Borrowing constraints and buy-to-let investment: An assignment model and empirical evidence*

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Abstract

We construct an assignment model in which buy-to-let investment appears as an equilibrium phenomenon as the result of borrowing constraints. We use this assignment model to predict which owner-occupied houses will be converted to rental houses, and test these predictions exploiting a series of contractions of the debt-service-to-income constraint in the Netherlands between 2012 and 2016. Using detailed micro-data and an event-study design, we find that tighter borrowing constraints causally explain a rise in the private rental sector. The series of contractions that ultimately affected 28 percent of households resulted in a cumulative increase of the private rental sector of 1.2 percentage points, which amounts to 21 percent of the overall increase in this sector by the end of 2016. These findings shed light on the role of borrowing constraints in explaining the rise of the private rental sector in many developed economies in the wake of the financial crisis.

Keywords: borrowing constraints, housing tenure, arbitrage, buy-to-let investment, assignment models.

JEL classifications: R31, R21, G51.

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1 Introduction

Private rental sectors have recently been growing in many developed economies.[1] The rise of the private rental sector has generally been perceived as coming at the expense of first-time buyers. Aspiring first-time buyers would experience more difficulty accessing the housing market, and thus miss out on the opportunity to acquire wealth via capital gains, self-amortizing mortgages and subsidies to homeownership. Homeownership rates would thus not fall because households prefer to rent, but because they find themselves borrowing constrained (Carozzi, 2019). Indeed, in the wake of the financial crisis, many countries have introduced borrowing constraints, coinciding with falling homeownership rates.

In this paper, we study whether tightening of constraints debt-service-to-income constraints, and specifically mortgage-payment-to-income constraints, can explain a rise of the private rental sector. Borrowing constraints have long been recognized to induce those whose choices are constrained, to rent rather than to own (Blickle and Brown, 2019; Kinghan et al., 2022; Linneman and Wachter, 1989)). However, identifying effects on impacted individuals is something very different from establishing effects on the aggregate housing market. The most important mechanism that prevents a simple aggregation is that borrowing constraints reduce house prices, potentially allowing other people to access the housing market (Rouwendal and Petrat, 2022; Vigdor, 2006). Consequently, the effect on the aggregate homeownership rate and the size of the private rental sector is not clear ex ante.

We develop an assignment model in which households that differ in income compete for a given number of houses that differ in quality. In this model, mortgage-payment-to-income constraints drive a wedge between households' willingness to pay and the price of housing. This wedge creates an arbitrage opportunity for buy-to-let (or keep-to-let) investors. Buy-to-let investors can buy houses at depressed prices and rent them out to borrowing-constrained households, because rental payments are not constrained by the ability to borrow. Buy-to-let investors thus allow households to spend more on housing than these households would be able to as homeowners. However, we show that increased spending on housing may not result in increased housing consumption, but may simply drive up housing costs.

The assignment model has clear predictions for which households in the income distribution will be affected by debt-service-to-income constraints, and, by virtue of the assignment, which owner-occupied houses will thus be converted to rental units and which will not. We exploit these predictions to discern the effects of a series of contractions of the debt-service-to-income constraint in the Nether-

[1] See Gabriel and Rosenthal (2015) for the U.S.; Byrne (2020) for the UK, Ireland and Spain; and Thiel and Zaunbrecher (2023) and below for the Netherlands.

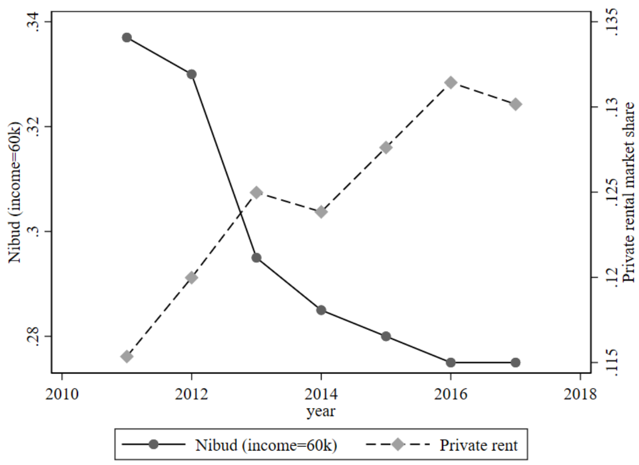

Figure 1: The change of NIBUD rule and private rental sector

Note: The National Institute for Budget Information (NIBUD) regulates the maximum share of a household's income that can be used for the mortgage payment. NIBUD rule differs based on income and interest rates. We show the NIBUD rule corresponding to the interest rate for that year when the income is 60K. We use year-level loans interest rate for house purchasing - pure new loans, and initial rate fixation is 5 to 10 years. See it on De Nederlandsche Bank (DNB). The share of private rental houses are calculated based on Statline.

lands between 2012 and 2016. In particular, we exploit that before August 2011 mortgage lenders could easily deviate from mortgage-payment-to-income constraints, which were expressed in the so-called NIBUD rule.[2] From August 2011 onwards, these guidelines became binding and such deviations were no longer possible as easily. Moreover, between 2012 and 2016, the NIBUD rule was tightened every consecutive year. Figure 1 shows the continued decline of the NIBUD rule between 2012 and 2016. The figure also shows a corresponding rise in the size of the private rental sector, consistent with our model.

To study the effect of tighter mortgage-payment-to-income constraints on the size of the private rental sector, we make use of the extensive register data available for the Netherlands. We consider the set of transactions that took place in the 19 months immediately preceding the date at which the constraint started to be enforced, i.e. all transactions between January 2010 and July 2011. Using the average interest rate in the month of the transaction and the income and mortgage loan of the buyer, we then determine which of these transactions would not have been possible if the mortgage-payment-to-income constraints present in the years between 2012 and 2016 would have prevailed. Consistent with the assignment model, we find that the transactions that would no longer have been possible are

[2]In the Netherlands debt-service-to-income guidelines are provided by NIBUD, an institute specialized in the study of consumer expenses. These guidelines are part of the Code of Conduct for mortgage underwriting used by the banks.

concentrated in one segment of the income distribution.[3]

Exploiting the predictions of the assignment model, we then define the transacted *houses* as belonging to treatment and control groups based on whether and when constraints became binding. The idea is that the houses bought in 2010 and the first half of 2011, would, if the stock of houses and the income distribution did not meaningfully change in between, in the equilibrium of later years be assigned to similar households. However, as the result of borrowing constraints, not all of these households would still be able to buy them, but some would have to rent them instead. The houses that the constrained households would otherwise have bought, provide profitable arbitrage opportunities for buy-to-let or keep-to-let investment. The assignment model also clarifies that such investment does not affect the likelihood of investment in the houses that unconstrained households would buy, despite the prediction that investment would also drive up price for these houses. In the terminology of the difference-in-difference literature, the stable unit treatment value assumption (SUTVA) holds, while the aggregate effect on the size of the rental sector can be identified.

Using a staggered difference-in-difference design, we find that owner-occupied houses are significantly more likely to be converted to rental housing from exactly the year onwards at which the corresponding 2010 or 2011 transaction would no longer have been possible, and not before. We find relatively homogeneous effects across event and calendar time groups: an initially owner-occupied house corresponding to a 'constrained transaction' has a 0.6 percentage points higher probability to become a rental unit.[4] Moreover, we find that the contractions affect buy-to-let investment and keep-to-let investment in equal measures. In addition, we find that the effects are entirely driven by houses that in 2010 and 2011 were bought by buyers younger than 36 years old, that is, by houses typically bought by first-time buyers.

Because the effects accumulate over time, the series of contractions that ultimately affected 28 percent of transactions resulted in a cumulative increase of the private rental sector of 1.2 percentage points, which amounts to 21 percent of the overall increase in this sector by the end of 2016. Because the event study coefficients show no sign of flattening off, we conjecture even larger cumulative effects after 2016 resulting from the contractions between 2012 and 2016 alone, but we cannot estimate these effects because the data suffer from a structural break after 2017.

Because houses that are bought in 2010 or 2011 and are already converted to rental units (although not necessarily transacted) by 2016 may constitute an unrep-

[3] Maybe surprisingly, and in contrast to Mabille (2022) for the U.S., we find no correlation between 'constrained transactions' and local house prices. Our assignment model therefore abstracts from a spatial dimension.

[4] The two-way fixed-effect estimator and the heterogeneity-robust estimator of Callaway and Sant'Anna (2021) result in an identical average treatment effect on the treated.

resentative sample of the housing stock, our most substantive robustness exercise considers a matched sample leveraging the registry of all houses in the Netherlands. For all houses transacted between January 2010 and July 2011, we find the house that is most similar in terms of housing characteristics within the same 6-digit postcode. We then define treated and control-group houses in the matched sample based the original sample, and find that our results continue to hold with only marginally smaller effect sizes. In addition, we perform two other robustness checks. First, we use a smaller sample to rule out anticipation effects from the announcement of the 2011 tightening in March 2011. Second, we use a larger sample that includes the transactions that were already constrained in 2010 or 2011, which we exclude in our baseline estimates.[5]

Our paper makes two contributions. Our first contribution is the introduction of borrowing constraints into a model that assigns heterogeneous households to a fixed housing stock that is also heterogeneous along one dimension. A given stock of housing is a realistic feature of housing markets in many urban areas that has important implications for the way the housing market functions. In particular, we show that borrowing constraints have very different effects when the housing stock is fixed compared to a model where the housing stock is malleable as in Muth (1969). Although the assumption of a fixed housing stock may be regarded as the opposite extreme of that of perfectly malleable housing, it is much closer to reality in urban housing markets, at least in the short or medium run.

We build on the model in Braid (1981), who analyzes only a rental market, while in our model the user cost replaces the rent in case housing is owner-occupied. We then introduce borrowing constraints into the model. The starting point of the analysis is a simple case in which there is only one group of households, all households face the same constraint and possess no wealth. We show that in this case the borrowing constraint has no impact on the allocation of households to housing – that is, each household lives in the same house as in the market without the borrowing constraint – and that user costs will be lower if the constraint is binding for some households with a lower income. In particular, if the households with the lowest incomes that are participating in the market experience a binding constraint, all households in the market will realize lower user cost of housing. This implies that the borrowing constraint will benefit some or even all households and harms none of them. This conclusion contrasts with the alternative (Muth) model with malleable housing in which borrowing constraints may harm some households and benefit no one. However, a paradoxical feature of the model is that constrained households are no longer in equilibrium, but have a marginal

[5]In the Appendix, we also consider a version of the assignment model in which households differ in their preference for housing relative to other consumption. We show that such preference heterogeneity results in an attenuation bias. Consequently, when households differ in their taste for housing, we would underestimate the effect of borrowing constraints on the size of the private rental sector.

willingness to pay for housing that exceeds its market price. They are thus willing to pay for relaxation of the constraint, even though this would imply that in the resulting equilibrium their welfare is lower.

These conclusions change if borrowing constraints differ among households, for instance because some have wealth that may be invested in owner-occupied housing to circumvent or soften the borrowing constraint. The reason is that in this more general situation the assignment rule may change due to the presence of borrowing constraints. That is, two households that are otherwise similar will be assigned to houses of different qualities if they face different borrowing constraints. As a consequence constrained households may now be assigned to houses of lower quality compared to the equilibrium without constraints, while unconstrained households may be assigned to housing of higher quality. The lower housing consumption of constrained households has a negative impact on their welfare. Still, it can be shown that in the more general situation with heterogeneous constraints or tastes, the borrowing constraints decrease the average housing price as long as they are binding for some households and do not increase the price for any level of housing quality. However, the welfare effects for constrained households are now ambiguous.

Finally, we show that binding borrowing constraints imply the possibility for arbitrage by buy-to-let investors. The reason is that the constrained households' willingness to pay for housing exceeds the marginal price it faces. Hence they are willing to pay more for higher quality housing than they are currently allowed to do. Buy-to-let investors can solve their problem by giving them access to the higher quality housing, albeit only as renters. These agents can buy the higher quality houses at a price that exceeds the market level and let them to (until then) constrained households at their marginal willingness to pay. With free entry of such investors and negligible intermediation costs, the market will move towards an equilibrium in which housing consumption and expenditure are the same as in the absence of binding borrowing constraints, but where all households experiencing binding constraints on the owner-occupied market have become tenants. The restrictions on housing consumption have effectively been removed, but so have the lower housing prices implied by the borrowing constraints.

Our second contribution is about the empirical effect of borrowing constraints on the size of the private rental sector. Our empirical strategy that focuses on houses rather than people allows us show that debt-service-to-income constraints have a significant effect on the aggregate size of the private rental sector. Indeed, the assignment model allows us to identify the houses that constrained and unconstrained households would want to live in without observing any characteristics of these households. In addition, it shows that spillovers do not contaminate the effect that we can isolate from comparing the houses that constrained and unconstrained households would buy. Moreover, and complementary to Carozzi (2019) and Hanson (2022) who consider only either keep-to-let or buy-to-let investment, respectively, we find that keep-to-let and buy-to-let investment are equally affected by the tightening of debt-service-to-income constraints.

Related Literature. Assignment models have been applied for a long time to allocation of workers over jobs, see Sattinger (1993) for a review. The first application of such a model to the housing market, as far as we know, is Braid (1981), which is the starting point of the model of the present paper. Braid studies a rental market in which houses differ in one-dimensional quality and households all have the same tastes, but may differ in income. The assignment model of Landvoigt et al. (2015) uses a generalized multi-period version in which households face a cash-on-hand constraint rather than a single period budget constraint. They expand their model to a more quantitative version in which heterogeneous households maximize intertemporal utility subject to an intertemporal budget constraint as well as a down-payment borrowing constraint for housing, which they then take to the data. However, they do not formally discuss how the heterogeneity of households and the presence of borrowing constraints affects the allocation of households over housing in equilibrium. [6]

The setup of the assignment model in Määttänen and Terviö (2014) and Määttänen and Terviö (2021), is similar to that in Landvoigt et al. (2015) in that they consider households facing a single-period constraint in which they have to distribute their wealth over owner-occupied housing and a composite consumption good. They start from an initial equilibrium allocation of the household population over the housing stock and then study the impact of change in the income distribution. In general such a change implies that some households move from one house to another and use the revenues from selling the initial house to help finance the next one. The analysis focuses on the impact of mean- and order-preserving spreads of the distribution which have the special feature of leaving each household's rank in the income distribution unchanged. In the assignment model such changes leave the allocation of households over houses unchanged - hence there is no trade in response to it - and only results in a change in house prices. That is, after the change in the income distribution for each household the preferred house is the same as before, but its price is now different. The authors show that if all incomes increase, all house prices will also increase. They note that Braid (1981)'s model is essentially a version of their model in which the price of the initial house is absent from the wealth constraint.

Few households have enough wealth to purchase a house without having to borrow. The availability of mortgage loans is therefore an important determinant of the homeownership rate. Banks usually impose restrictions on the size

[6] See also Piazzesi and Schneider (2016) where they set up a general framework for modelling the housing market.

of mortgage loans in order to limit the probability of default. They typically look at the loan-to-value ratio (LTV) and the ratio between the mortgage payments and household income at the time of purchase. An upper bound on the LTV implies a lower bound on the downpayment, amount of equity that has to be invested by the household. This constraint implies a hurdle for would-be homeowners that affects tenure choice (see Brueckner (1986)). With additional security, provided, for instance, by mortgage insurance, banks are willing to relax the downpayment constraint. The Netherlands is a relevant case, because there such insurance is offered under attractive conditions by a semi-government agency (NHG). With NHG insurance, households are able to finance the purchase of a house completely by a mortgage loan at a somewhat lower interest rate. However, to mitigate default risk, the mortgage payment-to-income ratio (PTI) is used as a yardstick that restricts the size of the loan that is available. Hence in the Netherlands the LTV constraint is of negligible importance for households qualifying for NHG, while the PTI constraint is binding for many. In the US and the UK, where mortgage insurance is usually expensive, the LTV or downpayment constraint is often the main hurdle. An important difference between the two is that the downpayment constraint forces many households to postpone ownership until enough has been saved to pay the downpayment, whereas the PTI constraint allows immediate access to homeownership albeit only for modest housing if income is relatively low.

Linneman and Wachter (1989) is one of the first empirical studies of the impact of borrowing constraints on tenure choice. The authors assume that the LTV and PTI constraints are simultaneously present. Simple elaboration of these constraints leads to maximum feasible values of the purchase prices of houses which depend on income and wealth. They estimate unrestricted housing demand by concentrating on households for which these constraints are not binding and use the estimation results to compute the gap between the price of desired and actual housing for the other households. This allows them to quantify the impact of both constraints. Interestingly, they go on to consider the impact of these constraints on tenure choice by estimating a discrete choice model and find that the frequency of renting is much higher among households that are unable to realize their desired housing consumption under owner-occupation. This suggests that such households have better possibilities of realizing their desired housing consumption on the rental market. One possible reason for this is that rents are lower than the user cost of comparable owner-occupied housing. From the point of view of economic theory this is somewhat implausible as one would expect arbitrage between both types of tenure.[7] A second explanation, in line with our analysis of borrowing constraints, holds that the rental market allows these households to realize, or at least come closer to, their desired housing consumption because there are no con-

[7] We should note that arbitrage arguments are less forceful on the housing market, which is in many respects imperfect, than on many asset markets, see Glaeser and Gyourko (2007).

straints on rent-to-income ratios. Duca et al. (2011) indeed show that borrowing constraints help to explain price-to-rent ratios.

Later literature on the impact of borrowing constraints does more carefully consider endogeneity and selection issues, which turn out to be non-trivial. For instance, Blickle and Brown (2019) study the impact of intra-family wealth transfers in Switzerland, a country with a large and diversified rental housing market. They find that such transfers lead to a sizable increase in the propensity to own but do not induce receivers to move to larger houses or more attractive locations. This suggest that the main effect of such transfers is the relaxation of borrowing constraints. Kinghan et al. (2022) exploit a policy reform in Ireland to provide direct empirical evidence that macroprudential LTV regulations reduce LTV ratios of affected borrowers. Both papers obtain causal identification by focusing on the effects for affected households, but as a result cannot address the effect of borrowing constraints on the homeownership rate or buy-to-let investment.

The most important mechanism that prevents a simple aggregation of individual effects is that borrowing constraints reduce house prices, as cross-country studies such as Akinci and Olmstead-Rumsey (2018) show. Lower house prices potentially allow other, unconstrained households to access the housing market. Vigdor (2006) is one of the few papers that considers such spillover effects of binding borrowing constraints. He develops a theoretical model that predicts that such constraints can depress house prices and points out that this implies that the relaxation of such constraints may have detrimental welfare effects for some households. Using veteran status as an instrument for better access to credit, he provides empirical evidence that supports this hypothesis. Using the same reform as Kinghan et al. (2022), Acharya et al. (2022) show that borrowing constraints result in spillovers in mortgage credit from low-income to high-income borrowers and from urban to rural areas. As a result, house price growth fell more for smaller properties in hot housing markets. None of these papers considers buy-to-let investment.

In an attempt to avoid econometric complications, Fuster and Zafar (2021) conduct a stated choice experiment in which respondents were asked their willingness to pay for housing, in case they would have to move to a different city with similar house prices on short notice. They find that current homeowners tend to choose similar houses, while current renters, who also have to imagine that they purchase a house, often opt for smaller houses. Lowering the required downpayment has a much larger impact on the willingness to pay of renters. If households are allowed to choose the downpayment, renters usually take lower values than owners. And if the respondents receive a large cash windfall, renters would spend a larger fraction of it on the downpayment. All these results point to the importance of borrowing constraints in tenure choice, but cannot straightforwardly be aggregated to equilibrium responses.

