Orbifolds
And Geometric Obstruction
to Scale Separation

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Verlinde^2 Symposium

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Amsterdam String Workshop

July 14, 2022

The talk I would like to give is not unrelated to the first work I did with the Verlinde's (and Dijkgraaf):

The Operator Algebra of Orbifold Models

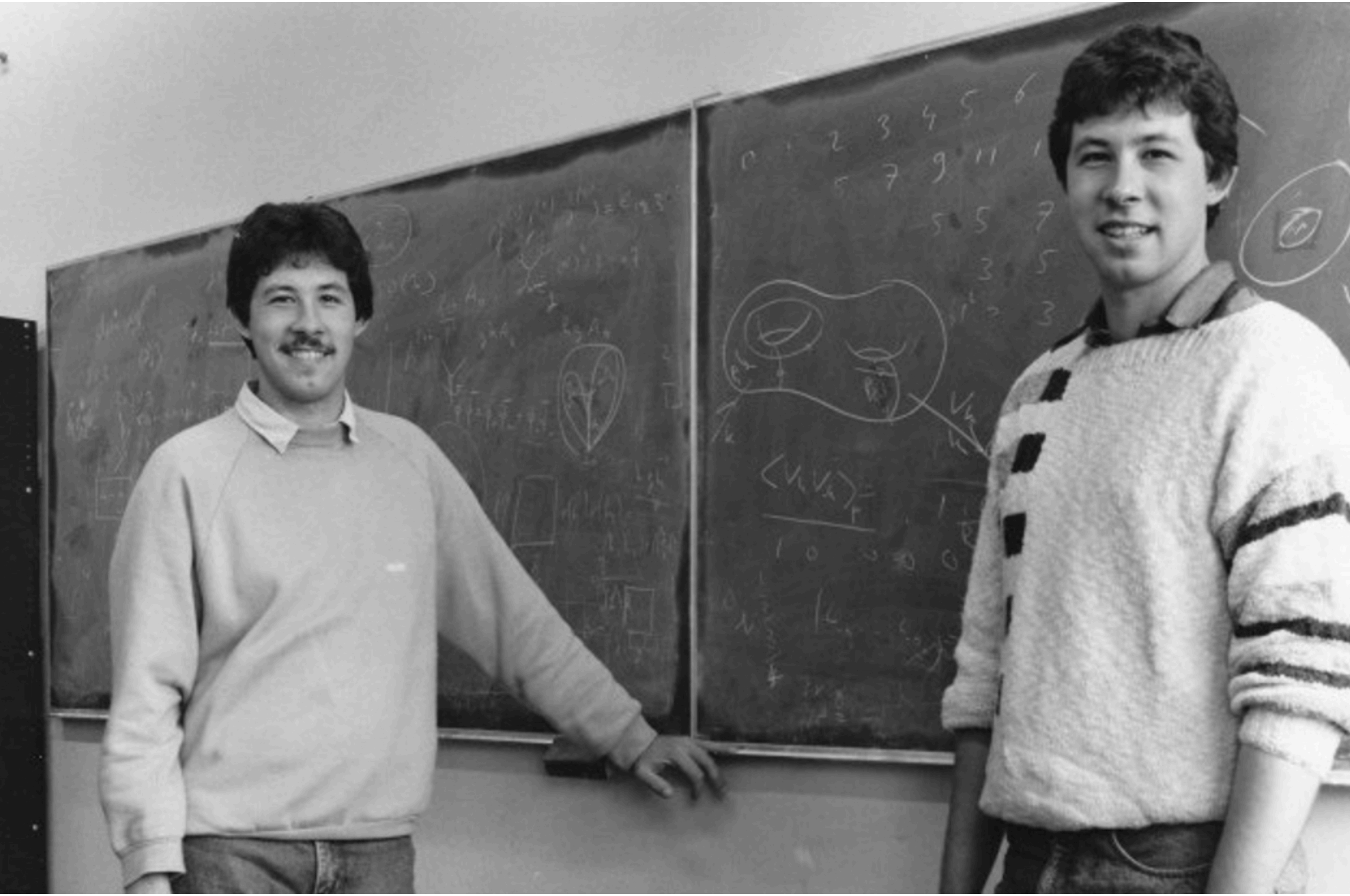
With

Robbert Dijkgraaf (Utrecht U.), Erik P. Verlinde (Princeton, Inst. Advanced Study), Herman L. Verlinde (Princeton U.) (Jan 6, 1989)

Published in: Commun.Math.Phys. 123 (1989) 485.

