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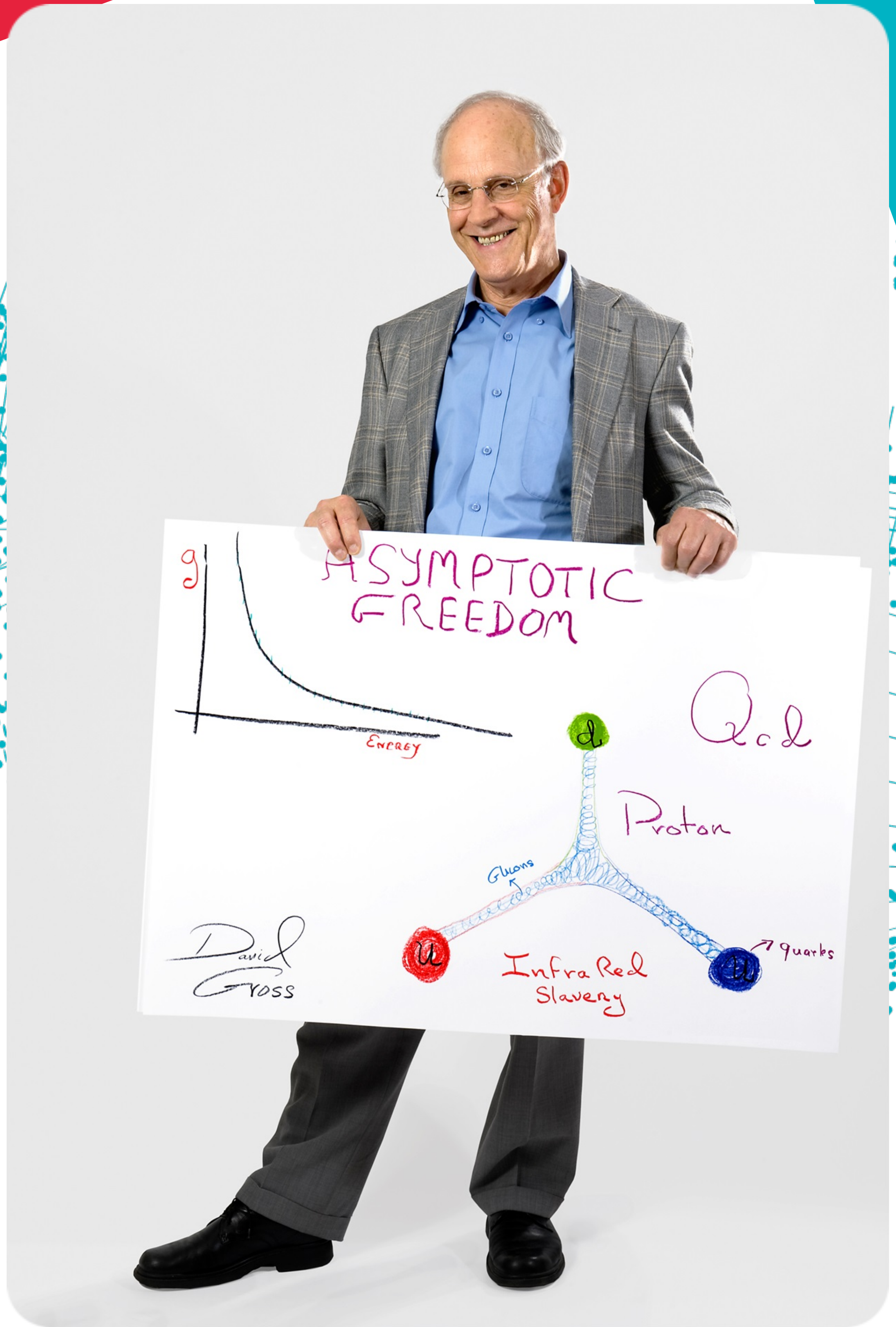


INTRODUCTION TO QUANTUM CHROMODYNAMICS

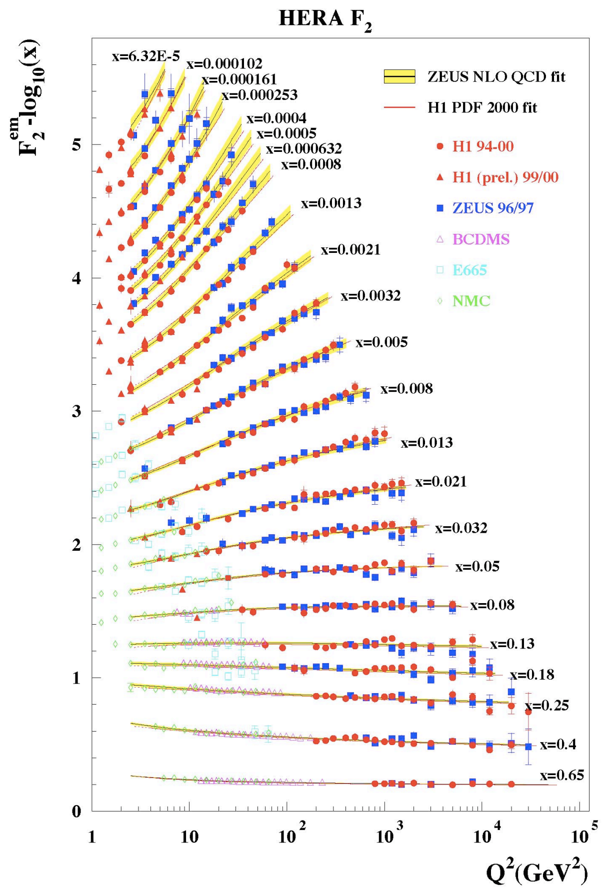
PARTICLE PHYSICS 2

Panos Christakoglou

SUMMARY

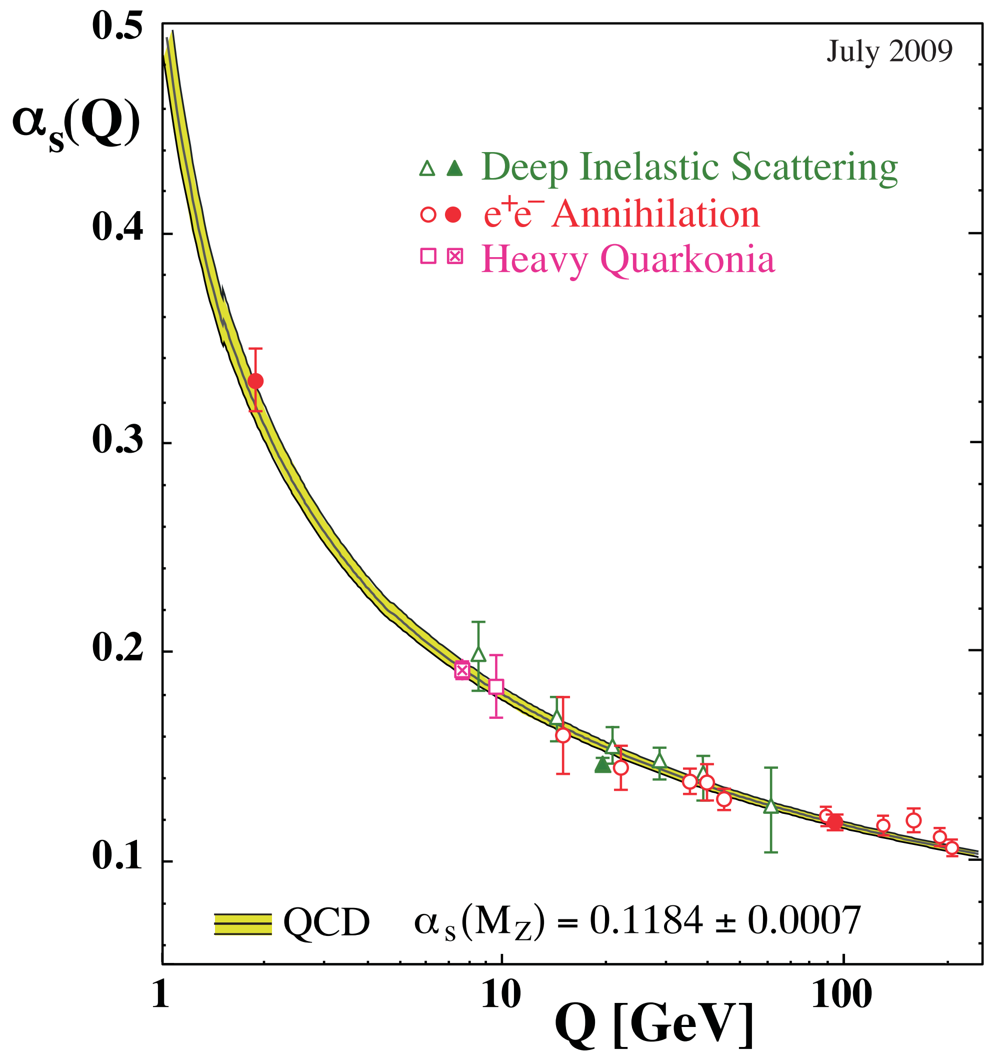
Last lecture

- Inelastic ep scattering
  - Structure of protons
  - Deep inelastic scattering experiments
  - Parton model
  - Parton distribution functions



Today's lecture

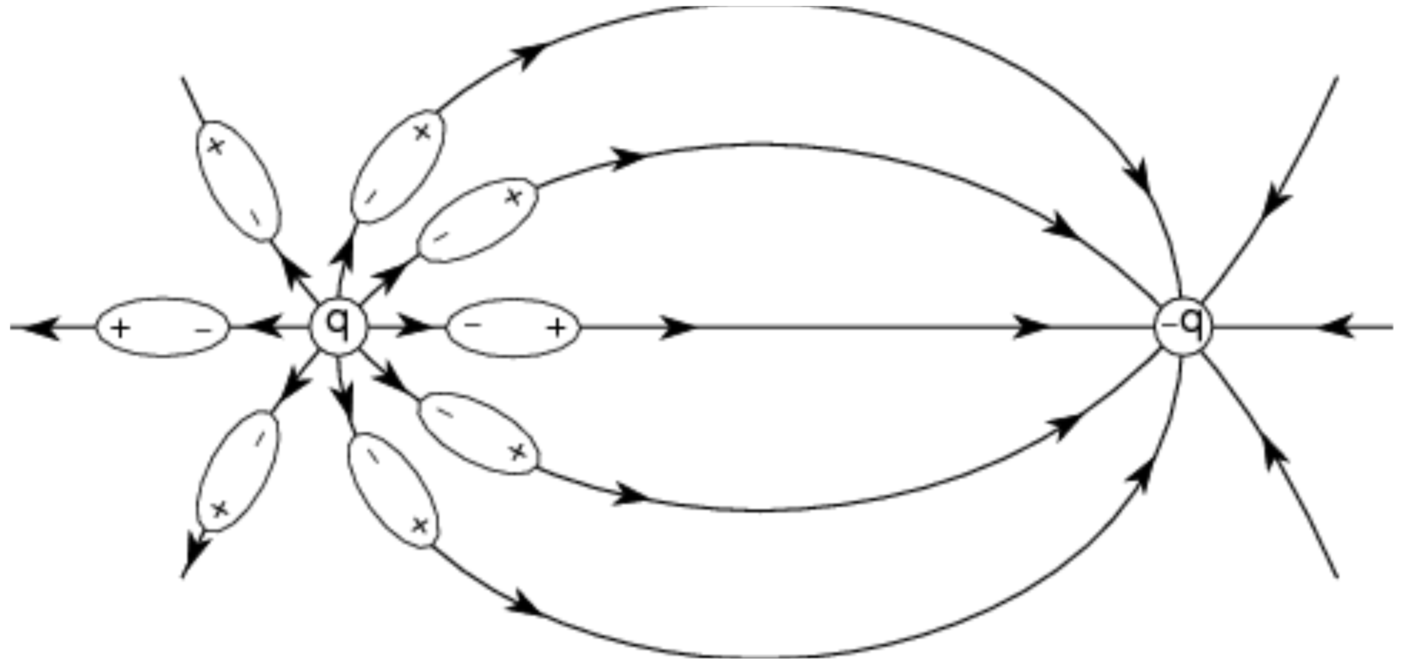
- Renormalisation
- The running of the coupling strength
  - Asymptotic freedom
- Confinement



