Quantum Cryptography Beyond QKD

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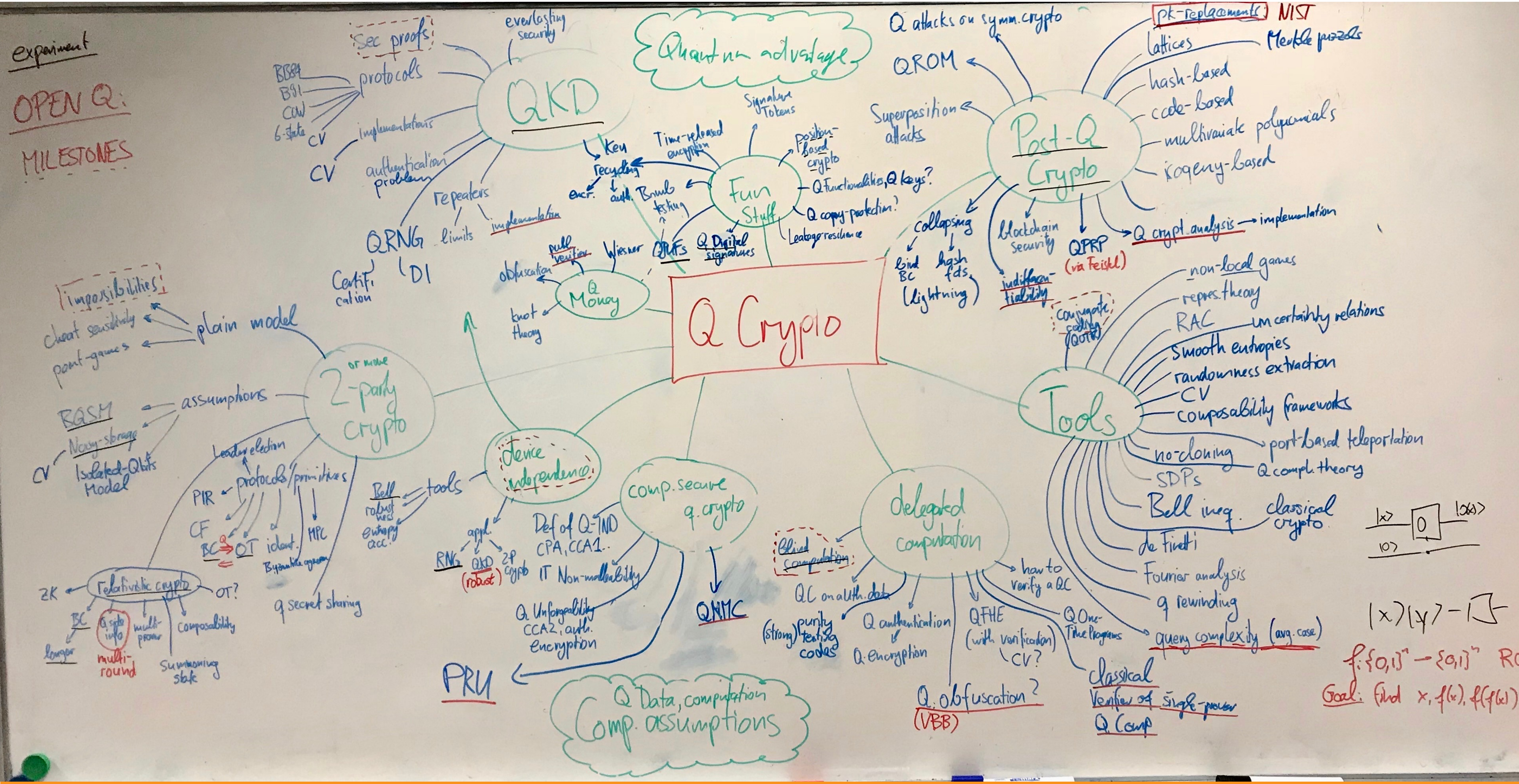
NWO Nederlandse Organisatie voor Wetenschappelijk Onderzoek

All material available on https://homepages.cwi.nl/~schaffne


experiment
OPEN Q:
MILESTONES
Quantum advantage
Q Crypto
QKD
Sec proofs
everlasting security
protocols
BB84
B91
COW
6-state
CV
implementations
CV
authentication problem
repeaters
limits
implementation
QRNG
DI
Certification
Fun Stuff
Signature Tokens
Time-released encryption
Key recycling
encr.
auth.
Bomb testing
position-based crypto
Q Functionalities, Q keys?
Q copy-protection?
Leakage resilience
QPUFs
Q Digital signatures
Wiesner
publ. verification
obfuscation
Q Money
knot theory
Q attacks on symm. crypto
QROM
Superposition attacks
Post-Q Crypto
pk-replacements
NIST
lattices
Merkle puzzles
hash-based
code-based
multivariate polynomials
isogeny-based
collapsing
bind BC
hash fcts
(lightning)
blockchain security
indifferentiability
QPRP
(via Feistel)
Q crypt. analysis
implementation
Tools
non-local games
repres. theory
RAC
uncertainty relations
smooth entropies
randomness extraction
CV
composability frameworks
port-based teleportation
no-cloning
Q compl. theory
SDPs
Bell ineq.
classical crypto
de Finetti
Fourier analysis
q rewinding
query complexity (avg. case)
conjugate coding (QOTP)
2-party crypto
or more
plain model
impossibilities
cheat sensitivity
point-games
assumptions
BQSM
Noisy-storage
CV
Isolated-Qbits Model
Leader election
protocols/primitives
PIR
CF
BC
OT
ident.
MPC
Byzantine agreement
q secret sharing
Relativistic crypto
ZK
OT?
BC
longer
Q side info
multi-round
multi-prover
composability
summoning state
device independence
tools
Bell
robustness
entropy acc.
appl.
RNG
QKD
(robust)
2P crypto
comp. secure q. crypto
Def of Q-IND
CPA, CCA1
IT Non-malleability
Q Unforgeability
CCA2, auth. encryption
QNMC
PRU
Q Data, computation
Comp. assumptions
delegated computation
Blind Computation
QC on auth. data
(strong) purity testing codes
Q authentication
Q encryption
QFHE
(with verification)
CV?
how to verify a QC
Q One-Time Programs
classical Verifier of single-prover Q Comp
Q obfuscation?
(VBB)
|x⟩
|0⟩
|f(x)⟩
|x⟩|y⟩
Goal: find x, f(x), f(f(x))

Quantum Cryptography Beyond QKD

- survey article with Anne Broadbent
- aimed at classical cryptographers

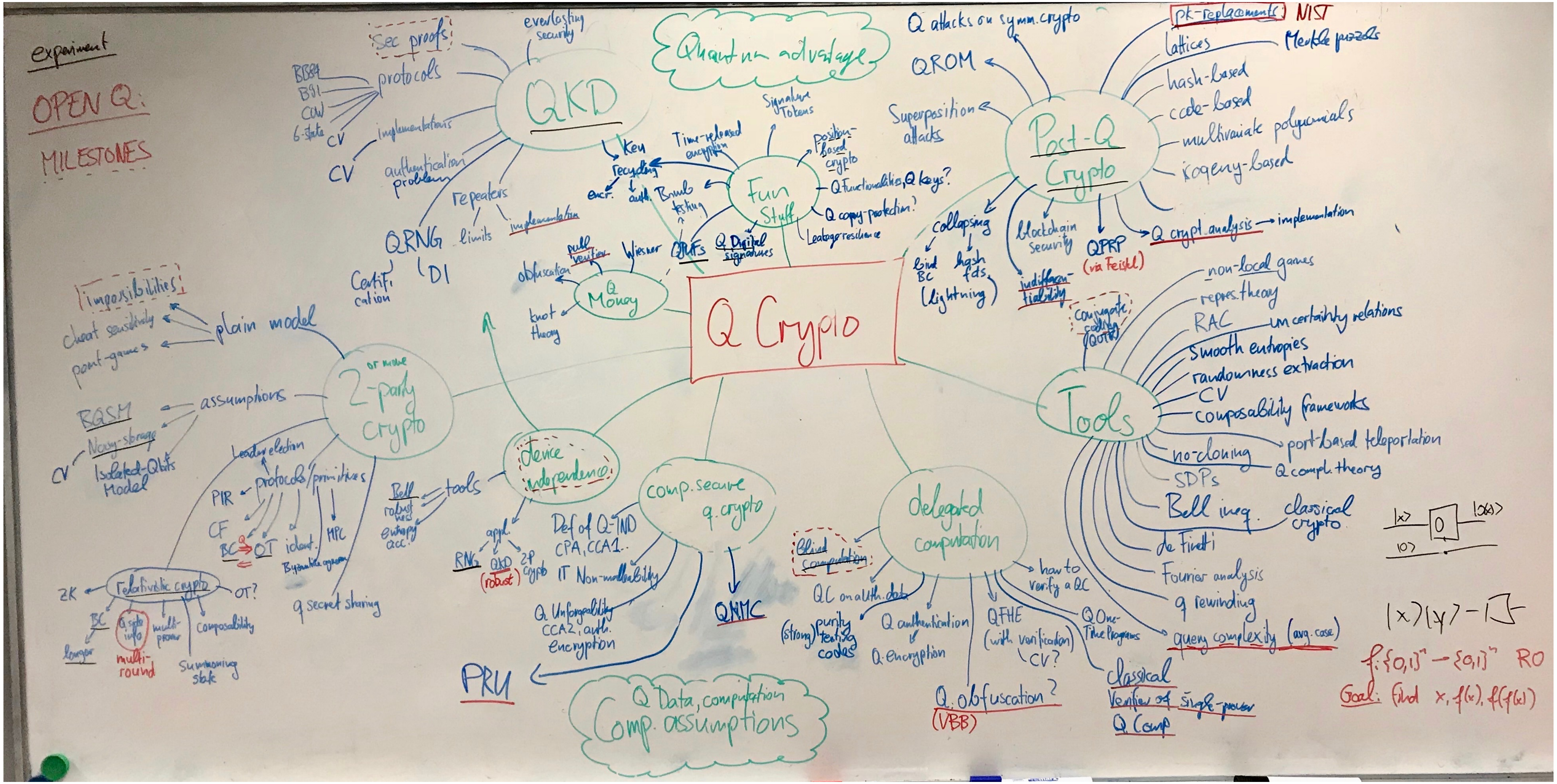
http://arxiv.org/abs/1510.06120
In Designs, Codes and Cryptography 2016

QCrypt Conference Series

- Started in 2011 by Christandl and Wehner
- Steadily growing since then:
  approx. 100 submissions, 30 accepted as contributions,
  330 participants in Cambridge 2017. This year: Shanghai, China
- It is the goal of the conference to represent the previous year's best results on quantum cryptography, and to support the building of a research community
- Trying to keep a healthy balance between theory and experiment
- Half the program consists of 4 tutorials of 90 minutes, 6-8 invited talks
- present some statistical observations about the last 4 editions

[QCrypt charter, QCrypt 2017 business meetings slides]

Overview

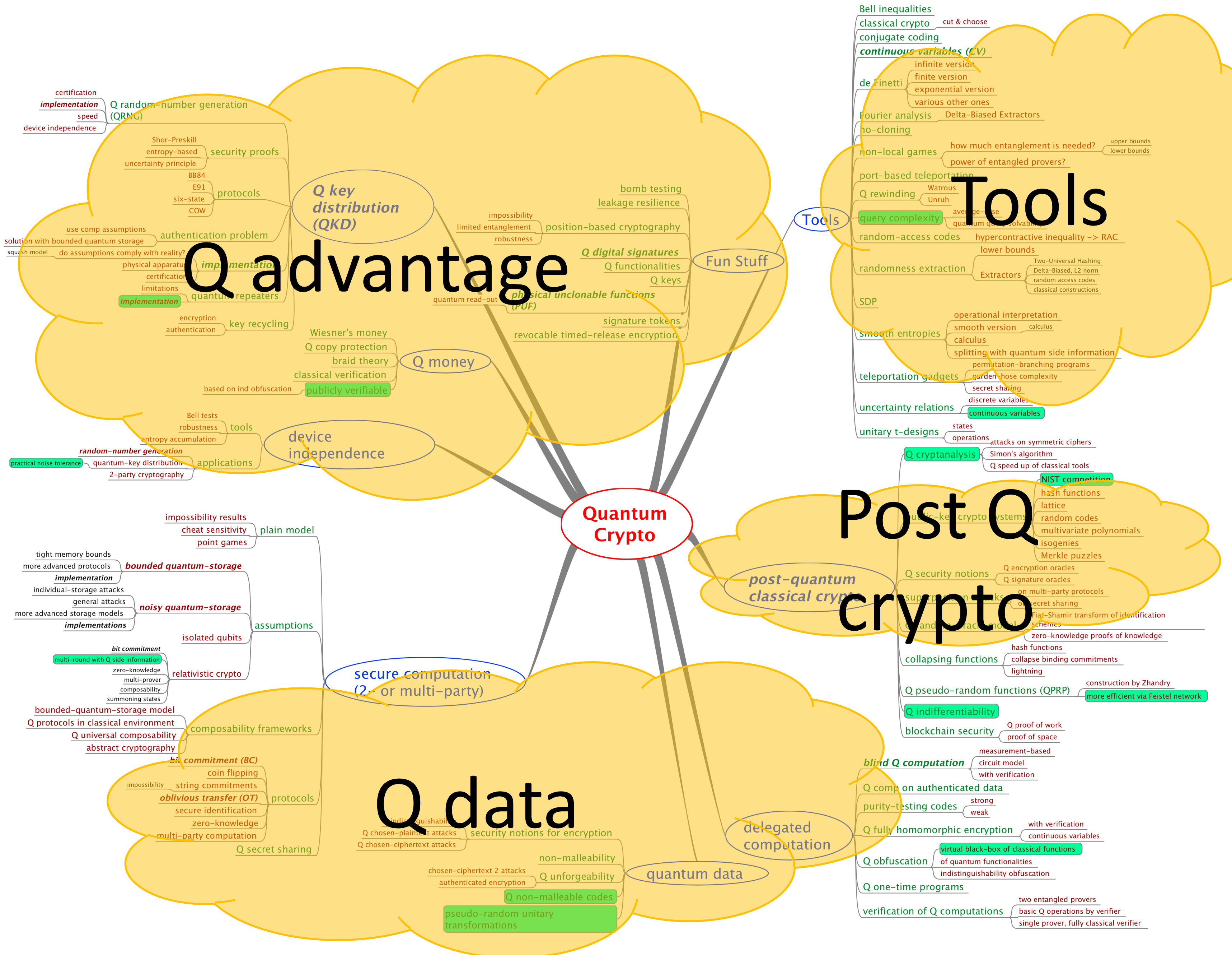


[thanks to Serge Fehr, Stacey Jeffery, Chris Majenz, Florian Speelman, Ronald de Wolf]

MindMap

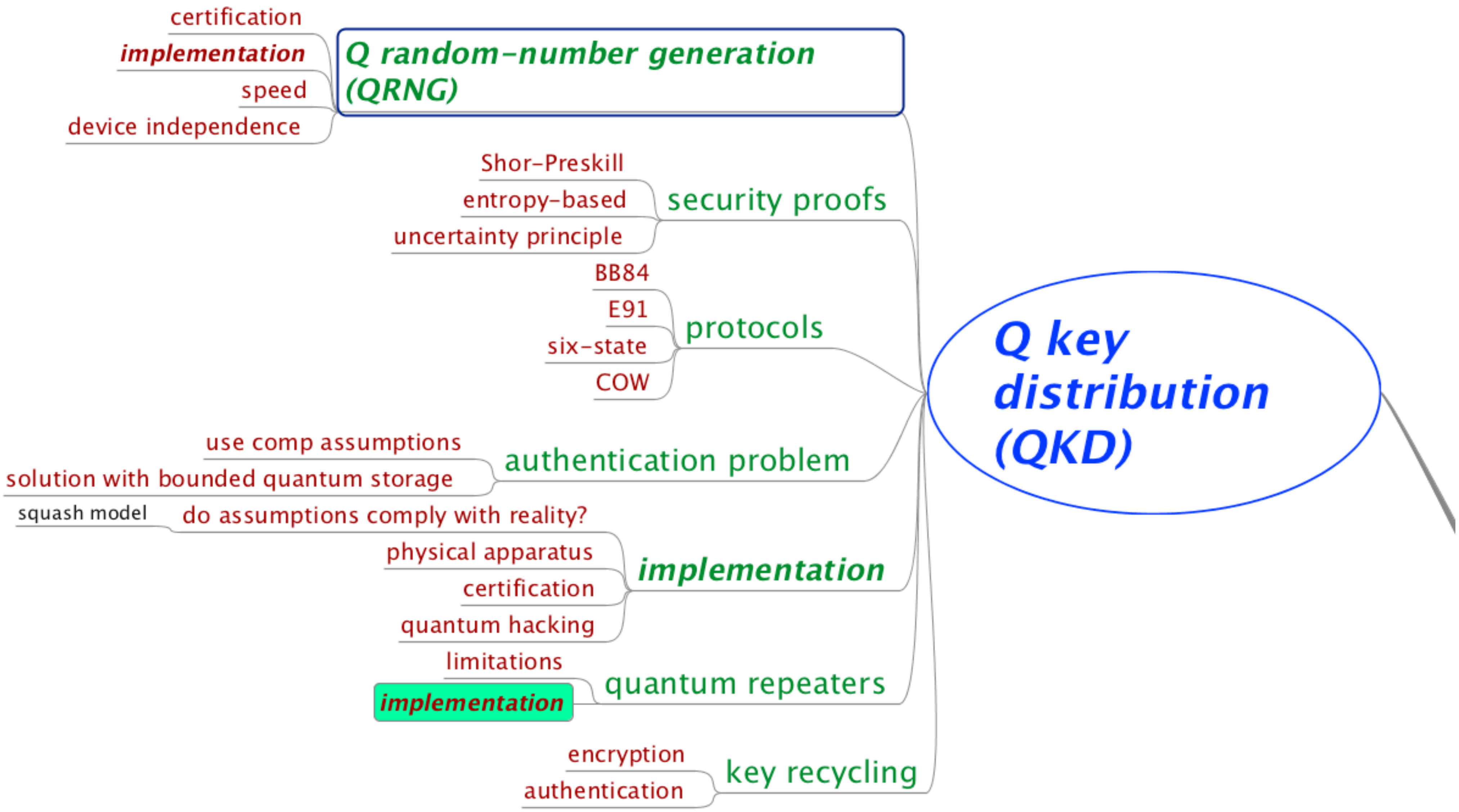
- ***experiments***
- Selection of **open questions**

- Fork me on github!

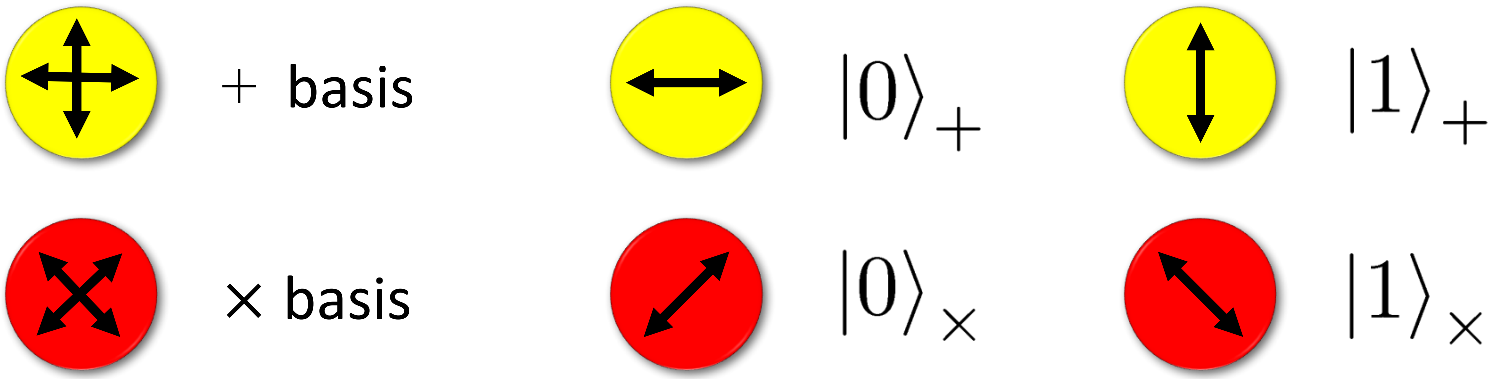


[https://github.com/cschaffner/QCryptoMindmap]

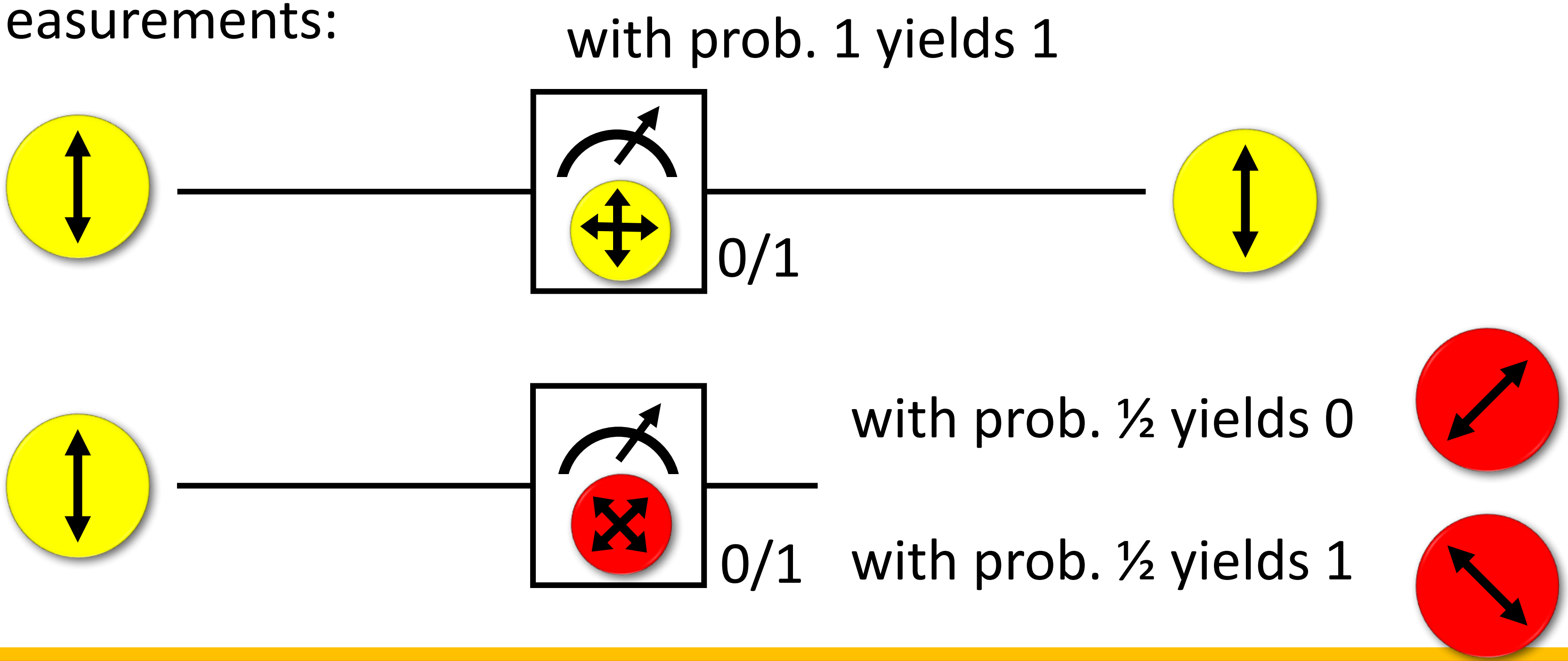
Quantum Key Distribution (QKD)



