Convex Analysis for Optimization - Exercise set 1

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Important note: you can certainly find the solutions to some of the below exercises. However, if you do so right at the start, you will not learn. So, try as much as you can on your own before searching for an existing solution.

Problem 1. Consider a convex set $C \subset \mathbb{R}^n$. Show that the set

$$\bar{C} = \{x \in \mathbb{R}^{n+1} : x = t(y, 1),\ y \in C,\ t \geq 0\}$$

is a convex cone.

Problem 2. Show that the closure of a convex set is a convex set as well.

Problem 3. Prove the basic rules of 'calculus of convexity' of sets, i.e., that the following are convex

- $\bigcap_i A_i$ is convex for any (possibly infinite) collection of convex sets A_i
- $A + B$ is convex
- $\mathbf{T}A$ is convex, where $\mathbf{T}$ is a linear transformation
- $\mathbf{T}^{-1}A$ is convex, where $\mathbf{T}$ is a linear transformation

where A, B are convex sets.

Problem 4. Show that each norm in a Euclidean space is a convex function. Recall that a norm is defined by three properties: $\|x + y\| \leq \|x\| + \|y\|$, $\|x\| = 0 \iff x = 0$ and $\|\alpha x\| = |\alpha|\|x\|$ for a scalar α.

Problem 5. Prove that the convex hull of a compact set is compact.

Problem 6. For a convex $S \subseteq \mathbf{R}^n$, assume its interior is non-empty. Prove that $\text{core}(S) = \text{int}(S)$. Hint: use the Line Segment Principle for $\text{int}(S)$.