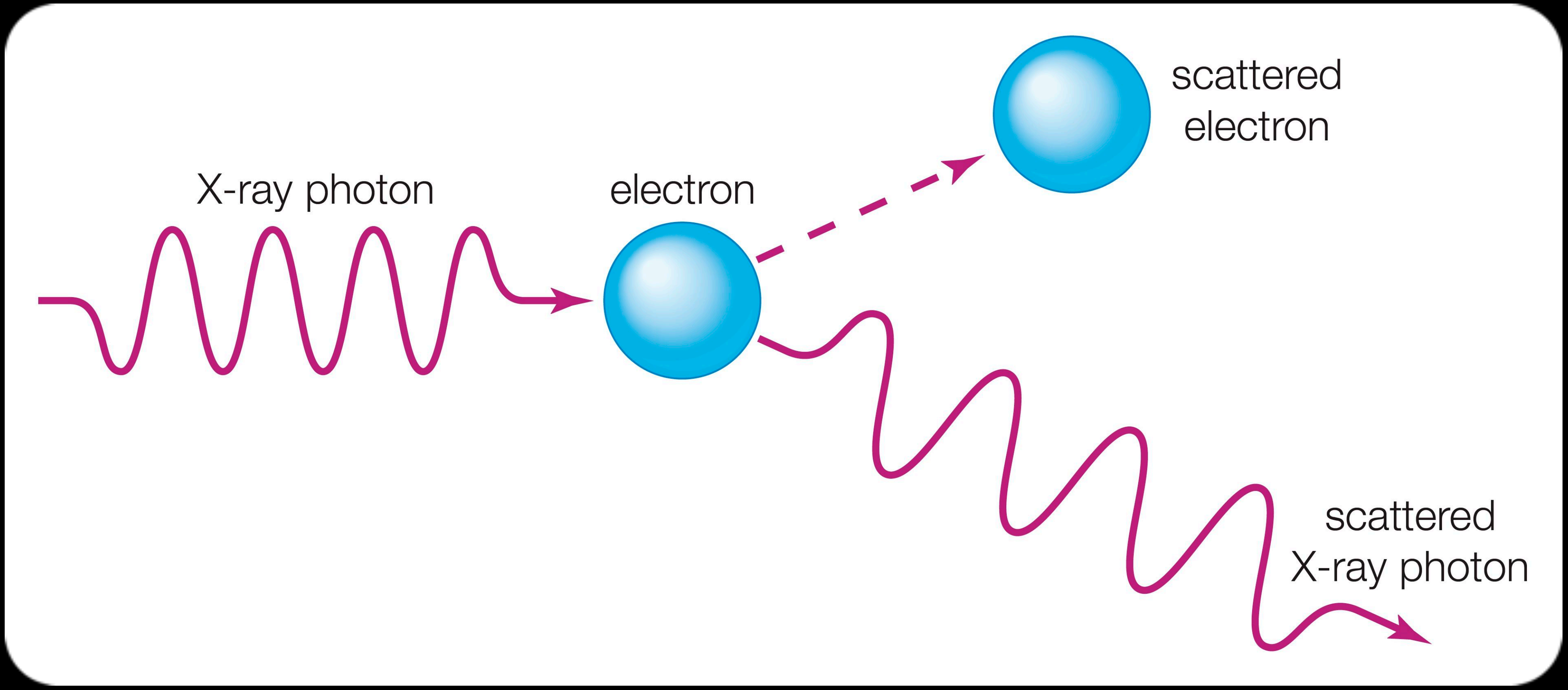
Electromagnetic interactions



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The Standard Model

Standard Model of

FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

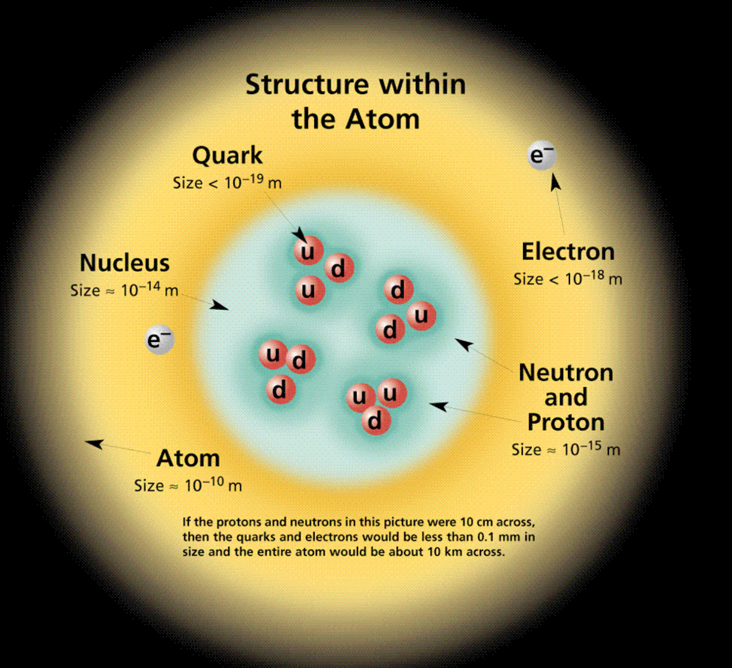
Leptons spin = 1/2		
Flavor	Mass GeV/c²	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

Spin is the intrinsic angular momentum of particles. Spin is given in units of ħ, which is the quantum unit of angular momentum, where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s = 1.05×10^{-34} J s.

Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c^2 (remember $E = mc^2$), where 1 GeV = 10^9 eV = 1.60×10^{-10} joule. The mass of the proton is 0.938 GeV/c^2 = 1.67×10^{-27} kg.



BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c²	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1		
Name	Mass GeV/c²	Electric charge
g gluon	0	0

Color Charge
Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically-charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and ***W*** and ***Z*** bosons have no strong interactions and hence no color charge.

Quarks Confined in Mesons and Baryons
One cannot isolate quarks and gluons; they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: **mesons** $q\bar{q}$ and **baryons** qqq.

Residual Strong Interaction
The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

PROPERTIES OF THE INTERACTIONS

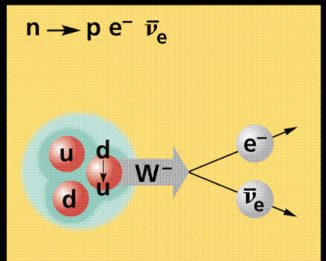
Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$ — Baryons are fermionic hadrons. There are about 120 types of baryons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c²	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

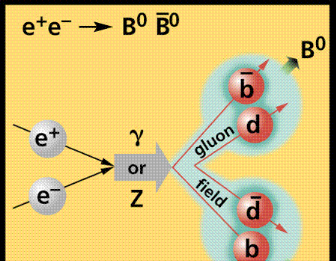
Mesons $q\bar{q}$ — Mesons are bosonic hadrons. There are about 140 types of mesons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c²	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

Matter and Antimatter
For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or – charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$, but not $K^0 = d\bar{s}$) are their own antiparticles.

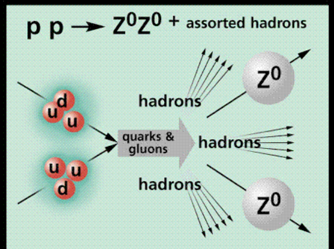
Figures
These diagrams are an artist's conception of physical processes. They are ***not*** exact and have ***no*** meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.



A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.



An electron and positron (antielectron) colliding at high energy can annihilate to produce B^0 and $\bar{B}^0$ mesons via a virtual Z boson or a virtual photon.



Two protons colliding at high energy can produce various hadrons plus very high mass particles such as Z bosons. Events such as this one are rare but can yield vital clues to the structure of matter.

The Particle Adventure
Visit the award-winning web feature *The Particle Adventure* at
http://ParticleAdventure.org

This chart has been made possible by the generous support of:
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BURLE INDUSTRIES, INC.

http://CPEPweb.org

The (complete) Standard Model Lagrangian

Source: Symmetry magazine

1

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2}ig_s^2(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu$$

2

$$-\partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac12\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac12 \partial_\mu A_\nu\partial_\mu A_\nu - \frac12\partial_\mu H\partial_\mu H - \frac12 m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac12\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h\left[\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac12(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac12 g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac12 g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \frac18 g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] - gMW^+_\mu W^-_\mu H - \frac12 g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac12 ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac12 g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)] + \frac12 g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac14 g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \frac14 g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-] - \frac12 g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac12 ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac12 g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac12 ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - g^1 s_w^2 A_\mu A_\mu\phi^+\phi^-$$

3

$$-\bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar\nu^\lambda\gamma\partial\nu^\lambda - \bar u^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j - \bar d^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_wA_\mu\left[-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac23(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \frac13(\bar d^\lambda_j\gamma^\mu d^\lambda_j)\right] + \frac{ig}{4c_w}Z^0_\mu\left[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar u^\lambda_j\gamma^\mu(\tfrac43 s_w^2 - 1 - \gamma^5)u^\lambda_j) + (\bar d^\lambda_j\gamma^\mu(1 - \tfrac83 s_w^2 - \gamma^5)d^\lambda_j)\right] + \frac{ig}{2\sqrt2}W^+_\mu\left[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right] + \frac{ig}{2\sqrt2}W^-_\mu\left[(\bar e^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right]$$

4

$$+\frac{ig}{2\sqrt2}\frac{m_e^\lambda}{M}\left[-\phi^+(\bar\nu^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar e^\lambda(1+\gamma^5)\nu^\lambda)\right] - \frac{g}{2}\frac{m_e^\lambda}{M}\left[H(\bar e^\lambda e^\lambda) + i\phi^0(\bar e^\lambda\gamma^5 e^\lambda)\right] + \frac{ig}{2M\sqrt2}\phi^+\left[-m_d^\kappa(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m_u^\lambda(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j)\right] + \frac{ig}{2M\sqrt2}\phi^-\left[m_d^\lambda(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j)\right] - \frac g2\frac{m_u^\lambda}{M}H(\bar u^\lambda_j u^\lambda_j) - \frac g2\frac{m_d^\lambda}{M}H(\bar d^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar u^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j)$$

5

$$+\bar X^+(\partial^2 - M^2)X^+ + \bar X^-(\partial^2 - M^2)X^- + \bar X^0\left(\partial^2 - \frac{M^2}{c_w^2}\right)X^0 + \bar Y\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar X^0X^- - \partial_\mu\bar X^+X^0) + igs_wW^+_\mu(\partial_\mu\bar YX^- - \partial_\mu\bar X^+Y) + igc_wW^-_\mu(\partial_\mu\bar X^-X^0 - \partial_\mu\bar X^0X^+) + igs_wW^-_\mu(\partial_\mu\bar X^-Y - \partial_\mu\bar YX^+) + igc_wZ^0_\mu(\partial_\mu\bar X^+X^+ - \partial_\mu\bar X^-X^-) + igs_wA_\mu(\partial_\mu\bar X^+X^+ - \partial_\mu\bar X^-X^-) - \frac12 gM\left[\bar X^+X^+H + \bar X^-X^-H + \frac{1}{c_w^2}\bar X^0X^0H\right] + \frac{1-2c_w^2}{2c_w}igM[\bar X^+X^0\phi^+ - \bar X^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + igMs_w[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + \frac12 igM[\bar X^+X^+\phi^0 - \bar X^-X^-\phi^0]$$

The (complete) Standard Model Lagrangian

[1]: Describes the gluon, the gauge boson which is the carrier of the strong force, and its interactions. Gluons come in eight types, interact among themselves and have a color charge.

[3]: This part of the equation describes how elementary matter particles interact with the weak force. This section also includes basic interactions with the Higgs field, from which some elementary particles receive their mass.

[4]: In quantum mechanics, there is no single path or trajectory a particle can take, which means that sometimes redundancies appear in this type of mathematical formulation. To clean up these redundancies, theorists use virtual particles they call ghosts.
This part of the equation describes how matter particles interact with Higgs ghosts, virtual artifacts from the Higgs field.

1

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g^c_\mu$$

2

$$- \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h [\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \frac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \frac{1}{8} g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] - gMW^+_\mu W^-_\mu H - \frac{1}{2} g \frac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \frac{1}{2} ig[W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2} g [W^+_\mu (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W^-_\mu (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)] + \frac{1}{2} g \frac{1}{c_w} (Z^0_\mu (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - ig \frac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + igs_w M A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z^0_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + igs_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{4} g^2 W^+_\mu W^-_\mu [H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \frac{1}{4} g^2 \frac{1}{c_w^2} Z^0_\mu Z^0_\mu [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+\phi^-] - \frac{1}{2} g^2 \frac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu \phi^- + W^-_\mu \phi^+) - \frac{1}{2} ig^2 \frac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) + \frac{1}{2} g^2 s_w A_\mu \phi^0 (W^+_\mu \phi^- + W^-_\mu \phi^+) + \frac{1}{2} ig^2 s_w A_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z^0_\mu A_\mu \phi^+ \phi^- - g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^-$$

3

$$- \bar{e}^\lambda (\gamma\partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda \gamma\partial \nu^\lambda - \bar{u}^\lambda_j (\gamma\partial + m_u^\lambda) u^\lambda_j - \bar{d}^\lambda_j (\gamma\partial + m_d^\lambda) d^\lambda_j + igs_w A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j \gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j \gamma^\mu d^\lambda_j)] + \frac{ig}{4c_w} Z^0_\mu [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}^\lambda_j \gamma^\mu (\frac{4}{3} s_w^2 - 1 - \gamma^5) u^\lambda_j) + (\bar{d}^\lambda_j \gamma^\mu (1 - \frac{8}{3} s_w^2 - \gamma^5) d^\lambda_j)] + \frac{ig}{2\sqrt{2}} W^+_\mu [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) e^\lambda) + (\bar{u}^\lambda_j \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d^\kappa_j)] + \frac{ig}{2\sqrt{2}} W^-_\mu [(\bar{e}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa} \gamma^\mu (1+\gamma^5) u^\lambda_j)] +$$

4

$$\frac{ig}{2\sqrt{2}} \frac{m_e^\lambda}{M} [-\phi^+ (\bar{\nu}^\lambda (1-\gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1+\gamma^5) \nu^\lambda)] - \frac{g}{2} \frac{m_e^\lambda}{M} [H (\bar{e}^\lambda e^\lambda) + i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}} \phi^+ [-m_d^\kappa (\bar{u}^\lambda_j C_{\lambda\kappa} (1-\gamma^5) d^\kappa_j) + m_u^\lambda (\bar{u}^\lambda_j C_{\lambda\kappa} (1+\gamma^5) d^\kappa_j] + \frac{ig}{2M\sqrt{2}} \phi^- [m_d^\lambda (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1+\gamma^5) u^\kappa_j) - m_u^\kappa (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1-\gamma^5) u^\kappa_j] - \frac{g}{2} \frac{m_u^\lambda}{M} H (\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2} \frac{m_d^\lambda}{M} H (\bar{d}^\lambda_j d^\lambda_j) + \frac{ig}{2} \frac{m_u^\lambda}{M} \phi^0 (\bar{u}^\lambda_j \gamma^5 u^\lambda_j) - \frac{ig}{2} \frac{m_d^\lambda}{M} \phi^0 (\bar{d}^\lambda_j \gamma^5 d^\lambda_j) +$$

5

$$\bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + igc_w W^+_\mu (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + igs_w W^+_\mu (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + igc_w W^-_\mu (\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + igs_w W^-_\mu (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z^0_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + igs_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) - \frac{1}{2} gM [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H] + \frac{1-2c_w^2}{2c_w} igM [\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w} igM [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + igM s_w [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2} igM [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]$$

[2]:Describes the interactions between bosons, particularly W and Z bosons.
Bosons are force-carrying particles, and there are four species of bosons that interact with other particles using three fundamental forces. Photons carry electromagnetism, gluons carry the strong force and W and Z bosons carry the weak force. The most recently discovered boson, the Higgs boson, is a bit different; its interactions appear in the next part of the equation.

[5]: This last part of the equation includes more ghosts. These ones are called Faddeev-Popov ghosts, and they cancel out redundancies that occur in interactions through the weak force.

The (complete) Standard Model Lagrangian

Note: Thomas Gutierrez, an assistant professor of Physics at California Polytechnic State University, transcribed the Standard Model Lagrangian for the web. He derived it from Diagrammatica, a theoretical physics reference written by Nobel Laureate Martinus Veltman. In Gutierrez's dissemination of the transcript, he noted a sign error he made somewhere in the equation.

