60 = Zikna(polyakov before firewalls)
60+60= Saturates the Moses Bound.

C = 1 Conformal Field Theories on Riemann Surfaces

Robbert Dijkgraaf (Utrecht U.), Erik P. Verlinde (Utrecht U.), Herman L. Verlinde (Utrecht U.)

Jul, 1987


Enlarge

Two Mysteries (at least)

ON INDUCED GRAVITY IN 2-d TOPOLOGICAL THEORIES

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S.ELITZUR and E.RABINOVICI *

[7] H.Verlinde, String theory and quantum gravity, ICTP, 1991; World Scient. p. 178 1992, Sixth Marcel Grossman Conference Meeting on General Relativity M.Sato, ed. (World Scientific, Singapore, 1992)

Persistent symmetry breaking at high temperatures

Eliezer Rabinovici

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Amsterdam July 14th 2022

arXiv no.: 2005.03676 + PRL 125,131603(2020)
With: Noam Chai, Soumyadeep Chaudhuri, Changha Choi, Zohar Komargodski, Michael Smolkin

arXiv no.: 2011.13981
With: Soumyadeep Chaudhuri, Changha Choi

arXiv no.: 2106.11323
With: Soumyadeep Chaudhuri

Why in this meeting? Test and Live

$$S(\lambda) = \frac{\Gamma(1+2i\lambda)\Gamma^2(\frac{1}{2}-4i\lambda)}{\Gamma(1-2i\lambda)\Gamma^2(\frac{1}{2}-2i\lambda)}$$

Send Eric and Herman Protected to test the Firewall or to the Sun

IDEE FIXE

VACUUM ENERGY IN CFTS WHICH POSSES FLAT DIRECTIONS

Conformal/scale invariance and the cosmological constant/vacuum energy

- A Zel'dovich type argument for a problem: consider QED in the 40s. One has a proton, an electron and a photon. One understands physics on a Rydberg scale R_Y. Ergo in an effective field theory, valid up to R_Y, the value of the cosmological constant would be at least R_Y^4. This is a problem.

- Loophole: an effective low energy theory needs to include all symmetries, be they either explicit or spontaneously broken. This is true also if the underlying theory of nature has no scale.

- Claim: in a conformal/scale invariant theory the value of the vacuum energy does not depend on the theory being spontaneously broken or not.

- The scale of the breaking of the symmetry does not affect the vacuum energy.

Illustrative example: consider

$$\mathcal{L} = \frac{1}{2}\partial_\mu\vec{\phi}\partial_\nu\vec{\phi} + \frac{1}{6N^2}g_6(\vec{\phi}^2)^3$$

where the fields $\vec{\phi}$ are in the vector representation of $O(N)$.

In the limit $N \to \infty$ (Bardeen, Bander, Moshe)

$$\beta_{g_6} = 0$$

(1/N corrections break conformality.)

$$\sigma = \vec{\phi}^2$$

$$V(\sigma) = f(g_6)|\sigma|^3$$

- Let's discuss the possible outcomes for a general $f(g_6)$.

(Amit, Rabinovici) ***(BARDEEN, RABINOVICI, SAERING)***

$f(g_6)$	$\|\langle\sigma\rangle\|$	S.B.	masses	V
$f(g_6) > 0$	0	No	0	0
$f(g_6) < 0$	∞	Yes, but ill defined	Tachyons or masses of order the cutoff	$-\infty$
$f(g_6) = 0$	0	No	0	0
$f(g_6) = 0$	$\neq 0$	Yes	Massless dilaton, N particles of equal mass	0

Are there D>3
Non SUSY systems
With flat quantum directions?

Or:

is there a NO GO Theorem?

And: the cosmological constant is small

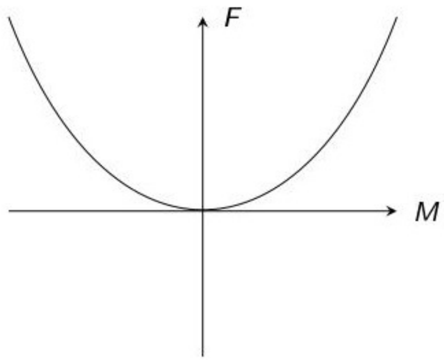
but not zero, where is the Dilaton?

Symmetry breaking: different phases

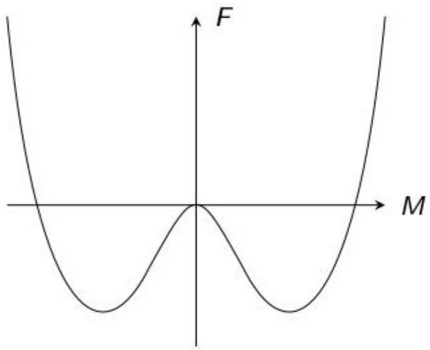
- Spontaneous breaking of global symmetries plays a very important role in characterizing the phases of systems.
- Such phases are typically determined by the value of an order parameter (say, M).
- $M = 0 \Rightarrow$ unbroken symmetry, disordered phase,
  $M \neq 0 \Rightarrow$ broken symmetry, ordered phase.
- The value of M is determined by the minimum of the free energy of the system.

Free energy for different phases

- The free energy (F) can behave in the following two ways near $M = 0$. They usually correspond to two distinct phases.

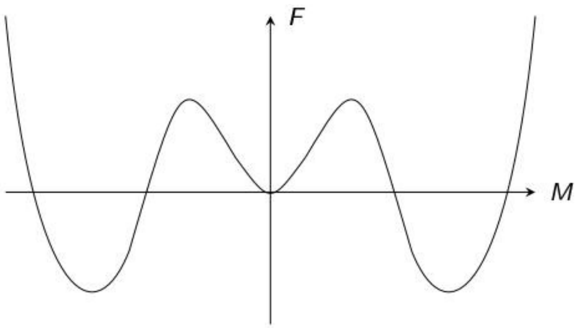


(a) symmetry-unbroken phase



(b) symmetry-broken phase

- It is also possible that although there is a local minimum at $M = 0$, the global minimum is at some nonzero M.



- In this case the symmetry is broken despite there being a local minimum at $M = 0$.

Free energy at different temperatures

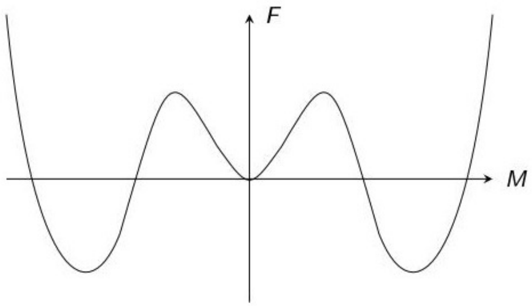
- At different temperatures the free energy can behave differently.
- The most trivial case would be one where the global minimum lies at $M = 0$ for all temperatures. Then the symmetry remains unbroken persistently.
- Another case is that of a symmetry being broken at low temperatures, but getting restored at some critical temperature (T_c).

  $\longrightarrow$ Familiar examples: demagnetization above Curie temperature, melting of solids, etc.

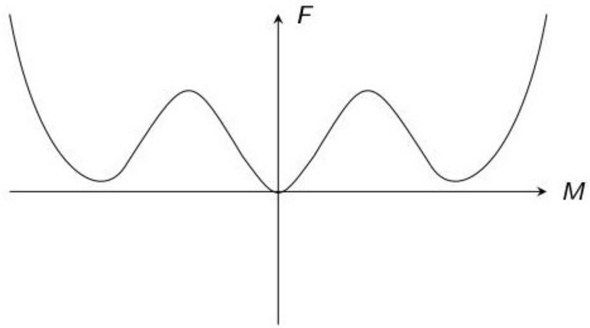
- Such a phase transition can be either first order or second order.

First and second order phase transitions: Usual scenario

- First order phase transition:

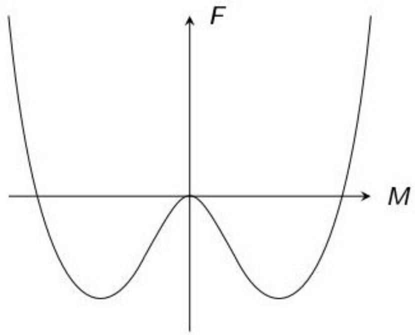


(a) $T < T_c$

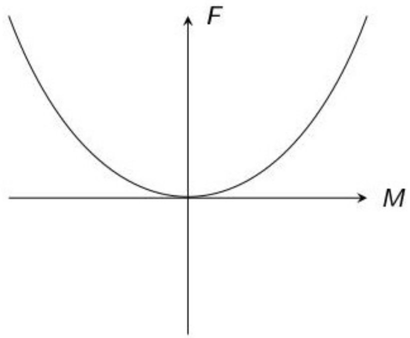


(b) $T > T_c$

- Second order phase transition:



(a) $T < T_c$



(b) $T > T_c$

Inverse symmetry breaking

- There are systems where a symmetry that is unbroken at sufficiently low temperatures gets broken at a nonzero critical temperature T_{c1}.

  $\longrightarrow$ Inverse symmetry breaking

  Eg. Rochelle salt (Potassium sodium tartrate), certain Liquid crystals,etc.