So in the talk today, I try to use orbifolds at least a little bit!

I want to first briefly say why this old paper has had a recent important application!



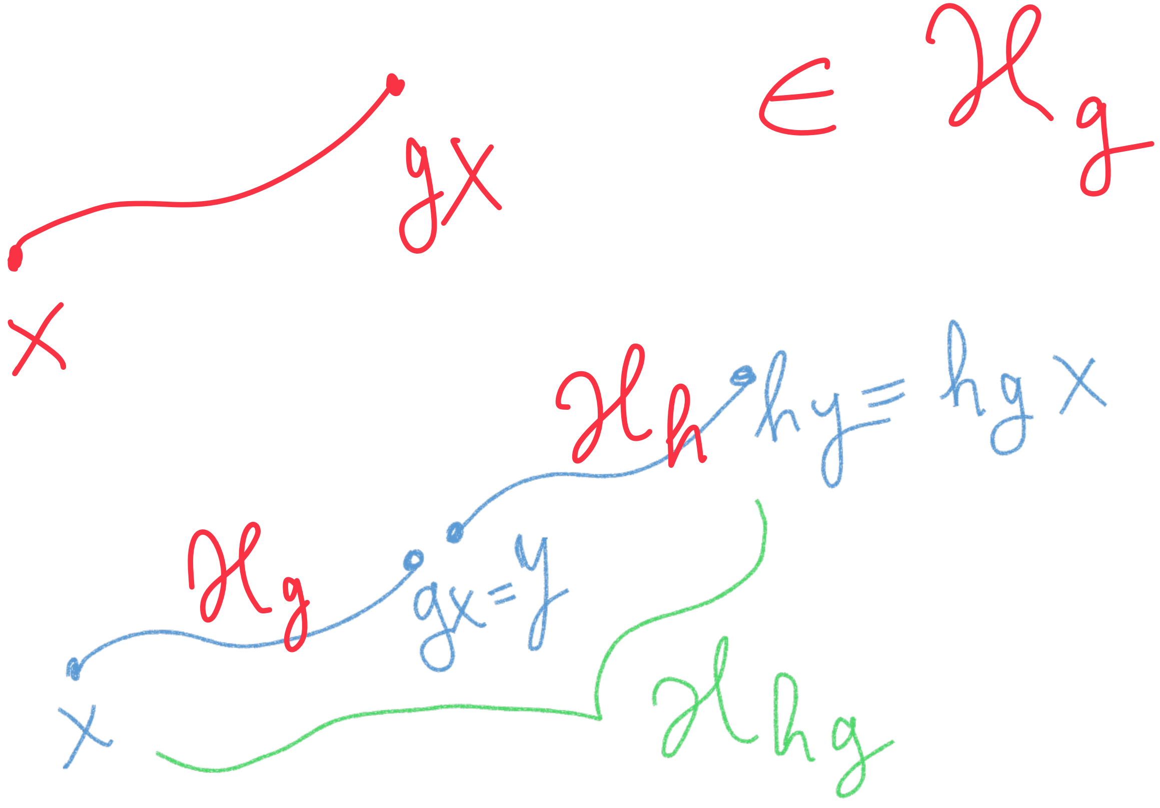
Non-invertible Global Symmetries

Typical global symmetries form groups with every element of the group admitting an inverse. However, recently it has been pointed out that this is not a complete list of symmetries. Indeed line operators in non-abelian Chern-Simons theories in 3d, and more generally non-cyclic Verlinde algebras exemplify this.

Another example of this (not unrelated to the above) is in the context of orbifolds:

Consider Strings propagating on M. Next consider strings on orbifold M/G where G is a discrete isometry of M.

This leads to symmetries:



But if we project to invariant subgroup each twisted sector is represented by a conjugacy class; So we get a ring structure involving multiplication of conjugacy classes:

$$C_i \times C_j = \sum_k N_{ij}^k C_k$$

Actually in our joint work with Verlinde's and Dijkgraaf we refined this to take into account the group action in each sector.

So as you can see, just like global symmetries there are selection rules when strings of different sectors interact. But which sector they end up is not completely determined for non-abelian groups, because the RHS involves a sum.

So we learn that any non-abelian orbifold gives examples of non-invertible symmetries.