1

$$
\begin{gathered}
-\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
\tfrac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -
\end{gathered}
$$

2

$$
\begin{gathered}
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H - \\
\tfrac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \tfrac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h [\tfrac{2M^2}{g^2} + \\
\tfrac{2M}{g} H + \tfrac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - i g c_w [\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)] - i g s_w [\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - \\
W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \tfrac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
\tfrac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2 A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha [H^3 + H \phi^0 \phi^0 + 2H \phi^+ \phi^-] - \\
\tfrac{1}{8} g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - \\
g M W^+_\mu W^-_\mu H - \tfrac{1}{2} g \tfrac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \tfrac{1}{2} i g [W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - \\
W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \tfrac{1}{2} g [W^+_\mu (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W^-_\mu (H \partial_\mu \phi^+ - \\
\phi^+ \partial_\mu H)] + \tfrac{1}{2} g \tfrac{1}{c_w} (Z^0_\mu (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - i g \tfrac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + \\
i g s_w M A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) - i g \tfrac{1-2c_w^2}{2c_w} Z^0_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
i g s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \tfrac{1}{4} g^2 W^+_\mu W^-_\mu [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \\
\tfrac{1}{4} g^2 \tfrac{1}{c_w^2} Z^0_\mu Z^0_\mu [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-] - \tfrac{1}{2} g^2 \tfrac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu \phi^- + \\
W^-_\mu \phi^+) - \tfrac{1}{2} i g^2 \tfrac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) + \tfrac{1}{2} g^2 s_w A_\mu \phi^0 (W^+_\mu \phi^- + \\
W^-_\mu \phi^+) + \tfrac{1}{2} i g^2 s_w A_\mu H (W^+_\mu \phi^- - W^-_\mu \phi^+) - g^2 \tfrac{s_w}{c_w} (2c_w^2 - 1) Z^0_\mu A_\mu \phi^+ \phi^- - \\
g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma\partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda \gamma\partial \nu^\lambda - \bar{u}^\lambda_j (\gamma\partial + m_u^\lambda) u^\lambda_j -
\end{gathered}
$$

3

$$
\begin{gathered}
\bar{d}^\lambda_j (\gamma\partial + m_d^\lambda) d^\lambda_j + i g s_w A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j \gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j \gamma^\mu d^\lambda_j)] + \\
\tfrac{ig}{4c_w} Z^0_\mu [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}^\lambda_j \gamma^\mu (\tfrac{4}{3} s_w^2 - \\
1 - \gamma^5) u^\lambda_j) + (\bar{d}^\lambda_j \gamma^\mu (1 - \tfrac{8}{3} s_w^2 - \gamma^5) d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}} W^+_\mu [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5) e^\lambda) + \\
(\bar{u}^\lambda_j \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}} W^-_\mu [(\bar{e}^\lambda \gamma^\mu (1+\gamma^5) \nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa} \gamma^\mu (1 + \\
\gamma^5) u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}} \tfrac{m_e^\lambda}{M} [-\phi^+ (\bar{\nu}^\lambda (1-\gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1+\gamma^5) \nu^\lambda)] -
\end{gathered}
$$

4

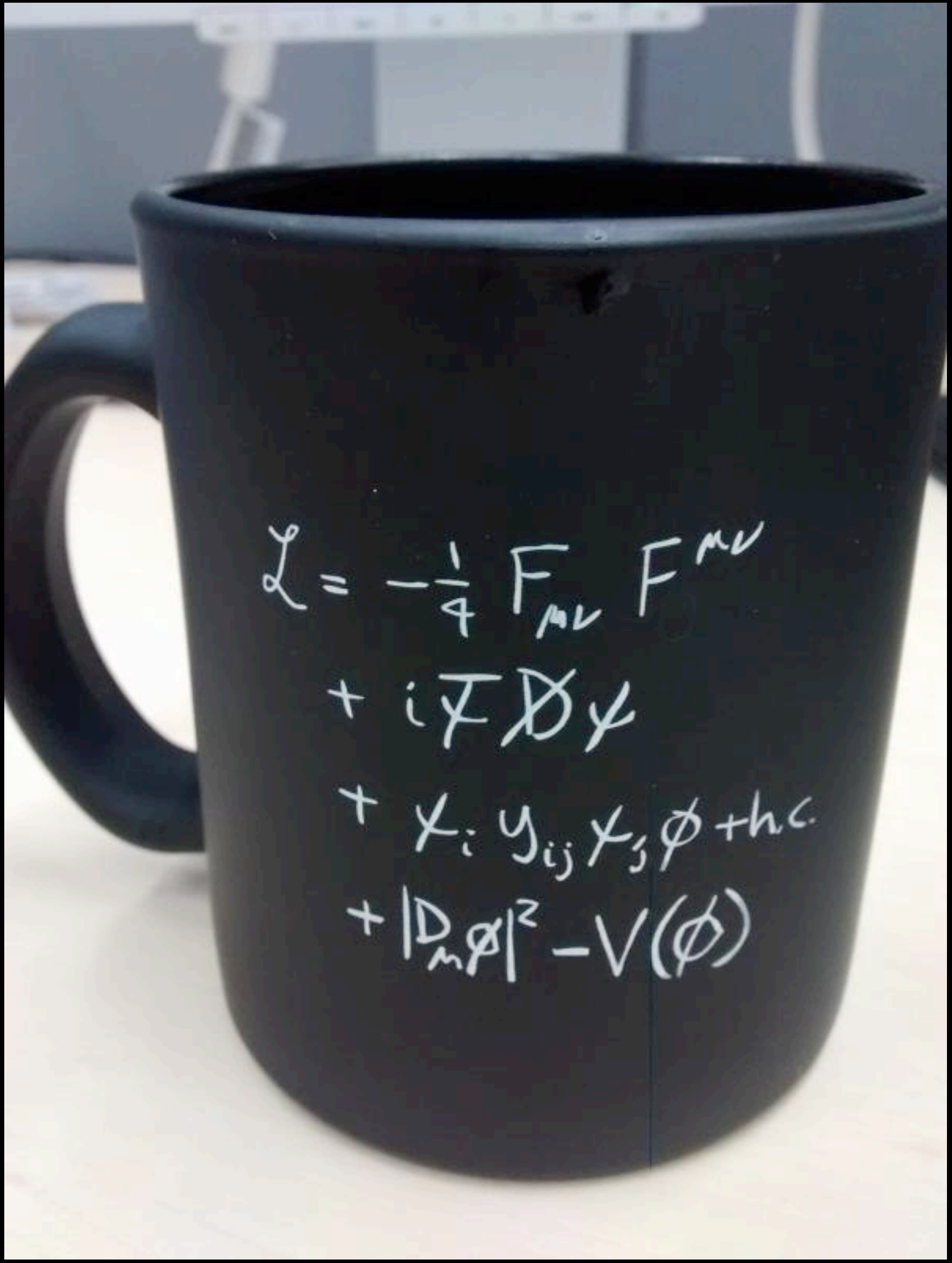
$$
\begin{gathered}
\tfrac{g}{2} \tfrac{m_e^\lambda}{M} [H (\bar{e}^\lambda e^\lambda) + i \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}} \phi^+ [-m_d^\kappa (\bar{u}^\lambda_j C_{\lambda\kappa} (1-\gamma^5) d^\kappa_j) + \\
m_u^\lambda (\bar{u}^\lambda_j C_{\lambda\kappa} (1+\gamma^5) d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}} \phi^- [m_d^\lambda (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1+\gamma^5) u^\kappa_j) - m_u^\kappa (\bar{d}^\lambda_j C^\dagger_{\lambda\kappa} (1 - \\
\gamma^5) u^\kappa_j] - \tfrac{g}{2} \tfrac{m_u^\lambda}{M} H (\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2} \tfrac{m_d^\lambda}{M} H (\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2} \tfrac{m_u^\lambda}{M} \phi^0 (\bar{u}^\lambda_j \gamma^5 u^\lambda_j) - \\
\tfrac{ig}{2} \tfrac{m_d^\lambda}{M} \phi^0 (\bar{d}^\lambda_j \gamma^5 d^\lambda_j) + \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 -
\end{gathered}
$$

5

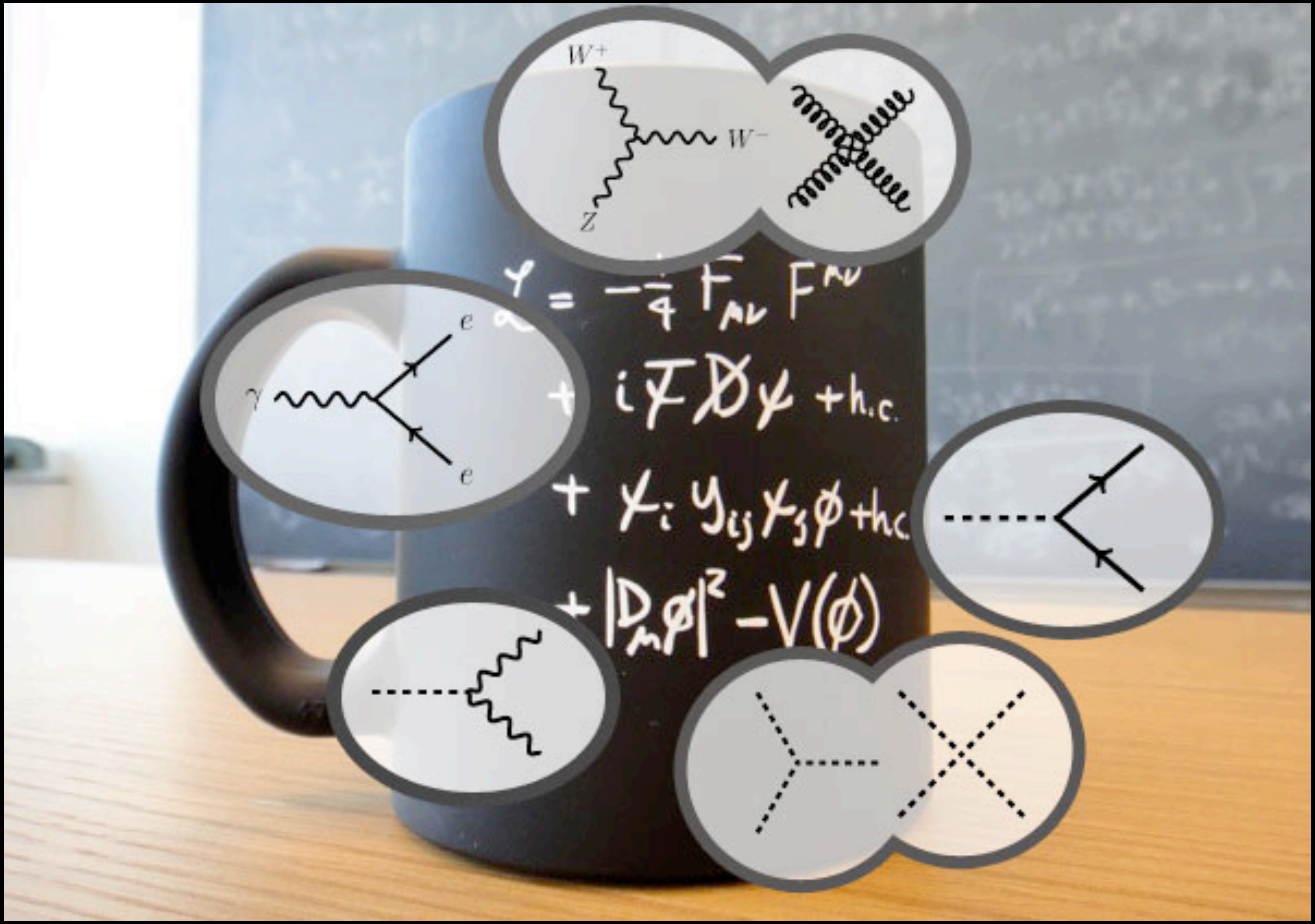
$$
\begin{gathered}
\tfrac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + i g c_w W^+_\mu (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + i g s_w W^+_\mu (\partial_\mu \bar{Y} X^- - \\
\partial_\mu \bar{X}^+ Y) + i g c_w W^-_\mu (\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + i g s_w W^-_\mu (\partial_\mu \bar{X}^- Y - \\
\partial_\mu \bar{Y} X^+) + i g c_w Z^0_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + i g s_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
\partial_\mu \bar{X}^- X^-) - \tfrac{1}{2} g M [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2} \bar{X}^0 X^0 H] + \\
\tfrac{1-2c_w^2}{2c_w} i g M [\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \tfrac{1}{2c_w} i g M [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \\
i g M s_w [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \tfrac{1}{2} i g M [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
\end{gathered}
$$

Good luck finding it!

The Standard Model Lagrangian (simplified…)



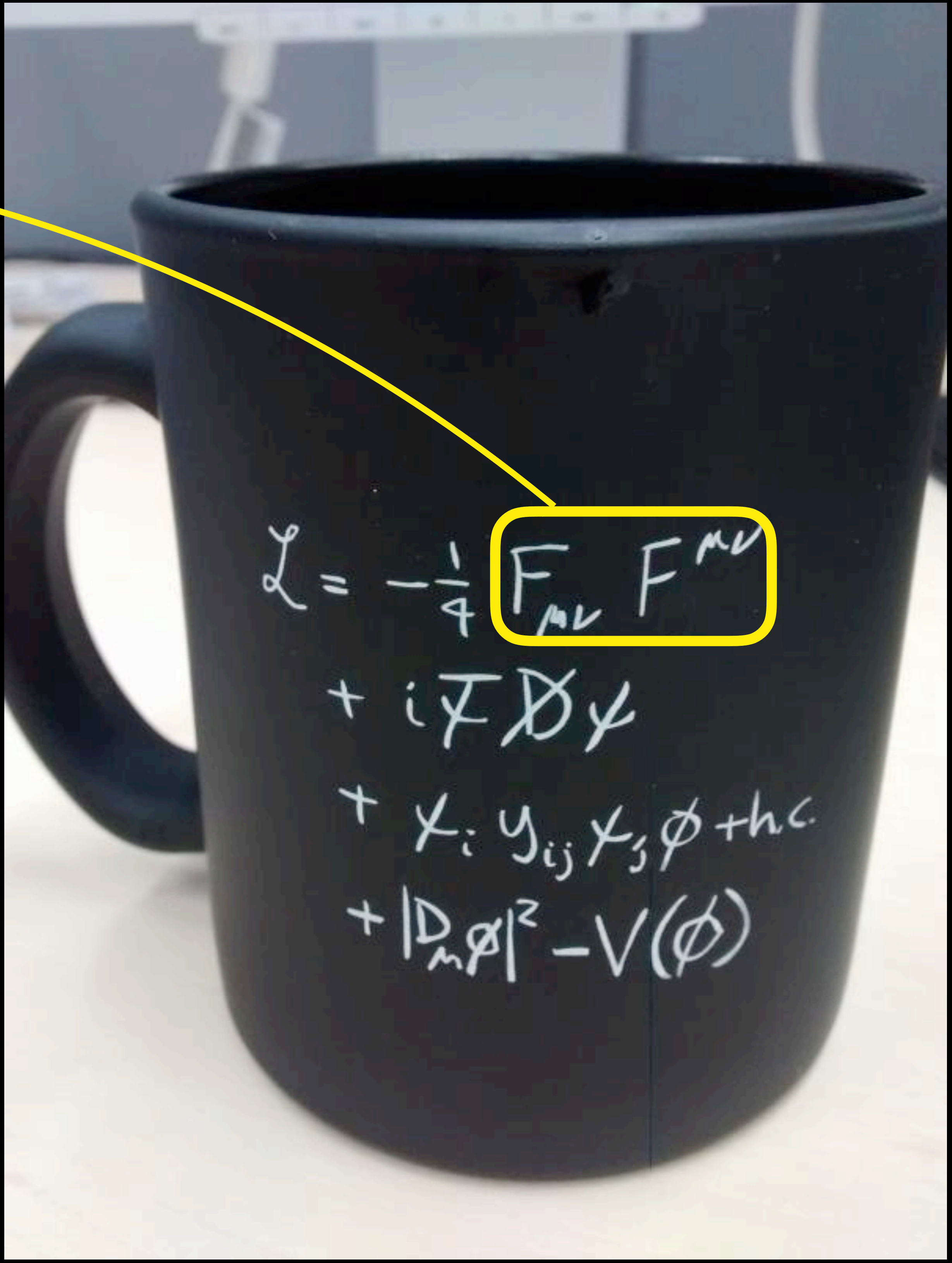
The Standard Model Lagrangian (simplified…)



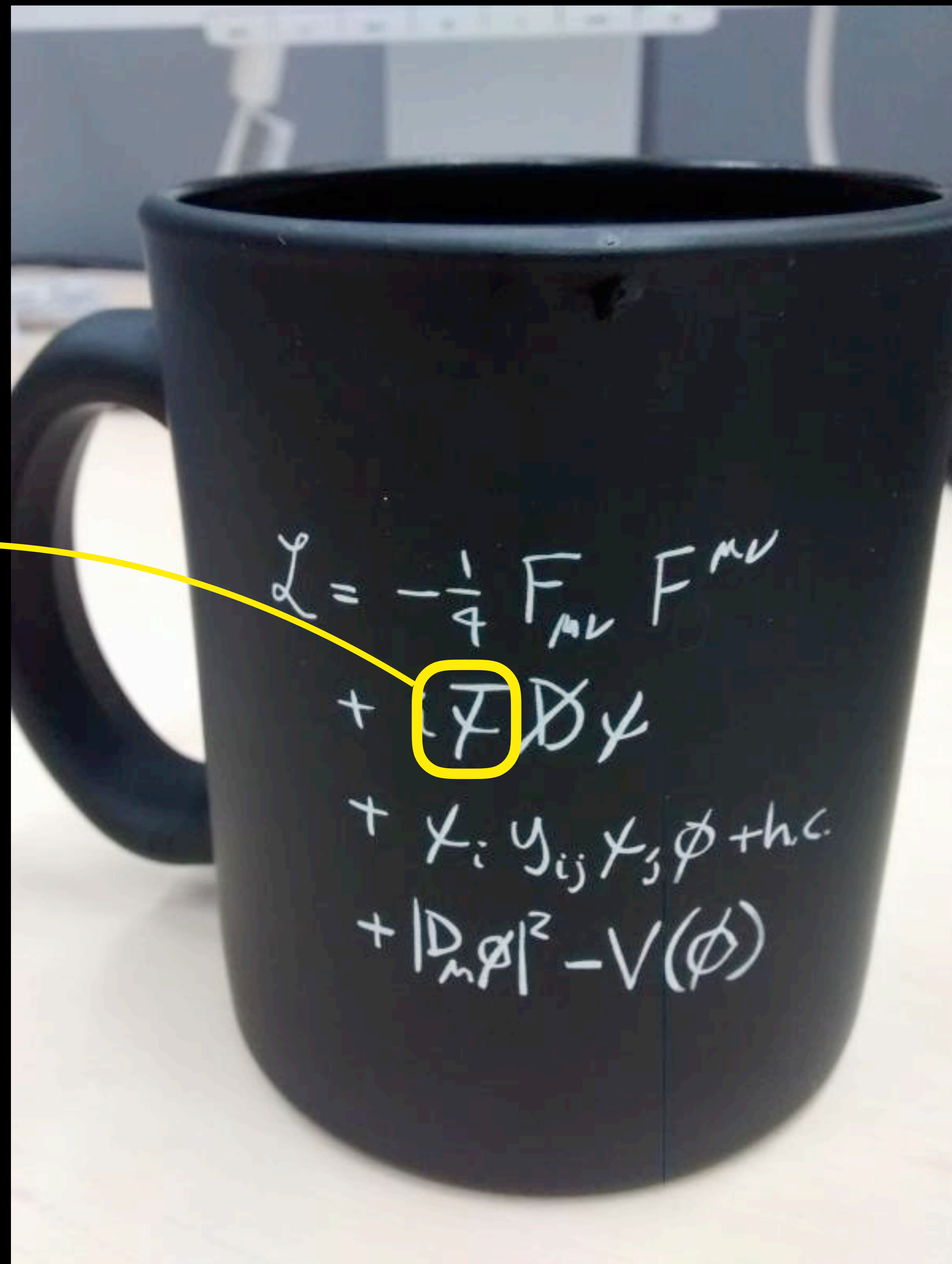
The Standard Model Lagrangian (simplified…)