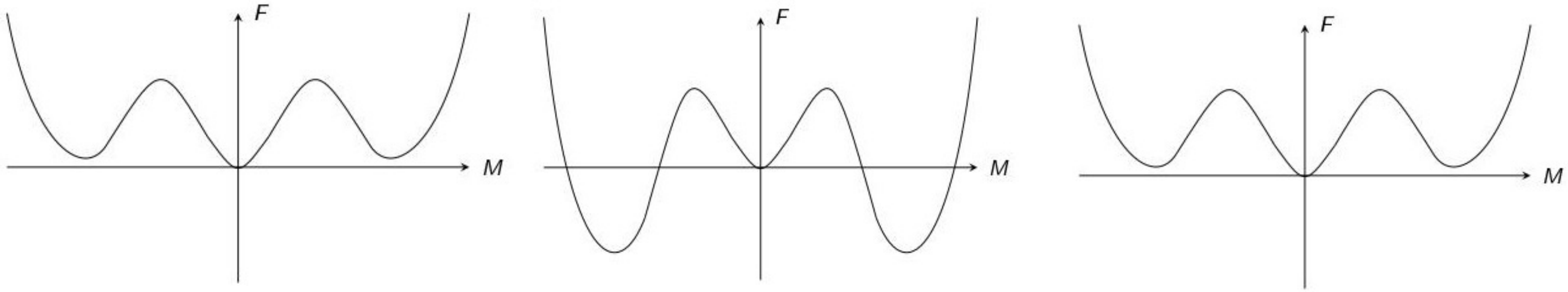
- The broken symmetry gets restored at an even higher critical temperature T_{c2}.

- For instance, in the case of the Rochelle salt, the crystalline structure changes as

$$\text{orthorhombic} \xrightarrow{-18^0 C} \text{monoclinic} \xrightarrow{24^0 C} \text{orthorhombic}$$

  The phase transitions are first order.

- At different temperatures the free energy behaves as



(a) $T < T_{c1}$ (b) $T_{c1} < T < T_{c2}$ (c) $T > T_{c2}$

Weinberg's model

- In all known cases symmetries are unbroken in the high temperature limit.

  Even if a symmetry is broken at low temperatures, it gets restored when the temperature is raised sufficiently.

- Question: Is this behavior universal?

- The universality of symmetry restoration was challenged by Weinberg in 1974(also Landau).

- He considered a 4-dimensional $Z_2 \times Z_2$-symmetric model consisting of two real scalars : ϕ_1 and ϕ_2 . A SINGLE GROUP DOES NOT EXHIBIT THIS BEHAVIOR!

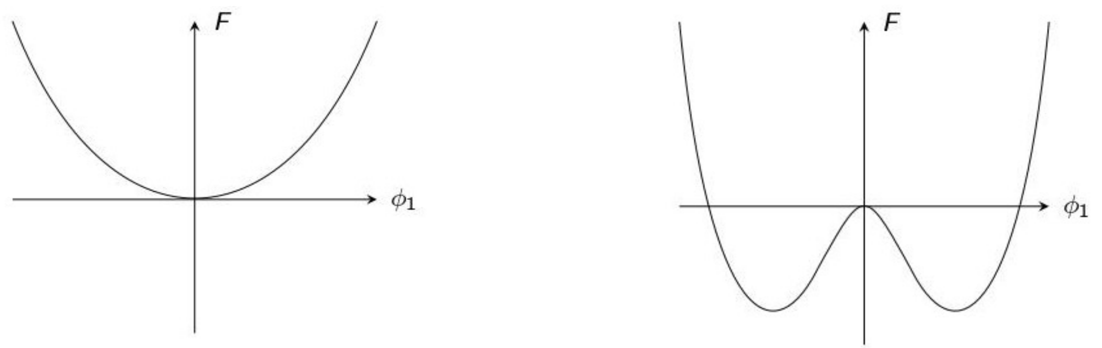
- The potential is given by

$$V = \frac{1}{2}\mu_i^2\varphi_i^2 + \frac{1}{4}g_{ij}\varphi_i^2\varphi_j^2$$

with $g_{12} = g_{21}$ and $\mu_i^2 > 0$.

Inverse symmetry breaking in Weinberg's model

- When $(3g_{11} + g_{12}) < 0$, the $\phi_1 \to -\phi_1$ symmetry is spontaneously broken for $T \gg \mu_{1,2}$.
- The $\phi_2 \to -\phi_2$ symmetry remains unbroken.
- The symmetry breaking occurs at a critical temperature T_c.
- The free energy (for $\phi_2 = 0$) at different temperatures behaves as



(a) $T < T_c$ (b) $T > T_c$

- A similar result was demonstrated by Weinberg for an $O(N) \times O(N)$ vector model as well.
  A SINGLE GROUP DOES NOT EXHIBIT THIS BEHAVIOR!

UV-incompleteness

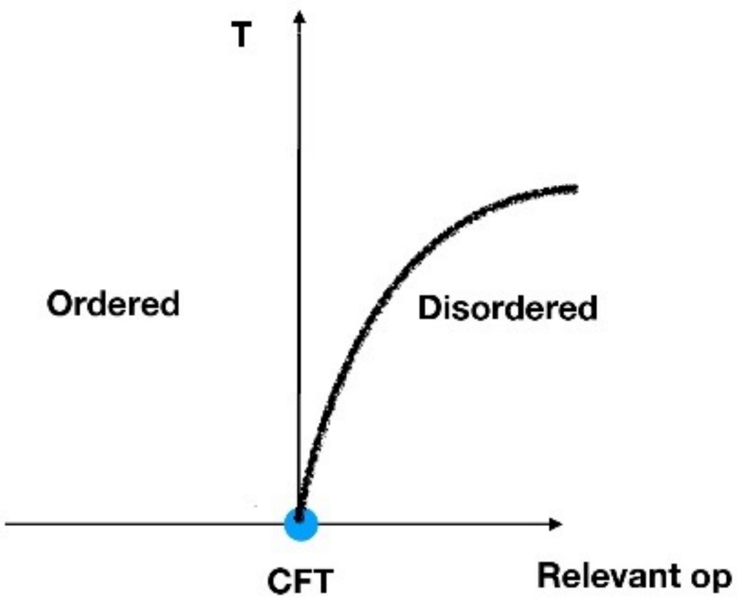
- The symmetry breaking in these models persists up to very high temperatures.
- However, the theories are UV-incomplete.

  $\longrightarrow$ Nothing conclusive can be said about symmetry restoration beyond a temperature scale.
- Many other works done subsequently suffer from the same limitation.

Our approach

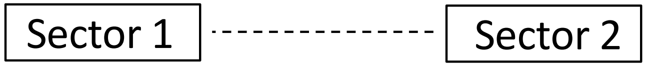
- We resolve this issue of UV-incompleteness by considering CFTs.
- Since a CFT has no intrinsic scale, if a(zero form) symmetry is broken at any nonzero temperature, it remains broken at all temperatures.
- I will describe certain Wilson-Fisher-like CFTs in fractional dimensions which exhibit such persistent symmetry breaking.
- The symmetry group in these models is $O(N_1) \times O(N_2)$.
  A SINGLE GROUP DOES NOT EXHIBIT THIS BEHAVIOR!
- IN the $N_{1,2} \to \infty$ limit different patterns of persistent symmetry breaking is possible.
- I will also discuss some effects of finite N corrections.
- I will mention certain 4-dimensional large N conformal gauge theories which also exhibit persistent symmetry breaking.
- I will also mention some marginally relevant deformations of these 4-dimensional large N CFTs. These deformations define systems which exhibit symmetry non-restoration in the high temperature limit.

Potential Phase diagram for persistent symmetry breaking- YES.



A common feature of these theories

- The Wilson-Fisher-like CFTs and the 4-dimensional models share the following common structure:



- There are two distinct sectors in the model.
  A SINGLE GROUP.....
  $\longrightarrow$ Each sector has its own global and gauge symmetry groups.

  $\longrightarrow$ The matter in each sector transforms only under the global and gauge symmetry groups corresponding to that sector.
- The matter in each sector is weakly coupled.
- In addition, there is a weak coupling between the two sectors (shown by the dashed line).
- This structure seems to be necessary to get persistent symmetry breaking.

  Up till now all models that have exhibited persistent symmetry breaking are of this kind. WHY???

Our model in $(4-\epsilon)$ dimensions

- Let us now look at a theory in $(4-\epsilon)$ dimensions which has the above structure.
- Consider two scalar fields ϕ_1 and ϕ_2 with m and $(N-m)$ real components respectively. Here $1 \ll m < N$.
- An action that is symmetric under $O(m) \times O(N-m)$:

$$S = \int d^d x \left(\tfrac{1}{2}\partial_\mu \varphi_i \partial^\mu \varphi_i - \tfrac{g_{ij}}{4N}\varphi_i^2 \varphi_j^2\right)$$

where $d = 4 - \epsilon$ with $\epsilon \ll 1$.