QED CHARGE SCREENING

In QED a charged particle is surrounded by a cloud of virtual photons and electron-positron pairs continuously pop in and out of existence

- Because of attraction and repulsion in case of an electron, the positrons of the pairs tend to be closer and “screen” its charge
  - This is called vacuum polarisation and is analogous to the polarisation of a dielectric medium
- This gives rise to the notion of an effective charge that becomes smaller at large distances
- One defines the β-function that is positive in QED (as we will see later)



$$\beta = -\frac{de(r)}{d(lnr)}$$

QCD CHARGE ANTISCREENING

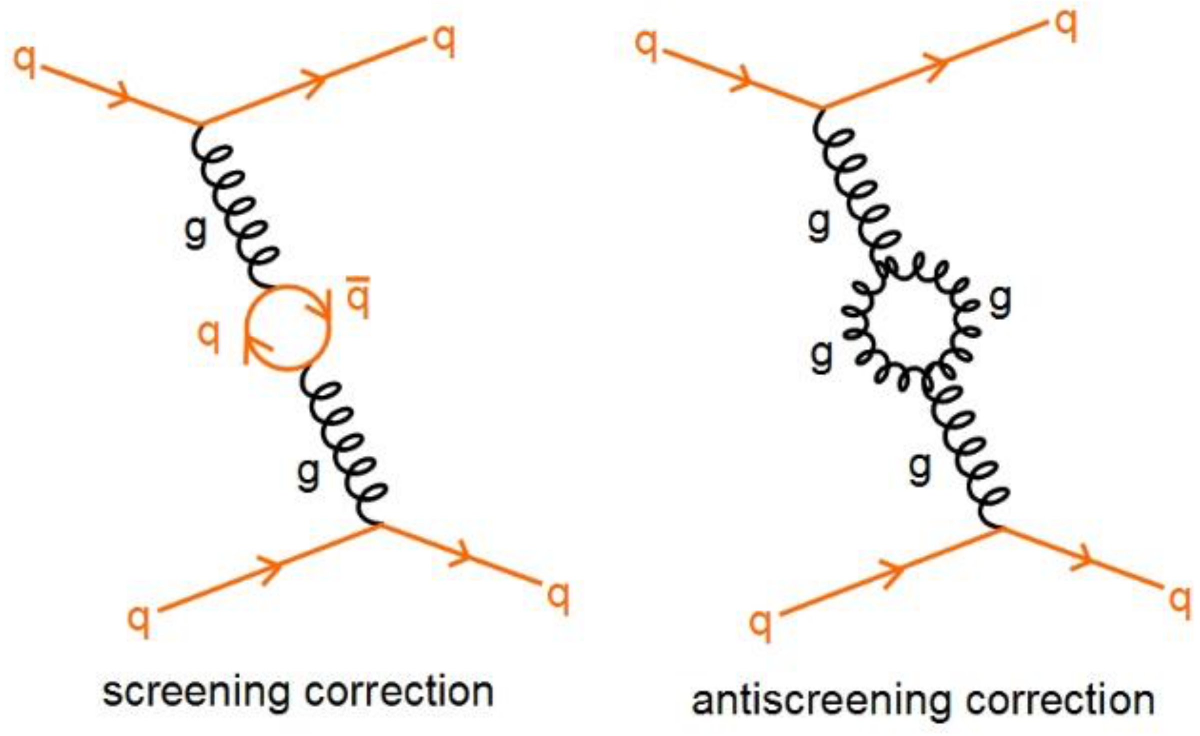
Likewise in QCD a quark is surrounded by quark-antiquark pairs and if this would be the only thing then the effect would be similar to the one we see in QED

- the strong coupling constant would be small at large distances

QCD CHARGE ANTISCREENING

However, due to the gluon self-coupling the vacuum is also filled by gluon pairs

- Due to the colour charge that quarks and gluons carry, the effective charge becomes larger with increasing energy
  - the β-function is negative in QCD
  - The effect is called anti-screening
- The negative contribution from the colour charge wins over the positive contribution of the normal charge and the quark colour charge
  - the strong coupling constant becomes small and short distances



THE RUNNING OF THE COUPLING STRENGTHS

Charge (anti-)screening in (QCD)QED leads naturally to the concept of the running coupling constant

In QED, screening leads to the increase of α at short distances or at large momentum transfers

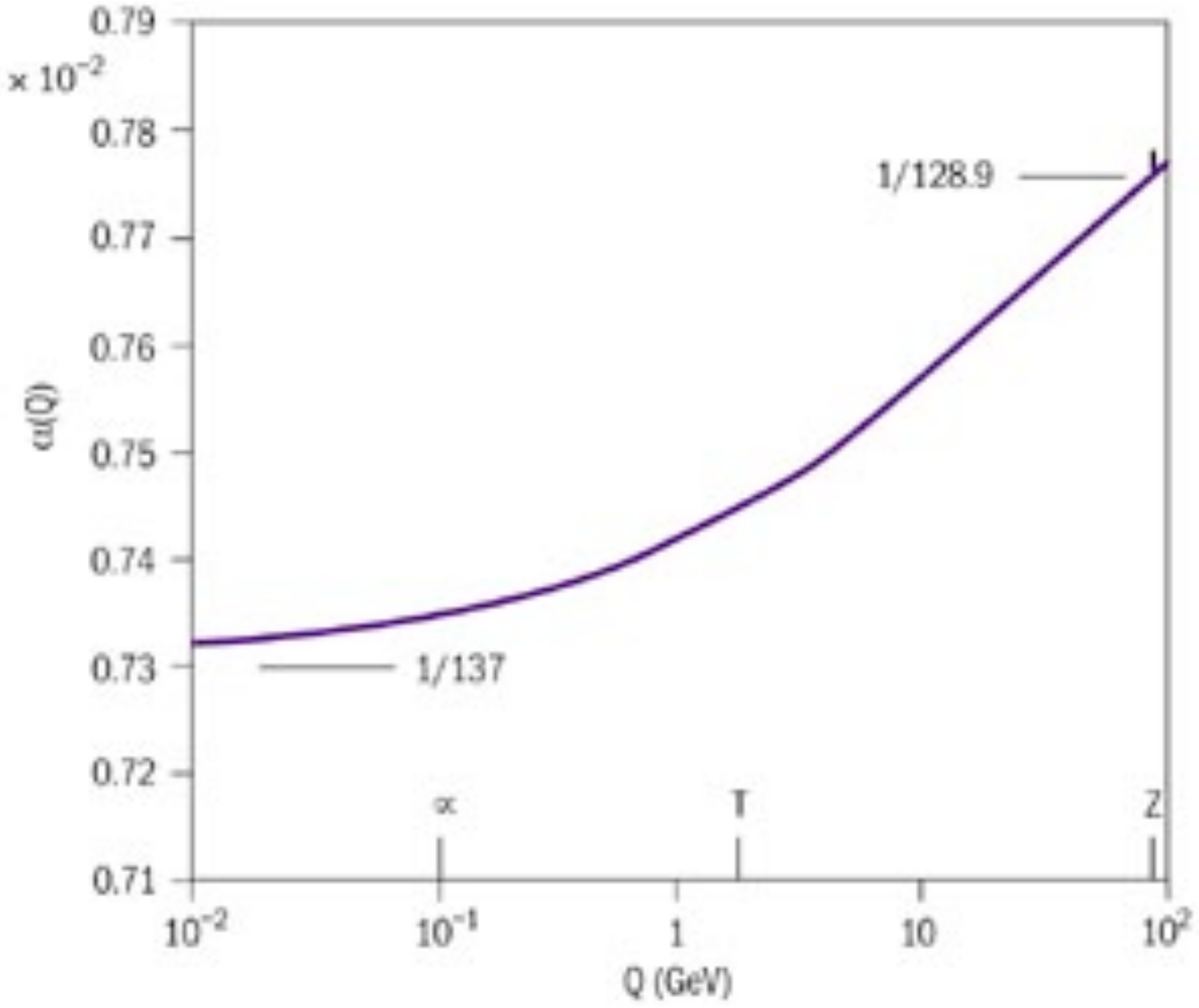
In QCD, anti-screening leads to the decrease of α_s at short distances or at large momentum transfers

