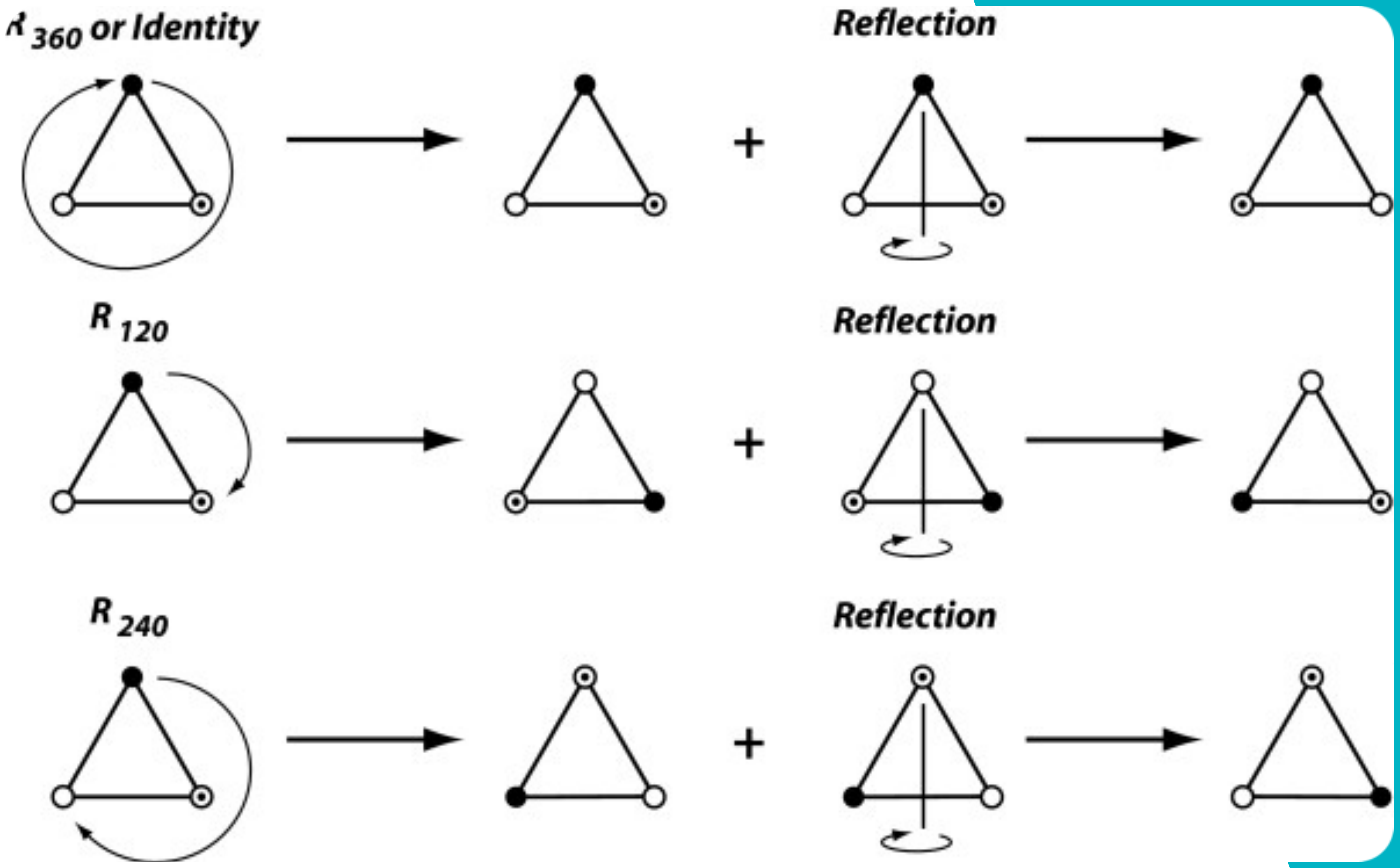
Nikhef



INTRODUCTION TO QUANTUM CHROMODYNAMICS

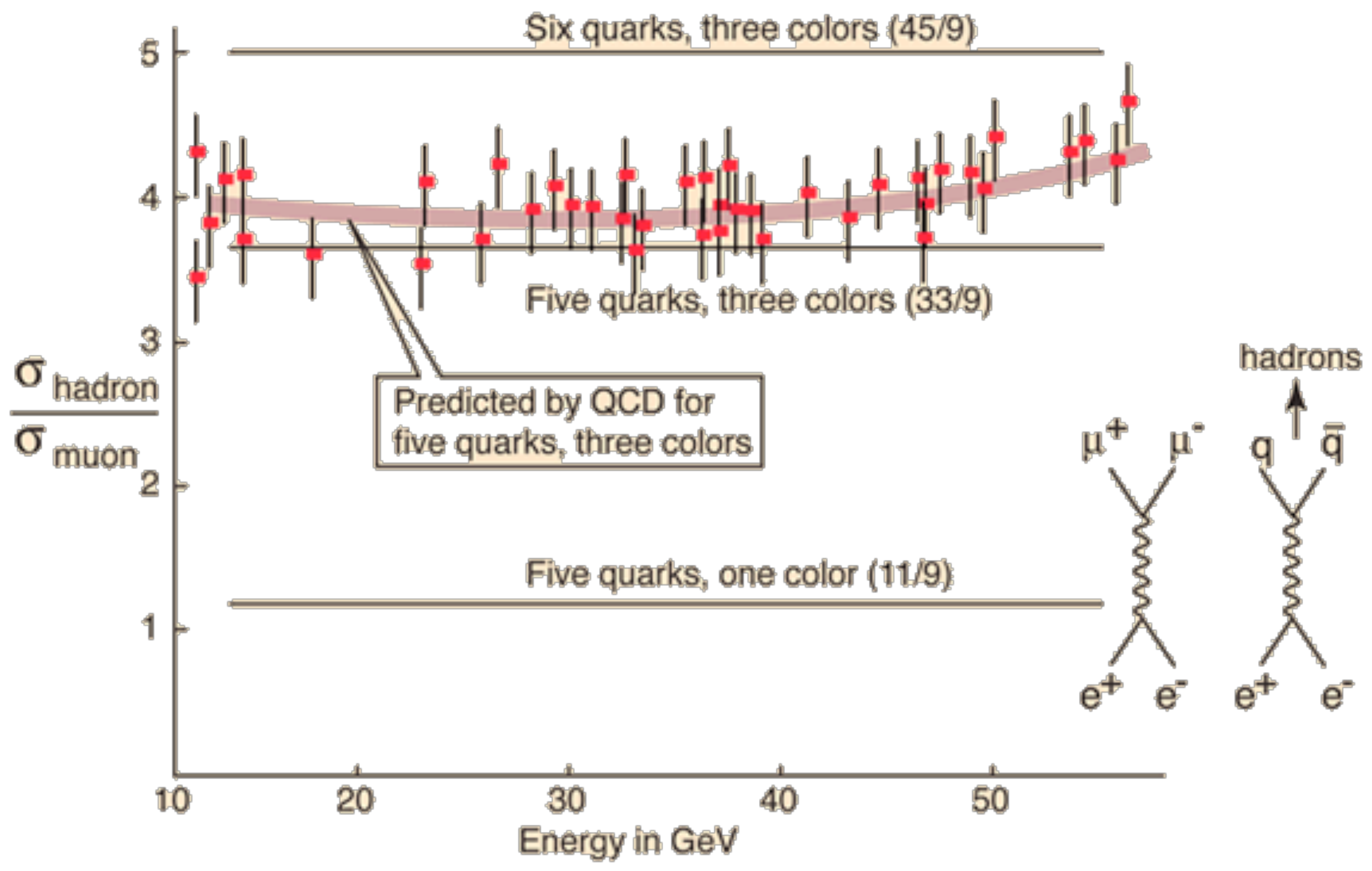
PARTICLE PHYSICS 2

Panos Christakoglou

SUMMARY

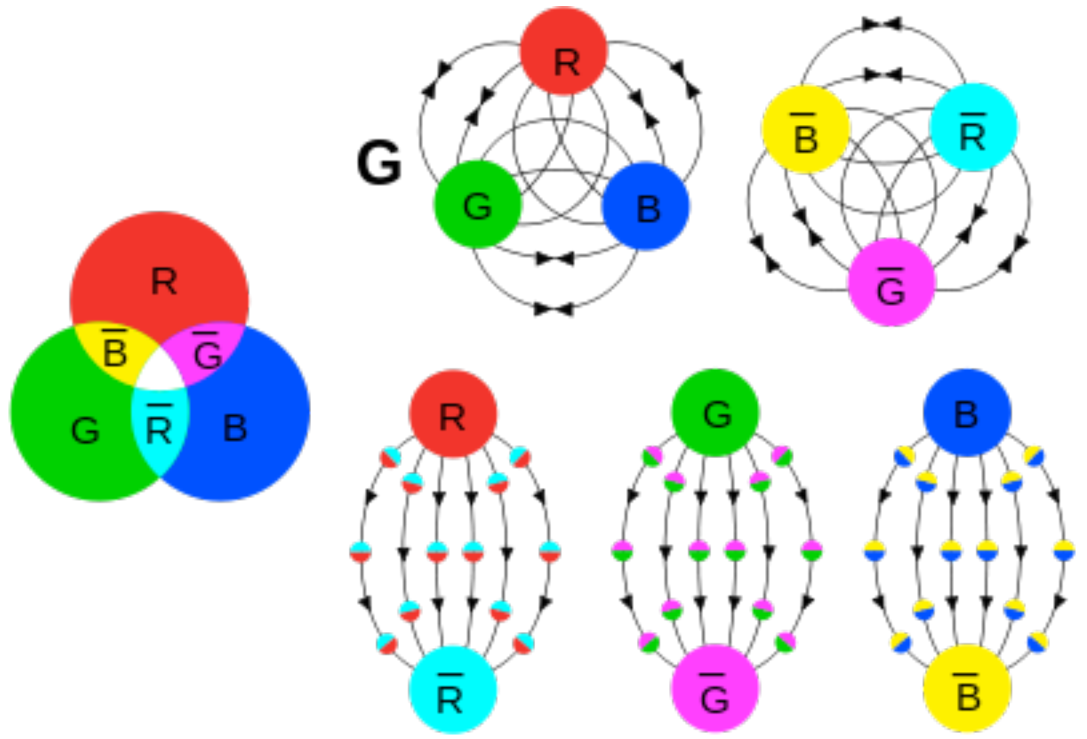
Last lecture

- Elements of Quantum Mechanics
- Categories of particles
- Quark model
- Experimental evidence of colour



Today's lecture

- Transformations
  - Local vs global
  - Discrete vs continuous
- Symmetries & conservation laws
  - Translations & rotations
- Introduction to groups
  - Generators
  - SO(3), SU(2), SU(3)



Nikhef

TRANSFORMATIONS

TYPES OF TRANSFORMATIONS