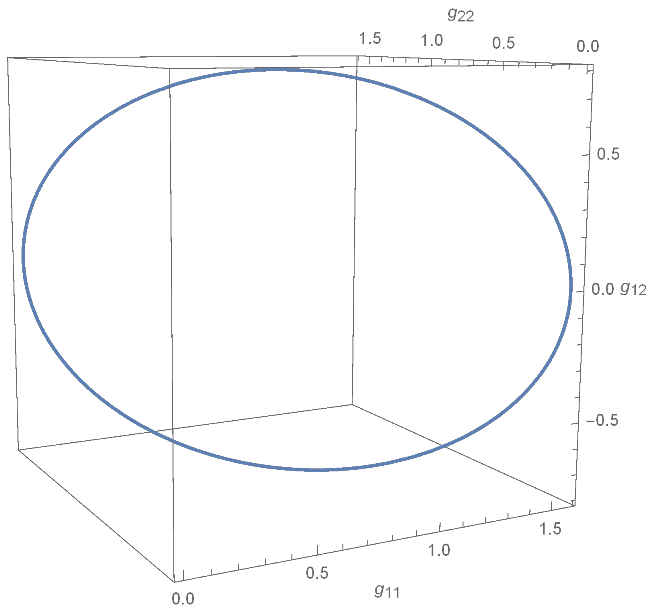
- I will first describe the case where $m, N \to \infty$ while m/N is fixed at a finite value.
- Then the case where m and N are large but finite, and $\epsilon \ll \frac{1}{N}$.

Equal rank case: conformal manifold in the planar limit

- First let us consider the case where the ranks of the two orthogonal groups are equal, i.e., $m = \frac{N}{2}$.
- In addition, let us for now set $N \longrightarrow \infty$.
- In this planar limit, the model has a conformal manifold which is given by

$$\det g_{ij} = 0,\ g_{11} + g_{22} = 16\pi^2\epsilon,$$

- This is a closed curve in the space of couplings



Conformal manifold for $\epsilon = 0.01$

Equal rank case, planar limit: moduli space of vacua at $T=0$

- For any point on this conformal manifold, one can consider the minima of the effective potential of the scalar field at zero temperature.

- These minima are given by

$$\begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \end{pmatrix} = \begin{pmatrix} \pm\sqrt{g_{22}/g_{11}} \\ 1 \end{pmatrix} \mu^{2-\epsilon} \quad \text{for} \quad \mathrm{sgn}(g_{12}) = \mp 1.$$

Here $\sigma_i^2 = \varphi_i^2/N$ and μ is an arbitrary energy scale.

- When $g_{12} \geq 0$, the only admissible vacuum is $\sigma_1^2 = \sigma_2^2 = 0$.

$\longrightarrow$ Both orthogonal groups are unbroken.

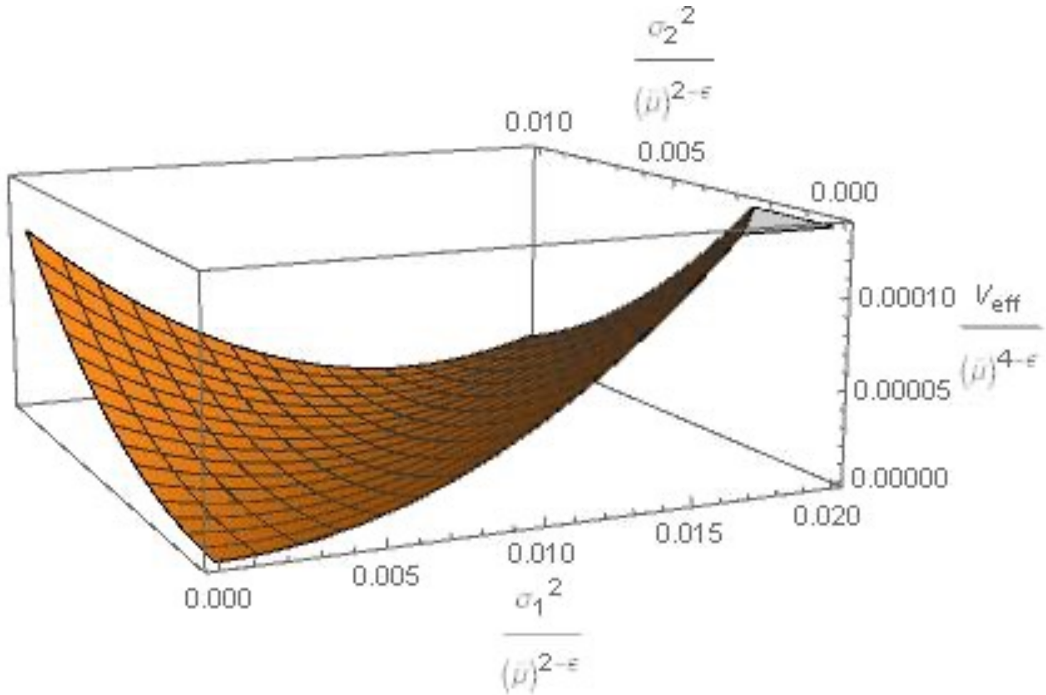
- When $g_{12} < 0$, we get a moduli space of vacua parametrized by μ !!!.

- At the $\mu = 0$ vacuum, $\sigma_1^2 = \sigma_2^2 = 0 \Rightarrow$ Both orthogonal groups are unbroken.

- At all other vacua, $\sigma_{1,2}^2 \neq 0 \Rightarrow$ Both orthogonal groups are spontaneously broken.

Equal rank case, planar limit: effective potential at T=0

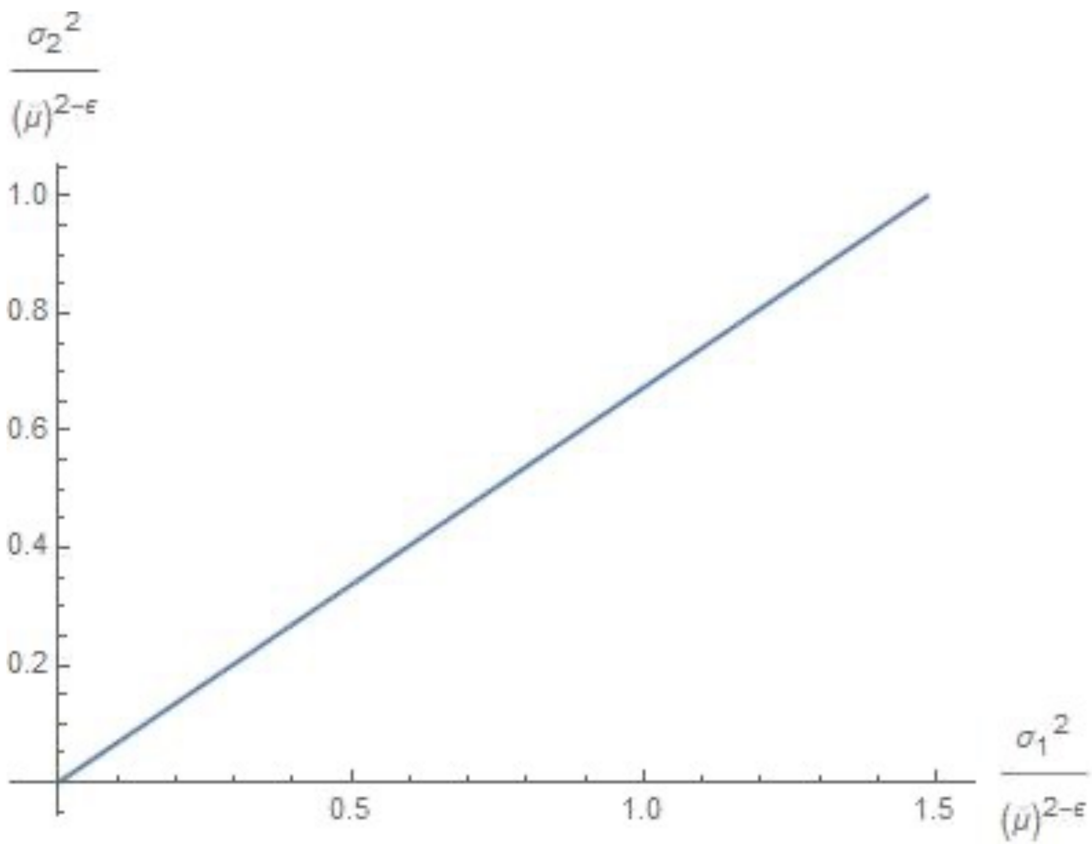
- The effective potential at T=0 for the fixed point $g_{11} = 5\,\pi^2\varepsilon$, $g_{22} = 11\,\pi^2\varepsilon, g_{12} = -\sqrt{g_{11}g_{22}}$ with $\varepsilon = 0.01$



- Here $\bar{\mu}$ is an arbitrary scale introduced to account for the dimensions of the different quantities.

Equal rank case, planar limit: moduli space of vacua at T=0

- The moduli space of vacua at T=0 for this fixed point:



Massless and massive modes in the symmetry-broken phase

- The spontaneous breaking of the orthogonal groups lead to the generation of massless Nambu-Goldstone bosons.
- The scale invariance is also broken due to the VEV $\Rightarrow$ presence of a dilaton.
- The dilaton corresponds to the flat direction of the potential.
- In addition to these massless modes, there is also a massive mode.
- This mass is generated due to the VEV.

Equal rank case, planar limit: moduli space of vacua at $T \neq 0$

- At nonzero temperatures, the orthogonal groups are unbroken when $g_{12} \geq 0$ just as in the $T = 0$ case.

- When $g_{12} < 0$, the thermal vacua are given by

$$\begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \end{pmatrix} = \begin{pmatrix} \sqrt{g_{22}/g_{11}} \\ 1 \end{pmatrix} \mu^{2-\epsilon} - \frac{c(\epsilon) T^{2-\epsilon}}{24} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

where

$$c(\epsilon) = \frac{6\Gamma(\frac{2-\epsilon}{2})\,\zeta(2-\epsilon)}{\pi^{\frac{4-\epsilon}{2}}} > 0 \quad \text{for } 0 < \epsilon < 1.$$

- Note that the moduli space of vacua survives at nonzero temperatures.