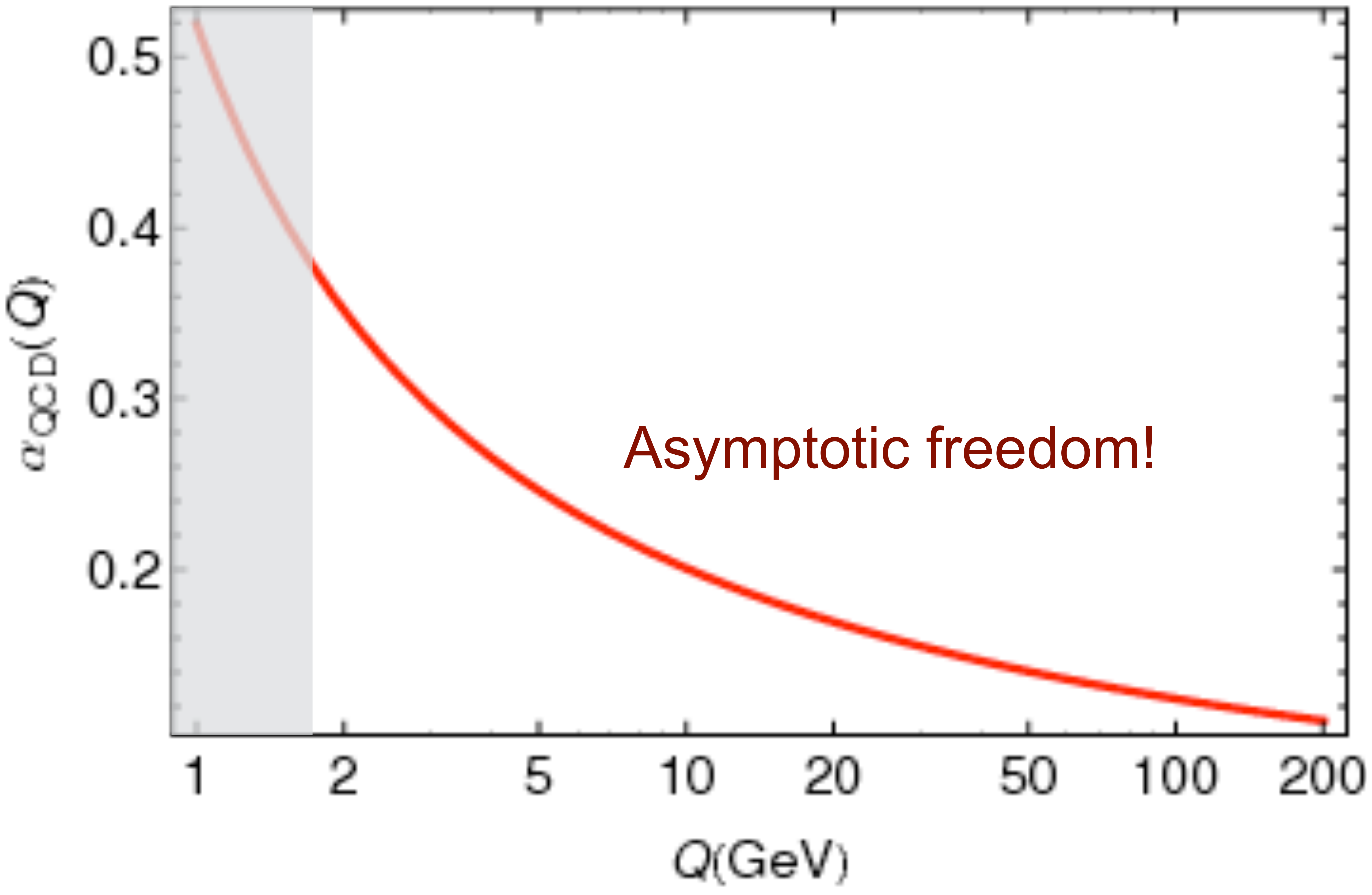
- Quarks fly as free particles within the hadron when probed at large Q ➡ **asymptotic freedom**!
- Since α_s is small at large Q, one can use perturbation theory and come up with predictions of e.g. DIS x-sections

At large distances (or small Q) α_s becomes large and the strong interaction is indeed strong

- it is impossible to isolate a quark from a hadron ➡ **confinement**!
- Although <u>**confinement**</u> is verified in lattice QCD, since it is not perturbative, <u>**it can't be proven from first principles**</u>

THE RUNNING OF THE COUPLING STRENGTHS





ASYMPTOTIC FREEDOM

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek

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The Nobel Prize in Physics 2004

David J. Gross
Prize share: 1/3

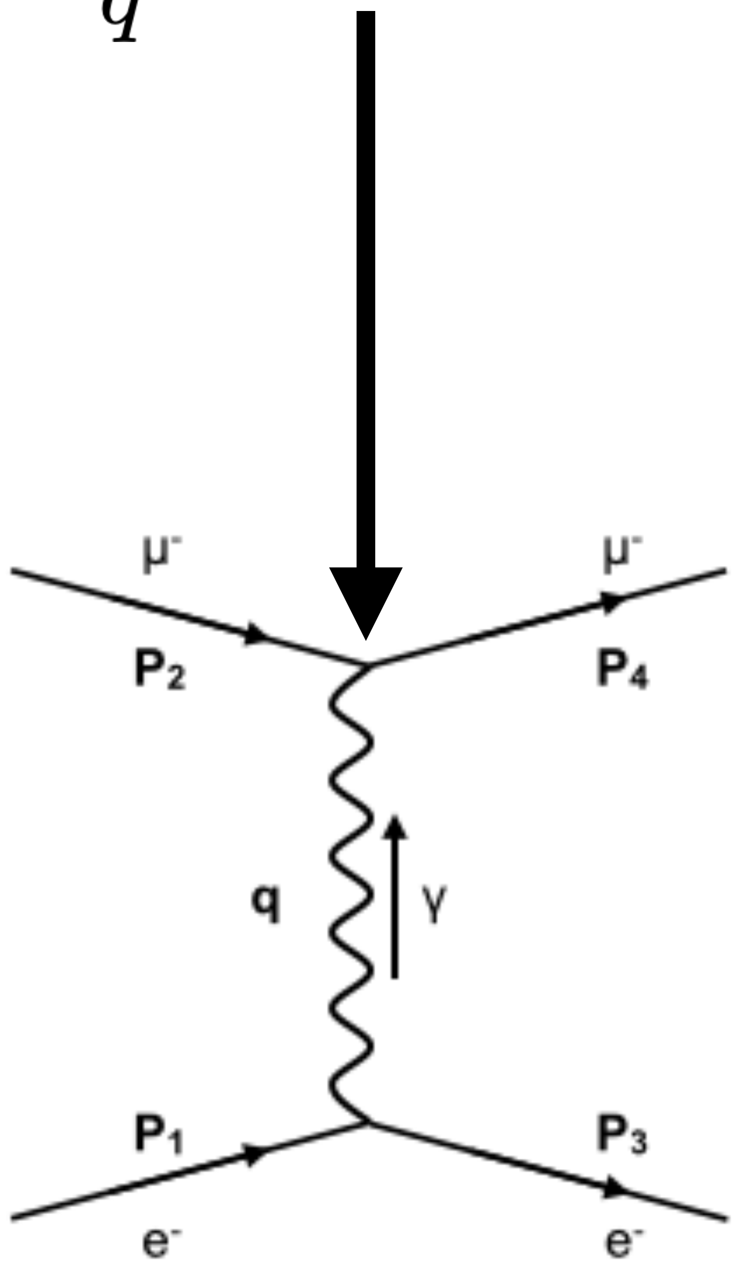
H. David Politzer
Prize share: 1/3

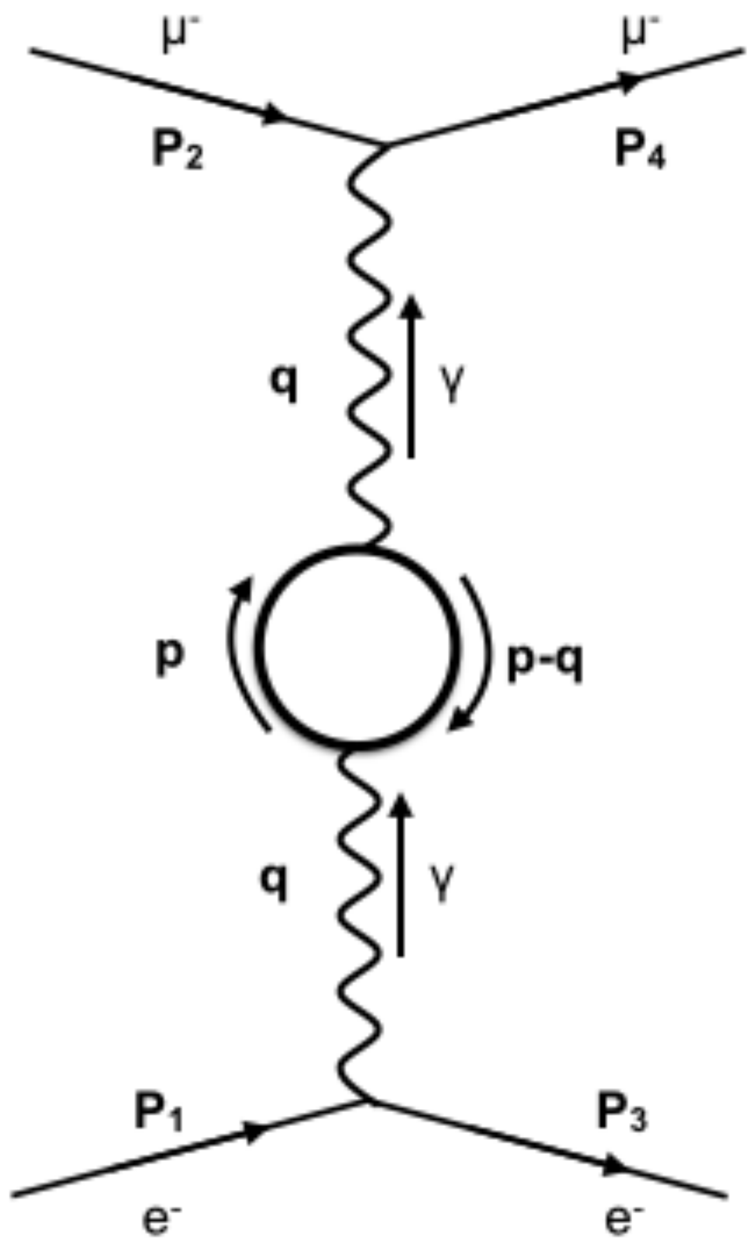
Frank Wilczek
Prize share: 1/3

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.

RENORMALISATION

$$M_{if} = \frac{g_e^2}{q^2}(\overline{u}_3\gamma^\mu u_1)(\overline{u}_4\gamma_\mu u_2)$$

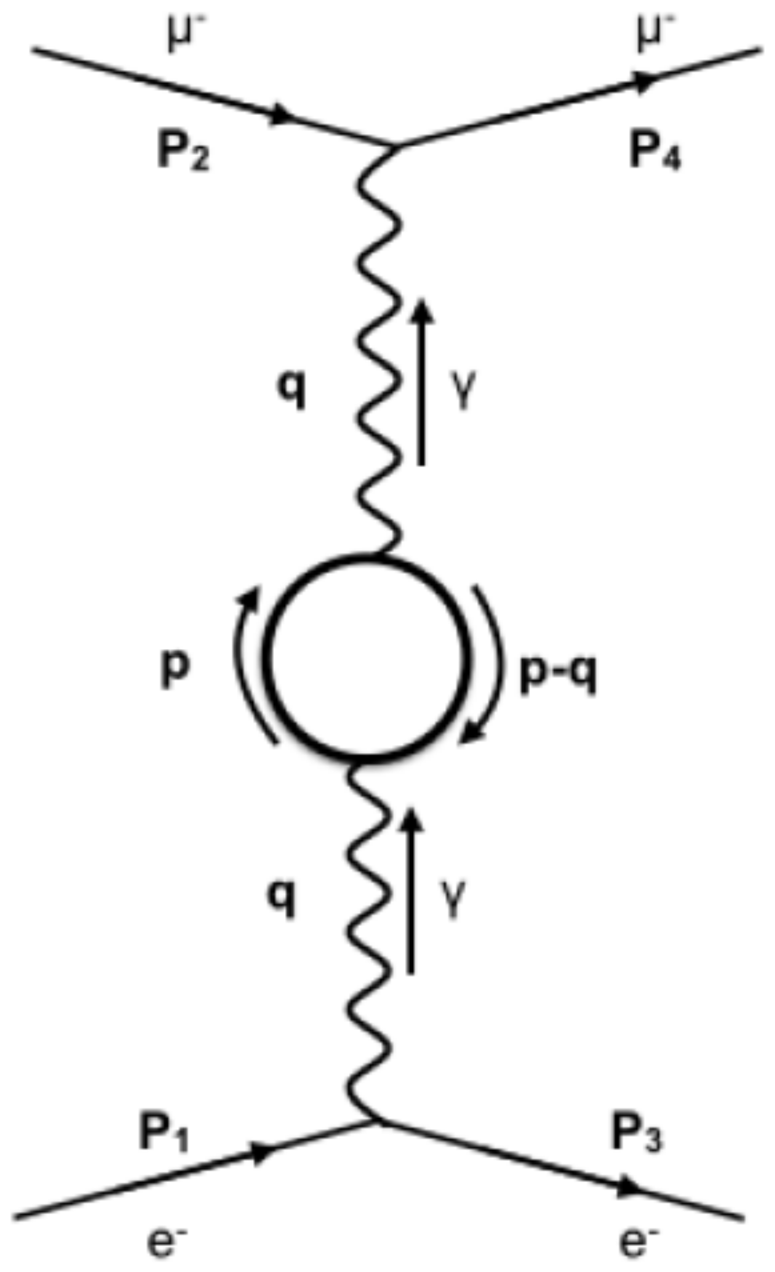




QED 1-LOOP CORRECTION

The $(-1)^n$ factor comes from the number of fermions in loops

- The matrix element contains the integral over d^4P due to higher order corrections
  - The four-momentum P is part of the loop and it is unrestricted
    - It can take any value
    - Since it is not observed, we need to integrate it out as for every internal line



$$M_{if} = -(-1)^1 g_e^2 [\overline{u}(\mathbf{P_3})\gamma^\mu u(\mathbf{P_1})]\left(-i\frac{g_{\mu\mu'}}{q^2}\right)\times$$