Field description $F_{\mu\nu}$

Photons, gluons
Maxwell equations, E- and B-fields, self interactions

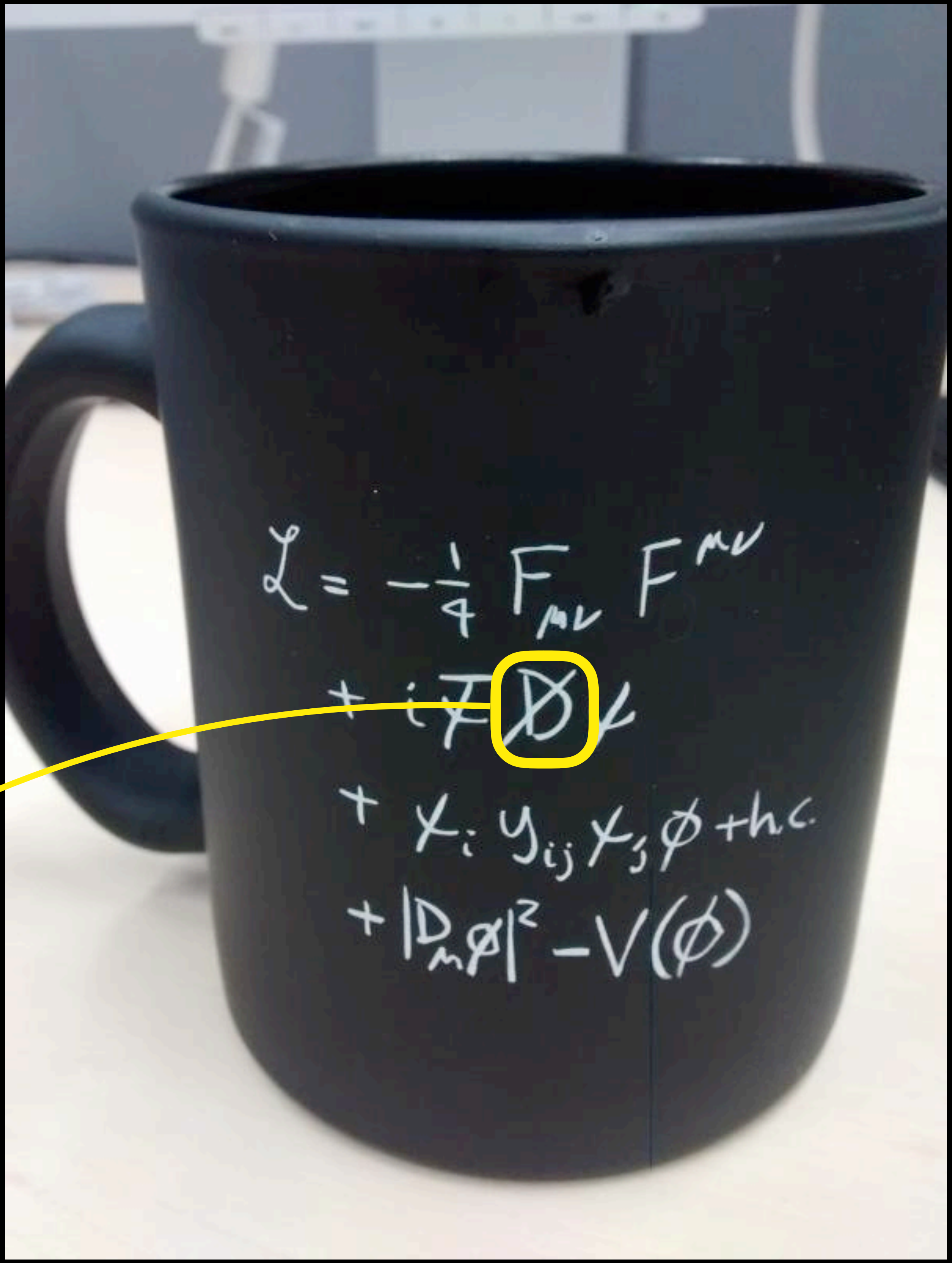


The Standard Model Lagrangian (simplified…)



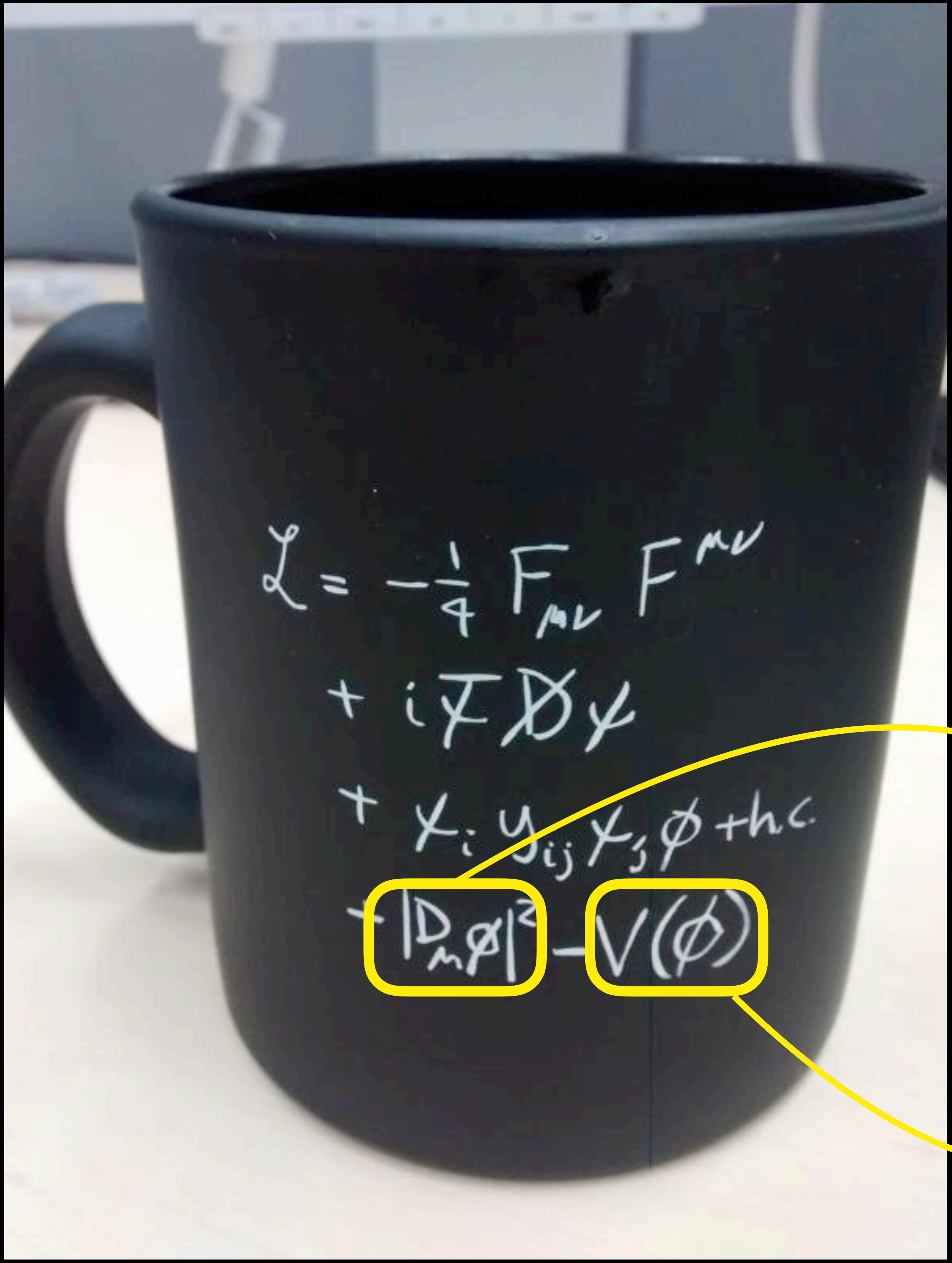
Particles ψ
normal matter, particles and antiparticles, quarks, leptons

The Standard Model Lagrangian (simplified…)



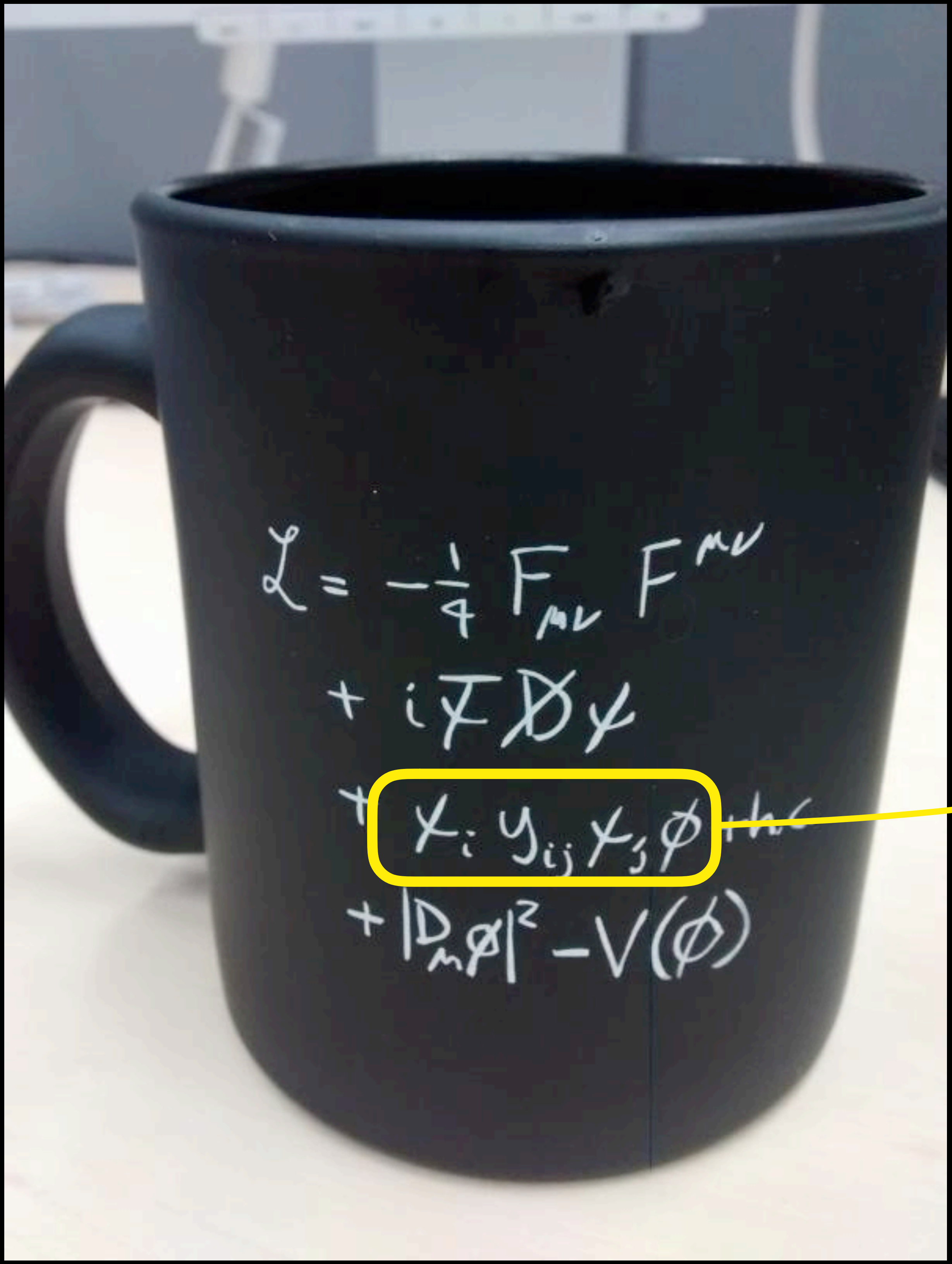
Interactions D
interactions between particles and fields

The Standard Model Lagrangian (simplified…)



Higgs field

The Standard Model Lagrangian (simplified…)



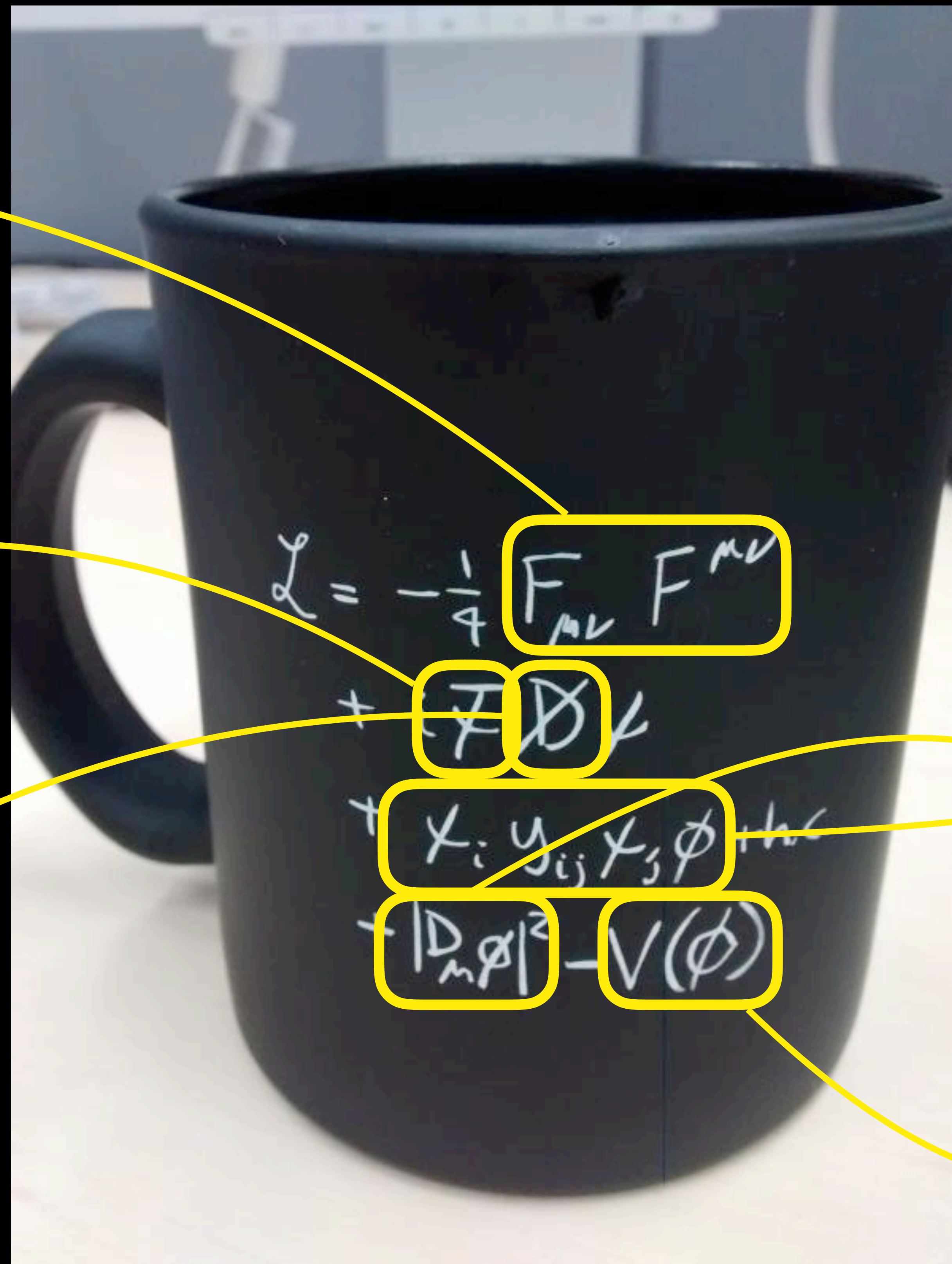
Mass ψψφ
how particles acquire mass by interacting with the Higgs field

The Standard Model Lagrangian (simplified…)

Field description $F_{\mu\nu}$
Photons, gluons
Maxwell equations, E- and B-fields, self interactions

Particles ψ
normal matter, particles and antiparticles, quarks, leptons

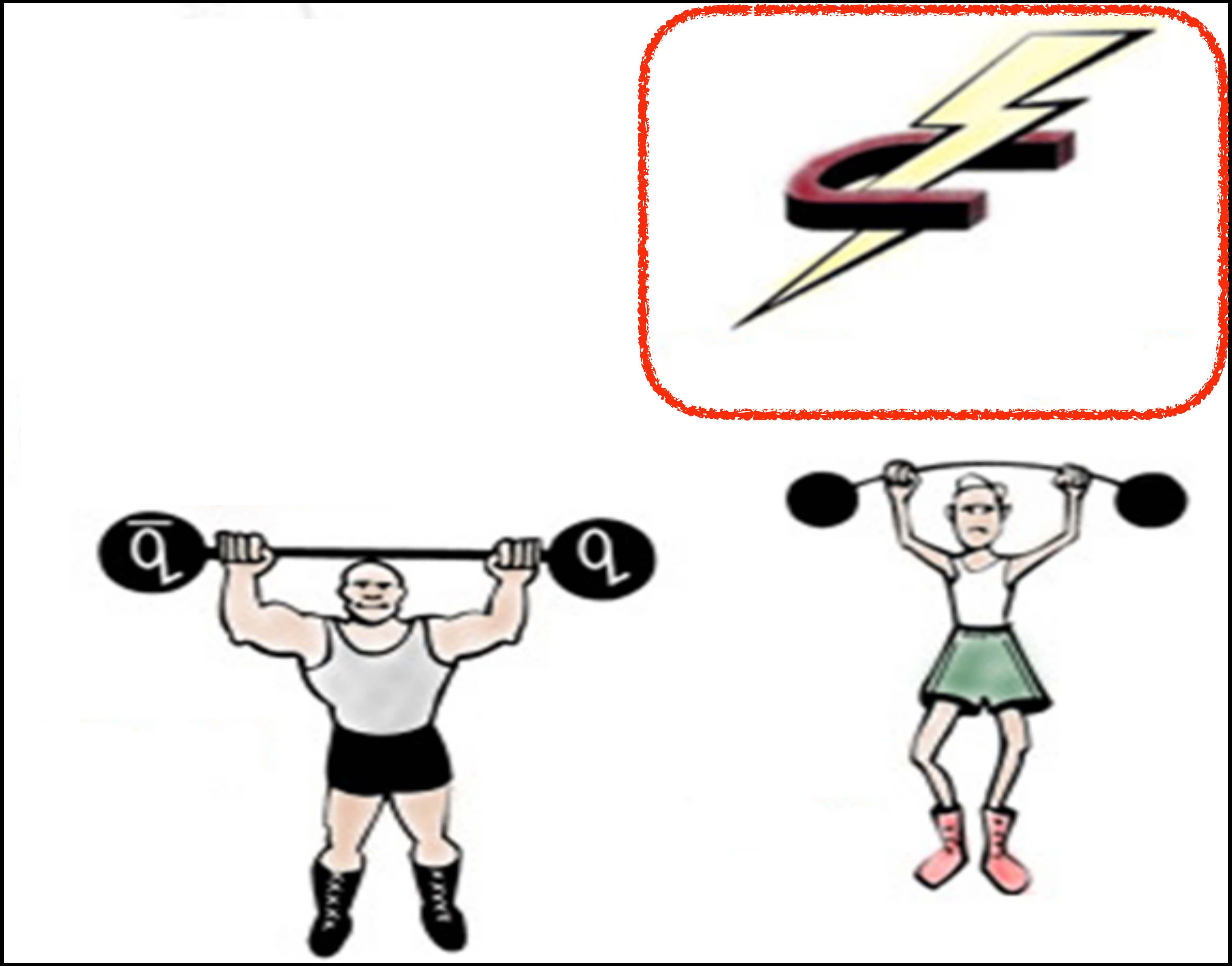
Interactions D
interactions between particles and fields