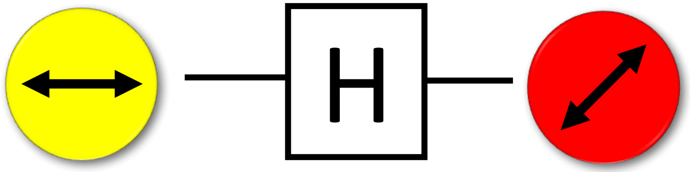
Quantum Mechanics



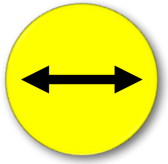
Measurements:

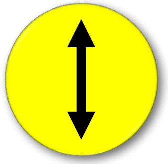


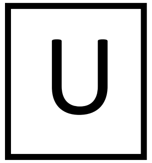
Quantum operations: $\boxed{U}$

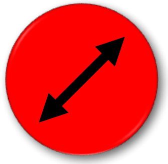


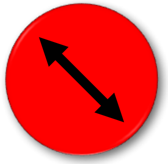
No-Cloning Theorem

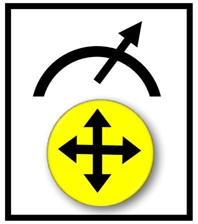
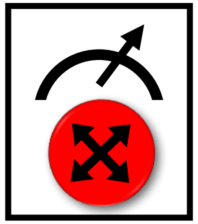
 $|0\rangle_+$

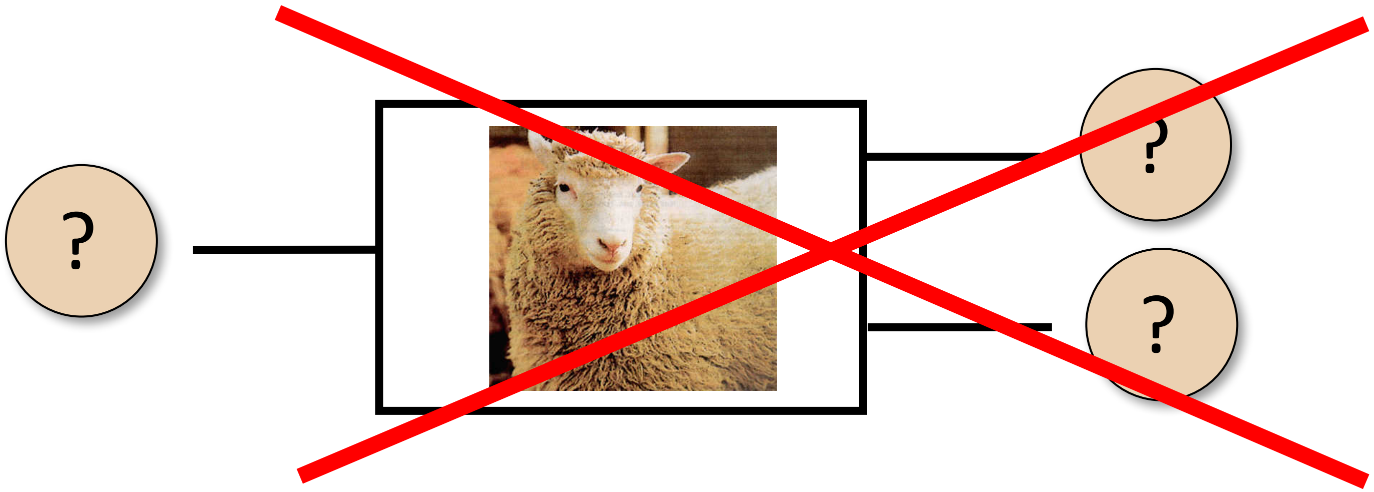
 $|1\rangle_+$

Quantum operations: 

 $|0\rangle_\times$

 $|1\rangle_\times$

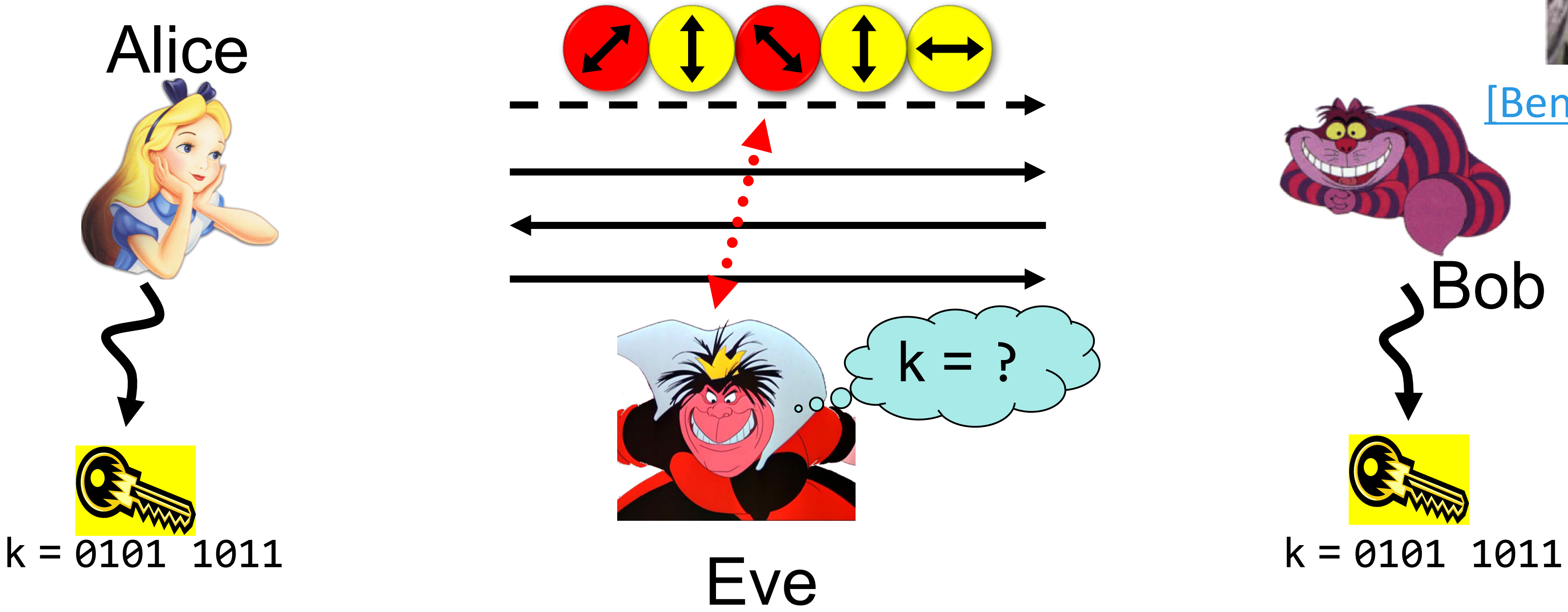
 



Proof: copying is a non-linear operation

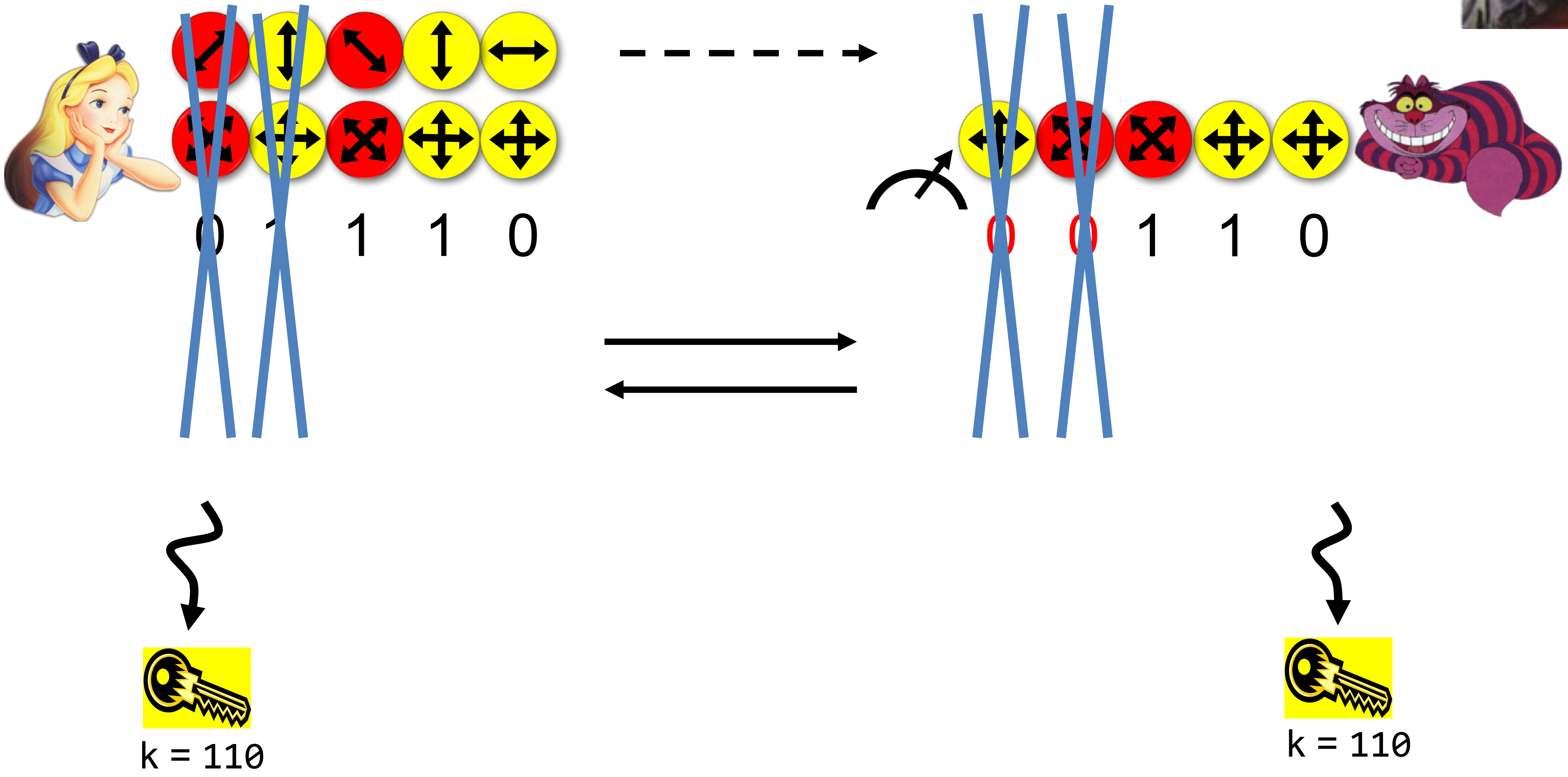
Quantum Key Distribution (QKD)

[Bennett Brassard 84]

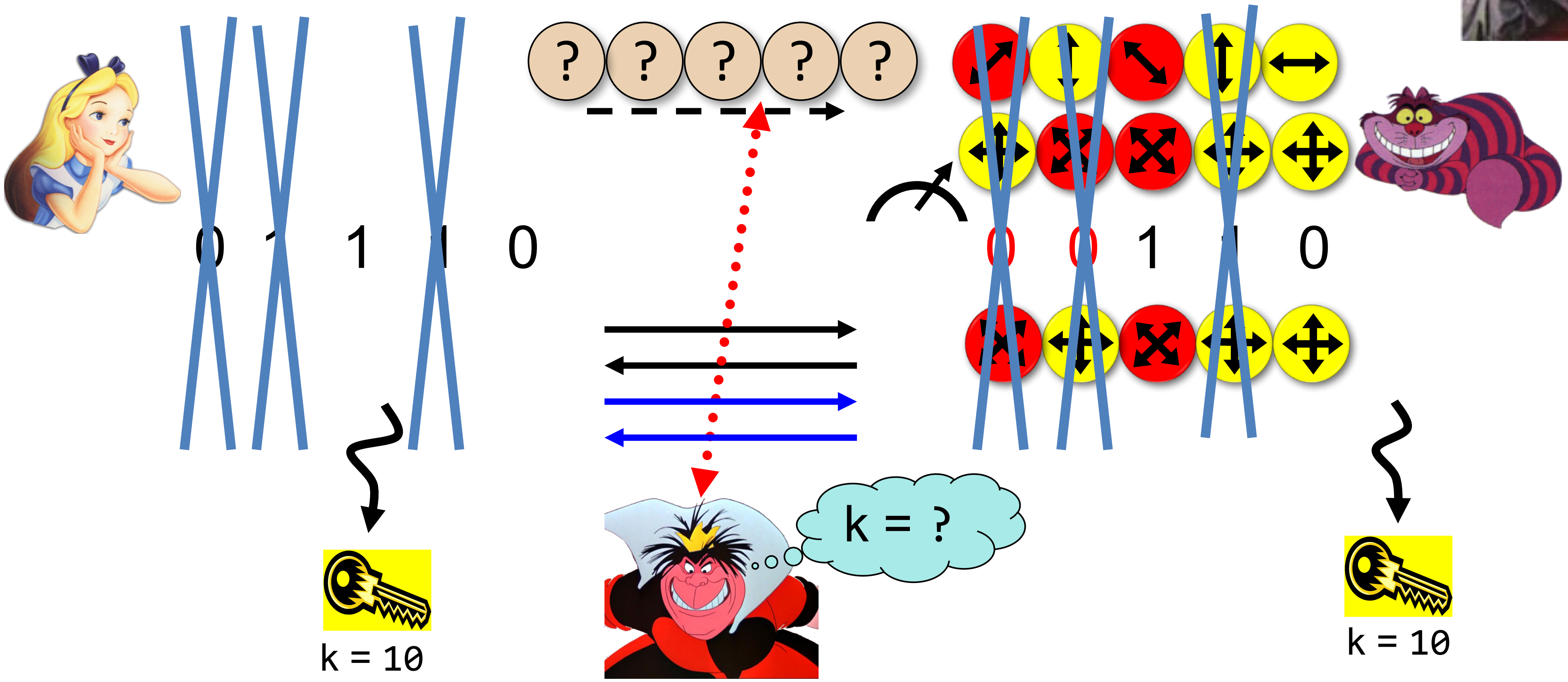


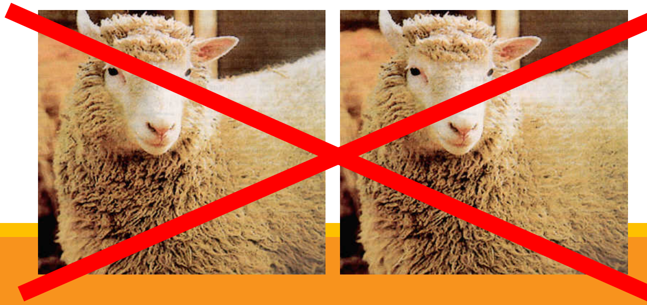
- Offers an quantum solution to the key-exchange problem which does not rely on computational assumptions (such as factoring, discrete logarithms, security of AES, SHA-3 etc.)
- Caveat: classical communication has to be authenticated to prevent man-in-the-middle attacks

Quantum Key Distribution (QKD)



Quantum Key Distribution (QKD)

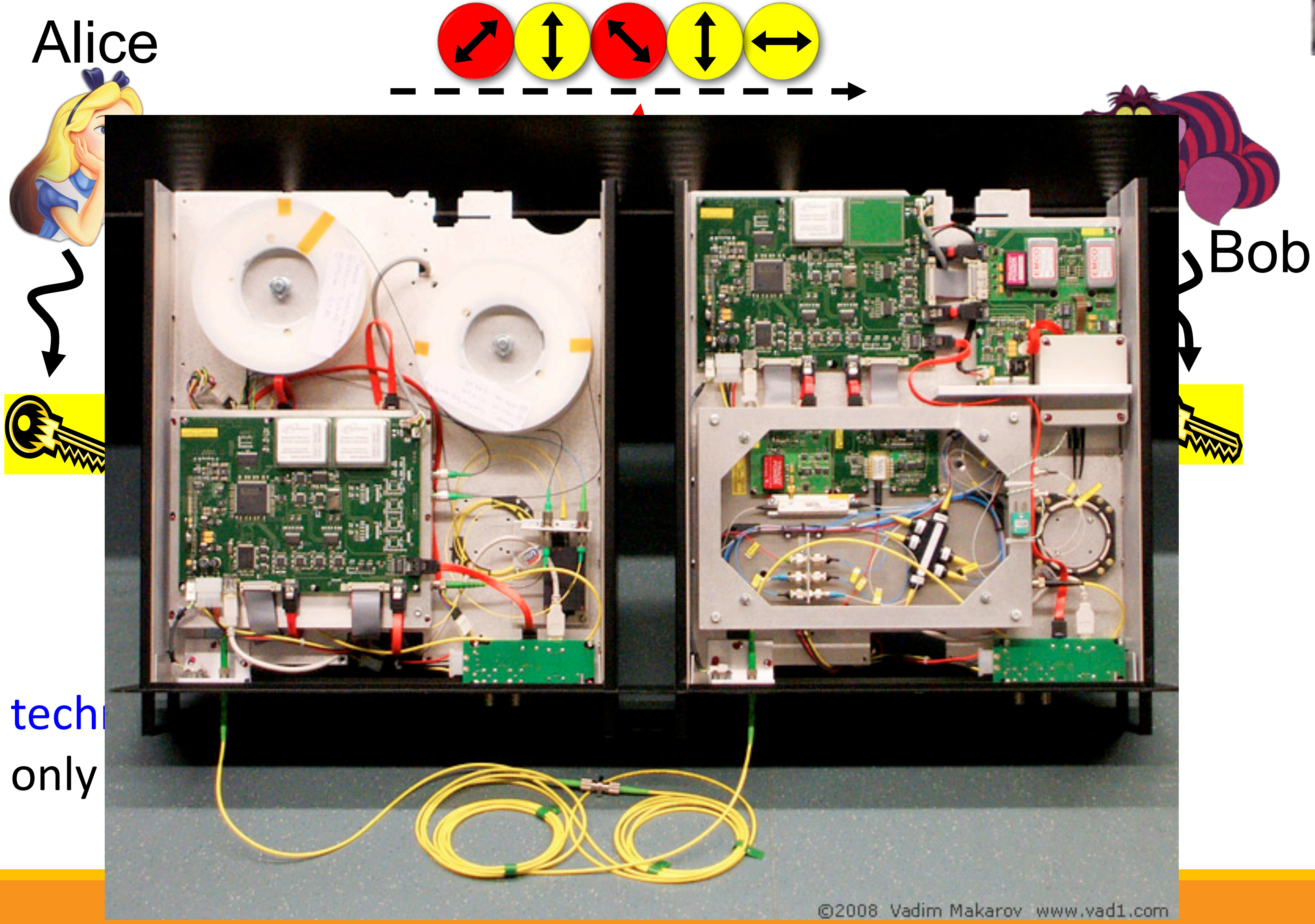




- Quantum states are unknown to Eve, she cannot copy them.
- Honest players can test whether Eve interfered.