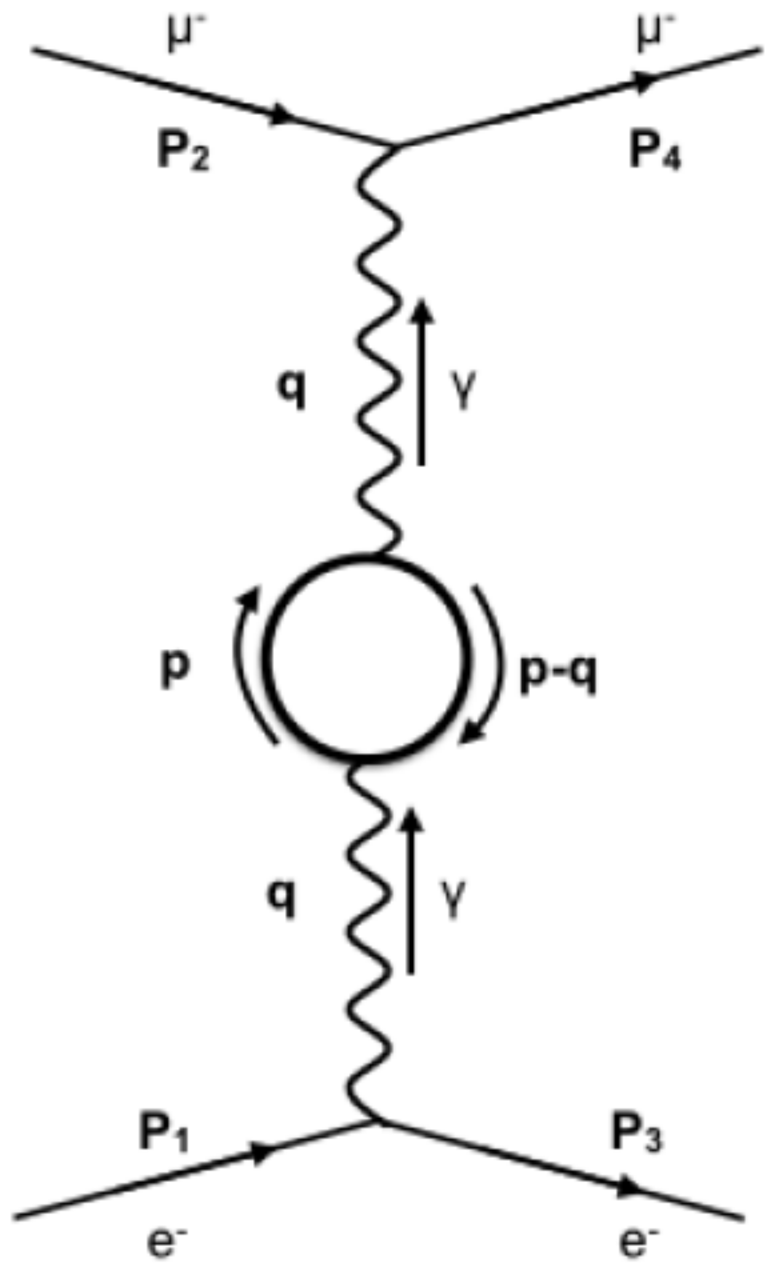
$$\int \frac{d^4p}{(2\pi)^4}\left[(ig_e\gamma^{\mu'})_{\kappa\lambda}\frac{i(\not{p}+m)_{\lambda\rho}}{p^2-m^2}(ig_e\gamma^{\nu'})_{\rho\tau}\frac{i(\not{p}-\not{q}+m)_{\tau\kappa}}{(p-q)^2-m^2}\times\right.$$

$$\left(-i\frac{g_{\nu\nu'}}{q^2}\right)[\overline{u}(\mathbf{P_4})\gamma^\nu u(\mathbf{P_2})]$$

QED 1-LOOP CORRECTION

This calculation can be considered as a modification of the lowest order diagram and in particular of its initial propagator

$$M_{if} = \frac{g_e^2}{q^2}(\overline{u}_3\gamma^\mu u_1)(\overline{u}_4\gamma_\mu u_2) \longleftarrow \left(\frac{-ig_{\mu\nu}}{q^2}\right)$$



$$M_{if} = -(-1)^1 g_e^2 [\overline{u}(\mathbf{P_3})\gamma^\mu u(\mathbf{P_1})] \left(-i\frac{g_{\mu\mu'}}{q^2}\right) \times$$

$$\int \frac{d^4p}{(2\pi)^4} \Big[(ig_e\gamma^{\mu'})_{\kappa\lambda} \frac{i(\not{p}+m)_{\lambda\rho}}{p^2-m^2} (ig_e\gamma^{\nu'})_{\rho\tau} \frac{i(\not{p}-\not{q}+m)_{\tau\kappa}}{(p-q)^2-m^2} \times$$

$$\left(-i\frac{g_{\nu\nu'}}{q^2}\right)[\overline{u}(\mathbf{P_4})\gamma^\nu u(\mathbf{P_2})]$$

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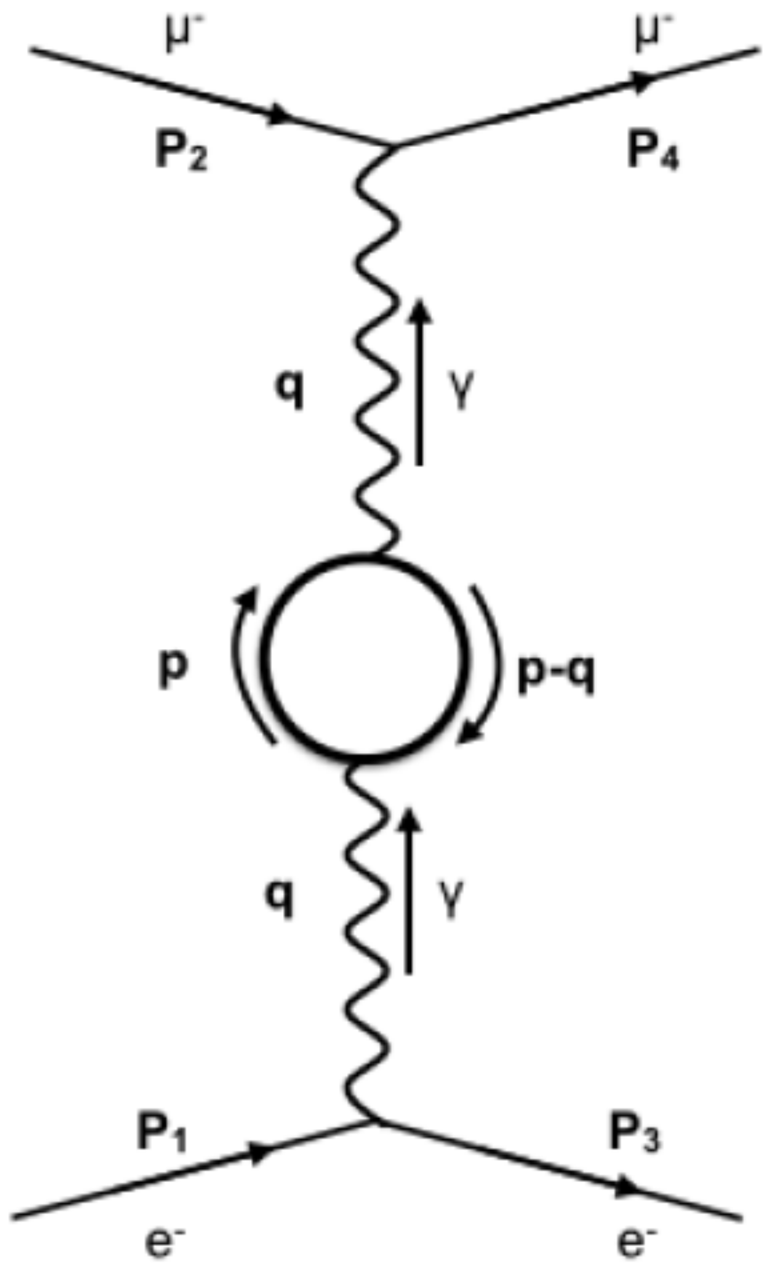
QED 1-LOOP CORRECTION

$$M_{if} = \frac{g_e^2}{q^2}(\overline{u}_3\gamma^\mu u_1)(\overline{u}_4\gamma_\mu u_2) \longleftarrow \left(\frac{-ig_{\mu\nu}}{q^2}\right)$$



$$-i\frac{g_{\mu\nu}}{q^2} \to -i\frac{g_{\mu\nu}}{q^2} + \left(-i\frac{g_{\mu\mu'}}{q^2}\right)I^{\mu'\nu'}\left(-i\frac{g_{\nu\nu'}}{q^2}\right) =$$

$$-i\frac{g_{\mu\nu}}{q^2} + \frac{(-i)}{q^2}I_{\mu\nu}\frac{(-i)}{q^2} = -i\frac{g_{\mu\nu}}{q^2} + (-i)^2\frac{I_{\mu\nu}}{q^4}$$

$$I_{\mu\nu} = (-1)^1\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[(ig_e\gamma_\mu)\frac{i(\not{p}+m)}{p^2-m^2}(ig_e\gamma_\nu)\frac{i(\not{p}-\not{q}+m)}{(p-q)^2-m^2}\right]$$

$$M_{if} = -(-1)^1 g_e^2[\overline{u}(\mathbf{P_3})\gamma^\mu u(\mathbf{P_1})]\left(-i\frac{g_{\mu\mu'}}{q^2}\right)\times$$

$$\int\frac{d^4p}{(2\pi)^4}\left[(ig_e\gamma^{\mu'})_{\kappa\lambda}\frac{i(\not{p}+m)_{\lambda\rho}}{p^2-m^2}(ig_e\gamma^{\nu'})_{\rho\tau}\frac{i(\not{p}-\not{q}+m)_{\tau\kappa}}{(p-q)^2-m^2}\times\right.$$

$$\left(-i\frac{g_{\nu\nu'}}{q^2}\right)[\overline{u}(\mathbf{P_4})\gamma^\nu u(\mathbf{P_2})]$$

