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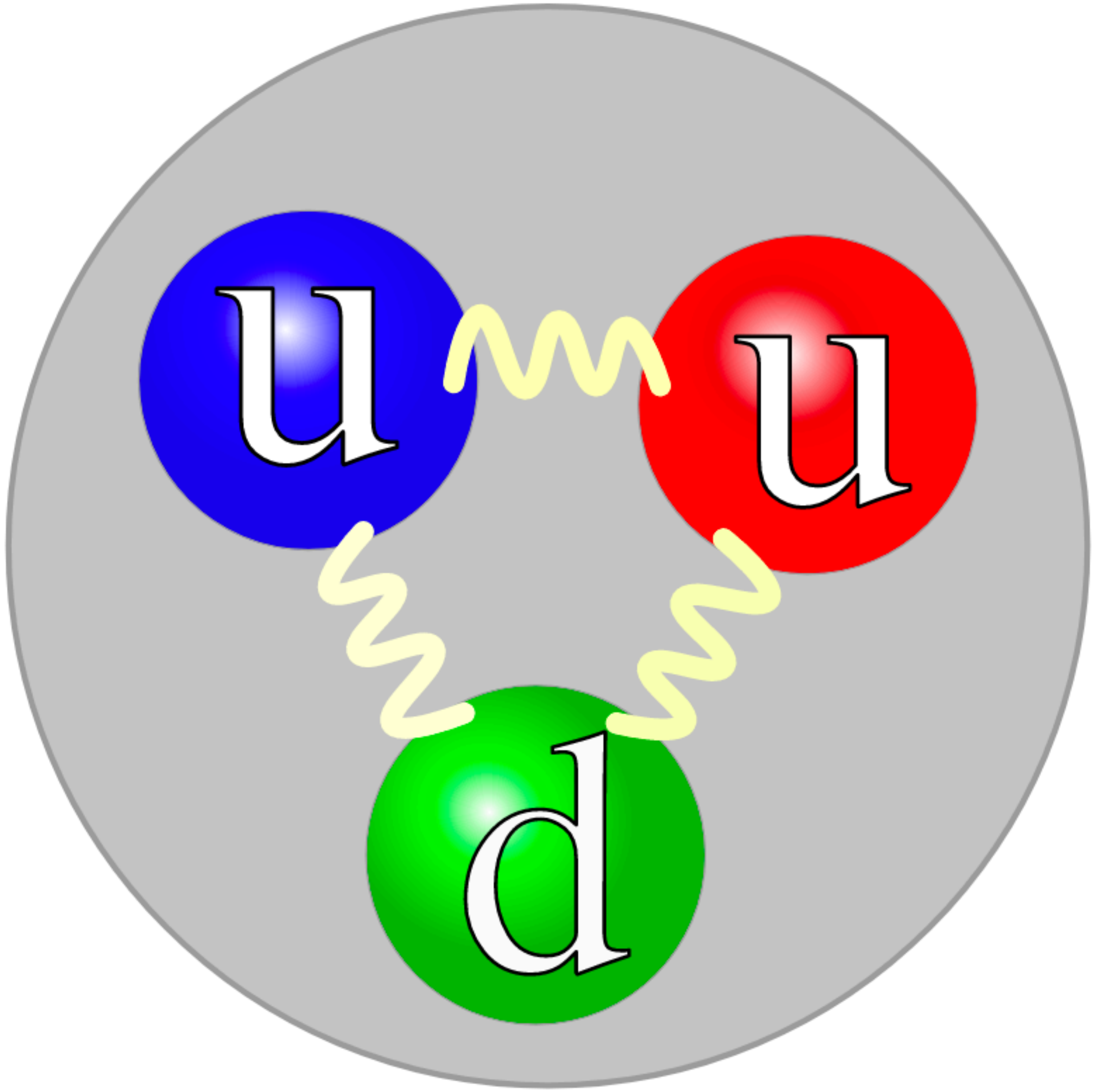