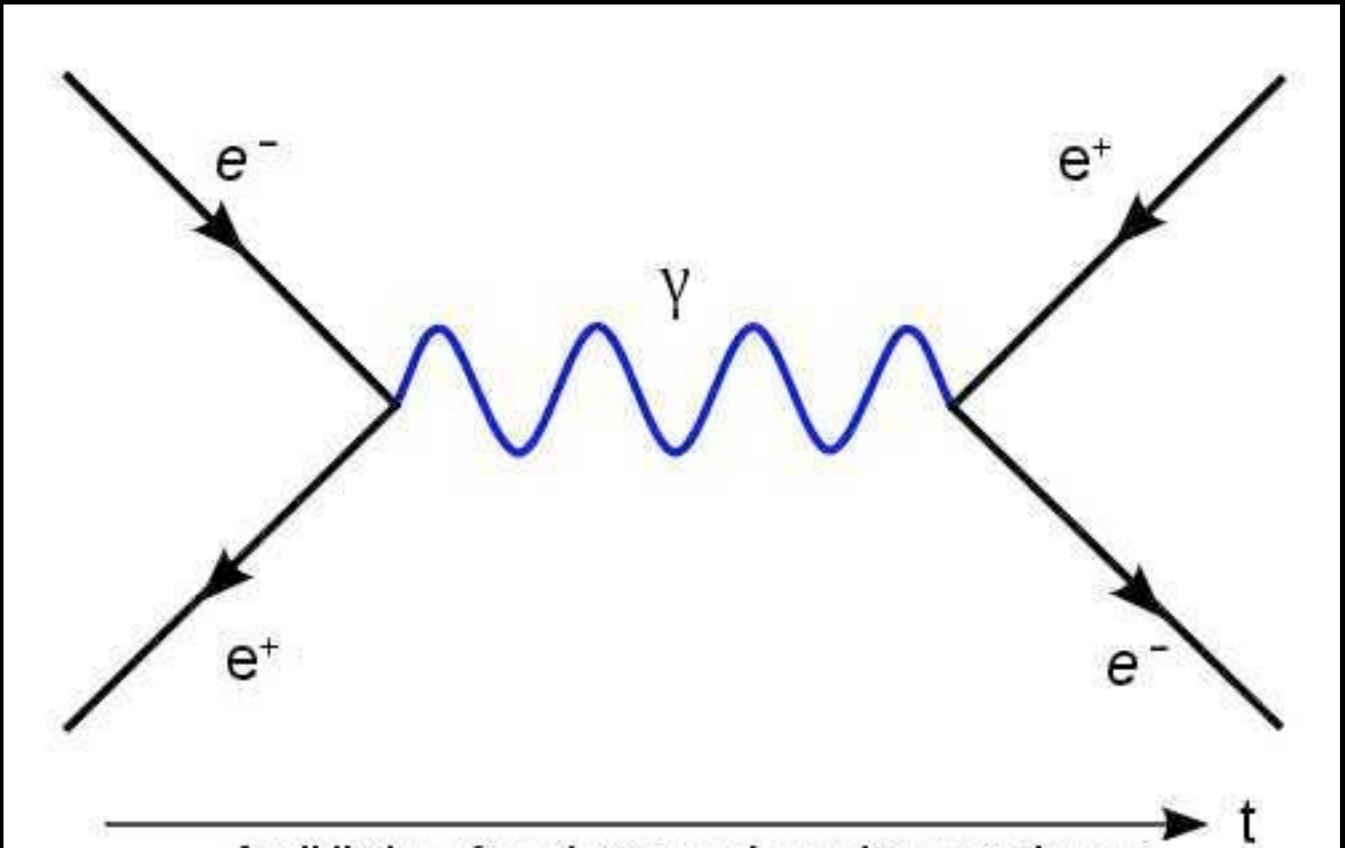
Mass ψψφ
how particles acquire mass by interacting with the Higgs field

Higgs field

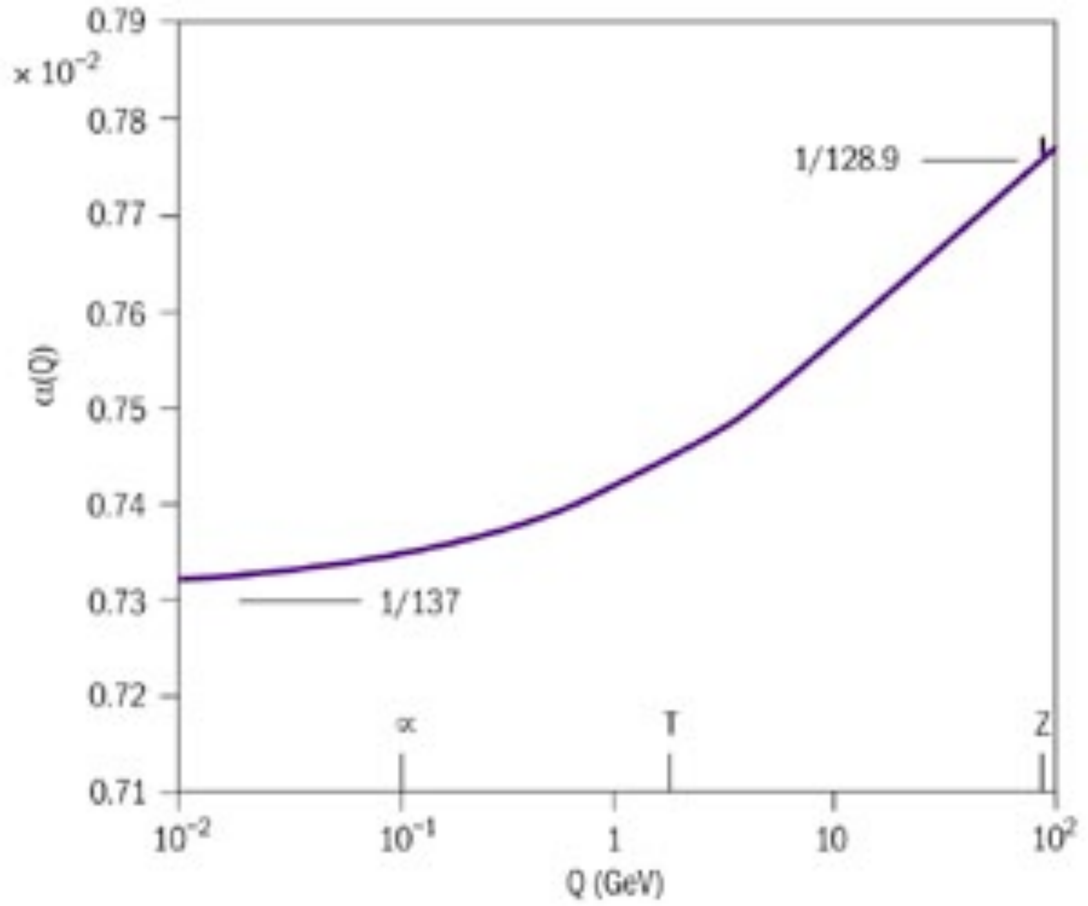
Forces and mediators



Quantum ElectroDynamics (QED)



Annihilation of an electron and a positron creating a photon which decays into a new electron positron pair



Fermi's golden rule

- **Decay** $a \longrightarrow b+c$
  - Decay width is reciprocal of decay time $\tau = \frac{1}{\Gamma}$
  - Total width is sum of partial widths $\Gamma_{tot} \sum \Gamma_i$
  - Branching fraction for certain decay mode $BP = \frac{\Gamma_i}{\Gamma_{tot}}$
  - Unit: inverse seconds

- **Scattering** $a+b \longrightarrow c+d$
  - Parameter of interest is "size of target", *cross section* σ
  - Total cross section is sum of possible processes $\sigma_{tot} = \sum \sigma_i$
  - Unit: inverse surface

- **Fermi's golden rule**

$$transition\text{-}rate = \frac{2\pi}{\hbar} |M_{if}|^2 \rho(E)$$

Fermi's golden rule (cont.)

Initial state

Final state

$|i\rangle = |\psi_i\rangle$

$|f\rangle = |\psi_f\rangle$

Fermi's golden rule gives the transition probability to go from the initial state ψ_i to the final state ψ_f

$$\Gamma = \frac{2\pi}{\hbar} |\langle f|H|i\rangle|^2 \rho(E_f) = \frac{2\pi}{\hbar} |M_{if}|^2 \rho(E_f)$$

Matrix element
contains dynamical information related to the interactions (E/M, weak, strong)

Density of states factor
contains kinematic information related to the masses, momenta etc

Maxwell equations

$$\vec{\nabla} \cdot \vec{E} = \rho \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{j}$$

We want to work in the relativistic limit

Introduce a mathematical tool in the form of a field: $A^{\mu} = (A^0, \vec{A})$

The electric and magnetic fields (E and B) are the physical fields, A^{μ} is not!!!

Rewrite the Maxwell equations in the form:

$$\partial_{\mu}\partial^{\mu}A^{\nu} - \partial^{\nu}\partial_{\mu}A^{\mu} = j^{\nu}$$

$$\partial_{\mu}F^{\mu\nu} = j^{\nu}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Gauge invariance

- The electric and magnetic fields are invariant under gauge transformations

$$A^0 \rightarrow A^{0'} = A^0 - \frac{\partial \Lambda}{\partial t}$$

$$A^\mu \rightarrow A^{\mu'} = A^\mu + \partial^\mu \Lambda$$

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \vec{\nabla}\Lambda$$

- Is physics the same for the fields A^μ and $A^{\mu'}$?
  - Yes if $\Psi \rightarrow \Psi' = e^{iq\Lambda}\Psi$
- What if the invariance is not requested to be global but local?
  - What if $\Lambda \rightarrow \Lambda(\vec{r}, t)$
  - How can we keep the Schrodinger equation still invariant under such transformation?
- Introducing the gauge-covariant derivative solves the problem

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + iqA_\mu$$

QED Lagrangian

$$(i\gamma_\mu\partial^\mu - m)\Psi = 0$$

- Dirac equation:

$$\mathscr{L} = \overline{\psi}\,(i\gamma_\mu D^\mu - m)\psi \;=\; \overline{\psi}\,(i\gamma_\mu\partial^\mu - m)\psi - qA_\mu\,\overline{\psi}\,\gamma^\mu\psi$$

$$= \mathscr{L}_{free} - \mathscr{L}_{int}$$

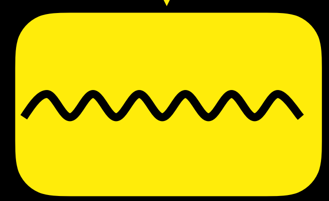
- Let's replace the derivative with the covariant derivative

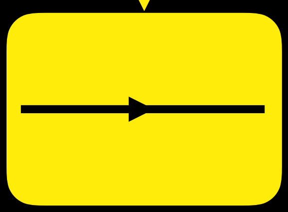
Gauge invariance leads to fields
and their interactions with particles

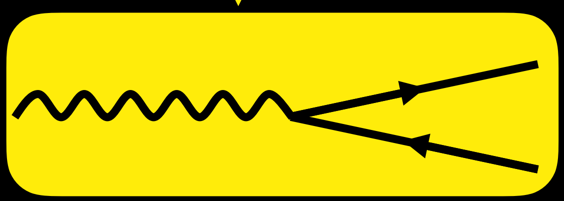
$$\mathscr{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\,(i\gamma_\mu\partial^\mu - m)\psi - qA_\mu\,\overline{\psi}\,\gamma^\mu\psi$$

QED Lagrangian (cont.)

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\,(i\gamma_\mu\partial^\mu - m)\psi - qA_\mu\,\overline{\psi}\,\gamma^\mu\psi$$







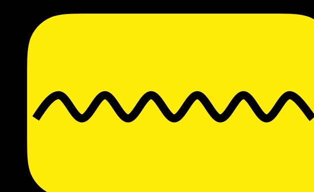
Free photon field A^μ

Fermion ψ with mass m

Interaction with coupling α

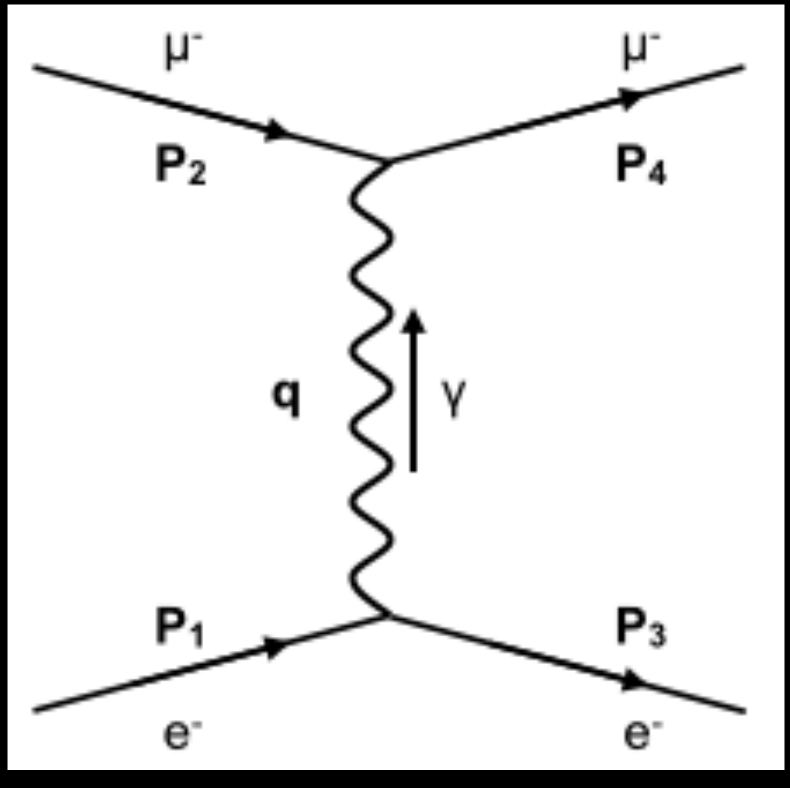
Symmetries and gauge tranformations (QED)

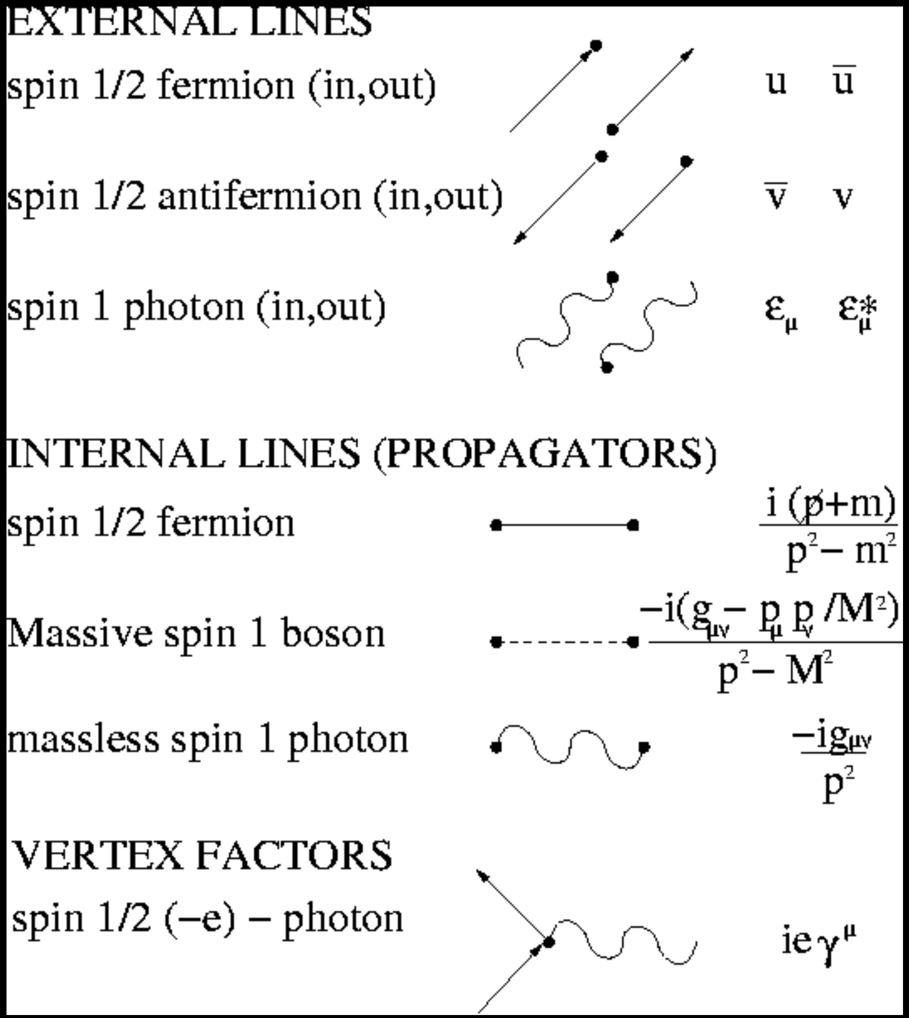
Conserved quantities	Gauge transformation	Symmetry group	Field
Charge: ψ	$\Psi \rightarrow \Psi' = e^{ia}\Psi$	U(1) (QED)	A_μ (photon)



Feynman rules for QED

- ✓ A tool to describe interactions and calculate processes (e.g. cross-sections)
- ✓ Convenient set of rules to calculate the matrix element and the density of state factor





