How to Backdoor (Classic) McEliece and How to Guard Against Backdoors

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Our Contribution

- First backdoor for McEliece-like systems
  - Applicable to Classic McEliece (uncompressed keys)
- Simple countermeasure for this backdoor
- First post-quantum secure backdoor

Backdoor

- Transformation of simplified McEliece-like cryptosystem
- Leaks secret information
  - Only accessible to adversary $\mathcal{A}$
- Indistinguishable from original system
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Goal

Use secret information to recompute the secret key of the user.

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Weak SETUP

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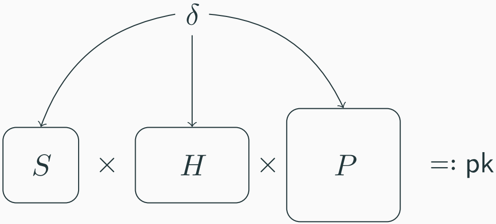
Vanilla McEliece

Key Generation

$$S \times H \times P =: \mathsf{pk}$$

Key Generation



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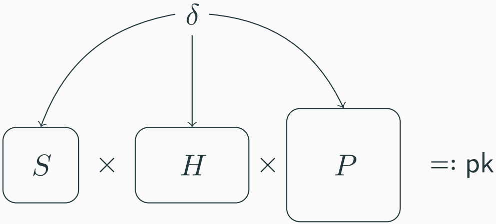
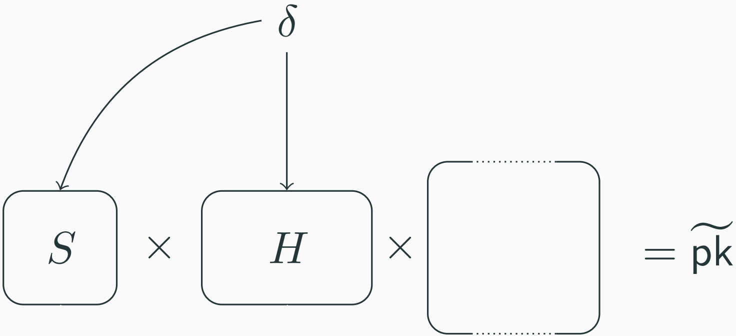


Goal of the Backdoor

Leak δ to $\mathcal{A}$, embedded in pk

Backdoor for Vanilla McEliece



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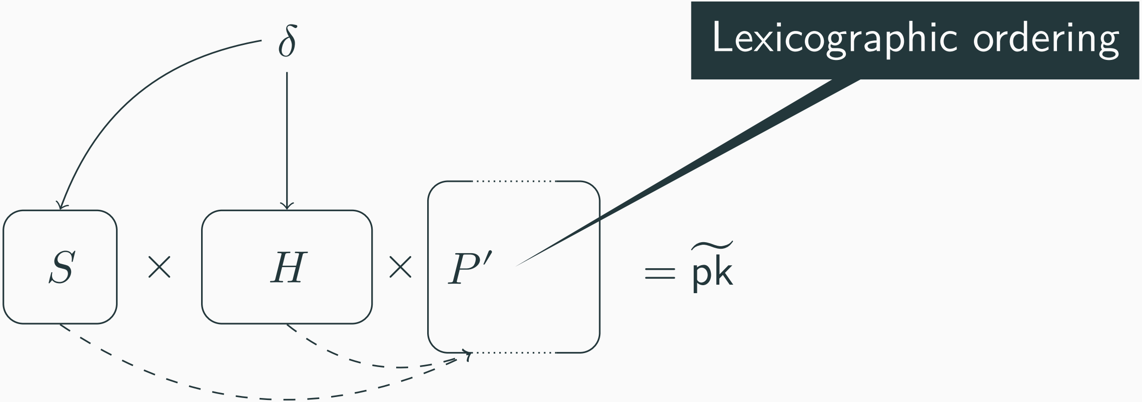

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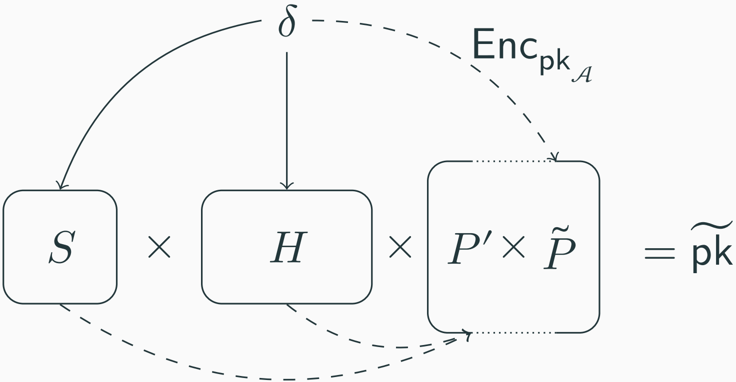