INTRODUCTION TO QUANTUM CHROMODYNAMICS

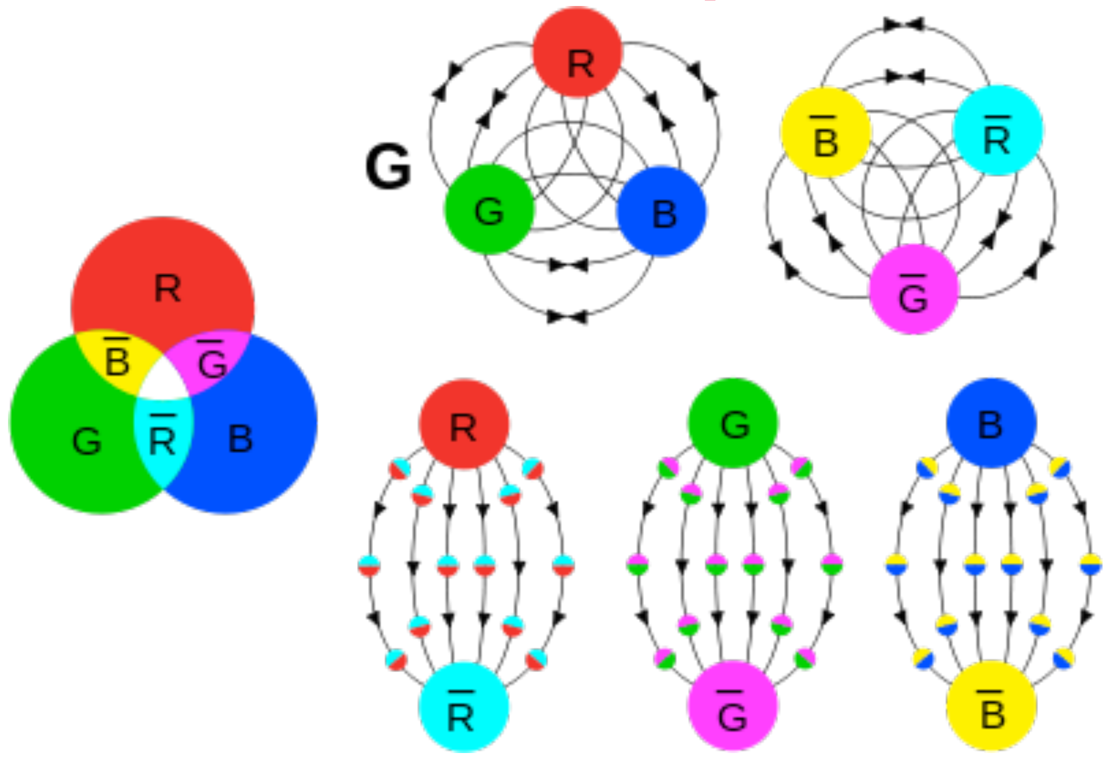
PARTICLE PHYSICS 2

Panos Christakoglou

SUMMARY

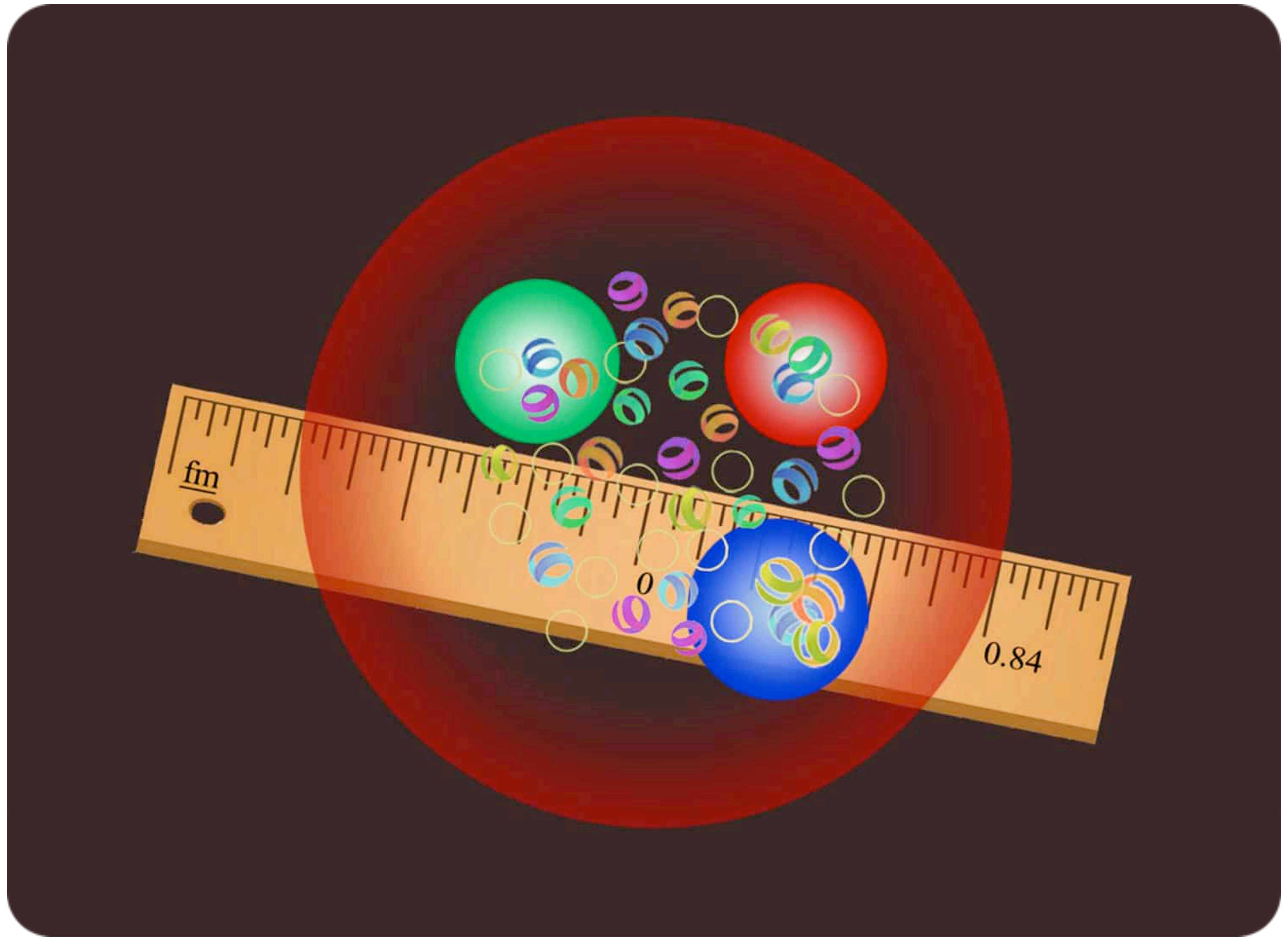
Last lecture

- Colour factors
- Interaction between quarks and antiquarks
  - Octet vs colour singlet configuration
- Interaction between quarks
  - Sextet vs triplet



Today's lecture

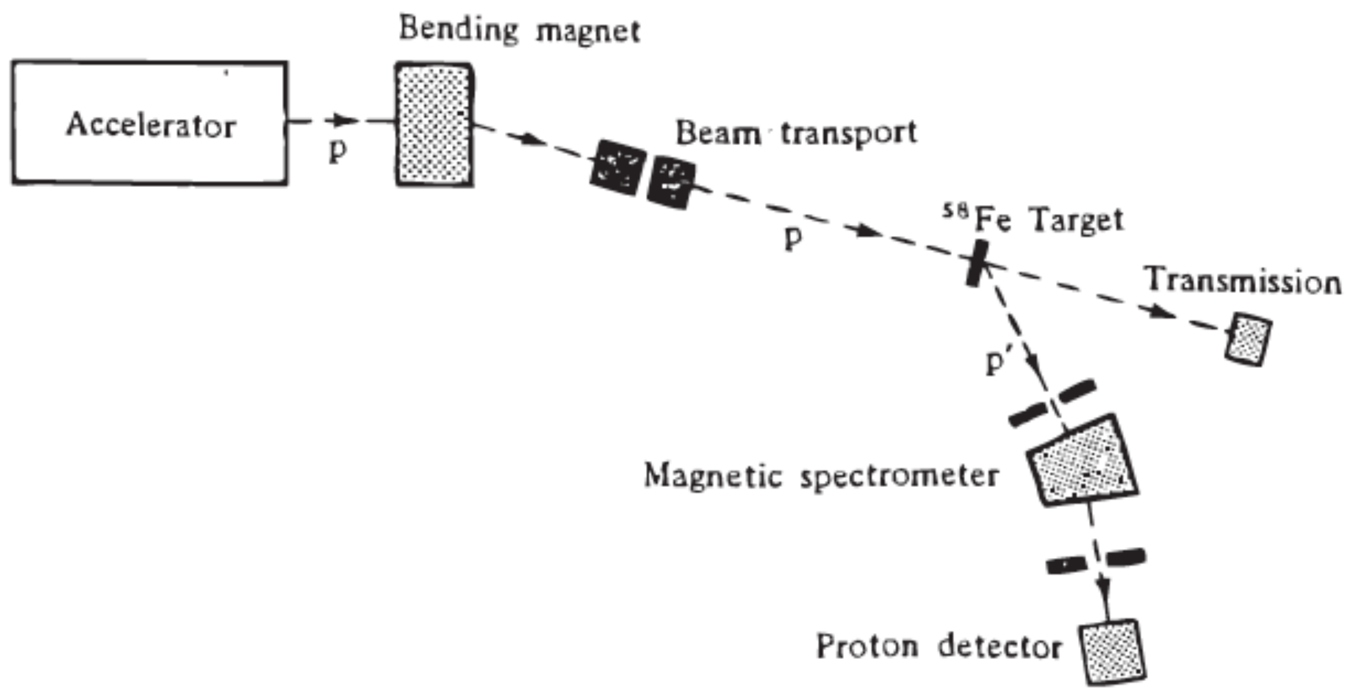
- Elastic electron-proton scattering
  - Form factors
  - Electric & magnetic charge distribution of a proton



ELASTIC SCATTERING VS SPECTROSCOPY

In spectroscopy, one angle is selected and the spectrum of the scattered particles is studied at this angle

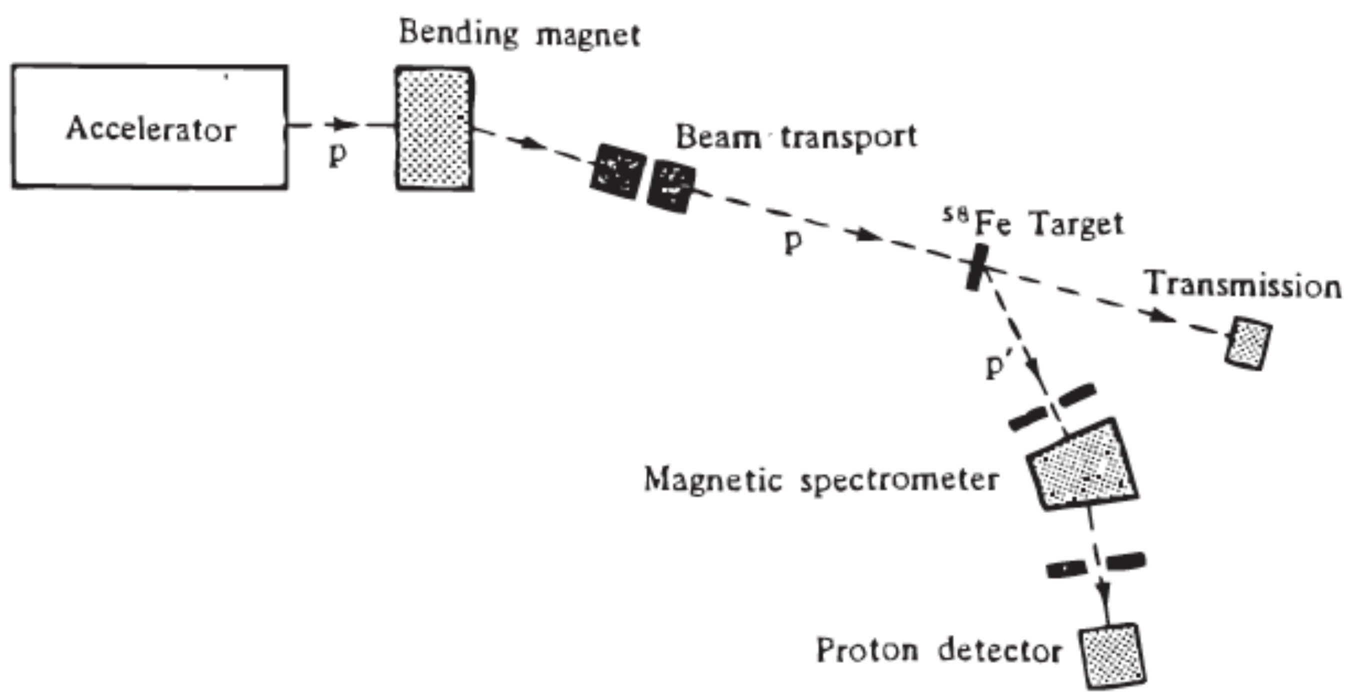
- In structure experiments, the detector configuration is very similar to the one used in spectroscopy experiments
  - the detectors look at the elastic peak



ELASTIC SCATTERING VS SPECTROSCOPY

The structure of particles is studied via either elastic or inelastic interactions between the particles of study and probes (beams) of incoming particles