Transformations are usually described by operators

- When applied on a system, they might leave it invariant → symmetry

- They can be categorised
  - Global (transformation by the same "amount" at all points of space-time) vs local (transformation by a different "amount" at various points of space-time)
  - Continuous vs discrete
    - Continuous: can be compact vs non-compact
    - Discrete: finite vs infinite mechanics

EXAMPLES OF DISCRETE TRANSFORMATIONS

Charge conjugation (C):

- Converts a particle to its anti-particle (i.e. sign-flip)

$$Q \to -Q$$

$$\hat{C}|\pi^+\rangle \to |\pi^-\rangle \neq \pm|\pi^+\rangle$$

$$\hat{C}|\pi^0\rangle = \pm|\pi^0\rangle \qquad \pi^0 \to \gamma\gamma$$

$$\hat{C}|\pi^0\rangle = +1|\pi^0\rangle$$

Parity (P):

- Converts right-handed systems to left-handed ones
- Vectors change sign but axial vectors remain unchanged

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$

$$\vec{x} \to -\vec{x}$$
$$\vec{P} \to -\vec{P}$$
$$\vec{L} = \vec{x} \times \vec{P} \to \vec{L}$$

Time reversal T

- Reverses the direction of motion of particles

Symmetries are part of the building blocks of particle physics. However their validity rests on **experimental verification**!!!

