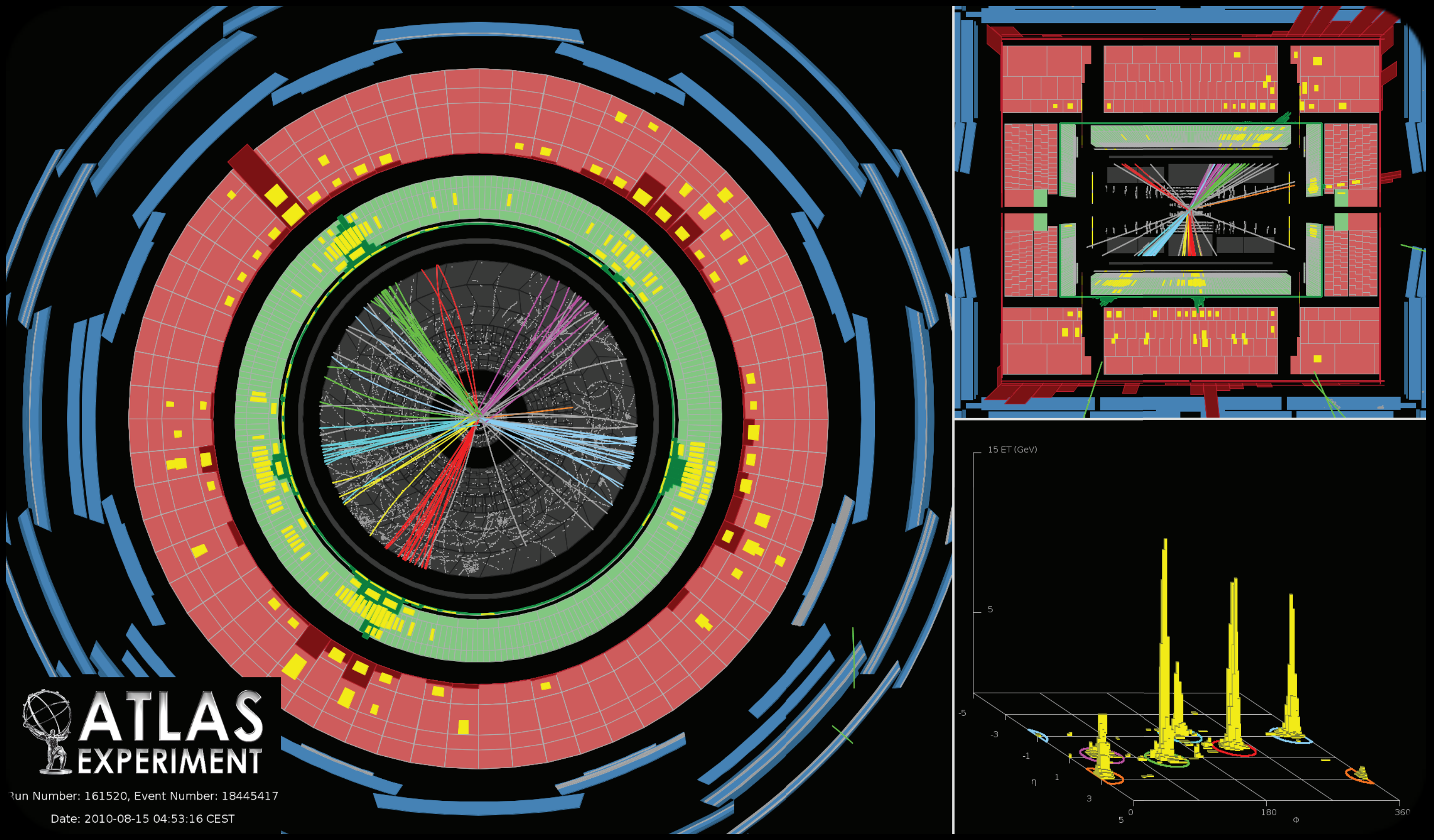
Strong interactions



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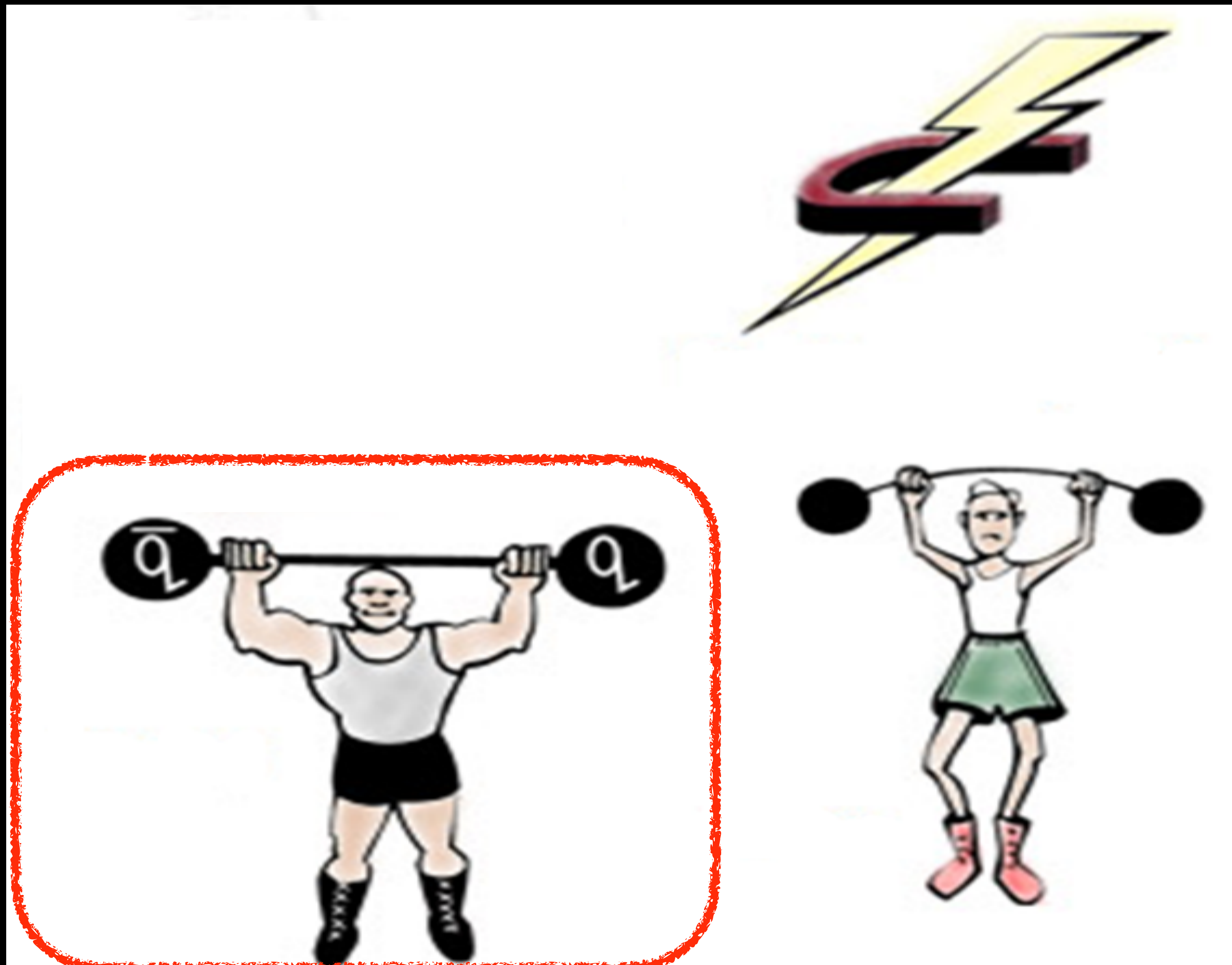