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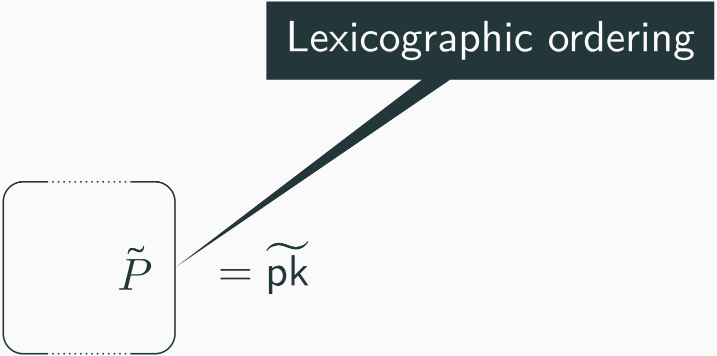

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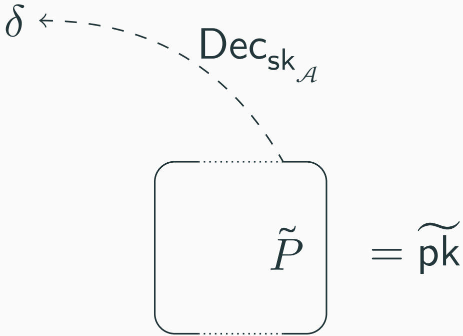

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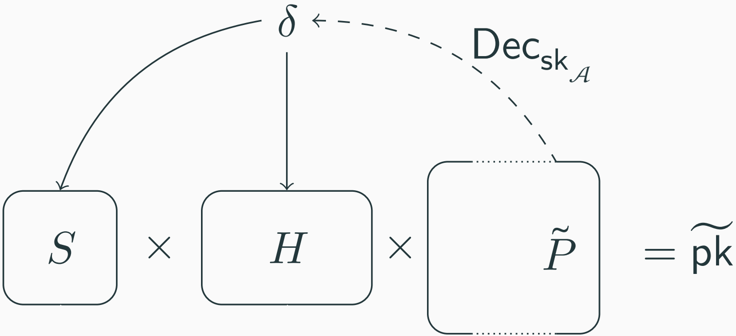

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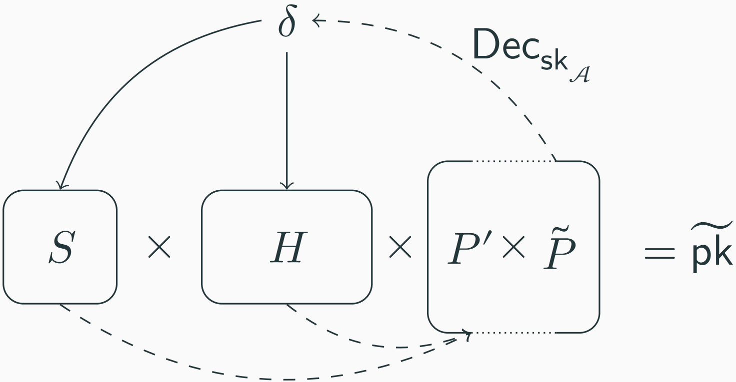

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Type of SETUP (Theorem 1)

This is a **strong SETUP**, assuming the ciphertext is indistinguishable from random ($\mathsf{IND\$} - \mathsf{CPA}$).

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Advice for Implementors (Theorem 2)

If δ is part of sk, only weak SETUPs are possible.

Classic McEliece

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Goppa Codes

$$H = \begin{pmatrix} \frac{1}{g(\alpha_1)} & \frac{1}{g(\alpha_2)} & \cdots & \frac{1}{g(\alpha_n)} \\ \frac{\alpha_1}{g(\alpha_1)} & \frac{\alpha_2}{g(\alpha_2)} & \cdots & \frac{\alpha_n}{g(\alpha_n)} \\ \vdots & & \ddots & \\ \frac{\alpha_1^{t-1}}{g(\alpha_1)} & \frac{\alpha_2^{t-1}}{g(\alpha_2)} & \cdots & \frac{\alpha_n^{t-1}}{g(\alpha_n)} \end{pmatrix}$$

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Data Rate

Target instance	Category	n	k	$\lceil \log_2(k!) \rceil$
`kem/mceliece348864`	1	3488	2720	27117
`kem/mceliece460896`	3	4608	3360	34520
`kem/mceliece6688128`	5	6688	5024	54528
`kem/mceliece6960119`	5	6960	5413	59332
`kem/mceliece8192128`	5	8192	6528	73316

Conclusion

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Open Question

What about other code-based schmes?

Advice for Implementors

Store δ to make keys verifiable.

Bibliography

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