[Bennett Brassard 84]

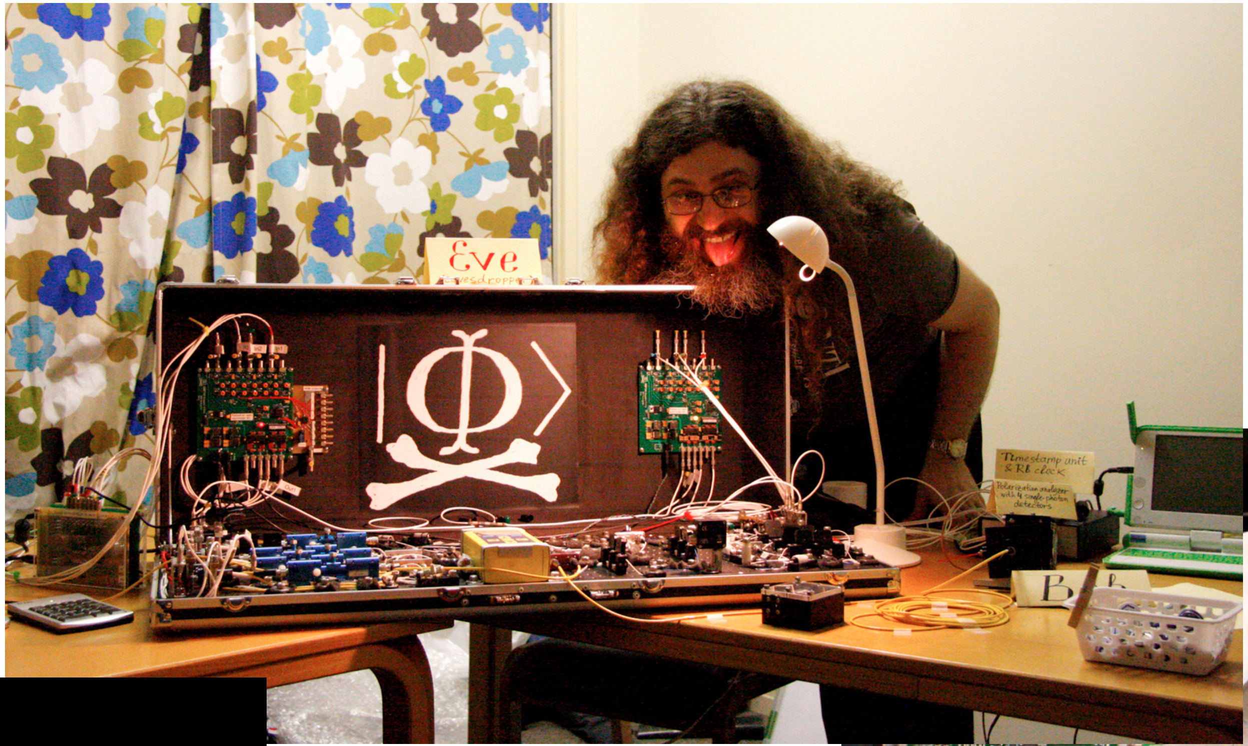
Quantum Key Distribution (QKD)

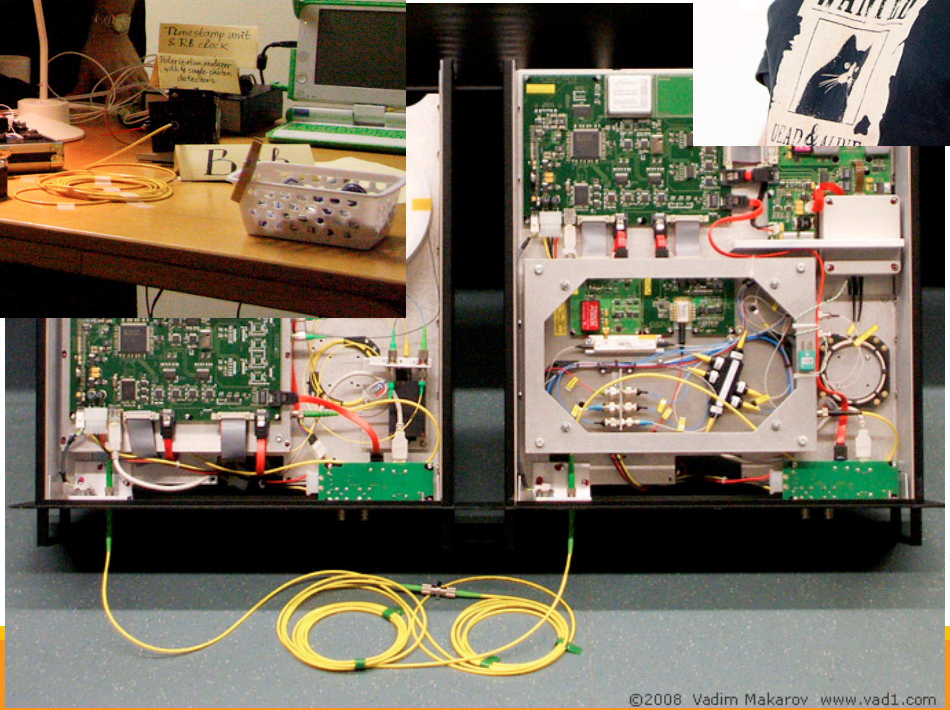


- tech
  only

Quantum Hacking

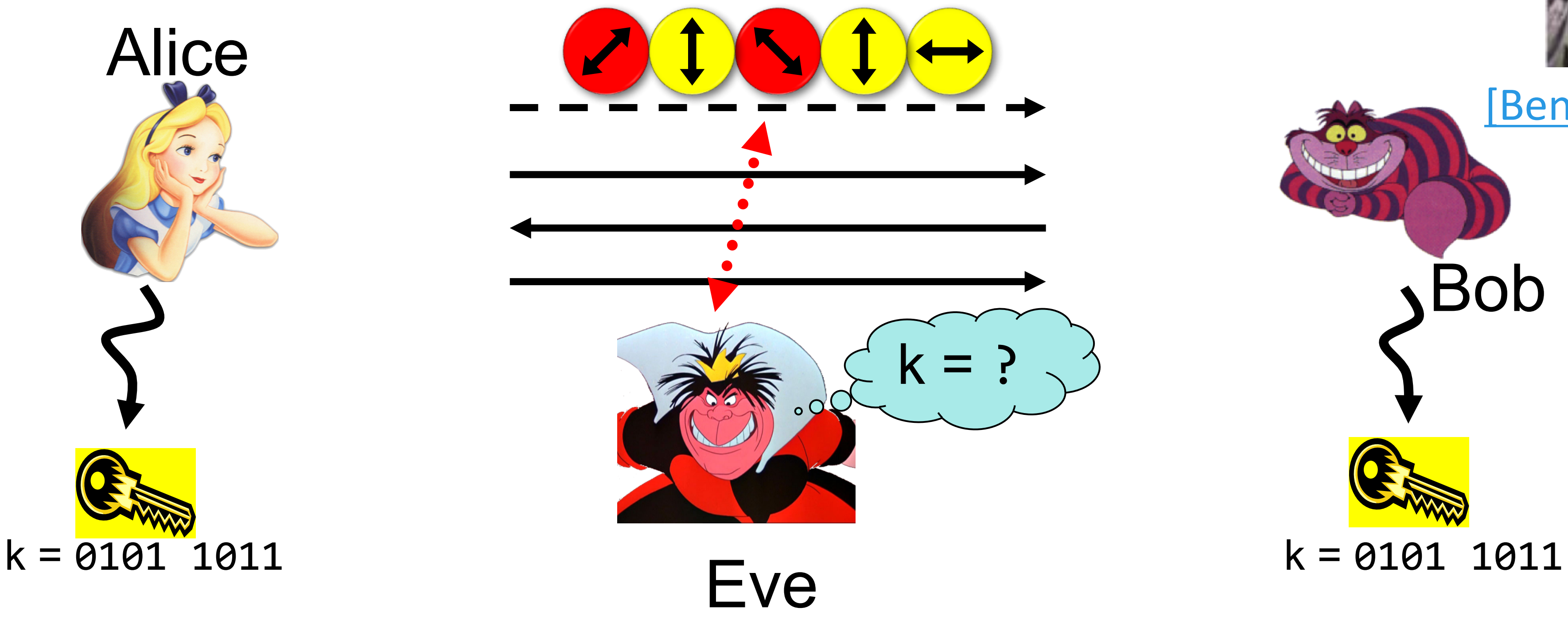
e.g. by the group of Vadim Makarov (University of Waterloo, Canada)





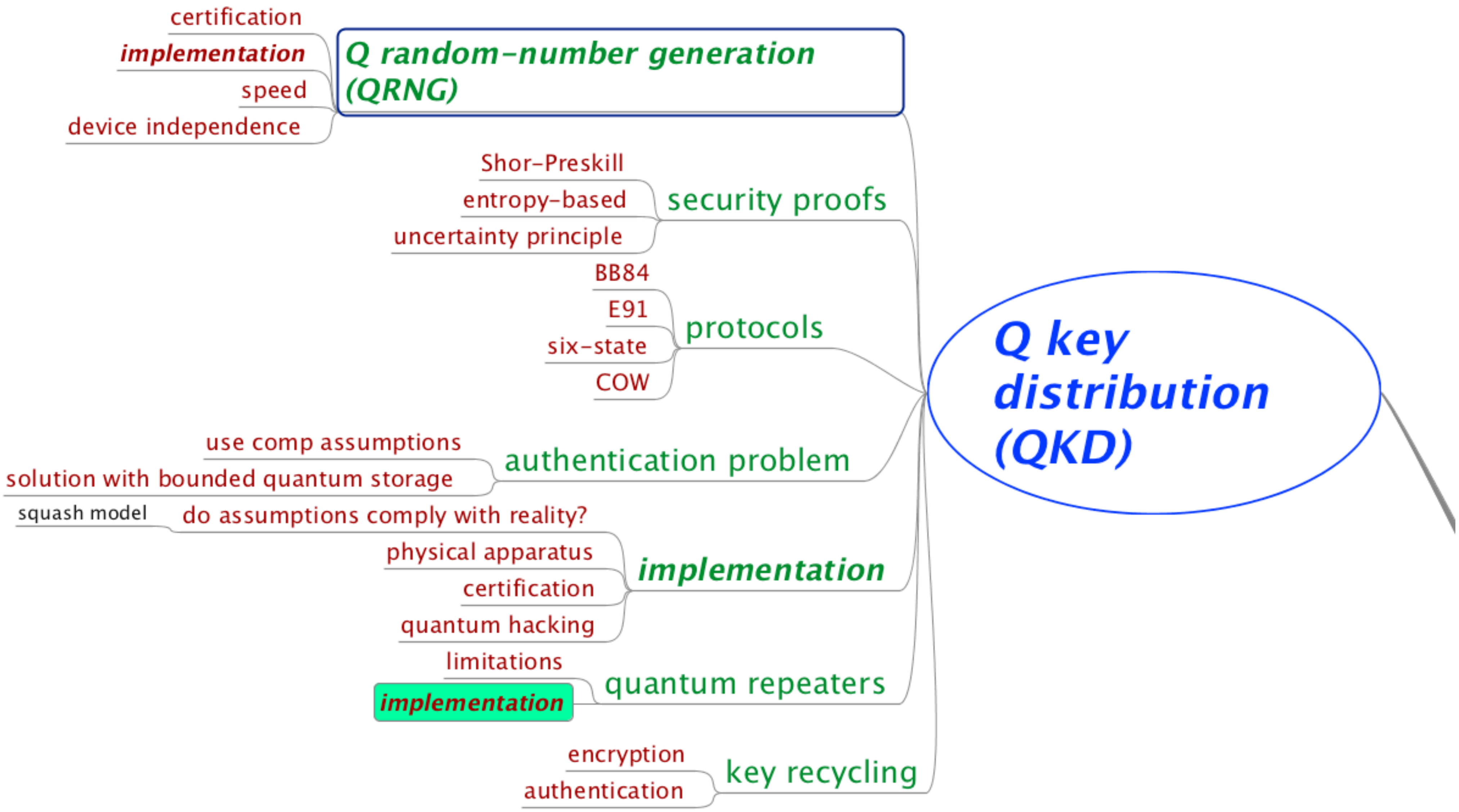
Quantum Key Distribution (QKD)

[Bennett Brassard 84]



- **Three-party scenario**: two honest players versus one dishonest eavesdropper
- **Quantum Advantage:** Information-theoretic security is provably impossible with only classical communication (Shannon's theorem about perfect security)

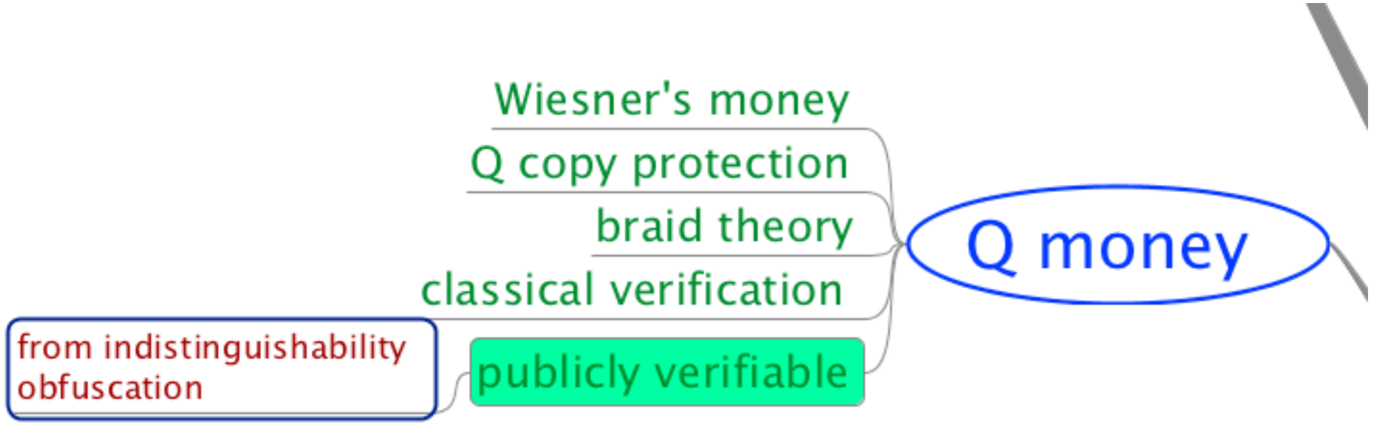
Quantum Key Distribution (QKD)



Conjugate Coding & Q Money

[Wiesner 68]

also known as **quantum coding** or **quantum multiplexing**



- Originally proposed for securing quantum banknotes (private-key quantum money)
- Adaptive attack if money is returned after successful verification
- Publicly verifiable quantum money is still a topic of active research, e.g. very recent preprint by Zhandry17

[Molina Vidick Watrous 13, Brodutch Nagaj Sattath Unruh 14]

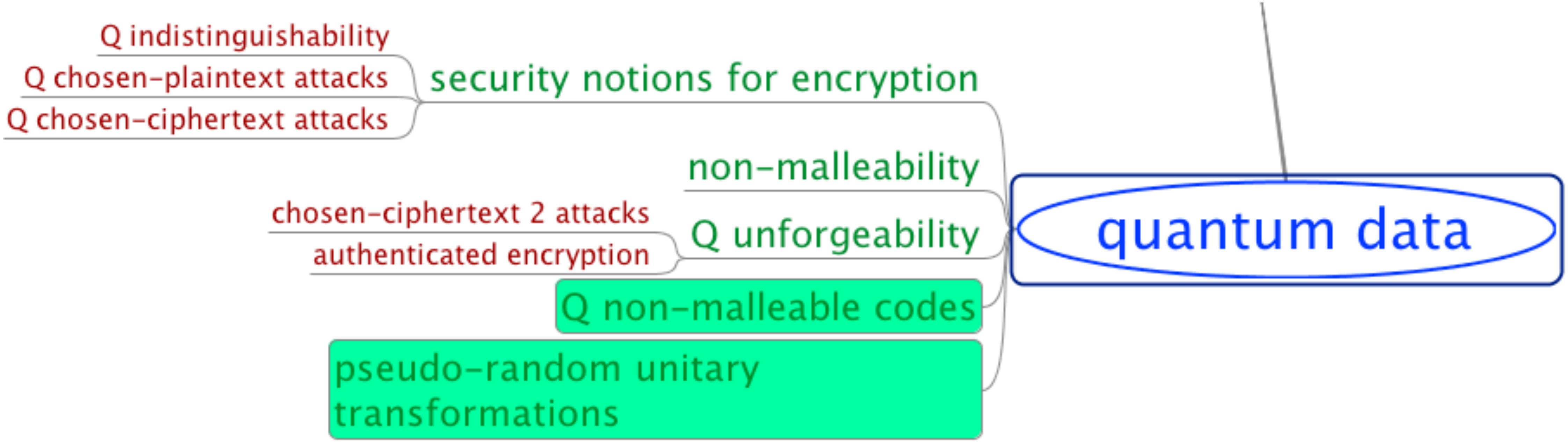
Computational Security of Quantum Encryption

http://arxiv.org/abs/1602.01441
at ICITS 2016

GORJAN ALAGIC, COPENHAGEN
ANNE BROADBENT, OTTAWA
BILL FEFFERMAN, MARYLAND
TOMMASO GAGLIARDONI, DARMSTADT
MICHAEL ST JULES, OTTAWA

CHRISTIAN SCHAFFNER, AMSTERDAM

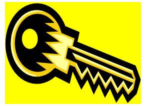
Computational Security of Quantum Encryption



Secure Encryption

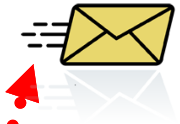
plaintext message m

Alice



Secret key sk

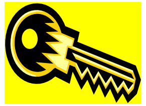
ciphertext $c = Enc_{sk}(m)$



Eve

$m = Dec_{sk}\ (c)$

Bob



Secret key sk

One-Time Pad:

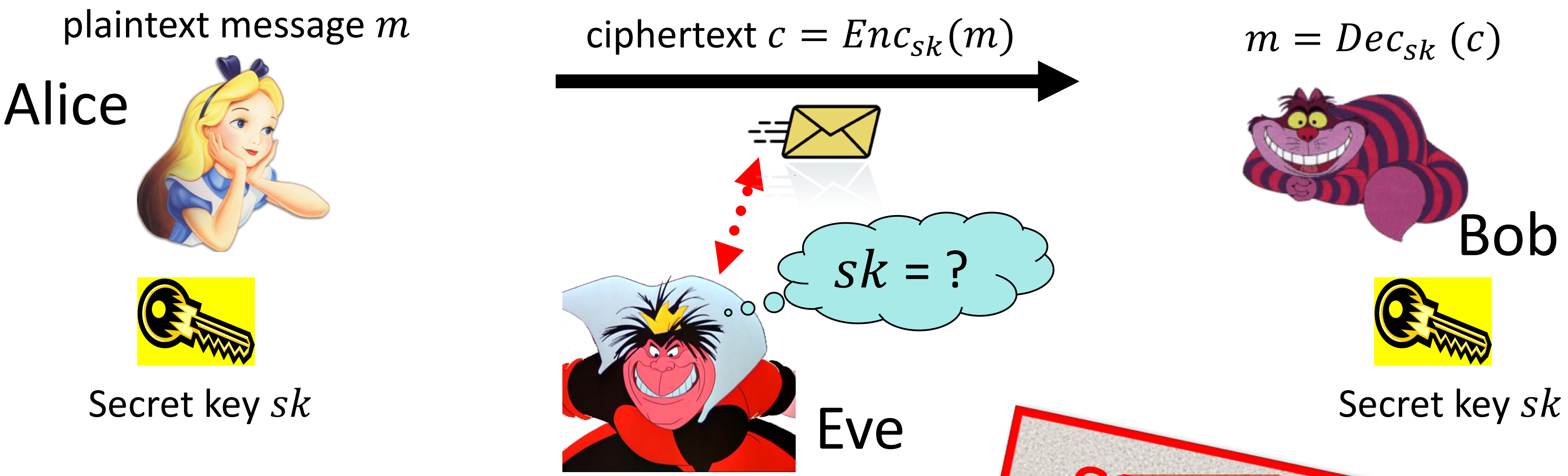
Classical: $c = Enc_{sk}(m) := m \oplus sk$, $Dec_{sk}(c) := c \oplus sk$

Quantum: $Enc_{a,b}(\rho_M) := X^a Z^b \rho_M Z^b X^a$
$Dec_{a,b}(\rho_C) := X^a Z^b \rho_C Z^b X^a$

SECURE

QOTP

Information-Theoretic Security



Perfect / information-theoretic security:

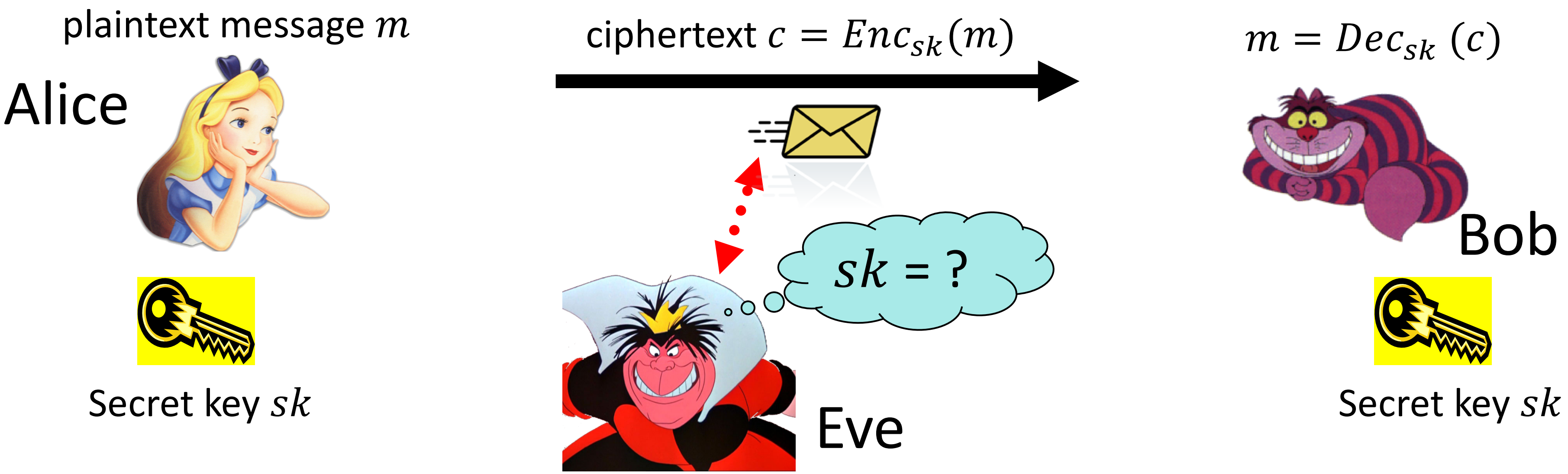
Ciphertext distribution P_C is statistically independent of message distribution P_M.

Theorem: Secret key has to be as large as the message.