Some papers have linked the decrease in homeownership that occurred after the Global Financial Crisis to tighter credit constraints, including Carozzi (2019) for the UK and Mabille (2022) for the USA. Carozzi (2019) argues that younger households could no longer afford to buy a house after downpayment constraints increased, while the house they would otherwise have occupied were kept by the owners to let them to the constrained households. He shows that transaction volumes decreased much more substantially in the lower part of the market, is is suggested by this point of view. Mabille (2022) develops a macro-spatial model in which a general contraction in credit supply has heterogeneous local implications due to differences in house prices and incomes. Hanson (2022) studies the substantial increase in activity of institutional investors on the housing markets of U.S. metropolitan areas. He develops a theoretical model in which households and institutional investors, who both maximize mean-variance utility functions, compete for housing. The households can also choose to rent, while the institutional investors have the possibility to invest in a different asset. He shows that all housing will be owner-occupied when the operator costs of institutional investors are higher than the marginal disutility of housing risk of households, if borrowing constraints are not important. However, when such restrictions are sufficiently tight, this may create opportunities for (institutional) buy-to-let investors to enter the market. The paper argues that this happened in the U.S. since 2012.

The analysis of Hanson (2022) has important similarities with that of the present paper, the most important being that investors enter in those parts of the markets that are most severely hit by the tightening of borrowing constraints. The interaction between the share of low-income households and borrowing constraints is used as an instrument for investor excess returns, which are then shown to have a significant impact on institutional investor activity. House prices and rents increase more in markets with the most intense presence of institutional investors. Hanson (2022) concludes that most of the excess returns realized by the institutional investors are due to appreciation rather than rental yields, which appear to be stable over time within geographical units. Our analysis differs from Hanson (2022) in the theoretical model, but also by including small buy-to-let investors (e.g. private persons) and keep-to-let, and by concentrating the empirical analysis on individual houses rather than aggregate spatial entities.

Thiel and Zaunbrecher (2023) is close to our paper in that they also study the growth of the private rental sector in the Netherlands in recent years in relation to borrowing constraints. They develop a structural model of the Dutch housing market in which tightened borrowing constraints cause a switch of households and houses from the owner-occupied to the private rental sector. Their primary interest in in the probability of young households to enter owner-occupation. They exploit a difference between the tightening of the borrowing restrictions for single and dual earners (closely related to one and multiple person households, respectively) to estimate that the probability to rent increases by 10.7%points if the borrowing capacity decreases by 1%. In the structural model heterogeneity among buy-to-let investors plays an important role. When the rent-to-price ratio increases less efficient investors can enter the market and the share of the private rental sector increases. They conclude that 20% of the increase in the private rental sector is due to the tightened borrowing constraints.

2 Institutional background

This section contains a brief discussion of recent developments in the Dutch housing market that triggered the analysis of the present paper. However, remember that buy-to-let investment occurs in many other housing markets and is possibly also related there to the impact of borrowing constraints.

In the second half of the 20^{th} century the Dutch housing market was characterized by a large social housing sector and a growing share of owner-occupied housing. Since social housing was subsidized and full mortgage deductibility lowered the user costs of owner-occupiers, there was little room left for the private rental sector. As a consequence, this part of the housing market, which was the most important one until the 1950s, was continuously shrinking over time.[8] A reasonable prediction in the 1990s would have been that the provision of short term housing for expats would be its only market niche in the long run. However, things have changed since then.

In the Netherlands house prices have been almost continuously increasing since the mid-1980s and, after a dip associated with the Global Financial Crisis and the euro crisis, reached unprecedented values. Since 2000, mortgage lenders have – under pressure of the government and consumers authorities, introduced a Code of Conduct that was tightened in 2011. Although the principle of the Code is 'comply or explain' its rules only allow for exceptions in specific cases since the tightening took place in 2011 and in more recent years some of its rules have been transformed into official government regulations that are strictly binding for these lenders. The most important aspect of the code of conduct is that home buyers should not be allowed to get a loan implying a mortgage payment to income ratio that threatens their possibility of realizing other necessary consumer expenses[9]. With high demand pressure and strongly increasing prices, the implied mortgage qualification constraint becomes binding for a growing share of households. As a result there emerged a concern for the position of (what are referred to as) medium income households, that is households that are not eligible for social housing, but

[8]The slow disappearance of the sector despite its lack of profitability was due in large part to extensive protection of the rights of existing tenants. They could not be forced to depart, while rent control ensured the prolonged attractiveness of continuation of the contract.

[9]In the Netherlands guidelines are provided by NIBUD, an institute specialized in the study of consumer expenses. These guidelines are part of the Code of Conduct for mortgage underwriting used by the banks.

whose borrowing capacity is not sufficient to allow them access to (a reasonable part of) the owner-occupied markets in large cities, especially Amsterdam. For such households, rental housing in the private sector is an obvious alternative.

It is perhaps not a coincidence that since the early 2000s the room for the private rental sector has increased. It was determined that the determination of rents that exceeded a particular threshold could be left to market forces. That is, rent control no longer referred to such housing. Moreover, tenant protection was reduced in this part of the market. Temporary contracts became common. Activities of private investors in rental housing, also from those of foreign origin, were welcomed by the national government. However, not only professional investors entered the market, many private individuals possessing some wealth decided to invest in rental housing, expecting a stable flow of revenues and on top of that the possibility to realize a handsome indirect return via continued price increases[10].

Indeed the private rental sector in the Netherlands has shown a remarkable revival in recent years. Van der Harst and de Vries (2019) show that the turning point occurred in 2012, one year after the tightening of the Code of Conduct. In the municipality of Amsterdam, where social housing dominated the market until the 1990s and owner-occupied housing reached a share of 30% only in the 2010, the growth of the private rental sector started to reduce the share of owner-occupied housing again in the period 2017-2019.[11] To illustrate, in 2019 47% of the recent movers in Amsterdam entered a house in the private rental sector.[12] The growth continued in more recent years. In 2021 more than half of the housing moves in Amsterdam ended in a private rental house.[13] This rapid increase in private rental housing caused concern among especially medium-income would-be homebuyers that they were outbid from market segments that until then were accessible to them.

3 Two models of housing market allocation

In this section we develop an analysis of tenure choice on a housing market with borrowing constraints. In subsection 3.1 the general setup of the housing market will be introduced. Subsection 3.2 discusses Muth (1960)'s model of the housing market which considers housing essentially as a conventional (in microeconomic textbooks) homogeneous commodity. This model is standard in the economic literature. In subsection 3.3 we take into account that housing is durable and that its stock is given, at least in the short run. The model discussed there is essentially

[10] Lankhuizen and Rouwendal (2020) report gross direct returns of 6% in the Amsterdam metropolitan area and somewhat lower values in the municipality of Amsterdam.

[11] From 32.4% in 2017 to 30.8%. See Berkers and Dignum (2020).

[12] Berkers and Dignum (2020)

[13] 54.6% while private rental housing now covers 30.5% of the housing stock, a larger share than the owner-occupied sector.

that of Braid (1981), although that study considers a rental market, while here the model is applied to owner-occupied housing.

3.1 The setup

We consider a market with a population of households who all have identical preferences over housing q and other consumption c that can be described by the utility function

$$u = u(q, c). \tag{1}$$

The utility function u is increasing in both arguments, quasi-concave and twice differentiable. Housing and other consumption are both normal goods.[14]

Households maximize utility subject to a budget constraint. The budget will be referred to as income, but it should really be interpreted as the amount of money the household is willing to spend on consumption (of housing and other goods) in the period we consider.[15] Households differ in incomes. The distribution of income is given by the strictly increasing and continuously differentiable function $F(y)$, which has positive support on the interval $[y^{min}, y^{max}]$. We denote the density function by $f(y)$. The total number of households equals B, where $B = F(y^{max})$.

Households maximize utility subject to the budget constraint

$$c + p(q) = y, \tag{2}$$

where $p(q)$ denotes the user cost of housing and y is the available budget. Think of the user cost $p(q)$ as a function of the sales price of the house. More specifically it is the product of the market value and the opportunity cost of the capital invested in the house, plus costs of maintenance and taxes minus the expected increase in value of the house. A common specification in the literature is: $p(q) = \gamma P(q) - E(\Delta P/(1+r))$, where $P(q)$ denotes the sales (transaction) price, γ reflects the various cost items (maintenance, insurance, taxes), ΔP is the (expected) change in the price of the house and r the rate of discount. In what follows, we focus on the user costs that equilibrate the market in the current period, without paying attention to its composition.[16] Note that it is not assumed that the user cost $p(q)$ (or

[14] A population with identical tastes is clearly restrictive. In Appendix A.2 we discuss an extension of the model to situations with households differing in tastes.

[15] This budget can be thought of as being determined in an intertemporal utility maximizing framework, jointly with savings. See, for instance, Deaton and Muellbauer (1980). The cash-on-hand constraint used by Landvoigt et al. (2015) and Piazzesi and Schneider (2016) is a variant of the intertemporal budget constraint. In this framework uncertainty about future house prices and the expected wealth effects of housing transactions can also be taken into account.

[16] Note that the first part of the user cost, $\gamma P(q)$, are mainly to out-of-pocket expenses like taxes, mortgage interest payments and maintenance, whereas expected price changes are not. The implication is that monetary outlays on housing can exceed the user cost when house prices are expected to increase. Borrowing restrictions, such as the mortgage qualification constraint discussed later in this paper refer to monetary expenses. Note also that there is in general not a one-to-one correspondence between users cost $p(q)$ and transaction price $P(q)$.

the transaction price $P(q)$) is linear in the amount of housing. That is, the marginal price of housing $\pi(q) = \partial p/\partial q$ may depend on the quantity of housing services consumed.[17]

Households have an outside option, for instance renting social housing or living in temporary housing. We include this in the model by assuming there exists a combination of housing consumption q^* and user cost p^* that is available for every household. Hence the reservation utility is $u^*(y) = u(q^*, y - p^*)$ and households will only participate in the (primary) housing market studied here if this offers them a higher utility than the outside option.

3.2 Malleable housing: Muth's model

In this subsection we disregard the durability of housing and assume that houses are supplied on a market with perfect competition according to a cost function $C(q)$. We focus on the case in which this function is linear in the housing quality:[18]

$$C(q) = bq. \tag{3}$$

The price of a house with quality q is then equal to

$$p(q) = bq, \tag{4}$$

which implies that the budget constraint is linear in housing consumption q. Maximization of utility subject to this constraint leads to a housing demand function

$$q = h(q, y). \tag{5}$$

We assume throughout that housing is a normal good: $\partial h/\partial y > 0$. In equilibrium all households are able to realize their demand for housing, which means that the distribution of housing reflects the distribution of income. More precisely, for any pair of households the one with the highest income will always consume more housing services.

Households will participate in the market if indirect utility $v(b, y) = u(h(b, y), y - bh(b, y))$ exceeds reservation utility u^*. The critical income y^c that is needed to participate in the housing market is defined implicitly by the equality: $v(b, y^c) = u^*(y^c)$. If this income level is smaller than y^{min} we take y^{min} as the critical income level.

For later reference, we note some other aspects of the equilibrium. All households with an income at least equal to y^c occupy a house. The household with the

[17]In later subsections we will encounter situations in which the function $p(q)$ is not differentiable at some points.

[18]This implies no loss of generality. If the cost is increasing in housing quality produced, we can define quality so that the amount of housing services is proportional to the production cost.

critical income inhabits a house of quality $q^{min} = h(b, y^c)$. Denoting the distribution of housing as $G(q)$, we must then have

$$G(h(b,y)) = F(y) - F(y^c). \tag{6}$$

Housing consumption and income are thus aligned in the sense that the ordering of households on the basis of their consumption of regular housing is identical to their ordering in the basis of their incomes. If both distributions are differentiable and we denote their derivatives as $f(y) = \partial F/\partial y$ and $g(q) = \partial G/\partial q$, we must in equilibrium have $g(q)dq = f(y)dy$, and since all households are on their demand curves, this implies that the slope of the Engel curve for housing is

$$\partial q/\partial y = f(y)/g(q). \tag{7}$$

An important advantage of Muth's model is that it can easily deal with heterogeneity of tastes, borrowing constraints and buy-to-let investors as will now be discussed. The reason is that the housing stock adjusts easily to the implied changes in demand. If households differ in preferences, they can simply order the amount of housing that is suitable for them at the prevailing price. Those with more intense preferences for housing simply order a larger house. To make things concrete, if there are G groups of households $g = 1...G$ and households in each group have identical tastes, but may differ in income, we can describe the market for each group in the same way as was done above. We get G market segments that have the same properties as the single one analyzed above.

Borrowing constraints impose a ceiling on housing demand. To analyze their impact we start by considering a simple situation in which housing expenditure is limited to a particular share μ of household income and there is no rental housing available. In this case, households experiencing a binding constraint must reduce their housing demand to the level that is compatible with the constraint. That is, borrowing constraints reduce housing consumption but have no impact on the price of housing. This remains true if we consider a more general model in which households may have some wealth available to invest in housing. They can use this wealth to relax the borrowing constraint and if their wealth is large enough, avoid it completely. Again, the constraints do not have any impact on housing prices, but the average amount of housing consumption is reduced as long as the constraint is binding for some households. No household will ever increase its housing consumption because other households are experiencing binding restrictions.

However, the presence of binding borrowing constraints provides arbitrage opportunities for investors in rental housing. By offering rental housing of the qualities that are preferred by restricted households, but out of reach for them due to the borrowing constraints, they can make a profit. The reason is that the constrained households have a marginal willingness to pay for housing that exceeds the marginal production cost b. If enough investment in rental housing is realized, and the additional cost of making the housing available as rental housing is negligible, the market will return to the equilibrium without borrowing constraints as far as housing consumption and expenditure are concerned, but all households experiencing binding borrowing constraints in the owner-occupied market have shifted to the private rental sector.[19]

3.3 Durable housing

In the previous subsection we have assumed that all houses are created in the period considered. In reality most of the housing dates back from the past and only a small percentage is constructed per period. To capture this, we will now consider the situation in which housing supply is completely fixed. That is, the housing stock – the distribution of houses, each with a given number of housing services – is given. This setup is similar to Braid (1981) who considered a rental market. Here we assume the market refers to the market for owner-occupied housing in which the user cost occupies the role of rent.

As in the previous section, houses are available in a continuum of varieties and the distribution function of the quality of housing is still denoted $G(q)$. However, in contrast with the previous section, G is now given. It is assumed to have support on an interval $[q^{min}, q^{max}]$ and to be strictly increasing and continuously differentiable. The number of houses is $S, S = G(q^{max})$. We assume that the number of households is at least equal to the number of houses: $B \geq S$. The critical value y^c is now thus determined by the condition that only S households can own a house:

$$B - F(y^c) = S. \tag{8}$$

In equilibrium, user costs will be such that households with incomes below the critical value y^c will choose the outside option. Households with higher incomes compete with each other for the available housing instead of ordering housing construction. The following lemma characterizes the resulting assignment of houses to households.

Lemma 1 (Assignment rule)**.** *In equilibrium, the assignment follows the continuous function*

$$y(q) = F^{-1}\left(F\left(y^c\right) + G(q)\right). \tag{9}$$

[19]There exists a version of Muth's model in which the total number of housing services in a market is fixed, while its distribution over housing units is flexible via linear conversion. See, for instance, section 10 of Piazzesi and Schneider (2016) for discussion. In this model the price per unit of housing services is still equal for housing of all qualities, but this price is longer determined by construction costs.

Proof. First, it is easy to see that $p(q)$ must be increasing and continuous in q.[20] That is, the user cost of housing is in the present framework a possibly nonlinear, but always increasing, function of the housing services offered by a house.[21] Second, since housing is a normal good, a household with a higher income will in equilibrium consume more housing than a household with a lower income.[22] These two observations imply the assignment rule. This function is continuous because (9) implies that the change in income dy/dq is

$$\frac{dy}{dq} = \frac{g(q)}{f(y)}, \tag{10}$$

where $g(q)$ and $f(y)$ are continuous and positive. □

In equilibrium the ranking of households on the basis of housing consumption thus corresponds to the ranking of households on the basis of income. However, in this setting there is no reason to suppose that housing expenditure in equilibrium will be proportional to the quality of housing services consumed, q. The main purpose of this section is to find the equilibrium user cost function $p(q)$ on this market.

The user cost function can be derived as follows. The household with the critical income must be indifferent between housing of the lowest quality and the outside option:

$$u(q^{min}, y^c - p(q^{min}) = u^*(y^c). \tag{11}$$

This equation pins down the value $p(q^{min})$, the user cost of housing of the lowest quality.

The following lemma characterizes the equilibrium user cost function.

Lemma 2 (Equilibrium prices)**.** *In equilibrium, the user cost function is given by the first-order differential equation*

$$\pi(q) = \frac{\partial p}{\partial q} = M(q, y(q) - p(q)), \tag{12}$$

with initial condition $p(q^{min})$ from (11), in which $M(q,c) = (\partial u/\partial q)/(\partial u/\partial c)$, and in which $y(q)$ follows from the assignment rule in (9).

[20] Suppose a house with a better quality is less expensive than that of a lower quality. Then there will be no household choosing the lower quality house. Suppose there is a discontinuity in the house price function. Then the marginal price of housing is infinitely high at the point of the discontinuity, which means that there will be no demand for housing with quality just above the point of discontinuity.

[21] The importance of the non-linearity of the housing price function from an empirical as well as a theoretical perspective was emphasized in Landvoigt et al. (2015) and Piazzesi and Schneider (2016)

[22] Consider a pair of households with different incomes and assume that the one with the highest income consumes less housing than the one with the lowest income. Then the high-income household is able to compensate the low-income household for moving to the lower quality house and still reach a higher utility. Hence they can engage in a transaction that is beneficial to both, which shows that that initial situation is incompatible with equilibrium. See Appendix A.1.1 for details.

Proof. The outside option pins down the user cost at q^{min} and y^c. The user cost at higher qualities is determined by the requirement that the slope of the price function, the marginal price of housing $\pi(q)$, must be equal to the marginal rate of substitution $M(q,c)$. $p(q)$ is differentiable because $M(q,c)$, $p(q)$, and $y(q)$ are continuous in their arguments. □

We can thus trace out the housing price function by making use of the equilibrium condition and the assignment rule. That is, starting from the critical income y^c, the lowest housing quality q^{min}, and its price $p(q^{min})$, (9) determines the income associated with each housing quality and then (12) determines the equilibrium price for each housing quality.

Let us now consider the special case in which the equilibrium price function is linear, as in Muth's model. This happens if the marginal price of housing is constant: $\partial\pi(q)/\partial q = 0$. It requires the marginal willingness to pay for housing to be constant as well. It can be shown (see Appendix A.1.2) that this implies that we must have

$$\frac{\partial M/\partial q}{\partial M/\partial(y-p)} = \pi - \frac{g(q)}{f(y)}, \tag{13}$$

for all possible combinations of q and $y - p(q)$. This condition requires that the distributions of income and housing are aligned to the marginal willingness to pay in a very specific way. In the model with malleable housing $g(q)$ adjusts so that (13) is always satisfied. However, with $g(q)$ fixed, we lose this flexibility.[23]

One way of interpreting this is that with a given stock of housing there needs to be equality of supply and demand for every housing quality. That is, instead of a single market for housing services that can be used to construct any desired quality of housing, we now have a continuum of markets for housing with given quality.[24] Equilibrium then requires a separate price for each sub-market. This is the reason that we cannot expect the price function $p(q)$ to be linear when the housing stock is given.[25]

3.4 Effects of an income increase compared

To appreciate the difference between Muth's model and the assignment model, it is useful to briefly discuss some comparative statics. What happens if all incomes increase by the same percentage so that the ranking of the households on the basis of their income remains unchanged? In Muth's housing market all households

[23]For instance, with Cobb-Douglas utility $u(q,c) = q^\alpha c^\beta$ the left-hand side of (13) is equal to $(y - p(q))/q$ and one may choose $p(q)$ for every q so that (13) is satisfied, but the first derivative of this price function will only by coincidence be equal to the constant π, implying a contradiction.

[24]Compare the plural in the title of Landvoigt et al. (2015).

[25]Our assumptions imply that the second order condition for utility maximization is satisfied with a linear budget constraint. With a nonlinear housing price function the budget constraint is also nonlinear. Appendix A.1.3 shows that the second-order condition remains satisfied.

increase their housing consumption, while the price per unit of housing remains unchanged. Moreover, some households formerly choosing the outside option will now order an owner-occupied house, thereby increasing the size of the housing stock.