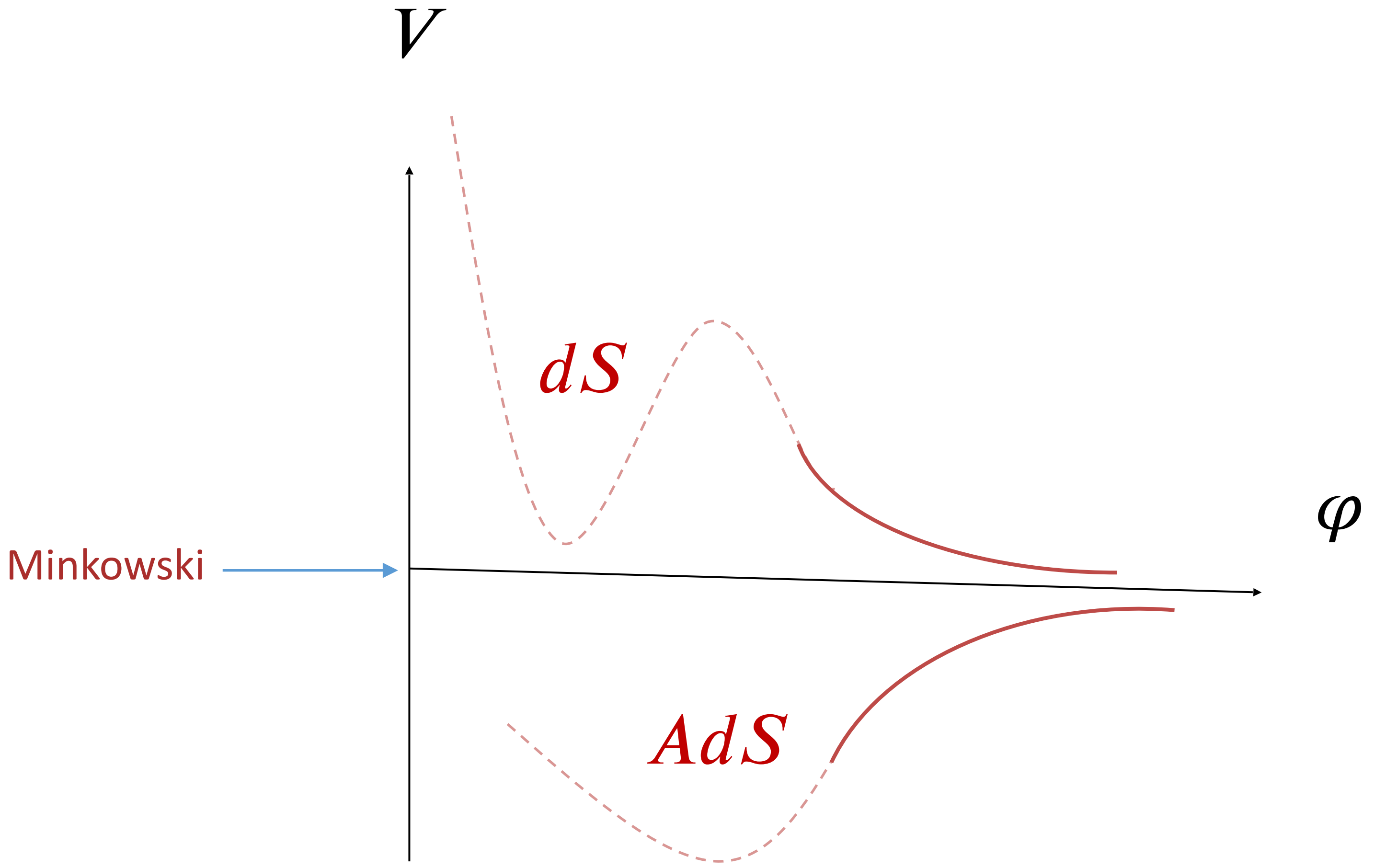
A question was raised as to whether non-invertible symmetries can be gauged. The answer is immediately clear from this construction: Since all symmetries in string theory are gauged we therefore learn that non-invertible symmetries realized as non-abelian orbifolds of compact manifolds automatically give examples of non-invertible gauge symmetries. Of course if we consider the same in a non-compact setup in string theory, we would get non-invertible global symmetries of the type studied in QFT's.