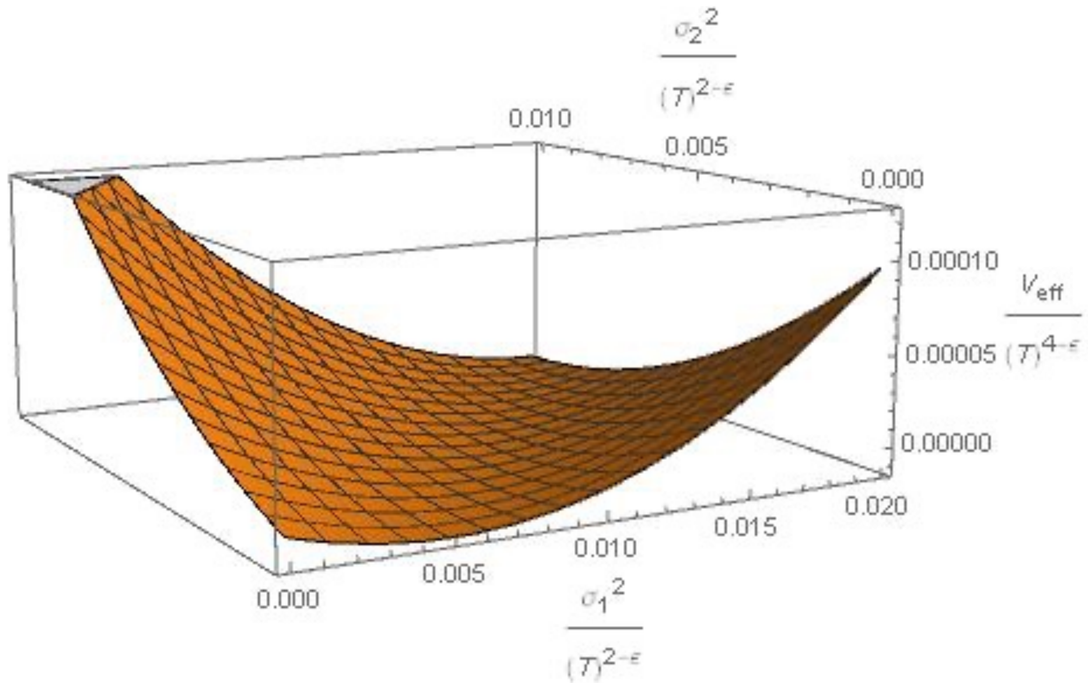
- However, it does not necessarily pass through the point $\sigma_1^2 = \sigma_2^2 = 0$.

- In cases where it does not pass through $\sigma_1^2 = \sigma_2^2 = 0$, at least one of the orthogonal groups is broken.

- This provides an example of persistent symmetry breaking in the $N \to \infty$ limit.

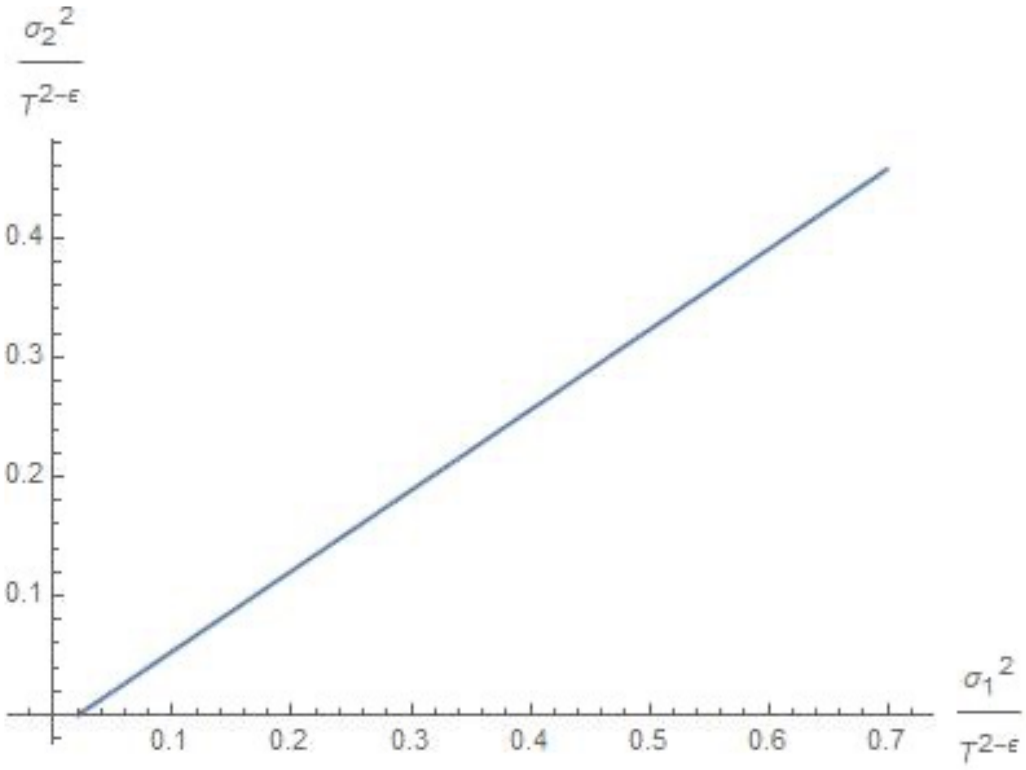
Equal rank case, planar limit: effective potential at T≠0

- The effective potential at $T \neq 0$ for the same fixed point:



Equal rank case, planar limit: moduli space of vacua at T $\neq$ 0

- The moduli space of vacua at T $\neq$ 0 for this fixed point:



Equal rank case, finite N corrections

- Let us now look at the case when N is large but finite, and $\epsilon \ll 1/N$.
- The finite N corrections lift the degeneracy in the fixed points.
- Only a discrete set of fixed points survive.
- There are fixed points where $g_{12} = 0$.

  $\rightarrow$ Each sector can be either at the Gaussian fixed point or the Wilson-Fisher fixed point.
- There is also a fixed point where $g_{11} = g_{22} = -g_{12} \approx 8\pi^2\epsilon$.
- As I have already mentioned, for the fixed points where $g_{12} \geq 0$, there is no symmetry breaking at any temperature.

Equal rank case, finite N corrections

- The only other fixed point is at $g_{12} \approx -8\pi^2\epsilon$, $g_{11} = g_{22} \approx 8\pi^2\epsilon$.

- Comment: In the $N \to \infty$ limit the moduli space of thermal vacua for a fixed point with $g_{11} = g_{22}$ is given by

$$\begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \end{pmatrix} = \left(\mu^{2-\epsilon} - \frac{c(\epsilon)T^{2-\epsilon}}{24}\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

  This space passes through the point $\sigma_1^2 = \sigma_2^2 = 0$.

- In fact, the finite N corrections lift the flat direction.

- Only the point $\sigma_1^2 = \sigma_2^2 = 0$ is left as the true thermal vacuum.

  $\longrightarrow$ No symmetry breaking at any temperature.

Unequal rank case: conformal manifold in the planar limit

- Next, let us consider the case where the ranks of the two orthogonal groups are unequal, i.e., $m \neq \frac{N}{2}$.

- Again, let us first set $N \longrightarrow \infty$ with $x \equiv m/N$ kept finite.

- In this limit, the model has a CONFORMAL MANIFOLD which is given by

$$\det g_{ij} = 0, \; x g_{11} + (1 - x) g_{22} = 8\pi^2 \epsilon \, .$$

- This traces an ellipse in the space of couplings.

Unequal rank case, planar limit: moduli space of vacua

- For the points on the conformal manifold, the minima of the effective potential of the scalar field at zero temperature are similar to those in the equal rank case.
- At nonzero temperatures, the orthogonal groups are unbroken when $g_{12} \geq 0$ just as in the equal rank case.
- When $g_{12} < 0$, the thermal vacua are given by