QED 1-LOOP CORRECTION

$$I_{\mu\nu} = (-1)^1 \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[(ig_e\gamma_\mu)\frac{i(\not{p}+m)}{p^2-m^2}(ig_e\gamma_\nu)\frac{i(\not{p}-\not{q}+m)}{(p-q)^2-m^2}\right]$$

The integral contains terms

$$\int P^3 dP/P^2$$

- The correction due to the loop diverges

$I_{\mu\nu}$ is a function of q^2 and can be written in its most general form as

$$I_{\mu\nu} = -ig_{\mu\nu}q^2 I(q^2) + q_\mu q_\nu J(q^2)$$

- where $I(q^2)$ and $J(q^2)$ are some unknown functions of q^2

The term containing $J(q^2)$ does not contribute to the matrix element

QED 1-LOOP CORRECTION - CUTOFF SCALE

$$I(q^2) = \frac{g_e^2}{12\pi^2}\left\{\int_{m_e^2}^{\infty}\frac{dz}{z} - 6\int_0^1 dz\, z(1-z)\ln\left[1-\frac{q^2}{m_e^2}z(1-z)\right]\right\}$$

$$\downarrow$$

$$I(q^2) = \frac{g_e^2}{12\pi^2}\left[\int_{m_e^2}^{\infty}\frac{dz}{z} - f\left(\frac{Q^2}{m_e^2}\right)\right]$$

$$\downarrow$$

$$f\left(\frac{Q^2}{m_e^2}\right) \equiv 6\int_0^1 dz\, z(1-z)\ln\left[1+\frac{Q^2}{m_e^2}z(1-z)\right]$$

Imposing a cutoff so that the first integral becomes finite:

$$\int_{m_e^2}^{\infty}\frac{dz}{z} \quad \rightarrow \quad \int_{m_e^2}^{M^2}\frac{dz}{z} = \ln\left(\frac{M^2}{m_e^2}\right)$$

$$\downarrow$$

$$I(q^2) = \frac{g_e^2}{12\pi^2}\left\{\ln\left(\frac{M^2}{m_e^2}\right) - f\left(\frac{Q^2}{m_e^2}\right)\right\}$$

RENORMALISATION

The modification of the propagator can now be written as

$$\frac{g_{\mu\nu}}{q^2} \rightarrow \frac{g_{\mu\nu}}{q^2} - \frac{iI_{\mu\nu}}{q^4} = \frac{g_{\mu\nu}}{q^2}[1 - I(q^2)]$$

Since the propagator comes always with a factor of g_e, the modification above can be considered as a loop correction to coupling constant

$$g_e^2 \rightarrow g_e^2[1 - I(q^2)] = g_e^2\left\{1 - \frac{g_e^2}{12\pi^2}\left[\ln\left(\frac{M^2}{m_e^2}\right) - f\left(\frac{Q^2}{m_e^2}\right)\right]\right\}$$

The first term is the normalised coupling constant

$$g_0^2 = g_e^2\left[1 - \frac{g_e^2}{12\pi^2}\ln\left(\frac{M^2}{m_e^2}\right)\right]$$

Setting $g_e^4 = g_0^4$

$$g_e^2 \rightarrow g_0^2 + \frac{g_e^4}{12\pi^2} f\left(\frac{Q^2}{m_e^2}\right) = g_0^2\left\{1 + \frac{1}{12\pi^2}\frac{g_e^4}{g_0^2} f\left(\frac{Q^2}{m_e^2}\right)\right\}$$

The cutoff is now absorbed inside g_0!

$$g_e^2 \rightarrow g_0^2\left\{1 + \frac{g_0^2}{12\pi^2} f\left(\frac{Q^2}{m_e^2}\right) + \mathrm{O}(g_0^4)\right\} \equiv g_R^2(Q^2)$$

GAUGE THEORIES ARE RENORMALISABLE

Our theory can not describe physics at asymptotically small distances or at extremely large energy scales

We must replace the divergent part of the calculation with a measurement

- Renormalisation!!!

The Nobel Prize in Physics 1999
Gerardus 't Hooft, Martinus J.G. Veltman

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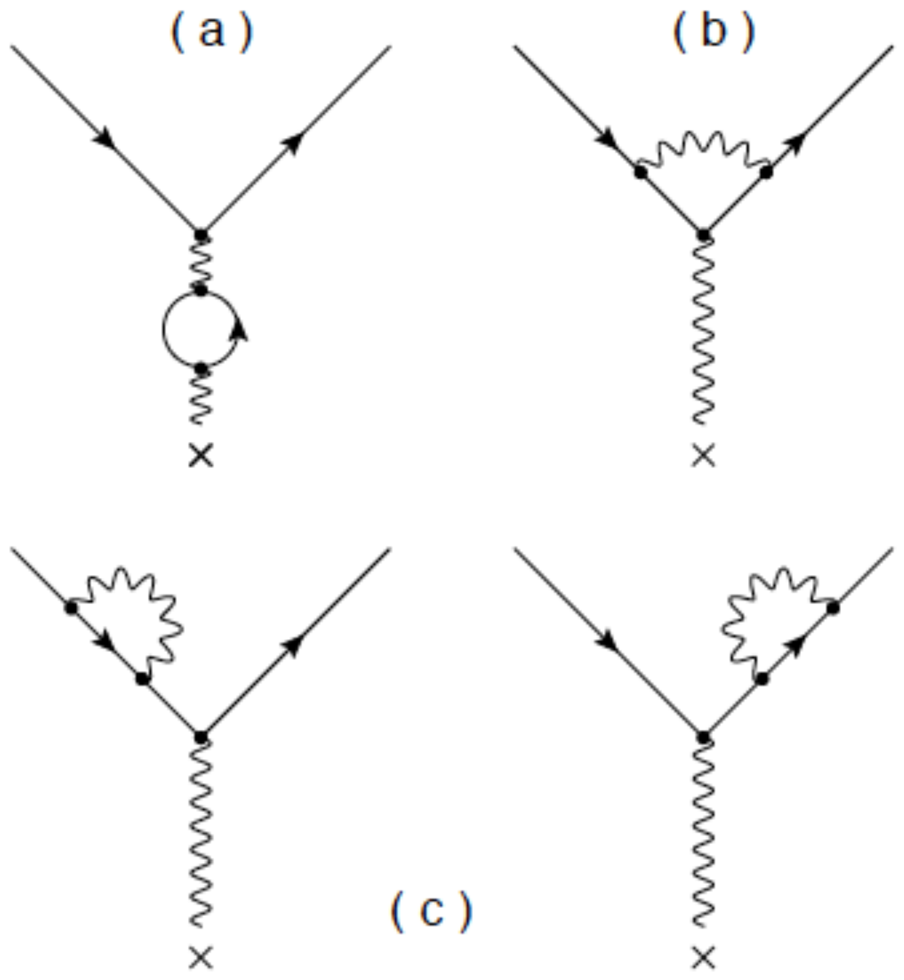
The Nobel Prize in Physics 1999

Gerardus 't Hooft
Prize share: 1/2

Martinus J.G. Veltman
Prize share: 1/2

The Nobel Prize in Physics 1999 was awarded jointly to Gerardus 't Hooft and Martinus J.G. Veltman *"for elucidating the quantum structure of electroweak interactions in physics"*

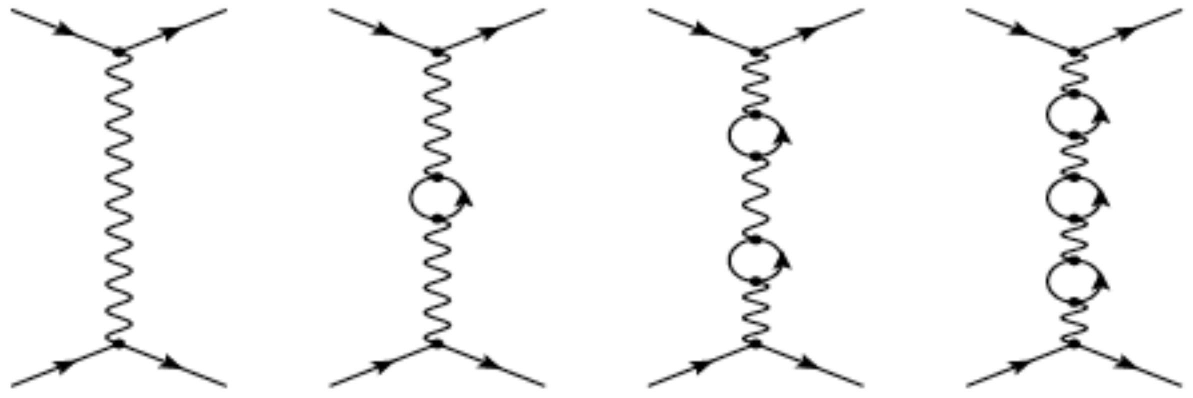
HIGHER ORDER CORRECTIONS



For 1-loop corrections:

$$\alpha(Q^2) = \alpha(0)\left\{1 + \frac{\alpha(0)}{3\pi}\ln\left(\frac{Q^2}{m_e^2}\right) + O(\alpha^2)\right\}$$

HIGHER ORDER CORRECTIONS



Considering higher order corrections introduces a series

$$1 + X + X^2 + X^3 + \cdots = \frac{1}{1-X}$$

For a full calculation

$$\alpha(Q^2) = \frac{\alpha(0)}{1 - [\alpha(0)/3\pi]\ln(Q^2/m_e^2)}$$

The expression blows up for

$$\ln(Q^2/m_e^2) = 3\pi/\alpha(0)$$

$$Q^2_{max} = 10^{280}\ \mathrm{MeV}^2$$

RENORMALISATION SCALE

Let's calculate the coupling constant not at $Q^2 = 0$ but at an arbitrary scale $Q^2 = \mu^2$ called the renormalisation scale

- Handy for QCD, since there we can not use the reference scale of $Q^2 = 0$

$$\alpha(Q^2) = \frac{\alpha(0)}{1 - [\alpha(0)/3\pi]\ln(Q^2/m_e^2)}$$

$$\frac{1}{\alpha(Q^2)} = \frac{1}{\alpha_0} - \frac{1}{3\pi}\ln\left(\frac{Q^2}{m_e^2}\right)$$

$$\frac{1}{\alpha(\mu^2)} = \frac{1}{\alpha_0} - \frac{1}{3\pi}\ln\left(\frac{\mu^2}{m_e^2}\right)$$

$$\frac{1}{\alpha(Q^2)} - \frac{1}{\alpha(\mu^2)} = -\frac{1}{3\pi}\left[\ln\left(\frac{Q^2}{m_e^2}\right) - \ln\left(\frac{\mu^2}{m_e^2}\right)\right] = -\frac{1}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)$$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - [\alpha(\mu^2)/3\pi]\ln(Q^2/\mu^2)} \quad \text{for} \quad m_e^2 \ll Q^2 < Q^2_{\max}$$