The rest of my talk today is based on the paper:

On Upper Bounds in Dimension Gaps of CFT's

T. Collins, D.Jafferis, CV, K. Xu and S.-T. Yau: 2201.03660 [hep-th]

Homogeneous solutions are of three types: Minkowski, AdS, dS



In principle we can obtain these by string compactification:

$$M_d \times K, \quad AdS_d \times K, \quad dS_d \times K$$

No reliable dS constructed in string theory (if it exists).

Minkowski space: many examples

scale of the internal space free parameter

AdS: Many solutions. But l_{AdS}, l_K are not independent:

Swampland AdS distance conjecture (LPV):

$$l_K \sim l_{AdS}^a \qquad a \geq 1$$

$a = 1$ is the strong form of this conjecture;

CFT dual for AdS, dimension given by

$$\Delta \sim m l_{AdS} \sim \frac{l_{AdS}}{l_K} \sim l_{AdS}^{1-a}$$

Note that if instead

$$a < 1$$

We get dimensions much larger than 1 in the limit of large AdS radius leading to large Dimension gap for CFT. If this were true, the mass scales of AdS excitations would be large enough and if there is strict gap, one could imagine getting dS by small SUSY breaking effect because all the massive excitations frozen out in AdS scale. This is thus an important question.

What do we know about dimension gaps in CFT's?

$\Delta \leq d$ relevant or marginal

$\Delta > d$ irrelevant

No CFT is known with no low lying conformal operators. The biggest known gap is for d=2, the Monstrous Moonshine construction, where the first non-zero $\Delta = 4.$

We may even relax the condition and ask
Could there be a CFT with a few low lying operators but with a large gap after that? None known!

Is this possible using holography? Related to whether the KK tower can be made heavy:

Focus on first massive spin 2.

Need to find first non-zero eigenvalue for scalar Laplacian

$$\Delta(\Delta - d) = m^2 = a\lambda$$

(a is order 1, and λ is eigenvalue of Laplacian).

We will consider holographic cases where branes probe singularities of internal manifold. This leads to

$$AdS_d \times K$$

where K is an Einstein manifold. It will have

More structure for various supersymmetric cases but all satisfy (where AdS length scale is set to 1) :

$$R_{ij} = (n-1)g_{ij}$$

Consider the case of $K = S^n$

Naively is seems it should be possible to have a very large first eigenvalue by considering quotients instead:

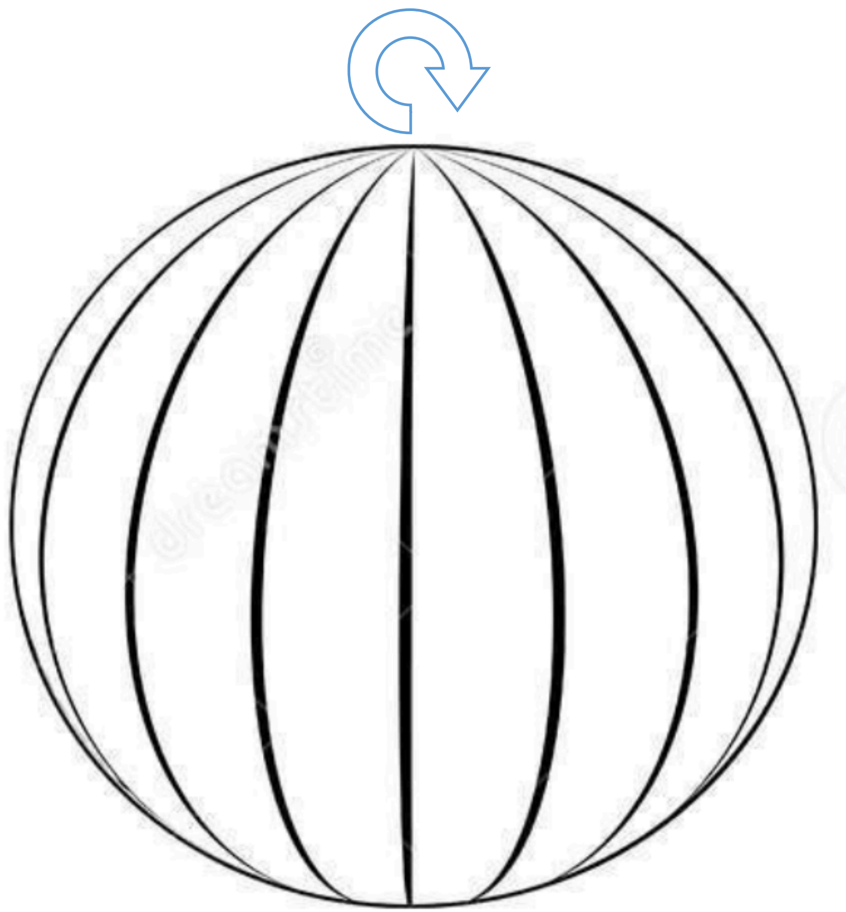
$K = S^n/G$ where G is a discrete subgroup of $SO(n+1)$