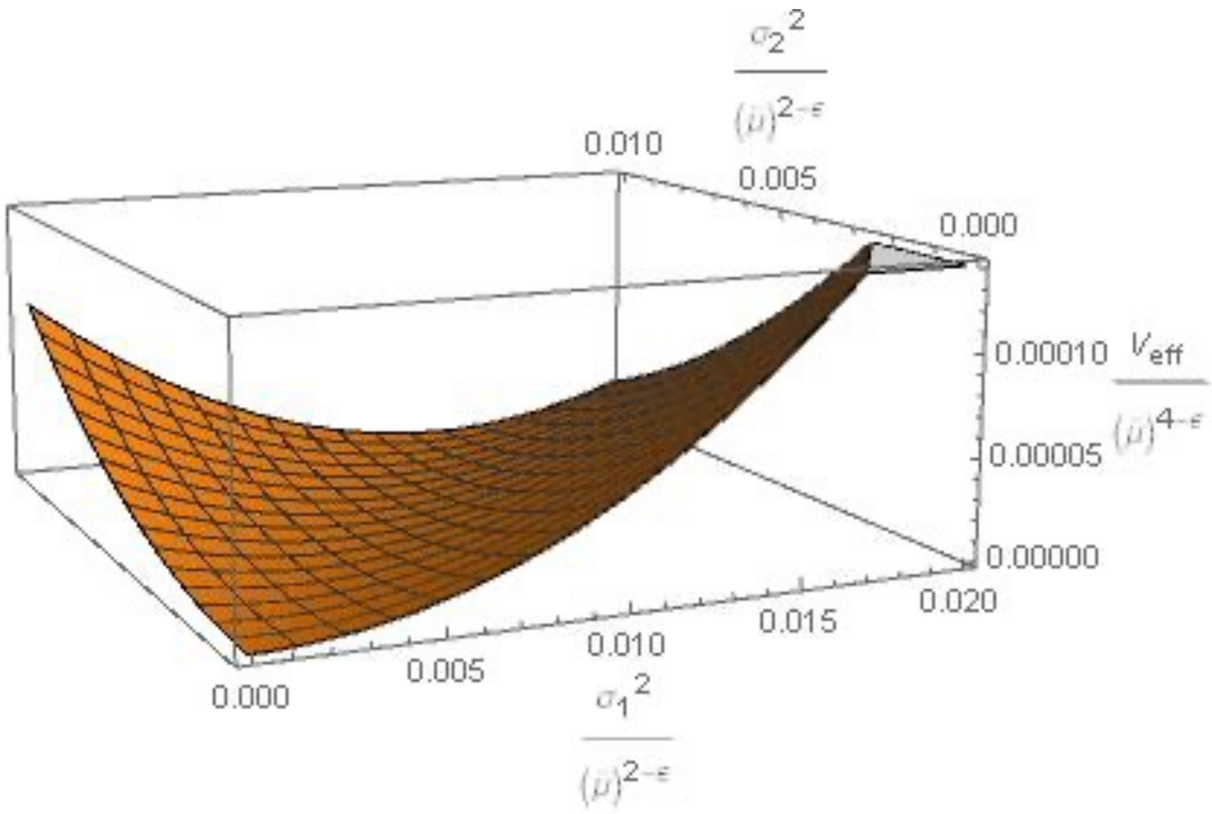
$$\begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \end{pmatrix} = \begin{pmatrix} \sqrt{g_{22}/g_{11}} \\ 1 \end{pmatrix} \mu^{2-\epsilon} - \frac{c(\epsilon) T^{2-\epsilon}}{12} \begin{pmatrix} x \\ 1-x \end{pmatrix} .$$

- If $\sqrt{g_{22}/g_{11}} \neq \frac{x}{1-x}$, the moduli space does not pass through $\sigma_1^2 = \sigma_2^2 = 0$.

  $\longrightarrow$ At least one of the orthogonal groups is broken.

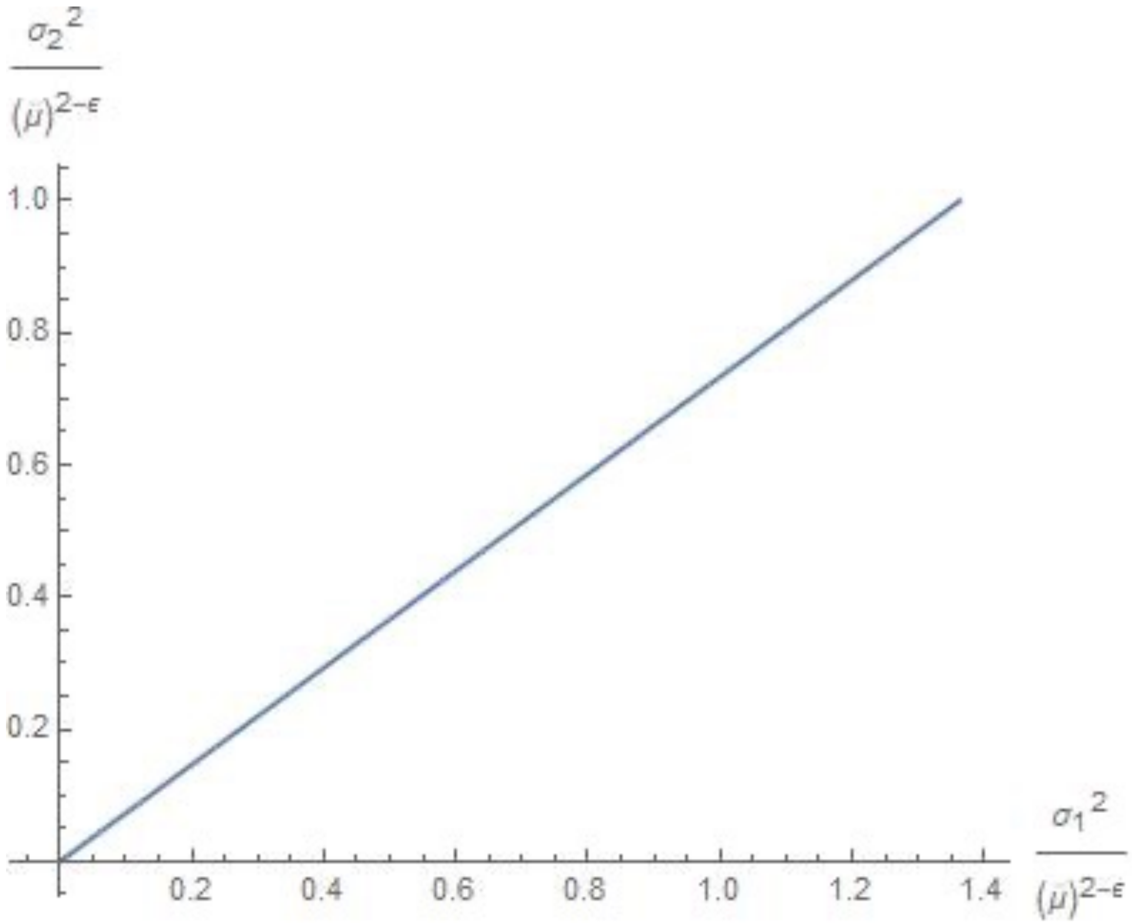
Unequal rank case, planar limit: effective potential at T=0

- The effective potential at T=0 for $x = 0.3, \varepsilon = 0.01$ and the fixed point $g_{11} = 5\,\pi^2\varepsilon,\ g_{22} = \frac{65}{7}\,\pi^2\varepsilon,\ \ g_{12} = -\sqrt{g_{11}g_{22}}$



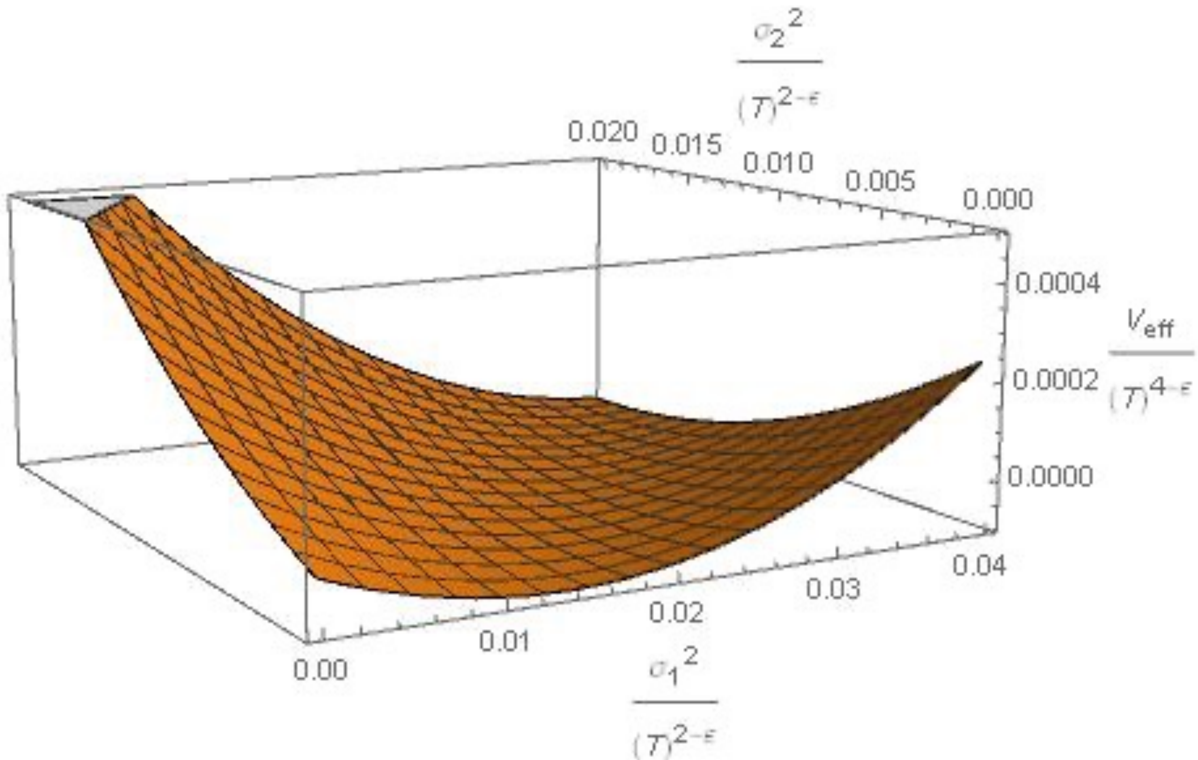
Unequal rank case, planar limit: moduli space of vacua at T=0

- The moduli space of vacua at T=0 for this fixed point:



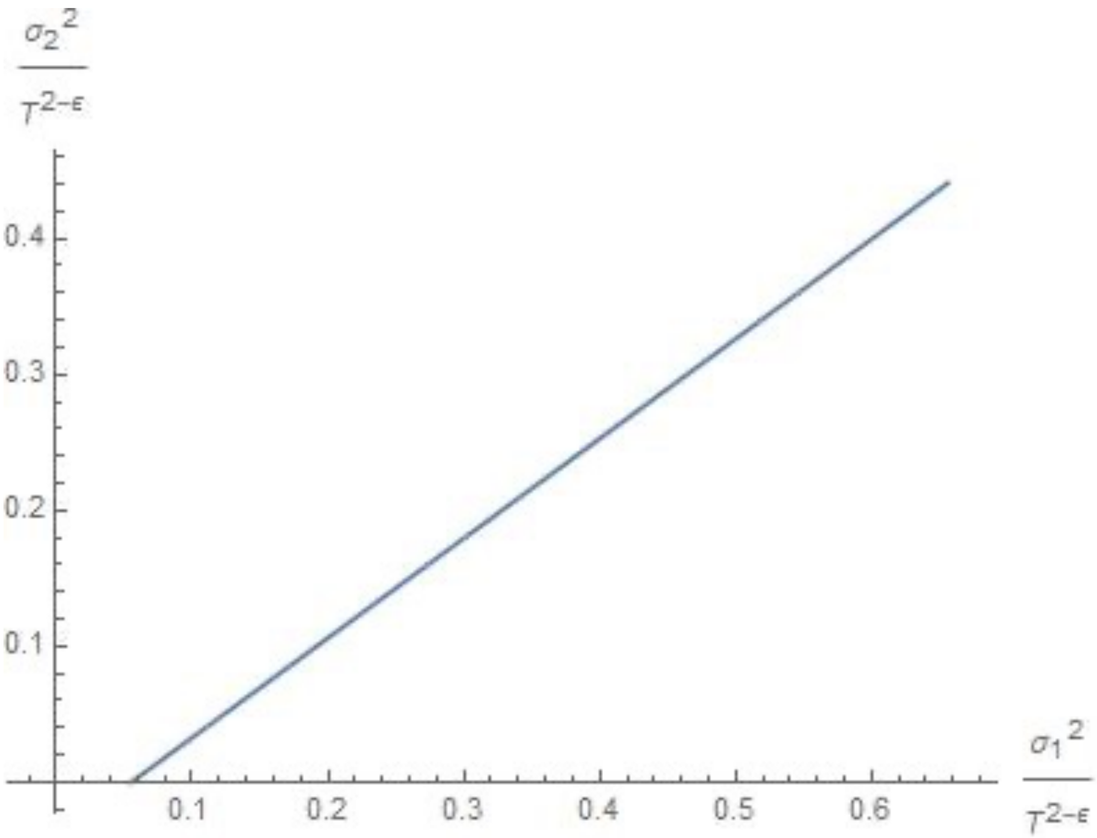
Unequal rank case, planar limit: effective potential at T≠0

- The effective potential at $T \neq 0$ for the same fixed point:



Unequal rank case, planar limit: moduli space of vacua at $T \neq 0$

- The moduli space of vacua at $T \neq 0$ for this fixed point:



Unqual rank case, finite N corrections

- As in the equal rank case, finite N corrections lift the degeneracy in the fixed points.

  $\longrightarrow$ Only a discrete set of fixed points survive.

- For the fixed points where $g_{12} \geq 0$, there is no symmetry breaking at any temperature.

- The only fixed point where $g_{12} < 0$:

$$g_{11} = \frac{1+\mathrm{sgn}\left(x-\frac{1}{2}\right)\sqrt{1-4x(1-x)t_*^2}}{x},$$

$$g_{22} = \frac{1-\mathrm{sgn}\left(x-\frac{1}{2}\right)\sqrt{1-4x(1-x)t_*^2}}{1-x},$$

$$g_{12} = 2\, t_* \, .$$

Here t_* is the only negative real solution of the equation:

$$4\, x(1-x)t^3 - 20\, x(1-x)t^2 + 3t + 9 = 0.$$

The other 2 solutions are complex.

Unqual rank case, finite N corrections

- The finite N corrections also lead to the existence of a unique thermal vacuum.
- Without loss of generality, we can take the first sector to have the large rank, i.e., $\frac{1}{2} < x < 1$.
- The thermal vacuum in this case:

$$(\sigma_1^2, \sigma_2^2) \approx \left(0\,, \frac{2t_*^2 x(1-x) + t_*(-2x^2+5x-3) - 3x}{12t_*(3-4x(1-x)t_*)} T^{2-\epsilon}\right).$$

- At $T = 0$ both the orthogonal groups are unbroken.
- At $T \neq 0$ the orthogonal group in the larger sector remains unbroken.
- At $T \neq 0$ the orthogonal group in the smaller sector is spontaneously broken.

$\longrightarrow$ Evidence of a symmetry-broken phase in a CFT at all nonzero temperatures.

Problem with unitarity and extension to d=3

- The existence of such symmetry-broken phase at all nonzero temperatures in $d = (4 - \epsilon)$ dimensional CFTs raises the question of whether this can be extended to $d = 3$.

- This is particularly relevant as such CFTs in fractional dimensions may not be unitary due to the possible existence of states with negative norms.
  (Hogervorst, Rychkov and van Rees, arXiv:1512.00013)

- However, in $d = 3$, there exists a general no-go theorem (Mermin-Wagner-Coleman-Hohenberg) which precludes symmetry breaking at nonzero temperatures in local QFTs.

- The large N analysis should break down as $\epsilon \longrightarrow 1$.(also 0)

Problem with unitarity and extension to d=3

- We can see that there is indeed such a breakdown from the thermal vacua in the $N \to \infty$ limit:

$$\begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \end{pmatrix} = \begin{pmatrix} \sqrt{g_{22}/g_{11}} \\ 1 \end{pmatrix} \mu^{2-\epsilon} - \frac{c(\epsilon)T^{2-\epsilon}}{12} \begin{pmatrix} x \\ 1-x \end{pmatrix} .$$

- The function $c(\epsilon)$ is given by

$$c(\epsilon) = \frac{6\Gamma(\frac{2-\epsilon}{2})\,\zeta(2-\epsilon)}{\pi^{\frac{4-\epsilon}{2}}}.$$

- $\zeta(2-\epsilon) \to \infty$ as $\epsilon \to 1$ leading to divergence of the thermal expectation values of the scalar fields.
- Thus, the analysis cannot be extended to $d = 3$.
- The question of whether thermal order in CFTs is consistent with unitarity is not resolved by these models.
- I will next discuss instances of persistent symmetry breaking in large N conformal gauge theories in $d = 4$ dimensions where there is no such problem with unitarity.

Summary so far

- We found an example of a model in fractional dimensions which demonstrates symmetry non-restoration.
  (Chai, Chaudhuri, Choi, Komargodski, Rabinovici and Smolkin, arXiv: 2005.03676)
  ("Symmetry Breaking at All Temperatures" published in PRL)
- This is an $O(N_1) \times O(N_2)$-symmetric vector model defined in $(4-\epsilon)$ dimensions.
  $(N_1, N_2 \gg 1,\ 0 < \epsilon \ll 1)$
- When $N_2 < N_1$, there exist Wilson-Fisher-like fixed points for which the $O(N_2)$-symmetry is broken at any nonzero temperature.
- It posses a Conformal Manifold.

  It has a flat direction, this also at non zero temperature, although not Supersymmetric this for infinite N.

  Since a CFT has no intrinsic scale, if a symmetry is broken at any nonzero temperature, it remains broken at all temperatures.

D=4 Gauge Theories with Persistent Symmetry Breaking

- We found examples of (3+1)-dimensional large N gauge theories which demonstrate symmetry non-restoration in the $N \to \infty$ limit.
- They are Banks-Zaks-like fixed points of these theories.
- An important feature of these theories in the planar ($N \to \infty$) limit:

  A conformal manifold that is a circle of fixed points in coupling

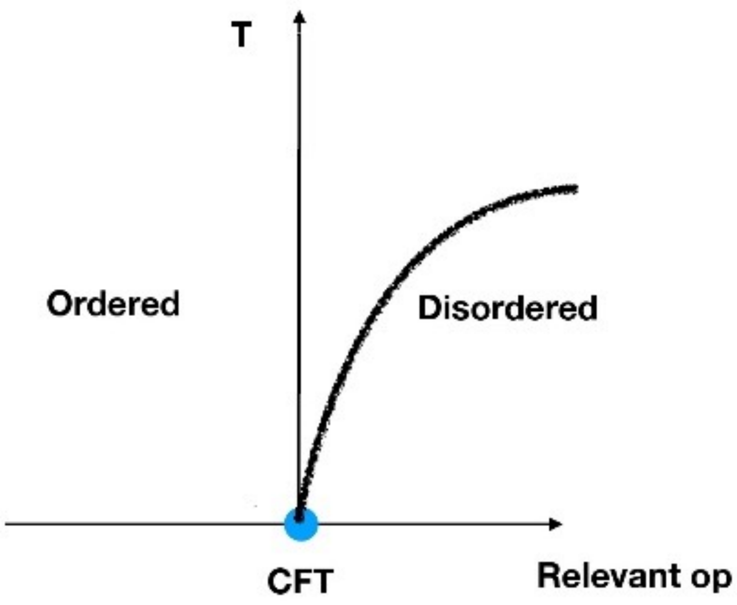
  space.
- In certain parameter regimes, a subset of points on this fixed circle demonstrates spontaneous breaking of a global symmetry at non-zero temperatures. → Thermal order.