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THE QED RUNNING COUPLING STRENGTH

$$\beta = -\frac{de(r)}{d(lnr)}$$

Writing the coupling constant as

$$\frac{1}{\alpha(Q^2)} = \frac{1}{\alpha(\mu^2)} - \frac{1}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)$$

And differentiating wrt to

$$t = \ln Q^2$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\alpha}\right) = -\frac{1}{\alpha^2}\frac{\mathrm{d}\alpha}{\mathrm{d}t} = -\frac{1}{3\pi}$$

Where we have introduced the β function

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} \equiv \beta(\alpha) = \frac{1}{3\pi}\alpha^2$$

$$\frac{\mathrm{d}\alpha(Q^2)}{\mathrm{d}\ln(Q^2)} \equiv \beta(\alpha) = -(\beta_0\alpha^2 + \beta_1\alpha^3 + \beta_2\alpha^4 + \cdots)$$

For 1-loop corrections:

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + \beta_0\,\alpha(\mu^2)\ln(Q^2/\mu^2)} \qquad \beta_0 = -\frac{1}{3\pi}$$

THE QCD RUNNING COUPLING STRENGTH

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \beta_0\, \alpha_s(\mu^2) \ln(Q^2/\mu^2)}$$

Nc: number of colours

n_f: number of flavours

$$\beta_0 = \frac{11N_c - 2n_f}{12\pi}$$

Introducing the scale parameter Λ aka QCD scale (~200-300 MeV)

$$\frac{1}{\alpha_s(Q^2)} = \frac{1}{\alpha_s(\mu^2)} + \beta_0 \ln\left(\frac{Q^2}{\mu^2}\right) \equiv \beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)$$

PROBING THE STRONG COUPLING STRENGTH

$$R = \frac{\sigma(e^- + e^+ \to \text{hadrons})}{\sigma(e^- + e^+ \to \mu^- + \mu^+)} = R_{EW}(Q)\left[1 + \delta_{QCD}(Q)\right]$$

where $R_{EW}(Q)$ is the purely electroweak prediction for the ratio and $\delta_{QCD}(Q)$ is the correction due to QCD effects. To keep the discussion simple, we can restrict our attention to energies $Q << M_Z$, where the process is dominated by photon exchange ($R_{EW} = 3\sum_q e_q^2$, neglecting finite-quark-mass corrections, where the e_q are the electric charges of the quarks):

$$\delta_{QCD}(Q) = \sum_{n=1}^{\infty} c_n \left[\frac{\alpha_s(Q^2)}{\pi}\right]^n + \mathrm{O}\left(\frac{\Lambda^4}{Q^4}\right)$$

A new global analysis using all available precision data of deep inelastic and related hard scattering processes includes recent measurements of structure functions from HERA and of the inclusive jet cross sections at the Tevatron. After analysis of experimental and theoretical uncertainties the authors obtain

$$\begin{aligned} \alpha_s(M_{Z^0}) = 0.1165 &\pm 0.002 \text{ (exp.)} \\ &\pm 0.003 \text{ (theo.)}\,, \end{aligned} \tag{7.2.4}$$

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EVIDENCE OF COLOUR

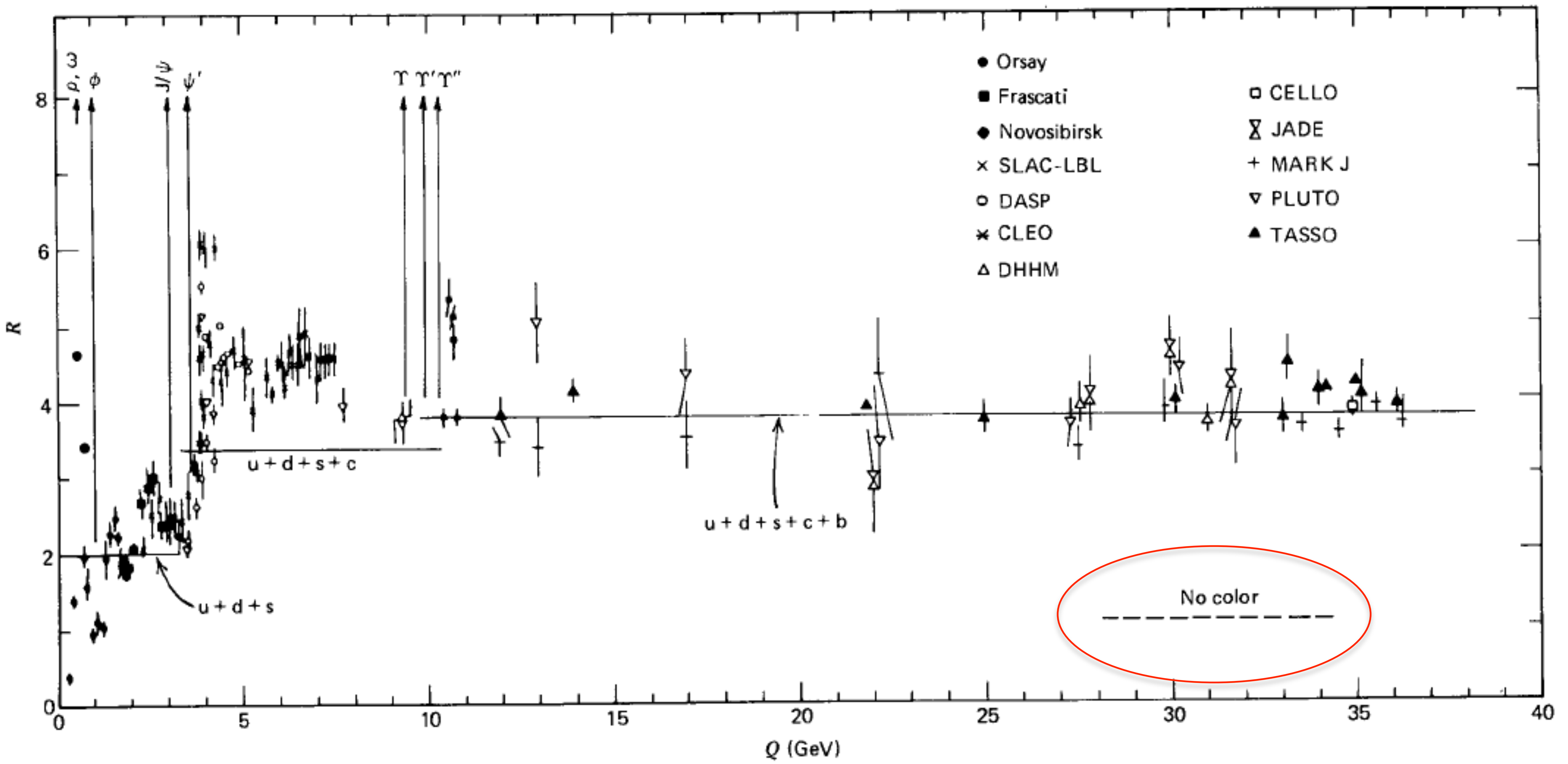
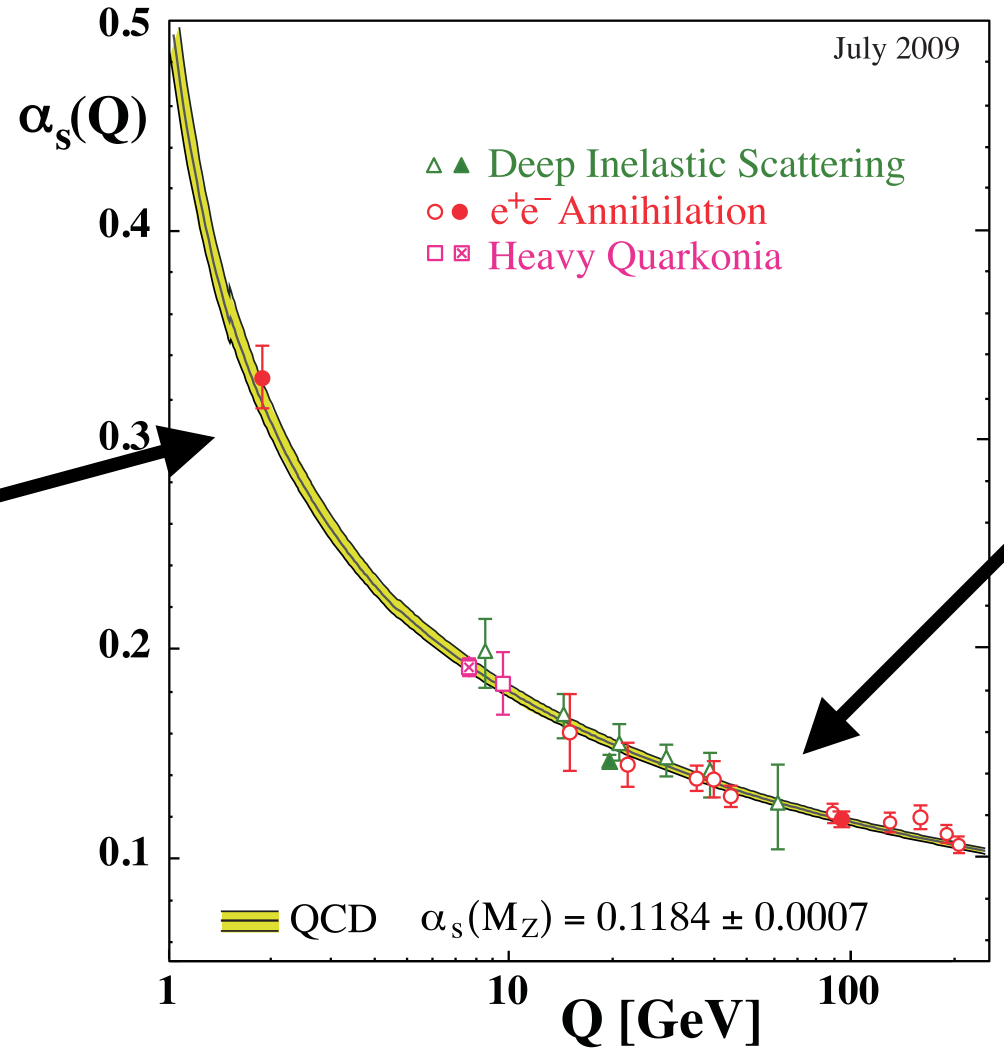


Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

THE STRONG COUPLING STRENGTH

Confinement
(e.g. hadrons)



Asymptotic freedom
(e.g. DIS)