Following the Feynman rules for QED we have:

$$\int \left[\overline{u}(\mathbf{P_3})(ig_e\gamma^\mu)u(\mathbf{P_1})\right]\left(\frac{-ig_{\mu\nu}}{q^2}\right)\left[\overline{u}(\mathbf{P_4})(ig_e\gamma^\nu)u(\mathbf{P_2})\right]\cdot\left[(2\pi)^4\delta^4(\mathbf{P_1}-\mathbf{P_3}-\mathbf{q})\right]\cdot\left[(2\pi)^4\delta^4(\mathbf{P_2}-\mathbf{P_4}+\mathbf{q})\right]\cdot\frac{d^4q}{(2\pi)^4}$$

$$= ig_e^2(2\pi)^4\int \overline{u}(\mathbf{P_3})\gamma^\mu u(\mathbf{P_1})\cdot\frac{g_{\mu\nu}}{q^2}\cdot\overline{u}(\mathbf{P_4})\gamma^\nu u(\mathbf{P_2})\cdot\delta^4(\mathbf{P_1}-\mathbf{P_3}-\mathbf{q})\cdot\delta^4(\mathbf{P_2}-\mathbf{P_4}+\mathbf{q})d^4q$$

But $\mathbf{P_1}-\mathbf{P_3}=\mathbf{q}$ and the previous can be written:

$$ig_e^2(2\pi)^4\overline{u}(\mathbf{P_3})\gamma^\mu u(\mathbf{P_1})\cdot\frac{g_{\mu\nu}}{(\mathbf{P_1}-\mathbf{P_3})^2}\cdot\overline{u}(\mathbf{P_4})\gamma^\nu u(\mathbf{P_2})\cdot\delta^4(\mathbf{P_1}+\mathbf{P_2}-\mathbf{P_3}-\mathbf{P_4})d^4q$$

To get the matrix element, one simply cancels the δ-function and gets rid of the imaginary factor:

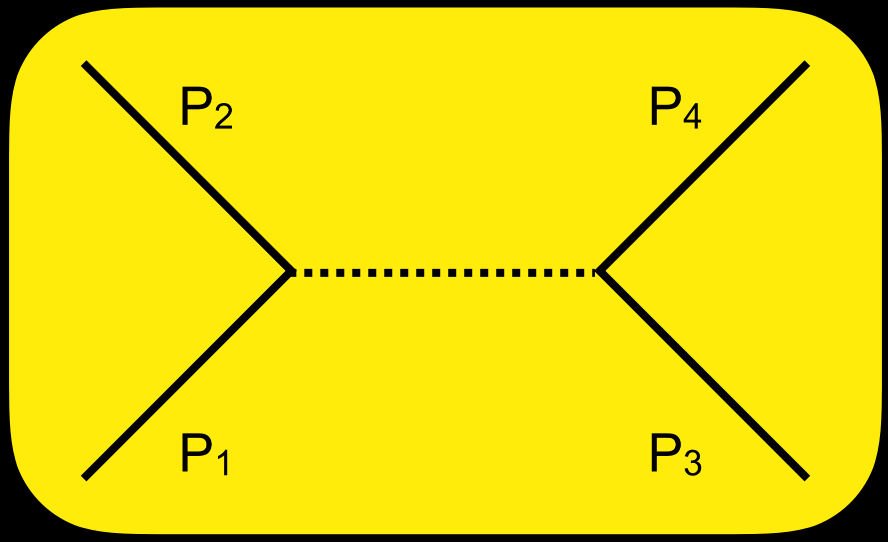
$$M_{if}=\frac{-g_e^2}{(\mathbf{P_1}-\mathbf{P_3})^2}\left[\overline{u}(\mathbf{P_3})\gamma^\mu u(\mathbf{P_1})g_{\mu\nu}\overline{u}(\mathbf{P_4})\gamma^\nu u(\mathbf{P_2})\right]\Leftrightarrow$$

$$M_{if}=\frac{-g_e^2}{(\mathbf{P_1}-\mathbf{P_3})^2}\left[\overline{u}(\mathbf{P_3})\gamma^\mu u(\mathbf{P_1})\overline{u}(\mathbf{P_4})\gamma_\mu u(\mathbf{P_2})\right] \qquad (3.2.1)$$

General Feynman diagrams

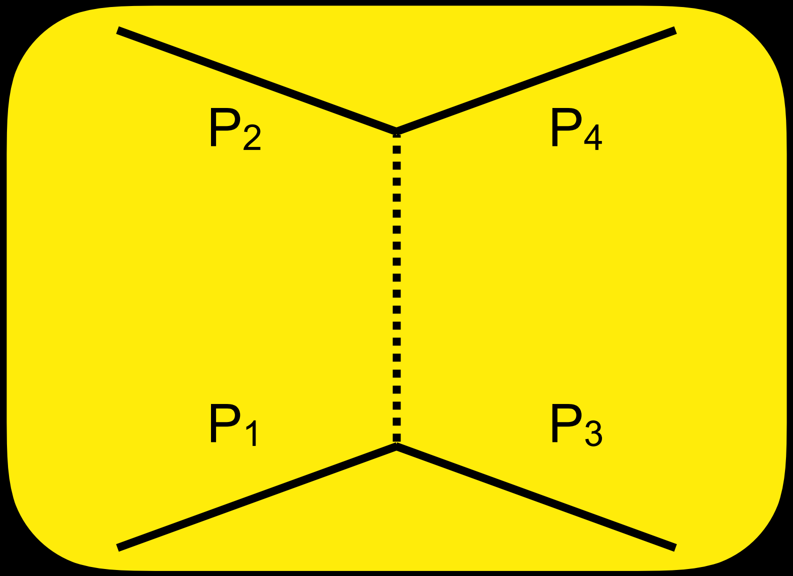
Mandelstam variables s, t, u

s-channel

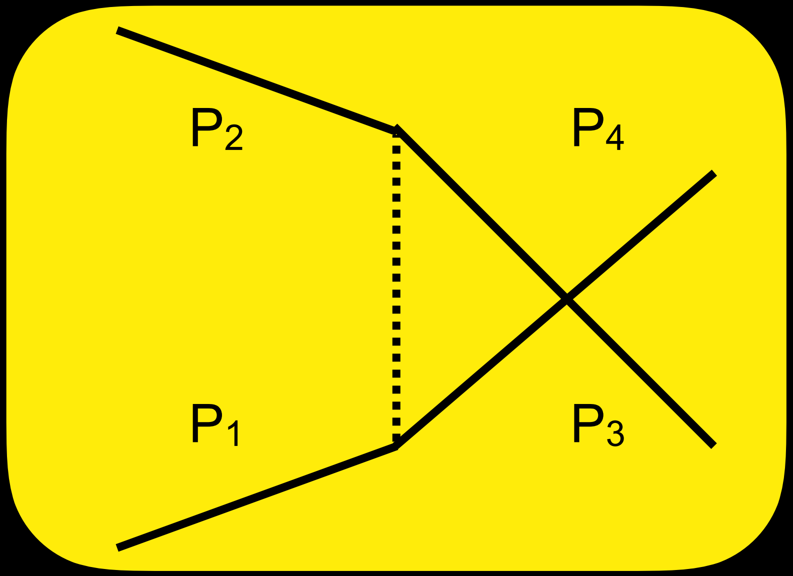


- s-channel: particles 1 and 2 join at the vertex and create particles 3 and 4 with the emission of an intermediate particle
  - resonances, unstable particles
- t-channel: particle 1 interacts with particle 2 via the emission of an intermediate particle; particle 1 then becomes particle 3 and particle 2 becomes particle 4
- u-channel: same as the t-channel but particles 3 and 4 are interchanged (important if the initial or/and final state particles are identical)

t-channel

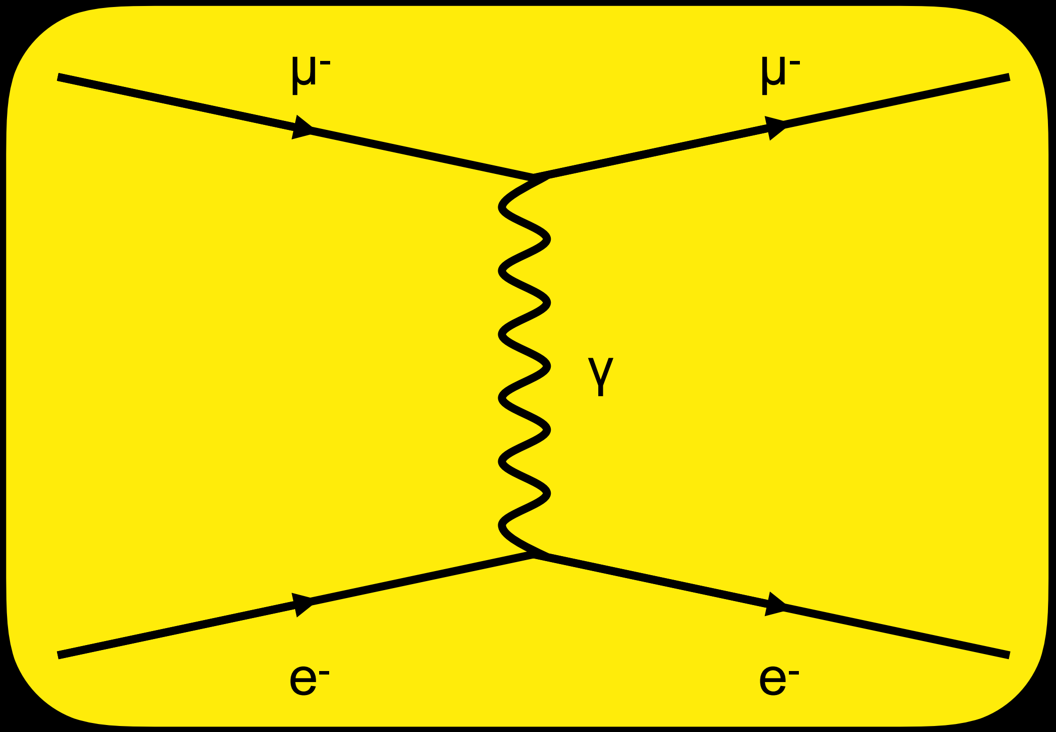


u-channel

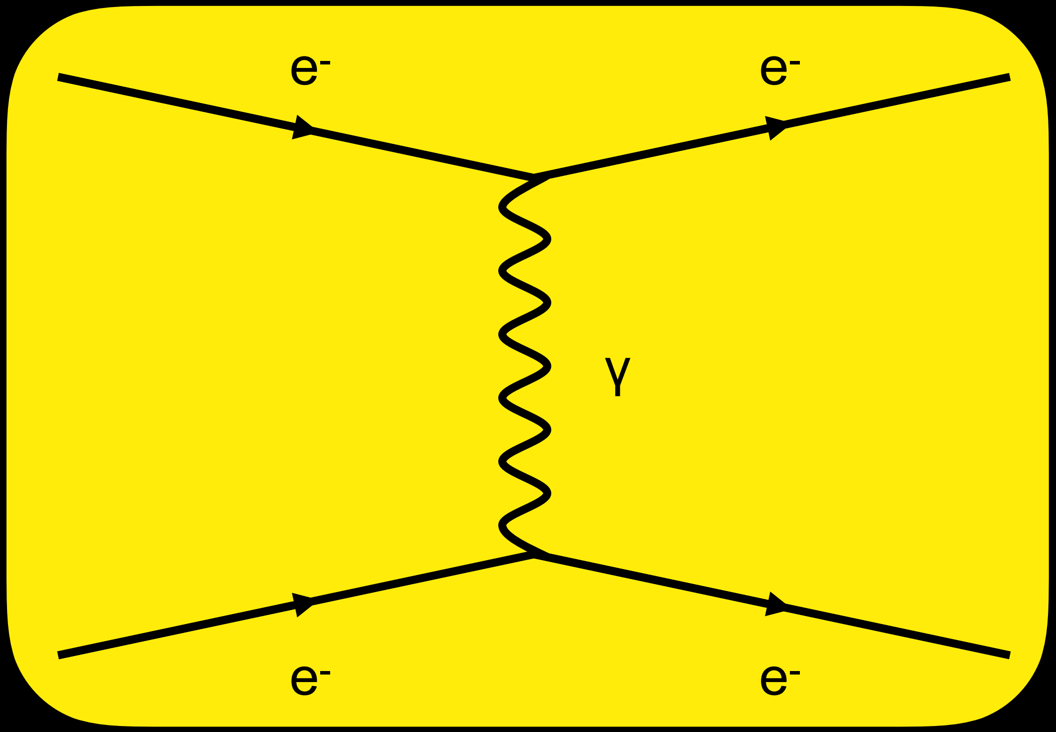


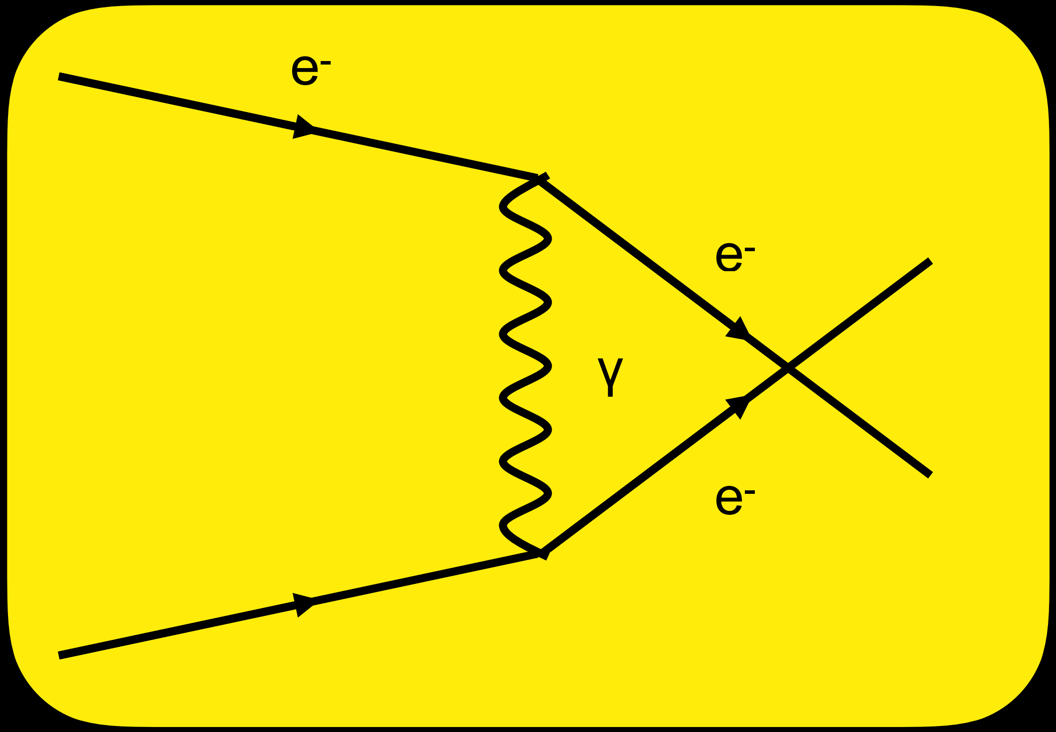
QED processes via Feynman diagrams (elastic processes)

e-μ scattering



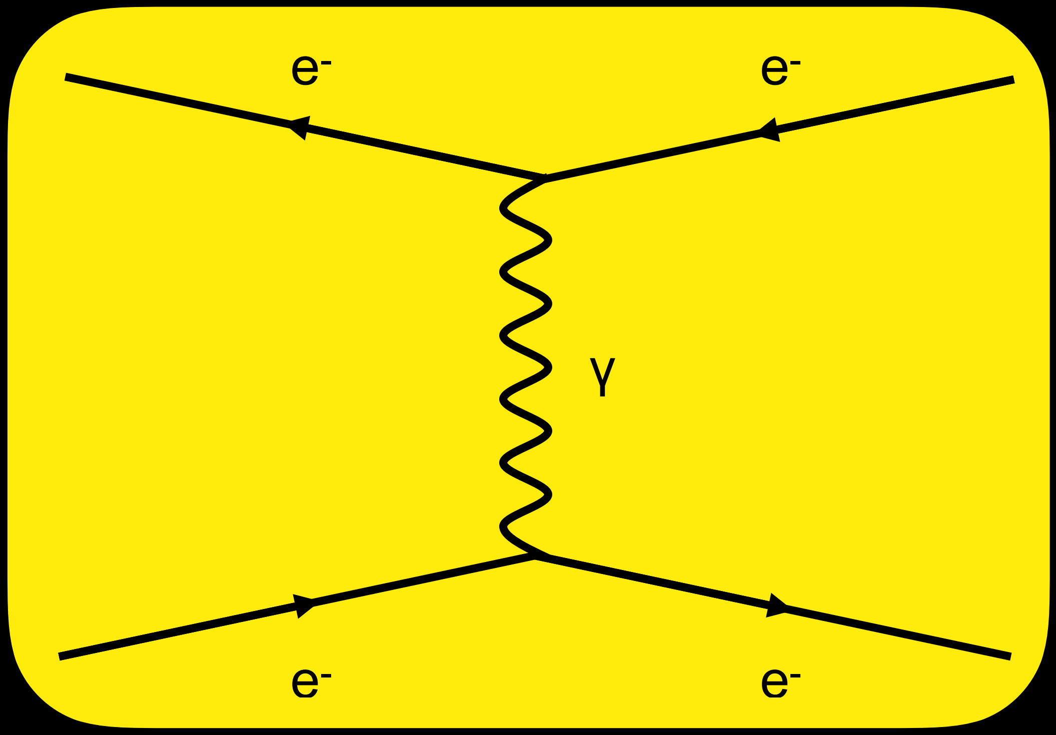
Moeller scattering

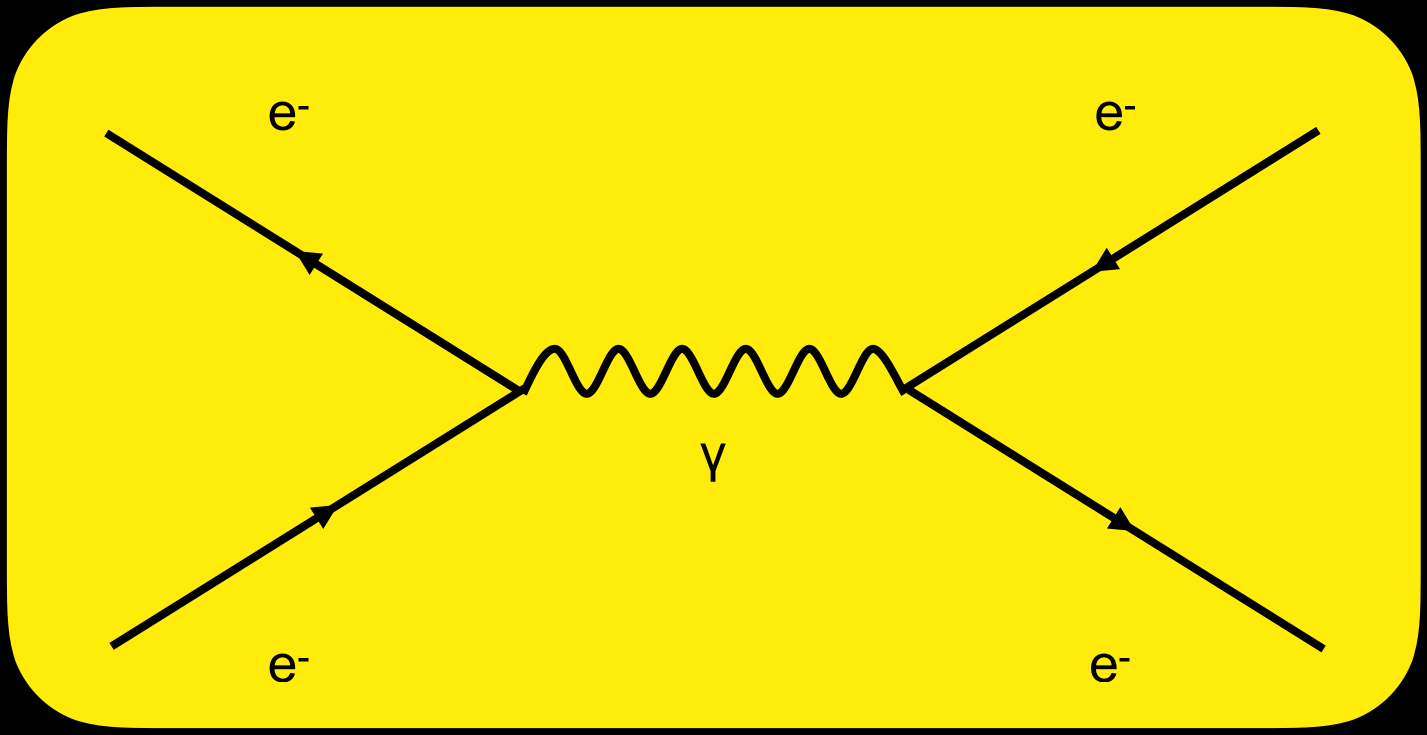




QED processes via Feynman diagrams (elastic processes)