  The SSB is accompanied by Higgsing of a subset of gauge bosons.

D=4 Gauge Theories with Persistent Symmetry Breaking

- There are marginally relevant scalar field deformations of these fixed points.
- These deformations are obtained by varying certain quartic couplings between the scalar fields alone in the model.
- In the $N \to \infty$ limit, the RG flows triggered by these deformations approach the points on the fixed circle in the UV regime.
- In the IR they approach a unique weakly coupled fixed point which lacks thermal order.
- Therefore, a subset of the systems defined by these RG flows undergo a transition from a disordered phase at low temperatures to an ordered phase at high temperatures.
- Using thermal perturbation theory, one can give an estimate of the corresponding critical temperature(see paper).

Potential Phase diagram for persistent symmetry breaking- YES.



Description of the models

- Let me now describe the models.(see paper for proof that single groups do not work)
- The gauge groups in these models have the structure
$$G = \prod_{i=1}^{2} G_i \times G_i$$
where G_i can be either $SO(N_{ci})$ or $SU(N_{ci})$.
- There are two different sectors distinguished by the index i.
- N_{c1} and N_{c2} can be unequal.
- $G_i = SO(N_{ci}) \rightarrow$ Real double bifundamental model,
$G_i = SU(N_{ci}) \rightarrow$ Complex double bifundamental model.

Description of the models

- The matter content in each sector comprises of

  1. 2 sets of massless fermions: each transforming in the fundamental representation of one of the G_i's and invariant under the other.

     $\rightarrow \psi_i^{(p)}$ and $\chi_i^{(p)}$ where p denotes a flavor index running from 1 to N_{fi}.

  2. An $N_{ci} \times N_{ci}$ matrix of massless scalar fields (Φ_i) transforming in the bifundamental representation of $G_i \times G_i$.

- Real double bifundamental model→Majorana fermions, real scalars.

  Complex double bifundamental model→Dirac fermions, complex scalars.

- The scalar fields interact with each other via both single trace and double trace quartic interactions.

- A double trace quartic interaction couples the scalar fields in the two sectors.

Description of the models

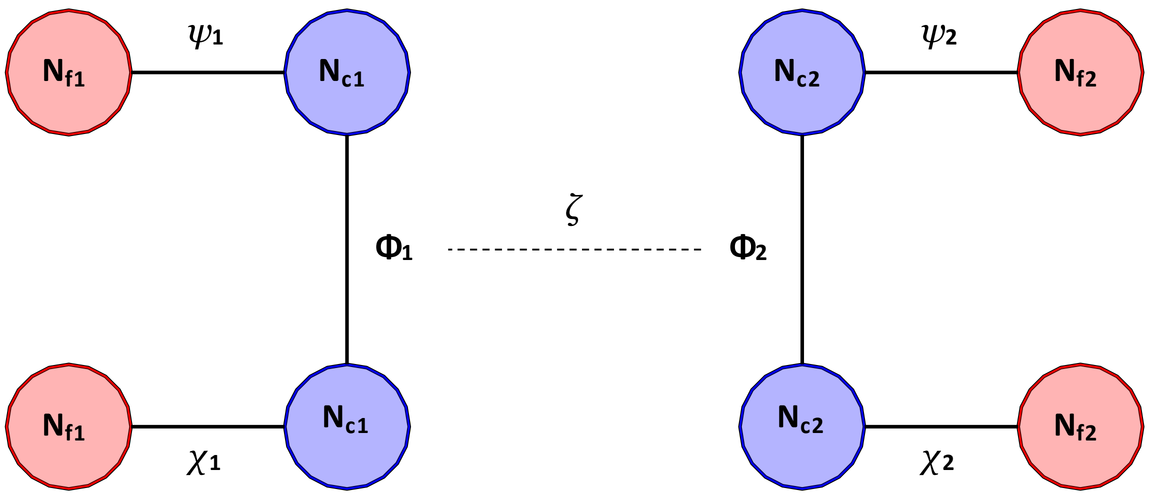


Figure: A schematic diagram of the field content of the double bifundamental models.

Description of the models

- To tame the UV divergences, we work with dimensional regularization and the $\overline{\text{MS}}$ scheme.
- The renormalized Lagrangian (suppressing gauge-fixing and ghost terms):

$$L = -\frac{1}{4}\sum_{i=1}^{2}\sum_{\alpha=1}^{2}(F_{i\alpha})_{\mu\nu}^{A}(F_{i\alpha})^{\mu\nu A} + i\kappa\sum_{i=1}^{2}(\overline{\psi_i}^{(p)})_{a_i}\gamma^{\mu}D_{\mu}(\psi_i^{(p)})_{a_i}$$
$$+ i\kappa\sum_{i=1}^{2}(\overline{\chi_i}^{(p)})_{j_i}\gamma^{\mu}D_{\mu}(\chi_i^{(p)})_{j_i} + \kappa\sum_{i=1}^{2}\mathrm{Tr}\,(D_{\mu}\Phi_i)^{\dagger}(D^{\mu}\Phi_i)$$
$$-\sum_{i=1}^{2}\tilde{h}_i\,\mathrm{Tr}\,(\Phi_i^{\dagger}\Phi_i\,\Phi_i^{\dagger}\Phi_i) - \sum_{i=1}^{2}\tilde{f}_i\,\mathrm{Tr}\,(\Phi_i^{\dagger}\Phi_i)\,\mathrm{Tr}\,(\Phi_i^{\dagger}\Phi_i)$$
$$-\ 2\tilde{\zeta}\,\mathrm{Tr}\,(\Phi_1^{\dagger}\Phi_1)\,\mathrm{Tr}\,(\Phi_2^{\dagger}\Phi_2)$$

where $\kappa = \frac{1}{2}$ for the RDB model, and $\kappa = 1$ for the CDB model.

- $F_{i\alpha}$ is the field strength corresponding to the gauge field $V_{i\alpha}$. The index α distinguishes the two G_i's in each sector.
- The gauge couplings corresponding to the two G_i's in the i^{th} sector are the same (say, g_i).

Large N limit

- The Veneziano limit:

$$N_{c1}, N_{c2} \to \infty, \quad \frac{N_{fi}}{N_{ci}} \to x_{fi}, \quad \frac{N_{c2}}{N_{c1}} \to r.$$

- In this limit, the couplings scale as

$$g_i^2 = \frac{16\pi^2\lambda_i}{N_{ci}}, \tilde{h}_i = \frac{16\pi^2 h_i}{N_{ci}}, \tilde{f}_i = \frac{16\pi^2 f_i}{N_{ci}^2}, \tilde{\zeta} = \frac{16\pi^2\zeta}{N_{c1}N_{c2}},$$

where λ_i, h_i, f_i and ζ are the 't Hooft couplings.

Global symmetries

- The global symmetries of interest in the RDB model are two $\mathbb{Z}_2$ symmetries, one for each sector (labeled by i):

$$\Phi_i \to T_i \Phi_i,$$
$$\psi_i^{(p)} \to T_i \, \psi_i^{(p)} \ \forall\, p \in \{1, \cdots, N_{fi}\},$$
$$(V_{i1})_\mu \to T_i (V_{i1})_\mu \, T_i^{-1},$$

where T_i is an $N_{ci} \times N_{ci}$ diagonal matrix of the following form:

$$\mathrm{T}_i \equiv \mathrm{diag}\{-1, 1, \cdots, 1\}.$$

- The global symmetries of interest in the CDB model are two $U(1)$ symmetries, one for each sector (labeled by i):

$$\Phi_i \to (T_i)_\theta \Phi_i,$$
$$\psi_i^{(p)} \to (\mathrm{T}_i)_\theta \psi_i^{(p)} \ \forall\, p \in \{1, \cdots, N_{fi}\},$$
$$(V_{i1})_\mu \to (\mathrm{T}_i)_\theta (V_{i1})_\mu (\mathrm{T}_i)_\theta^{-1},$$

where $(\mathrm{T}_i)_\theta \equiv \mathrm{diag}\{e^{i\theta}, 1, \cdots, 1\}$.

- Each of these transformations maps one set of gauge-equivalent configurations to another

Order parameters

- Order parameters for the symmetries:

$$<[\det \Phi_i]>$$

where the brackets indicate that it is a renormalized operator.

- The order parameters are expectation values of these “baryonic"
- operators. The symmetries are global ‘baryon symmetries’ in the

models.

Spontaneous breaking of the baryon symmetries in a thermal state

- We employ perturbation theory to determine whether the baryon symmetries are broken in thermal states.

- Disclaimer: Thermal perturbation theory is generally plagued by infrared problems which require re-summation of certain diagrams. This typically modifies the subleading terms.

  Here we will focus on the leading order terms. We expect the IR issues to not affect the results based on the leading order calculations.

- At the leading order, the order parameters are related to the thermal expectation values of the scalar fields as
$$<[\det \Phi_i]> \approx \det < \Phi_i >.$$

- When $< \Phi_i > \neq 0$, it can be brought to a diagonal form with nonzero entries by gauge transformations.