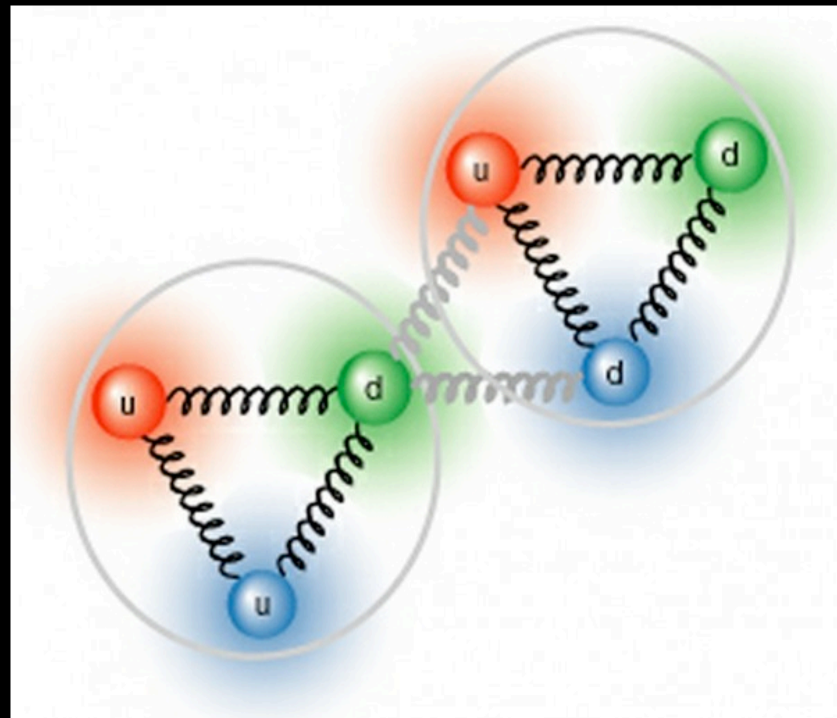
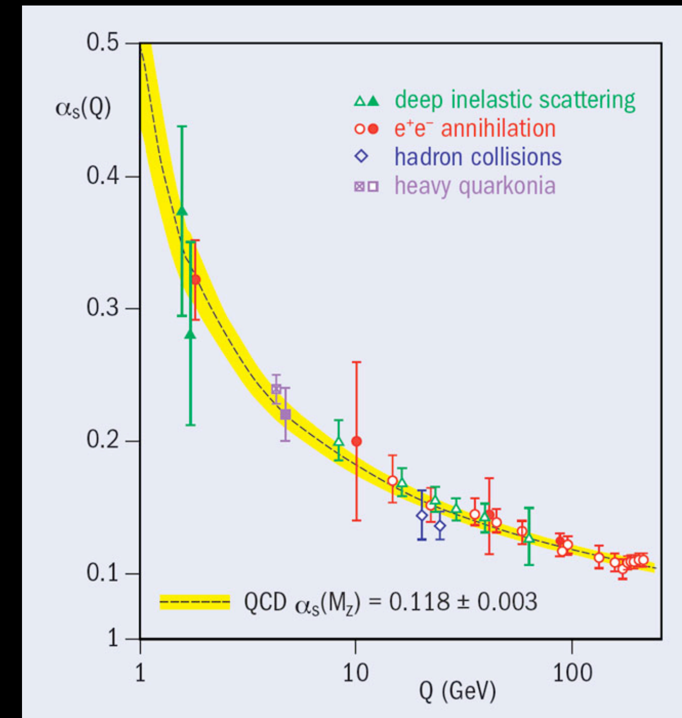
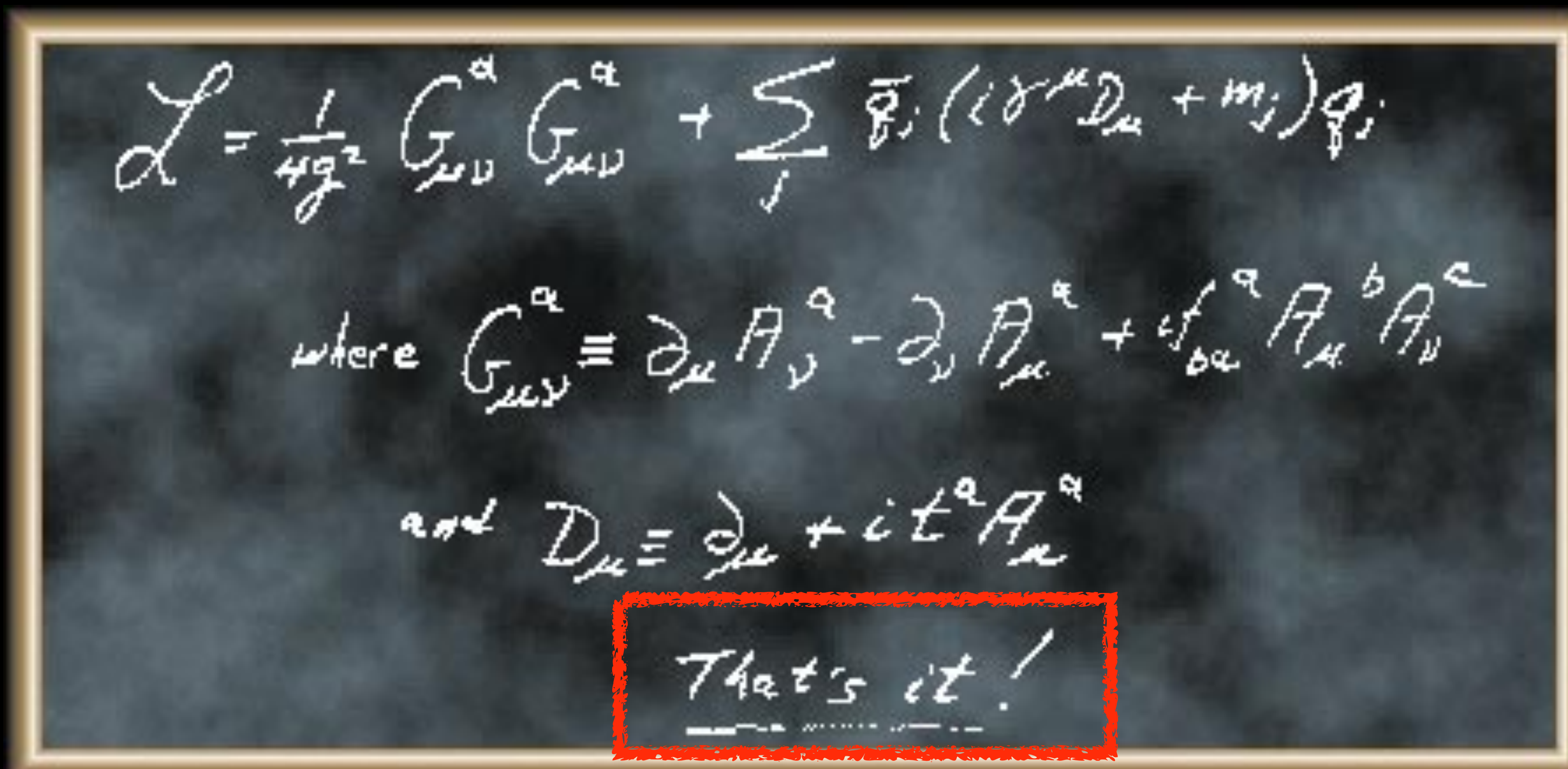
Forces and mediators