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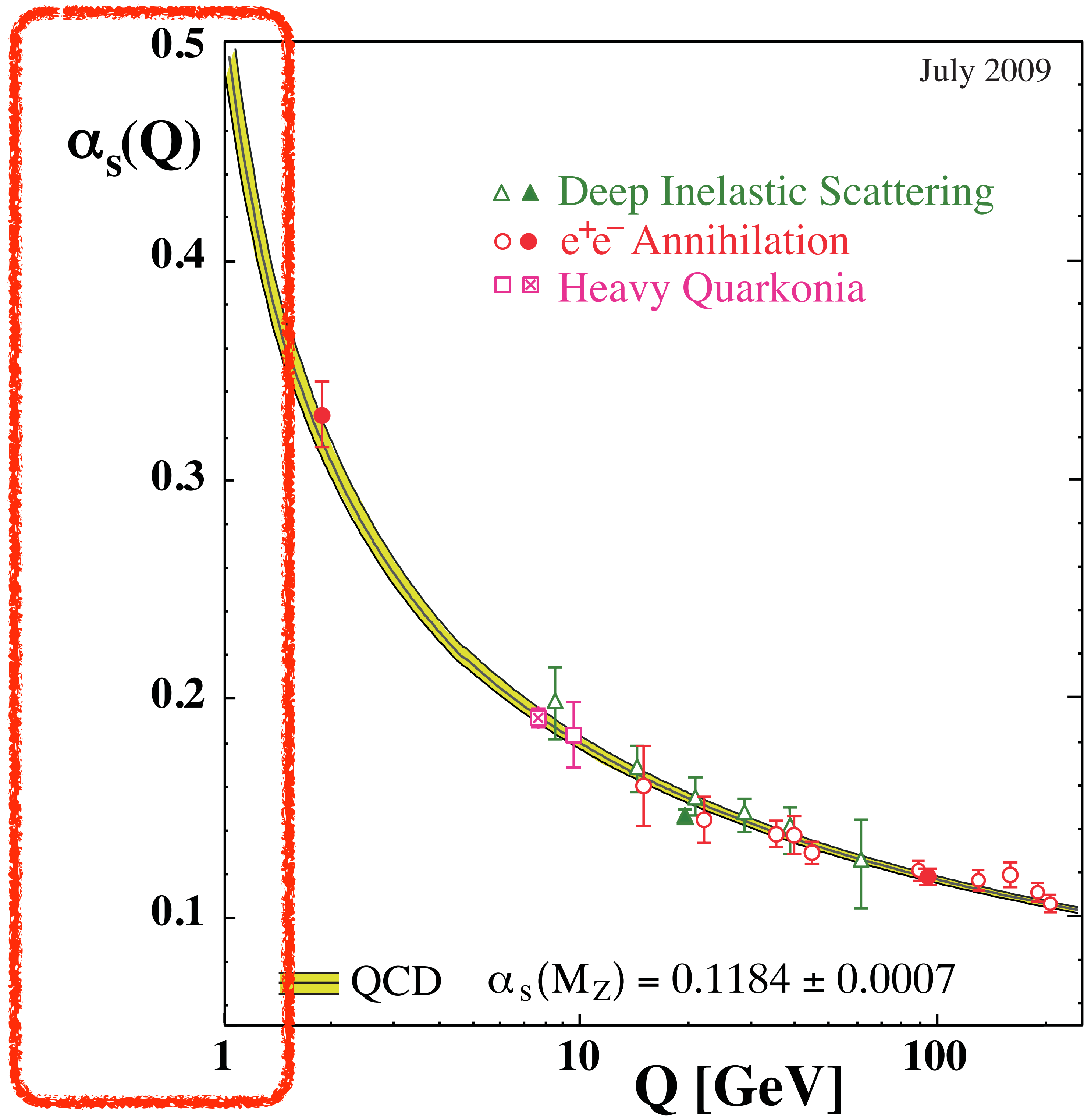
CONFINEMENT

For small values of momentum transfer, large spatial range*, the strong interactions are exceptionally strong

- a_s >>
  - Quarks are bound together within hadrons
  - We can not find quarks and gluons moving freely in nature



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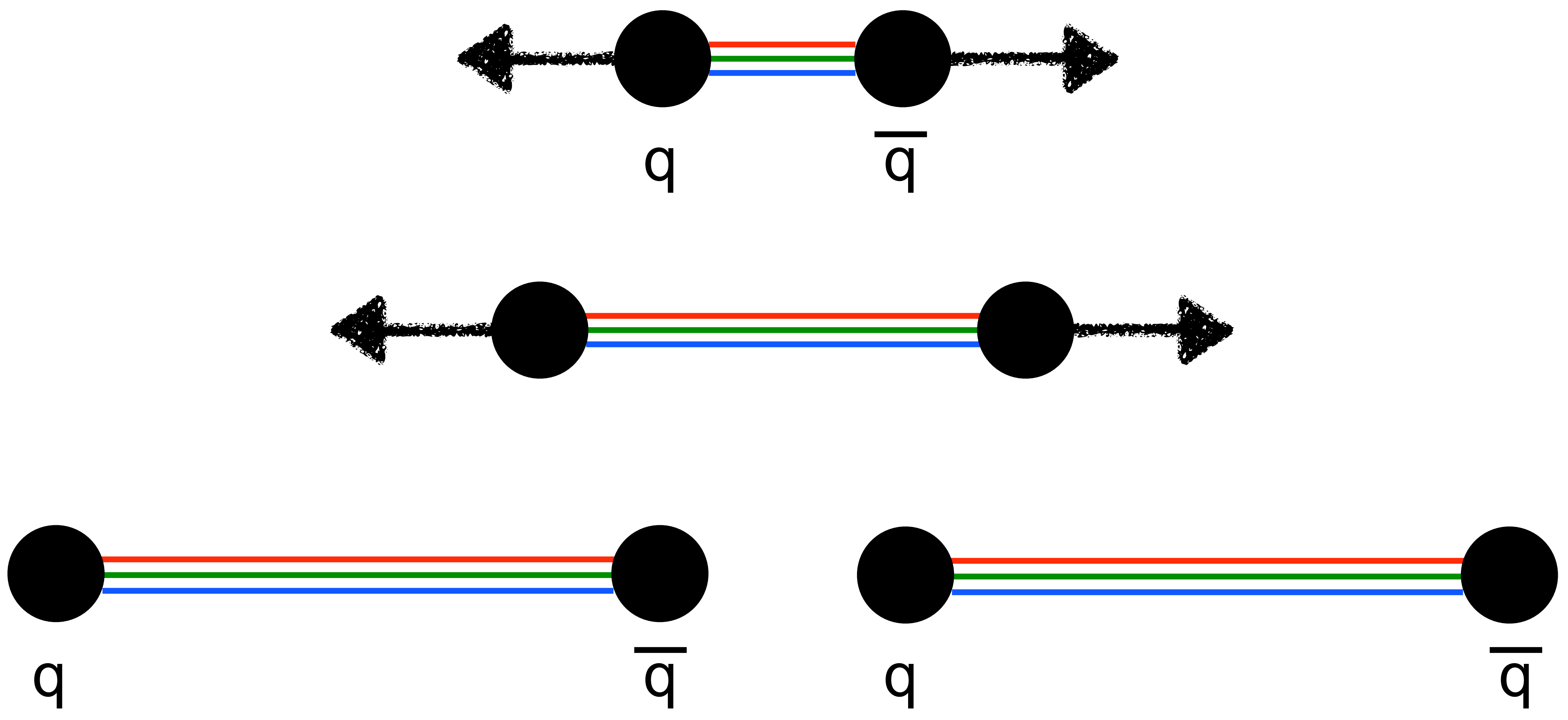
CONFINEMENT

Breaking the colour string that connects a pair of quark and antiquark is not easy

- At some point, it is energetically more favourable to create a new pair of quarks and antiquarks
  - Quarks and gluons can not move freely

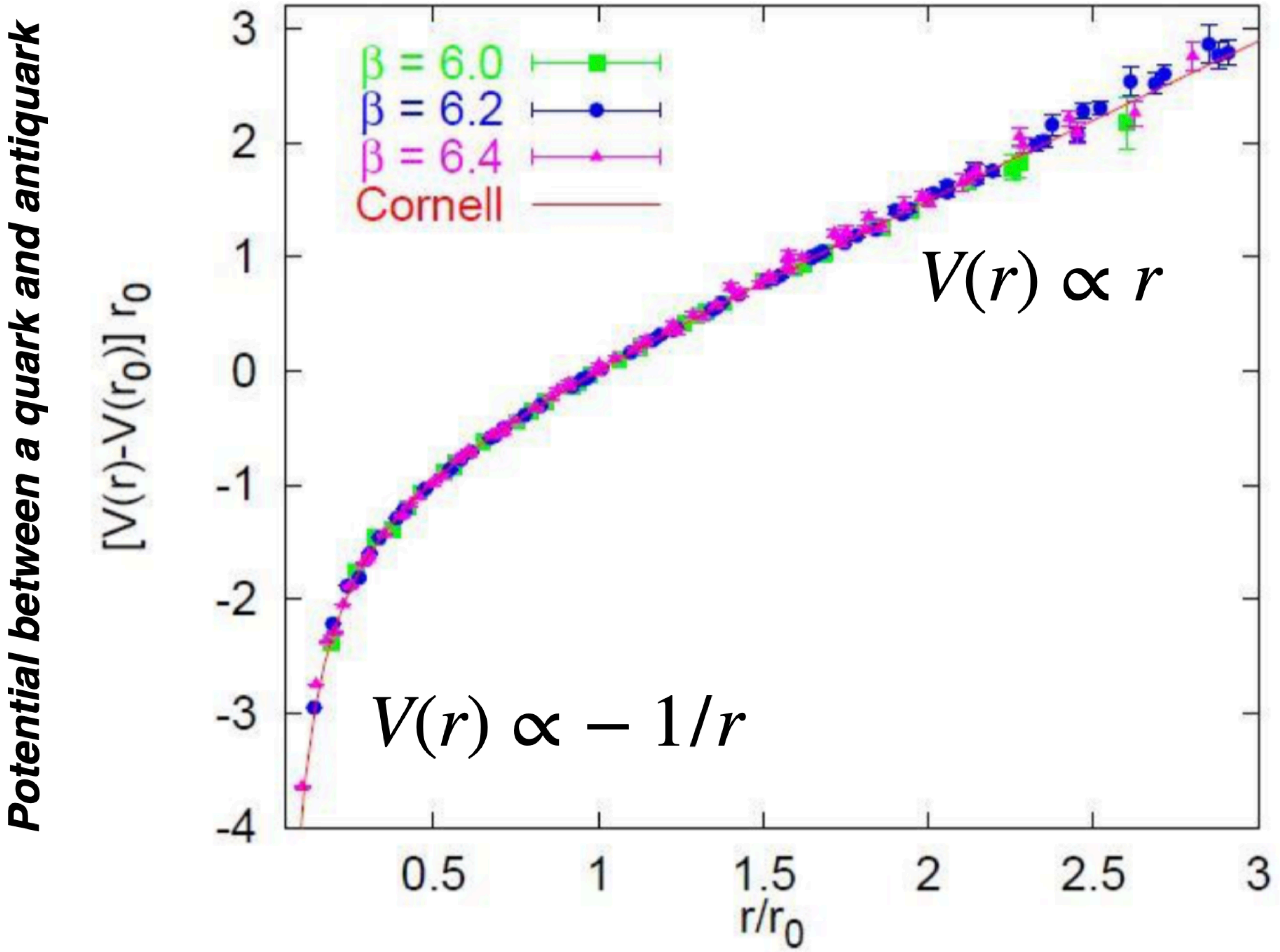


CONFINEMENT

The strong interaction is therefore a confining force: only hadrons which are in a colour-singlet state are physically allowed

A crucial property of colour is that it leads to a **confining force at large distances**

Since the interaction strength increases with the distance, we **cannot completely separate two quarks apart** since that would require an infinite force