Bhabha scattering



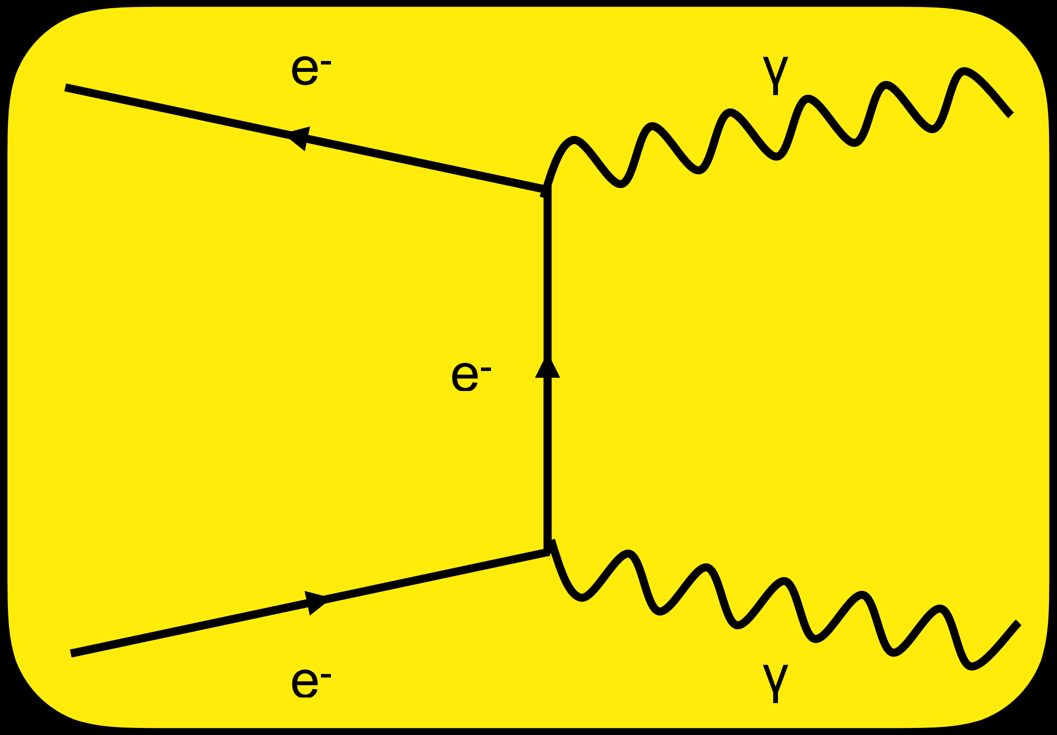


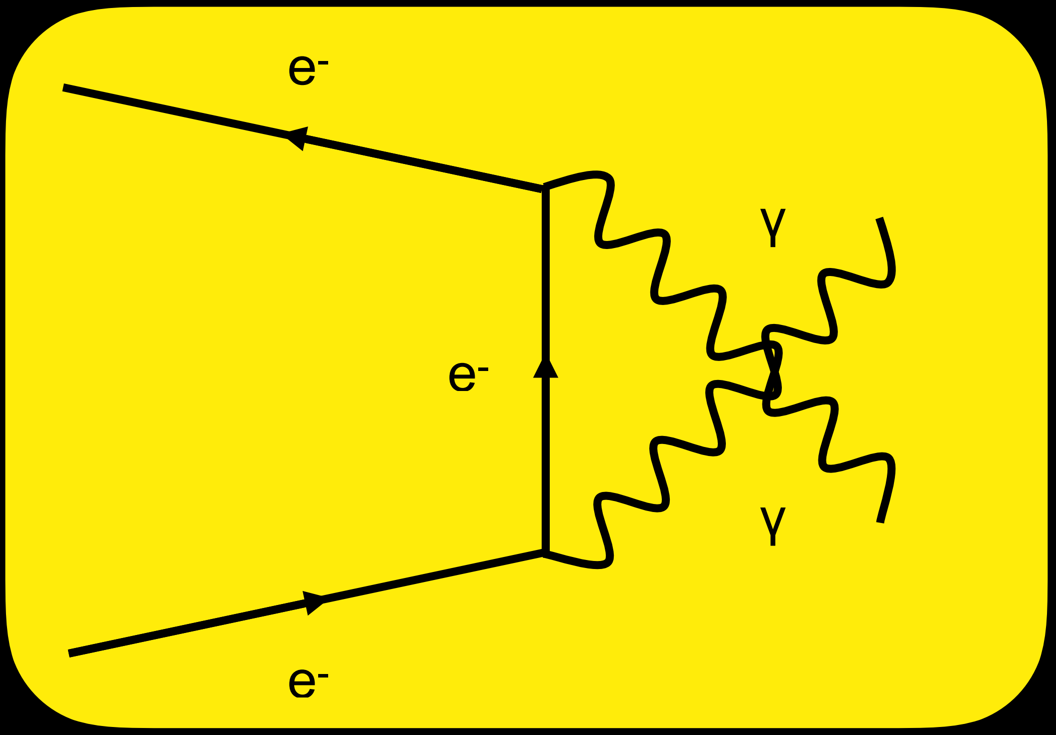
Scattering process

Annihilation process

QED processes via Feynman diagrams (inelastic processes)

Pair annihilation





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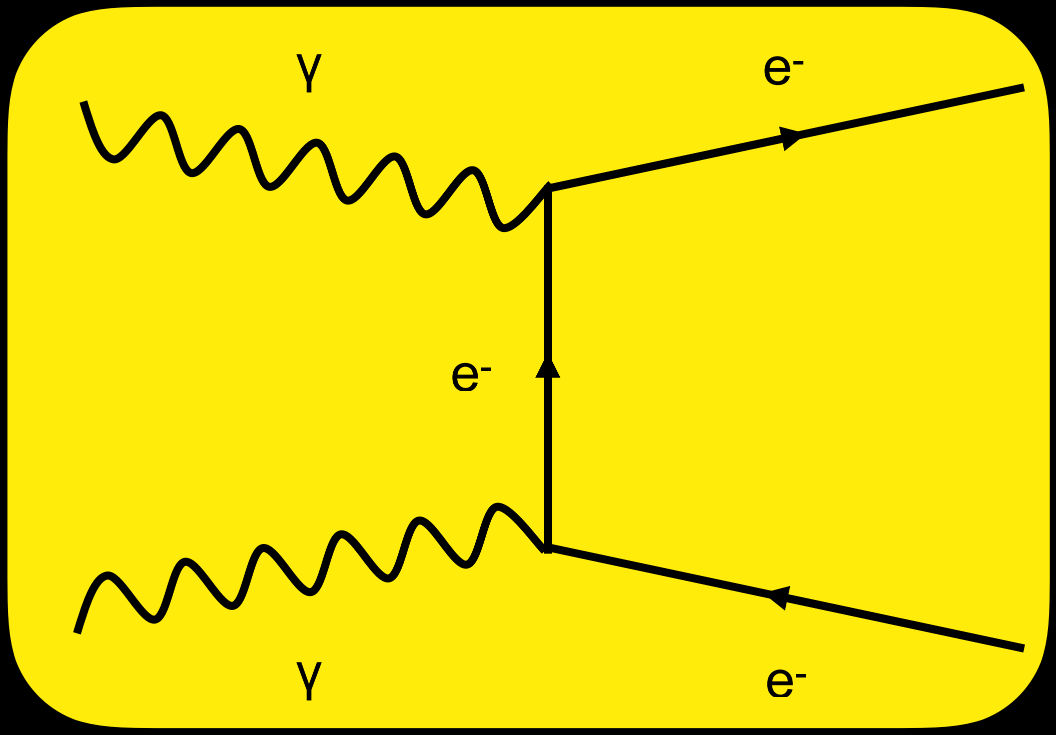
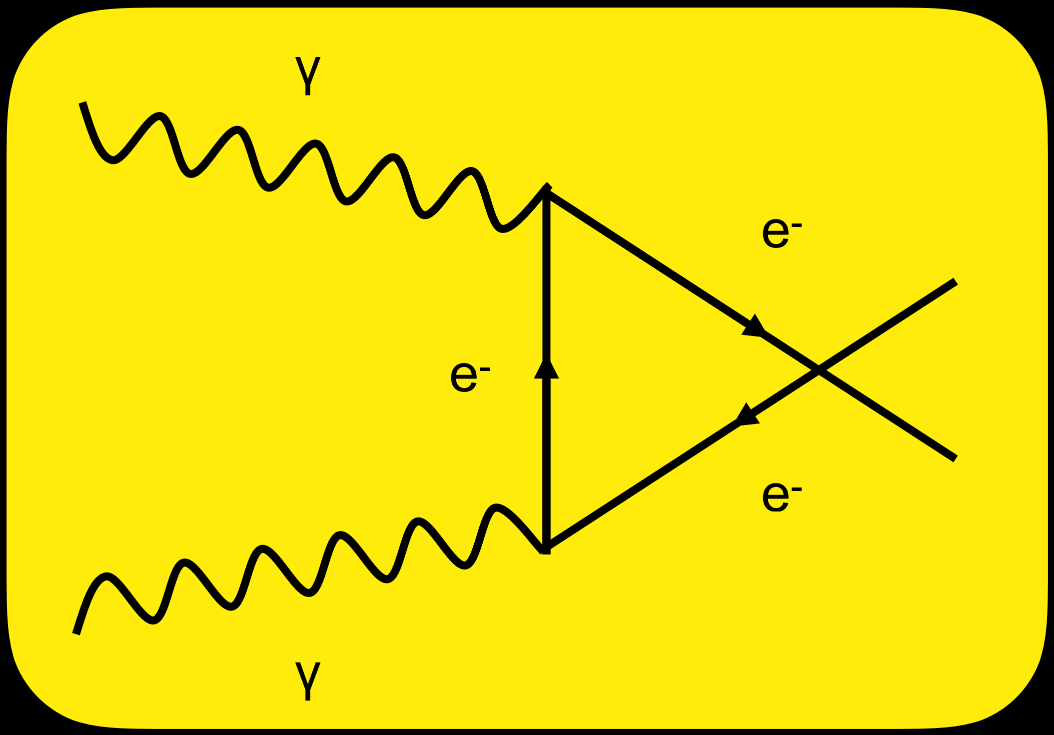
QED processes via Feynman diagrams (inelastic processes)

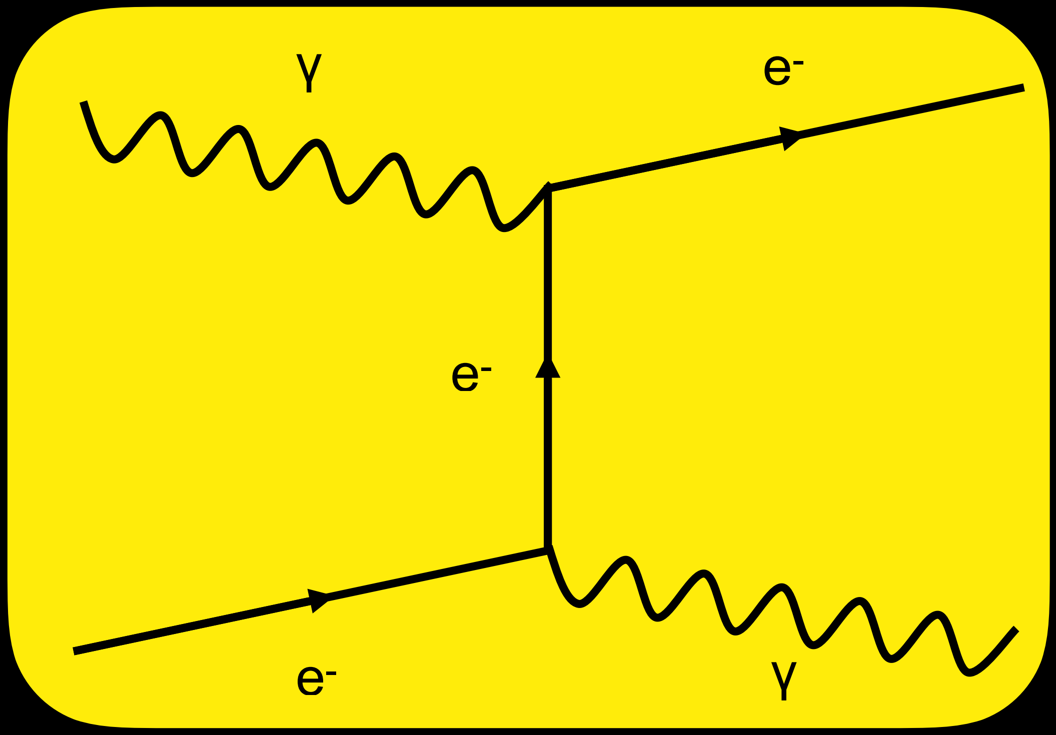
Pair production

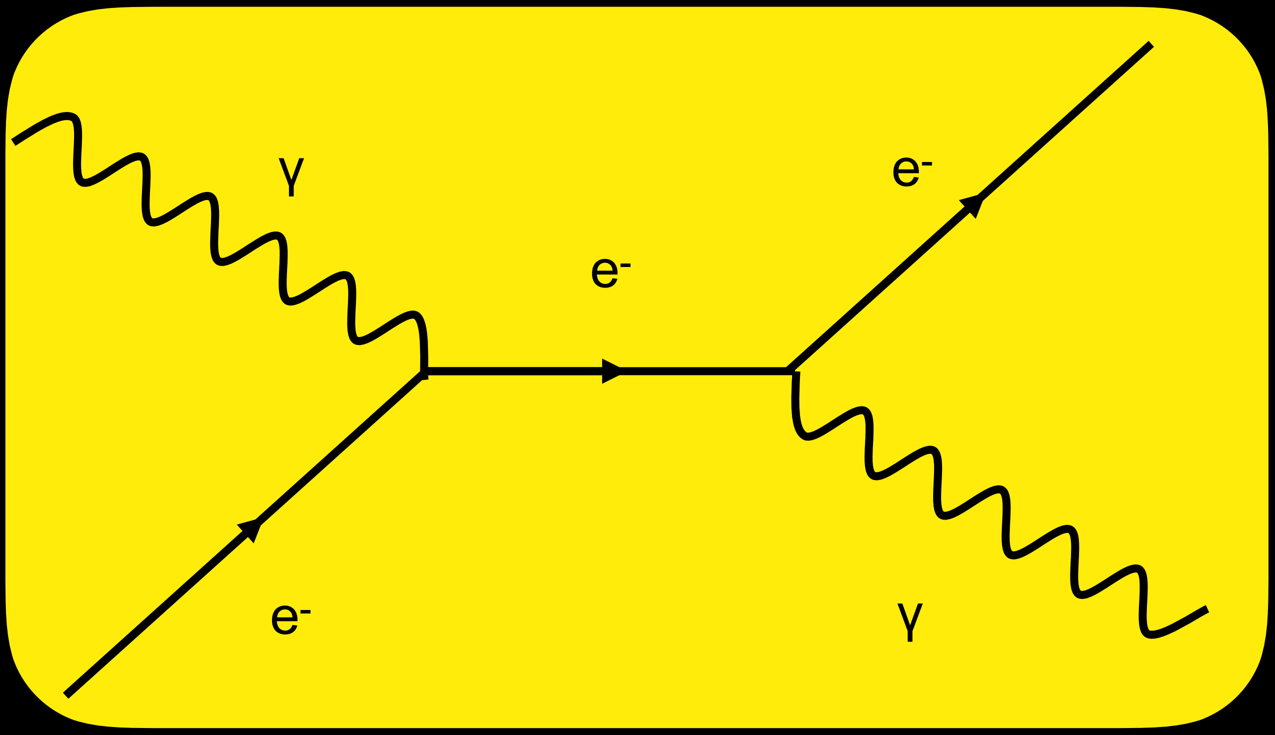




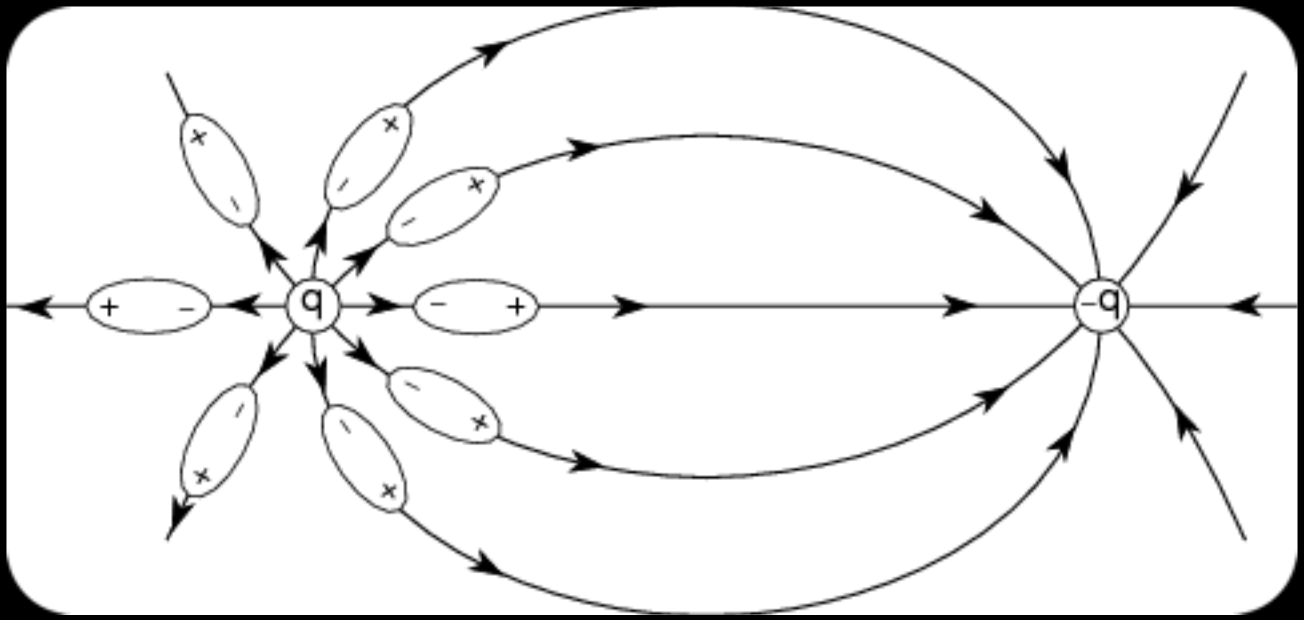
QED processes via Feynman diagrams (inelastic processes)