Quantum ChromoDynamics (QCD)







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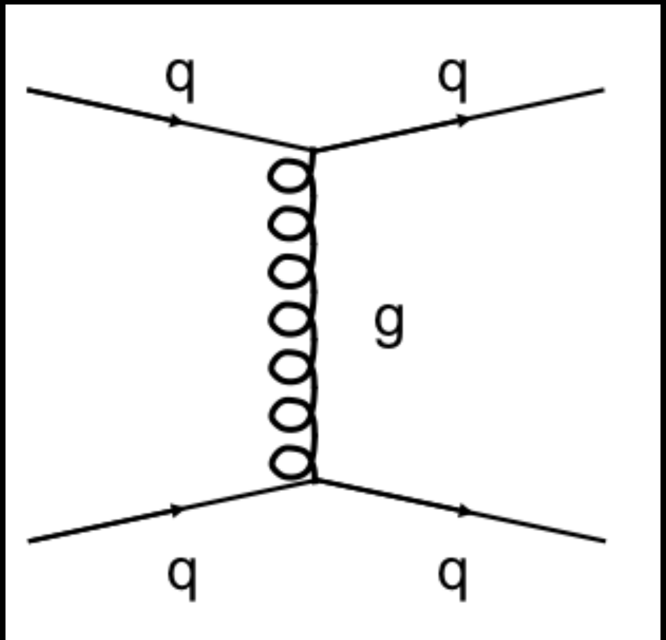
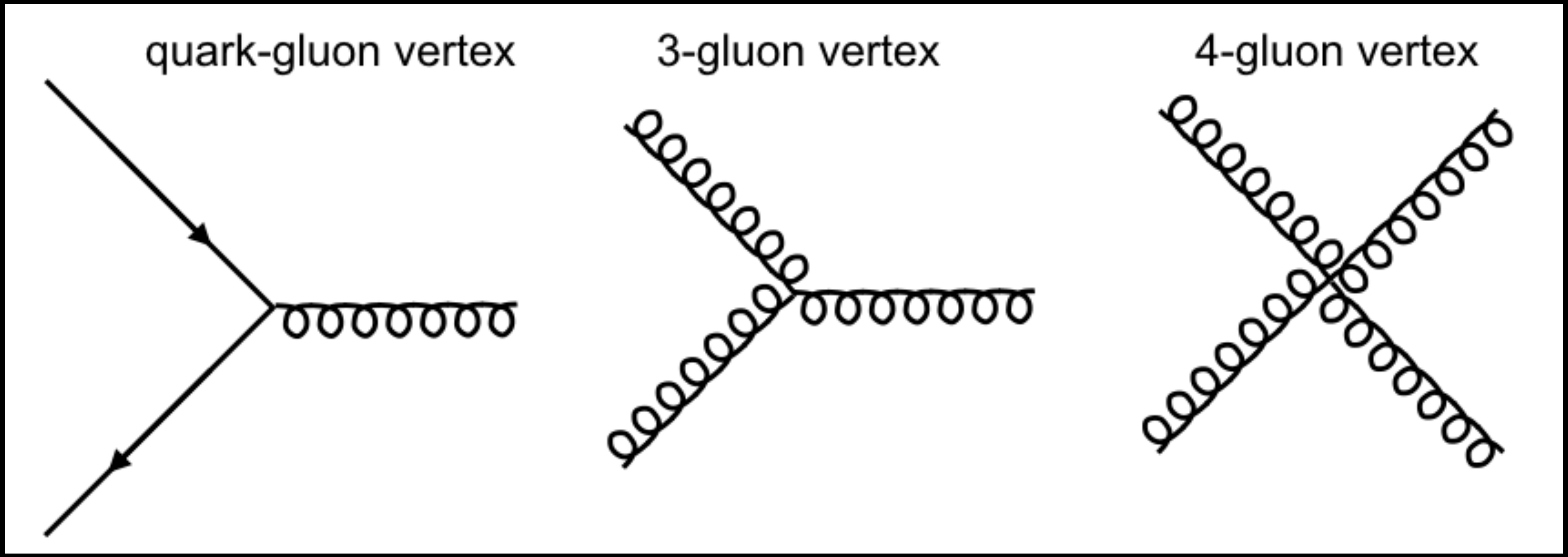
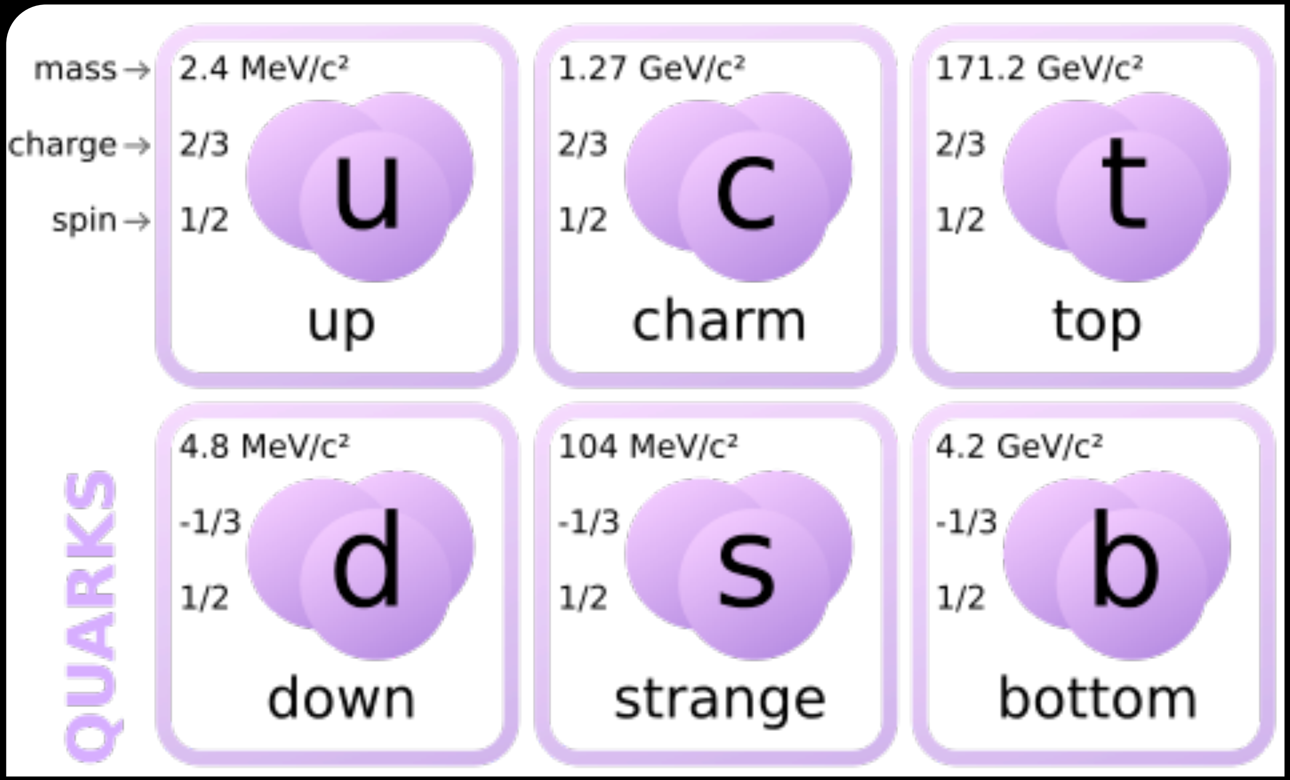
Particles and interactions: strong interactions