Now consider the assignment model. Here the size of the housing stock remains unchanged. Since the critical income increases, the price of housing of the lowest quality increases.[26] All households occupying a house in the given stock in the initial situation want to increase their housing consumption and bid up the price of housing. However, since the stock of housing is given and their position in the income distribution did not change, they end up in the same house, while paying a higher user cost. The reason is that housing of minimum quality has become more expensive, while on top of that the marginal willingness to pay for housing has increased for every value of housing consumption. Because of the latter effect, the price (user cost) increase will itself be increasing in q. This contrasts sharply in the market with malleable housing, but it seems to describe rather well what happens in many urban housing markets when household incomes increase.

4 Borrowing constraints

The next step is the introduction of borrowing constraints. We start in subsection 4.1 by considering a uniform restriction on the share of user cost in household income and generalize this in subsection 4.2 to an arbitrary distribution of maximum purchase prices or user costs that may depend on household income. Throughout this section we assume all households have the same preferences.

4.1 A mortgage qualification constraint

The analysis thus far has assumed that households are not restricted in their choice behavior, except by the budget constraint. Many households need a mortgage loan to finance the purchase of their house and mortgage payments are an important element of their user cost. Lenders usually impose restrictions on the size of these loans. As discussed in the introduction, in the Netherlands the ratio of mortgage payment to income is the most important indicator used by the lenders, and we will now consider the implications of such a constraint. In this subsection we do so in a simple way: we impose that user cost can at most be equal to a fraction μ of income for all households:[27]

$$p(q) \leq \mu y. \tag{14}$$

We refer to this restriction as the mortgage qualification constraint.

[26] The reason is that the normality of housing implies that the willingness to pay for an increase in housing consumption from q^* to q^{min} increases.

[27] We will later consider cases in which households can face different constraints.

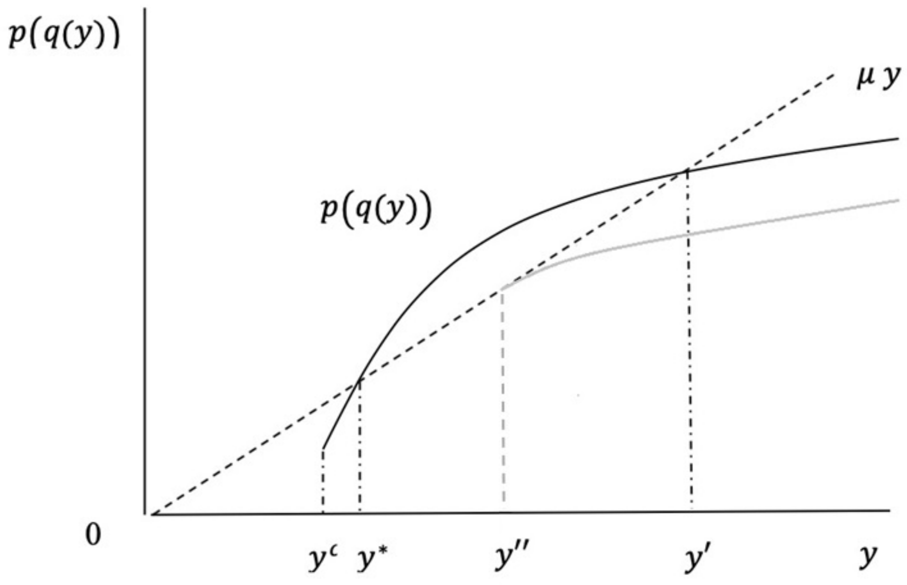

Figure 2: The mortgage qualification constraint and the price of housing

To set the stage and get a first impression of the impact of constraint (14) on house prices, we consider the housing price as a function of income. The assignment rule (9) is a continuous and increasing relationship between housing quality and income and we write its inverse as $q(y)$. Using this, we derive the user cost of housing as a function of income, $p(q(y))$. Since the equilibrium housing price is increasing in quality and, according to the assignment rule, quality is increasing in income, the user cost $p(q)$ must also be increasing in income, with slope

$$\frac{dp}{dy} = \pi(q)\frac{dq}{dy} = \pi(q)\frac{f(y)}{g(q(y))}. \tag{15}$$

Figure 2 illustrates the relationship between user cost and income and the mortgage qualification constraint (14). The equilibrium price function in the initial situation (without borrowing constraints) is $p(q(y))$. In the situation pictured, the constraint is binding for incomes between y^* and y': for such incomes the user cost implied by $p(q(y))$ is higher than μy. This picture may suggest that after the introduction of the constraint the equilibrium price function will coincide with the constraint on the interval $[y^*, y']$ while for incomes higher than y' the initial price function is still valid. As will be shown now, this conjecture is not valid. Instead, the constrained price function will start departing from the borrowing constraint from some $y'' < y'$ onwards, as illustrated by the grey curve in Figure 2.

We assume that initially the market is in equilibrium and there is no borrowing constraint, and consider what changes if such a constraint is imposed. We will refer from now on to the price function derived in the previous section for the situation without borrowing constraints as $p^m(q)$ and to the one that is relevant with the mortgage qualification constraint present as $p^{bc}(q)$. We will first consider the case in which the borrowing constraint starts to bind at $y^* \geq y^c$, similar to the situation in Figure 2, and present the case at which the borrowing constraint already binds at y^c afterwards.

Proposition 1. *Consider the introduction of a borrowing constraint that starts to bind at $y^* \geq y^c$: $p^m(q^{min}) \leq \mu y^c$, and y^* is the smallest $y \geq y^c$ such that in the right-sided neighborhood of y^*, $p^m(q(y)) > \mu y$. Define y'' as the smallest $y > y^*$ for which in the right-sided neighborhood of y'', $M(q(y), (1-\mu)y) f(y)/g(q(y)) < \mu$ if that occurs, and as y^{max} otherwise. Then,*

- *The assignment rule $q(y)$ does not change;*
- *$p^{bc}(q(y)) = p^m(q(y))$ for $y \in [y^c, y^*]$, and $p^{bc}(q(y)) < p^m(q(y))$ for $y \in (y^*, y^{max}]$, so that utility is the same for all households with $y \leq y^*$ and higher for all households with $y > y^*$:*
  1. *$p^{bc}(q(y)) = \mu y$ for $y \in [y^*, y'']$;*
  2. *$p^{bc}(q(y))$ is described by $\pi^{bc}(q(y)) = M(q(y), y - p^{bc}(q(y)))$ with initial condition $p^{bc}(q(y'')) = \mu y''$, for $y \in [y'', y^{**}]$ if y^{**} exists, and for $y \in [y'', y^{max}]$ otherwise, in which y^{**} is the smallest $y > y''$ such that in the right-sided neighborhood of y^{**}, $p^{bc}(q(y)) > \mu y$.*
- *If y^{**} exists, the constraint binds the constrained price function again, and 1. and 2. apply recursively with y^{**} replacing y^* and y'' redefined accordingly.*

Proof. Assume for now that the assignment rule does not change as the result of the mortgage qualification constraint. Then the minimum income of owner-occupiers is the same as in the equilibrium without a borrowing constraint, and because the borrowing constraint becomes binding only later, $p^{bc}(q^{min}) = p^m(q^{min})$. We can now follow the logic of Lemma 2 to show that until income y^*, where the borrowing constraint becomes binding, and the associated housing quality $q(y^*)$, the functions $p^{bc}(q)$ and $p^m(q)$ will coincide.

At income y^*, $p^m(q)$ crosses the borrowing constraint. Using (15), it must thus be that $\pi^m(q(y^*)) f(y^*)/g(q(y^*)) > \mu$. A household with an income slightly higher than y^* would thus like to spend a larger income share than μ on housing, but is restricted. The slope of $p^{bc}(q)$ is then thus smaller than $p^m(q)$ and equal to

$$\frac{\partial p^{bc}}{\partial q} = \mu \frac{g(q(y))}{f(y)} < M(q(y), y - p^{bc}(q(y))). \tag{16}$$

Now consider two cases. First, the constraint remains binding. Then (16) continues to describe the slope of $p^{bc}(q)$ and thus $p^{bc}(q(y)) = \mu y$ for all $y \in [y^*, y^{max}]$. Then it follows immediately that $p^{bc}(q(y)) < p^m(q(y))$ for $y > y^*$. Second, the constraint stops to bind: there exists some $y'' > y^*$ for which $p^{bc}(q(y'')) = \mu y''$ and $M(q(y''), y'' - p^{bc}(q(y''))) f(y'')/g(q(y'')) = \mu$, after which the left-hand side becomes smaller. Then $p^{bc}(q(y)) = \mu y$ only for $y \in [y^*, y'']$.

To see that this function does not describe the constrained price function all the way up to y', note that at income y', where the constraint is again just binding, the marginal willingness to pay for housing is smaller than $\mu g(q(y))/f(y)$. Consequently, the constraint cannot be binding at income y'. Hence there must be a lower income, y'', for which this constraint stops being restrictive, unless the constraint is binding up to y^{max}. The marginal willingness to pay for housing is a continuous function of income, so the function $p^{bc}(q(y))$ will not have a kink at y'', which defines y''.

For incomes higher than y'', the slope of $p^{bc}(q)$ is thus equal to the marginal willingness to pay for housing, as long as the constraint does not become binding again, at which point the slope is again given by $\mu g(q(y))/f(y)$. Irrespective of whether that happens, it follows that $p^{bc}(q(y)) < p^m(q(y))$ for $y > y''$.

The households with $y \geq y''$ pay a lower price for the same housing they would have occupied in the unconstrained market equilibrium and they are satisfied with their situation, as the marginal willingness to pay for housing equals the marginal price they face. The households with $y \in (y^*, y'')$ would like to overbid richer households, but the borrowing constraints prevent them from doing so. The households with $y \in [y^c, y^*]$ still face the marginal price that equals their marginal willingness to pay for housing, so are satisfied too, and will not be overbid by households with $y < y^c$. We conclude that the assignment rule does not change.

Because $p^{bc}(q(y))$ and $p^m(q(y))$ already started diverging for $y > y^*$, prices are lower for $y > y^*$. Because the assignment is the same, it follows that utility is the same for all households with $y \leq y^*$ and higher for all households with $y > y^*$.

□

Summarizing, we have shown that the function $p^{bc}(q(y))$ coincides with $p^m(q(y))$ until this function hits the borrowing constraint. Then $p^{bc}(q(y))$ follows the borrowing constraint until, if that happens, the marginal willingness to pay for housing is so low that households prefer to spend less on housing than is allowed by this constraint. This happens at an income y'' that is lower than the income y' at which $p^m(q(y))$ crosses the borrowing constraint. There is a kink in $p^{bc}(q(y))$ at income y^* but not at y''.

Households with income between y^* and y'' want to consume more housing, but are unable to realize this desire. Due to the binding borrowing constraint they pay less for the same house they would have occupied in the unconstrained market equilibrium, so that their utility will be higher. The utility of households with income largher than y'' will also be higher, but their marginal willingness to pay for housing will equal the marginal price, unless the borrowing constraint binds again. The assignment rule of the unconstrained market equilibrium remains valid, because no household can reach a higher utility by deviating from this rule.

Now consider the introduction of a borrowing constraint that already binds at q^c. The following corollary shows that this sitation closely follows the description above.

Corollary 1. *Consider the introduction of a borrowing constraint that already binds at* y^c*:* $p^m(q^{min}) > \mu y^c$*. Define* y'' *as the smallest* $y \geq y^c$ *for which in the right-sided neighborhood of* y''*,* $M(q(y),(1-\mu)y)f(y)/g(q(y)) < \mu$ *if that occurs, and as* y^{max} *otherwise. Then,*

- *The assignment rule* $q(y)$ *does not change;*
- $p^{bc}(q(y)) < p^m(q(y))$*, so that utility is higher for all households:*
  1. $p^{bc}(q(y)) = \mu y$ *for* $y \in [y^c, y'']$*;*
  2. $p^{bc}(q(y))$ *is described by* $\pi^{bc}(q(y)) = M(q(y), y - p^{bc}(q(y)))$ *with initial condition* $p^{bc}(q(y'')) = \mu y''$*, for* $y \in [y'', y^*]$ *if* y^* *exists, and for* $y \in [y'', y^{max}]$ *otherwise, in which* y^* *is the smallest* $y > y''$ *such that* $p^{bc}(q(y)) = \mu y$*.*
- *If* y^* *exists, the constraint binds the constrained price function again, and Proposition 1 applies.*

What happens if the mortgage qualification constraint is relaxed? Consider again the situation pictured in Figure 2. Households for whom the constraint is no longer binding will attempt to increase their housing consumption until the constraint binds again, or until they are on their housing demand function. However, if housing supply does not adjust, all households will stay in the same house, which will have become more expensive. If in the new situation the constraint is no longer binding for any household, the market returns to the equilibrium price function $p^m(q(y))$. If some households are still constrained, then in the new equilibrium the interval for which the constraint is binding will be a smaller. Prices will increase for all households with an income higher than y^*. The welfare of all these households will decrease, since their housing consumption and income do not change. However, the number of constrained households (for whom the borrowing constraint is binding) will be smaller than with the tighter constraint.

Note that the results of this section are sensitive to the assumption that all households have to borrow all the money needed for purchasing their houses. If some households with a given income experience a binding credit constraint while others own some wealth and are willing to invest it in their houses, the latter group may not experience a binding credit constraint while the former group does. In such a case the allocation of households over the housing stock will be affected by the borrowing constraint, as will be discussed below.

4.2 General borrowing constraints

In this subsection we consider a more general situation in which households can experience borrowing constraints of a general nature. The borrowing constraint is a household-specific maximum imposed on the purchase price of a house.[28] Since there is a one-to-one relationship between purchase price and user cost, we include this in our model as a maximum user cost ρ. We assume that ρ has positive support on $[\rho^{min}, \rho^{max}]$ for some $\rho^{min} \geq 0$ and $\rho^{max} \leq \infty$.

Households are characterized by their income and maximum user cost, an ordered pair (y, ρ). We denote the simultaneous distribution of income and maximum user cost as $F(y, \rho)$. $F(y, \rho)$ is thus the number of households with income at most equal to y who can bid at most ρ for a house. The corresponding density is $f(y, \rho)$. In the previous subsection we discussed a case in which ρ is a function of income, $\rho = \mu y$. This is a special case of the situation considered here, in which the density $f(y, \rho)$ is only positive for $\rho = \rho(y)$. The discussion that follows considers a different and much more general case in which $f(y, \rho)$ is a continuous function of its two arguments. Such a situation is compatible with the mortgage constraint discussed in the previous section if households may have wealth that can be used, possibly in addition to a mortgage loan to finance a house.

The ρ that is relevant for a particular household should be interpreted as the user cost that the household can afford. One relevant situation is that in which the mortgage qualification constraint of the previous section holds. Denoting the user cost permitted by the mortgage loan now as $\rho^l (= \rho^m(y)) = \mu y$, for households without any wealth the relevant constraint is still $p \leq \mu y$. If the household has wealth that can be used to help finance the house, there is a second part of the user cost, to be denoted ρ^w which is a function of the household's wealth w. The constraint is now that the actual user cost p is at most equal to the sum $\rho = \rho^m + \rho^w$.

For simplicity, one may assume that user cost is proportional to the purchase price of a house, with the constant of proportionality equal to the mortgage interest rate, which is equal for all households. If it is further assumed that the opportunity cost of wealth is equal to this interest rate there is a conveniently simple relationship between the purchase price of a house and its user cost. The discussion below refers to this simplified case.

However, we note that the analysis is also relevant for other cases. For instance, if there is a down-payment constraint instead of a mortgage qualification constraint, the household must have enough wealth to pay a share σ that equals one minus the maximum loan-to-value ratio. Using the assumptions of the previous paragraph, the maximum user cost may be determined as follows. The down-payment constraint is: $\sigma P < w$, where P denotes the purchase price. Multiplication of both sides of the inequality by the mortgage interest rate and dividing by σ gives: $rP < rw/\sigma$. The left-hand side of this inequality is the user cost. The right hand side gives the maximum ρ of the user cost in this situation, which is now independent of income. This shows that the model discussed below is as relevant

[28]This household-specific maximum can be conditional on the mortgage underwriting rules and the mortgage interest rate.

for situations with a down-payment constraint as with the mortgage qualification constraint of the previous section.

The supply side of the market is unchanged. The number of households assigned to a house must therefore be equal to the number of houses that is available. These households must have an income that is at least is as high as the critical level at which housing of the lowest quality is consumed and a maximum user cost that is larger than that of the lowest quality housing. There are thus potentially two groups of households demanding housing of minimum quality: (i) those with a maximum user cost $\rho = p(q^{min})$ and income $y \geq y^c$, and (ii) those with income $y = y^c$ and maximum user cost $\rho \geq p(q^{min})$:

$$F(y^{max}, \rho^{max}) - F(y^c, \rho^{max}) - F(y^{max}, p(q^{min})) + F(y^c, p(q^{min})) = S, \tag{17}$$

in which the last term shows up to avoid double-counting.

The value of $p(q^{min})$ is determined in the same way as before, namely by the condition in (11) that a household with the critical income must be indifferent between the housing of minimum quality and the outside option. Because housing is a normal good, $p(q^{min})$ is an increasing function of the critical income y^c, and the number of households with a maximum user cost above $p(q^{min})$ must thus be a decreasing function of y^c. It follows that the left-hand side of (17) is a decreasing function of y^c and that y^c is uniquely determined.

To trace out the user cost function we consider what happens at a combination of income y, quality q and an associated user cost $p(q)$. The idea is that all households with an income lower than y or a maximum user cost lower than $p(q)$ either have been assigned a house, or will not participate in the housing market. The supply of housing of quality q is $g(q)dq$ and this must be equal to the demand. Demand originates both from households experiencing borrowing constraints and from those who do not, so that

$$g(q)dq = f^{bc}(y, p(q))dp + f^{uc}(y, p(q))dy, \tag{18}$$

in which $f^{bc}(y, p(q))$ is the density of households who have not been assigned a house but are constrained at user cost $p(q)$,

$$f^{bc}(y, p(q)) = \int_{y}^{y^{max}} f(y, p(q))dy, \tag{19}$$

while $f^{uc}(y, p(q))$ is the density of unconstrained households choosing a house with quality q,

$$f^{uc}(y, p(q)) = \int_{p(q)}^{\rho^{max}} f(y, \rho)d\rho. \tag{20}$$

Figure 3 illustrates. The box indicates the combinations of income $y \in [y^{min}, y^{max}]$ and maximum user costs $\rho \in [\rho^{min}, \rho^{max}]$ for which the distribution $F(y, \rho)$ has positive support. The housing price is given as a function of income. It starts at the

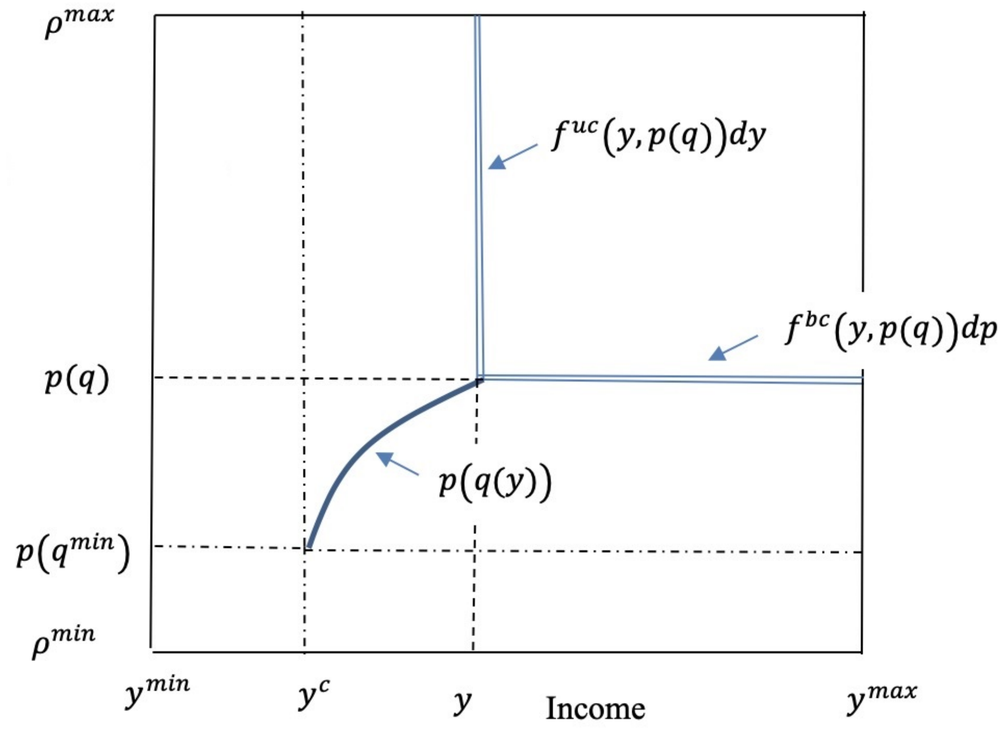

Figure 3: Constrained and unconstrained housing demand

critical income and is shown until some higher y corresponding to housing demand q that commands price $p(q)$. The two narrow (blue) rectangles indicate the demand for housing at this point. The vertical one refers to unconstrained households, that is households with income y who are able to bid at least $p(q)$. The horizontal box refers to constrained households, who can just afford to bid $p(q)$ but cannot afford more expensive housing because of a borrowing constraint. Total demand for housing of quality q is equal to the number of households whose combinations of income and maximum loan belong to these two boxes.