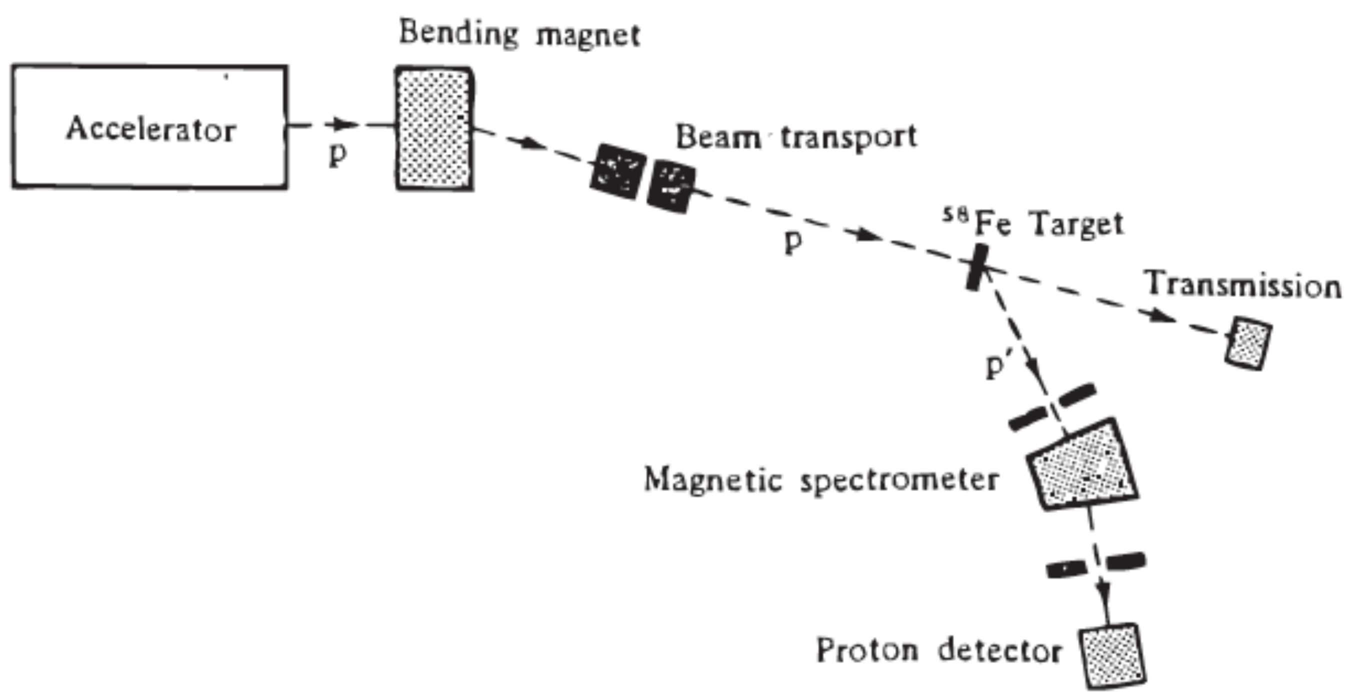
- Elastic scattering experiments have provided a significant part of the information we now know about the structure of subatomic particles
- Elastic scattering experiments very similar to the spectroscopy ones



ELASTIC SCATTERING VS SPECTROSCOPY

Elastic scattering experiments very similar to the spectroscopy ones

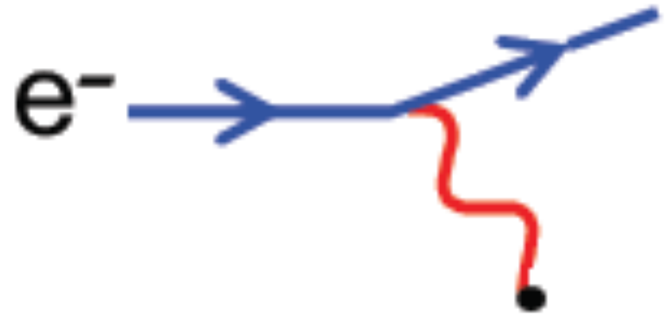
- the intensity of the elastic peak is measured as a function of the scattering angle
  - the intensity changes with angle because of the different recoil of the target particles
  - the observed intensity is translated into a differential cross-section $d\sigma/d\Omega$
  - information about the structure of a particle is then deduced from the cross-section



THE INTERNAL STRUCTURE OF THE PROTON

We start off with a “kid’s microscope” with low resolution i.e. small momentum transfers $\Longrightarrow$ low energy

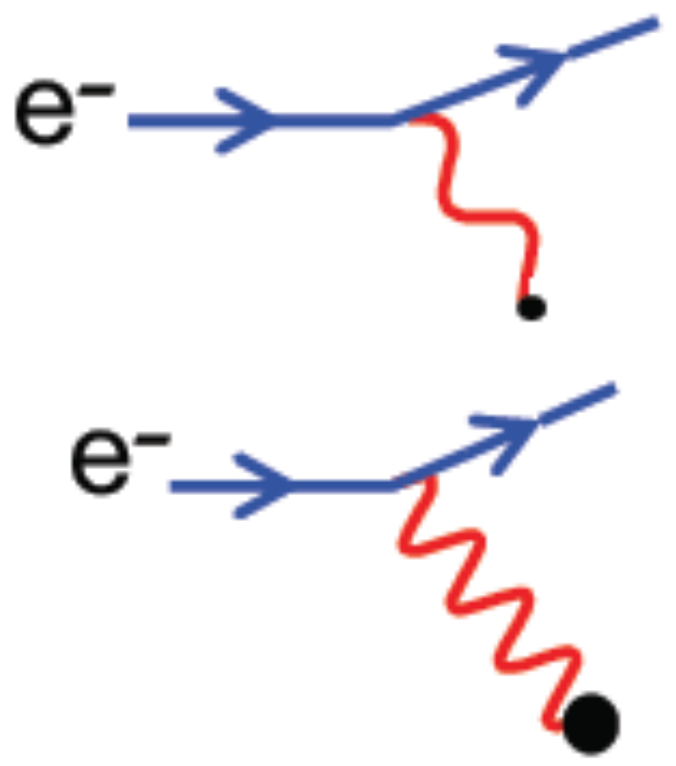
- able to detect the existence of a static electric potential



THE INTERNAL STRUCTURE OF THE PROTON

Increasing the energy leads to better resolution

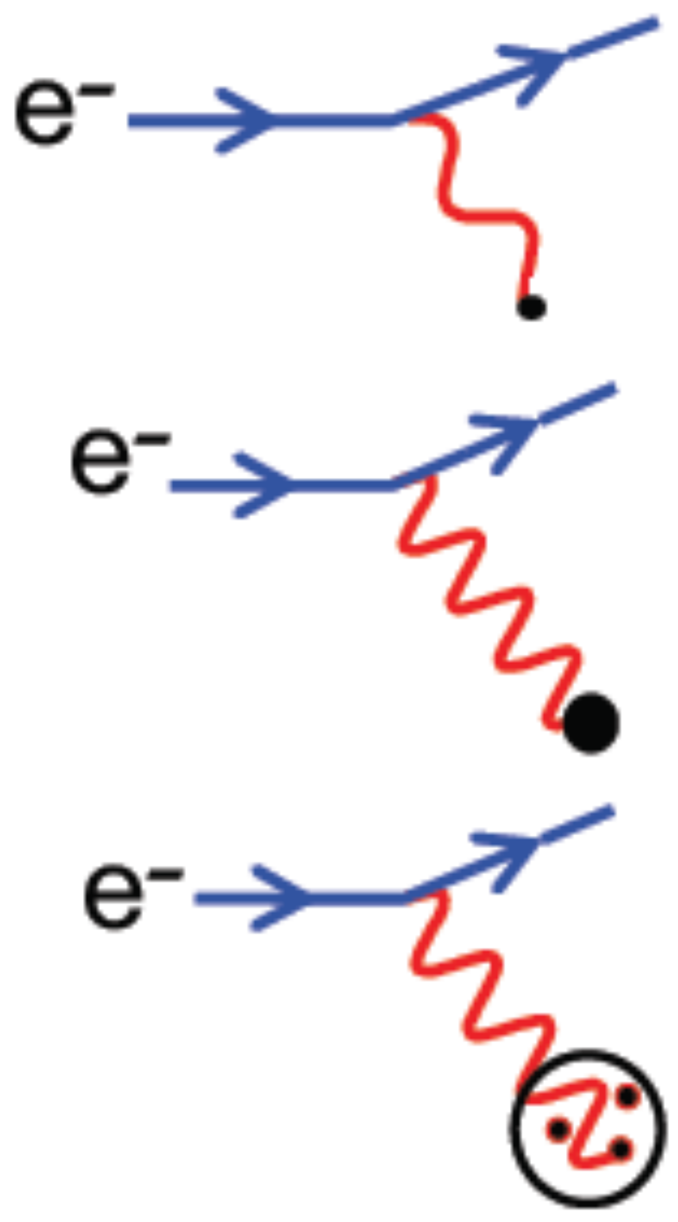
- our target has a sizeable charge distribution



THE INTERNAL STRUCTURE OF THE PROTON

For large momentum transfers

- our target has internal structure i.e. valence quarks
- Electron-proton inelastic scattering is seen as an electron-quark elastic scattering process