- ✓ Mediated by the gluon (g)
- ✓ Strongest interaction in the standard model
- ✓ Acts on gluons and quarks (and hadrons)
- ✓ Responsible for binding composite particles made of quarks and gluons (e.g. protons)

Quarks


mass
charge
spin
QUARKS
2.4 MeV/c²
2/3
1/2
u
up
1.27 GeV/c²
2/3
1/2
c
charm
171.2 GeV/c²
2/3
1/2
t
top
4.8 MeV/c²
-1/3
1/2
d
down
104 MeV/c²
-1/3
1/2
s
strange
4.2 GeV/c²
-1/3
1/2
b
bottom

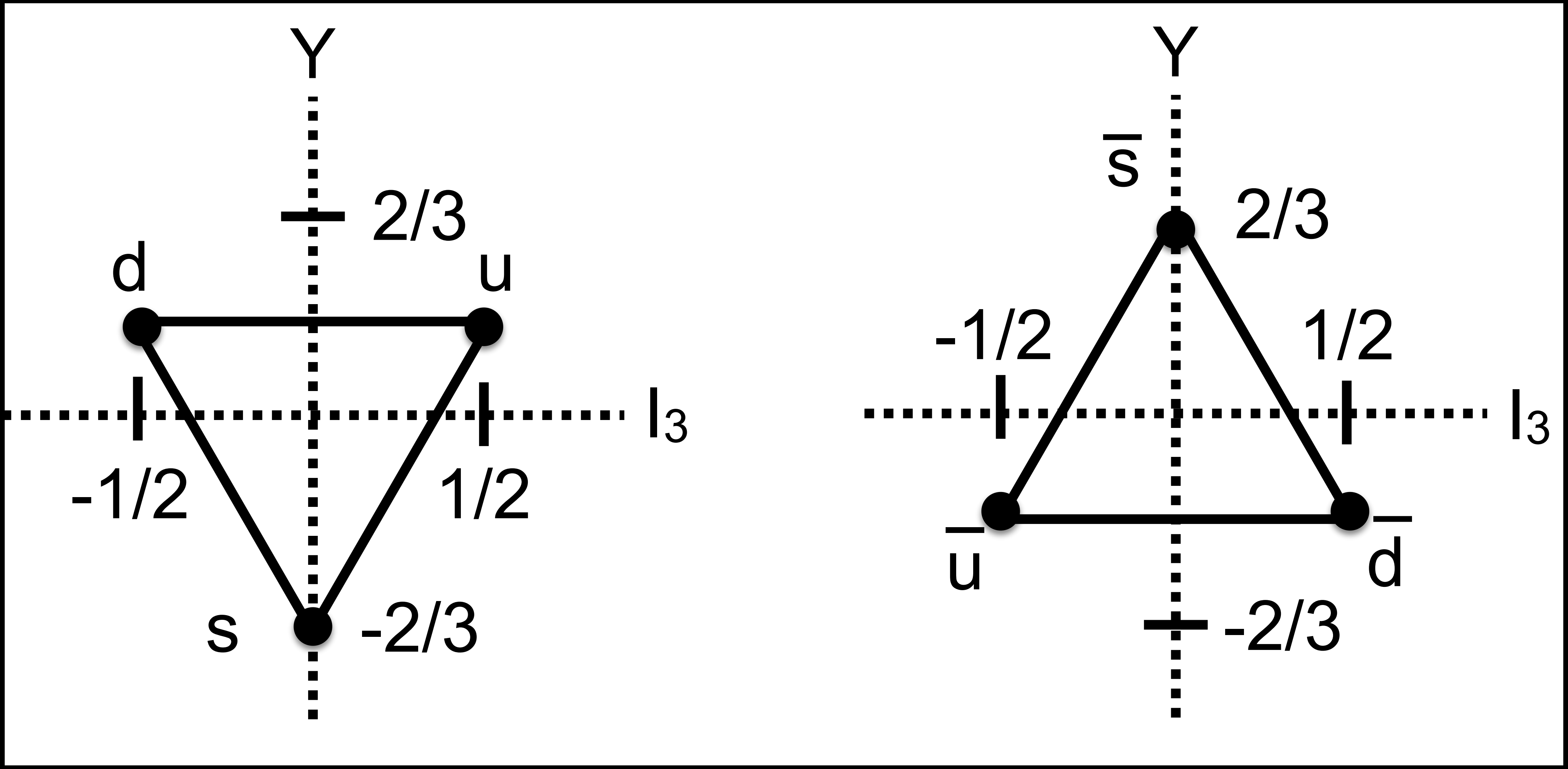
Quarks

Table 15.1: Additive quantum numbers of the quarks.

	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	-1	0	0	0
C – charm	0	0	0	$+1$	0	0
B – bottomness	0	0	0	0	-1	0
T – topness	0	0	0	0	0	$+1$

Quark model

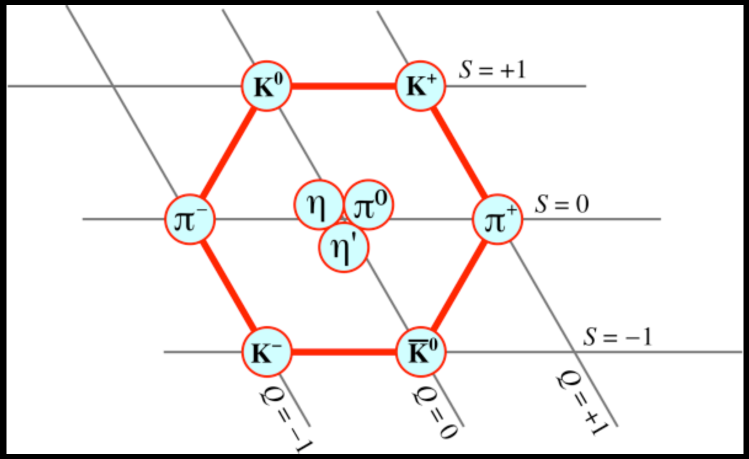
- A pattern for classifying hadrons based on their quantum numbers
- All mesons are made of a combination of a quark and an anti-quark
- All baryons are made of a combination of three quarks
- The basic representation of SU(3) is a triplet