Observe that unconstrained households will only choose the combination $(q, p(q))$ of housing quality and user cost if the first-order condition (12) holds. Using the definition of the marginal price $\pi(q) = dp/dq$, participation of unconstrained households at $(q, p(q))$ thus implies

$$dp = M(q, y - p(q))dq. \tag{21}$$

Substituting (21), we can rewrite (18) as

$$[g(q) - f^{bc}(y, p(q))M(q, y - p(q))]dq = f^{uc}(y, p(q))dy. \tag{22}$$

Since the right-hand side is non-negative, the expression in square brackets on the left-hand side must also be non-negative. If this is the case, there are enough houses offering quality q available for all constrained and unconstrained households interested. This corresponds to a 'mixed' equilibrium in which a given type of housing is inhabited by both types of households, a situation that did not occur with the uniform mortgage qualification constraint studied in the previous subsection.

If the expression in square brackets on the left-hand side of (22) is negative, (21) does not hold at $(q, p(q))$ and a mixed equilibrium is not feasible. In this situation, there are so many households with a binding borrowing constraint at $p(q)$ that they consume all housing with quality q and nothing is left for unconstrained households. The second term on the right-hand side of (18) thus disappears ($f^{uc} = 0$), and instead we have

$$g(q)dq = f^{bc}(y, p(q))dp. \tag{23}$$

The slope of the user cost function will now be determined by the densities of housing and of borrowing-constrained households in such a way that all constrained households are exactly on their constraint:

$$\pi(q)(= \frac{dp}{dq}) = \frac{g(q)}{f^{bc}(y, p(q))}. \tag{24}$$

Note that (23) allows the density of borrowing-constrained households to be larger than the density of housing. If many borrowing-constrained households are clustered in a particular price-quality range, they may occupy all housing for a range of qualities, as happened in the previous subsection. Unconstrained households will be put 'on hold' until all the constrained households have been served. @@Constrained@@ households can be put on hold, because constrained household that are assigned to q no longer compete for higher-quality houses. Because constrained households are forced to accept lower housing consumption, unconstrained households are enabled to consume housing of higher quality relative to the unconstrained equilibrium. If there is a quality range where all houses are occupied by borrowing-constrained households, the marginal price is also lower than the marginal willingness to pay for unconstrained households interested in this housing.[29]

As will be clear by now, the equilibrium with credit constraints in the model of the present subsection differs substantially from that in the previous subsection with a uniform mortgage qualification constraint. In particular, the assignment rule in the constrained equilibrium now differs from that in the unconstrained one. Households experiencing a binding borrowing constraint will in general consume less housing than unconstrained households with the same income level. Some of them may even be pushed out of the housing market, while other households with lower incomes but less tight borrowing constraints will be able to enter. However, with the generalized borrowing constraints of the present subsection it is still true

[29] To see this, note that in this range the expression in square brackets on the left-hand side of (22) is negative implying that $\frac{g(q)}{f^{bc}(y,p(q))} < M(q, y - p(q))$ and use (24).

that - relative to the corresponding equilibrium without constraints, house prices will be lower.

The key observation here is that the income of unconstrained households demanding housing of any quality q will never be higher than in the unconstrained equilibrium and will be lower if some borrowing constraints are binding at prices below $p(q)$. To see this, consider first the critical income. This will never be higher than in the unconstrained equilibrium and therefore $p(q^{min})$ will never be higher than in the unconstrained equilibrium. If the critical income is lower in the constrained equilibrium, the price for housing of minimum quality will also be lower. Moreover, the marginal price at $p(q^{min})$ will also be lower. In the present version of the model, constrained households will consume less housing than unconstrained households with the same income level. This implies that unconstrained households will at least consume the same housing quality than in the unconstrained equilibrium and thus that their marginal willingness to pay for housing is lower than in the unconstrained equilibrium. Houses of all qualities will be inhabited by households that have at most the same income as in the unconstrained equilibrium. Hence the marginal price of housing will be at most equal to that in the unconstrained equilibrium. Since we have already drawn a similar conclusion for the price level of housing of minimum quality, it follows that for all levels of housing quality the price will at most be equal to that in the unconstrained equilibrium. If some households are driven out of the market by the borrowing constraints, the equilibrium housing price will be lower for all quality levels. If this is not the case, but some households are forced to accept a lower housing quality by the borrowing constraints, then the prices for all higher quality levels are lower than they would be in the unconstrained equilibrium.

On the other hand, it is easy to verify that the price of housing of quality q or lower will not be affected by the presence of binding borrowing constraints that refer exclusively to households assigned to houses of higher quality. However, this appears to be an exceptional situation because binding borrowing constraints push households toward lower quality housing and should therefore be expected to concentrate constrained demand at the low-quality range. We may conclude that the presence of borrowing constraints that are binding for some households will never result in higher house prices for unconstrained households and will result in strictly lower (total and marginal) house prices, and therefore increased housing consumption for unconstrained households with actual user costs higher than the maximum user cost of some constrained other households.[30] The first conclusion is in line with the analysis of the previous subsection, but the second is a substantial deviation. The benefits of the unconstrained households are related

[30]Note that these constrained households may either consume lower quality housing or may have been pushed out of the market because they are not even allowed to occupy housing of minimum quality.

to the lower housing consumption of the constrained households, which suggests that the constraints reduce utility for constrained households.

Consider a borrowing-constrained household. The house they would have preferred in the situation without borrowing constraints is no longer available to them, notwithstanding the lower price. Instead, they had to accept a lower level of housing services at a total price that is low enough for their marginal willingness to pay for housing to exceed the actual marginal price. Their utility will certainly be lower than they could have reached at the currently prevailing user cost function (should their borrowing constraint be relaxed while those of all others remained in place), but it is unclear if their utility is also lower than in the situation without borrowing constraints for any household. If their constraint is mild, in the sense that housing consumption is modestly reduced, the net effect may be positive, as is always the case with a uniform mortgage qualification constraint, but if the constraint forces them to reduce their housing consumption substantially, it will be negative. Hence the welfare effect of the borrowing constraints on the constrained households is ambiguous in the situation studied in the present subsection.

5 Buy-to-let investors

Let us consider what happens if buy-to-let investors enter the market. We assume that buy-to-let investors do not experience borrowing constraints. They have access to capital that allows them to buy any house, but they need a return that exceeds γ. As before, household utility only depends on q and c, not on tenure type.

If buy-to-let investors buy houses at the prevailing market price $P^m(q)$ when no borrowing constraints are present, they will be able to let these house against a rent $p^m(q)$, which offers the investor a return of exactly γ. Because this return is not sufficiently attractive to trigger investments in the housing market, buy-to-let investors will not enter a market like the one described in Section 3.3.

However, the situation is different when some households experience a binding borrowing constraint. As we have seen above, households restricted by a borrowing constraint have a marginal willingness to pay for housing that exceeds the marginal price. Then there are households willing to pay more than the user cost $p^{bc}(q)$ as rent if this offers them the possibility to consume more housing than they are able to do in owner-occupied housing with the borrowing constraint present. Hence the return of the buy-to-let investor will be higher than γ.

The model thus predicts that buy-to-let investors will enter the market when borrowing constraints are binding for some households. We argue below that this unleashes an arbitrage process that ends when user costs are equal to the user costs in the equilibrium without borrowing constraints.

Consider the situation in Figure 4, in which the prevailing user costs are af-

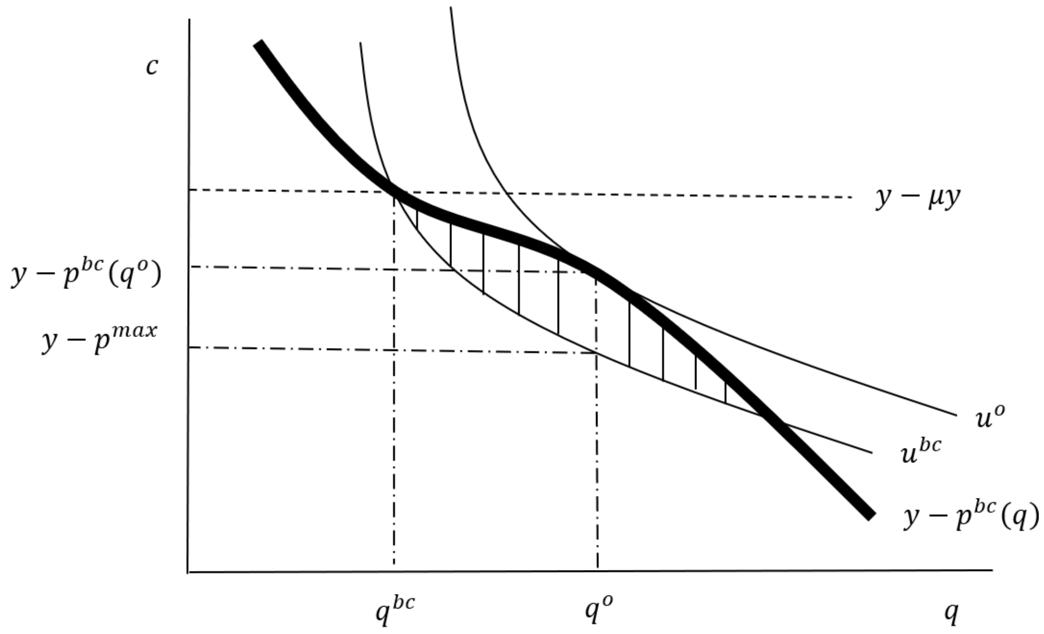

Figure 4: Borrowing constraints and profitable buy-to-let

fected by the presence of borrowing constraints. The bold line indicates the budget constraint of a household, that is the difference between income and the user cost of housing, $y - p^{bc}(q)$. The household also faces a maximum user cost μy, and the resulting borrowing constraint is indicated by the dashed line. As a result, the highest utility that the household can achieve is u^{bc} and it chooses to consume quality q^{bc}.

However, in the absence of a binding borrowing constraint, but assuming for now that prices remain fixed, the household would choose housing consumption q^o. A single buy-to-let investor could then purchase a house of quality q^0 at the prevailing market price, finance it with user cost $p(q^o)$ and offer it to the borrowing-constrained household at a rent between $p(q^o)$ and p^{max}. This investment would offer the investor a return above γ and it would give the household the possibility to increase its utility by consuming the quality it would consume without borrowing constraints. In fact, any offer to rent a house implying that the household reaches a combination of housing and other consumption somewhere in the shaded area means a possibility to improve utility relative to the present state of constrained owner-occupied housing consumption for the household.

However, competition between buy-to-let investors would drive up prices. While price increases only contribute to the incentives to invest in the housing market, they may affect the quality of housing that is available to a household, irrespective of whether it is owning or renting. The following proposition characterizes the equilibrium with buy-to-let investment.

Proposition 2. *The assignment, user cost function and welfare in the equilibrium with borrowing constraints and buy-to-let investment are equal to the assignment, user cost function and welfare of the equilibrium without borrowing constraints.*

Proof. The investor charging the highest rent will be able to pay most to obtain the house, so competition ensures that in equilibrium buy-to-let investors charge the highest rent that renters would still choose to pay. Because households do not care about tenure type and investors can arbitrage away the impact of borrowing constraints, the highest rent equals the highest user cost that households would be willing to pay. The highest user cost will be paid by the household with the highest marginal willingness to pay. As a result, the equilibrium assignment follows the ranking of Lemma 1 and the user cost function follows the differential equation of 2. Because the assignment and the user cost function are identical, welfare is also identical to the equilibrium without borrowing constraints. □

The activity of buy-to-let investors thus drives up housing prices from a situation of borrowing constraints. The prices of houses for which demand was depressed by borrowing constraints increase until the possibility of profit-making buy-to-let activities has disappeared. In this situation, all households who were initially borrowing-constrained avoid the implied restriction by moving to rental housing. The allocation of housing over households is identical to that in a pure owner-occupied market without borrowing constraints as a result of the arbitrage of buy-to-rent investors.[31]

To see how this this equilibrium with buy-to-let investment plays out in Figure 4, first consider the case of uniform borrowing constraints as considered in Section 4.1. In this case, we know from Proposition 1 that the assignment is not affected by the presence of borrowing constraints, but that prices are lower for households with income levels higher than the income level at which the borrowing constraints starts to bind, compared to the situation without borrowing constraints. Buy-to-let investment thus does not affect the assignment in the equilibrium with borrowing constraints either, and only drives up prices.

In the equilibrium with buy-to-let investment, the household in Figure 4 would thus still consume q^{bc} and would not choose to rent q^0 or any other housing quality. User costs and rents would be higher, so that the budget constraint would lie below the bold line. At q^{bc}, this lower budget constraint would be tangent to an indifference curve corresponding to a lower utility than u^{bc}. Indeed, uniform borrowing constraints increase utility for all constrained households and those with higher incomes, and this utility gain is lost when buy-to-let investment drives up prices.

In the case of general borrowing constraints as considered in Section 4.2, constrained households may suffer from the presence of borrowing constraints. If this is the case, then poorer, unconstrained households consume the housing quality

[31]In a model in which borrowing constraints have macro-prudential benefits, such constraints may nevertheless be useful as they protect households (and banks) against the risks associated with mortgage default, at least to the extent that buy-to-let investors are better able to carry these risks.

they would otherwise have consumed. Then buy-to-let investment would allow a household consuming q^{bc} to increase its housing consumption, for instance to q^0. Prices would still increase, so that u^0 would not be attainable, but some utility below u^0, and above u^{bc} if borrowing constraints decreased their utility, would be. Of course, the poorer, unconstrained households would then not be willing to pay the higher user costs, and would choose to consume lower-quality housing, losing the utility gain that resulted from borrowing constraints.

6 Empirical work

6.1 Strengthened borrowing constraints and buy-to-let activity

In this section, we exploit the predictions of the assignment model to make sense of a series of contractions of the debt-service-to-income constraint in the Netherlands between 2012 and 2016. In particular, we test that buy-to-let investors enter the part of the housing stock that is left by households facing tightened borrowing constraints.

A main element of the Dutch Code of Conduct for mortgage loans is that households are, as a rule, only allowed to have mortgage loans that leave them enough income for other necessary spending categories. A coarse rule of thumb is that the net mortgage payments should not exceed 30% of net income. However, in the Dutch system this rule is formalized and the details are elaborated each year by an independent institute for expenditure research, NIBUD, that annually produces a detailed table indicating the maximum amount that can be borrowed by households without other debt and a specific income.

Before 2011 this 'NIBUD rule' was applied as a formal qualification requirement for the national mortgage guarantee (NHG). On top of that they were part of the Code of Conduct of Dutch mortgage providers, where they had the status of a guideline. Borrowing more than the NIBUD rule allowed could, when NHG was not required, be justified by general arguments like the expected higher than average increase in future incomes for the higher educated or plans for investments in the home that would increase its value. However, after the new version of the Code was released in the course of 2011, the 'comply or explain' principle was applied in a more rigorous way, requiring case-specific reasons for each exception. In the years that followed the formalization of the rules for mortgage lending continued and parts of the Code became elements of the national Dutch law system, applying to everyone. The context of this policy reform has been discussed in Section 2.

The significance of this development for our paper is that the gradual further tightening of the NIBUD rule that occurred after 2011 had direct implications for the actual borrowing capacity of all households, and - according to the assignment model - to the possibilities of buy-to-let investors to enter the market.

Figure 5 shows the evolution of the maximum mortgage payment to income ratio (MPTI) for four income levels from the start of 2008 until the end of 2021. The vertical axis shows the maximum MP2I. The Figure shows a relative large decrease in the maximum MP2I in January 2013, and smaller but still significant changes in first month of the next three years. In 2017 and later years there are modest relaxations of the maximum MP2I for some income levels.

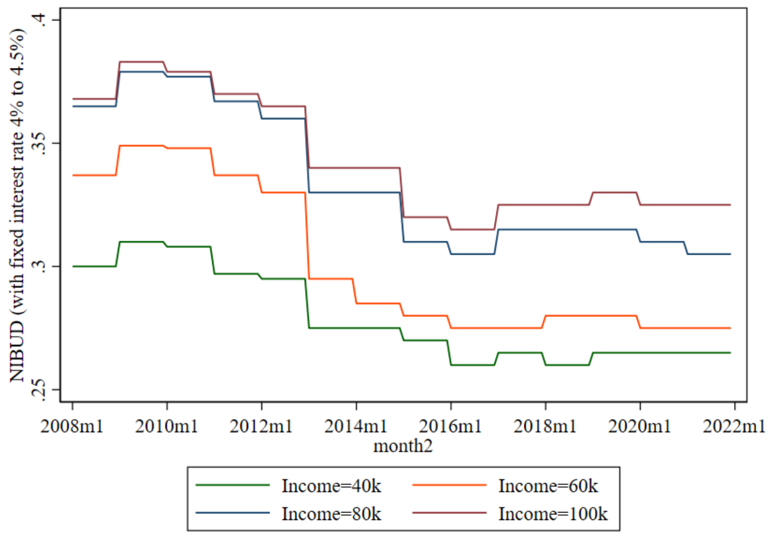

Figure 5: The evolution of NIBUD rule with fixed interest rate

Note: The graph plot the evolution of NIBUD rule with fixed interest rate. We keep the interest rate in the interval of 4.5% to 5% (interest rate prevailing in 2010 and 2011) to avoid NIBUD change is driven by interest rate change. The data comes from National Institute for Budget Information (NIBUD).

The combination of tightened borrowing restrictions and stricter enforcing makes it likely that some of the housing transactions realized before the tightening of the Code of Conduct in 2011 would have been impossible afterwards. The assignment model developed in the previous sections then suggests that the houses involved in such transactions had a larger probability of being bought by buy-to-let investors than others. We will investigate this in detail in the next subsections.

6.2 Data and sample selection

6.2.1 Data

We employ several data sources for our empirical analysis. First of all, we combine the housing transaction datasets of the Kadaster and NVM (Dutch Association of Real Estate Agents). The properties involved in these transactions are our primary focus. We know the properties that were traded and the changes in ownership status of those properties in subsequent years. We also know the individuals and households to which these transactions correspond.

Then, we use rich micro-data from Statistics Netherlands to construct our sample. We have unique address identifiers of houses from GBA-Adresobject and unique person and household identifiers of buyers from GBA-persoon and GBA-huishoudens for each transaction. Through household assets information Vehtab, we match households' total mortgage loans to each buyer and house. Through Inkomen en bestedingen, we obtain gross income of the household involved in the transaction. We obtain houses' ownership status through Eigendomtab [32].

We define the mortgage payment (MP) and mortgage payment to income ratio (MPTI) using, respectively,

$$MP = ML * \frac{r(1+r)^360}{(1+r)^360-1}, \tag{25}$$

and

$$MPTI = \frac{MP}{Income} * 12, \tag{26}$$

in which, the ML refers to the total mortgage loan. The r refers to the monthly interest rate, which is equal to the annual interest rate divided by 12. The $Income$ refers to yearly household gross income. Figure 6 shows the frequency distribution of the MPTI for the transactions we used. It has a relatively fat right tail, suggesting that a non-negligible share of the transactions did not satisfy the NIBUD rules, perhaps associated with their lenient enforcement at the time. However, even in these years the large majority of the transactions (more that 80%) satisfy the NIBUD rule. By comparing the MPTI with the NIBUD rule in subsequent years, we can determine whether the house is subject to borrowing constraints in subsequent years.