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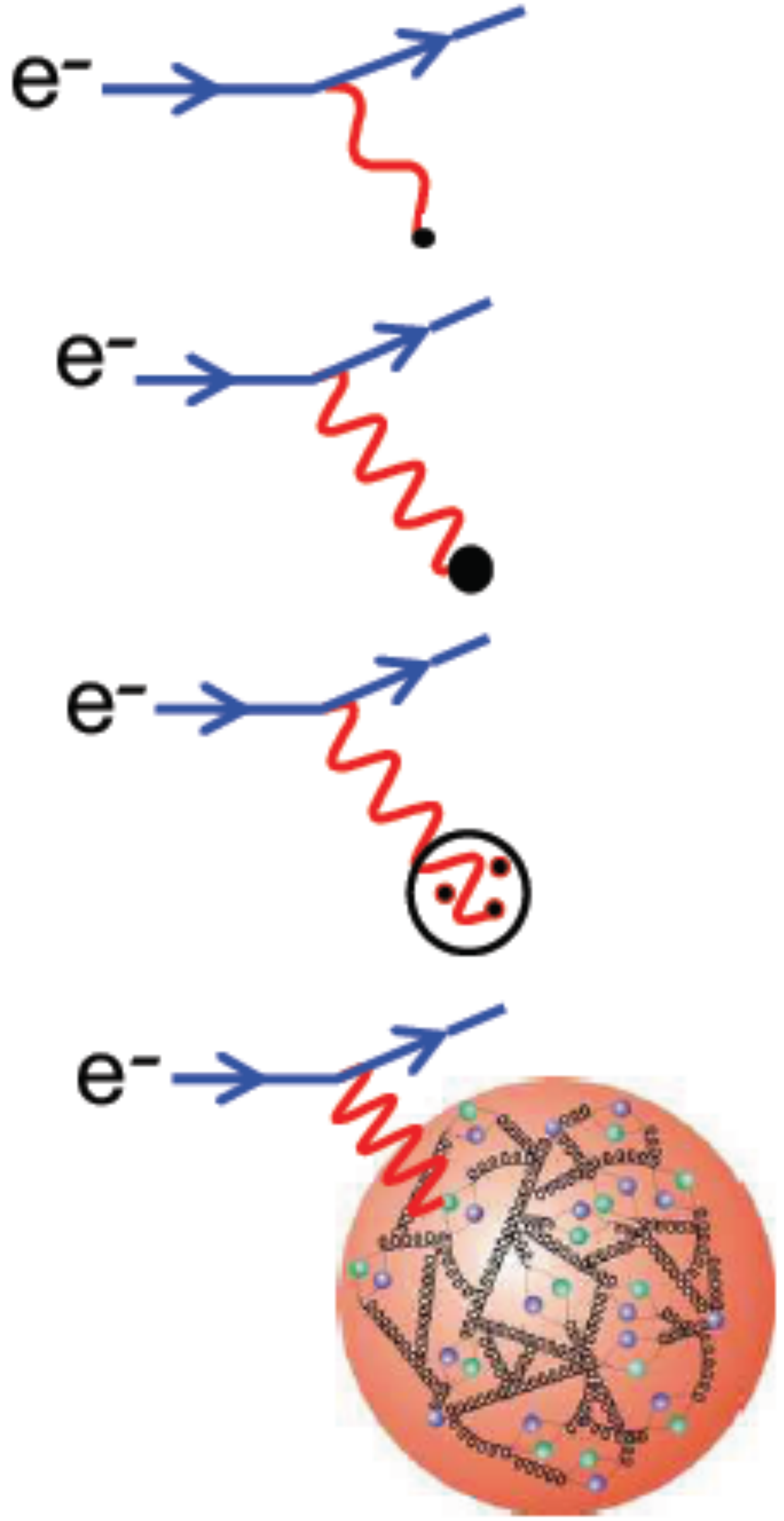
THE INTERNAL STRUCTURE OF THE PROTON

For even larger momentum transfers

- The internal structure of our target is even richer than we thought
- not only valence but also sea quarks and gluons!
- Introduce the parton distribution functions



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RUTHERFORD SCATTERING

Elastic scattering of α-particles by the Coulomb field of the nucleus of charge Ze

- The cross section can be computed, with the same results, either classically or quantum-mechanically

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{4m^2(Z_1 Ze^2)^2}{q^4}.$$

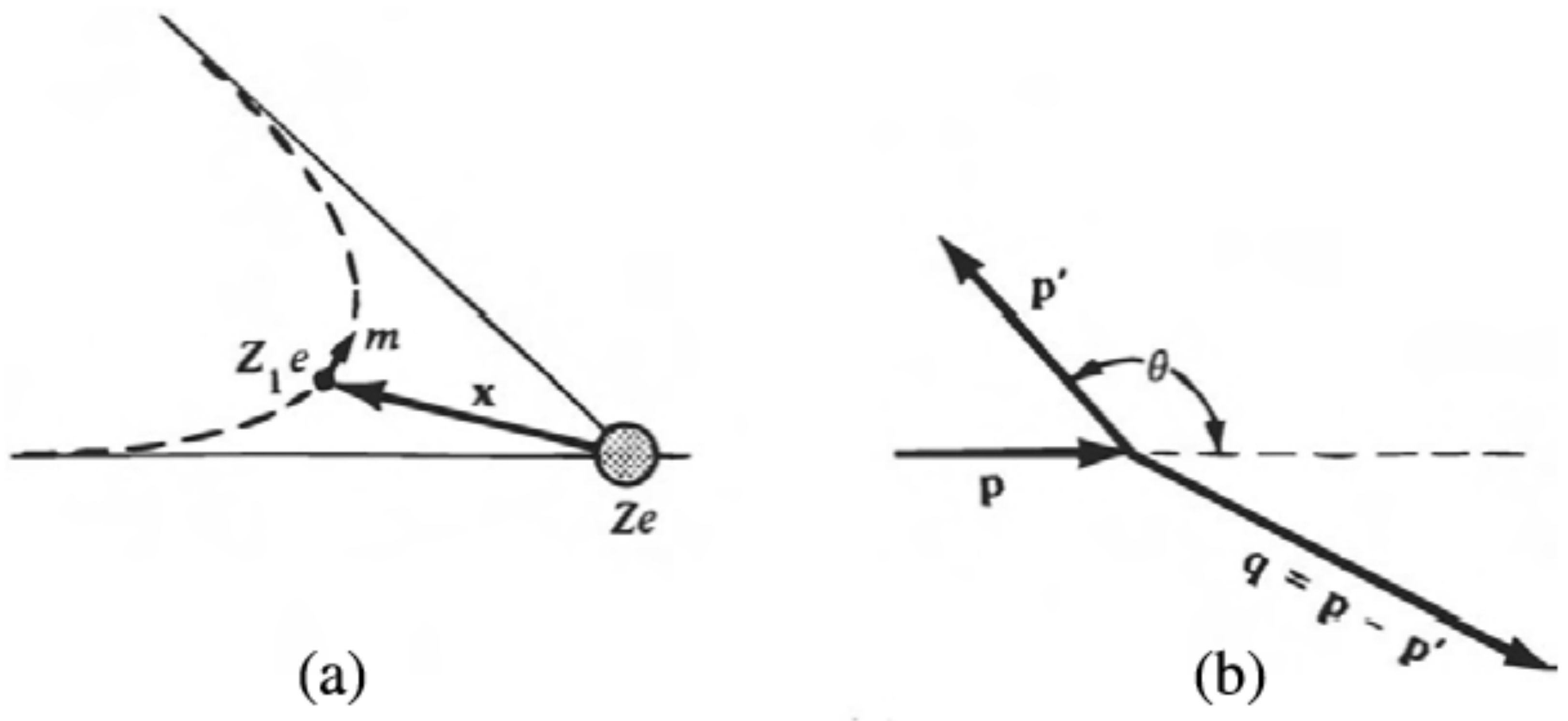


Figure 6.1: Rutherford scattering. (a) Classical trajectory of a particle with charge Z_1e in the field of a heavy nucleus with charge Ze. (b) Representation of the collision in momentum space.

RUTHERFORD SCATTERING

The Rutherford scattering is the low-energy limits of $e^{-}p$ scattering

- In this case the electron energy is sufficiently low that is considered as non-relativistic
- Also the kinetic energy of the recoiling proton is negligible compared to its rest mass
- In this case the proton can be considered as a fixed, point-like source of 1/r electrostatic potential
- The cross-section is calculated from scattering theory by using the first order terms in the perturbation expansion

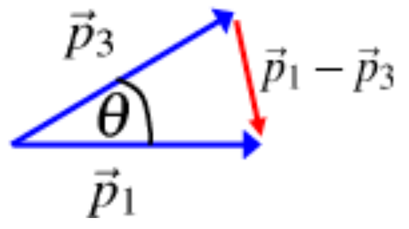
RUTHERFORD SCATTERING

Rutherford scattering:

- the proton recoil can be neglected and the electron is non-relativistic
- The differential cross-section is given then by

$$\langle|M_{if}|\rangle^2 = \frac{16 m_p^2 m_e^2 e^4}{q^4}$$

$$q^2 = (P_1 - P_3)^2 = 4|\vec{P}|^2 \sin^2(\theta/2)$$



$$\langle|M_{if}|\rangle^2 = \frac{m_p^2 m_e^2 e^4}{|\vec{P}|^4 \sin^4(\theta/2)}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{1}{m_p + E_1 - E_1\cos\theta}\right)^2 \langle|M_{if}|\rangle^2$$

MOTT SCATTERING

The Mott scattering is the limit where the electron is relativistic but the proton recoil can still be negligible

These conditions apply when $m_e << E << m_p$

The matrix element is given this time by

$$\langle |M_{if}| \rangle^2 \approx \frac{e^4}{4E^2 \sin^4(\theta/2)} \cos^2(\theta/2) \, \frac{E'}{E}$$