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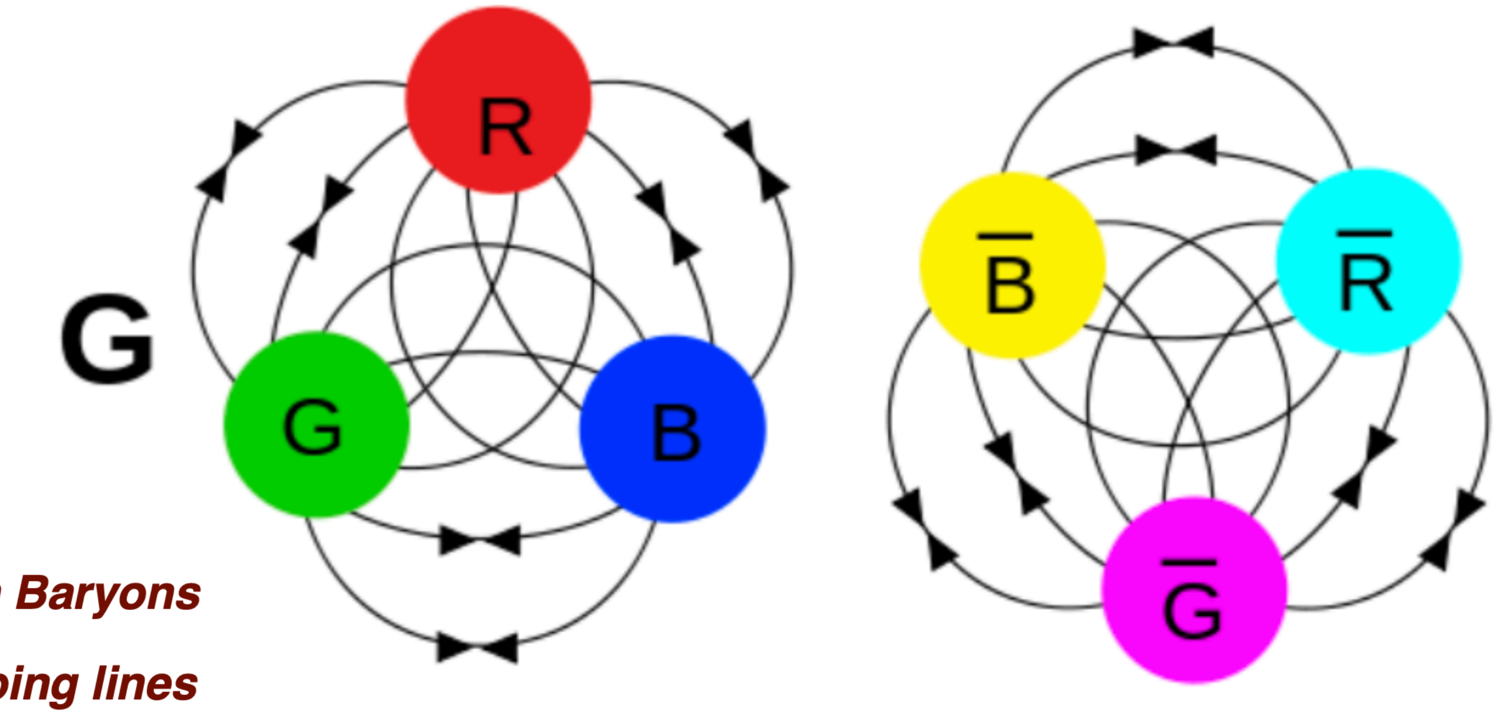
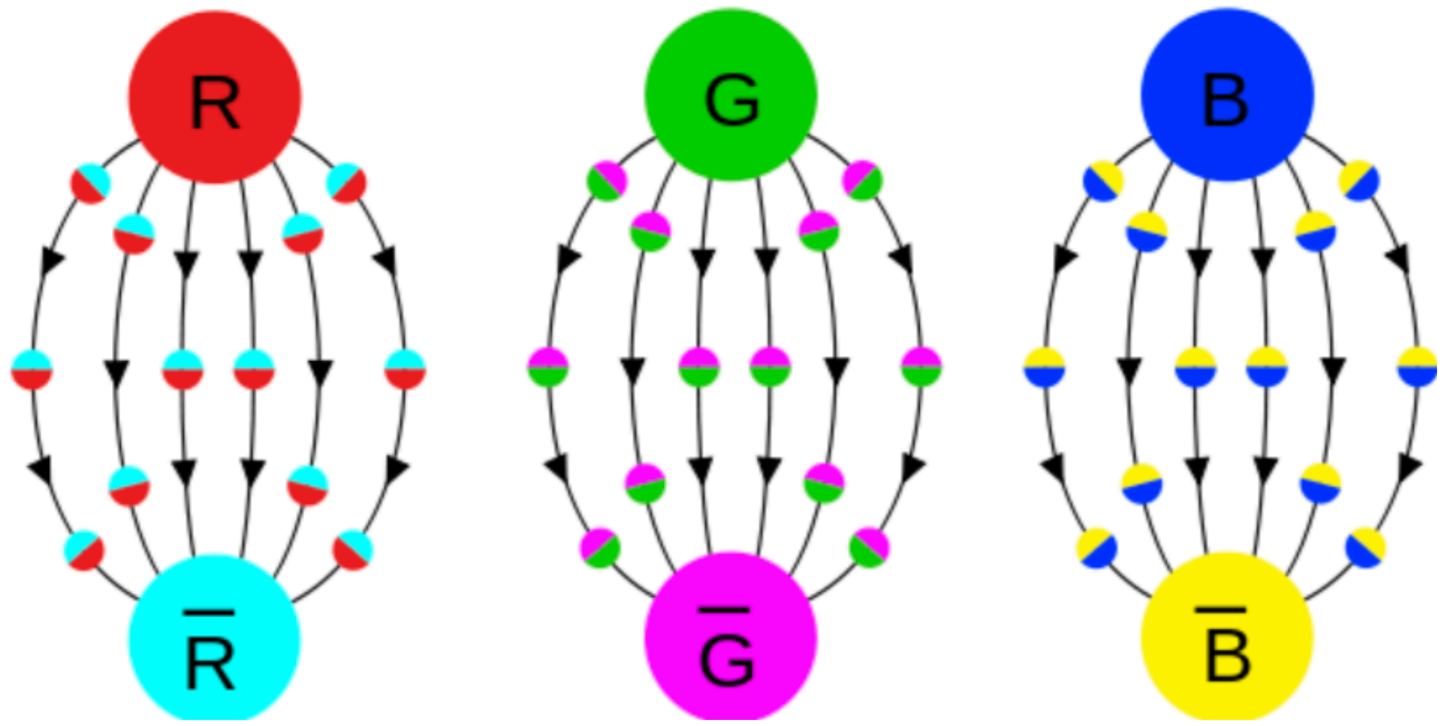
CONFINEMENT

The strong interaction is therefore a **confining force**: only hadrons which reside in a **color-singlet state** are physically allowed



Color fields in Baryons

Note no outgoing lines



Color fields in Mesons

Note no outgoing lines

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Thank you for your attention!

LATEX

\beta = -\frac{de(r)}{d(lnr)}

M_{if} = \frac{g_e^2}{q^2}(\overline{u}_3\gamma^{\mu}u_1)(\overline{u}_4\gamma_{\mu}u_2)

\Big(\frac{-ig_{\mu\nu}}{q^2}\Big)

\int P^3dP/P^2

LATEX

```
\sum_{i=1}^N\int xf_i(x)dx = 1
\frac{d^2\sigma}{d\hat{t}} = \frac{2\pi a^2e_i^2}{\hat{s}^2}\Big(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\Big)
\Big(\frac{d\sigma}{d\Omega}\Big)_{cm} = \frac{a^2}{2s}\Big(\frac{s^2 + u^2}{t^2}\Big)
\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi a^2}{Q^4}\Big[1 + (1 - y)^2\Big]\sum_{i=1}^Ne_i^2f_i(x)
d\sigma = \sum_{i=1}^N d\hat{\sigma}(\hat{s},\hat{t},\hat{u})f_i(x)dx
\hat{s} = 2xpk = xs
\hat{t} = (k - k^{'})^2 = t
\hat{u} = -2xpk^{'} = xu
y = \frac{Pq}{Pk} = \frac{Q^2}{(s - m_p^2)x}
F_2(x) = \sum_{i=1}^Ne_i^2xf_i(x)
\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi a^2}{Q^4}\frac{[1 + (1 - y)^2]}{2x}F_2(x)
\frac{1}{x}F_2^p(x) = \sum_{i=1}^Ne_i^2f_i^p(x)
\frac{1}{x}F_2^p(x) = \Big(\frac{2}{3}\Big)^2[u^p(x) + \overline{u}^p(x)] + \Big(\frac{1}{3}\Big)^2[d^p(x) + \overline{d}^p(x)] + \Big(\frac{1}{3}\Big)^2[s^p(x) + \overline{s}^p(x)]
\frac{1}{x}F_2^n(x) = \Big(\frac{2}{3}\Big)^2[u^n(x) + \overline{u}^n(x)] + \Big(\frac{1}{3}\Big)^2[d^n(x) + \overline{d}^n(x)] + \Big(\frac{1}{3}\Big)^2[s^n(x) + \overline{s}^n(x)]

u^p(x) = d^n(x) = u(x)      u_{\rm v}(x) = u(x) - \overline{u}(x)        u_{\rm s}(x) = 2\overline{u}(x)  u(x) + \overline{u}(x) = u_{\rm v}(x) + u_{\rm s}(x)
d^p(x) = u^n(x) = d(x)      d_{\rm v}(x) = d(x) - \overline{d}(x)        d_{\rm s}(x) = 2\overline{d}(x)
s^p(x) = s^n(x) = s(x)      s_{\rm v}(x) = s(x) - \overline{s}(x) = 0    s_{\rm s}(x) = 2\overline{s}(x)
\int_0^1u_{\rm v}(x) dx = 2
\frac{1}{x}F_2^p(x) = \frac{1}{9}[4u_{\rm v}(x) + d_{\rm v}(x)] + \frac{4}{3}S(x)
\frac{1}{x}F_2^n(x) = \frac{1}{9}[u_{\rm v}(x) + 4d_{\rm v}(x)] + \frac{4}{3}S(x)
\int_0^1d_{\rm v}(x) dx = 1
\sum_{i=1}^N\int_0^1xf_i(x)dx \approx 0.5
S(x) \equiv u_{\rm s}(x) = \overline{u}_{\rm s}(x) = d_{\rm s}(x) = \overline{d}_{\rm s}(x) = s_{\rm s}(x) = \overline{s}_{\rm s}(x)
\sum_{i=1}^N\int_0^1xf_i(x)dx + \int_0^1xg(x)dx = 1

F_2^{ep}(x) = \sum_qxe_q^2[q(x) + \overline{q}(x)]
F_2^{ep}(x) = x\Big[\frac{4}{9}(u + \overline{u}) + \frac{1}{9}(d + \overline{d})\Big]
F_2^{en}(x) = x\Big[\frac{1}{9}(u + \overline{u}) + \frac{4}{9}(d + \overline{d})\Big]
F_2^{eN}(x) = \frac{5}{18}F_2^{\nu N}(x)
F_2^{ep}(x) - F_2^{en}(x) = \frac{1}{3}x[u_{\rm v}(x) - d_{\rm v}(x)]
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