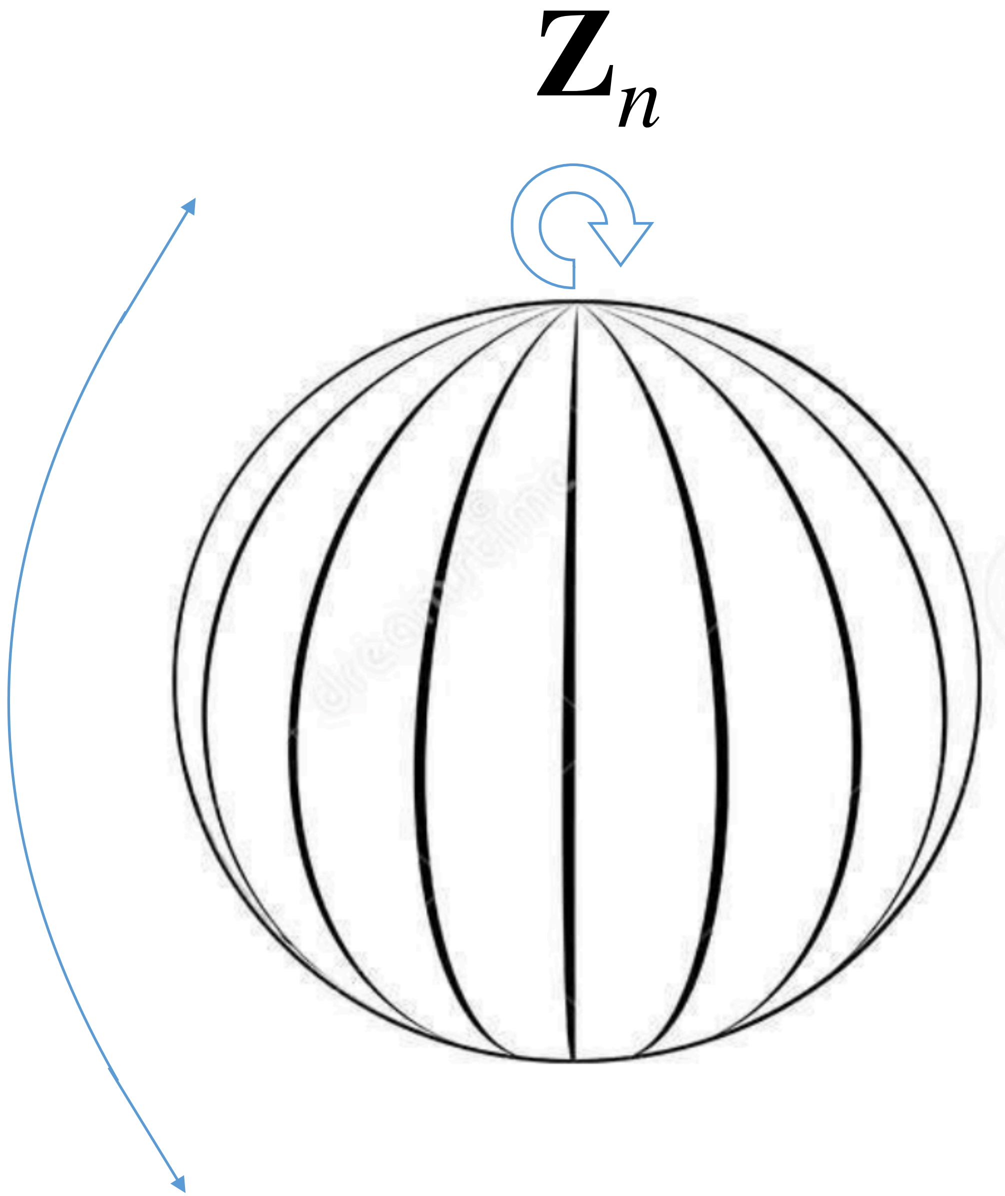
We can choose a large order for the discrete group and make the volume go to zero without changing the curvature: $l_K \sim 1/|G|^{1/n} \to 0$ as $|G| \to \infty$

From this one would be led to conclude $\lambda_1 \sim \frac{1}{l_K} \to \infty$

This turns out to be false! We find a universal bound on λ_1 which only depends on the dimension of K, namely n.