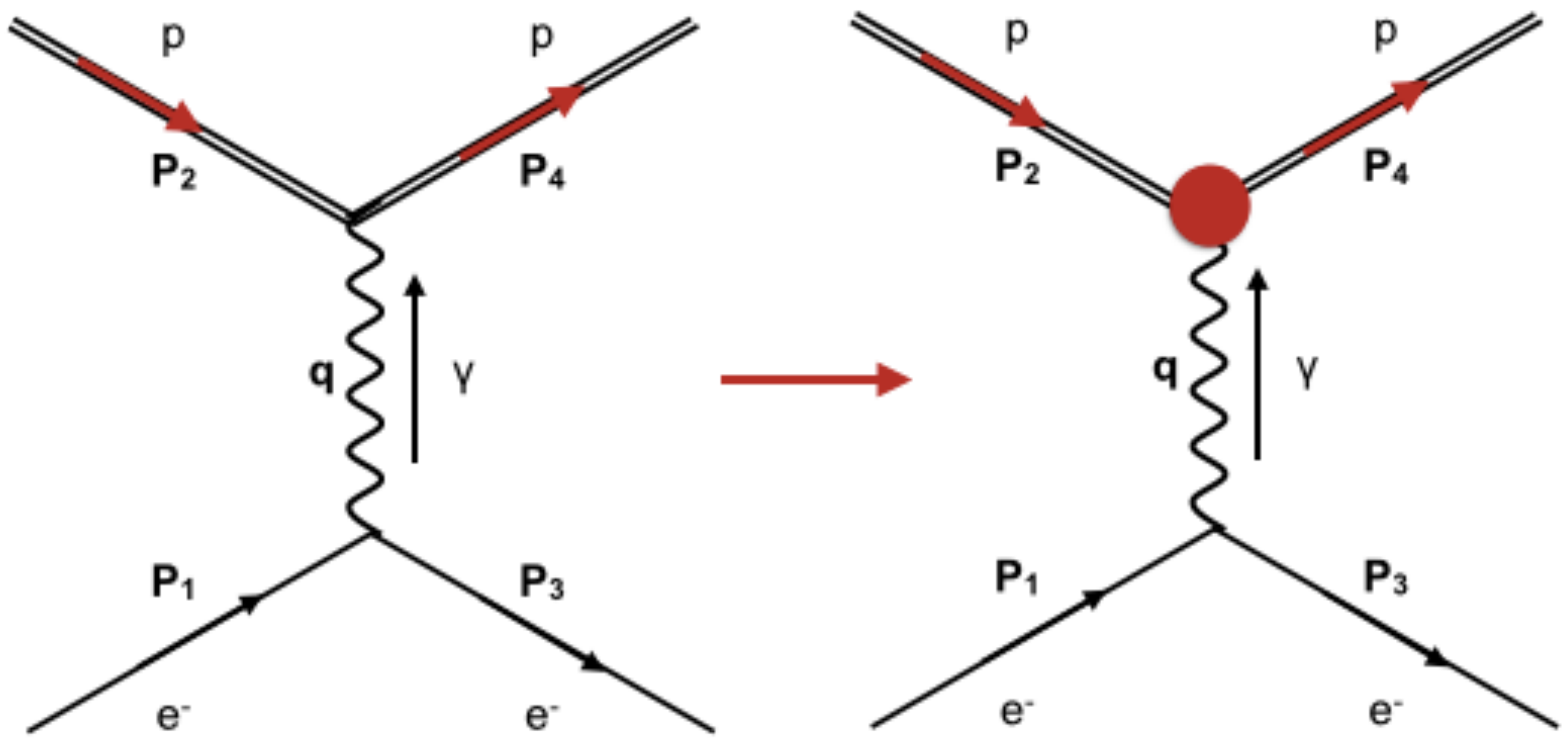
while the differential cross-section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{a^2}{4E^2 \sin^4(\theta/2)} \cos^2(\theta/2) \frac{E'}{E}$$

FORM FACTORS

Form factors are introduced to account for the fact that some particles are not elementary but have internal structure

- The electron and in general leptons are elementary particles and thus are ideal tools to probe the internal structure of particles
  - A typical example of such kind of interaction is the electron-proton elastic scattering

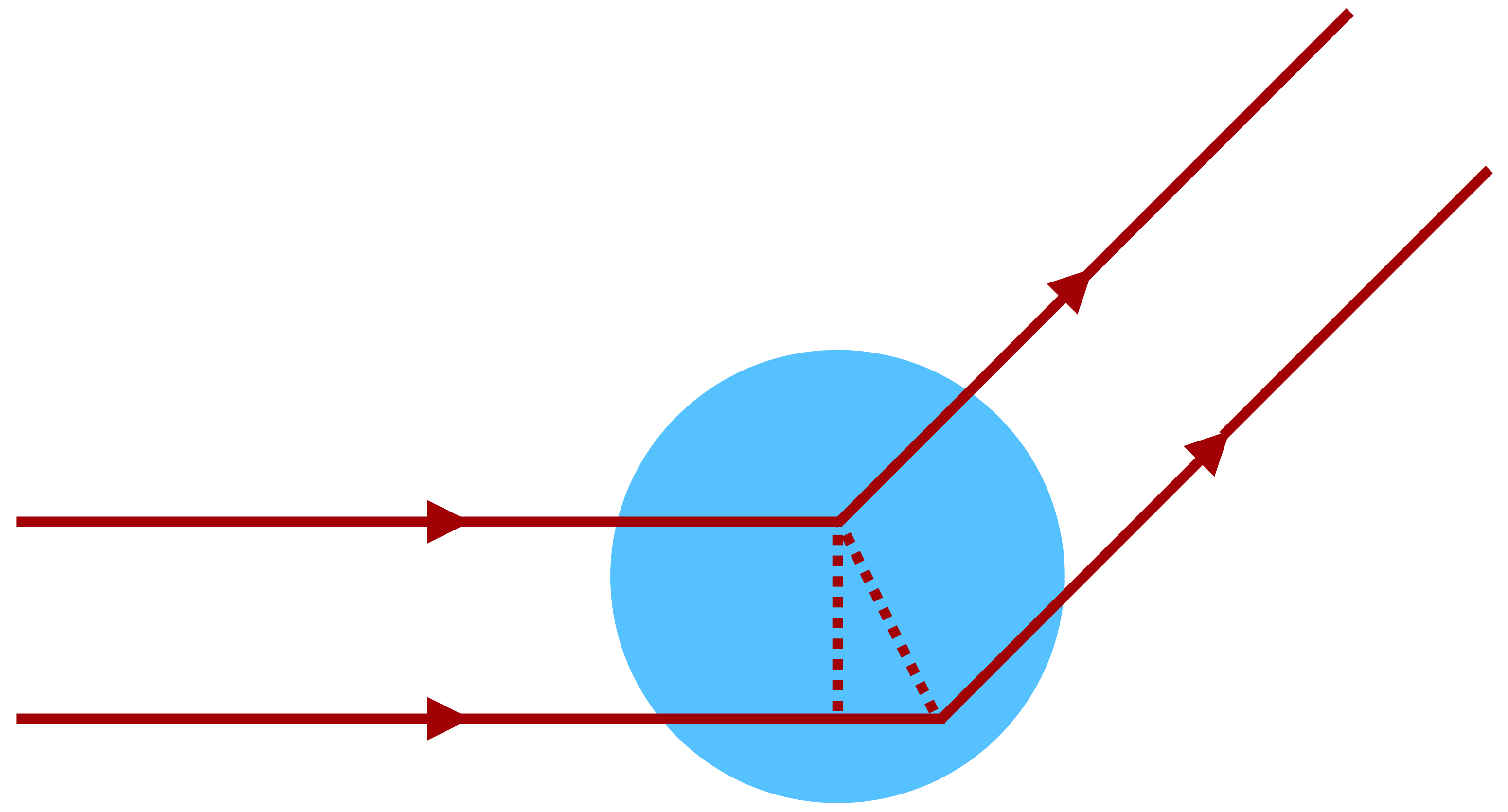


FORM FACTORS

Form factors act similar to diffraction of plane waves in optics

- The finite size of the scattering centre introduces a phase difference between plane waves "scattered from different points in space"
  - If the wavelength is long compared to the size of the object to be probed, all waves are in phase and $F(q^2) = 1$
- For a point-like particle the form factor is unity

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \rightarrow \frac{a^2}{4E^2 \sin^4(\theta/2)} \cos^2(\theta/2) |F(q^2)|^2 \frac{E'}{E}$$



FORM FACTORS

The connection between the form factor and the density distribution is given by a Fourier transform of the probability density

$$F(q^2) = \int d^3\rho(\vec{r})e^{i\vec{q}\vec{r}/\hbar}$$

Experiments measure $F(q^2)$ for various values of the square of momentum transfer q^2 and fit the data points to recover the continuum

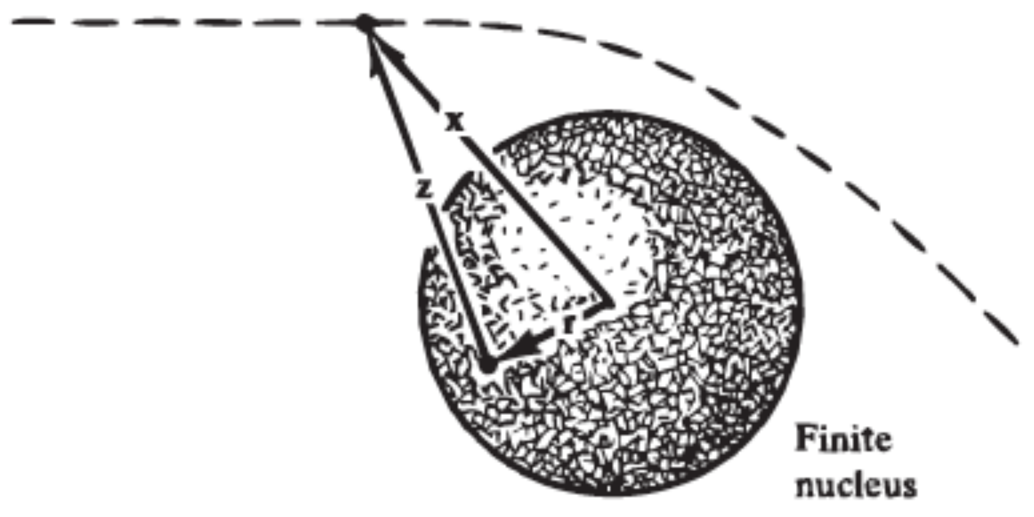


Figure 6.2: Scattering of a spinless electron by a spinless nucleus with extended charge distribution.