6.2.2 Sample selection

We performed the following sample selection procedure.

First, we retained samples of transactions completed between January 2010 and July 2011 in the Kadaster dataset. The years 2010 and 2011 are two years when NIBUD was relatively stable, and it was the last two years before NIBUD started to decline. We did not use transactions after July 2011 to avoid the impact of change of Code of Conduct on loan-to-value ratio, which would interfere with our results. Then, we only retain samples that match the buyer's household income and mortgage loan. Only from these samples can we calculate MPTI to determine whether the property violates the NIBUD rule in subsequent years. We only retained those houses that were owner-occupied at the end of 2011. At this point, we have a sample of around 120k unique properties.The descriptive of the house characteristics of these sample are presented in Table A1.

In our baseline sample, we also removed those samples that had violated the NIBUD rule in 2011 (always treated), which has around 18k observations. In our

[32] This dataset changed definition or statistical methods in 2010 and 2018, so we focus on the comparable period from 2011 to 2017. This was also the time period when NIBUD dropped sharply.

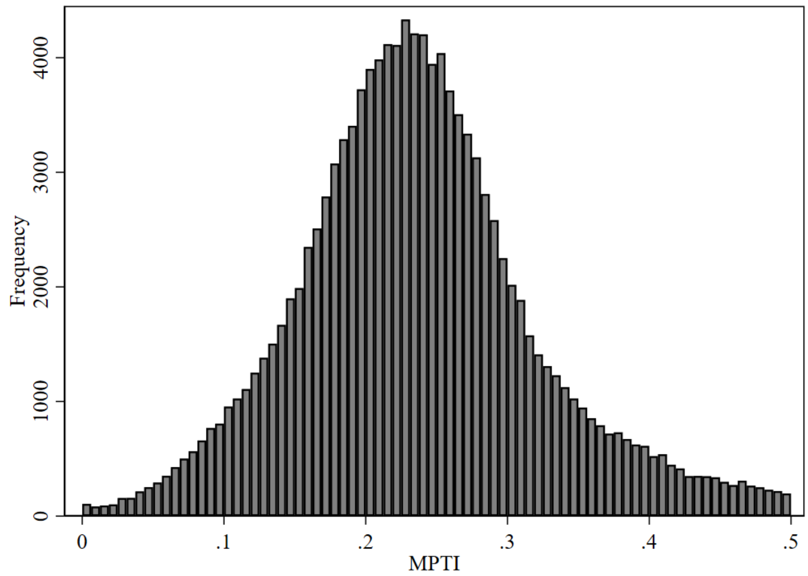

Figure 6: Distribution of MPTI of the 2010 to 2011.7 sample

Note: The graph is the frequency histogram of MPTI based on sample summarized in Table A1 We retain all owner-occupied properties transacted in 2010 and before July 2011. We drop the observations with MPTI larger than 0.5 or equal to 0.

baseline sample, we have over 100k properties (see the descriptive in Table A2). We transformed the baseline housing sample into a panel dataset from 2011 to 2016. Our regression will be conducted based on this panel dataset. The reason for choosing this time period is that the property ownership dataset changed its statistical method in 2011 and 2018, making the data before and after not comparable. In 2017, the NIBUD rule increased, causing some observations to withdraw from the treatment group (see it in Figure 1 and 7), which may lead to biased estimation.

We do some robustness checks by conducting different sample selection procedures. We keep always treated observations, extend the time period to 2017, and only use transactions in 2010 and first two months in 2011. Results are reported in our robustness check section 6.4.3. Since the transaction sample we used may be rather special (houses transacted in a specific period), we also matched the houses in the sample to houses with similar characteristics in the baseline sample, to verify whether borrowing restrictions had the same effect on them. We will elaborate this in our empirical results section 6.4.4.

6.3 A staggered difference-in-differences approach

We would like to introduce a diff-in-diff strategy to test if the houses involved in transactions before the tightening of the Code of Conduct that were no longer feasible in subsequent years were indeed more exposed to buy-to-let activity.

For those transactions for which this rule were satisfied at the time of realization, we could moreover check if they would still have been in accordance with the

NIBUD rule in the years that followed (until 2016) if everything else would have remained constant. That is, we ask the question if the same transaction would have been feasible if the income of the purchasing household, the price of the property and the mortgage interest rate would still be the same as in 2010 or 2011, while we use the NIBUD rules of later years. This provides us with a useful granular indicator of a binding borrowing constraint for households that are the likely buyers of such houses, which does not depend on market developments after 2011. According to our theory exactly these houses are exposed to buy-to-let activity.

We use a panel for the years 2010-2016 in our baseline analyses, which ownership status is observed for every house in every year. Ownership status of houses is indicated in our data as a categorical variable with categories: (i) owned by housing association, (ii) owner-occupied, (iii) owned by a private landlord and (iv) unknown. Houses in the third category can be owned by a private person as well as by a firm, for instance an institutional investor. We define a dummy variable to indicate whether the house is a private rent house in that year. Since our sample was all owner-occupied in late 2011, we actually saw a shift from owner-occupied to private rent. In the period considered here there are many switches between private rental and owner-occupied and *vice versa*. As noted above, many of the switches from owner-occupied to private rental resulted from buy-to-let or keep-to-let activities, while many switches in the other direction were from less expensive rent-controlled housing to owner-occupation.

We define whether a house enters the treatment group in a year based on whether the house's fixed MPTI (in 2010 or 2011) violates the NIBUD rule of that year. What we are concerned about is whether buyers with the same income and the same interest rate can still buy a house of the same price when the NIBUD rule changes. When controlling for year fixed effects, it can also be interpreted as, we assume that there is a common trend in the growth of interest rates, housing prices, income, etc over years. All houses were originally in the control group (in 2011) and were owner-occupied. With the gradual decline of the NIBUD rule, many houses began to enter the treatment group, and some houses were converted to private rentals. In other words, we define the year in which a house first violates the NIBUD rule as the treatment year.

Figure 7 shows the share of the transactions observed in 2010 and 2011 that violate the NIBUD rules in later years. It shows a continuing deepening and staggered timing of treatment, with the exception of 2017. This is why we set the base time period as 2011 to 2016. The increase is particularly large in 2013, which is in line with the decrease of NIBUD rule. Figure 8 describes the distribution of household income of baseline sample and treated sample. This suggests that households subject to borrowing constraints are concentrated among low-income households, which has a perfect match with our assignment model.

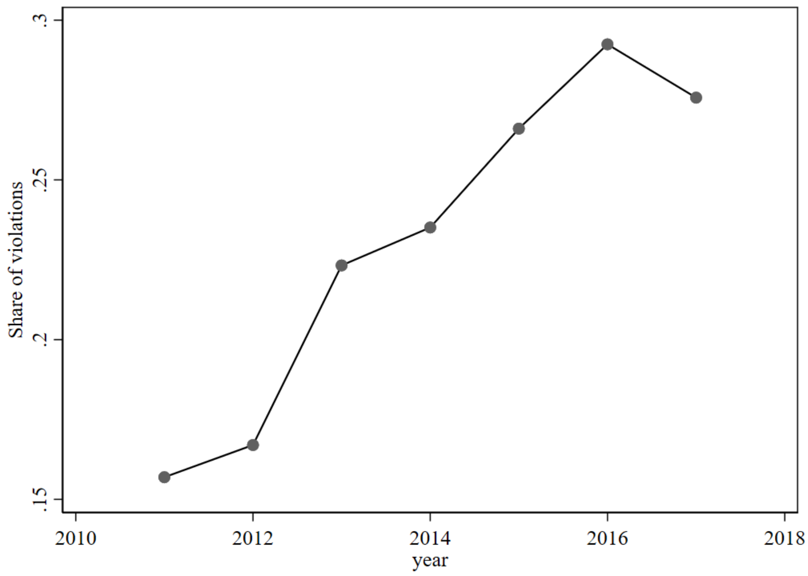

Figure 7: Share of properties violating NIBUD rule over year

Note: The graph shows the share of houses violating NIBUD rule over years. This shows both how the treatment group grew, and the treatment distribution over year.

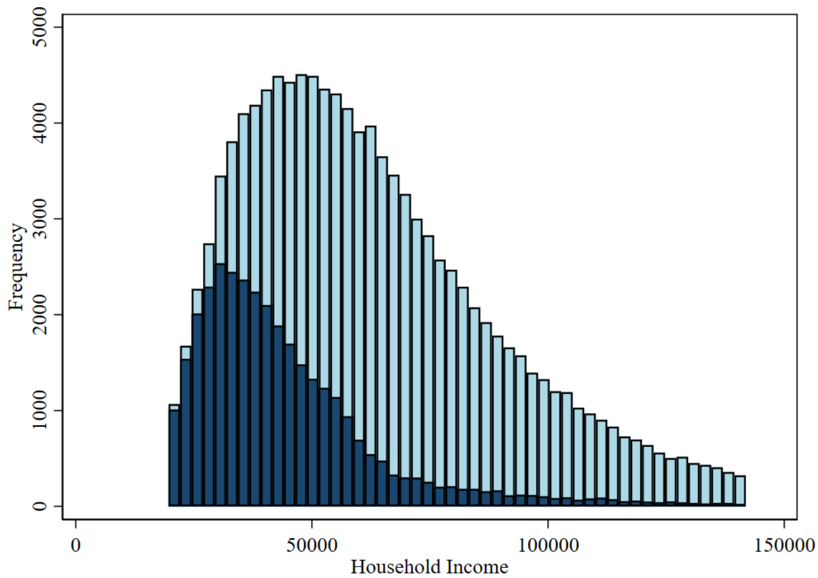

Figure 8: Income distribution of whole sample and treated sample

Note: The graph is the histogram of household income. The lighter bar is the income distribution of households in the baseline sample, and the darker bar is the income distribution of households subject to financing constraints from 2011 to 2017.

6.4 Empirical results

6.4.1 Baseline estimation

We employed the DID strategy at property-year level. With the panel dataset constructed in the previous section, we start our estimation with a canonical two-way fixed effect specification:

$$Private\_rent_{it} = \alpha + \beta Violate_{it} + \gamma_i + \phi_t + \epsilon_{it}. \tag{27}$$

In this equation, $Private\_rent_{it}$ is a dummy variable indicating whether house i is a private rental house in year t. $Violate_{it}$ is a dummy variable to indicate whether the $MPTI_i$ of house i is larger than $NIBUD_t$ in year t. If $MPTI_i$ is larger than $NIBUD_t$, we define $Violate_{it} = 1$, otherwise, 0. γ_i indicates property fixed effects and ϕ_t indicates year fixed effects, which capture the time-invariant property characteristics and common trend of properties over year, respectively. ϵ_{it} indicates the error term. As we have no time-variant covariates at property level, we also tried controlling for property type (apartment or not) multiplied by the linear year trend[33]. We assume that there are different time trends in ownership status for apartments and single-family houses. This is a plausible assumption, considering that apartments are a important component of the private rental sector.

In all estimations we use property and year fixed effects, as indicated in the equation. However, to take into account possible local trends in buy-to-let activity, we also estimate specifications with municipality*year and (postcode area)*year fixed effects.[34] Controlling for local trends at a small geographical scale may be useful because it is known that buy-to-let investment activities vary substantially within cities.

Table 1 presents the results of our baseline estimation. These results capture the average treatment effects on the treated (ATT). We find the expected positive and significant coefficients in all specifications. The estimators show that houses subject to borrowing constraints have a 0.6% to 0.9% higher probability of becoming a private rental property than houses that do not violate NIBUD. We conclude that our empirical analysis confirms a key result of our assignment model: tighter borrowing constraints open up profitable arbitrage possibilities for buy-to-let investors in those parts of the market where demand is depressed.

We then discuss the magnitude of the treatment effects starting with sample mean. In the treatment group, the average probability of a house being privately rented over the years was 2.6%, initially 0, and reaching 5.7% by the end of 2016.

[33] We also try to control for nonlinear time trends, and the estimates are almost the same as those under linear trends. To maintain brevity, we do not present it.

[34] We use the 4 position postcode areas defined by the 4 first digits of the Dutch postcode. The 4,770 areas defined in this way cover the whole Netherlands and consist of a small number of contiguous streets.

Our estimates show that the probability of a house entering the private rental sector increases by 0.6% to 0.9%, which is a large figure relative to the sample mean. In addition, in the following section 6.4.3, we used the new DID estimator and obtained a cumulative treatment effect of 1.2% by the end of 2016. In other words, by the end of 2016, the private rental sector share in the treatment group sample was 5.7%, of which 1.2% was contributed by borrowing constraints, accounting for approximately 21%. This suggests that borrowing constraints have strong explanatory power for the growth of the private rented sector. In addition, with the linear year trend, it is intuitive that apartments have a greater probability of becoming private rental housing.

Table 1: Baseline dif-in dif estimates

	(1)	(2)	(3)	(4)	(5)	(6)
Private rent	TWFE	Pc4 trend	Gem trend	TWFE	Pc4 trend	Gem trend
violate	0.009***	0.007***	0.008***	0.006***	0.006***	0.006***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
apartment*year				0.016***	0.014***	0.015***
				(0.000)	(0.000)	(0.000)
Observations	609,360	607,542	609,360	609,360	607,542	609,360
R-squared	0.393	0.417	0.399	0.401	0.419	0.404
Property F.E.	Y	Y	Y	Y	Y	Y
Year F.E.	Y	Y	Y	Y	Y	Y
Pc4*Year F.E.		Y			Y	
Gem*Year F.E.			Y			Y

Notes: *** $p<0.01$, ** $p<0.05$, * $p<0.1$. Standard errors are clustered at Pc4*year level. Column (1) shows the estimation results specified in equation 27, column (2) includes pc4*year fixed effects, and column (3) includes municipality*year fixed effects. Columns (4) to (6) include the interaction of whether the property is an apartment and the year trend.

We then examined differences in treatment effects between houses sold to first-time buyers (FTB) and those sold to non-FTB. We expect that houses sold to FTB may have a greater probability of entering the private rental market when they violate NIBUD rules. Table 2 presents the results, which are in line with our expectations. The effect we measure in our baseline results is largely due to houses sold to FTBs, which is consistent with the view that financing constraints promoted the growth of the private rental sector.

6.4.2 Event study

The key assumption for identifying ATT is that the parallel trend assumption holds, which allows us to interpret the control group as a counterfactual of the treatment

Table 2: Heterogeneity of first time buyer

Private rent	(1) TWFE	(2) Pc4 trend	(3) Gem trend	(4) TWFE	(5) Pc4 trend	(6) Gem trend
ftb*violate	0.016***	0.013***	0.014***	0.010***	0.010***	0.011***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
violate	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
apartment*year				0.016***	0.014***	0.015***
				(0.000)	(0.000)	(0.000)
Observations	609,360	607,542	609,360	609,360	607,542	609,360
R-squared	0.394	0.417	0.399	0.401	0.419	0.404
Property F.E.	Y	Y	Y	Y	Y	Y
Year F.E.	Y	Y	Y	Y	Y	Y
Pc4*Year F.E.		Y			Y	
Gem*Year F.E.			Y			Y

Notes: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are clustered at Pc4*year level. Column (1) shows the estimation results specified in equation 27, column (2) includes pc4*year fixed effects, and column (3) includes municipality*year fixed effects. Columns (4) to (6) include the interaction of whether the property is an apartment and the year trend. $ftb = 1$ if the property is sold to a buyer younger than or equal to 35 in 2010 or 2011.

group. The parallel trend assumption is untestable, but we can build some confidence that it holds if we observe a parallel pre-trend of treatment and control group. We conducted an event study to check the parallel trend and to investigate the dynamic effects of borrowing constraint. We carried out event studies by estimating:

$$Private\_rent_{it} = \alpha + \sum_{\tau=-5,\tau\neq-1}^{\tau=4} \beta_\tau \mathbb{I}(T_i - t = \tau) + \gamma_i + \rho_p\phi_t + \epsilon_{it}. \tag{28}$$

T_i denotes the year in which house i first violated the NIBUD rule. $\rho_p\phi_t$ denotes the 4 digit postcode times year fixed effect. The results are presented in Figure 9. The plots show that there are no significant differences between the treatment and control groups in the private rented sector before violating the NIBUD rule. The parallel pre-trend makes us trust that the parallel trend assumption is likely to hold. After violating the NIBUD rule, however, properties subject to borrowing restrictions start to have a significant greater probability of entering the private rental sector. We also saw a cumulative treatment effects over period. After a house violates NIBUD rules for the first time, the probability of it becoming a private rent becomes greater and slowly increases.

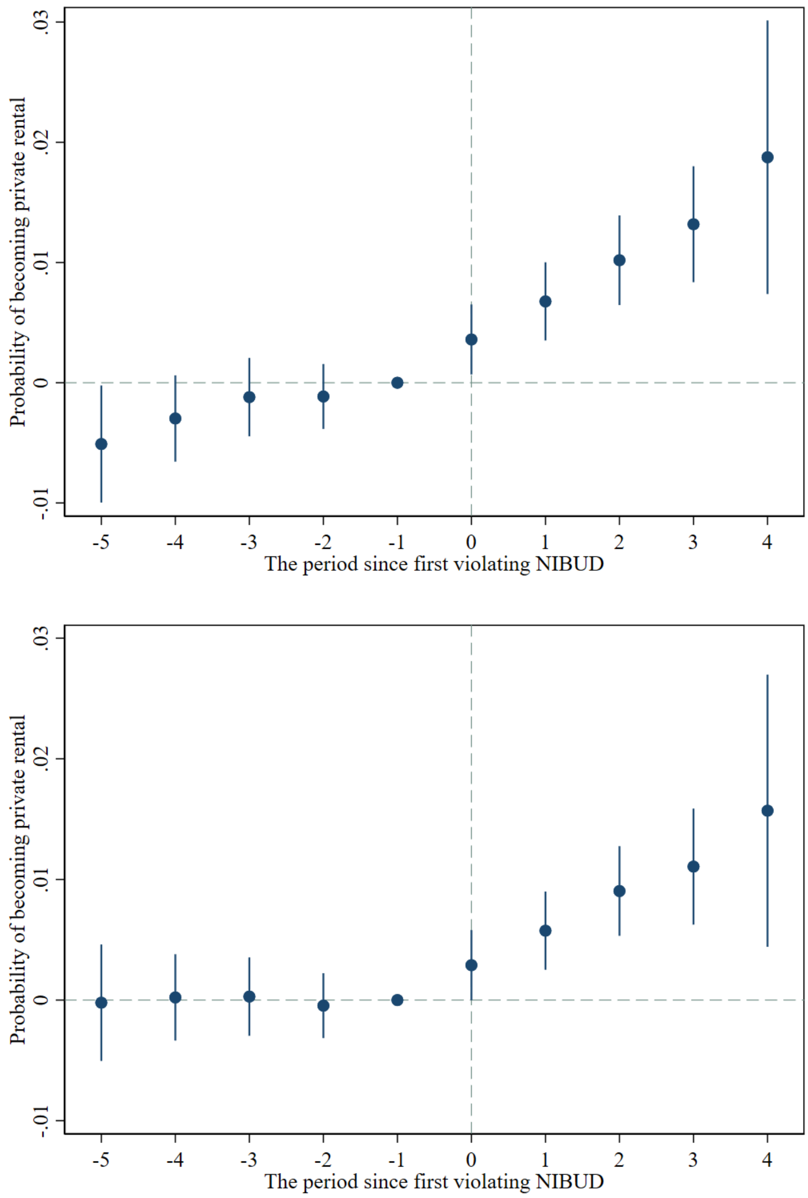

Figure 9: Event study

Note: This figure is a coefficient plot and 95% confidence interval of the estimation results in equation 28. Standard errors are clustered at Pc4*year level. The graph above is the coefficient plot with Pc4*year and without apartment*year trend, which is corresponding to column (2) in Table 1. The graph below is the coefficient plot with Pc4*year and apartment*year trend, corresponding to column (5) in Table 1.