- So, when $< \Phi_i > \neq 0$, we have $\det < \Phi_i > \neq 0 \Rightarrow <[\det \Phi_i]> \neq 0$.

- This means that the baryon symmetry in the i^{th} sector is spontaneously broken.

Spontaneous breaking of the baryon symmetries in a thermal state

- To determine whether $<\Phi_i>\neq 0$, one can consider the thermal effective potential of the scalar fields.

- Up to leading order in perturbation theory, this potential has quadratic terms and quartic terms .

- The Quadratic terms are negative in some region of the conformal manifold.

  The Quartic terms:

  $$V_{quartic} = \sum_{i=1}^{2} \tilde{h}_i \, \mathrm{Tr}\,({\Phi_i}^{\dagger}\Phi_i \, {\Phi_i}^{\dagger}\Phi_i) + \sum_{i=1}^{2} \tilde{f}_i \, \mathrm{Tr}\,({\Phi_i}^{\dagger}\Phi_i)\, \mathrm{Tr}\,({\Phi_i}^{\dagger}\Phi_i) + 2\tilde{\zeta}\mathrm{Tr}\,({\Phi_1}^{\dagger}\Phi_1)\, \mathrm{Tr}\,({\Phi_2}^{\dagger}\Phi_2)\,.$$

- For the theory to have a stable vacuum, we need these quartic terms to be bounded from below.

- This is indeed the case for the fixed points of the double bifundamental models.

Effective potential at T=0 for a point on the fixed circle

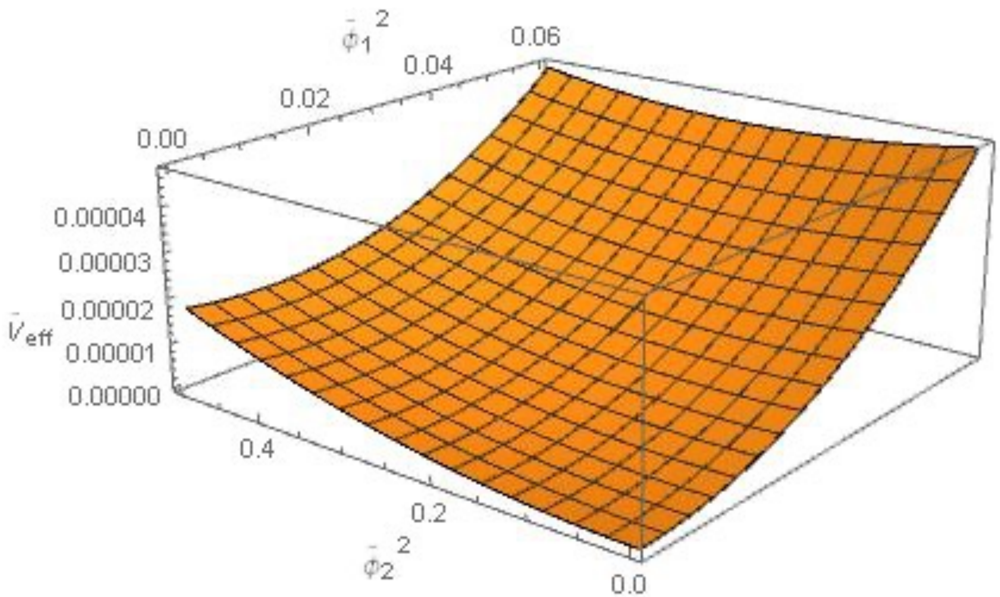
- Consider the fixed point with the following parameters:

$$r = 0.1,\ \lambda = 0.01, h_1 = h_2 = h = (\frac{3-\sqrt{6}}{16})\,\lambda,$$

$$f_p = \frac{\sqrt{6}}{8}\lambda,\ f_m = 0.0994\,\lambda,\ \zeta = -0.1\lambda\,,$$

  where $f_{p,m} = \frac{f_1 \pm f_2}{2}$.

- The effective potential is shown in the following figure:



Minimum of the potential at T=0: Absence of symmetry breaking

- The minimum of the potential is at $\widetilde{\varphi}_1{}^2 = \widetilde{\varphi}_2{}^2 = 0$.

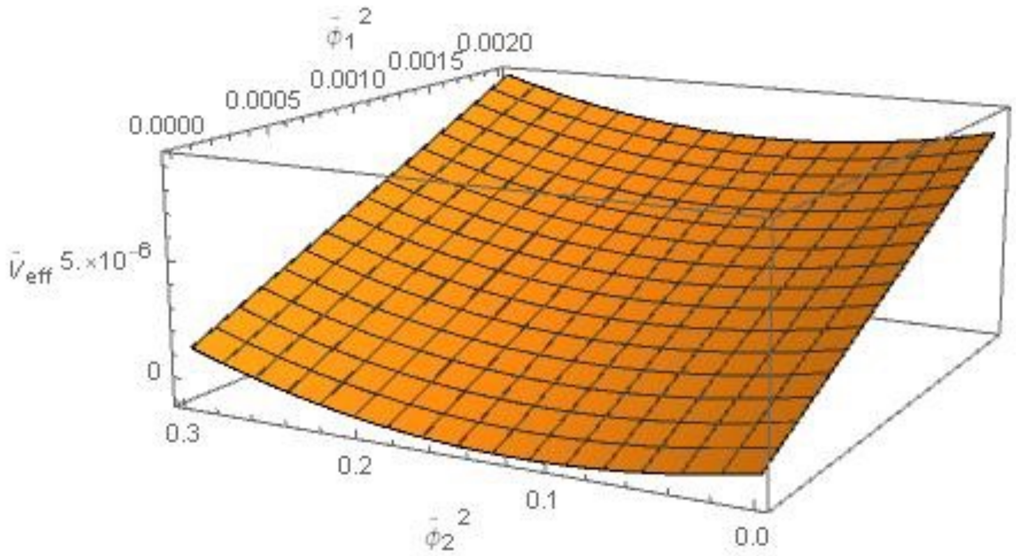
- This means that at T=0, the baryonic symmetries in both the sectors are unbroken.

Thermal effective potential for the fixed point

- Substituting the forms of the 1-loop thermal masses, we get

$$\widetilde{V_{eff}} = \left(h + \frac{f_p+f_m}{2} + \frac{r\zeta}{2} + \frac{3}{16}\lambda\right)\widetilde{\varphi_1}^2 + \left(hr^2 + \frac{(f_p-f_m)r^2}{2} + \frac{r\zeta}{2} + \frac{3}{16}\lambda r^2\right)\widetilde{\varphi_2}^2$$
$$+ 3h\widetilde{\varphi_1}^4 + 3h\, r^2\, \widetilde{\varphi_2}^4 + 3(f_p + f_m)\widetilde{\varphi_1}^4 + 3(f_p - f_m)r^2\, \widetilde{\varphi_2}^4 + 6\zeta\, r\, \widetilde{\varphi_1}^2\, \widetilde{\varphi_2}^2,$$

- This thermal effective potential is shown in the following figure:



Minimum of the thermal effective potential: Persistent symmetry breaking

- The minimum of the thermal effective potential is at $\widetilde{\varphi}_1{}^2 = 0,\ \widetilde{\varphi}_2{}^2 \approx 0.12$.

- This means that at nonzero temperatures, the baryonic symmetry in the first sector remains unbroken, while the baryonic symmetry in the second sector is spontaneously broken.

  →Persistent symmetry breaking.

  AND ONE FINDS A MASSLESS GB FOR THIS BREAKING(N INFINITY ISSUE)

Summary

- In this part of the talk, I discussed examples of (3+1)-dimensional large N gauge theories where certain global symmetries are spontaneously broken in the high temperature limit.
- I showed that these models have conformal manifolds (fixed circles) in the planar limit.
- Under appropriate conditions, an angular interval on this fixed circle exhibits symmetry breaking at nonzero temperatures.
- The broken symmetry is $\mathbb{Z}_2$ in the RDB model and $U(1)$ in the CDB
- model. The SSB is accompanied by Higgsing of a subset of gauge bosons.
- I also showed that there are marginally relevant deformations of these CFTs which lead to an IR fixed point that lacks thermal order.
- These systems undergo a transition from a disordered phase to an ordered phase as the temperature is increased.
-

N INFINITE FLAT POTENTIALS IN NON SUPERSYMMETRIC THEORIES ALMOST UP TO D=4 AND
CONFORMAL MANIFOLDS ALSO AT D=4 EXIST !

PERSISTENT SYMMETRY BREAKING OCCURS.

MOSTLY N IS INFINITE.

Future directions

- How does the story change under finite N corrections?

  Some perturbative computations up to 2-loops were done for the RDB

  model. They suggest that the fixed circle may survive under finite N

  corrections.

  Even if the degeneracy is lifted, do any of the fixed points demonstrating thermal order survive?

- How do the finite N corrections affect the RG flows.
  Are there other finite N CFTs which demonstrate SSB at nonzero temperatures?

  → See the recent work arXiv:2106.09723 by Chai, Dymarsky and Smolkin.

- Can one identify some universal feature of this class of CFTs?

- Finally, it would also be interesting to find holographic CFTs which show thermal order. In the dual theory of gravity, this would mean violation of the 'no-hair' theorem for black holes.

ברכות ליום הולדת - יום הולדת שמח - מזל טוב - בלונים בצורת לבבות אדומים ולבנים

Thanks for all I learned from you over the years!

Wishing you a healthy

120+120