Highly impractical, e.g. for encrypting a video stream…

[Shannon 48, Dodis 12, Ambainis Mosca Tapp de Wolf 00, Boykin Roychowdhury 03]

Computational Security



Threat model:

- Eve sees ciphertexts (eavesdropper)
- Eve knows plaintext/ciphertext pairs
- Eve chooses plaintexts to be encrypted
- Eve can decrypt ciphertexts

Security guarantee:

c does not reveal sk

c does not reveal the whole m

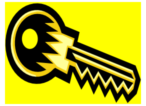
c does not reveal any bit of m

c does not reveal “anything” about m

Semantic Security

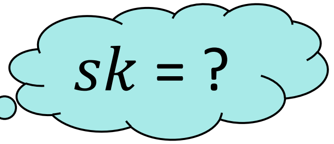
plaintext message m

Alice



Secret key sk

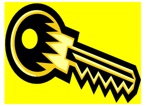
ciphertext $c = Enc_{sk}(m)$



Eve

$m = Dec_{sk}\,(c)$

Bob

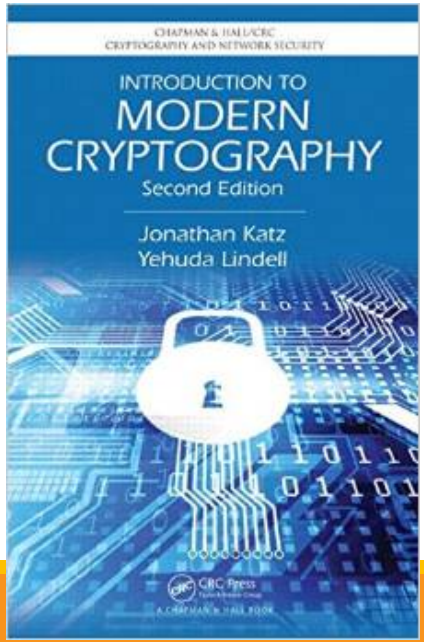


Secret key sk

DEFINITION 3.12 *A private-key encryption scheme* (Enc, Dec) *is* semantically secure in the presence of an eavesdropper *if for every* PPT *algorithm $\mathcal{A}$ there exists a* PPT *algorithm $\mathcal{A}'$ such that for any* PPT *algorithm* Samp *and polynomial-time computable functions f and h, the following is negligible:*

$$\left| \Pr[\mathcal{A}(1^n, \mathsf{Enc}_k(m), h(m)) = f(m)] - \Pr[\mathcal{A}'(1^n, |m|, h(m)) = f(m)] \right|,$$

where the first probability is taken over uniform $k \in \{0,1\}^n$, m output by $\mathsf{Samp}(1^n)$*, the randomness of $\mathcal{A}$, and the randomness of* Enc*, and the second probability is taken over m output by* $\mathsf{Samp}(1^n)$ *and the randomness of $\mathcal{A}'$.*



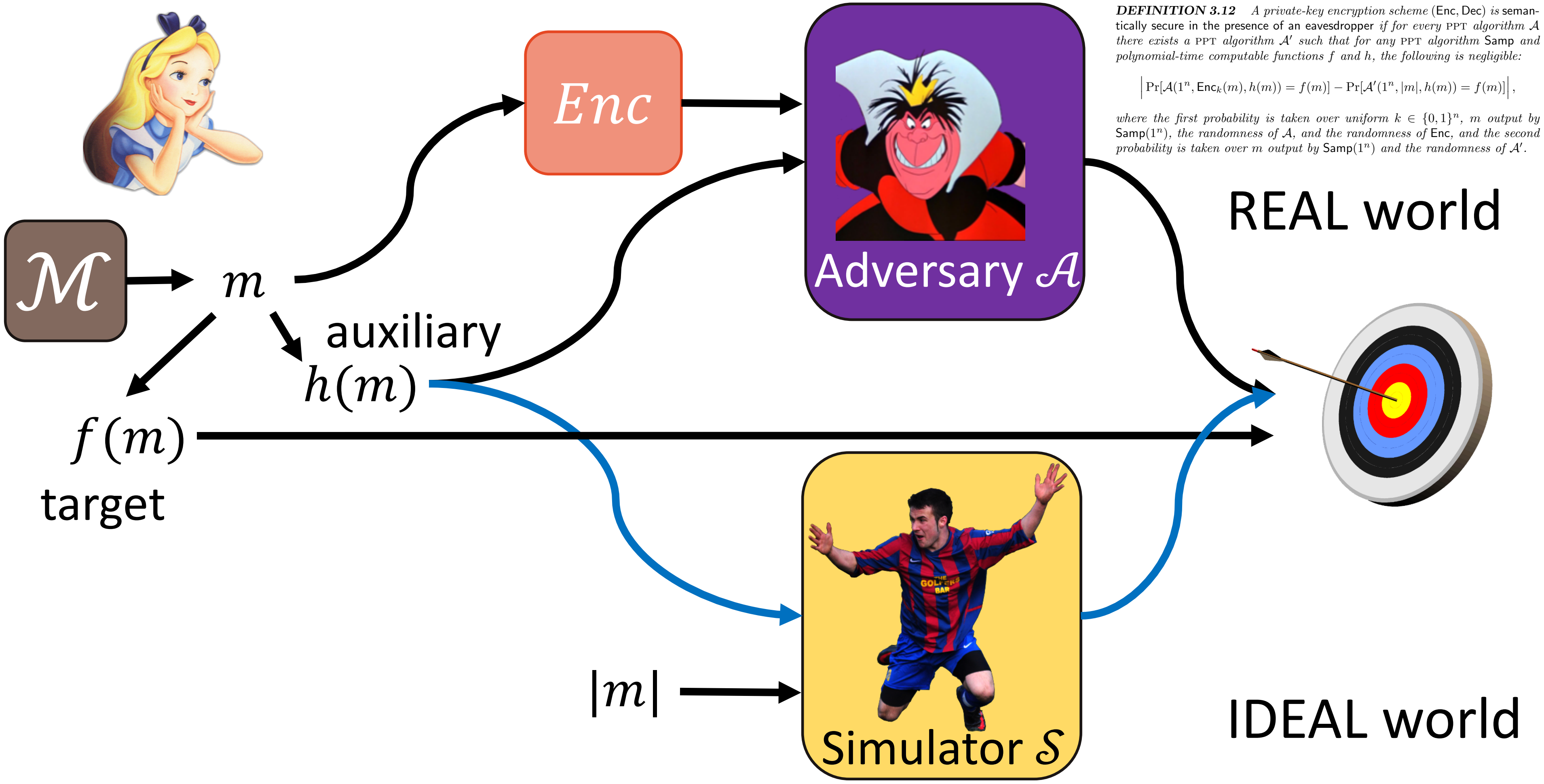
[Goldwasser Micali 84] leading to Turing-Award (Nobel price for CS)

Classical Semantic Security

DEFINITION 3.12 *A private-key encryption scheme* (Enc, Dec) *is* semantically secure in the presence of an eavesdropper *if for every* PPT *algorithm* $\mathcal{A}$ *there exists a* PPT *algorithm* $\mathcal{A}'$ *such that for any* PPT *algorithm* Samp *and polynomial-time computable functions* f *and* h, *the following is negligible:*

$$\left|\Pr[\mathcal{A}(1^n, \mathsf{Enc}_k(m), h(m)) = f(m)] - \Pr[\mathcal{A}'(1^n, |m|, h(m)) = f(m)]\right|,$$

where the first probability is taken over uniform $k \in \{0,1\}^n$, m *output by* $\mathsf{Samp}(1^n)$, *the randomness of* $\mathcal{A}$, *and the randomness of* Enc, *and the second probability is taken over* m *output by* $\mathsf{Samp}(1^n)$ *and the randomness of* $\mathcal{A}'$.



Definition (SEM): $\forall \mathcal{A}\ \exists \mathcal{S} : \forall(\mathcal{M}, h, f)$

$$\Pr[\mathcal{A}(Enc_k(m), h(m)) = f(m)] \approx \Pr[\mathcal{S}(|m|, h(m)) = f(m)]$$

[Goldwasser Micali 84] leading to Turing-Award (Nobel price for CS)

Classical Indistinguishability

$PrivK^{eav}$

Challenger

$\mathcal{A}$

$\xleftarrow{\ m\ }$

$b \leftarrow \{0,1\}$

$$c = \begin{cases} Enc_{sk}\left(0^{|m|}\right) \text{ if b=0} \\ Enc_{sk}(m) \text{ if b=1} \end{cases} \xrightarrow{\ c\ }$$

$\mathcal{A}$ wins iff $b = b'$ $\xleftarrow{\ b'\ }$

Definition (IND): $\forall \mathcal{A}:\ \Pr[\mathcal{A} \text{ wins } PrivK^{eav}] \leq \frac{1}{2} + \ negl(n)$

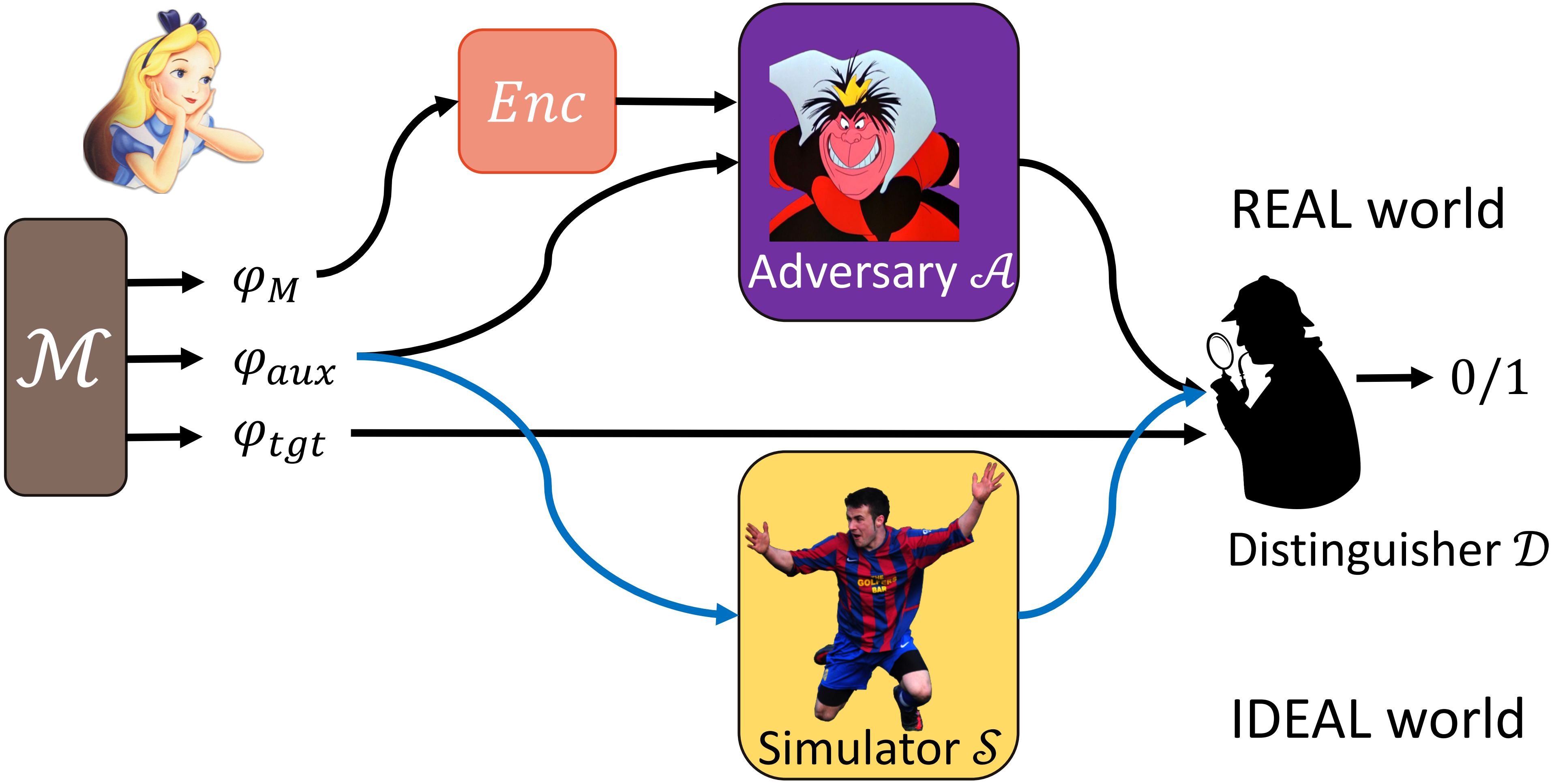
Theorem: SEM ⇔ IND

[Goldwasser Micali 84] leading to Turing-Award (Nobel price for CS)

Our Contributions

1. Formal definition of Quantum Semantic Security
2. Equivalence to Quantum Indistinguishability
3. Extension to CPA and CCA1 scenarios
4. Construction of IND-CCA1 Quantum Secret-Key Encryption from One-Way Functions
5. Construction of Quantum Public-Key Encryption from One-Way Trapdoor Permutations

Quantum Semantic Security



Definition (QSEM): $\forall \mathcal{A}\ \exists \mathcal{S}\ \forall (\mathcal{M}, \mathcal{D})$:

$$\Pr[\mathcal{D}(\mathrm{REAL}) = 1] \approx \Pr[\mathcal{D}(\mathrm{IDEAL}) = 1]$$

Quantum Indistinguishability

$QPrivK^{eav}$

Challenger 

 $\mathcal{A}$

$\xleftarrow{\rho_M}$

$b \leftarrow \{0,1\}$

$\rho_C = \begin{cases} Enc_{sk}(|0\rangle) \text{ if b=0} \\ Enc_{sk}(\rho_M) \text{ if b=1} \end{cases}$ $\xrightarrow{\rho_C}$

$\mathcal{A}$ wins iff $b = b'$ $\xleftarrow{b'}$

Definition (QIND): $\forall \mathcal{A}\colon\ \Pr[\mathcal{A} \text{ wins } QPrivK^{eav}] \leq \frac{1}{2} + negl(n)$

Theorem: QSEM $\Leftrightarrow$ QIND

Chosen-Plaintext Attacks (CPA)

$QPrivK^{cpa}$

Challenger 

$\xleftarrow{\rho_M}$

$\xrightarrow{Enc_{sk}(\rho_M)}$

 $\mathcal{A}$

$b \leftarrow \{0,1\}$

$\xleftarrow{\rho_M}$

$$\rho_C = \begin{cases} Enc_{sk}(|0\rangle) \text{ if b=0} \\ Enc_{sk}(\rho_M) \text{ if b=1} \end{cases}$$

$\xrightarrow{\rho_C}$

$\mathcal{A}$ wins iff $b = b'$ $\xleftarrow{b'}$

Definition (QIND-CPA): $\forall \mathcal{A}:\ \Pr[\mathcal{A} \text{ wins } QPrivK^{cpa}] \leq \frac{1}{2} + negl(n)$

Theorem: QSEM-CPA ⇔ QIND-CPA

Fact: CPA security requires **randomized encryption**

Chosen-Ciphertext Attacks (CCA1)

$QPrivK^{cca}$

Challenger

ρ_C ←

$Dec_{sk}(\rho_C)$ →

$\mathcal{A}$

$b \leftarrow \{0,1\}$

ρ_M ←

$$\rho_C = \begin{cases} Enc_{sk}(|0\rangle) \text{ if b=0} \\ Enc_{sk}(\rho_M) \text{ if b=1} \end{cases}$$

ρ_C →

ρ_M ←

$Enc_{sk}(\rho_M)$ →

$\mathcal{A}$ wins iff $b = b'$

b' ←

Definition (QIND-CCA1): $\forall \mathcal{A}$: $\Pr[\mathcal{A} \text{ wins } QPrivK^{cca}] \leq \frac{1}{2} + negl(n)$

Theorem: QSEM-CCA1 $\Leftrightarrow$ QIND-CCA1

Fact: QSEM-CCA1 $\overset{\neq}{\Rightarrow}$ QIND-CPA $\overset{\neq}{\Rightarrow}$ QIND,

stronger adversaries yield stronger encryption schemes

Our Contributions

✓ Formal definition of Quantum Semantic Security

✓ Equivalence to Quantum Indistinguishability

✓ Extension to CPA and CCA1 scenarios

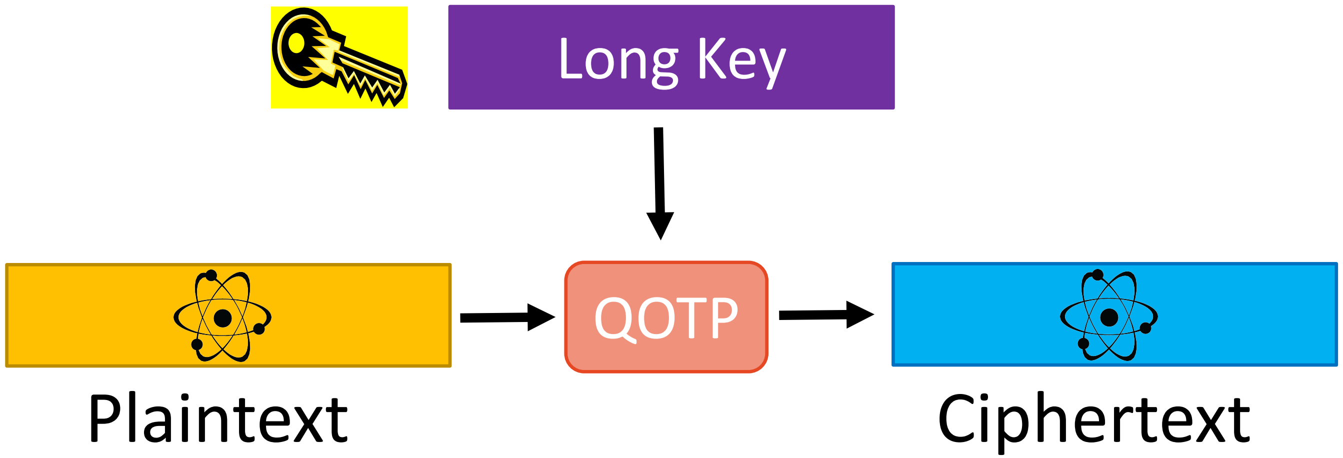
4. Construction of IND-CCA1 Quantum Secret-Key Encryption from One-Way Functions
5. Construction of Quantum Public-Key Encryption from One-Way Trapdoor Permutations

Quantum Secret-Key Encryption

Goal: build CCA1-secure quantum secret-key encryption

Ingredients:

quantum one-time pad (QOTP)



Not even CPA secure, scheme is not randomized!

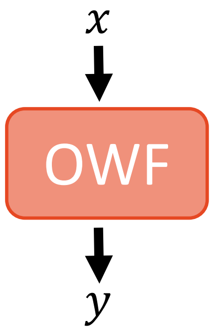
Quantum Secret-Key Encryption

Goal: build CCA1-secure quantum secret-key encryption

Ingredients:

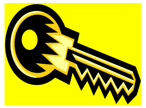
quantum one-time pad (QOTP)

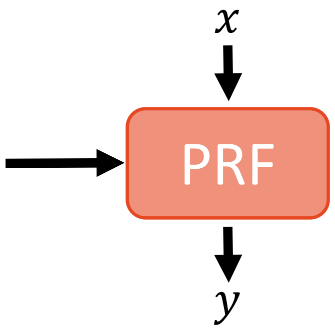
quantum-secure one-way function (OWF)



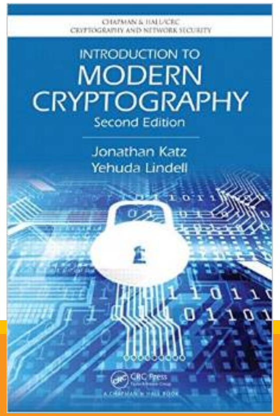
$f: x \mapsto y$ easy to compute, but hard to invert even for quantum adversaries, e.g. lattice-problems, …

Theorem: One-Way Function $\Longrightarrow$ Pseudo-Random Function





$\{f_k: x \mapsto y\}_k$ is indistinguishable from random function if key k is unknown



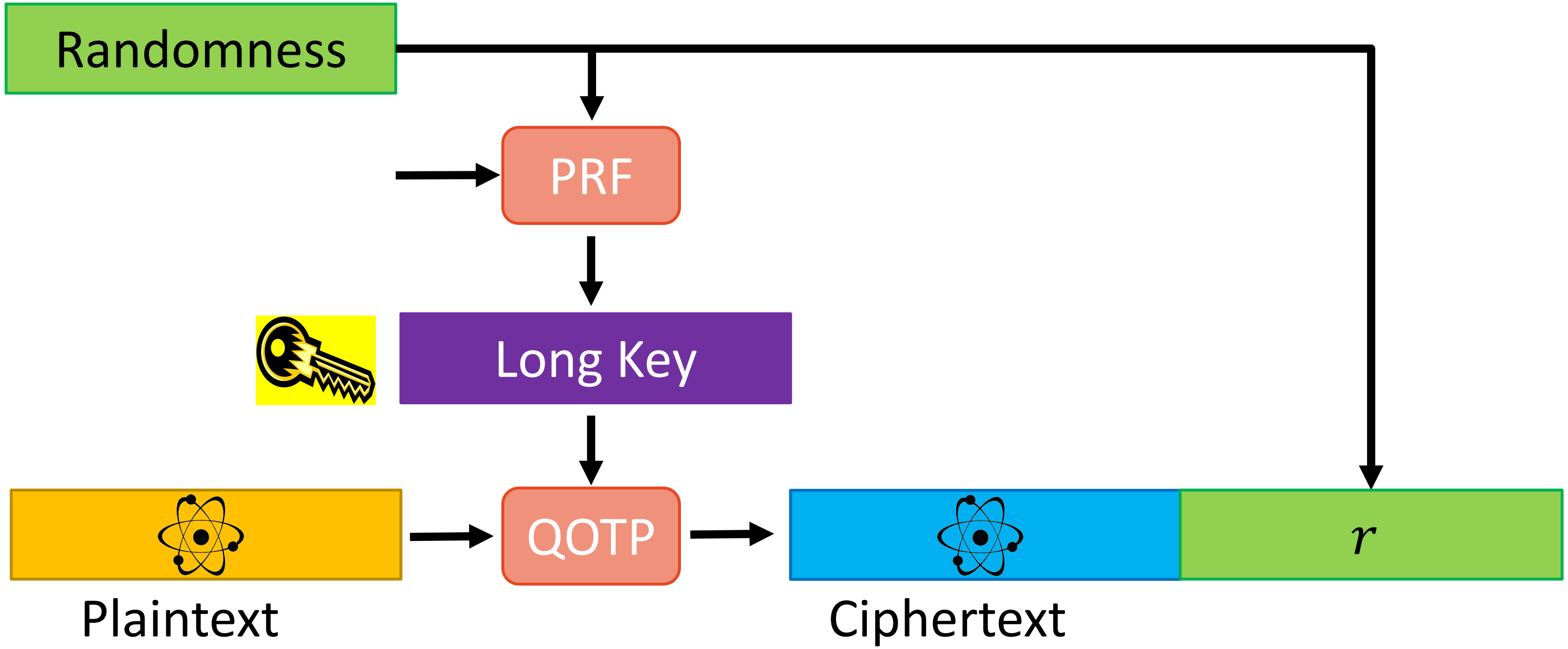
Quantum Secret-Key Encryption

Goal: build CCA1-secure quantum secret-key encryption

Ingredients:

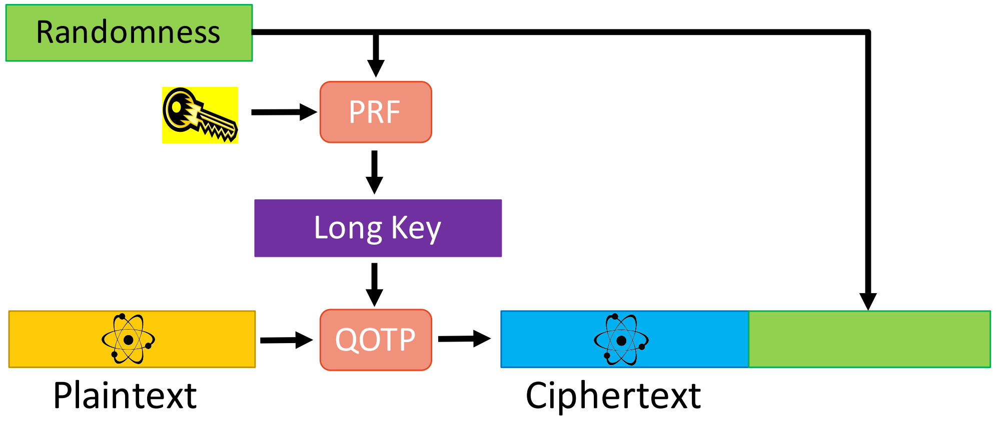
quantum one-time pad (QOTP)

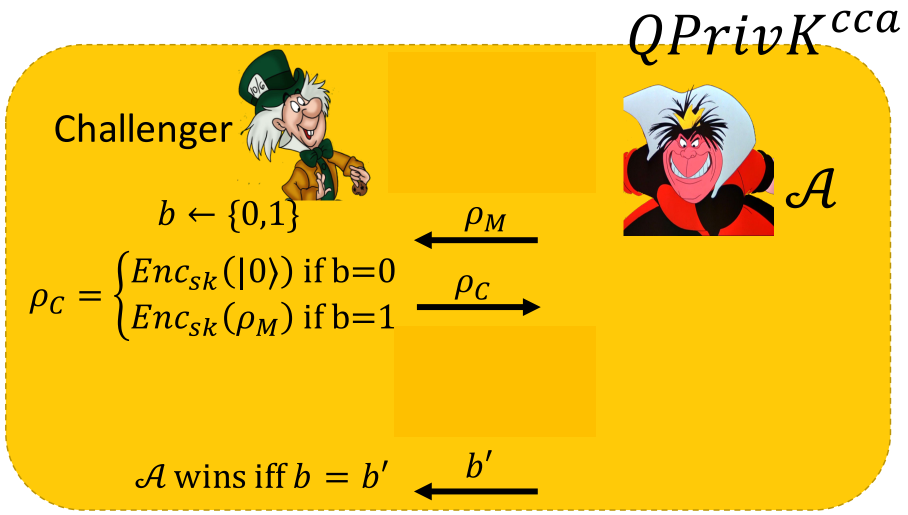
quantum-secure one-way function (OWF) $\Longrightarrow$ PRF



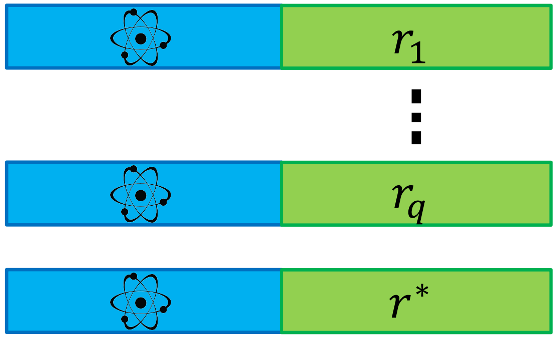
Classical version: [Goldreich Goldwasser Micali 85]

Intuition of CCA1 security





1. Replace pseudo-random function with totally random function
2. Encryption queries result in polynomially many ciphertexts with different randomness:
3. With overwhelming probability the randomness of the challenge ciphertext will be different from previous r's.



Our Contributions

✓Formal definition of Quantum Semantic Security

✓Equivalence to Quantum Indistinguishability

✓Extension to CPA and CCA1 scenarios

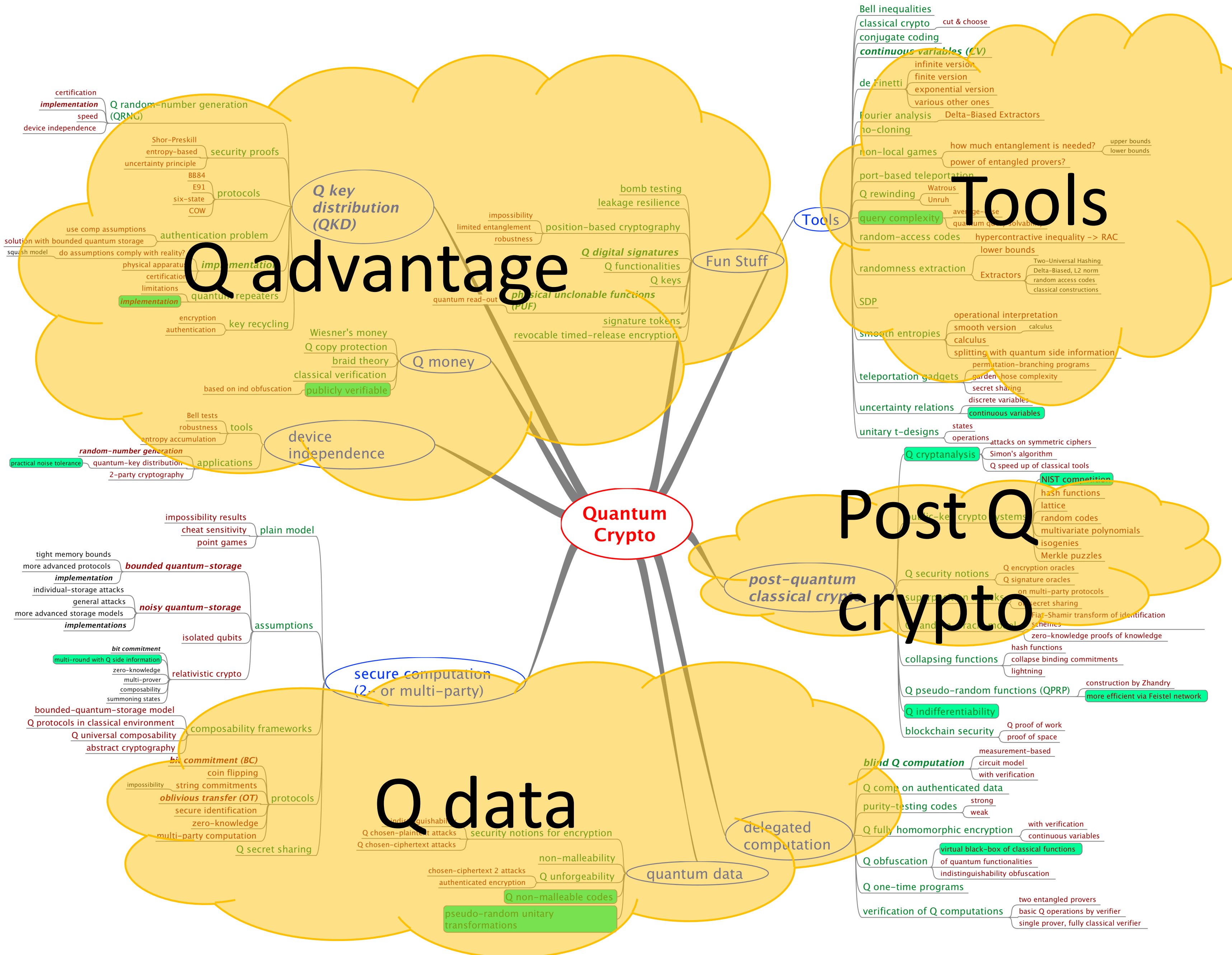
✓Construction of IND-CCA1 Quantum Secret-Key Encryption from One-Way Functions

5. Construction of Quantum Public-Key Encryption from One-Way Trapdoor Permutations

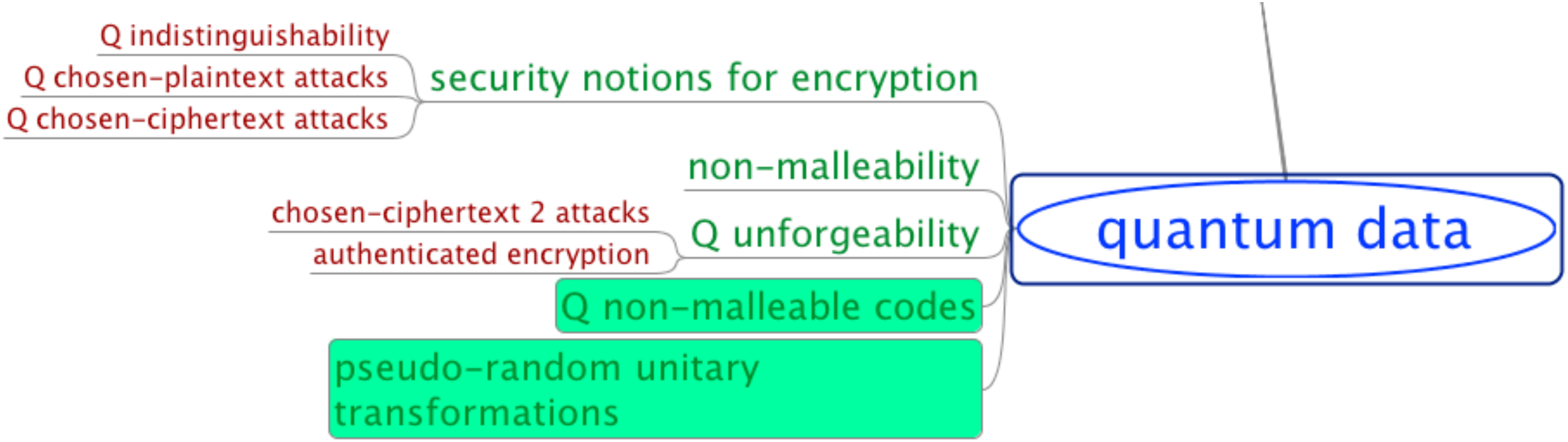
MindMap

- ***experiments***
- Selection of open questions

- Fork me on github!

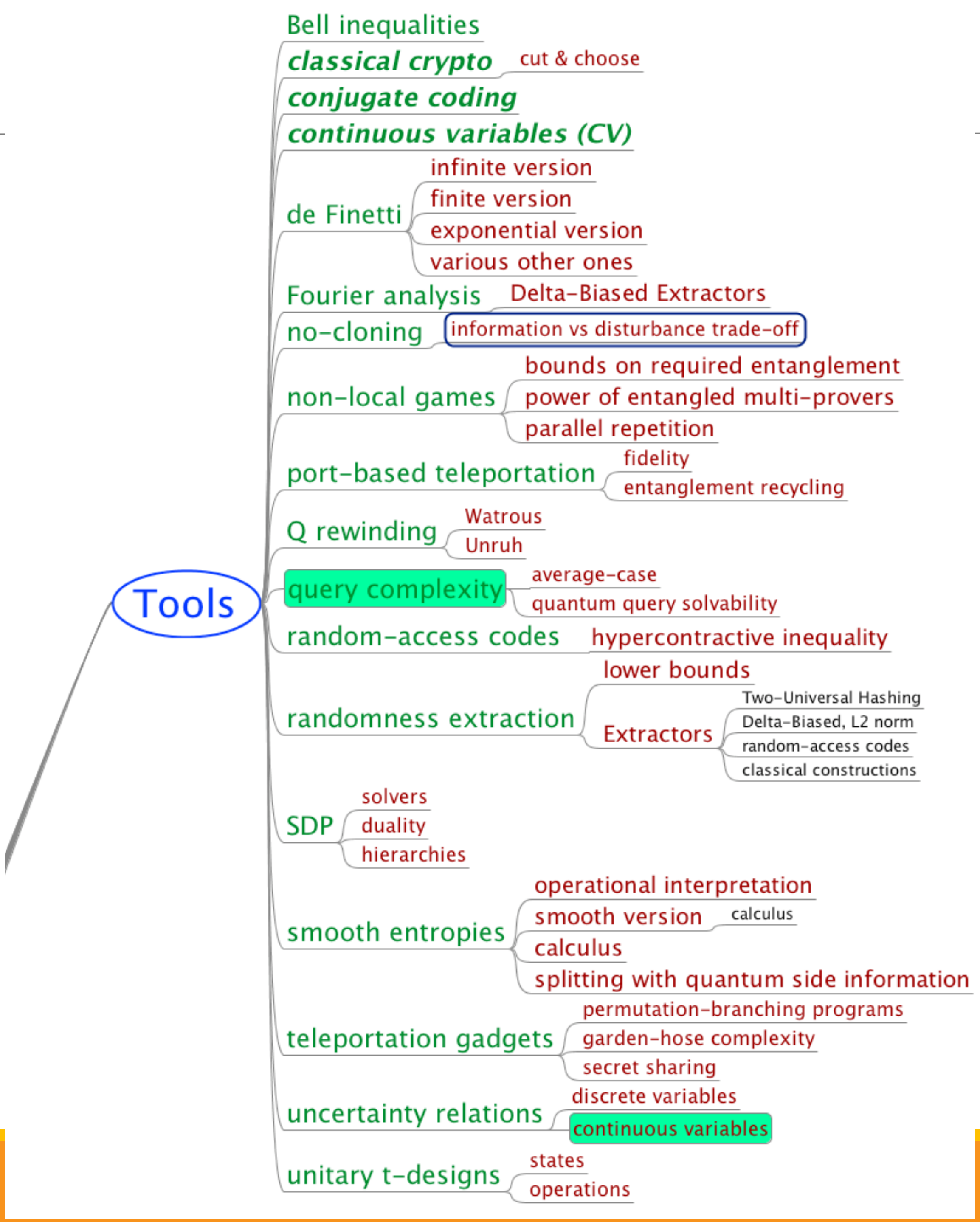


[https://github.com/cschaffner/QCryptoMindmap]



quantum data
security notions for encryption
Q indistinguishability
Q chosen-plaintext attacks
Q chosen-ciphertext attacks
non-malleability
Q unforgeability
chosen-ciphertext 2 attacks
authenticated encryption
Q non-malleable codes
pseudo-random unitary transformations

Tools


Tools
Bell inequalities
classical crypto
cut & choose
conjugate coding
continuous variables (CV)
de Finetti
infinite version
finite version
exponential version
various other ones
Fourier analysis
Delta-Biased Extractors
no-cloning
information vs disturbance trade-off
non-local games
bounds on required entanglement
power of entangled multi-provers
parallel repetition
port-based teleportation
fidelity
entanglement recycling
Q rewinding
Watrous
Unruh
query complexity
average-case
quantum query solvability
random-access codes
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randomness extraction
lower bounds
Extractors
Two-Universal Hashing
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calculus
calculus
splitting with quantum side information
teleportation gadgets
permutation-branching programs
garden-hose complexity
secret sharing
uncertainty relations
discrete variables
continuous variables
unitary t-designs
states
operations

Open Query-Complexity Question

- Let $f: \{0,1\}^n \rightarrow \{0,1\}^n$ be a random function
- **Goal:** Given quantum oracle access to f, output a "chain of values" $x, f(x), f(f(x))$
- **Observation:** easy to do with 2 classical queries
- **Question:** Prove hardness with a single quantum query
- **More interesting:** Prove hardness with polynomially many non-adaptive quantum queries
- **Classical hardness:** straightforward
- **Partial result:** iterated hashing analyzed by Unruh in context of revocable quantum timed-released encryption

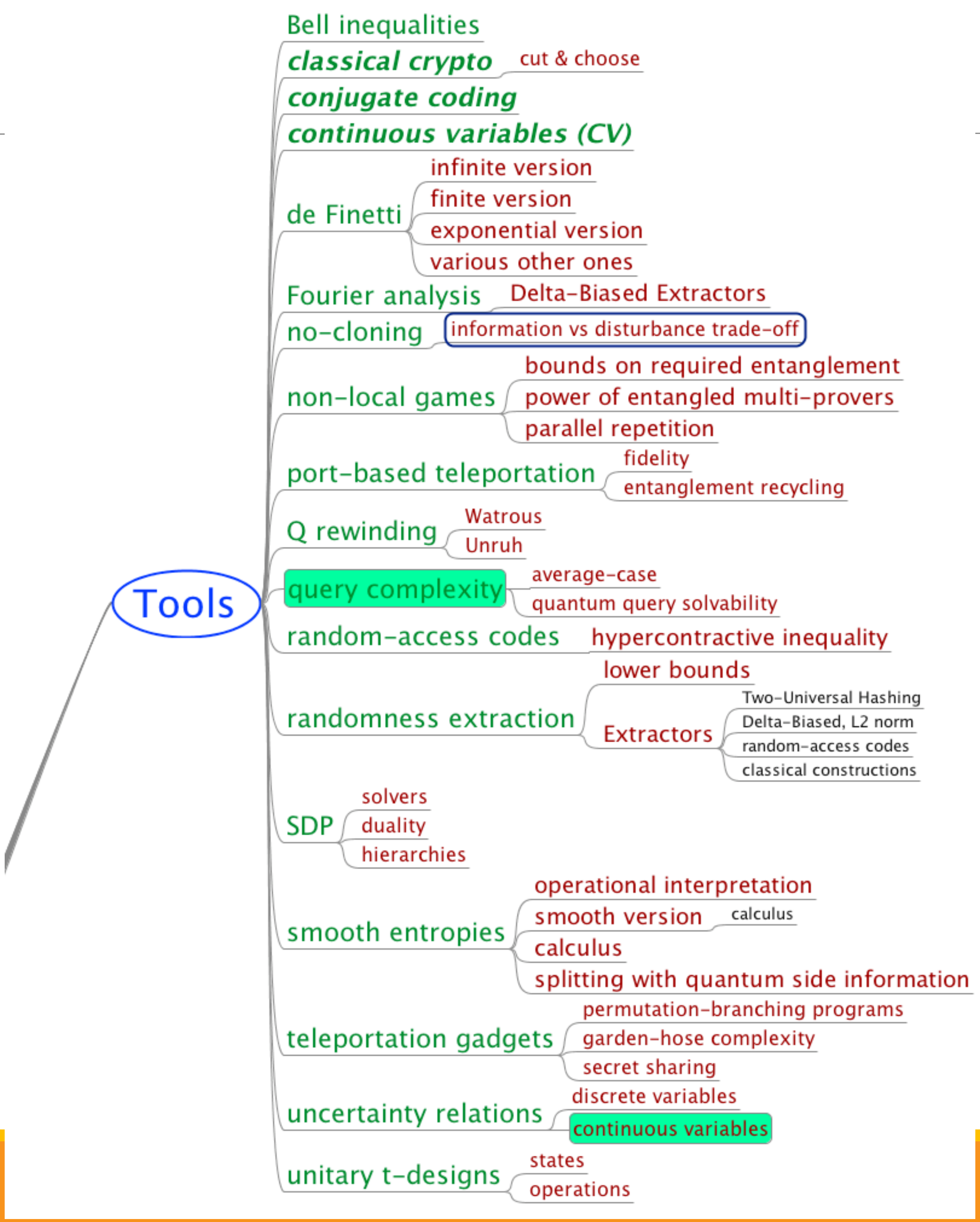
[question by Serge Fehr 17, Unruh 13]

Quantum Query Solvability

- Notion introduced by Mark Zhandry at QuICS workshop 2015: https://www.youtube.com/watch?v=kaS7OFAm-6M
- Often, quantum query-complexity bounds are given in the form: "$\Theta(g(N))$ queries are required to solve a problem with success probability 2/3 (in the worst case)"
- For crypto, it would be way more useful to have: "Given q quantum queries, the maximal success probability is $\Theta(g(q,N))$, in the average case"
- Example: Given a function $F: [N] \rightarrow \{0,1\}$, find x such that $F(x) = 1$.
- Q query-complexity answer: $\Theta\left(N^{1/2}\right)$ by (optimality of) Grover search
- But is the success probability $\Theta\left(q/N^{1/2}\right)$, $\boxed{\Theta(q^2/N)}$, or $\Theta(q^4/N^2)$?
- Matters for efficiency when choosing crypto parameters in order to get tiny security errors

[Zhandry 15]

Tools



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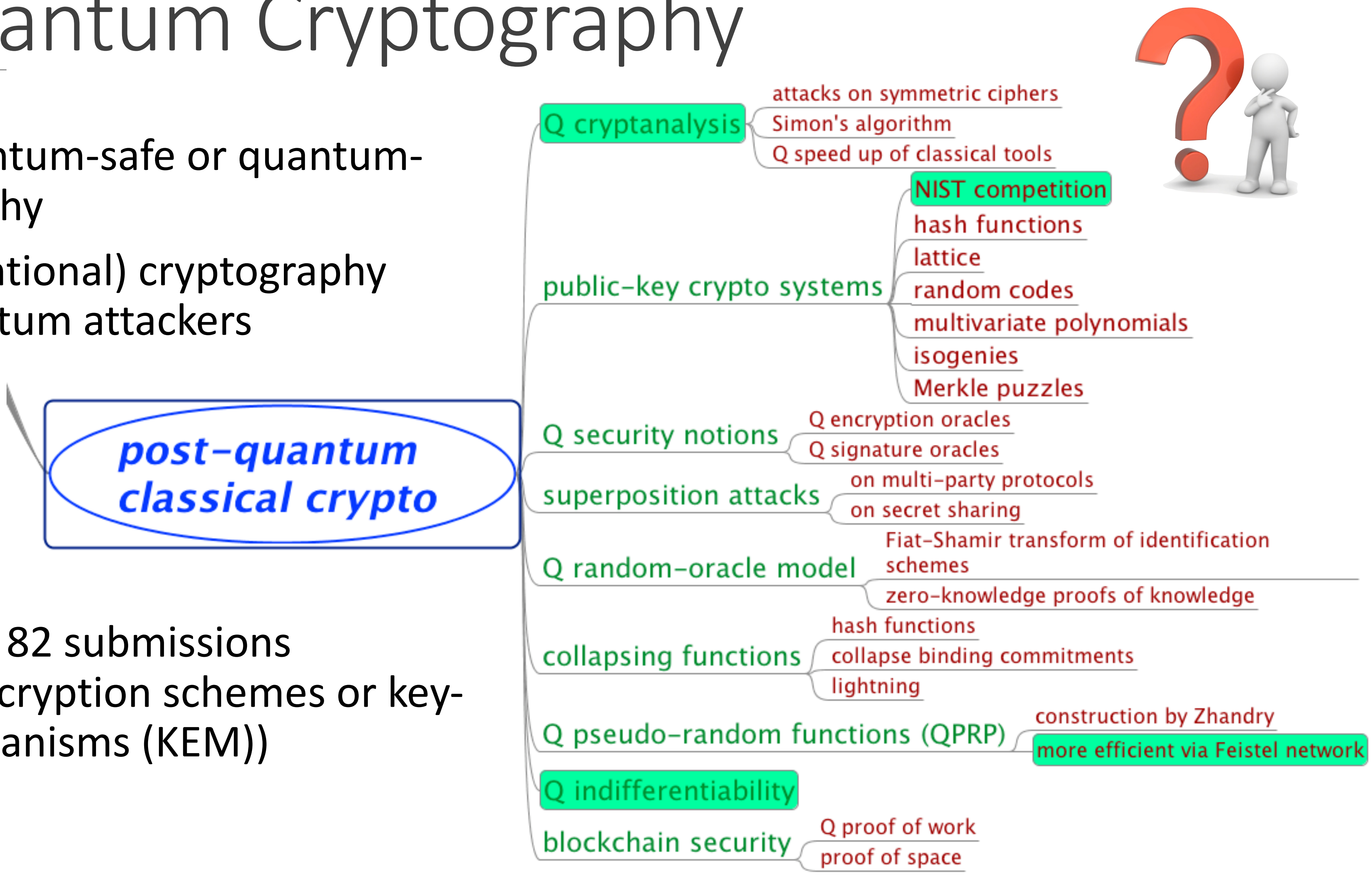
Post-Quantum Cryptography