$\mathbf{Z}_n$




$\mathbf{Z}_n$

For S^3: $\lambda_1 \in [3,168], D \in (0.32,\pi] \rightarrow \Delta_1 \in [3,14]$

Achieved by icosahedral subgroup
$G \subset SU(2)_{diag.} \subset SU(2)_L \times SU(2)_R$

For S^5 : $\lambda_1 \in [5,32], D \in (0.96,\pi] \rightarrow \Delta_1 \in [5,8]$

For S^7 : $\lambda_1 \in [7,40], D \in (0.84,\pi] \rightarrow \Delta_1 \in [3.5,5]$
Achieved again by icosahedral subgroup
$G \subset SU(2)_{diag.} \subset SU(2) \times SU(2) \subset SU(4)$

There is a similar bound for all sphere quotients of any dimension depending only on dimension.

Another class of examples, D3 branes probing CY 3-fold singularities, 4d N=1. Leads to Sasaki-Einstein manifolds. For Fermat type singularities:

$f(z) = \sum_{i=0}^{3} z_i^{a_i} = 0$ the biggest gap for holomorphic eigenvalues $(a_i) = (13,11,3,2) \rightarrow \Delta_1 = 202$

For M2 branes probing CY 4-folds, leads to 3d N=2.

For CY 4-fold fermat type $f(z) = \sum_{i=0}^{4} z_i^{a_i} = 0$

the biggest gap for holomorphic eigenvalues

$(a_i) = (85,83,7,3,2) \rightarrow \Delta_1 = 6975$

More generally for CY n-fold Fermat type the one with highest gap in holomorphic eigenvalue the biggest gap is achieved with

$$a_n = 2, \;\; a_k = \prod_{l>k} a_l + 1 \;\; (k > 1), \;\; a_1 = 2\prod_{l>1} a_l - 1, \;\; a_0 = 2\prod_{l>1} a_l + 1$$

Using the classification of regular singularities for CY 3-folds we have shown that for all of them there is a universal upper bound on the first eigenvalue of the scalar Laplacian.
Studying all these examples leads us to

Conjecture:

The diameter of an n-dimensional Einstein manifold is bounded from below and the first non-vanishing eigenvalue of scalar Laplacian bounded from above, where the bounds depend only on n.

From the worldsheet perspective probing singular CY, this leads to a conjecture:

Consider the corresponding world sheet theory probing singular CY. We get an arbitrary (2,2) SCFT in d=2 such that $\hat{c} < n$ -the balance of central charge is fixed by linear dilaton.

Then the holomorphic eigenvalue gap not being too large in the bulk translates to a condition for the world sheet theory chiral fields.

$$\frac{q}{n - \hat{c}} \leq C_n$$

where C_n is a universal constant which depends only on n.

Let q be the R-charge of the lightest chiral operator.

Then the holomorphic eigenvalue gap not being too large in the bulk translates to a condition for the world sheet theory chiral fields.

$$\frac{q}{n-\hat{c}} \leq C_n$$

where C_n is a universal constant which depends only on n.

