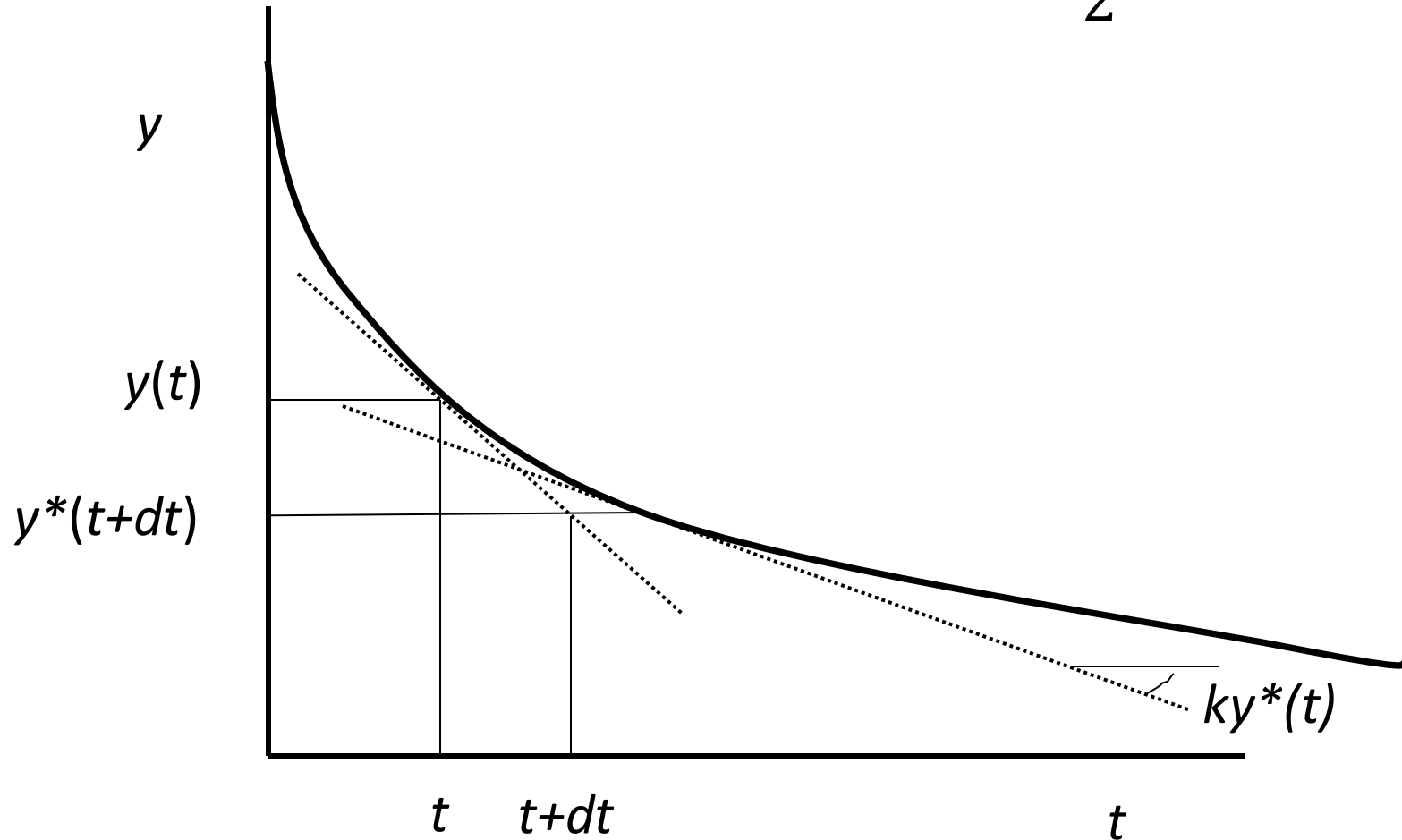
# Heun

$$y(t + \Delta t) = y(t) + \Delta t \cdot \frac{f(y(t), t) + f(y^*(t), t)}{2}$$



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$$y(t + \Delta t) = y(t) + \Delta t \cdot \frac{f(y(t), t) + f(y^*(t), t)}{2}$$

$$y(t + \Delta t) = y(t) + \Delta t \cdot \frac{ky + ky^*}{2}$$

$$y(t + 60) = 0.02 + 60 \cdot \frac{(-0.002 \cdot 0.02) + (-0.002 \cdot 0.0176)}{2}$$

$$y(t + 60) = 0.017744$$

# Runge-Kutta (1)

$$k_1 = \Delta t \cdot f(y(t), t)$$

$$k_2 = \Delta t \cdot f\left(y(t) + \frac{1}{2} k_1, t\right)$$

$$k_3 = \Delta t \cdot f\left(y(t) + \frac{1}{2} k_2, t\right)$$

$$k_4 = \Delta t \cdot f(y(t) + k_3, t)$$

$$y(t + \Delta t) = y(t) + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

# Runge-Kutta (1)

$$k_1 = 60 \cdot -0.002 \cdot 0.02$$

$$k_1 = -0.0024$$

$$k_2 = 60 \cdot -0.002 \cdot (0.02 + \frac{1}{2}(-0.0024))$$

$$k_2 = -0.002256$$

## Runge-Kutta (2)

$$k_3 = 60 \cdot -0.002 \cdot (0.02 + \frac{1}{2}(-0.002256))$$

$$k_3 = -0.00226464$$

$$k_4 = 60 \cdot -0.002 \cdot (0.02 + (-0.00226464))$$

$$k_4 = -0.0021282432$$

$$y(t + 60) = 0.017738413$$

# Comparison

Analytical	0.0177384
Euler	0.0176000
Heun	0.0177440
Runge-Kutta	0.0177384

# Week 3 – Spatial models

Section 2.3 from the study guide

- Reading materials
- eLectures
- Working group on Wednesday

# Working group (Section 2.3.5 in study guide)

To prepare for the session:

- Create a group in Blackboard, available at Course Content -> Working Groups (3 persons)
- Listen to the e-Lecture, study materials from reader
- Find an article from the literature on cellular automata or self organisation
- Prepare an 8 minute presentation (one per group)
- Upload presentation to cloud drive or use usb drive
- Details: refer to study guide



# Working Group Session

- Two sub-groups, two different rooms
- Schedule: announcement in Blackboard
- Each group gives 8 minute presentation + 4 minute discussion

# Week 2 – Computer Labs

- Map Algebra
- Dynamic Modelling

Finish by the end of next week

# Week 4 – Short Paper

Section 2.3.6 from the study guide

Group of 2 students

Due Feb 28th

You could start already now if you prefer to work ahead

# Personal project / case study

Groups can be 3 or 4 students

No effect on grades (I expect more work / output from larger groups)