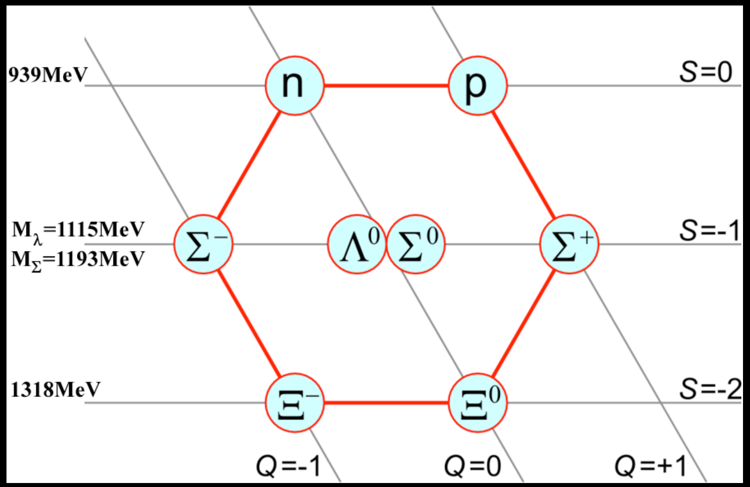


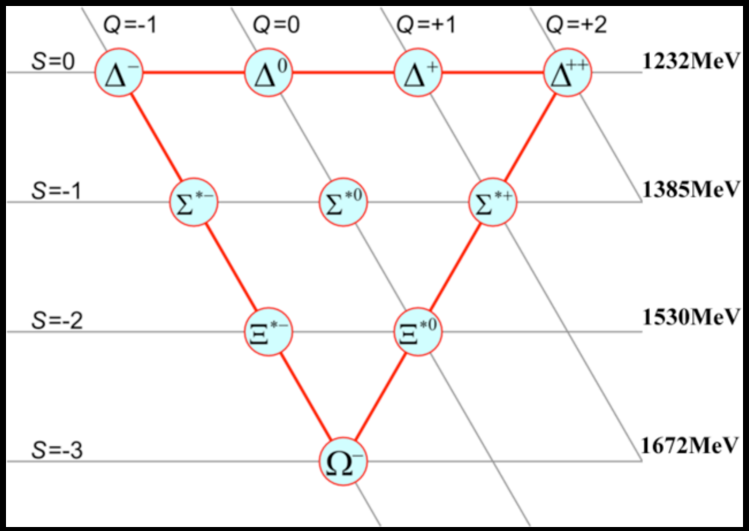
Quark model

- Gell-Mann and Zweig (1964)
  - All multiplets can be explained if you assume that hadrons are composite particles built from more elementary constituents: the quarks and antiquarks
    - Baryons are made of three quarks (Antibaryons are made of three antiquarks)
    - Mesons are made of a quark and an antiquark combination
- First quark model consisted of the three lightest quarks (and antiquarks)

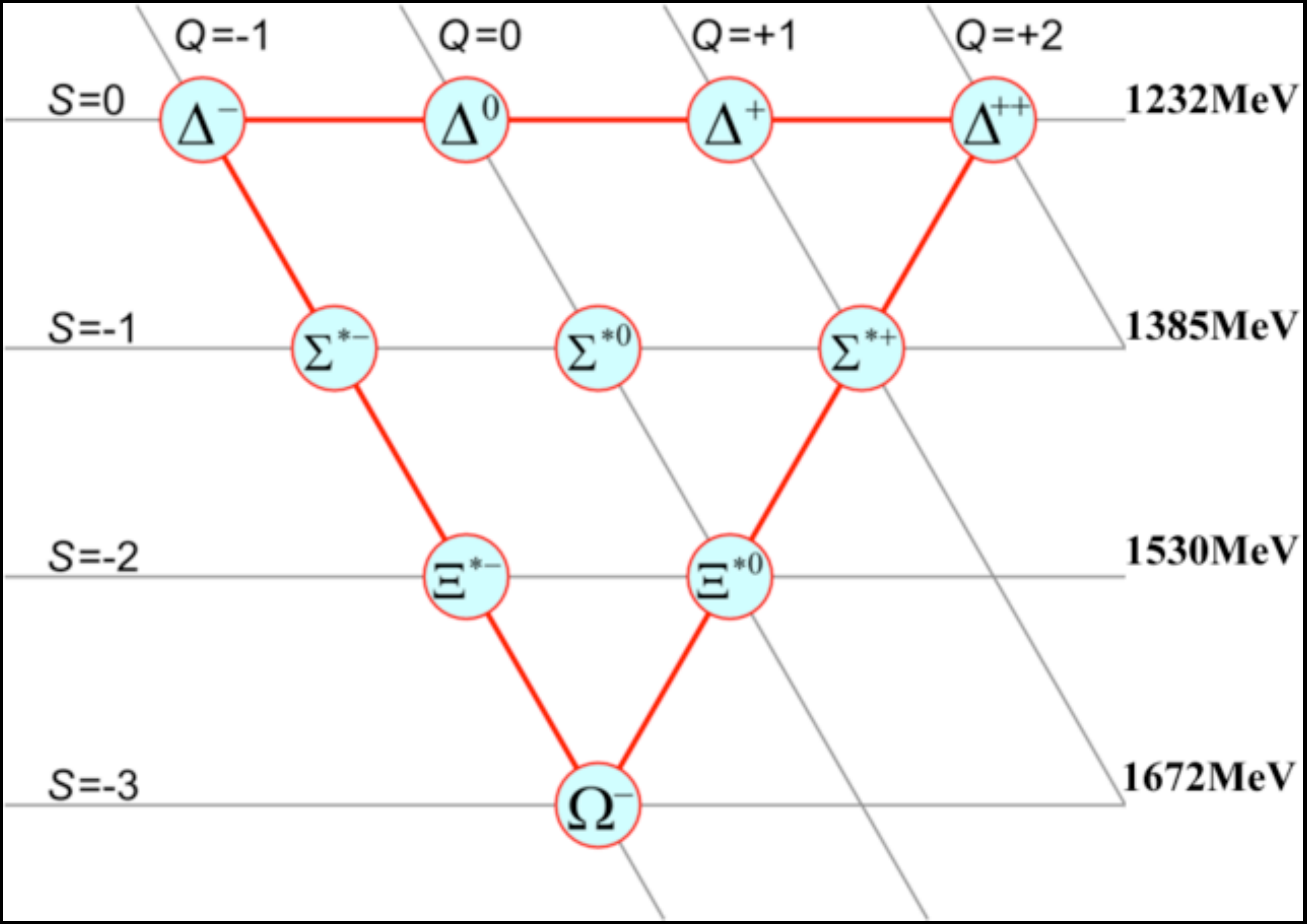
up down strange







Baryons


Q=-1
Q=0
Q=+1
Q=+2
S=0
S=-1
S=-2
S=-3
Δ⁻
Δ⁰
Δ⁺
Δ⁺⁺
Σ*⁻
Σ*⁰
Σ*⁺
Ξ*⁻
Ξ*⁰
Ω⁻
1232MeV
1385MeV
1530MeV
1672MeV

Problem of the quark model

How can a baryon like the Δ^{++} (uuu) or the Ω^- (sss) given the Pauli principle?