Compton scattering





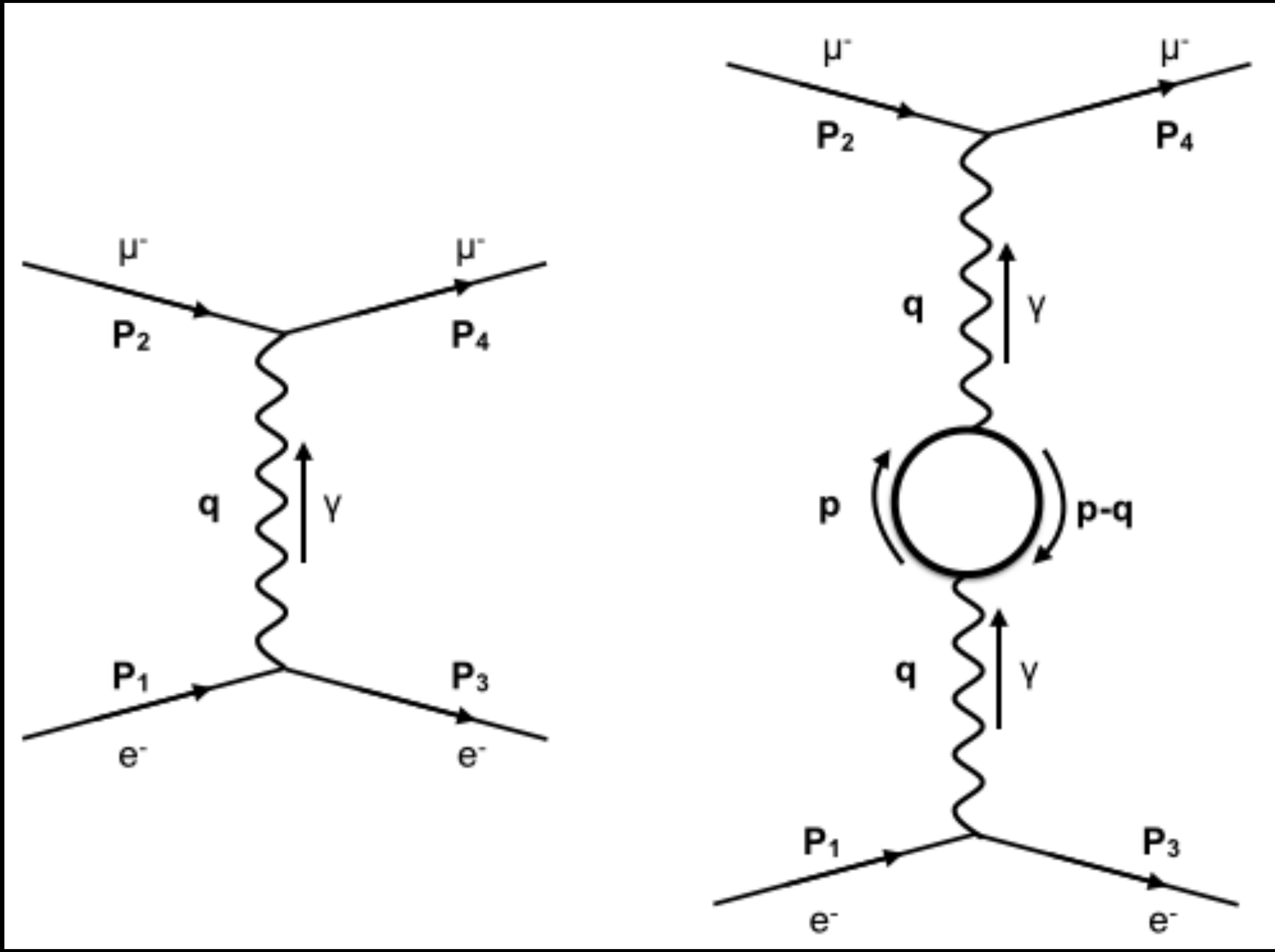
Charge screening in QED



- In QED a charged particle is surrounded by a cloud of virtual photons and electron-positron pairs continuously pop in and out of existence
- Because of attraction and repulsion in case of an electron, the positrons of the pairs tend to be closer and "screen" its charge
  - This is called vacuum polarisation and is analogous to the polarisation of a dielectric medium
- This gives rise to the notion of an effective charge that becomes smaller at large distances
- One defines the β-function that is positive in QED (as we will see later)

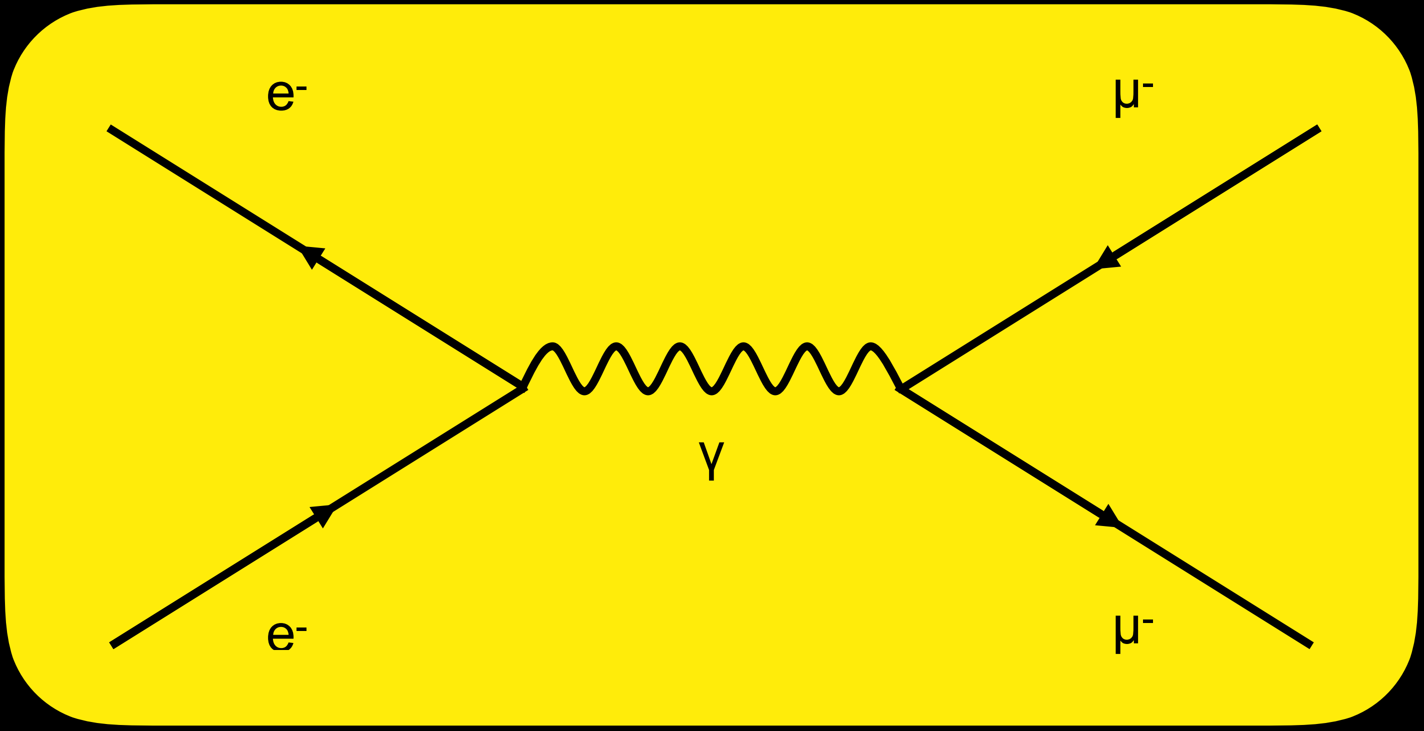
$$\beta = -\frac{de(r)}{d(\ln r)}$$

QED higher order corrections



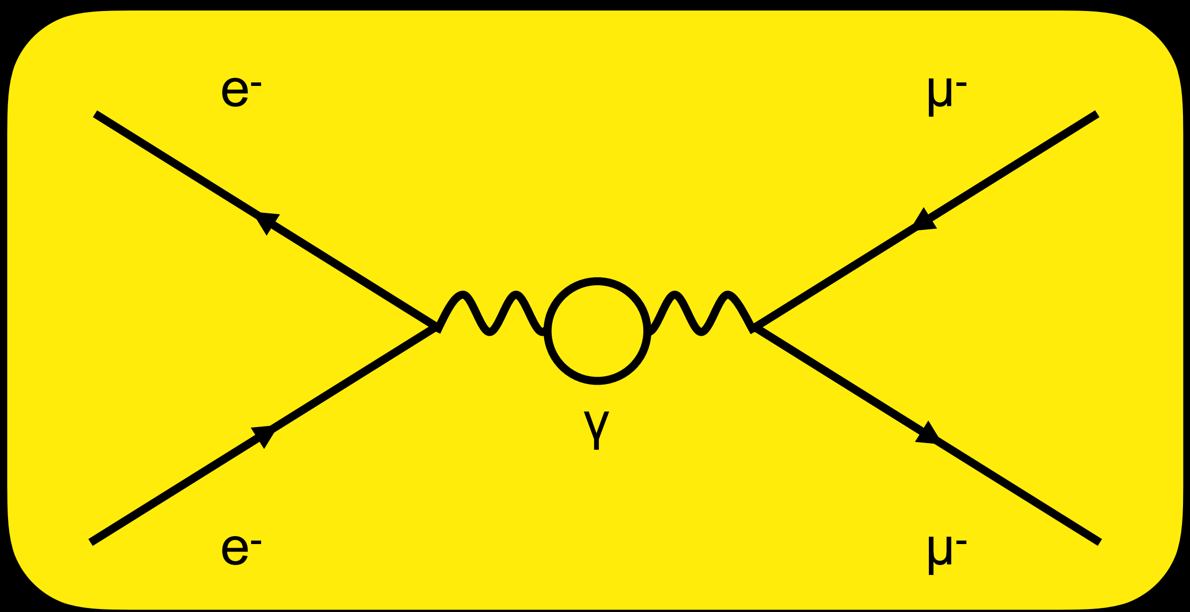
QED higher order corrections (cont.)

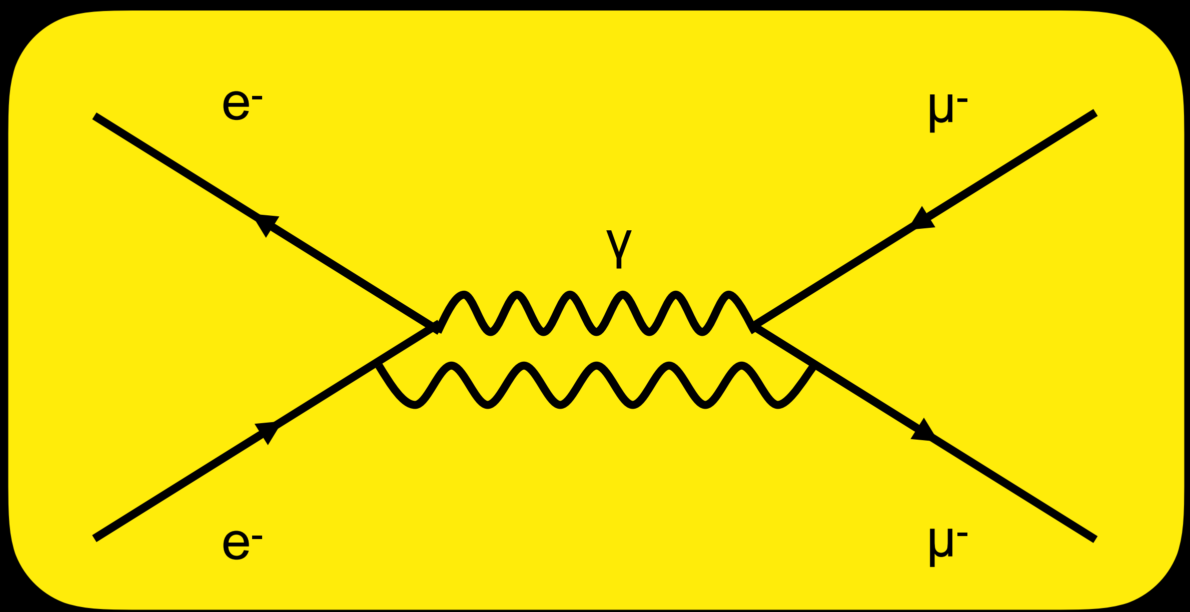
Lower order diagram

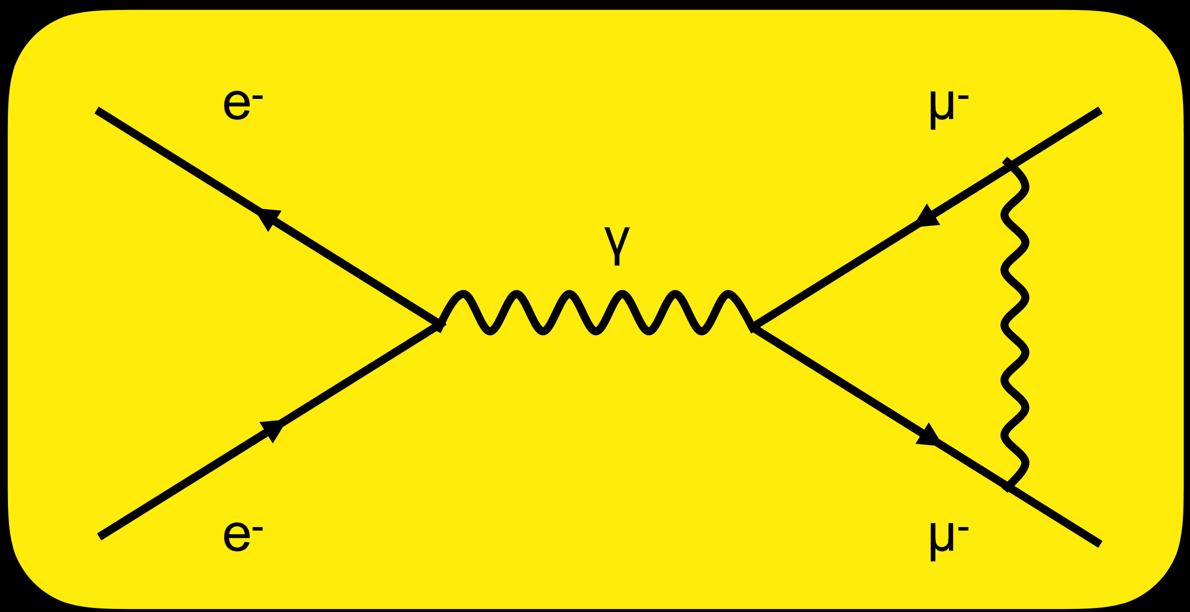


QED higher order corrections (cont.)