6.4.3 Robustness checks

We conduct several robustness check to strengthen our baseline estimates.

First, we adopt some prevailing "heterogeneity-robust" diagnostics for staggered treatment timing (In our case, the house could start to violate NIBUD rule in any year between 2011 and 2016). A recent econometric literature pointed out that the two-way fixed effect estimator can be interpreted as a weighted average estimator of each staggered treatment effect (de Chaisemartin and D'Haultfœuille, 2020; Goodman-Bacon, 2021). If there were heterogeneity of treatment effects over time and units, we may get negative weights which make the TWFE estimator hard to interpret.

We calculate and plot the weights of each treatment following de Chaisemartin and D'Haultfœuille (2020). Figure A2 shows that all the weights are non-negative and there is little variation. Moreover, we use the heterogeneity-robust estimator of Callaway and Sant'Anna (2021). The estimator is obtained by estimating all 2x2 DID estimators and aggregation. The event-study plots are presented in Table A3. We also decompose the treatment effect by group entering treatment in different years. The Figure A4 presents the results that the treatment effects are positive for all groups, however, only significant in group 2013. It suggests that estimated treatment effects are dominated by the sample entering the treatment group in 2013. The aggregated treatment effect of the borrowing constraints using the new estimator is 0.006 (with controlling for apartment*year trend), which is as same as our baseline estimates.

We also use alternative samples to ensure that the results are not sensitive to sample selection. We first keep the houses violating NIBUD rule in 2011, which are always treated. The estimates are exhibited in Table A4 and Figure A5. We also extend our sample to 2017, although there are some houses withdraw from treatment group in 2017. The estimates are exhibited in Table A5 and Figure A6. In still another check, we only retain houses which were transacted in 2010 and first 2 month in 2011. As the change of code of conduct is announced in February 2011, this avoids including possible anticipation effects. We show the estimates in Table A6 and Figure A7. The estimates reached when using these different samples give slightly different estimates of ATT, ranging from 0.005 to 0.011. This is consistent with our baseline results, ranging from 0.006 to 0.009, with little variation. We therefore conclude that our conclusions are not sensitive to sample composition.

6.4.4 Matching sample

In the previous subsection we always restricted the analysis to houses transacted just before the change in the Code of Conduct of 2011. However, it may be argued that is an idiosyncratic set of houses, for example, may because dwelling that are frequently traded are over-represented. This may cause these houses to be more prone to trades or change ownership status, thus biasing our estimates. To address this issue we reconstruct the sample by using for properties similar to these transaction samples. That is, we assume that houses with characteristics that are similar to those in the sample used thus far, also have a similar probability of becoming treated in the period 2012-2016. We use location, type, price and floor area to construct this matched sample (e.g. other apartments in the same condominium) and continue to use the MPTI of the property in the original sample to determine whether the new property violates NIBUD. Our hypothesis is that the ownership status of the matched property is as likely to switch from owner-occupied to private rental at that of the original property.

We conducted a following matching procedure. First of all, we make sure the new property has same 6-digit post code and type with baseline property. For many-to-many matching of the same pc6 and type, we only retain the properties that are most similar to the baseline property in terms of floor area, evaluation price, and construction year. At this time, we have obtained a 1-to-1 exact match, but there are still some characteristics of the matched samples that are quite different from the baseline properties. We therefore only retain matching samples whose absolute value of characteristics gap is within the 95% percentile. Of course, we also ensure that all matched properties are owner-occupied by the end of 2011. We ended up with around 400k observations of matching houses. Descriptive statistics of the characteristics of these houses are shown in Table A3. Their characteristics differ little from the baseline sample (Table A2), showing that we really found similar houses of the baseline sample.

The estimates using matching sample are presented in Table 3 and Figure 10. The estimated coefficients for the matched sample remain significantly positive, with parallel trend and coefficients ranging from 0.005 to 0.007. This suggests that the higher probability of being converted to the private rental sector was not limited to houses transacted in 2010 or the first half of 2011 but occurred for all houses with similar characteristic in the entire housing stock.

6.4.5 Buy-to-let or keep-to-let

The primary concern of this article is whether borrowing constraints trigger buy-to-let activity. So far, although we have seen growth in the private rental market related to borrowing constraints, this may be caused by either buy-to-let or keep-to-let. To distinguish them, we match ownership status transitions of the properties to transaction records in Kadaster dataset. When a property is private rent in a year and has transaction records from July 2011 onwards to the end of that year, we define it as private rent with buy-to-let (BTL). In contrast, we define it as private rent with keep-to-let (KTL) if the property is private rental property but there is not transaction record.

The estimators using BTL and KTL respectively as the explained variables are

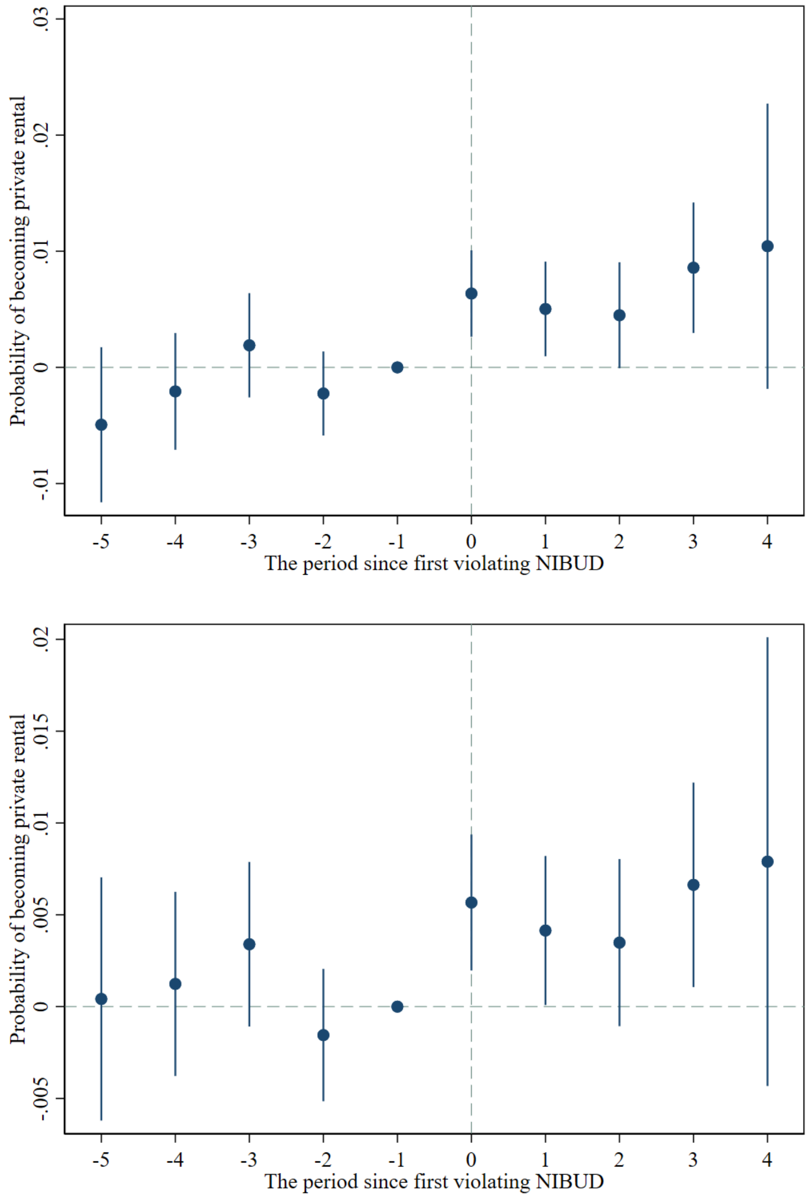

Figure 10: Event study of the matching sample

Note: This figure is a coefficient plot and 95% confidence interval of the estimation results of equation 28 using matched sample. Standard errors are clustered at Pc4*year level. The graph above is the coefficient plot without apartment*year, and the graph below is the coefficient plot with apartment*year.

Table 3: Matching sample results

Private rent	(1) TWFE	(2) Pc4 trend	(3) Gem trend	(4) TWFE	(5) Pc4 trend	(6) Gem trend
violate	0.007***	0.007***	0.007***	0.005***	0.005***	0.005***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
apartment*year				0.015***	0.014***	0.015***
				(0.000)	(0.001)	(0.000)
Observations	405,936	403,794	405,930	405,936	403,794	405,930
R-squared	0.447	0.473	0.451	0.451	0.475	0.454
Property F.E.	Y	Y	Y	Y	Y	Y
Year F.E.	Y	Y	Y	Y	Y	Y
Pc4*Year F.E.		Y			Y	
Gem*Year F.E.			Y			Y

Notes: *** $p<0.01$, ** $p<0.05$, * $p<0.1$. Standard errors are clustered at Pc4*year level. Column (1) shows the estimation results specified in equation 27, column (2) includes pc4*year fixed effects, and column (3) includes municipality*year fixed effects. Columns (4) to (6) include the interaction of whether the property is an apartment and the year trend.

shown in the Table 4. The results indicate that violating the NIBUD rule resulted in significant and identical positive effects for BTL and KTL. Half of the growth in the private rental sector due to borrowing restrictions has come from BTL and half from KTL.

The bottom line of this empirical section is that our results confirm the prediction derived from the assignment model: tighter borrowing constraints stimulate BTL activities.

7 Conclusion

This paper provides an analysis of the interaction between binding borrowing constraints and buy-to-let investment behavior in the context of housing markets where the housing stock can be considered as given, at least in the short run. Although naive households may mortagge-payment-to-income constraints, they imply lower house prices and it is shown that in a simple benchmark case their impact on utility is positive or zero, implying a Pareto-improvement. In a more general setting, borrowing constraints still decrease house prices, and improve welfare for all unrestricted households, while they may decrease welfare of the most restricted households.

Binding borrowing restrictions open up possibilities for profitable arbitrage by buy-to-let investors. By offering the houses preferred by the restricted households as rental housing, they allow them to reach the same level of housing consumption

Table 4: Borrowing constraint and buy-to-let activity

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A:	Private rent with transaction					
	TWFE	Pc4 trend	Gem trend	TWFE	Pc4 trend	Gem trend
violate	0.004***	0.004***	0.004***	0.003***	0.003***	0.003***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
apartment*year				0.008***	0.007***	0.008***
				(0.000)	(0.000)	(0.000)
Observations	609,360	607,542	609,360	609,360	607,542	609,360
R-squared	0.343	0.366	0.349	0.350	0.369	0.353
Panel B:	Private rent without transaction					
	TWFE	Pc4 trend	Gem trend	TWFE	Pc4 trend	Gem trend
violate	0.004***	0.004***	0.004***	0.003***	0.003***	0.003***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
apartment*year				0.009***	0.006***	0.007***
				(0.000)	(0.000)	(0.000)
Observations	609,360	607,542	609,360	609,360	607,542	609,360
R-squared	0.396	0.416	0.400	0.399	0.417	0.401
Property F.E.	Y	Y	Y	Y	Y	Y
Year F.E.	Y	Y	Y	Y	Y	Y
Pc4*Year F.E.		Y			Y	
Gem*Year F.E.			Y			Y

Notes: *** $p<0.01$, ** $p<0.05$, * $p<0.1$. Standard errors are clustered at Pc4*year level. Column (1) shows the estimation results specified in equation 27, column (2) includes pc4*year fixed effects, and column (3) includes municipality*year fixed effects. Columns (4) to (6) include the interaction of whether the property is an apartment and the year trend. The explained variable in Panel A is that private rentals with transaction records; the explained variable in Panel B is that private rentals without transaction records.

as in the case without borrowing constraints, albeit at a higher price than is relevant in the situation when borrowing constraints are binding. In the equilibrium with free entry of buy-to-let investors the allocation of housing over households is the same as in the situation without borrowing constraints.

We use this assignment model to predict which owner-occupied houses will be converted to rental houses, and test these predictions exploiting a series of contractions of the debt-service-to-income constraint in the Netherlands between 2012 and 2016. The empirical analysis showed that in the Netherlands switches from the owner-occupied to private rental sector were significantly more prevalent among houses that had been involved in transactions that became infeasible after the tightening of the the borrowing constraints. The series of contractions that ultimately affected 28 percent of households resulted in a cumulative increase of the private rental sector of 1.2 percentage points, which amounts to 21 percent of the overall increase in this sector by the end of 2016.

These findings show that the borrowing constraints that have been introduced in many countries in the wake of the financial crisis, have likely significantly contributed to the observed rise of the private rental sectors in many developed economies. However, our findings suggest that that there is still substantial explanatory power left for other factors.

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Appendix

A Model Extension

A.1 Derivation and second-order condition

A.1.1 Higher income households consume more housing

In the assignment model the marginal price of housing is not given. To show that housing consumption is still increasing in income, recall that normality of housing implies that the marginal willingness to pay for housing $M(q,c)$ is increasing in c for given q. Consider two households, 1 and 2, with incomes y_1 and y_2, $y_1 > y_2$ and housing consumption q and q_2, $q_1 < q_2$. It will be shown that both households can benefit from switching houses. Household 1 reaches a higher utility, u_1, than household 2, u_2. The willingness to pay of household 1 for the larger house can be written as:

$$WTP = \int_{q_1}^{q_2} M(q, c(q, u_1))dq \tag{A1}$$

where $c(q, u_1)$ denotes the value of other consumption that keeps the household on its initial indifference curve when housing consumption is q. Similarly, we can write the minimum required compensation (willingness to accept) of household 2 for the smaller house as:

$$WTA = \int_{q_1}^{q_2} M(q, c(q, u_2))dq \tag{A2}$$

Where the interpretation of $c(q, u_2)$ is analogous. Since $M(q, c(q, u_1)) > M(q, c(q, u_2))$ for all q, we must have $WTP > WTA$, which implies that both households can reach a higher utility level if they switch houses. Figure A1 illustrates.

A.1.2 Derivation of equation (13)

Note that the slope of the housing price function is always equal to the marginal willingness to pay of the households that have been assigned to the houses with the quality considered, see (11). Starting from the total derivative of the marginal rate of substitution, we can therefore derive:

$$\begin{aligned} d\pi &= \frac{\partial M}{\partial q}dq + \frac{\partial M}{\partial(y-p)}d(y-p) \\ &= \frac{\partial M}{\partial q}dq + \frac{\partial M}{\partial(y-p)}(dy - \pi dq) \\ &= [\frac{\partial M}{\partial q} - \pi\frac{\partial M}{\partial(y-p)}]dq + \frac{\partial M}{\partial(y-p)}dy \\ &= [\frac{\partial M}{\partial q} - \pi\frac{\partial M}{\partial(y-p)}]dq + \frac{\partial M}{\partial(y-p)}\frac{g(q)}{f(y)}dq. \end{aligned} \tag{A3}$$

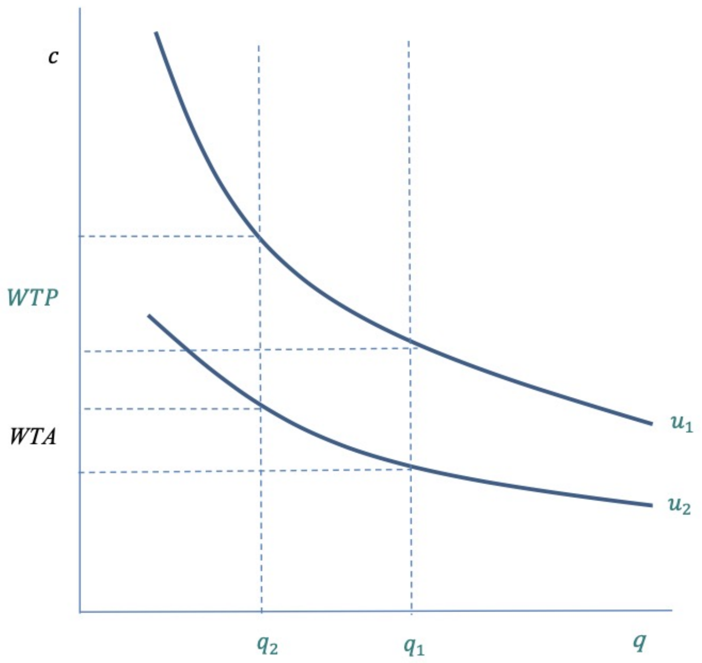


Figure A1: Illustration of switching houses

The right-hand side of the first line is the total derivative of the marginal willingness to pay. The second line uses the definition of the marginal price of housing. The third line is a rearrangement of terms. The fourth line uses the assignment rule (10). Re-writing the last line gives:

$$\frac{d\pi}{dq} = [\frac{\partial M}{\partial q} - \pi \frac{\partial M}{\partial (y-p)}] + \frac{\partial M}{\partial (y-p)} \frac{g(q)}{f(y)} \tag{A4}$$

It is now easy to verify that (13) holds if $d\pi/dq = 0$.

A.1.3 The second-order condition

The first-order condition (12) requires that the slop of the budget line is equal to that of the indifference curve. The second-order condition requires that a move along the budget line starting from the point where the first-order condition is satisfied results in a lower utility. This is the case if the budget line is locally less convex than the indifference curve. We show here that this is always the case if the assignment rule is followed.

By the definition of the marginal willingness to pay, it must be true on an indifference curve that:

$$dc = -Mdq.$$

Now how M changes along an indifference curve:

$$\begin{aligned} dM &= \frac{\partial M}{\partial q}dq + \frac{\partial M}{\partial(y-p)}dc \\ &= \frac{\partial M}{\partial q}dq - \frac{\partial M}{\partial(y-p)}Mdq \\ &= [\frac{\partial M}{\partial q} - \pi\frac{\partial M}{\partial(y-p)}]dq. \end{aligned} \tag{A5}$$

The first line is the total derivative of M, now with the notation c for $y - p(q)$. The second line imposes a move along the indifference curve and the third line uses the equality between the marginal price and the marginal willingness to pay for housing. The expression in square brackets in (A5) is the second derivative of the indifference curve with reversed sign.

In A.1.3 we considered the second derivative of the equilibrium housing price function, $d\pi/dq$, also with reversed sign, see (A3). The second-order condition is satisfied if the budget line is less convex than the indifference curve in the optimum, that is if:

$$-\frac{d\pi}{dq} \leq -\frac{dM}{dq_{u,constant}} \tag{A6}$$

Comparison of the two equations makes clear that this is equivalent $\frac{\partial M}{\partial(y-p)}\frac{g(q)}{f(y)} > 0$, which is true because housing is normal and demand is only expressed for existing housing by existing households, implying that both $g(q)$ and $f(y)$ are positive. Condition (A6) can alternatively be formulated as:

$$\frac{d\pi}{dq} > [\frac{\partial M}{\partial q} - \pi\frac{\partial M}{\partial(y-p)}]. \tag{A7}$$

Consider the housing of minimal quality q^{min}. For any given price $p(q^{min})$ we set, there will (within limits imposed by the income distribution and tastes) be critical incomes for both groups at which households are indifferent between the outside option and living in the housing of minimal quality. However, the marginal willingness to pay for housing will differ. Allocate the housing to the group with the highest marginal willingness to pay for housing. We can then use the method proposed in section 3.2 to trace out the housing price function for this group. At some quality it may be the case that there are households of the other group opting for such housing. The marginal willingness to pay for housing of both groups is identical and if the relevant indifference curves are locally similar so that condition (A7) is satisfied, there will be a segment of mixed housing occupation.

A.2 Taste differences

A.2.1 Introduction

The assumption that all households have the same tastes appears unrealistic and we will therefore in this section consider what changes in the model if we allow heterogeneity in tastes. More specifically, we assume that there are $n \geq 2$ groups of households. Within each group all households have the same tastes, but their incomes differ. We use a super-fix $i = 1...n$ to refer to groups. Incomes belong to group-specific intervals $[y^{(i,min)}, y^{(i,max)}]$ and the income distributions $F^i(y^i)$ are differentiable and increasing on that interval. The total number of households in all groups exceeds the number of houses. Moreover, the housing distribution $G(q)$ is differentiable and increasing on that same interval. For each group the same regularity assumptions hold as were assumed in the main text for the situation with a single group. Hence there would exist a market equilibrium for each of the groups separately if the housing stock were smaller than the number of households in this group. We refer to the situation just defined as the assignment model with multiple groups.