Δ^{++} (uuu) u↑ u↑ u↑ Intrinsic spin: symmetric $\left|\frac{3}{2},+\frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$

Quarks: symmetric $|uuu\rangle$

Ω^- (sss) s↑ s↑ s↑ Intrinsic spin: symmetric $\left|\frac{3}{2},+\frac{3}{2}\right\rangle = \uparrow\uparrow\uparrow$

Quarks: symmetric $|sss\rangle$

Half-integer spin particle ⇒ fermion that obeys the Fermi-Dirac statistics: anti-symmetric wave-functions

Solution: Introduce a new quantum number ⇒ **COLOUR**

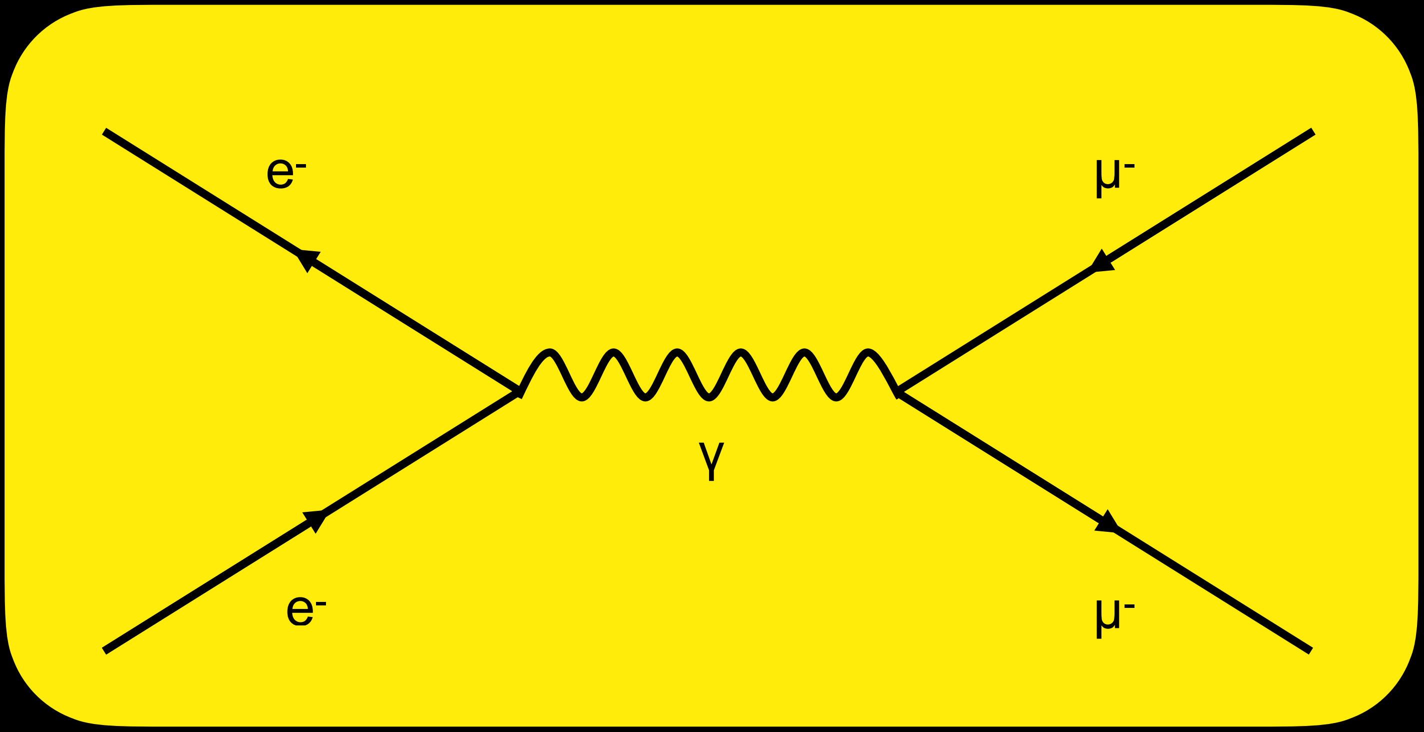
- ✓ Comes with three flavours: RED, GREEN, BLUE
- ✓ Applicable for quarks not leptons!

All naturally occurring particles come in **colour singlet** states: **invariant under rotations in colour space**

Δ^{++} (uuu) u↑ u↑ u↑ Ω^- (sss) s↑ s↑ s↑

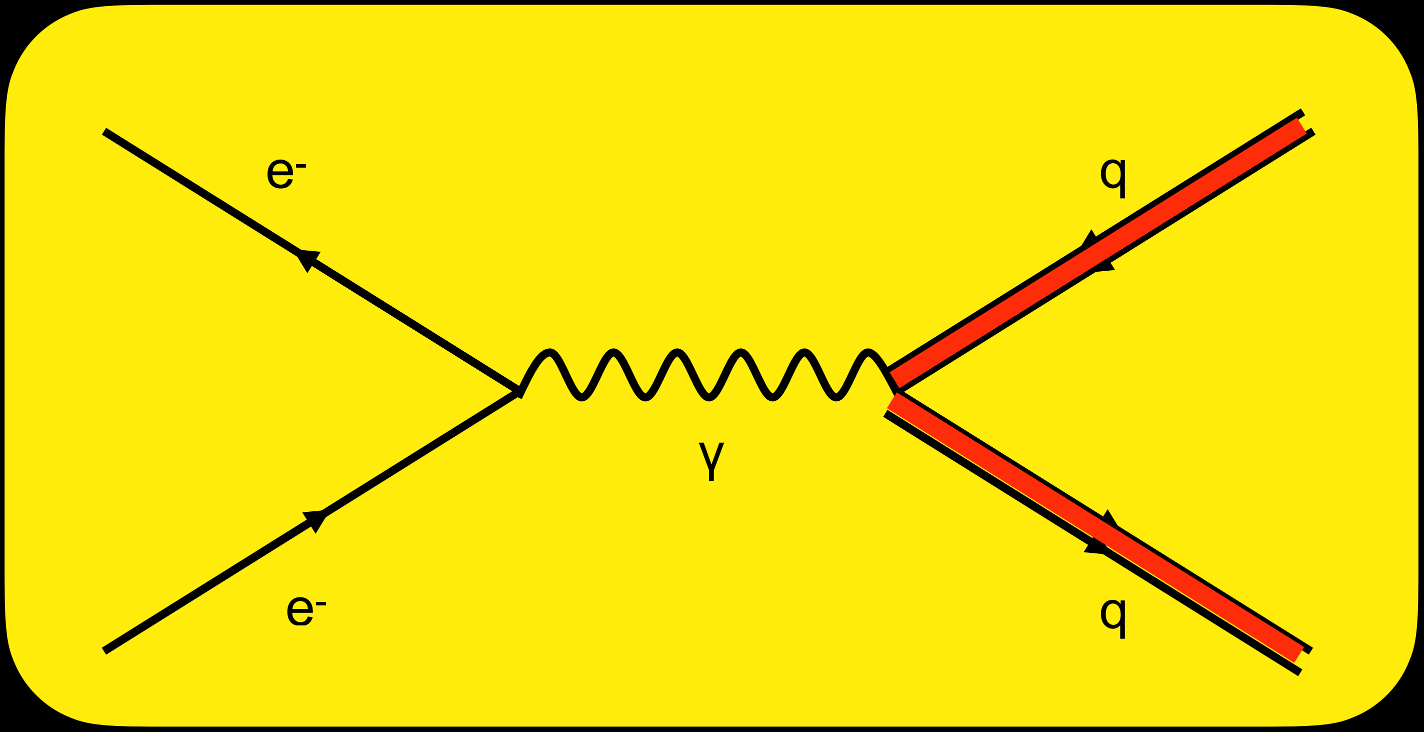
Evidence for colour

- Existence of colour via QED processes
  - The first one (i.e. muon production via electron-positron scattering) is well controlled experimental and is used as a reference



Evidence for colour (cont.)