ELASTIC FORM FACTORS OF NUCLEONS

Nucleons are not point like particles

- They have an internal structure, containing a combination of uud (protons) and udd (neutrons)
- The best way to explore the charge and current distributions of nucleons is again via the elastic scattering with electron beams
- For protons, one can use a liquid hydrogen target in the path of an electron beam and determine the differential cross-section of the scattered electrons
- For neutrons things become more complicated as there is no neutron target
    - One relies on deuteron targets and subtract the effect of protons
    - This usually leads to significantly large uncertainties in the measurements
- The form factors for spineless particles is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} |F(q^2)|^2$$

- This formula needs to be generalised for spin-1/2 particles with internal structure

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ELASTIC FORM FACTORS OF NUCLEONS

The form factor in the previous formula describes the electric charge distribution, and thus $F(q^2)$ is called the electric form factor

- A proton has also a magnetic moment with its “magnetisation” being distributed over the volume of the nucleon and is described by the magnetic form factor
- The generalisation of the previous formula is given by the Rosenbluth equation

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{ep}} = \frac{a^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + bG_M^2}{1+b} \cos^2(\theta/2) + 2bG_M^2 \sin^2(\theta/2) \right)$$

$$b = \frac{-q^2}{4m_p^2}$$

  - where G_E and G_M are the electric and magnetic form factors and they are both a function of q^2
  - m_p is the mass of the nucleon, θ is the scattering angle between the incoming and outgoing electrons with energies E_1 and E_3, respectively
  - q is the momentum transfer to the nucleon

ELASTIC FORM FACTORS OF NUCLEONS

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{ep}} = \frac{a^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + bG_M^2}{1+b} \cos^2(\theta/2) + 2bG_M^2 \sin^2(\theta/2) \right)$$

$$\downarrow$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{ep}} = \frac{a^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \cos^2(\theta/2) \left(\frac{G_E^2 + bG_M^2}{1+b} + 2bG_M^2 \tan^2(\theta/2) \right)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{ep}} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left(\frac{G_E^2 + bG_M^2}{1+b} + 2bG_M^2 \tan^2(\theta/2) \right)$$

At low values of $q^2 \rightarrow b \sim 0$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{ep}} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \approx G_E^2(q^2)$$

At high values of $q^2 \rightarrow b \gg 1$

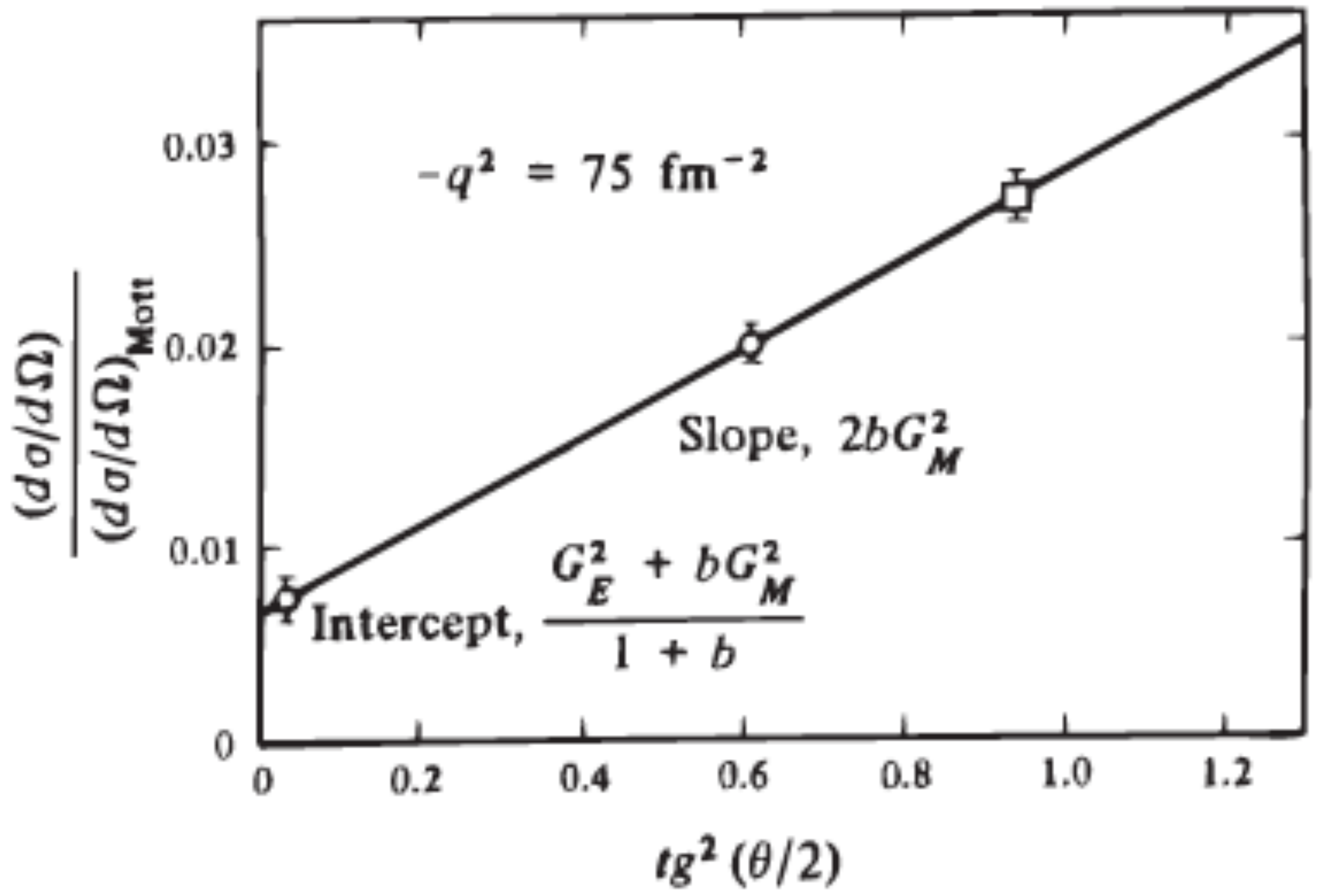
$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{ep}} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \approx \left(1 + 2b\tan^2(\theta/2)\right) G_M^2(q^2)$$

In general we are sensitive to both form factors

ELASTIC FORM FACTORS OF NUCLEONS

To extract the form factors, we use this formula $$\left(\frac{d\sigma}{d\Omega}\right)_{\text{ep}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left(\frac{G_E^2 + bG_M^2}{1+b} + 2bG_M^2 \tan^2(\theta/2)\right)$$

- and plot the ratio of the measured cross-section for a given q^2 value over the Mott cross-section as a function of $\tan^2(\theta/2)$
- The slope gives the factor multiplying $\tan^2(\theta/2)$ from where we extract G_M^2
- The intercept on the y-axis gives the other factor from where we extract G_E^2 values of $q^2 \rightarrow b \sim 0$



