Complete Bell state analyser for photonic qubits using *semi-demolition* or *entangled non-demolition* measurements

A Kozubov[1], A Gaidash[1], G Miroshnichenko[1],

[1]ITMO University, Saint-Petersburg 197101, Russia

Abstract

In this paper we present for the first time two possible techniques for deterministic two-step complete Bell state analyzer of optical (polarization) qubits using *semi-demolition* or *entangled non-demolition* measurements. Main difference to a prior studies in the field is that we do not use hyperentanglement or representation of the Bell states as concatenated Greenber–Horne–Zeilinger (C-GHZ) state to provide the discrimination. We demonstrate two different approaches for complete Bell state measurement based on different types of filtration. In *entangled non-demolition* measurement we allocate two pairs of the states from each other as the filtration process. The approach can be based on the utilization of cubic (Kerr) nonlinearity and auxiliary mode. In *semi-demolition* measurement two states are unambiguously discriminated and hence destroyed; however two other states passes the filter without modification. The measurement destroys the single photon subspace in every mode and preserves the superposition of zero and two photons. It can be realized with discrete photodetection based on microresonator with atoms. Such filtration can be considered as quadratic nonlinearity just as any measurement. The most significant about this approach is that we do not transform the initial states using any type of filtration based on different nonlinearities.

Bell state analyser with entangled non-demolition measurement

States that one needs to distinguish can be described as:

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(a_{H_1}^\dagger a_{V_2}^\dagger \pm a_{V_1}^\dagger a_{H_2}^\dagger)|0\rangle, \quad |\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(a_{H_1}^\dagger a_{H_2}^\dagger \pm a_{V_1}^\dagger a_{V_2}^\dagger)|0\rangle,$$

where $a_{H_1}^\dagger, a_{V_1}^\dagger$ and $a_{H_2}^\dagger, a_{V_2}^\dagger$ are the creation operators for horizontal and vertical modes of the first and the second spatial modes respectively, $|0\rangle$ is the vacuum state. We rewrite our initial states in spatial mode notation. Further we express creation operators, basis vectors, and considered Bell states respectively according to the described transformation as follows:

$$a_{H1}^\dagger = a_1^\dagger, \quad a_{V1}^\dagger = a_2^\dagger, \quad a_{H2}^\dagger = a_3^\dagger, \quad a_{V2}^\dagger = a_4^\dagger \ |H\rangle_1 = |1\rangle_1, \quad |V\rangle_1 = |1\rangle_2, \quad |H\rangle_2 = |1\rangle_3, \quad |V\rangle_2 = |1\rangle_4$$

$$|\Psi^+\rangle = |B_1\rangle = \frac{1}{\sqrt{2}}(a_1^\dagger a_4^\dagger + a_2^\dagger a_3^\dagger)|0\rangle, \quad |\Psi^-\rangle = |B_2\rangle = \frac{1}{\sqrt{2}}(a_1^\dagger a_4^\dagger - a_2^\dagger a_3^\dagger)|0\rangle,$$

$$|\Phi^+\rangle = |B_3\rangle = \frac{1}{\sqrt{2}}(a_1^\dagger a_3^\dagger + a_2^\dagger a_4^\dagger)|0\rangle, \quad |\Phi^-\rangle = |B_4\rangle = \frac{1}{\sqrt{2}}(a_1^\dagger a_3^\dagger - a_2^\dagger a_4^\dagger)|0\rangle.$$

It is obvious that pair $\{|B_1\rangle, |B_2\rangle\}$ is unambiguously distinguishable from pair $\{|B_3\rangle, |B_4\rangle\}$. Nevertheless there is no distinguishability between the states inside the pairs. Let us introduce the filtering operation that discriminates between described pairs. Filtering is performed using operators that have filtered states as their eigenvectors with an eigenvalue that equals to one. Filtration operations can be performed by QND measurements using cross-Kerr nonlinearity. QND measurement in each mode separately it will destroy the entanglement and thus one cannot discriminate between states inside the pair. In order to avoid entanglement destruction we propose the next filtering:

$$\hat{F}_{12} = |1\rangle_1\langle 1| \otimes |1\rangle_4\langle 1| + |1\rangle_2\langle 1| \otimes |1\rangle_3\langle 1|,$$
$$\hat{F}_{34} = |1\rangle_1\langle 1| \otimes |1\rangle_3\langle 1| + |1\rangle_2\langle 1| \otimes |1\rangle_4\langle 1|.$$