- Also known as: quantum-safe or quantum-resistant cryptography
- Classical (i.e. conventional) cryptography secure against quantum attackers

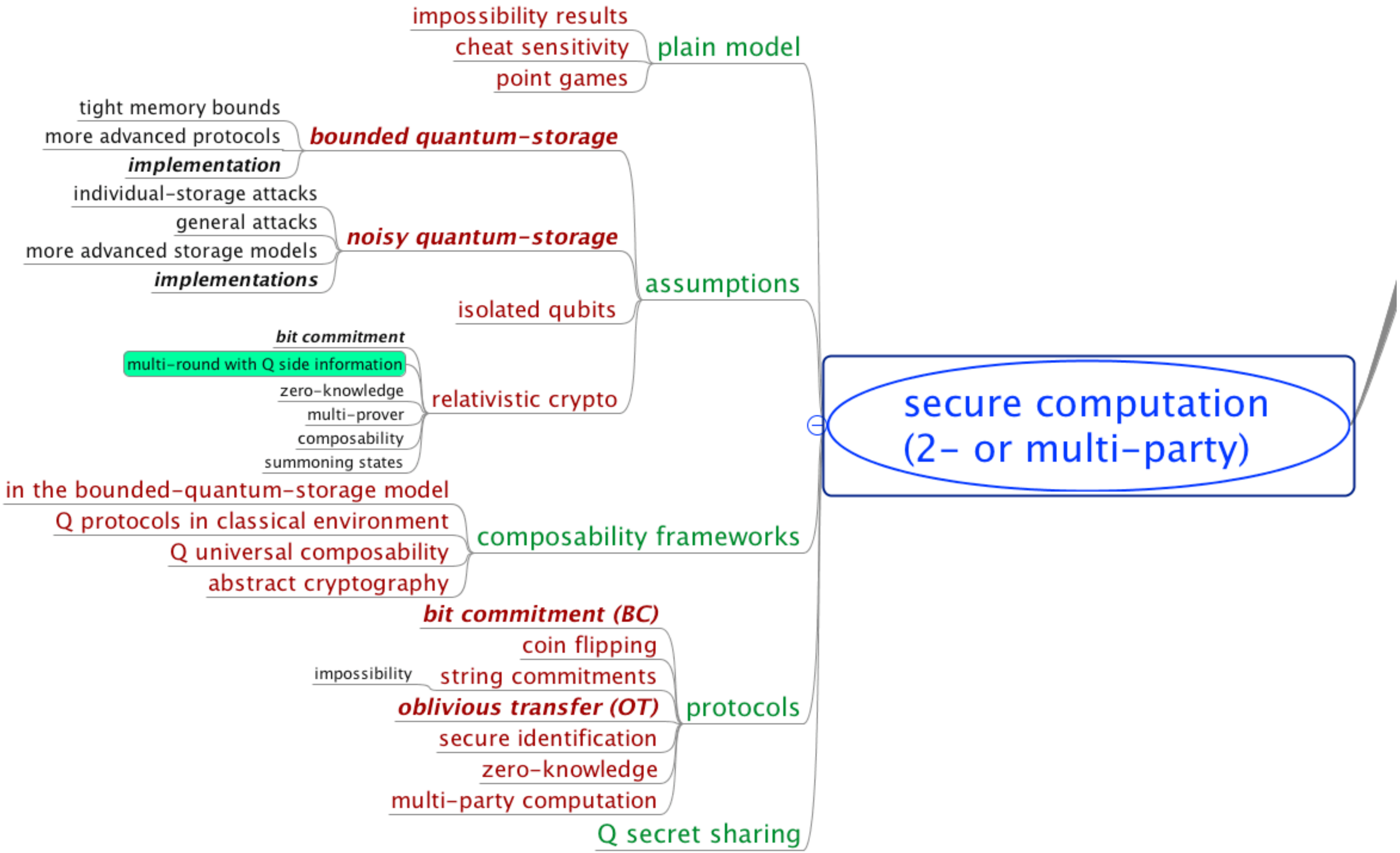


- NIST "competition": 82 submissions (23 signature, 59 encryption schemes or key-encapsulation mechanisms (KEM))

[https://csrc.nist.gov/Projects/Post-Quantum-Cryptography]

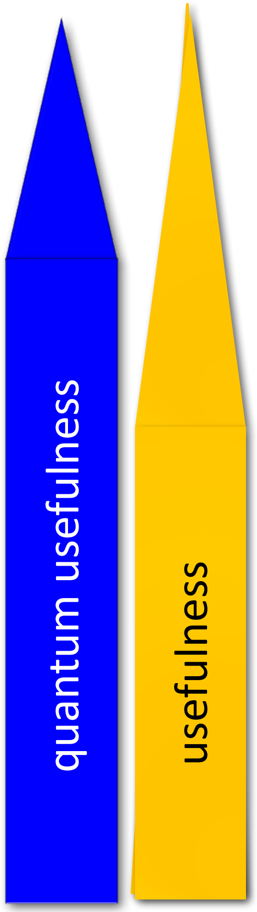
Observations from QCrypts 2014-17

- Rough classification of contributed, invited and tutorial talks
- QKD is the most developed branch of Q crypto, closest to implementation
- When looking at experimental talks: mostly QKD and (closely) related topics
- Tools and post-quantum crypto are consistently of interest
- 2-party crypto was en vogue in 2014/15, not anymore in 2016/17
- Taken over by delegated computation and authentication, started in 2016
- 2016/17: DI has made a comeback
- Long tail: lots of other topics


secure computation (2- or multi-party)
plain model
impossibility results
cheat sensitivity
point games
assumptions
bounded quantum-storage
tight memory bounds
more advanced protocols
implementation
noisy quantum-storage
individual-storage attacks
general attacks
more advanced storage models
implementations
isolated qubits
relativistic crypto
bit commitment
multi-round with Q side information
zero-knowledge
multi-prover
composability
summoning states
composability frameworks
in the bounded-quantum-storage model
Q protocols in classical environment
Q universal composability
abstract cryptography
protocols
bit commitment (BC)
coin flipping
string commitments
impossibility
oblivious transfer (OT)
secure identification
zero-knowledge
multi-party computation
Q secret sharing

Secure Two-Party Cryptography

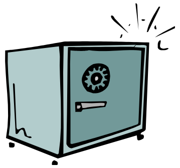
- Information-theoretic security
- No computational restrictions

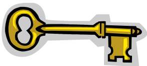


- Coin-Flipping

$\Uparrow \quad \not\Downarrow$

- Bit Commitment





$\Uparrow \quad \not\Downarrow \quad \Downarrow$

- Oblivious Transfer

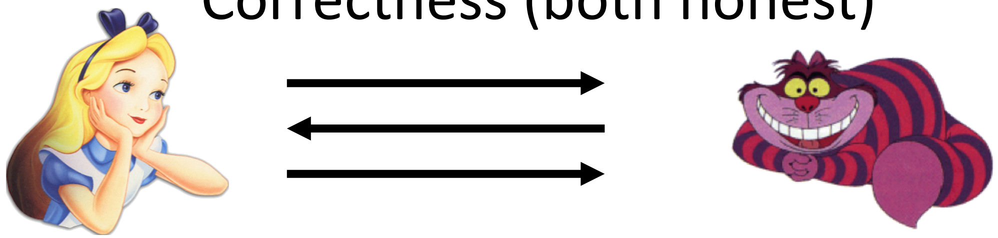
$\Uparrow \quad \Downarrow$

- 2-Party Function Evaluation

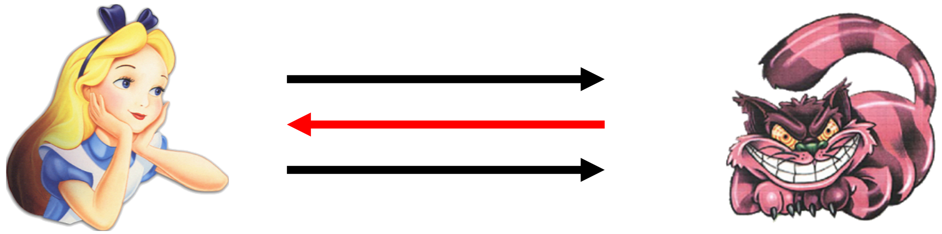
$\Updownarrow$

- Multi-Party Computation (with dishonest majority)

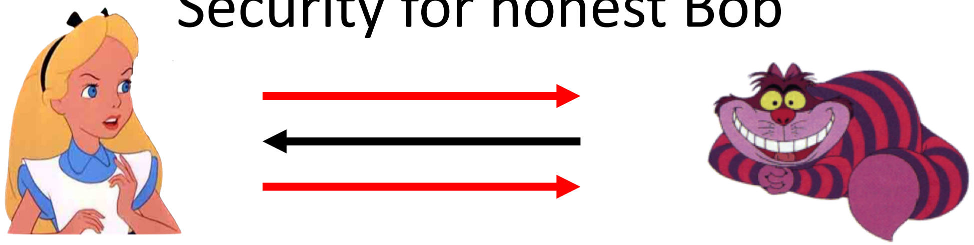
Correctness (both honest)



Security for honest Alice



Security for honest Bob



[Blum 83, Kilian 88]

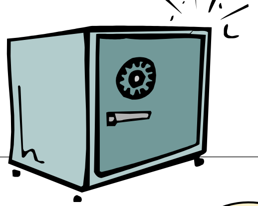
Coin Flipping (CF)

- Strong CF: No dishonest player can bias the outcome
- Classically: a cheater can always obtain his desired outcome with prob 1
- Quantum: [Kitaev 03] lower bounds the bias by $\frac{1}{\sqrt{2}} - \frac{1}{2} \approx 0.2$

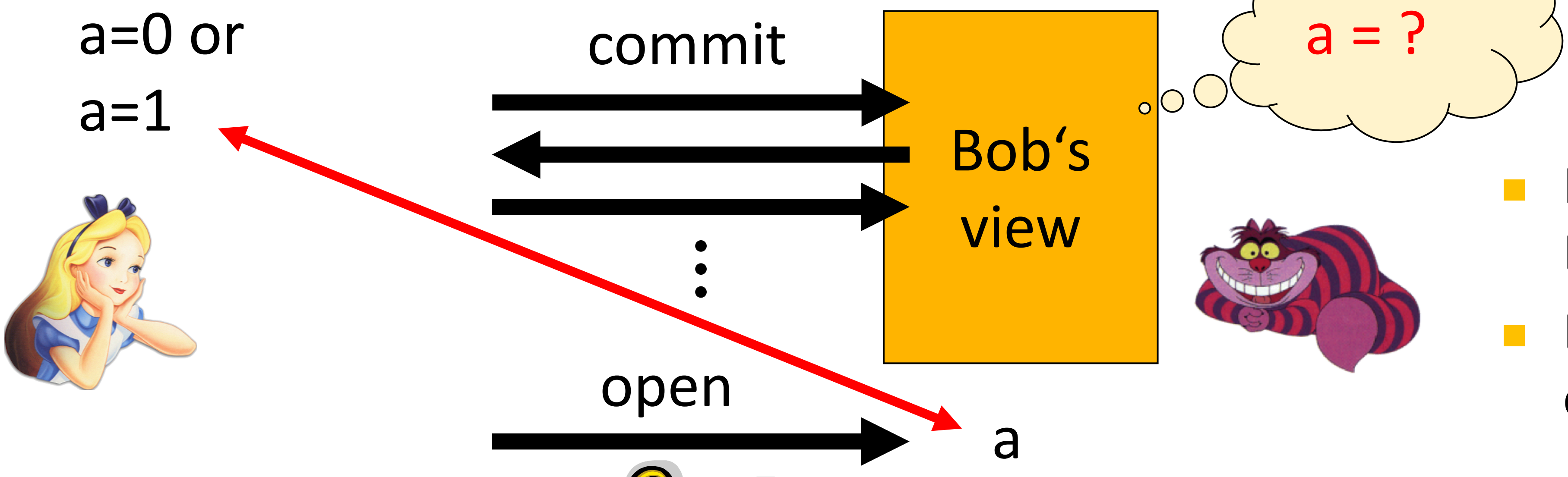
  [Chailloux Kerenidis 09] give optimal quantum protocol for strong CF with this bias

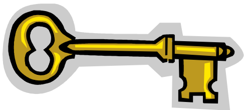
- Weak CF ("who has to do the dishes?"): Alice wants heads, Bob wants tails
- [Mochon 07] uses Kitaev's formalism of point games to give a quantum protocol for weak CF with arbitrarily small bias $\varepsilon > 0$
- [Aharonov Chailloux Ganz Kerenidis Magnin 14] reduce the proof complexity from 80 to 50 pages… explicit protocol?

Bit Commitment (BC)



- Two-phase (reactive) protocol:





- Hiding: even dishonest Bob does not learn a
- Binding: dishonest Alice cannot change her mind

- Classically: impossible
- Quantum: believed to be possible in the early 90s
- shown impossible by [Mayers 97, LoChau 97] by a beautiful argument (purification and Uhlmann's theorem)
- [Chailloux Kerenidis 11] show that in any quantum BC protocol, one player can cheat with prob 0.739. They also give an optimal protocol achieving this bound. Crypto application?

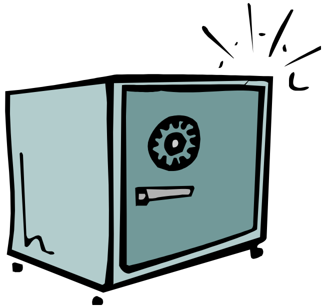
[Brassard Crepeau Jozsa Langlois: A quantum BC scheme provably unbreakable by both parties, FOCS 93]

Bit Commitment ⇒ Strong Coin Flipping

a=0 or

a=1

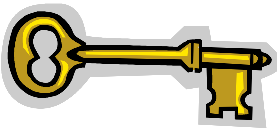
a



b

b=0 or

b=1



a = b

a ≠ b

[Blum 83]

Oblivious Transfer (OT)

Example One: A means for transmitting two messages either but not both of which may be received.

- 1-out-of-2 Oblivious Transfer: $s_0, s_1 \rightarrow$ OT $\leftarrow c$; OT $\rightarrow s_c$
- Rabin OT: (secure erasure) $s \rightarrow$ ROT $\rightarrow s \,/\, \perp$
- Dishonest Alice does not learn choice bit
- Dishonest Bob can only learn one of the two messages
- These OT variants are information-theoretically equivalent (homework! 😉)
- OT is symmetric [Wolf Wullschleger at EuroCrypt 2006, only 10 pages long]
- 1-2 OT ⇒ BC:

commit

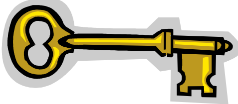
$a = 0$ or $a = 1$

$r_1, a \oplus r_1 \rightarrow$ OT $\leftarrow c_1 \in_R \{0,1\}$; OT $\rightarrow s_{c_1}$

$r_2, a \oplus r_2 \rightarrow$ OT $\leftarrow c_2 \in_R \{0,1\}$; OT $\rightarrow s_{c_2}$

$r_3, a \oplus r_3 \rightarrow$ OT $\leftarrow c_3 \in_R \{0,1\}$; OT $\rightarrow s_{c_3}$

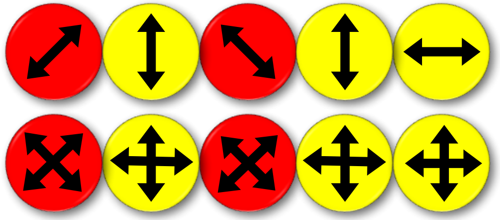
open

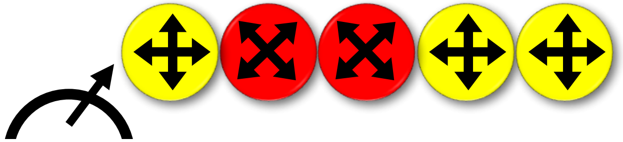


$a, r_1, r_2, \ldots$

Quantum Protocol for Oblivious Transfer

$s_0 \rightarrow$ OT $\leftarrow c$
$s_1 \rightarrow$ OT $\rightarrow s_c$





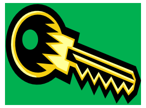
0 1 1 1 0

0 0 1 1 0

$I_c = \{3,4,5\}, I_{1-c} = \{1,2\}$

I_0, I_1

f_0, f_1



$k_0 = f_0(01)$

$k_1 = f_1(110)$

$t_0 = s_0 \oplus k_0$
$t_1 = s_1 \oplus k_1$

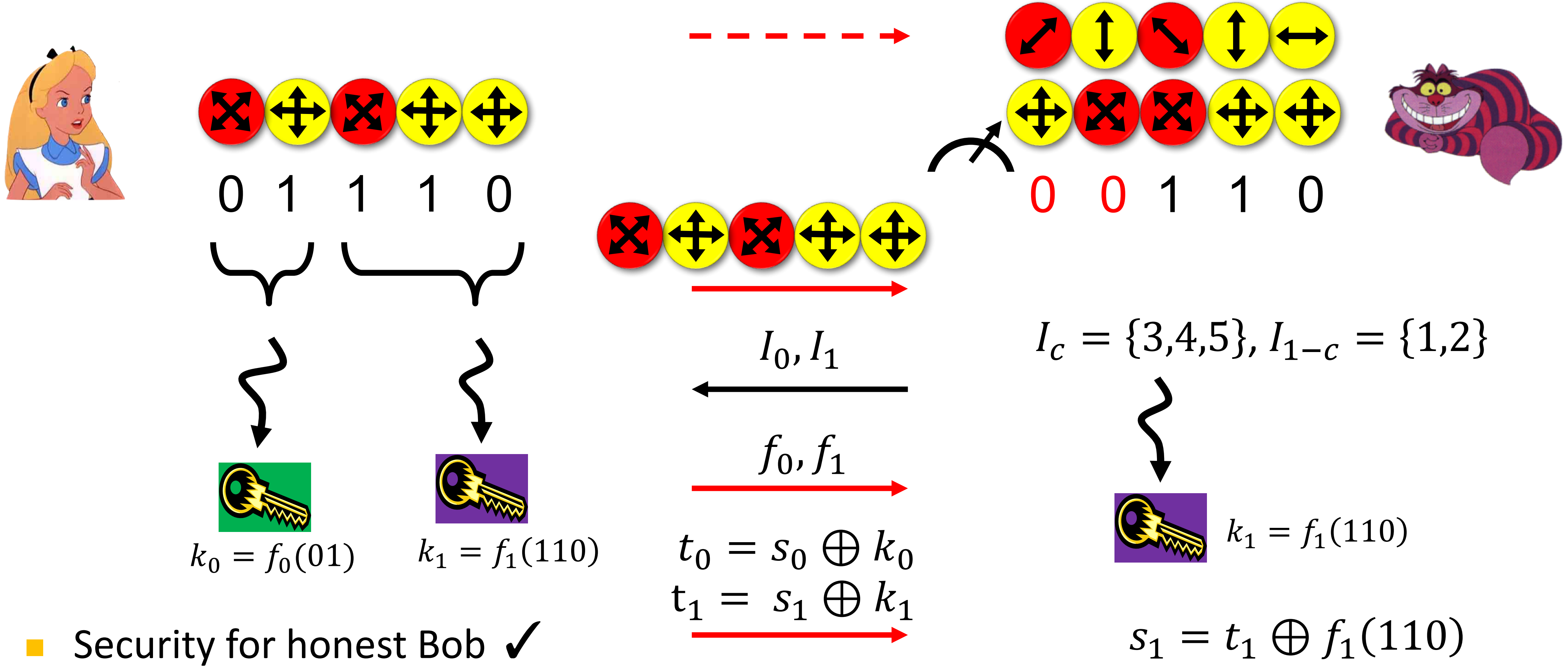
$k_1 = f_1(110)$

$s_1 = t_1 \oplus f_1(110)$

Correctness ✓

Quantum Protocol for Oblivious Transfer

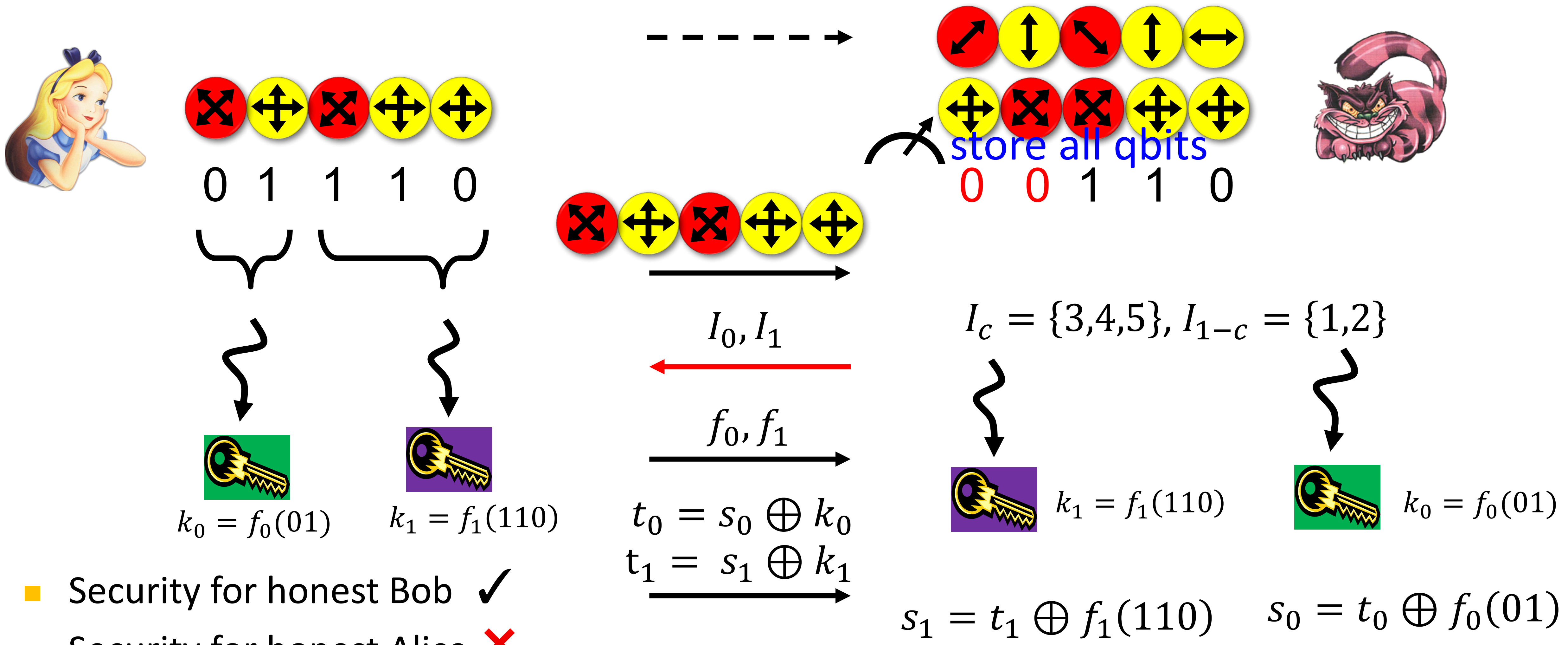
$s_0 \rightarrow$ OT $\leftarrow c$
$s_1 \rightarrow$ OT $\rightarrow s_c$