To provide some intuition for the results that follow, it is helpful to recall the familiar graphical analysis associated with Rosen (1974) in which households that differ in tastes or incomes maximize utility while taking a hedonic price function, referring to a differentiated good with a single characteristic, as given. Different households in general choose different positions, but it is possible that households with different tastes and incomes choose the same position on the hedonic price function. The necessary condition for optimal choice is that the marginal willingness to pay for the characteristic equals its marginal price, while the sufficient condition is that the household's indifference curve is more convex than the budget constraint. Market equilibrium requires that all varieties present in the market will be chosen by some households and that for each variety there is equilibrium between supply and demand. This is the situation considered here, for the case in which there is a finite number of groups whose members have identical tastes, but differ in income.

We assume that for each group housing and other consumption are normal goods. As in the case with homogeneous households, the hedonic price function must be increasing and continuous. For each group of households housing consumption and expenditure are increasing in income and the marginal willingness to pay for housing is equal to its marginal price.

The main issue that arises is that not all households consume all the housing qualities that are available in the market. There are two possible reasons for this. The first is that the marginal willingness to pay for housing quality does not equal its marginal price for any household in a given group. The second is that the second- order condition is violated. This means that, in general, households of a certain group do not necessarily consume all available qualities.

To provide a formal analysis, we continue to denote the housing price function as $p(q)$ and now denote the critical income level of group $i, i = 1...n$ as $y^{(i,c)}$. A market equilibrium is a housing price function $p(q)$ defined on the interval $[q^{min}, q^{max}]$ that leads to equality of supply and demand at each quality level. It follows then that every q in the relevant interval must be the optimal choice for households in at least one group. That is, there must be at least one combination (i, y) with $y \in [y^{(i,min)}, y^{(i,max)}]$ for which q is the optimal choice.

In what follows we will prove the existence of an equilibrium by using a fixed point argument. We will first show how we can still trace out the hedonic price function starting from an arbitrary price $p(q^{min})$ for housing of minimum quality, generalizing the procedure used in the case of homogeneous households. The starting point is arbitrary because we cannot determine the critical income level of all groups a priori. Although the procedure that we develop results in an equilibrium for all the housing qualities for which we define the hedonic price function, we may end up in a situation in which all households disregarding the outside option are allocated to a house, but some housing is left, or a situation in which all houses are filled, while there are still some households left. The fixed point argument is then used to show that there is one price for the minimum quality housing at which the number of households disregarding the outside option is exactly equal to the number of available houses.

A.2.2 Starting the allocation and price determination process

We start by determining a closed interval for the price of housing of minimum quality, given the quality and user cost of the outside option. To be able to do so, we assume:

Assumption A1. For a sufficiently high price, every house hold in every group prefers the outside option over the housing of minimum quality.

This assumption requires that even at very high levels of income households will refer the outside option to all housing qualities if their prices are sufficiently high. If this assumption holds, there are certainly prices for the houses of minimum quality for which all households in all groups prefer the outside option to a house of minimum quality. Since all houses have to be occupied, this price, and any higher price, is incompatible with a market equilibrium. Now let p^{max} be the highest lower bound the prices at which no demand is expressed for housing of minimum quality. At this price there may be households in one or more groups that are indifferent between the outside option and the housing of minimum quality. These will be the maximum households in these groups.

If we set the price for the housing of minimum quality equal to p^* we are sure that no demand for the outside option will exist. The reason is that the minimum quality of housing is by assumption larger than that of the outside option.

Now observe that the equilibrium price for housing of minimum quality must necessarily lie in the closed and bounded interval $[p^*, p^{max}]$.

Assumption A2. For every $p(q^{min})$ with $p^* \leq p(q^min) \leq p^{max}$ there is at least one group in which there is a households for which the willingness to pay to switch from the outside option to minimum quality housing is equal to $p(q^{min})$.

This is a continuity assumption.

Next we select an arbitrary price, call it p^s from the compact interval $[p^*, p^{max}]$ and consider what happens if we use this as the starting point of a market equilibrium. We therefore set $p(q^{min}) = p^s$. We know that households in each group will be allocated to houses such that a higher income implies a higher quality house. So we look for the lowest income household in each group that is willing to occupy the housing of minimum quality. This will often be a household that is indifferent between the outside option and this housing quality, but it can also be the lowest income level of a group, while the households with this income strictly prefer the minimum quality housing to the outside option. For some groups there may be no households preferring the housing of minimum quality to the outside option at the prevailing price, but assumption B2 ensures that there is at least one group with households earning a particular income that are indifferent between the outside option and the housing of minimum quality. There is therefore a non-empty set of group-specific critical income levels at the chosen price for housing of minimal quality.

Now select from this set the group with the lowest value of the marginal willingness to pay M at the housing of minimum quality and the price p^s. There may be two or even more groups with this lowest marginal willingness to pay, but suppose for the moment that there is only one. Now start tracing out the housing cost function in the same way as we did in section 3.3, but check continuously if there are households in other groups that have the same marginal willingness to pay at the price-quality combination at hand. As soon as this occurs, we stop the process and switch to a different one discussed below.

By starting the determination of the housing cost function in this way, we put all the other groups that have households preferring the minimum quality housing at price p^s on hold. Since they have a higher marginal willingness to pay for the housing of minimal quality they are also willing to pay more for higher quality housing. In terms of the familiar Rosen-diagram: their optimal location on the housing cost function (which is under construction) corresponds to a higher quality.

It is possible that the construction process of the housing cost function proceeds without meeting households of a second group having the same marginal willingness to pay for housing before either all households in the group are allocated or all houses are occupied. In the latter case the construction process ends. The housing cost function that has been constructed does not correspond to a market equilibrium and we discuss later on what the next step will be.

In the former case, we continue the construction of the housing cost function from the housing quality and price at which the household with the maximum income has been allocated. The procedure is restarted in exactly the same way, but with one group of households (the group that has already been allocated to housing) less. If necessary, this restarting procedure is repeated until either all households preferring a house over the outside option are allocated or all houses are occupied. If both events happen simultaneously, we have reached a market equilibrium. If not, we continue the procedure in the way discussed later in this section.

A.2.3 When multiple groups demand the same houses

Let us now consider what happens if households from two or more groups express demand for the same houses. That is, for a given housing quality and at a given price, households belonging to two or more groups have the same marginal willingness to pay for housing (recall again the Rosen picture). Moreover, the second-order condition for optimal housing demand also has to be satsified.

The assignment of households to housing is now described by a generalization of (10):

$$g(q)dq = \sum_i \delta^i(q) f^i(y) dy^i \tag{A8}$$

In this equation $\delta^i(q)$ is a 0-1 variable indicating that households of type i chose houses with quality q. Hence $\delta^i(q) = 1$ if the first-order condition is satisfied for an income $y \varepsilon [y^{i,min}, y^{i,max}]$ for which the second-order condition is also satisfied. The equation states that all available houses of quality q will be occupied by households that reach their optimal housing demand there.

This equation does not yet make clear how the houses are distributed over the households. To address this issue we consider the change in the first-order condition (12) that occurs if we move to a slightly higher housing quality. If households of group i continue to express demand at this higher quality, the equality between marginal willingness to pay for housing and the marginal price of housing must be maintained. This requires: $d\pi/dq = dM^i/dq$. Elaboration of this condition gives:

$$\frac{d\pi}{dq} = [\frac{\partial M^i}{\partial q} - \pi \frac{\partial M^i}{\partial(y-p)}] + \frac{\partial M^i}{\partial(y-p)} \frac{dy^i}{dq} \tag{A9}$$

The second-order condition for optimal housing demand requires that the budget line is less convex than the indifference curve at the point where both are tangent. Formally, the condition, which is discussed in A.1.3, requires:[35]

$$\frac{d\pi}{dq} \geq [\frac{\partial M^i}{\partial q} - \pi \frac{\partial M^i}{\partial(y-p)}] \tag{A10}$$

If the first-order condition is satisfied, but the second-order condition fails, households express no demand for housing of quality q but they may demand housing of lower or higher quality. If the first as well as the second order condition

[35]If the q is at the border of the interval for which group i expresses housing demand, $d\pi/dq$ should be interpreted as the limit of $d\pi/dq$ when housing consumption approaches the border from inside the interval.

is satisfied, equation (A9) implies that $dy^i/dq > 0$. Now observe that the change in the marginal price of housing is the same for all households, whereas the evolution of the marginal willingness top pay for housing as income changes may differ over the groups. The changes in income associated with the movement to a higher housing quality for the different groups must therefore be aligned to each other. To see how this works, we solve (A9) for dy^i/dq and substitute the result into (A8). This gives us an expression for the change in the marginal price of housing that is compatible with equilibrium:

$$\frac{d\pi}{dq} = \frac{\sum_i \delta^i(q) f^i(y^i) (\frac{\partial M^i}{\partial(y^i-p)})^{-1} [\frac{\partial M^i}{\partial q} - \pi \frac{\partial M^i}{\partial(y^i-p)}] + g(q)}{\sum_i \delta^i(q) f^i(y^i) (\frac{\partial M^i}{\partial(y^i-p)})^{-1}} \tag{A11}$$

The change in the marginal price implied by (A11) can now be used to find the change in income for each group of households that expresses demand. Equations (A9) and (A11) thus give us the possibility to trace out the housing price function and the allocation of households from a given starting point.

This analysis makes clear how we can continue to trace out the demand function when households with different tastes express demand for housing of the same quality. It also makes clear that households of one or more groups will stop to express housing demand even though the households with the maximum incomes of these groups are not yet allocated. The only reason why this can happen is that the second order condition is no longer satisfied. In such a case the households can always reach a higher utility by putting their demand on hold. Unless all available houses are allocated to households before all households preferring a house to the outside alternative are served, they will be allocated to a higher quality house through the continuation of the allocation and price determination process.

A.2.4 Existence of a market equilibrium

We have described a procedure to compute a housing cost function and the associated allocation of households starting from an arbitrary price in the compact set $[p^*, p^{max}]$. The resulting price function and household allocation are compatible with a market equilibrium except for one aspect: we may run out of houses before we have allocated all households, or we may have allocated all these households to houses before the whole housing stock is used. That is, we may end up with excess demand or supply. Now define a disequilibrium indicator D on the basis of the situation reached when the allocation algorithm stops as follows: if all households that prefer a house to the outside alternative are allocated, D equals minus the number of houses that are still vacant; if all houses are allocated, D equals the number of unallocated households that refer a house to the outside option. Now we can define a mapping m of the compact interval $[p^{min}, p^{max}]$ into itself as:

$m(p) = p^*$ if $p + D < p^*$

$m(p) = p^{max}$ if $p + D > p^{max}$
$m(p) = p + D$ otherwise.

The mapping m is continuous. Application of Brouwer's fixed point theorem proves the existence of a market equilibrium:

Proposition 3. *If Assumptions A1 and A2 hold, there exist a market equilibrium in the assignment model with multiple groups.*

A.2.5 A special case

It may be noted that two different aspects of heterogeneity are important in the analysis offered above. One is that households of different groups may realize the same marginal utility of housing at different income levels, the other is that the convexity of the indifference curves (the first derivative of the marginal willingness to pay for housing) may be different even though households belonging to different groups have the same marginal willingness to pay for housing. It is possible that the first aspect is relevant, while the second is not. That situation occurs if the indifference curves of households belonging to different groups are parallel to each other, that is if $u^i(c, q) = u(\theta^i + c, q)$. Two households belonging to different groups choose exactly the same location on the hedonic price function if the difference between their incomes is equal to the difference between their θ s.

When a particular value of housing consumption q is chosen by households of group i with income y, that same value of housing consumption will also be chosen by households of group j with income $y - \theta^i + \theta^j$. Note that the share of households of the two groups may still vary over housing qualities, depending on their income distributions.

To illustrate consider a population where all households have a linear demand function for housing, with a group-specific intercept:

$$q = \alpha^i + \beta\pi + \gamma y, \tag{A12}$$

differences in the constant term α^i have a similar impact on the demand for housing as differences in income. Hence in equilibrium households with different income levels will occupy the same houses, as we see in reality. Note that the differences in the intercept may be related to differences in household characteristics.

A.3 Intertemporal utility maximization with a binding borrowing constraint

A.3.1 Introduction

The model discussed in the main text refers to a single period, but we showed that it can be embedded in an intertemporal utility maximization framework if total (lifetime) utility is the sum of period specific utilities in the absence of borrowing constraints. In this part of the Appendix we show that the situation discussed in section 4.1 where households may experience a binding borrowing constraint but do not invest (part of) their income in home equity can also be consistent with intertemporal utility maximization.

We consider households living over a finite number of periods $1...T$ and earning a constant income e. They start their life without any wealth. The sales price of housing of quality q is $P(q)$. To finance the purchase of a house, interest-only mortgage loans up to size M are available. The associated interest rate is i and there are no other costs associated with housing. Households are therefore able to finance housing quality up to a limit q^m that is determined by the equality $P(q^m) = M$ through a mortgage loan. There is no bequest motive.

Household preferences are additively separable: $U = \Sigma_{t=1}^{T} R^t u(q, c)$. Households have the possibility to save, but they can only borrow via a mortgage loan. The mortgage interest rate i equals the household's rate of time preference, $R = 1/(1+i)$ Housing consumption can be financed by the mortgage loan, but also by investing in housing. There is no bequest motive. In the optimum the marginal rate of substitution will be equal to

$$\frac{\partial u}{\partial q} \Big/ \frac{\partial u}{\partial c} = i \frac{\partial P(q^o)}{\partial q}, \tag{A13}$$

in which q^o denotes the optimal housing consumption. As indicated in the main text, without borrowing constraints consumption expenditure is exactly equal to earnings e in each period while the marginal rate of substitution is equal to the marginal user cost.

A.3.2 A finite lifetime

Now assume that there is a binding borrowing constraint: $q^m < q^o$. Is it possible that households accept this borrowing constraint? Or will they invest in housing to better approach the optimal consumption path without the constraint? To find an answer, we suppose that a household initially consumes housing quality q^* in each period and spends the difference between earnings and mortgage payments $e - iM$ on other consumption. Can it improve its lifetime utility by investing in housing? If the household invests in housing, it will increase its housing consumption in the period in which it invests as well as in all subsequent periods. It will therefore be best to start as early as possible, in period 0. If the larger housing consumption is realized until the end of the households' life, the impact on utility will be

$$\frac{dU}{dq} = \frac{\partial u}{\partial q} \Big(\sum_{t=1}^{T} R^t \Big) - \frac{\partial u}{\partial c} \frac{\partial P(q^*)}{\partial q}. \tag{A14}$$

The first term on the right-hand side gives the benefits of the increased housing consumption over the total lifetime of the household, the second gives the associated cost in terms of delayed other consumption. For the investment to be worthwhile, the expression on the right-hand side has to be positive. This is equivalent to the requirement

$$\frac{\partial u}{\partial q} \Big/ \frac{\partial u}{\partial c} > \frac{1}{\sum_{t=1}^{T} R^t} \frac{\partial P(q^*)}{\partial q}. \tag{A15}$$

Comparing this to (A13) shows that the slope of the indifference curve at housing quality q^* must be strictly larger than that in the equilibrium without borrowing restrictions, which is certainly the case. Indeed $\frac{1}{\sum_{t=1}^{T} R^t}$ converges to i when T becomes large. As long as T is finite, (A15) requires the slope of the households indifference curve to be strictly larger than it is at the optimal level of housing consumption q^o to make investment in home equity worthwhile. Put differently, if the maximum housing consumption allowed by the borrowing constraint, q^m is not too far below q^m the household will accept the borrowing constraint. The reason is that the investment in additional housing provides returns for a limited number of periods only, while there is no bequest motive, so the household cannot realize the full return of the investment. Since the household still wants to smooth consumption, c will in this situation be equal to $e - iM$, which is larger than in the optimum without a borrowing constraint.

A.3.3 A stochastic lifetime

An alternative assumption is that the consumer's lifetime is uncertain. For instance, there may be a constant survival probability S in each year. Expected lifetime utility then becomes: $U = \Sigma_{t=1}^{T} R^t u(q, c)$. The optimal allocation per period in the absence of credit constraints does not change. The condition for investment in housing to be attractive, becomes

$$\frac{\partial u}{\partial q} \Big/ \frac{\partial u}{\partial c} > \frac{1}{\sum_{t=1}^{\infty} Q^t R^t} \frac{\partial P(q^*)}{\partial q}. \tag{A16}$$

Since Q is smaller than 1, this condition is stronger than (A13), so we reach again the conclusion that the household will accept the borrowing constraint as long as q^m is not too far below q^o.

A.3.4 Some other cases

If the households' rate of time preference exceeds the interest rate, it wants to increase consumption expenditure to a higher total than e in the first period. However, the borrowing constraint prevents housing consumption to exceed q^m. Moreover, the present value of the returns on a marginal investment in home equity are now lower than before. The conclusions reached above are therefore strengthened when the rate of time preference exceeds the (mortgage interest rate). One may consider the possibility that the household chooses the outside option in the first period(s) of its life and then saves so as to be able to reach a higher level of housing consumption afterwards. The reference life cycle path for consumption now refers to consuming q^* and $e - p^*$ in the outside alternative, while switching to q^m and $e - iM$ afterwards. Note that this reference path of life-cycle consumption implies an unambiguously lower lifetime utility than the one considered before in which the households moves immediately to the level of housing consumption at which the borrowing constraint is binding.

Investing in home equity now means that other consumption in the first periods drops to a level that is lower than $e - p^*$. The returns on this investment now realize only in the periods after the outside option is left and are therefore strictly smaller than in the cases considered above. It may happen that the cost is also smaller. The reason is that the marginal utility of other consumption may be lower when the consumer is in the outside option than when it moves immediately to a highest quality house allowed by the mortgage loan. This is not necessarily the case, since housing consumption is also different in the outside option. The sign of the cross-derivative of the utility function with respect to housing and other consumption is therefore important. If it is positive, the lower housing consumption further decreases the marginal utility of other consumption and the cost of investing in home equity. If it is negative, the opposite happens and the cost of such an investment decreases less, or becomes even higher than in the cases studied above.

Note that in this case it is not sufficient that a marginal investment in home equity is profitable relative to to the reference case. The additional lifetime utility generated by the investment in home equity must be sufficient to overcome the difference between the utility of choosing the outside option in one or more periods early in life and that of moving immediately to a house with the maximum quality allowed by the borrowing constraint.

B Empirical Tables and Figures

B.1 Tables

Table A1: Descriptive of the all owner-occupied properties

Variable	Obs	Mean	Std.Dev.	5%	95%
woz price	118,630	234178	112755	114000	463000
constr. year	120,464	1967	29.23	1910	2005
size	120,318	114.5	42.65	57	195
type	120,465	0.726	0.446	0	1
Sale price	120,465	233989	115810	112500	469000
income	120,465	69015	44405	26801	141814
MPTI	120,465	0.236	0.0839	0.103	0.393
loan	120,465	234151	126566	97500	491860
buyer age	120,465	35.61	11.13	23	58

Notes: This is the house characteristic description of the entire sample transacted in 2010 or first 7 months in 2011.

Table A2: Descriptive of the baseline properties

Variable	Obs	Mean	Std.Dev.	5%	95%
income	101,560	74158	45335	148205	148205
buyer age	101,560	35.97	11.05	23	58
loan	101,560	232669	122057	95000	473000
MPTI	101,560	0.211	0.0621	0.0965	0.299
woz price	100,061	238053	112567	116000	466000
constr. year	101,559	1967	29.14	1910	2005
size	101,439	116.1	42.29	59	196
type	101,560	0.742	0.437	0	1
Sale price	101,560	237973	115694	114500	472500

Notes: This is the house characteristic description of the baseline sample after dropping always treated sample and non-owner-occupied properties.

Table A3: Descriptive of the matching properties

Variable	Obs Mean	Std.Dev.	5%	95%	
constr. year	67,965	1970.134	36.776	1920	2005
size	67,965	116.655	38.583	64	182
woz price	67,965	239291.9	107423.7	124000	438000
type	67,965	0.790	0.407	0	1

Notes: This is the characteristic description of the new house sample after matching.