The result of non-demolition measurements is recorded as the total phase of coherent state $|\alpha\rangle$ stored in auxiliary mode. As a result of the Kerr interaction, the coherent state is transformed and acquires the total phase:

$$|\alpha\rangle \to |\alpha \exp(i \cdot \Theta)\rangle, \ \Theta = \Theta_1 + \Theta_2 + \Theta_3 + \Theta_4.$$

Without loss of generality we may assume that phases related to channels 1 and 3 are $\Theta_1 = \Theta_3 = \pi/2$ and related to channels 2 and 4 are $\Theta_2 = \Theta_4 = -\pi/2$. Thus $|B_1\rangle, |B_2\rangle$ are filtered when $\Theta = 0$ and $|B_3\rangle, |B_4\rangle$ when $\Theta = \pm\pi$. Indeed the state after filtration are as follows:

$$|B_{1,2}\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_4 \pm |1\rangle_2|1\rangle_3) \otimes |\alpha\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_4 \otimes |\alpha\rangle \pm |1\rangle_2|1\rangle_3 \otimes |\alpha\rangle) \to$$
$$\to \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_4 \otimes |\alpha \cdot \exp(i(\Theta_1 + \Theta_4))\rangle \pm |1\rangle_2|1\rangle_3 \otimes |\alpha \cdot \exp(i(\Theta_2 + \Theta_3))\rangle) = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_4 \pm |1\rangle_2|1\rangle_3) \otimes |\alpha\rangle$$
$$|B_{3,4}\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_3 \pm |1\rangle_2|1\rangle_4) \otimes |\alpha\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_3 \otimes |\alpha\rangle \pm |1\rangle_2|1\rangle_4 \otimes |\alpha\rangle) \to$$
$$\to \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_3 \otimes |\alpha \cdot \exp(i(\Theta_1 + \Theta_3))\rangle \pm |1\rangle_2|1\rangle_4 \otimes |\alpha \cdot \exp(i(\Theta_2 + \Theta_4))\rangle) = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_3 \pm |1\rangle_2|1\rangle_4) \otimes |-\alpha\rangle$$

Using well-known relations between input and output creation operators for beam splitters one can express the states as follows:

$$|B_1\rangle = \frac{1}{\sqrt{2}}(|1\rangle_2|1\rangle_4 - |1\rangle_1|1\rangle_3), \qquad |B_3\rangle = \frac{1}{\sqrt{2}}(|1\rangle_3|1\rangle_4 + |1\rangle_1|1\rangle_2),$$
$$|B_2\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_4 + |1\rangle_2|1\rangle_3). \qquad |B_4\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_3 + |1\rangle_2|1\rangle_4).$$
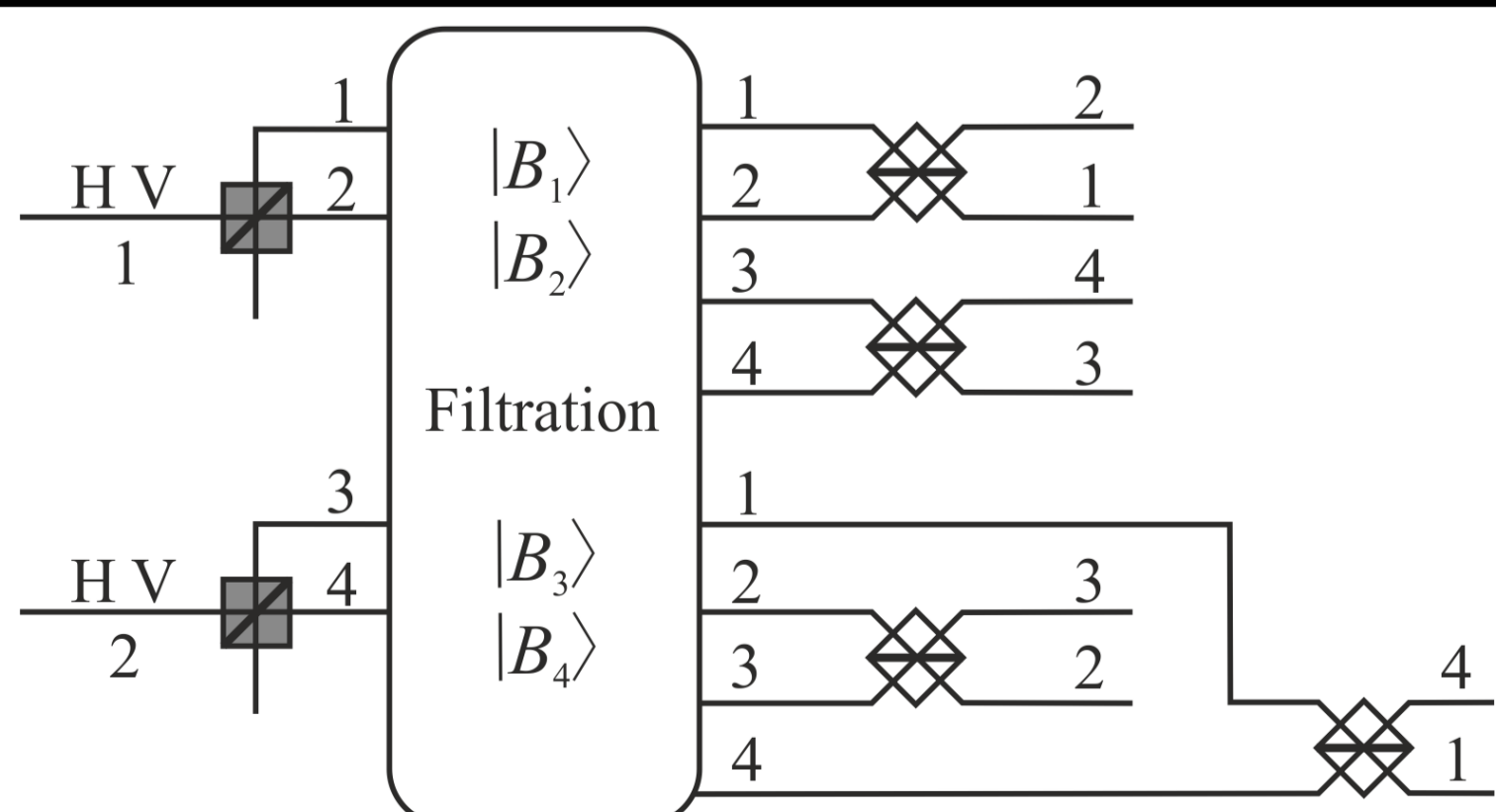
Bell state analyser with semi-demolition measurement

Using the notation introduced in the previous part one can rewrite the Bell states as follows:

$$|\Psi^+\rangle = |B_1\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_2 - |1\rangle_3|1\rangle_4), \quad |\Psi^-\rangle = |B_2\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_3 - |1\rangle_2|1\rangle_4),$$
$$|\Phi^+\rangle = |B_3\rangle = \frac{1}{\sqrt{2}}(|2\rangle_1 + |2\rangle_2 - |2\rangle_3 - |2\rangle_4), \quad |\Phi^-\rangle = |B_4\rangle = \frac{1}{\sqrt{2}}(-|2\rangle_1 + |2\rangle_2 - |2\rangle_3 + |2\rangle_4)$$