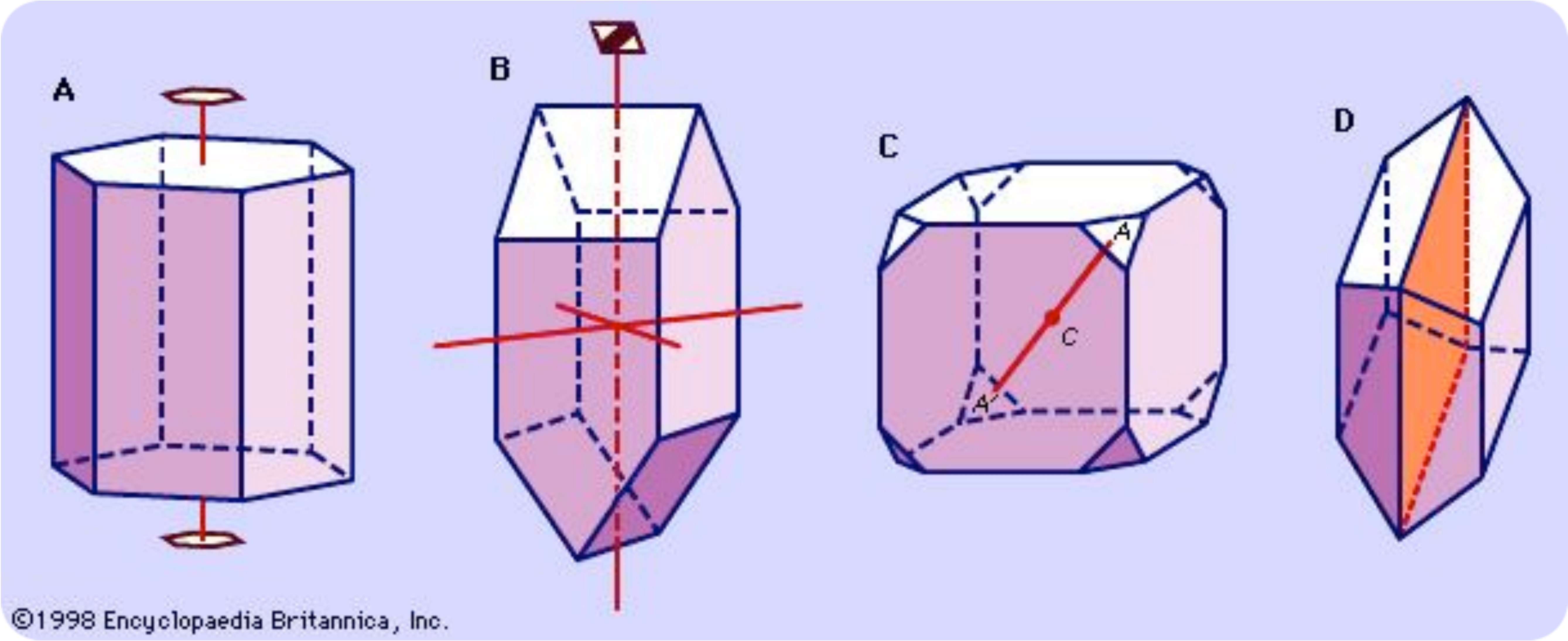
Nikhef

SYMMETRIES

ABOUT SYMMETRIES

Symmetries and invariances are important notions in physics

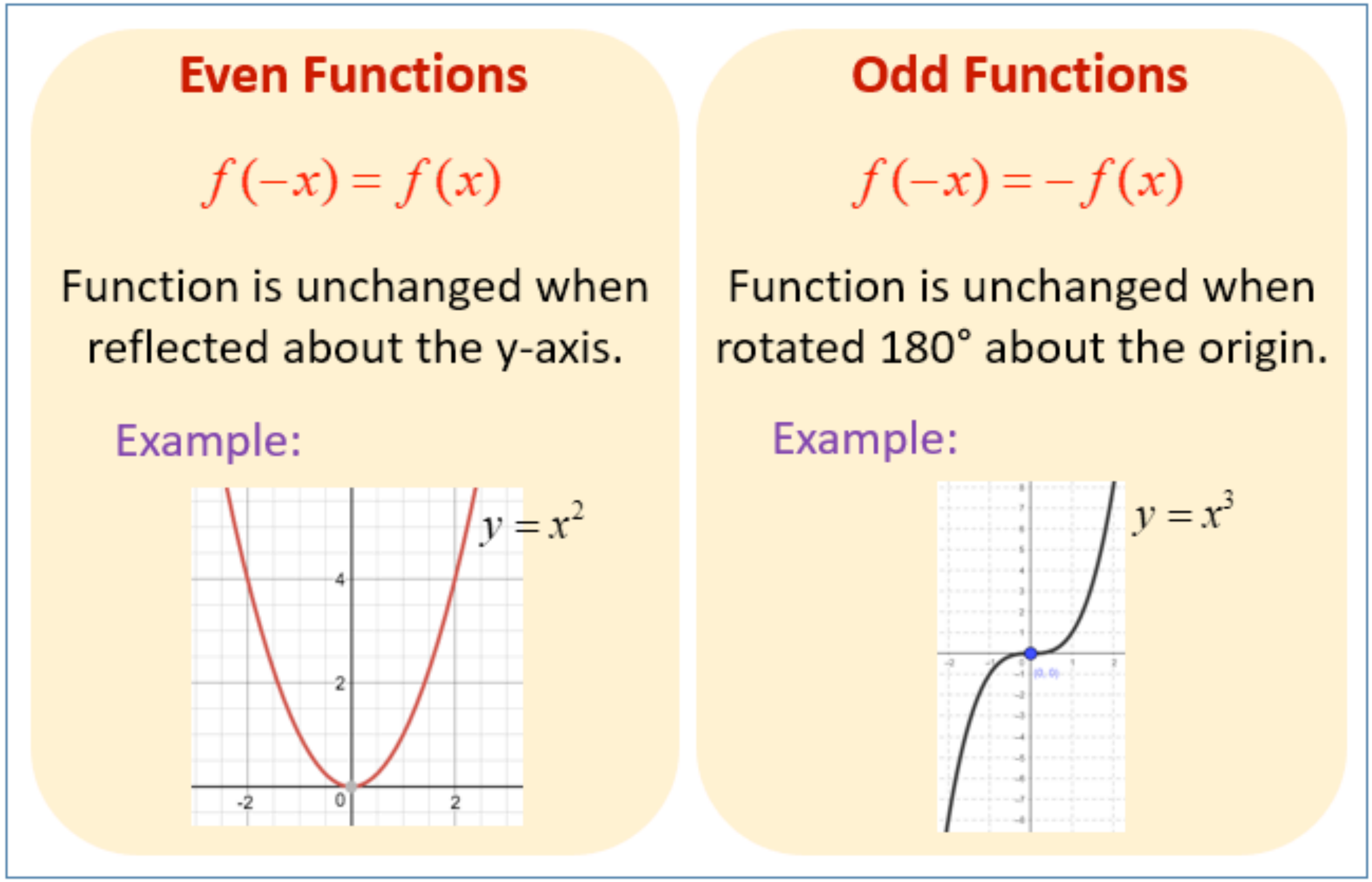
- They describe how a system remains unaltered under a given transformation
- We will focus on dynamical symmetries of motion and not on static symmetries e.g. as in crystals



FUNCTIONS AND SYMMETRIES OF A SYSTEM

A physical system is characterised by its quantum numbers and the equation of motion

Even if we don't know the inner dynamics of the systems e.g. its microscopic structure, we could deduce valuable information by observing its macroscopic behaviour



SYMMETRIES AND CONSERVATION LAWS

A system is normally described by its Lagrangian

- The Lagrangian can be found from first principles or can be deduced through the conservation laws of the system
- Noether's theorem connects symmetries with conservation laws
  - "Every symmetry in nature yields a conservation law and inversely every conservation law reveals an underlying symmetry
    - Momentum conservation: invariance under a translation in space
    - Angular momentum conservation: invariance under rotation in space

Emmy Noether
(1882 - 1935)

SYMMETRY OPERATORS

When is an operator called “Symmetry operator”?

Consider a transformation U that takes a state ψ and transforms it into a ψ’

- The wave functions are normalised: $\langle \Psi^{'} | \Psi^{'} \rangle = \langle \Psi | \Psi \rangle = 1$
- The transformation needs to be unitary $\hat{U}\hat{U}^{\dagger} = I$

We call U a symmetry operator if the new state ψ’ obeys the same Schrodinger equation as the initial wave function ψ

A symmetry operator is unitary and commutes with the Hamiltonian $[\hat{U}, \hat{H}] = 0$

WHEN IS AN OBSERVABLE CONSERVED?