Table A4: Keep always treated sample

Private rent	(1) TWFE	(2) Pc4 trend	(3) Gem trend	(4) TWFE	(5) Pc4 trend	(6) Gem trend
violate	0.007***	0.006***	0.006***	0.005***	0.004***	0.005***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
apartment*yearr				0.017***	0.014***	0.016***
				(0.000)	(0.000)	(0.000)
Observations	722,790	721,092	722,790	722,790	721,092	722,790
R-squared	0.395	0.416	0.401	0.403	0.419	0.406
Property F.E.	Y	Y	Y	Y	Y	Y
Year F.E.	Y	Y	Y	Y	Y	Y
Pc4*Year F.E.		Y			Y	
Gem*Year F.E.			Y			Y

Notes: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are clustered at Pc4*year level. Column (1) shows the estimation results specified in equation 27, column (2) includes pc4*year fixed effects, and column (3) includes municipality*year fixed effects. Columns (4) to (6) include the interaction of whether the property is an apartment and the year trend. The regression results in this table are based on the baseline sample and the sample of properties that violated the NIBUD rule in 2011.

Table A5: Sample period from 2011 to 2017

Private rent	(1) TWFE	(2) Pc4 trend	(3) Gem trend	(4) TWFE	(5) Pc4 trend	(6) Gem trend
violate	0.011***	0.009***	0.010***	0.007***	0.007***	0.007***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
apartment*year				0.018***	0.015***	0.017***
				(0.000)	(0.000)	(0.000)
Observations	710,920	708,799	710,920	710,920	708,799	710,920
R-squared	0.377	0.403	0.385	0.387	0.407	0.390
Property F.E.	Y	Y	Y	Y	Y	Y
Year F.E.	Y	Y	Y	Y	Y	Y
Pc4*Year F.E.		Y			Y	
Gem*Year F.E.			Y			Y

Notes: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are clustered at Pc4*year level. Column (1) shows the estimation results specified in equation 27, column (2) includes pc4*year fixed effects, and column (3) includes municipality*year fixed effects. Columns (4) to (6) include the interaction of whether the property is an apartment and the year trend. The regression results in this table are based on panel data from 2011 to 2017. Some properties withdrew from the treatment group in 2017, which may cause estimation bias. However, we still include this to prove the robustness of our baseline results.

Table A6: Shorter sample

Private rent	(1) TWFE	(2) Pc4 trend	(3) Gem trend	(4) TWFE	(5) Pc4 trend	(6) Gem trend
violate	0.008***	0.007***	0.008***	0.005***	0.005***	0.005***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
apartment*year				0.017***	0.014***	0.016***
				(0.000)	(0.000)	(0.000)
Observations	444,012	441,900	444,000	444,012	441,900	444,000
R-squared	0.397	0.425	0.404	0.405	0.428	0.409
Property F.E.	Y	Y	Y	Y	Y	Y
Year F.E.	Y	Y	Y	Y	Y	Y
Pc4*Year F.E.		Y			Y	
Gem*Year F.E.			Y			Y

Notes: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are clustered at Pc4*year level. Column (1) shows the estimation results specified in equation 27, column (2) includes pc4*year fixed effects, and column (3) includes municipality*year fixed effects. Columns (4) to (6) include the interaction of whether the property is an apartment and the year trend. The results in this table are based on transaction samples from 2010 and the first two months of 2011. This is to avoid the anticipated effect of change of the Dutch mortgage code of conduct announced in February 2011.

B.2 Figures

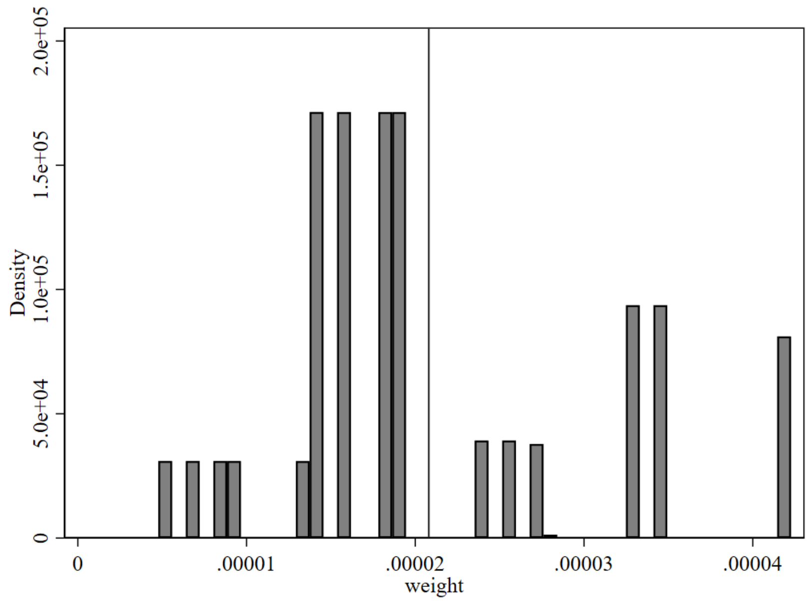


Figure A2: Weight distribution of each treatment

Notes: This is the weight associated with each treatment. The vertical line is the average weight. This is calculated through the command *twowayfeweights* in STATA 16.0.

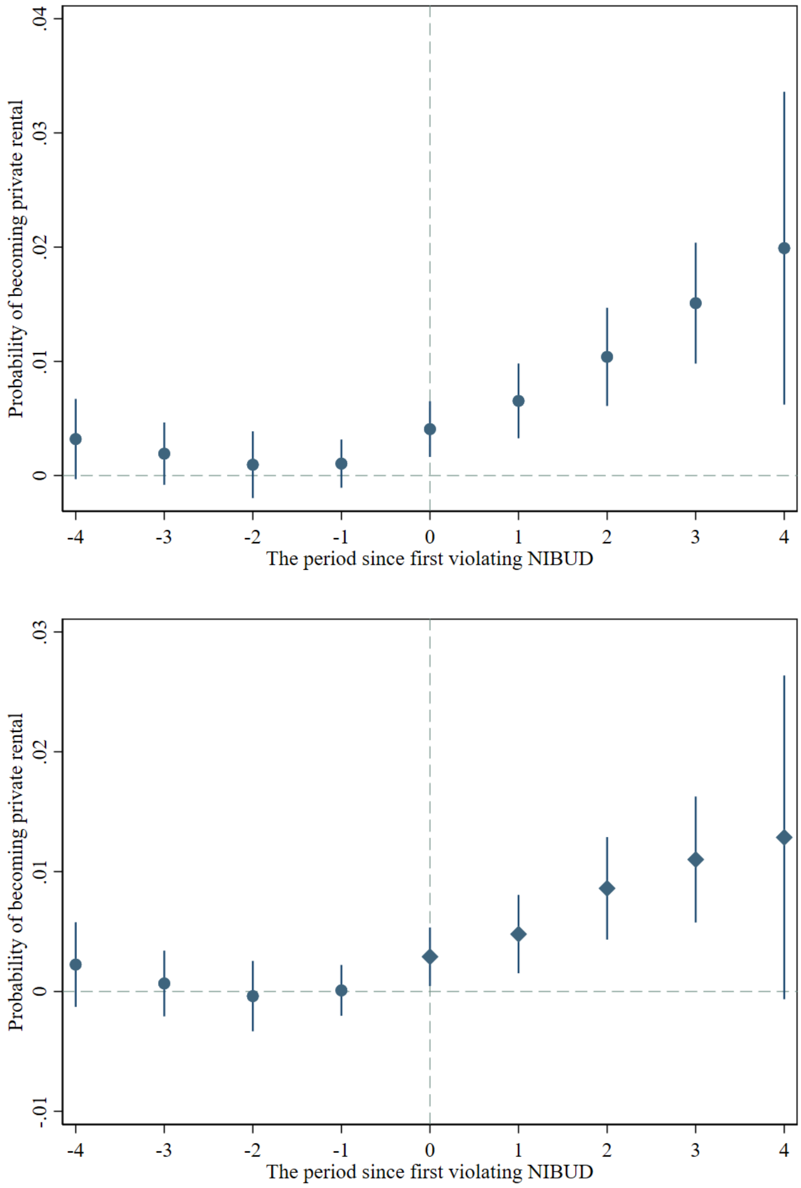


Figure A3: Event study using CSDID

Note: This figure is a coefficient plot and 95% confidence interval of the estimation results in equation 28. Standard errors are clustered at house level. The graph above is the coefficient plot without apartment*year, and the graph below is the coefficient plot with apartment*year. The graph is estimated by csdid, with both never treated and not-yet-treated as control group.

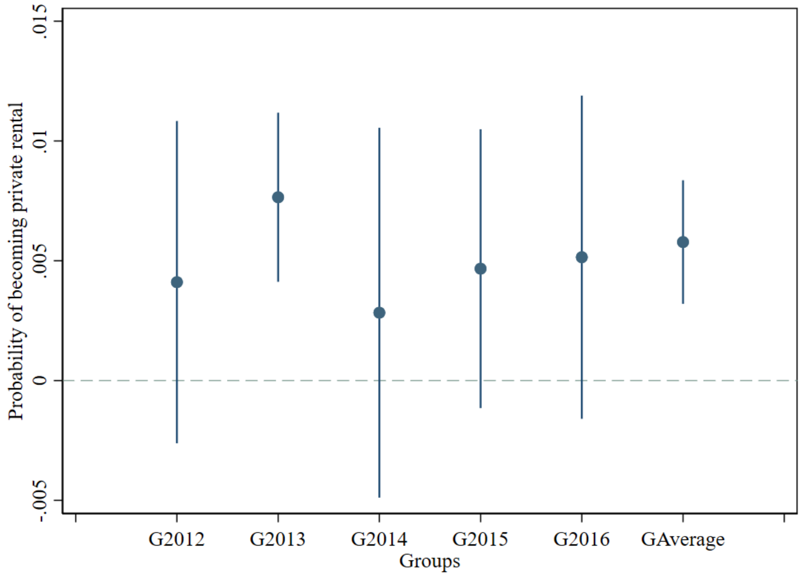


Figure A4: Treatment effects by group

Note: This is the treatment effect decomposed by group according to the year of entering the treatment. The aggregated average treatment effect is 0.006, which is the same as the baseline estimate.

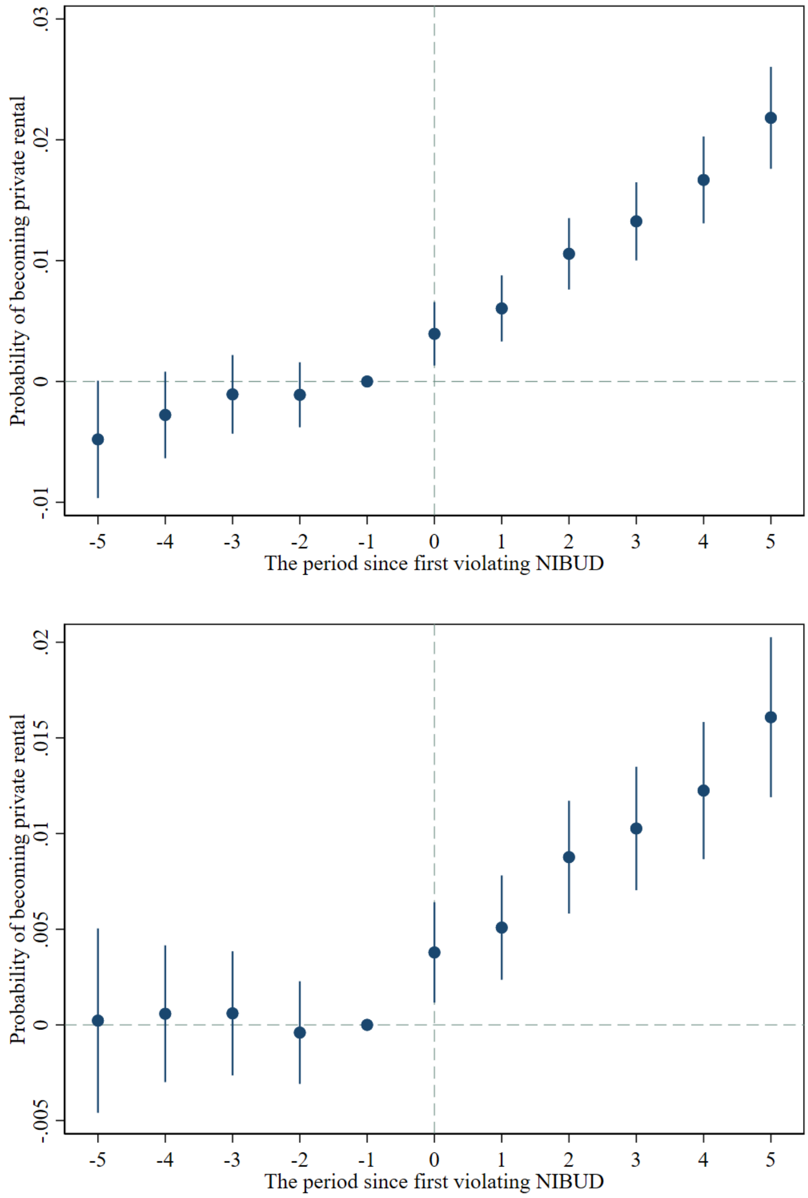


Figure A5: Event study of keeping always treated sample

Note: This figure is a coefficient plot and 95% confidence interval of the estimation results in equation 28. Standard errors are clustered at Pc4*year level. The graph above is the coefficient plot without apartment*year, and the graph below is the coefficient plot with apartment*year. The plots are based on the baseline sample and the sample of properties that violated the NIBUD rule in 2011.

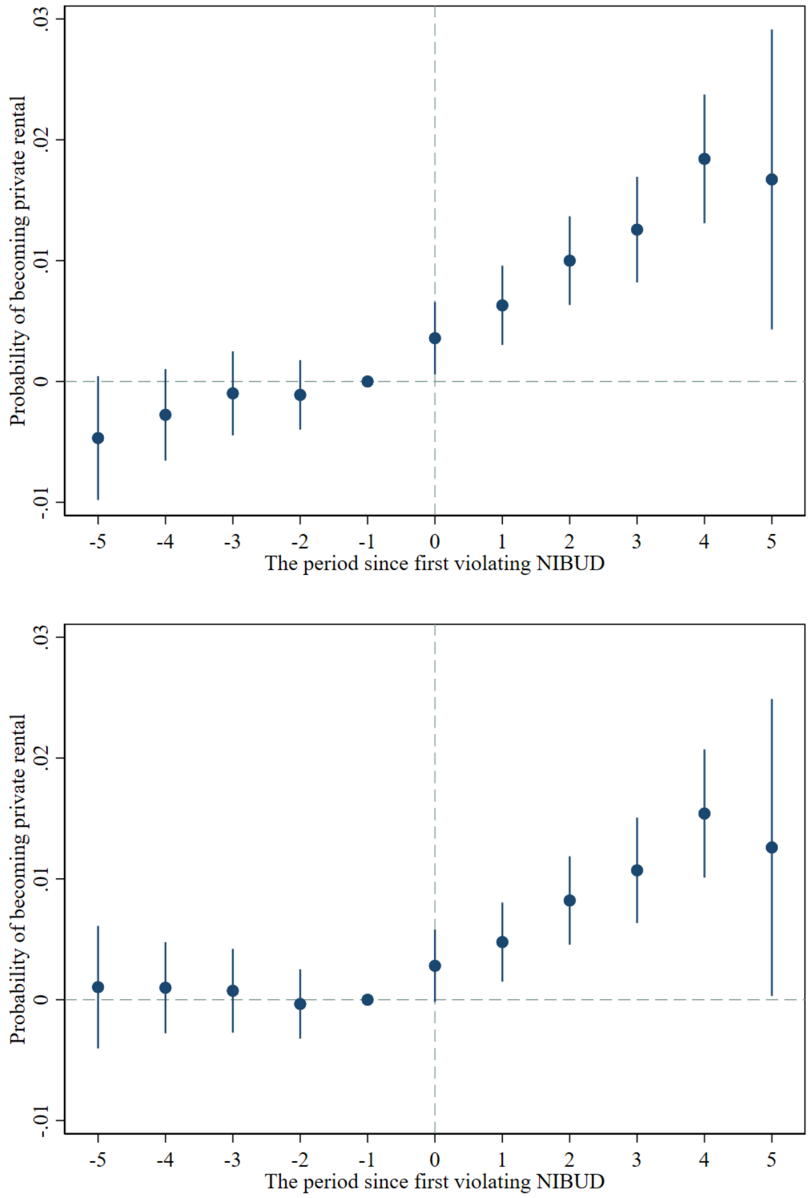
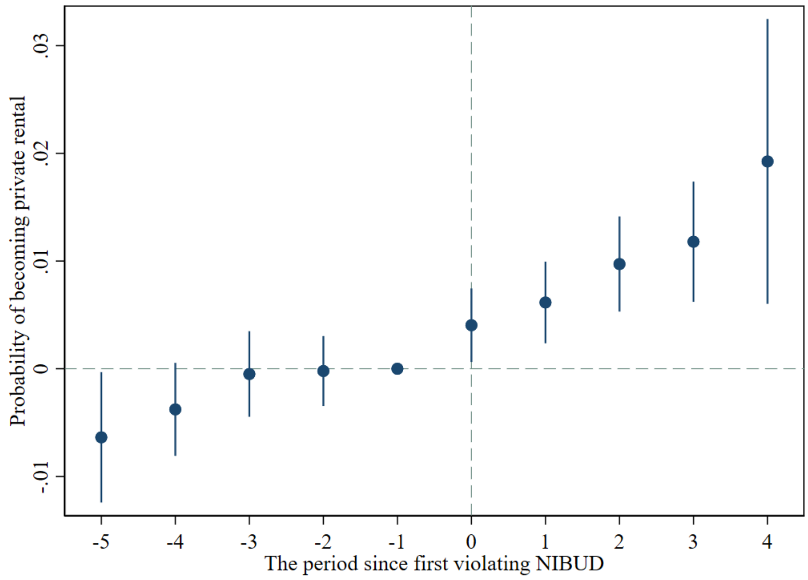


Figure A6: Event study of sample period from 2011 to 2017

Note: This figure is a coefficient plot and 95% confidence interval of the estimation results in equation 28. Standard errors are clustered at Pc4*year level. The graph above is the coefficient plot without apartment*year, and the graph below is the coefficient plot with apartment*year. The plots are based on panel data from 2011 to 2017.



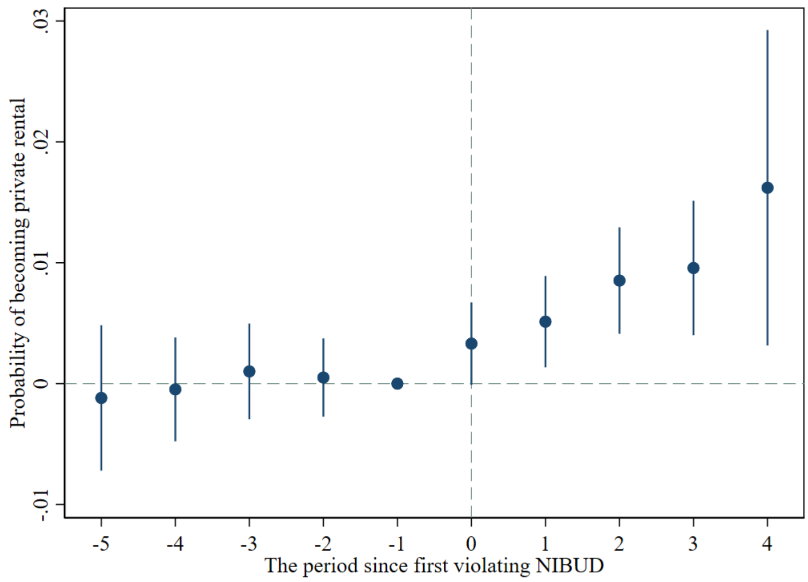


Figure A7: Event study of shorter sample

This figure is a coefficient plot and 95% confidence interval of the estimation results in equation 28. Standard errors are clustered at Pc4*year level. The graph above is the coefficient plot without apartment*year, and the graph below is the coefficient plot with apartment*year. The plots are based on transaction samples from 2010 and the first two months of 2011.