- How can we probe colour via a QED process?
  - The second process (i.e. quark production via electron-positron scattering) can not actually be observed in nature

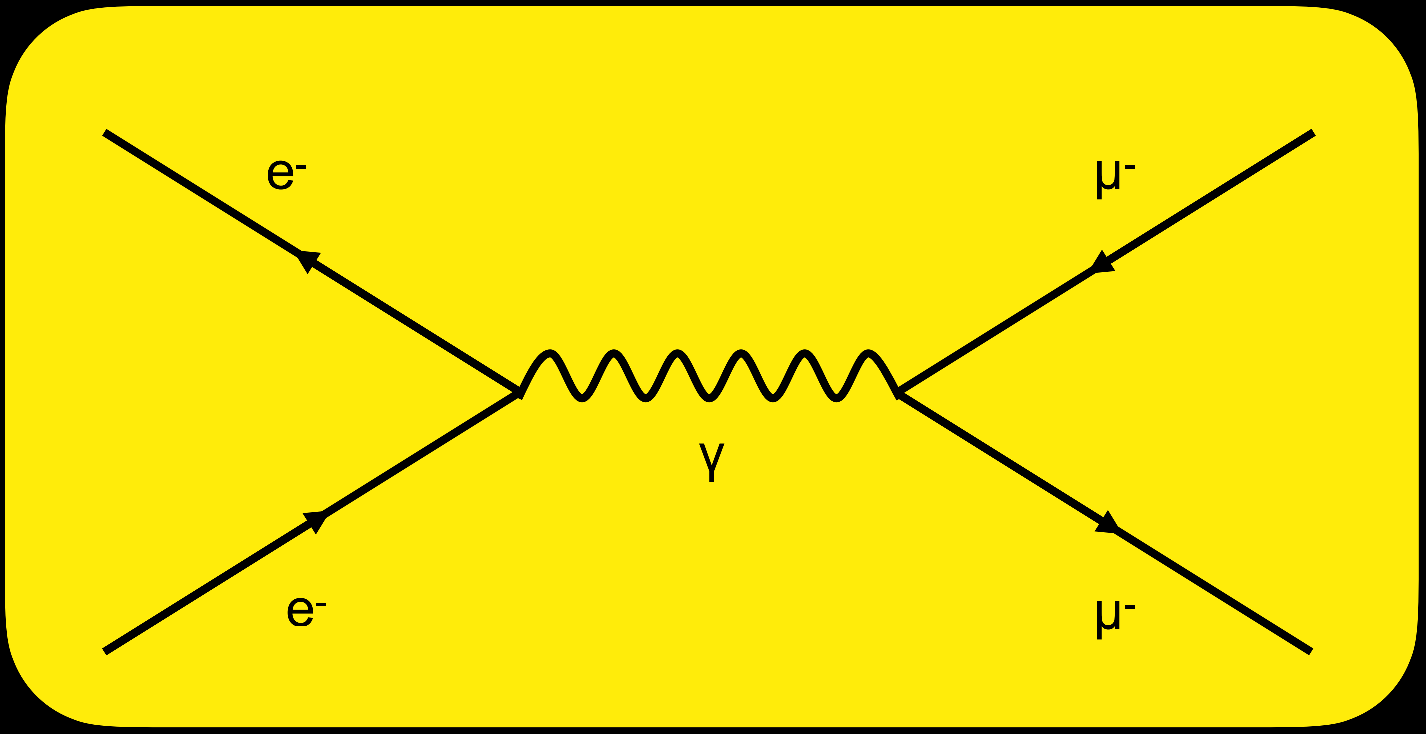


Evidence for colour (cont.)

- How can we probe colour via a QED process?
  - The second process (i.e. quark production via electron-positron scattering) can not actually be observed in nature
    - These quarks do not fly free for long (i.e. they can fly as “free” within the size of a hadron)
    - They fragment producing additional q-qbar pairs that when combined form hadrons
      - This is a QCD-type of process
    - The final state particles are detected as a collimated spray of hadrons ➡JETS
    - Due to energy-momentum conservation, these jets emerge in a back-to-back topology
- Any difference between the cross-sections of these two processes from a naive scaling with the square of the charge will signal the existence of additional factors

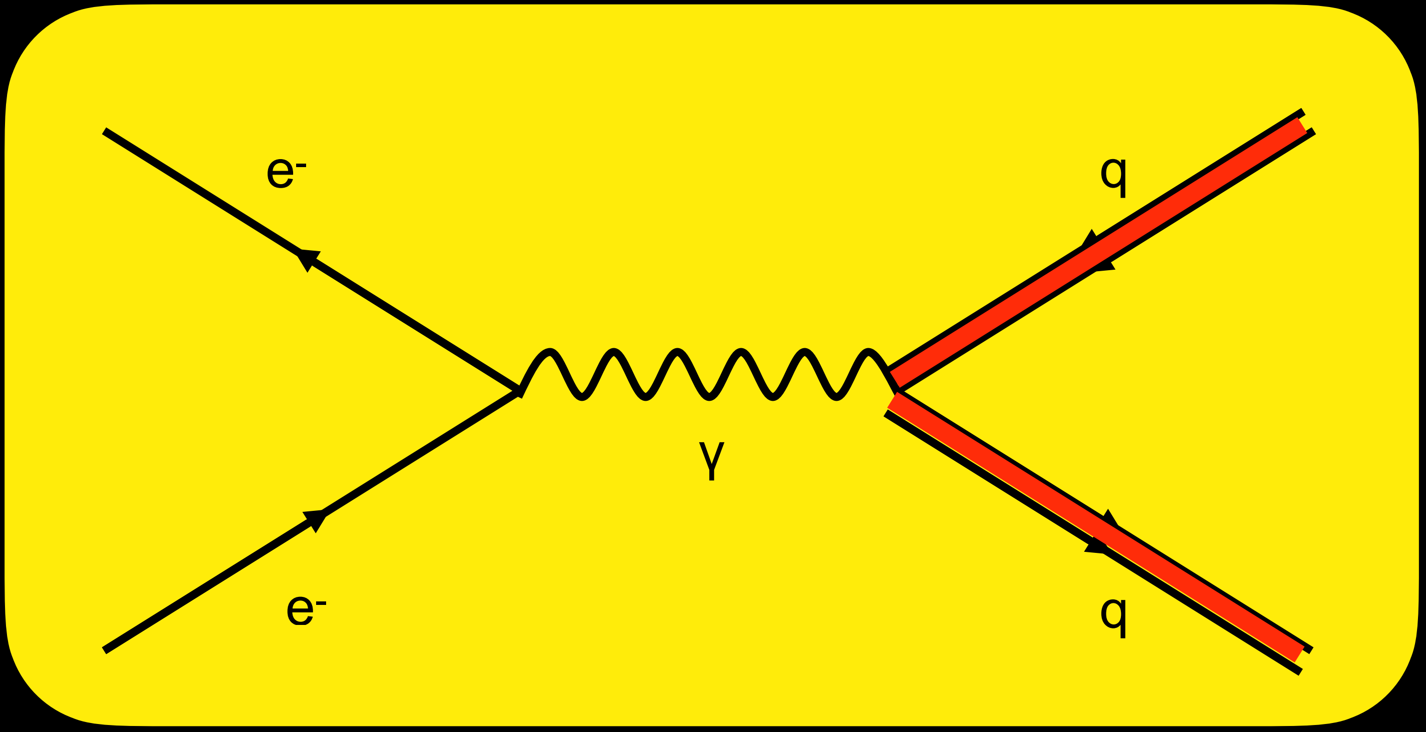
Evidence for colour (cont.)