- Security for honest Bob ✓

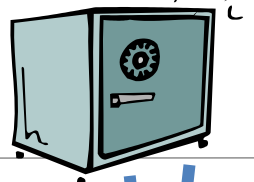
[Wiesner 61, Bennett Brassard Crepeau Skubiszewska 91]

Quantum Protocol for Oblivious Transfer

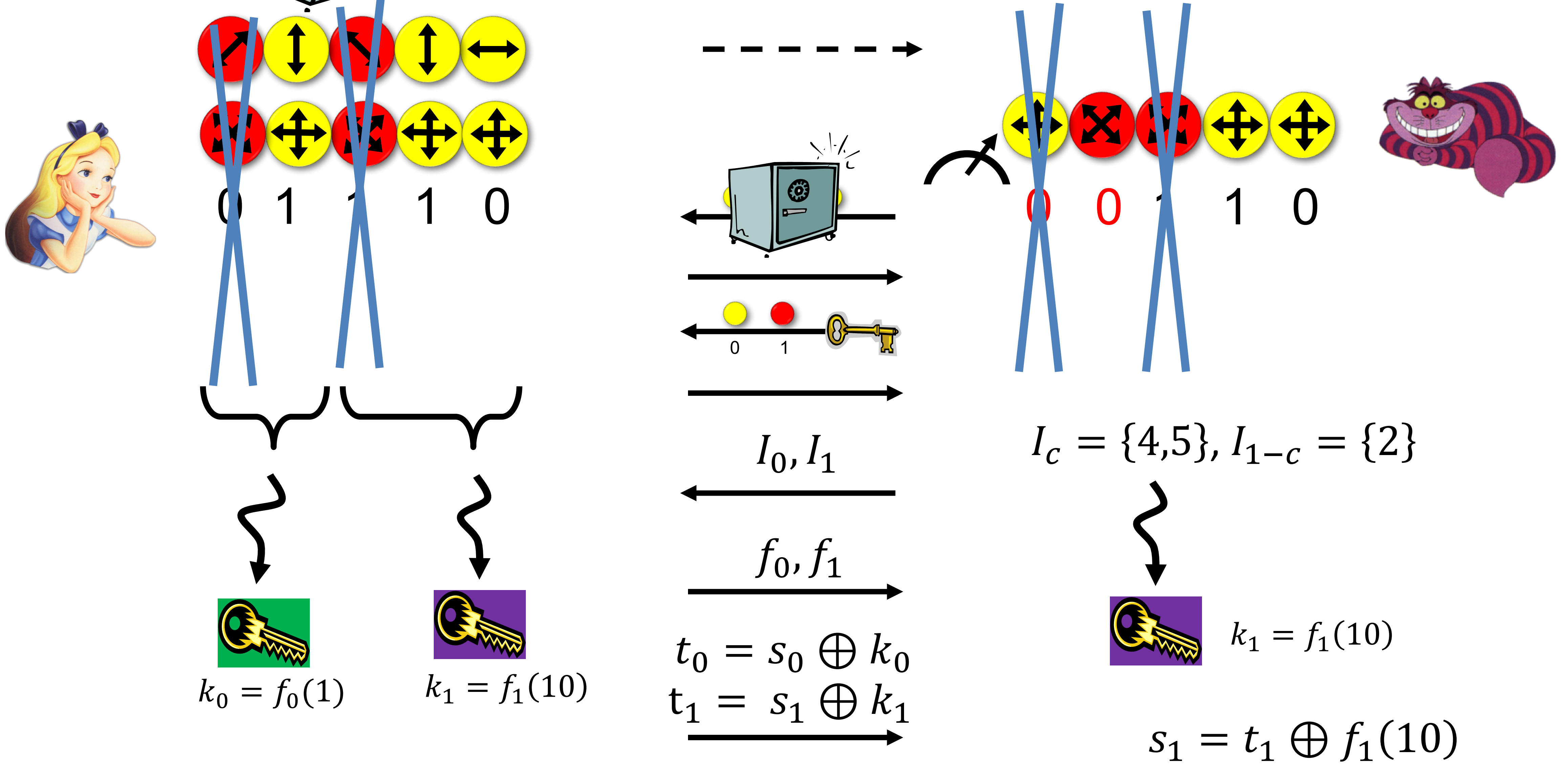


- Security for honest Bob ✓
- Security for honest Alice ✗

[Wiesner 61, Bennett Brassard Crepeau Skubiszewska 91]



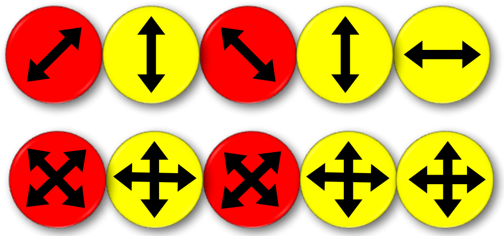
BC ⇒ Oblivious Transfer



[Bennett Brassard Crepeau Skubiszewska 91, Damgaard Fehr Lunemann Salvail Schaffner 09, Unruh 10]

Limited Quantum Storage

$s_0 \rightarrow$ OT $\leftarrow c$
$s_1 \rightarrow$ OT $\rightarrow s_c$



0 1 1 1 0

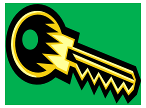
store all qbits

wait 1 sec

I_0, I_1

$I_c = \{3,4,5\}, I_{1-c} = \{1,2\}$

f_0, f_1



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$t_0 = s_0 \oplus k_0$
$t_1 = s_1 \oplus k_1$

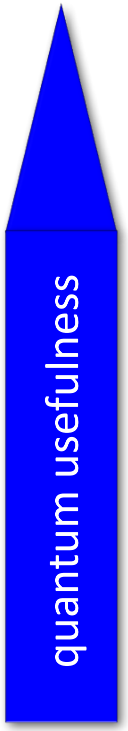
$k_1 = f_1(110)$

$s_1 = t_1 \oplus f_1(110)$

[Damgaard Fehr Salvail Schaffner 05, Wehner Schaffner Terhal 09]

Summary of Quantum Two-Party Crypto

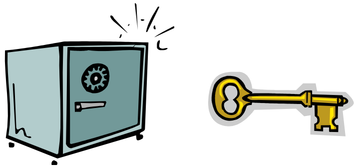
- Information-theoretic security
- No computational restrictions



- Coin-Flipping

$\Uparrow$ $\not\Downarrow$

- Bit Commitment

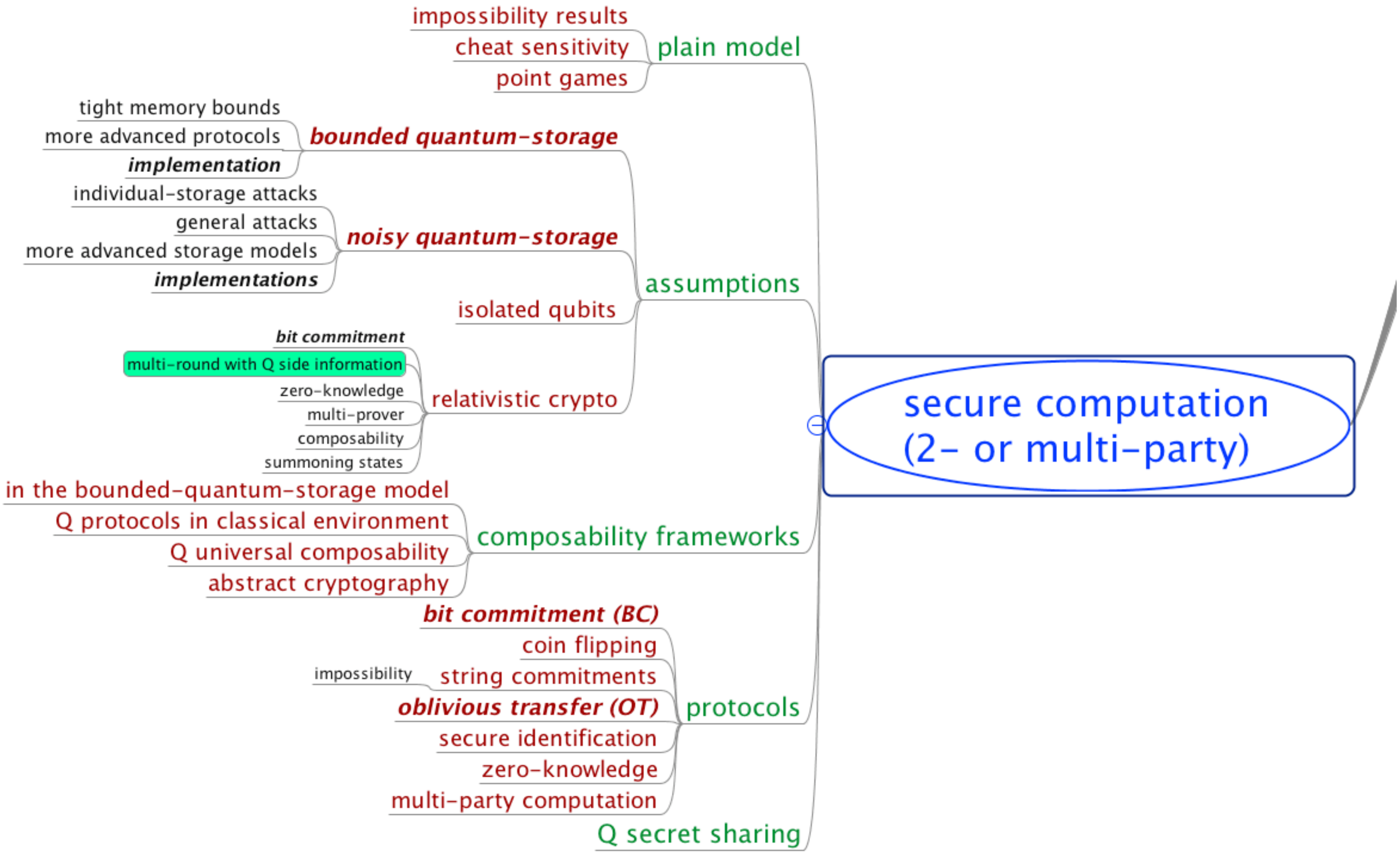


$\Uparrow$ $\not\Downarrow$ $\Downarrow$

- Oblivious Transfer

$\Uparrow$ $\Downarrow$

- 2-Party Function Evaluation



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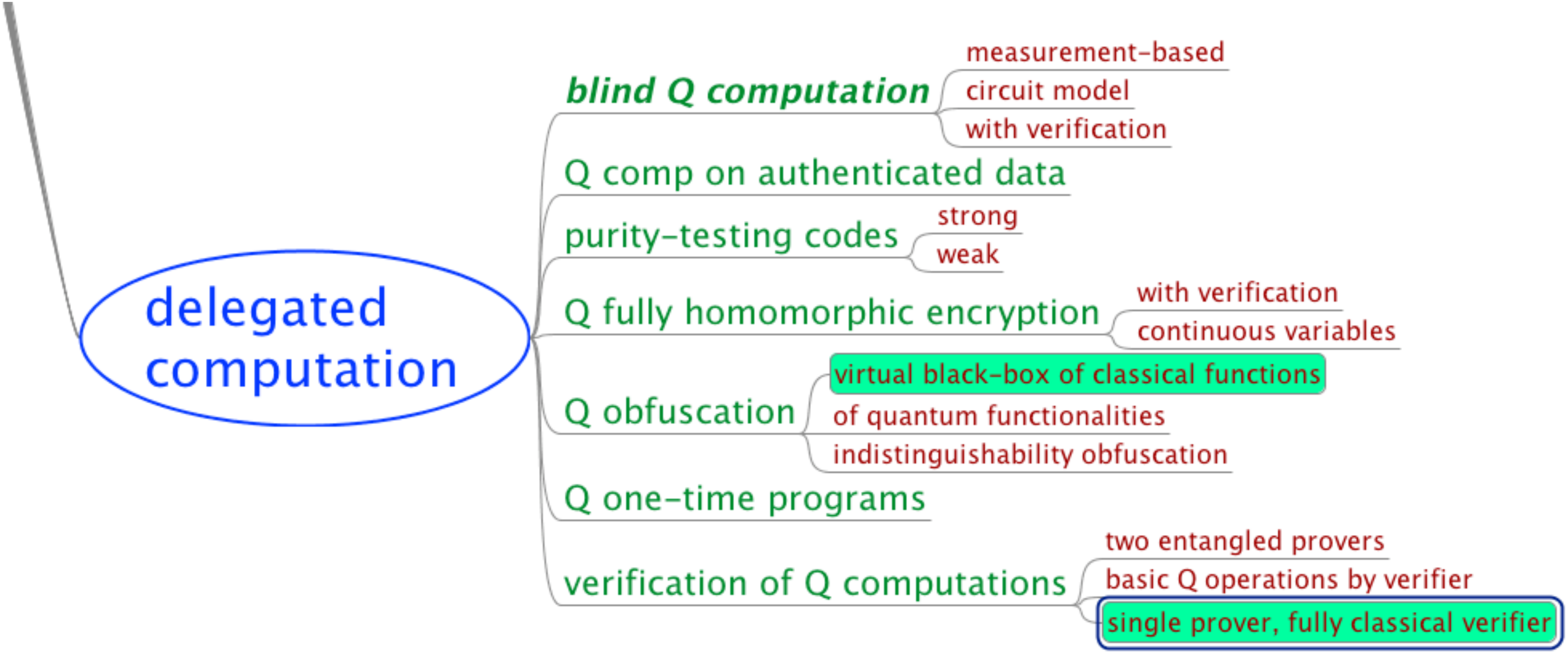
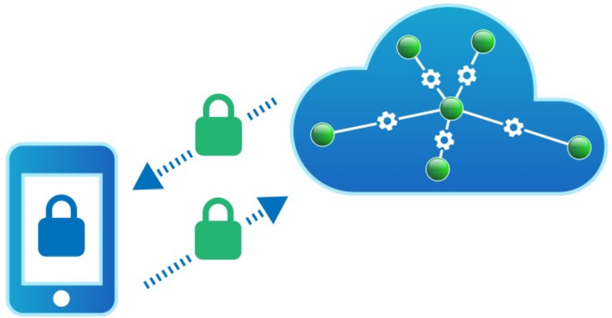
Q secret sharing

Delegated Q Computation



Delegated Computation



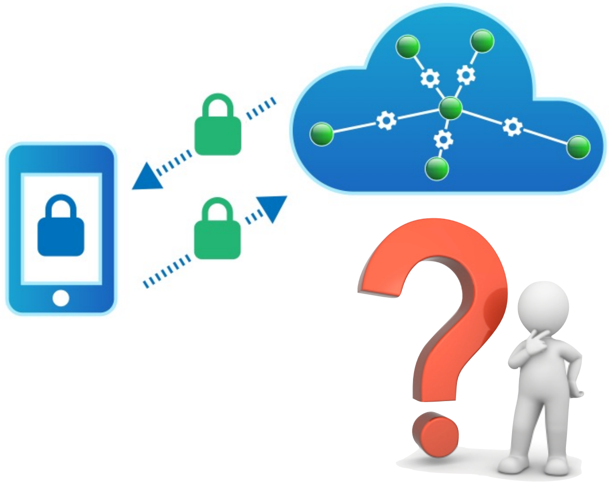
- QCloud Inc. promises to perform a BQP computation for you.
- How can you securely delegate your quantum computation to an untrusted quantum prover while maintaining privacy and/or integrity?
- Various parameters:
  1. Quantum capabilities of verifier: state preparation, measurements, q operations
  2. Type of security: blindness (server does not learn input), integrity (client is sure the correct computation has been carried out)
  3. Amount of interaction: single round (fully homomorphic encryption) or multiple rounds
  4. Number of servers: single-server, unbounded / computationally bounded or multiple entangled but non-communicating servers

[Childs 05, …. see e.g. Broadbent 17 or Fitzsimons 16 for overviews and references]

Image: Tremani / TU Delft

Classical Verification of Q Computation

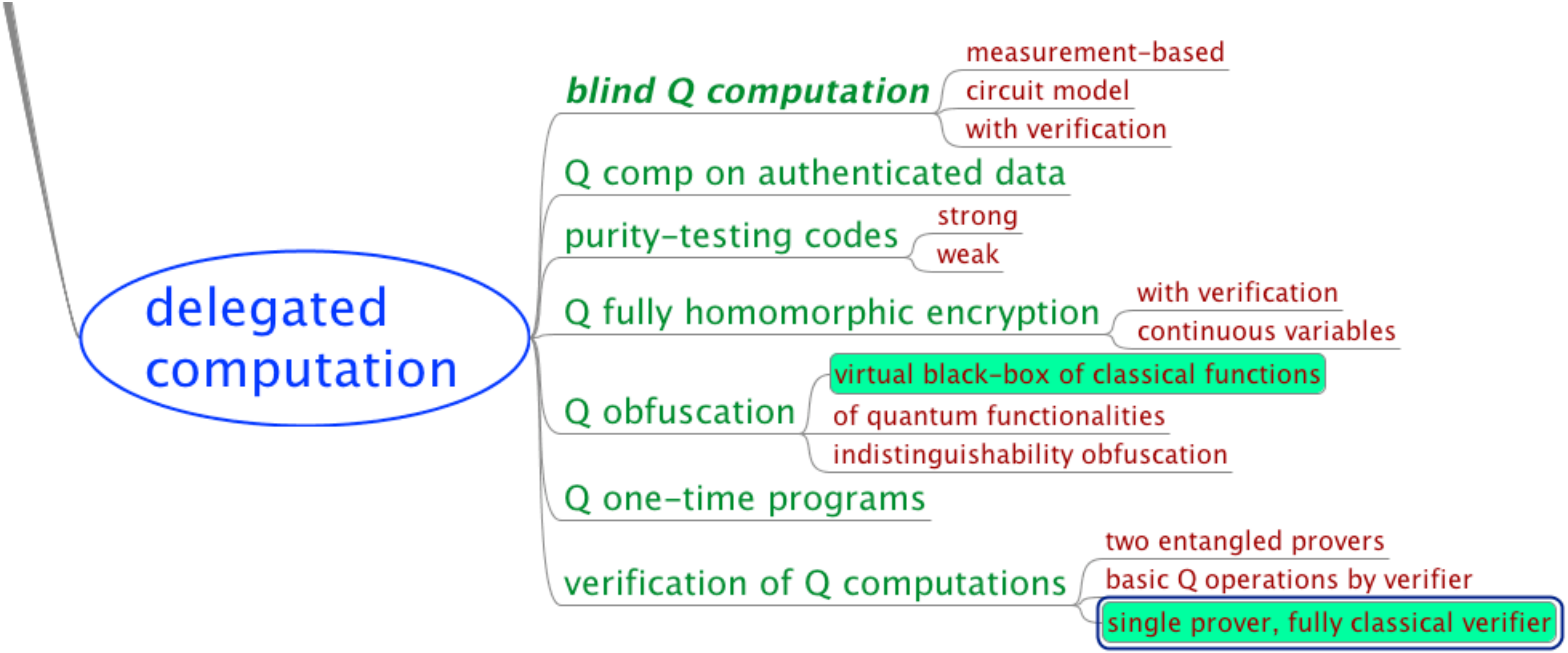
- QCloud Inc. promises you to perform a BQP computation
- How can a **purely classical verifier** be convinced that this computation actually was performed?