where $|1\rangle_j$ and $|2\rangle_j$ are single- and two-photon states in j^{th} spatial mode respectively. First two states can be easily distinguished since the distributions of single photons over four channels differ. It can be provided with Kraus operators:

$$K_1^\dagger K_1 = |B_1\rangle\langle B_1| = \left(K_{12}^\dagger - K_{34}^\dagger\right) = (K_{12} - K_{34}), K_2^\dagger K_2 = |B_2\rangle\langle B_2| = \left(K_{13}^\dagger - K_{24}^\dagger\right) = (K_{13} - K_{24})$$

$$K_{12} = \frac{1}{\sqrt{2}}|0\rangle_1\langle 1| \otimes |0\rangle_2\langle 1|, \qquad K_{13} = \frac{1}{\sqrt{2}}|0\rangle_1\langle 1| \otimes |0\rangle_3\langle 1|,$$
$$K_{34} = \frac{1}{\sqrt{2}}|0\rangle_3\langle 1| \otimes |0\rangle_4\langle 1|, \qquad K_{24} = \frac{1}{\sqrt{2}}|0\rangle_2\langle 1| \otimes |0\rangle_4\langle 1|$$

It is impossible to distinguish between states $|B_3\rangle$ and $|B_4\rangle$ by the distribution of two-photon states over four channels. However, it is possible to construct the Kraus operator K_3 which provides filtering of the states $|B_3\rangle$ and $|B_4\rangle$ without their destruction:

$$K_3|B_3\rangle = |B_3\rangle, \quad K_3|B_4\rangle = |B_4\rangle, \quad K_3|B_1\rangle = K_3|B_2\rangle = 0,$$
$$\sum_{j=1}^{3} = K_j^\dagger K_j = \sum_{m=1}^{4}|B_m\rangle\langle B_m| = \mathbb{I}$$

The Kraus operators are constructed from the set of projective operators in each mode:

$$k_1(\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle) = -i\beta|0\rangle, \quad k_0(\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle) = \alpha|0\rangle + \gamma|2\rangle.$$

where k_j is corresponding Kraus operator that acts in every channel. Thus the Kraus operators for introduced measurement can be described as follows:

$$K_1 = k_1^{(2)}k_1^{(4)}k_0^{(1)}k_0^{(3)} + k_1^{(1)}k_1^{(3)}k_0^{(2)}k_0^{(4)}, \quad K_2 = k_1^{(1)}k_1^{(4)}k_0^{(2)}k_0^{(3)} + k_1^{(2)}k_1^{(3)}k_0^{(1)}k_0^{(4)},$$
$$K_3 = U^\Phi\left[k_0^{(1)}k_0^{(1)}k_0^{(3)}k_0^{(4)}\right], \quad K_x = U^x\sqrt{A_x},$$

where U is arbitrary unitary operator. Due to this uncertainty one can choose any suitable unitary operator. Thus one can provide any possible configuration of the states after the implementation of POVM operator. In our case the unitary operation can be realized using two 50 : 50 beam splitters connecting channels 1 and 2, 3 and 4 respectively. The state transformation during the filtration and appropriate unitary operation on the first step can be described as:

$$|B_3\rangle \xrightarrow{U^\Phi} \frac{1}{2\sqrt{2}}\left(|2\rangle_1 + |2\rangle_2 - |2\rangle_3 - |2\rangle_4\right), \quad |B_4\rangle \xrightarrow{U^\Phi} \frac{1}{2\sqrt{2}}(|1\rangle_1|1\rangle_2 - |1\rangle_3|1\rangle_4).$$