For example for diagonal (2,2) minimal models, for n=1,

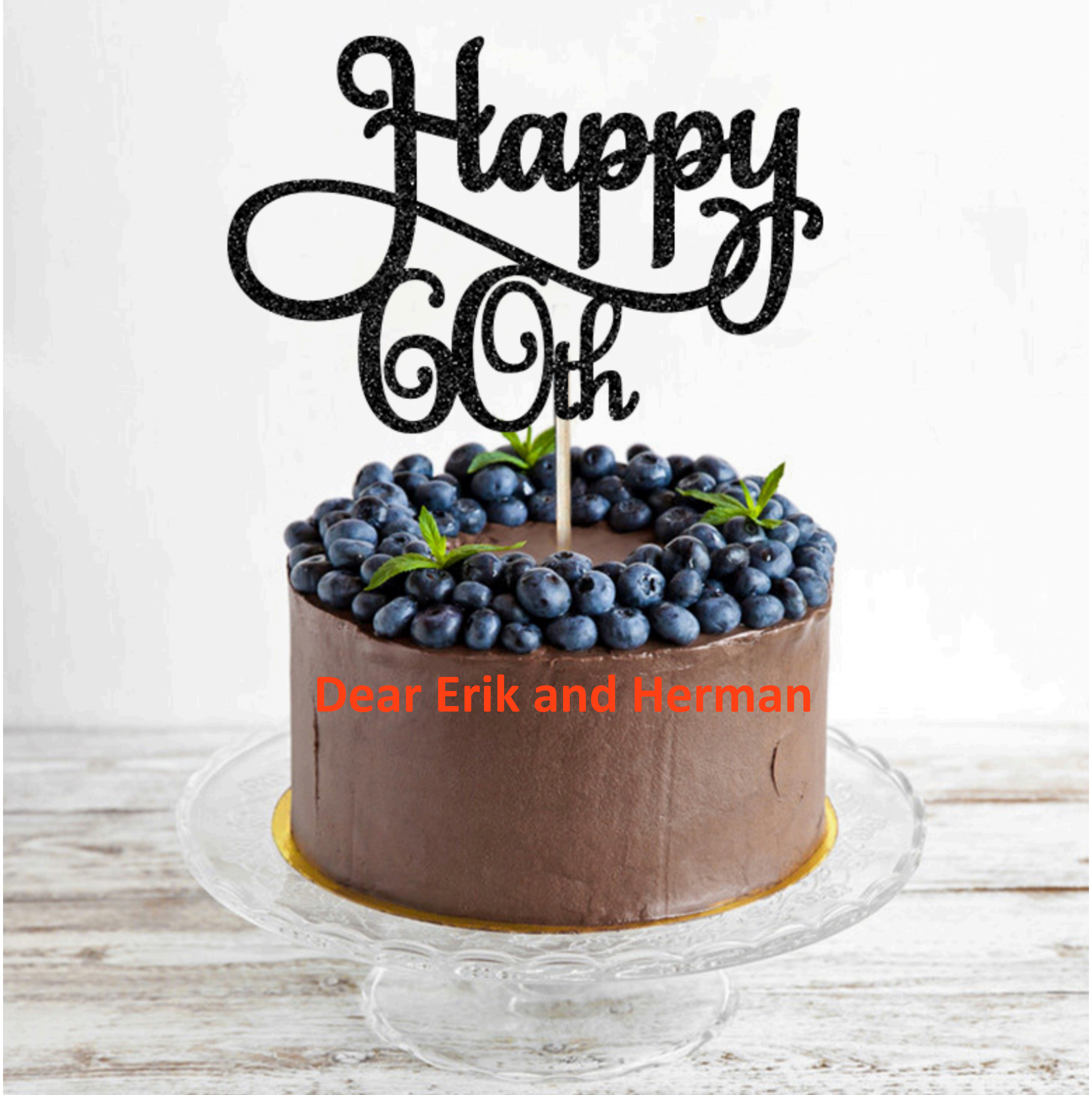
$\hat{c} = 1 - \frac{2}{m}$, with smallest $q = \frac{1}{m}$

We get $$\frac{\frac{1}{m}}{1-(1-\frac{2}{m})} = \frac{1}{2} \leq C_1$$

The biggest gap is achieved for the E_8 minimal model with $W = x^3 + y^5 + z^2$, leading

$$\frac{q}{n - \hat{c}} \leq C_n$$

$$q = \frac{1}{5},\ \hat{c} = \frac{14}{15} \Rightarrow \frac{\frac{1}{5}}{1 - \frac{14}{15}} = 3 = C_1$$


Happy
60th
Dear Erik and Herman

One may think that this statement is rather restrictive and applies only to the case of branes probing singularities and not to more complicated cases like DGKT. Indeed DGKT would correspond to a counter example to CFT with no large gap conjecture. Even though our conjecture does not directly apply to this, Cribiori et. al. dualize some DGKT-like models to pure M-theory geometry with G-flux, leading to Einstein manifolds. The naive reasoning may suggest scale separation (using generic length scales) —however our conjecture does apply to this class and would again suggest no scale separation
(naive estimates assumed isotropic geometries).