- Partial solutions:
  1. Using interactive protocols with quantum communication between prover and verifier, this task can be accomplished, using a certain minimum quantum ability of the verifier. [Fitzsimons Kashefi 17, Broadbent 17, AlagicDulekSpeelmanSchaffner17]
  2. Using two entangled, but non-communicating provers, verification can be accomplished using rigidity results [ReichardtUngerVazirani12]. Recently made way more practical by [ColadangeloGriloJefferyVidick17]
- Indications that information-theoretical blind computation is impossible [AaronsonCojocaruGheorghiuKashefi17]

[see Broadbent 17 or Fitzsimons 16 for overview and more complete references]

Delegated Q Computation

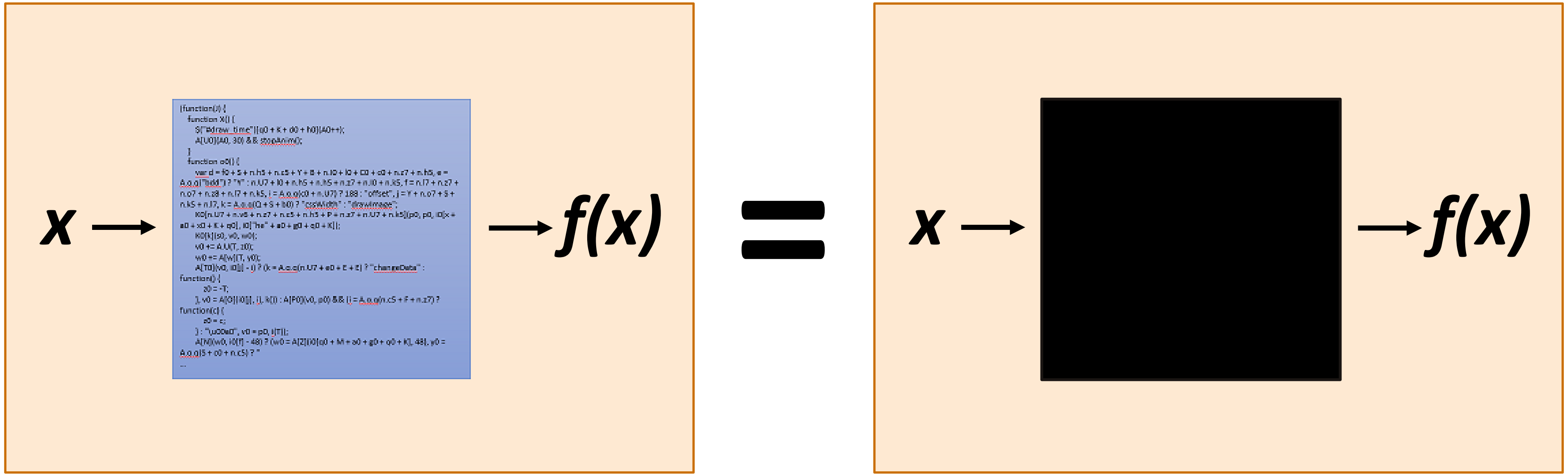


Black-Box Obfuscation

Idea: an obfuscator is an algorithm which rewrites programs, such that

1. efficiency is preserved;
2. input-output functionality is preserved;
3. output programs are hard to understand: *"If something is efficiently learnable from reading the code, then it is also efficiently learnable purely from input-output behavior."*

"black-box obfuscation"



Classical Obfuscation

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"black-box obfuscation"

Formal:

A black-box obfuscator *O* is an algorithm which maps circuits *C* to circuits *O(C)* such that:

1. efficiency-preserving: $|\mathcal{O}(C)| \leq \mathrm{poly}(|C|)$
2. functionality-preserving: $f_{\mathcal{O}(C)} = f_C$
3. virtual black-box: for every poly-time *A* there exists a poly-time *S* such that

$$\left|\Pr[\underbrace{\mathcal{A}(\mathcal{O}(C)) = 1}_{\text{learn something by reading circuit}}] - \Pr[\underbrace{\mathcal{S}^{f_C}(\bar{1}) = 1}_{\text{learn same thing from input-output}}]\right| \leq \mathrm{negl}(|C|)\,.$$

Classical Obfuscation

Why care? Lots of applications:

1. **Protecting IP:** obfuscate before publishing (already done, but ad-hoc);
2. **Secure patching:** revealing what is being patched exposes unpatched machines;
3. **Public-key crypto:** private-key encryption → public-key encryption:

$$k_{\text{decrypt}} := k \qquad k_{\text{encrypt}} := \mathcal{O}(\text{Enc}_k)\,.$$

4. **One-way functions:** choose delta-function circuit, make obfuscator's coins part of input;
5. **FHE:** encryption → fully-homomorphic encryption:

$$k_{\text{eval}} := \mathcal{O}(\text{Enc}_k \circ U \circ \text{Dec}_k)$$

universal circuit

"top of the crypto scheme hierarchy"

Bad news: classical black-box obfuscation is impossible [Barak et al '01].

Other definitions? "Computational indistinguishability" (first schemes proposed in 2013);

Quantum Obfuscation

A quantum obfuscator *O* is a (quantum) algorithm which rewrites quantum circuits, and is:

1. efficiency-preserving: $|\mathcal{O}(C)| \leq \mathrm{poly}(|C|)$
2. functionality-preserving: $\|U_C - U_{\mathcal{O}(C)}\| \leq \mathrm{negl}(|C|)$

quantum polynomial-time algorithm

3. virtual black-box: for every QPT A there exists a QPT S such that

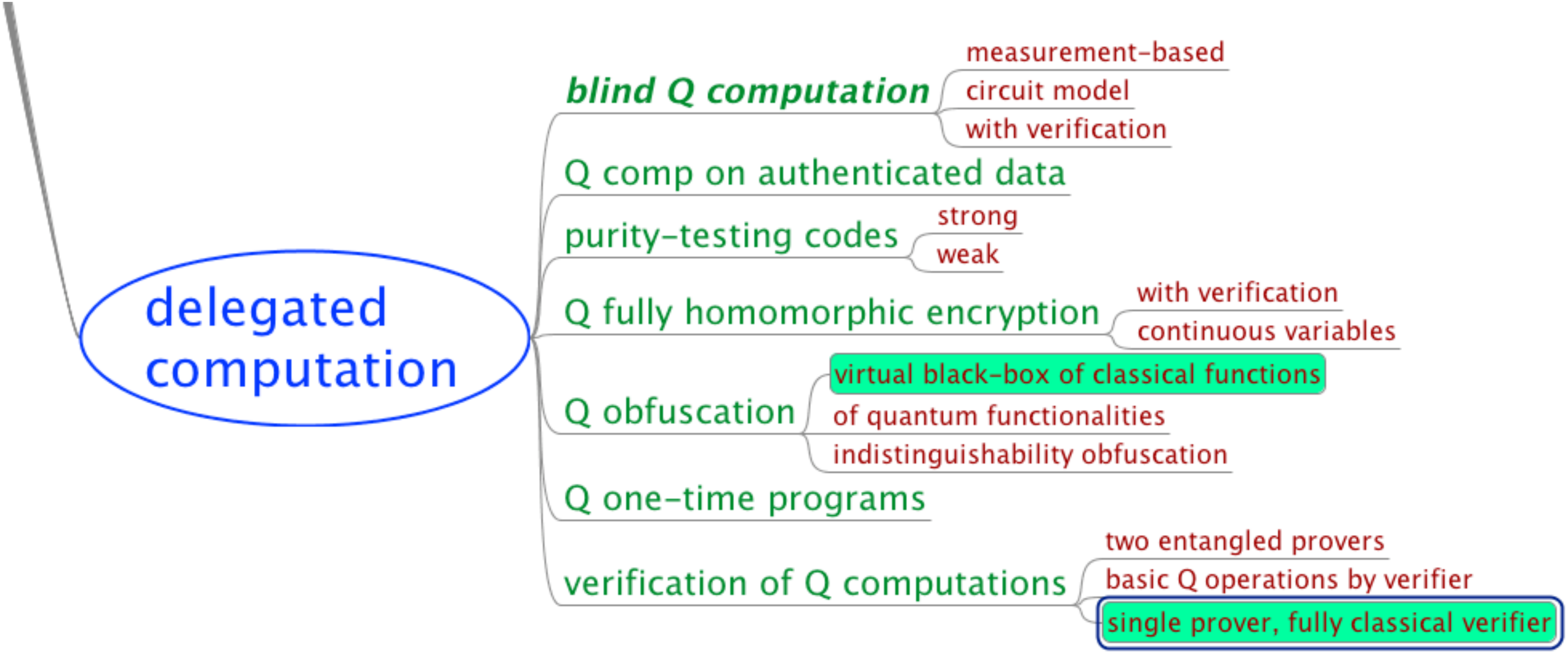
$$|\Pr[\mathcal{A}(\mathcal{O}(C)) = 1] - \Pr[\mathcal{S}^{U_C}(\bar{1}) = 1]| \leq \mathrm{negl}(|C|)\,.$$

Obfuscation	Input	Output	Adversary	Possibility?
Black-box	Quantum circuit	Quantum circuit	QPT	Impossible
Black-box	Quantum circuit	Quantum state (reusable)	QPT	Impossible
Black-box	Quantum circuit	Quantum state (uncloneable)	QPT	Open
Statistical I.O	Quantum circuit	Quantum state	QPT	Impossible
Computational I.O	Quantum circuit	Quantum state	QPT	Open

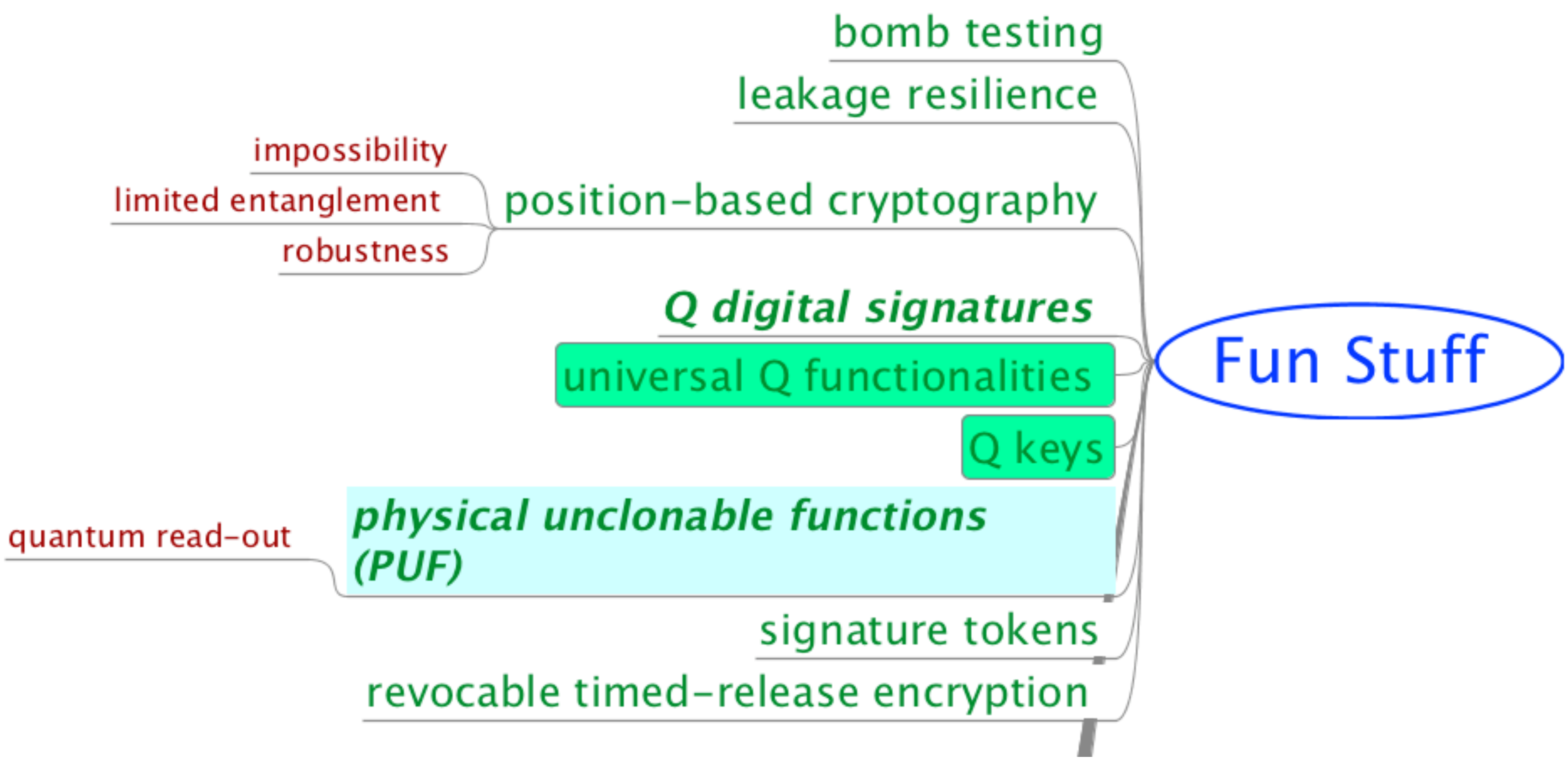
1. construct a black-box quantum obfuscator (that outputs states that cannot be reused);
2. construct a computational indistinguishability quantum obfuscator (that outputs circuits);

[Alagic Fefferman 16, slide by Gorjan Alagic, thanks a lot!]

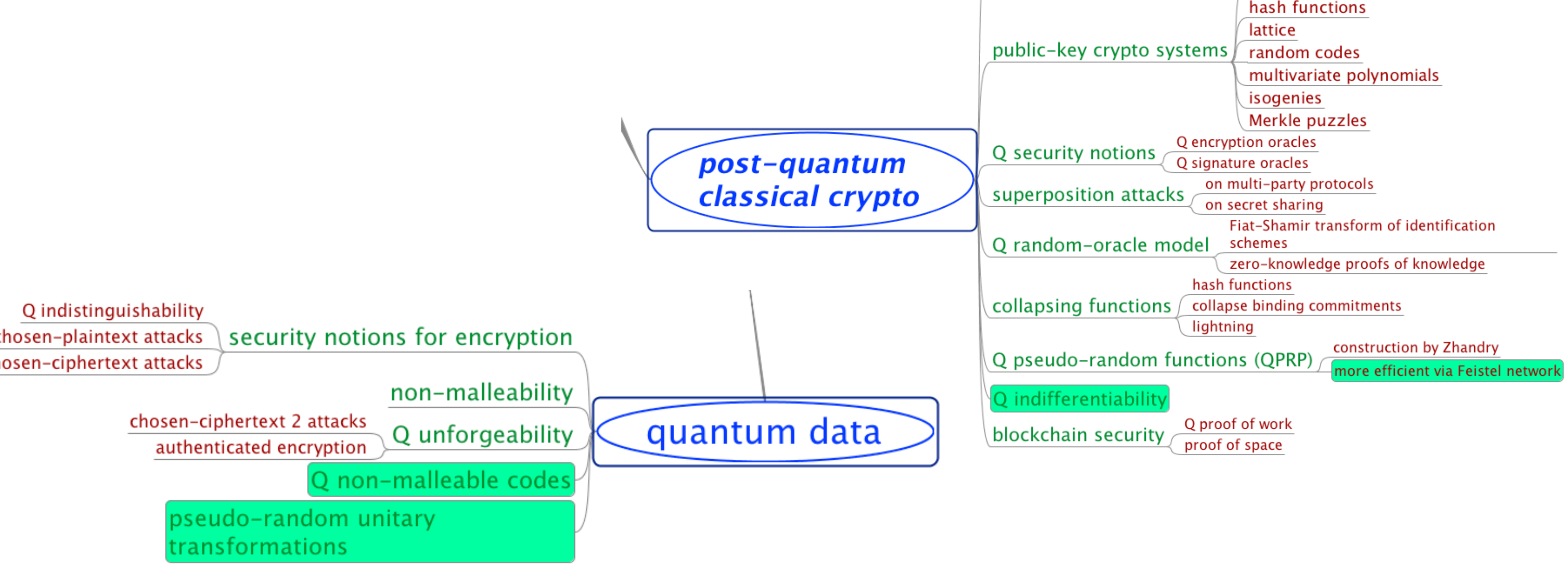
Delegated Q Computation



More Fun Stuff

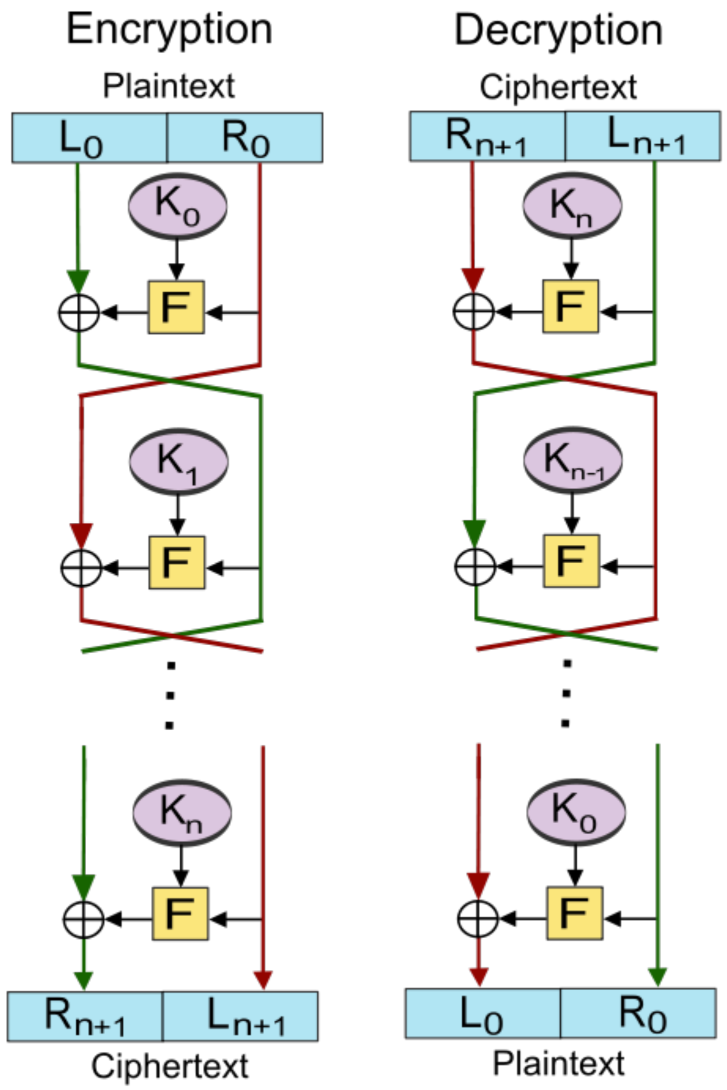


Pseudorandom Operations



[https://csrc.nist.gov/Projects/Post-Quantum-Cryptography]

Pseudorandom Permutation from Function



- Feistel network
- If F is a (pseudo)random function, the 3-round Feistel function H_3 is a pseudo-random permutation.
- Question: Show that 4-random Feistel H_4 is a quantum-secure pseudo-random permutation

For any QPT A, we want

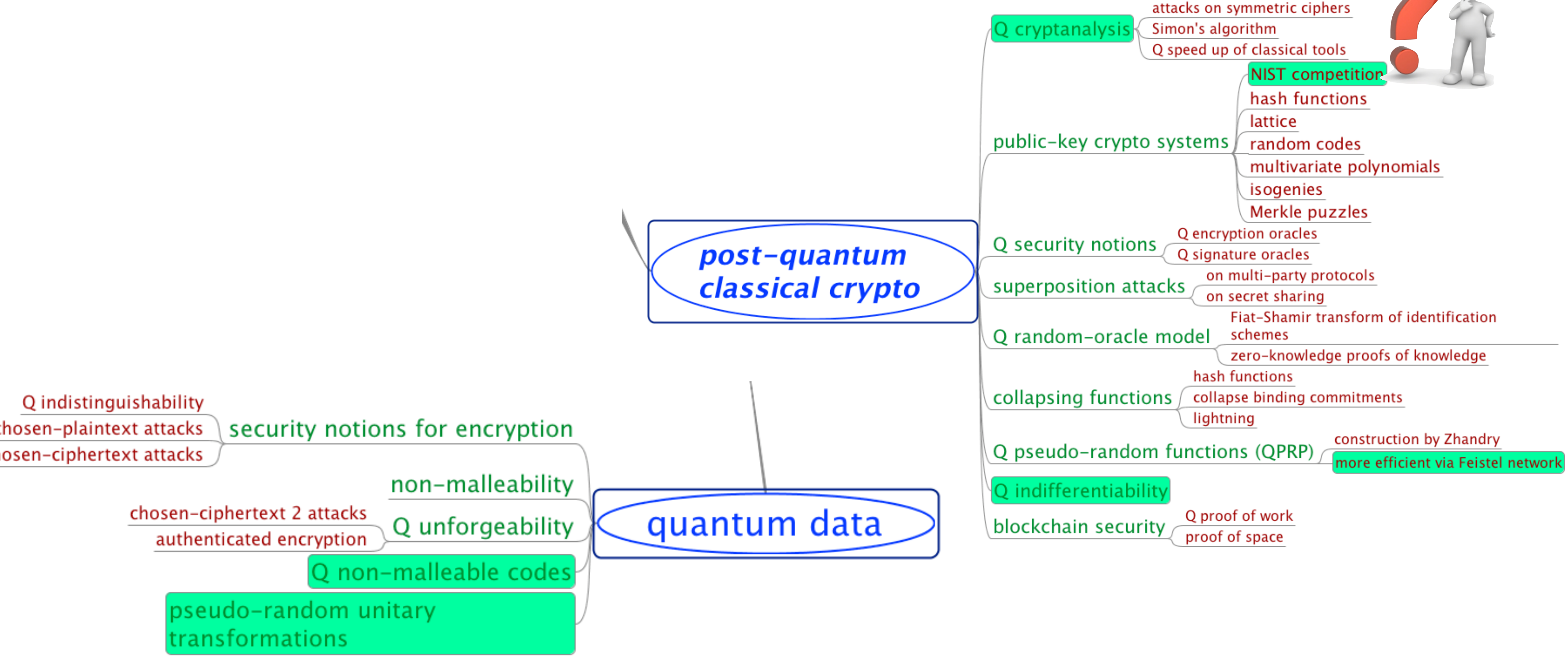
$$|\Pr[A^{|H_4>,|H_4^{-1}>}(1^n) = 1] - \Pr[A^{|rnd>,|rnd^{-1}>}(1^n) = 1]| < negl(n$$

- Partial result: Quantum attack based Simon's algorithm can distinguish 3-round Feistel H_3 from random function.

- Quantum pseudo-random unitaries?

[Kuwakado Morii 10, Ji Liu Song 17]

Pseudorandom Operations



[https://csrc.nist.gov/Projects/Post-Quantum-Cryptography]

Thank you!

https://github.com/cschaffner/QCryptoMindmap

http://arxiv.org/abs/1510.06120
In Designs, Codes and Cryptography 2016

- Thanks to all friends and colleagues that contributed to quantum cryptography and to this presentation.

Questions