FIG.1. *Principle scheme for Bell state measurement with filtration operation based on entangled non-demolition measurement. With grey and white colours polarization and 50:50 beam splitters are denoted respectively.*
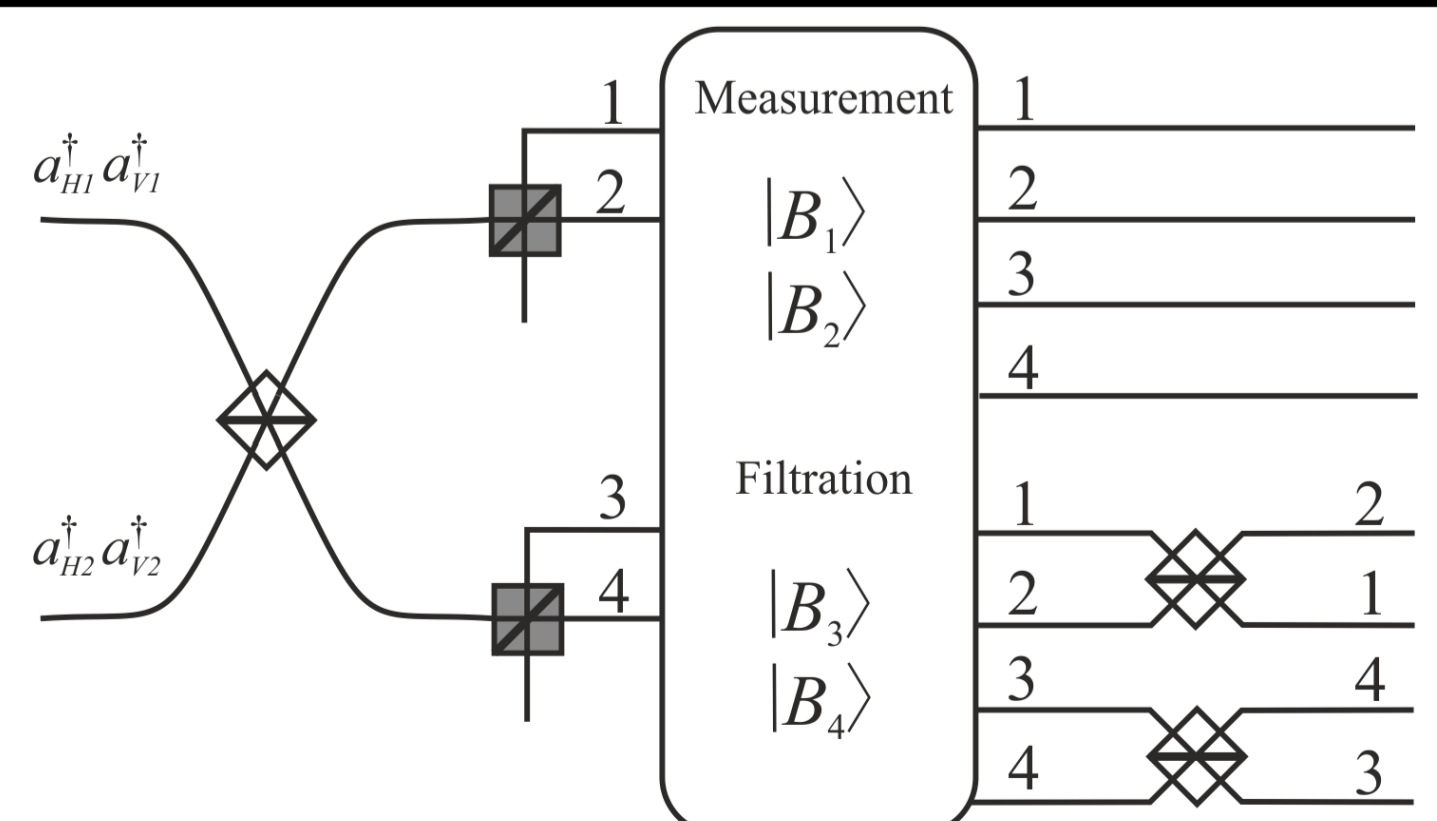

FIG. 2. *Principle scheme for Bell state measurement with filtration operation based on semi-demolition measurement. With grey and white colours polarization and 50:50 beam splitters are denoted respectively.*

References
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