ELASTIC FORM FACTORS OF NUCLEONS

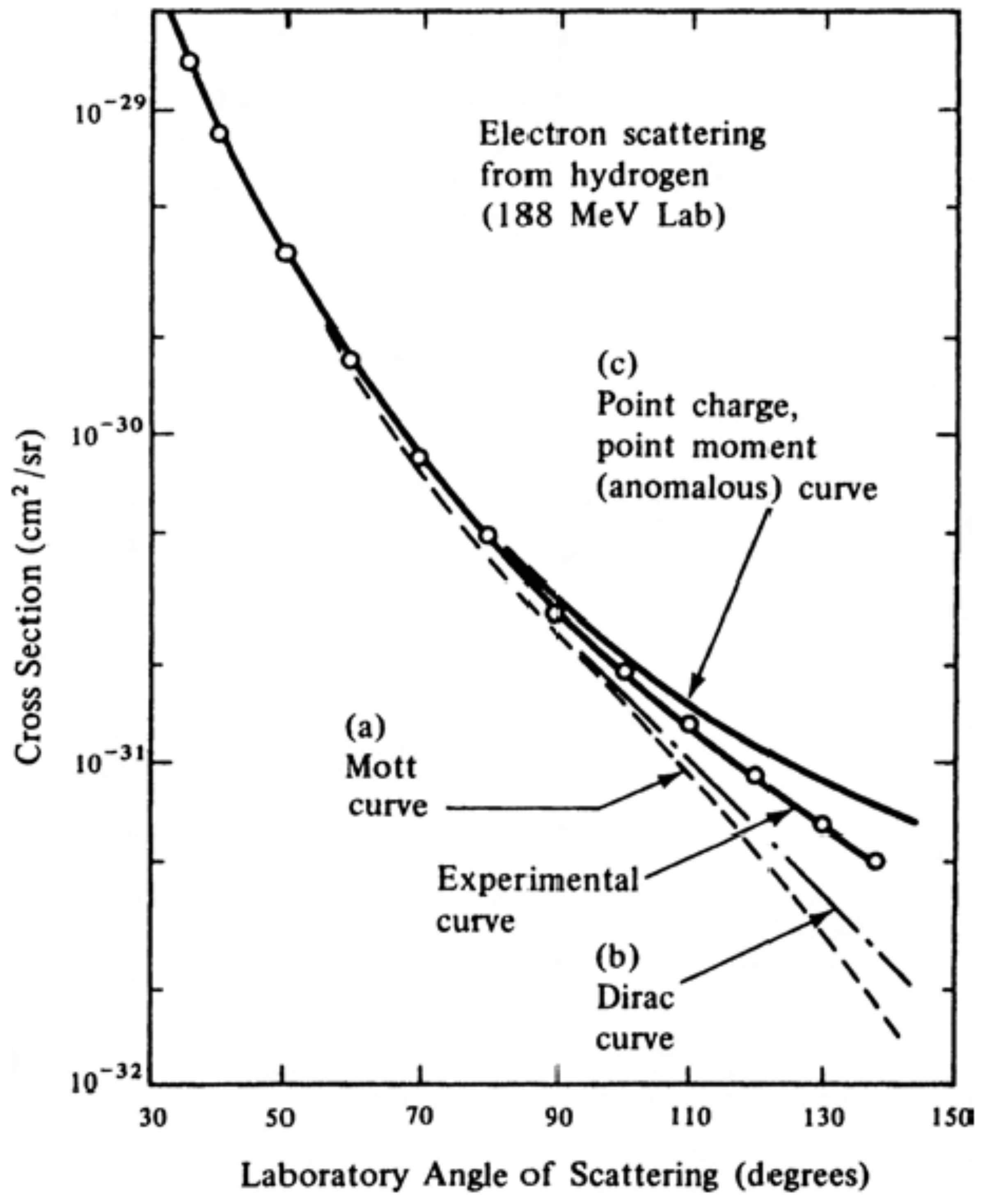


Figure 6.11: Electron–proton scattering with 188 MeV electrons. [R. W. McAllister and R. Hofstadter, *Phys. Rev.* **102**, 851 (1956).] The theoretical curves correspond to the following values of G_E and G_M: Mott (1;0), Dirac (1;1), anomalous (1;2.79).

Conclusion: Nucleons are not point like particles!

NOBEL PRIZE 1961

The Nobel Prize in Physics 1961
Robert Hofstadter, Rudolf Mössbauer

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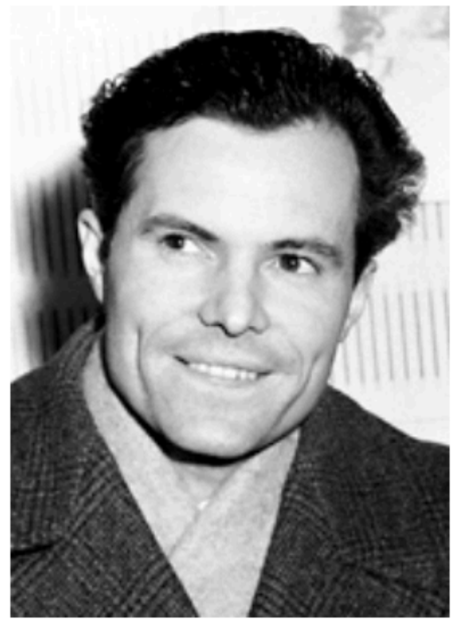
The Nobel Prize in Physics 1961

Robert Hofstadter
Prize share: 1/2



Rudolf Ludwig Mössbauer
Prize share: 1/2

The Nobel Prize in Physics 1961 was divided equally between Robert Hofstadter *"for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons"* and Rudolf Ludwig Mössbauer *"for his researches concerning the resonance absorption of gamma radiation and his discovery in this connection of the effect which bears his name"*.

ELASTIC FORM FACTORS OF NUCLEONS

For the static case we have

$$G_E(q^2 = 0) = \frac{Q}{e} \qquad G_M(q^2 = 0) = \frac{\mu}{\mu_N}$$

- where Q and μ are the charge and magnetic moment of nucleons
- More specifically, for protons and neutrons we have

$$G_E^p(q^2 = 0) = 1 \qquad G_E^n(q^2 = 0) = 0$$

$$G_M^p(q^2 = 0) = 2.79 \qquad G_M^n(q^2 = 0) = -1.91$$

ELASTIC FORM FACTORS OF NUCLEONS

The magnetic form factor of a proton is described fairly well by a dipole function for low values of q^2

- The proton charge and magnetic distributions are quite different

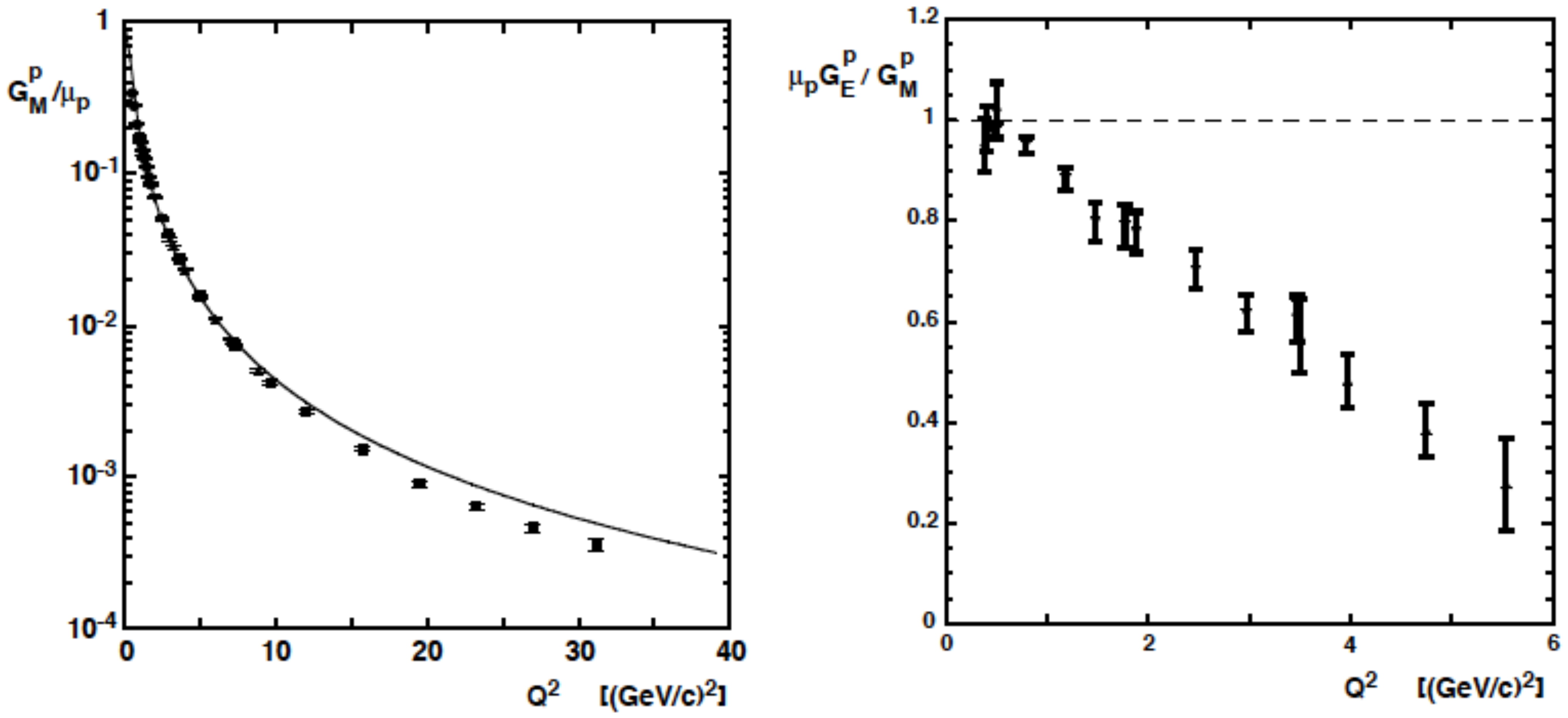


Figure 6.13: Left: Magnetic form factor for the proton plotted against the squared momentum transfer $|q|^2$. The different symbols correspond to different experiments. The 'dipole' function –described in the text and shown as a continuous line– describes the G_M data quite accurately below $|q|^2 \approx 10\ (\mathrm{GeV/c})^2$. Right: G_E/G_M. The distributions of charge and magnetism in the proton are quite different. [See C.Hyde-Wright and K. de Jager, *Annu. Rev. Nucl. Part. Sci.* **54**, 217 (2004).]