First order diagrams

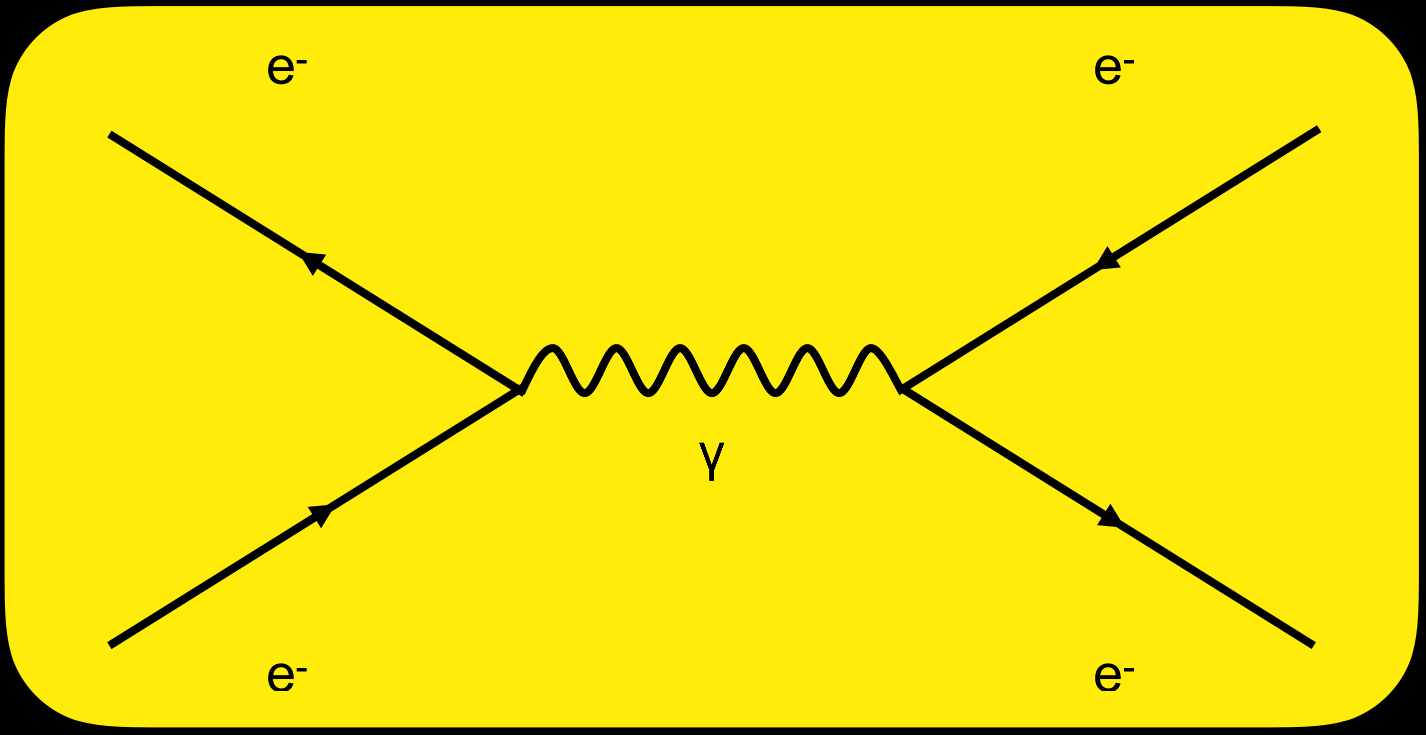






QED higher order corrections: Bhabha scattering

$e^+e^- \to e^+e^-$
annihilation process

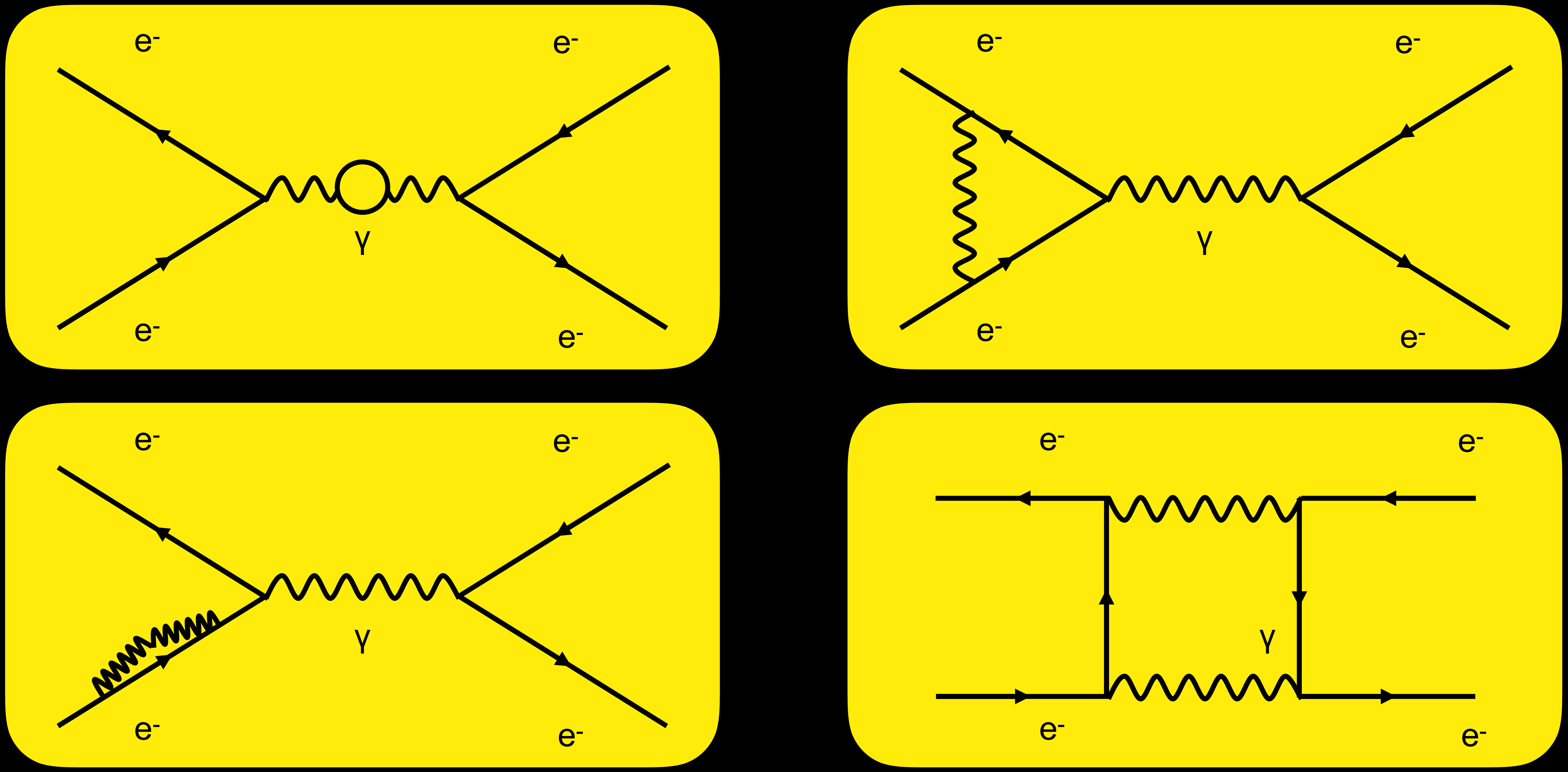


each vertex contributes a factor $\sqrt{\alpha}$ to the cross-section: $d\sigma/d\Omega \sim \alpha$

Note that
$\alpha \sim 1/137 << 1$

QED higher order corrections: Bhabha scattering (cont.)

$e^+e^- \rightarrow e^+e^-$
First order loops

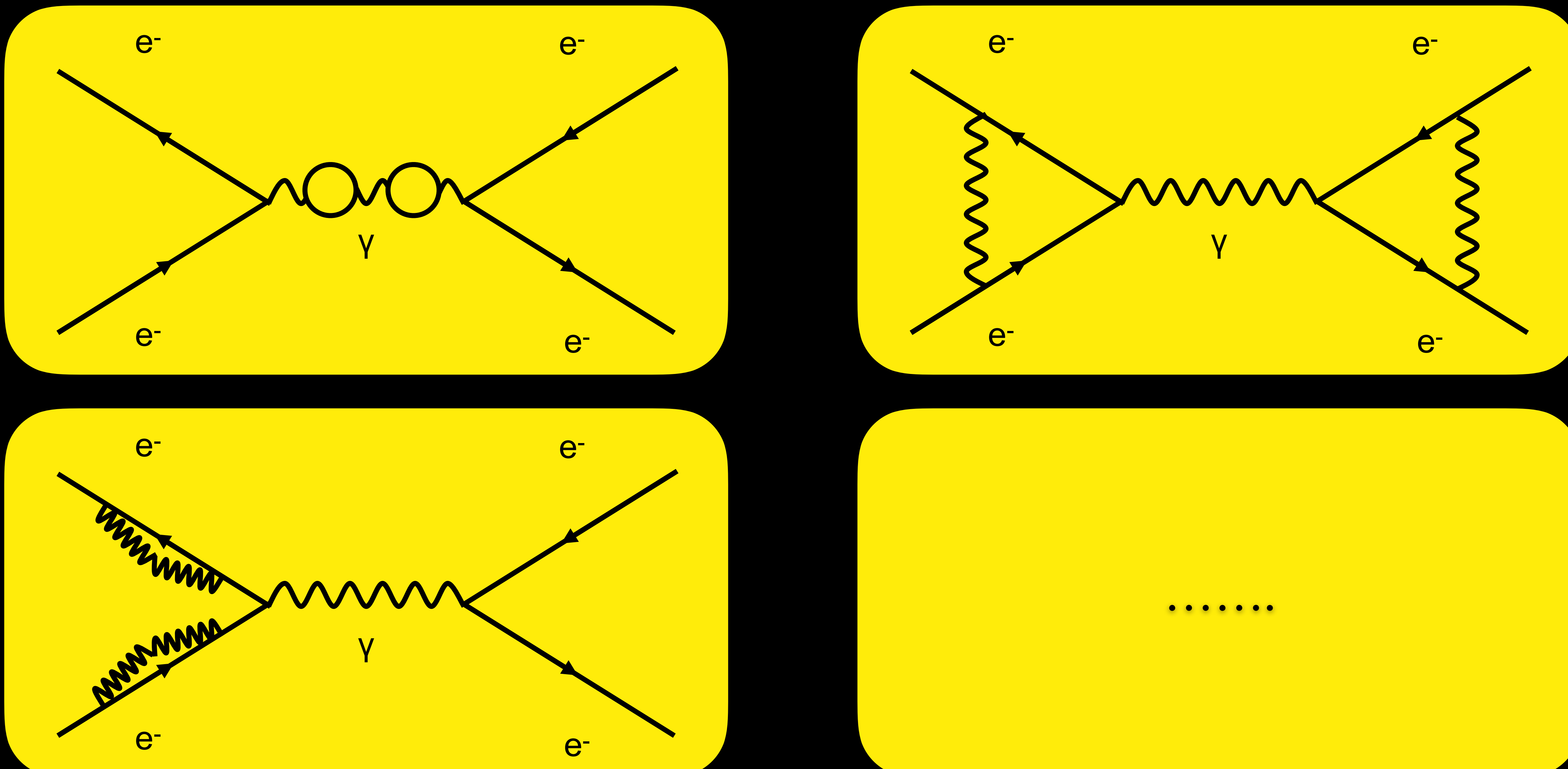


each vertex contributes a factor $\sqrt{\alpha}$ to the cross-section: $d\sigma/d\Omega \sim \alpha^2$

Note that
$\alpha \sim 1/137 << 1$

QED higher order corrections: Bhabha scattering (cont.)

$e^+e^- \rightarrow e^+e^-$
Second order loops



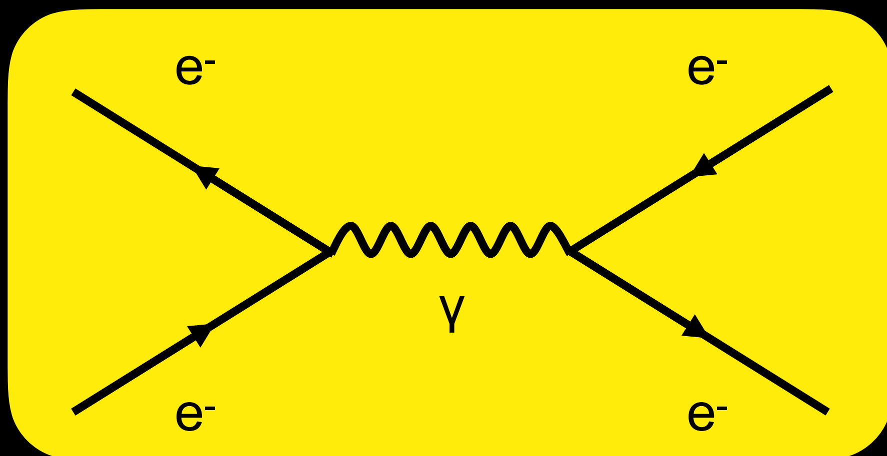
each vertex contributes a factor $\sqrt{\alpha}$ to the cross-section: $d\sigma/d\Omega \sim \alpha^3$

Note that
$\alpha \sim 1/137 << 1$

QED higher order corrections: Bhabha scattering (cont.)

- The cross section is a sum of contributions from all possible orders
  - the highest the order the less significant the contribution (~α=1/137)

$$\frac{d\sigma}{d\Omega} =$$



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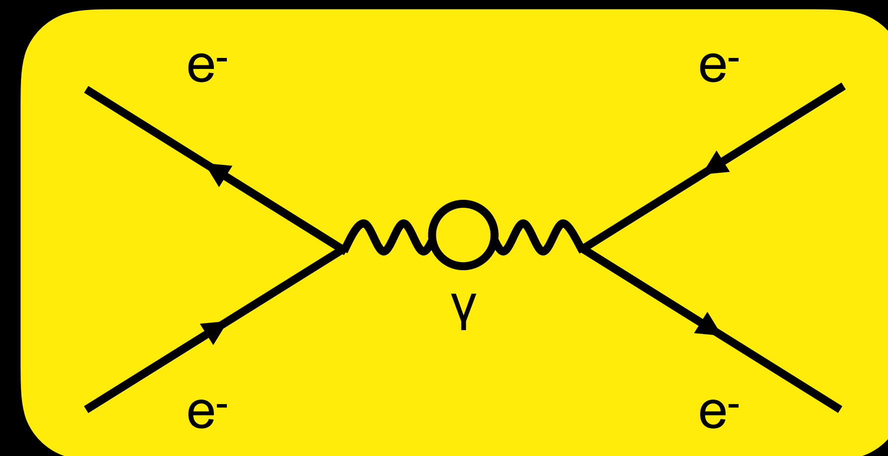

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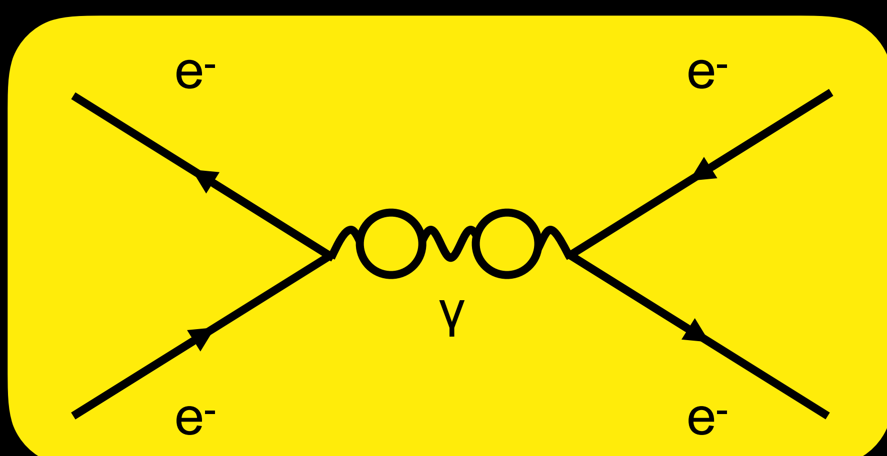

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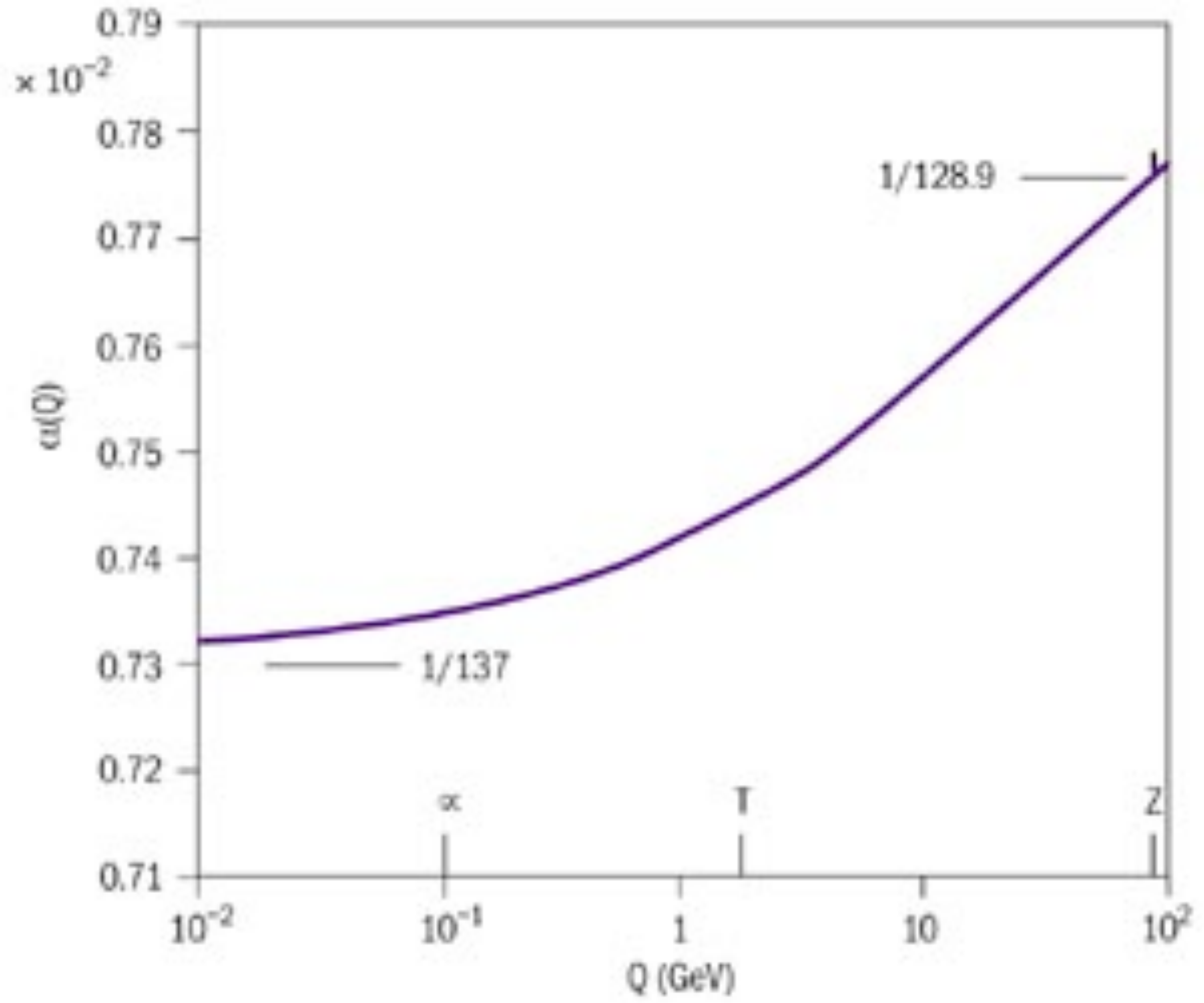
QED coupling constant

- Strength of E/M interactions reflected in the coupling constant α
- QED coupling constant $\alpha=e^2/4\pi$ is not a constant but depends on the distance
- QED coupling constant depends on the scale i.e. the momentum transfer Q

$$a(Q^2)=\frac{a(\mu^2)}{1-\frac{a(\mu^2)}{3\pi}\log\left(\frac{Q^2}{\mu^2}\right)}$$



QED coupling constant (const.)