Observables are always quantities that are represented by Hermitian operators

$$\hat{F} = \hat{F}^\dagger$$

- The expectation value is

$$\langle \hat{F} \rangle = \langle \Psi | \hat{F} | \Psi \rangle$$
$$\langle \hat{F} \rangle^* = \langle \Psi | \hat{F}^\dagger | \Psi \rangle$$

- An observable constant of motion F is Hermitian and commutes with the Hamiltonian

$$[\hat{F}, \hat{H}] = 0$$

INTRODUCTION TO GROUPS

CONTINUOUS TRANSFORMATIONS

They are unitary by definition but not necessarily Hermitian

- They rely on one or more continuous parameters so that
  - rotation by an angle α

$$|\Psi'\rangle = \hat{U}(\alpha)|\Psi\rangle$$

- These transformations can be written as a succession of infinitesimal deviations from the identity

$$\hat{U}(\alpha) = \lim_{n\to\infty}\left(\hat{I} + \frac{i\alpha}{n}\right)^n = e^{i\alpha\hat{F}}$$

- F is called the generator of U
- The generator of a unitary operator is Hermitian
- The generator of a symmetry operator commutes with the Hamiltonian
  - If U is a symmetry operator that commutes with H, then its generator is a Hermitian operator that also commutes with H

INTRODUCTION TO GROUPS

A group G is a collection of elements or operators a_1, a_2,…,a_n

- They have defined laws describing how one can combine any of the two elements with an operator e.g. “x” fulfilling the following conditions
  - **Closure**: For each two elements of G, their product is also an element of G

$$a_i \times a_j = a_k \in G$$

  - **Associativity**: Combining two elements is associative

$$(a_i \times a_j) \times a_k = a_i \times (a_j \times a_k)$$

  - **Identity element**: Every group has an identity element e such that for all elements of the group

$$a_i \times e = e \times a_i = a_i$$

  - **Inverse element**: For all elements in G there is a unique element such that

$$a_i \times (a_i)^{-1} = (a_i)^{-1} \times a_i = e$$

INTRODUCTION TO GROUPS

A group G is a collection of elements or operators a_1, a_2,…,a_n

- When a group consists of elements that any of two commute, then the group is called **Abelian**, otherwise non-Abelian
  - U(1) is an abelian group (QED is an abelian gauge theory)
  - SU(3) is a non-abelian group (QCD is a non-abelian gauge theory)
    - Fundamental implications!!! (Wait for next lecture)

- When a group contains finite number of elements n, then it is called finite group of order n

LIE GROUPS

In these lectures we focus on continuous transformations described by continuous groups, whose elements are labeled by continuous numbers (a_1, a_2,…,a_n) and by the relevant generators $g(a_1, a_2,…,a_n)=g(a)$

These transformations are usually represented by a set of matrices

- Coordinate transformations not only in space-time but also internal
  - isospin in SU(2), colour in SU(3)
  - Rotations in 3d form the SO(3) group with 3 real numbers as elements
  - Lorentz transformations form a group with six real numbers
    - 3 numbers for rotations and 3 for velocity transformation

LIE GROUPS

In these lectures we focus on continuous transformations described by continuous groups, whose elements are labeled by continuous numbers (a_1, a_2,…,a_n) and by the relevant generators $g(a_1, a_2,\ldots,a_n)=g(a)$

These transformations are usually represented by a set of matrices

- Parameterisations are arranged such that the 0th element is the identity element
- The closure requirement for a continuous group reads

$$g(a) \times g(b) = g[\gamma(a, b)]$$

  - where γ is a continuous function of α and b
  - If γ is an analytic function of α and b then the group is called a **Lie group**

Nikhef

GENERATORS OF TRANSFORMATIONS

Let us consider a group of transformations defined by

$$x_i^{'} = f_i(x_q, ..., x_n; a_1, ...a_n)$$

- x_i are the coordinates on which the transformation acts
- a_i are the elements (i.e. real numbers) of the transformation

By convention the identity element is a=0 such that

$$x_i^{'} = f_i(x; 0)$$

A transformation in the neighbour of the identity reads

$$dx_i^{'} = \sum_{j=1}^{n} \frac{\partial f_i}{\partial a_j} da_j$$

The generators of the transformation are found by considering a change in a function f(x) and are given by

$$\hat{X}_\nu = i \sum_{j=1}^{n} \frac{\partial f_j}{\partial a_\nu} \frac{\partial}{\partial x_j}$$

Nikhef

Thank you for
your attention!