ELASTIC FORM FACTORS OF NUCLEONS

The magnetic form factor of a neutron is described fairly well by a dipole function for low values of q^2

- The neutron charge distribution is quite small

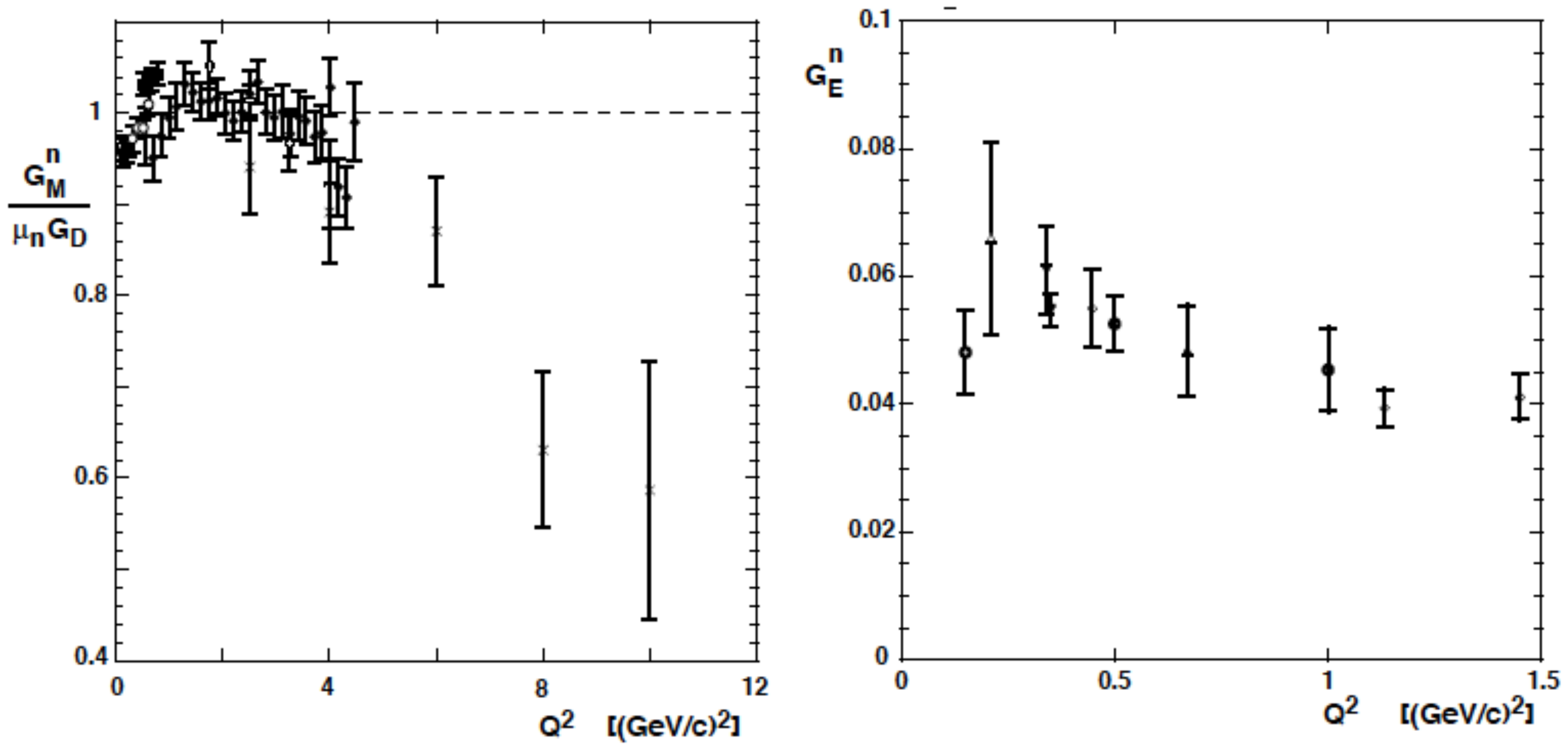


Figure 6.14: Magnetic (left) and electric (right) form factors for the neutron. Here we show the magnetic form factor divided by the dipole formula. The magnetic form factor shows rough agreement with the dipole formula for $|q|^2 < 5$ (GeV/c)^2. [See C.Hyde-Wright and K. de Jager, *Annu. Rev. Nucl. Part. Sci.* **54**, 217 (2004).]

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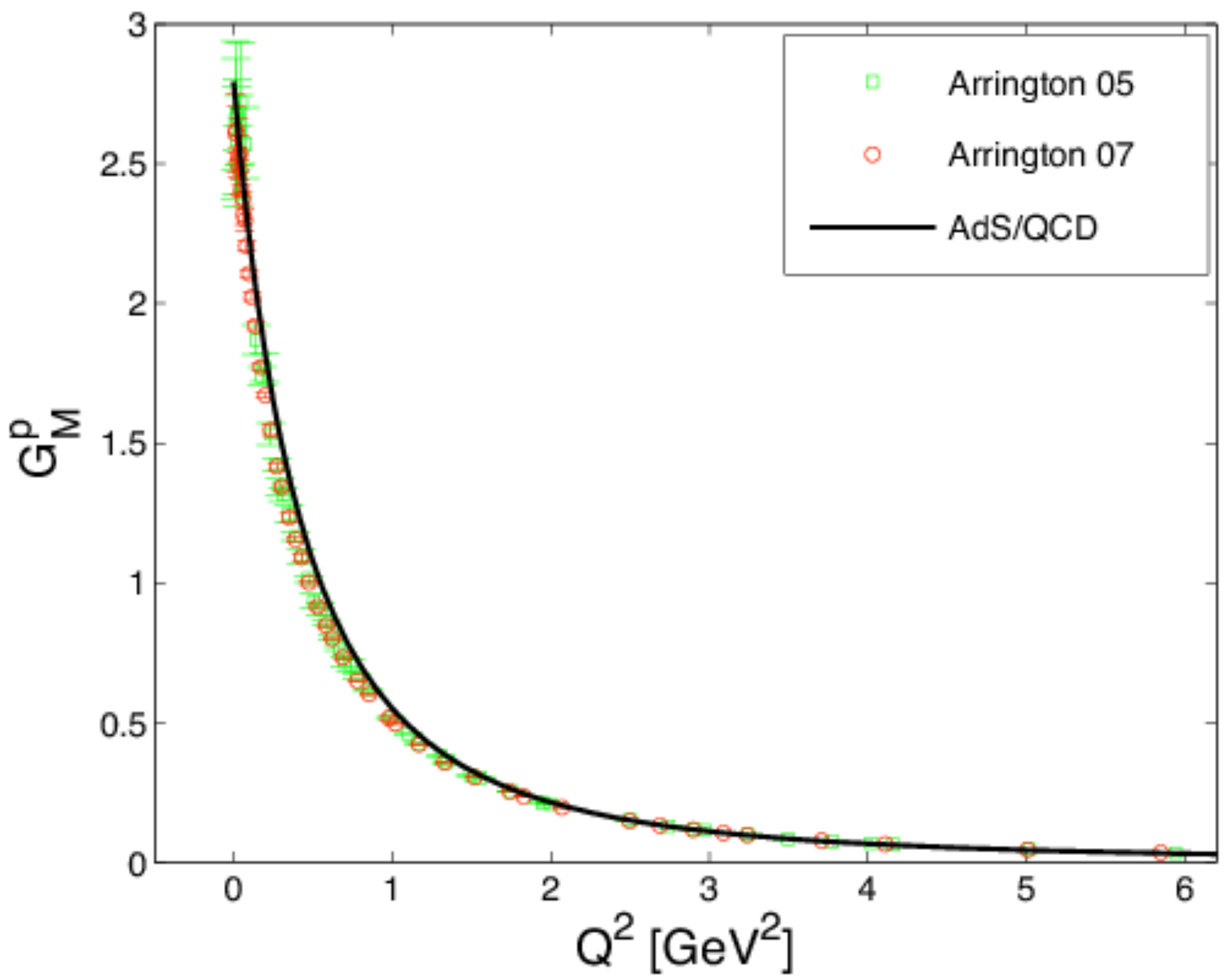
PROTON FORM FACTORS



$$G_E(q^2) = \left(1 - \frac{q^2}{0.71}\right)^{-2}$$

$$\langle r^2 \rangle = 6\left(\frac{dG_E(q^2)}{dq^2}\right)_{q^2=0} = (0.81 \cdot 10^{-13}\mathrm{cm})^2$$

Thank you for
your attention!

LATEX

```
\Big(\frac{d\sigma}{d\Omega}\Big)_{\rm Mott} \rightarrow \frac{a^2}{4E^2\sin^4(\theta/2)}\cos^2(\theta/2)|F(q^2)|^2

F(q^2) = \int d^3 \rho(\vec{r}) e^{i\vec{q}\vec{r}/\hbar}

\frac{d\sigma}{d\Omega} = \Big(\frac{d\sigma}{d\Omega}\Big)_{\rm Mott} |F(q^2)|^2

\Big(\frac{d\sigma}{d\Omega}\Big)_{\rm ep} = \frac{a^2}{4E^2\sin^4(\theta/2)}\frac{E_3}{E_1}\Big(\frac{G_E^2 + bG_E^2}{1 + b}\cos^2(\theta/2) + 2bG_M^2\sin^2(\theta/2)\Big)

\Big(\frac{d\sigma}{d\Omega}\Big)_{\rm ep} = \frac{a^2}{4E^2\sin^4(\theta/2)}\frac{E_3}{E_1}\cos^2(\theta/2) \Big(\frac{G_E^2 + bG_E^2}{1 + b} + 2bG_M^2\tan^2(\theta/2)\Big)

\Big(\frac{d\sigma}{d\Omega}\Big)_{\rm ep} = \Big(\frac{d\sigma}{d\Omega}\Big)_{\rm Mott} \Big(\frac{G_E^2 + bG_E^2}{1 + b} + 2bG_M^2\tan^2(\theta/2)\Big)

\Big(\frac{d\sigma}{d\Omega}\Big)_{\rm ep} / \Big(\frac{d\sigma}{d\Omega}\Big)_{\rm Mott} \approx \Big(1 + 2b\tan^2(\theta/2)\Big)G_M^2(q^2)
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