QED process



Final state
particles that can be seen in nature

QED process

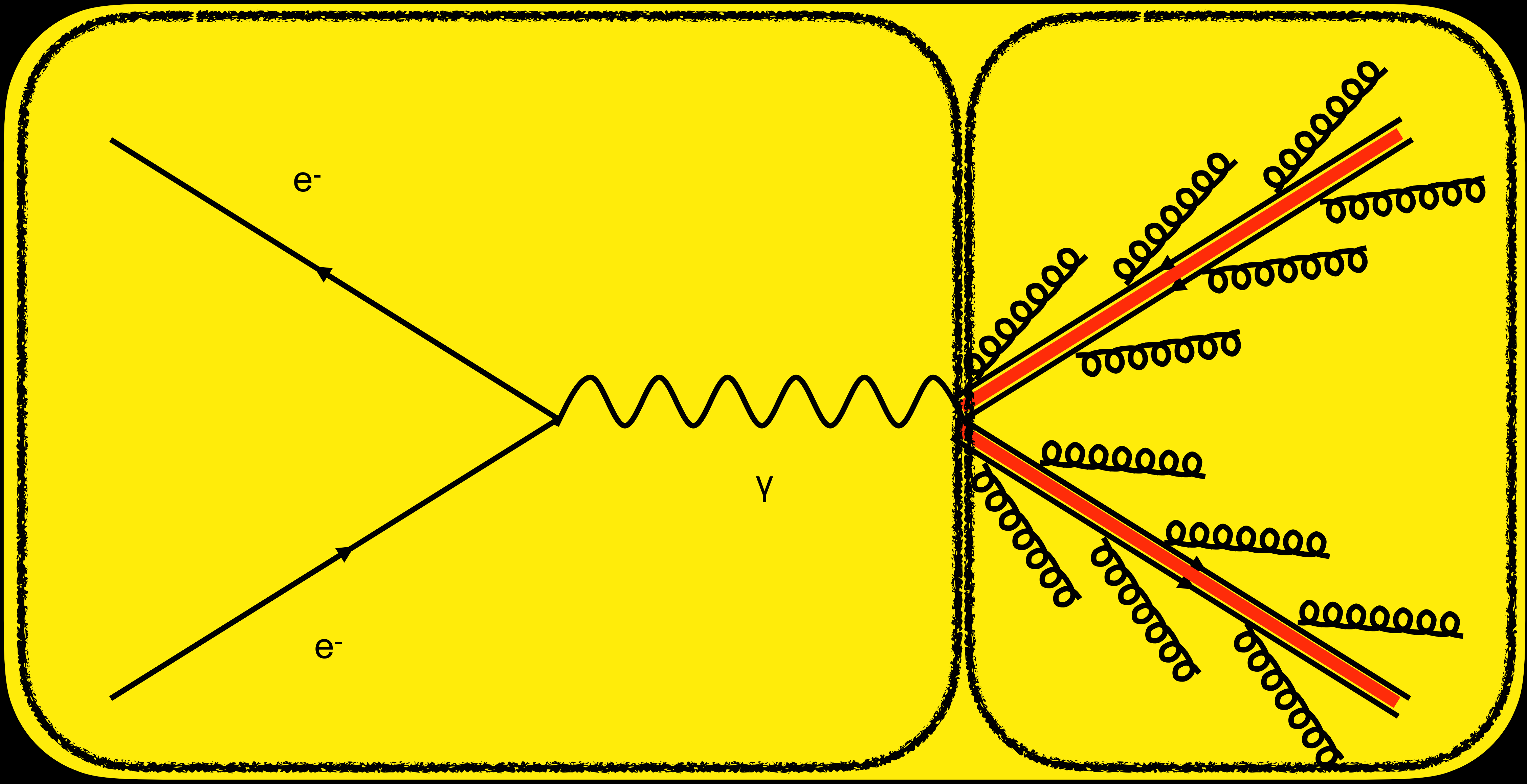


Final state
particles that do not fly free in nature

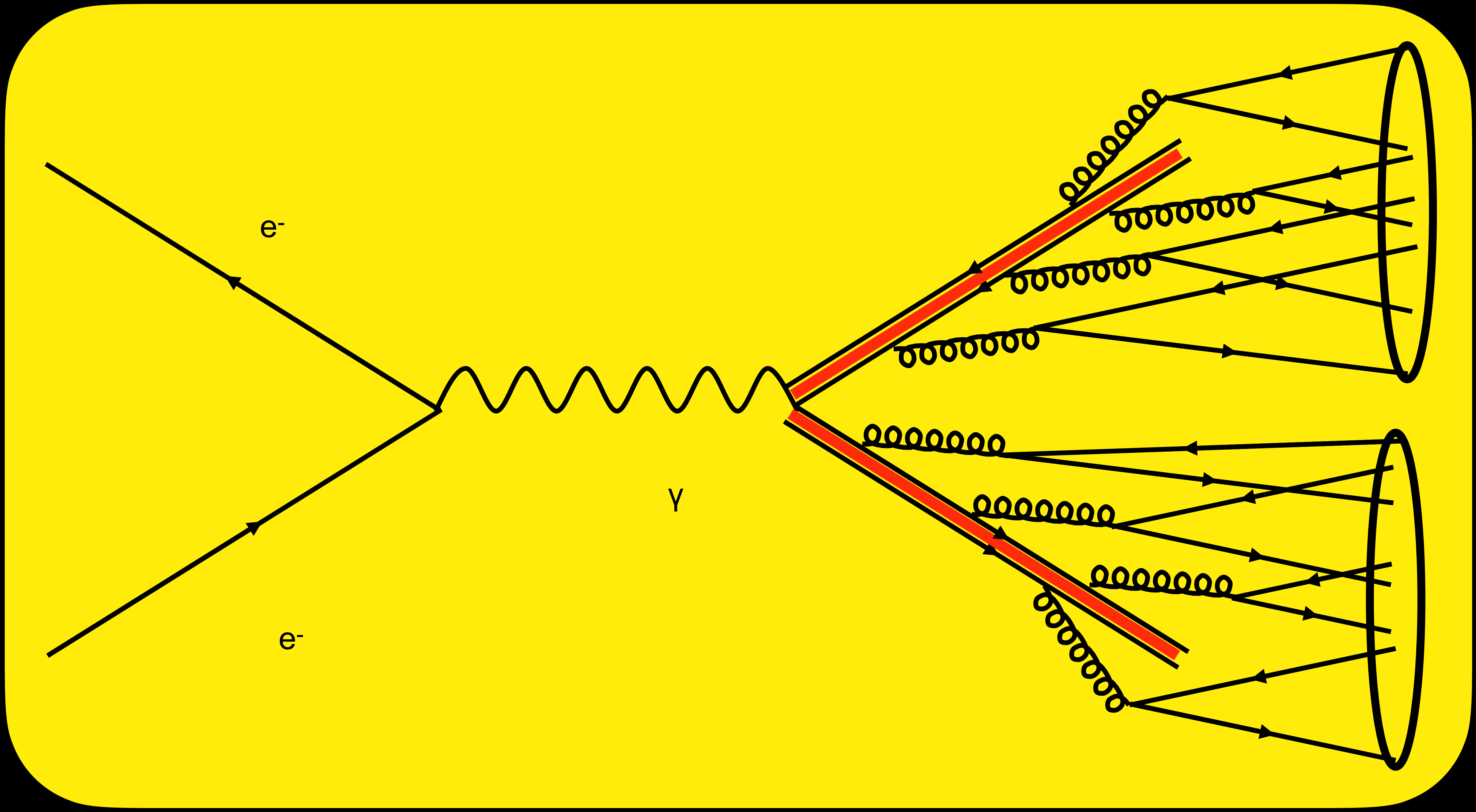
Evidence for colour (cont.)

QED process

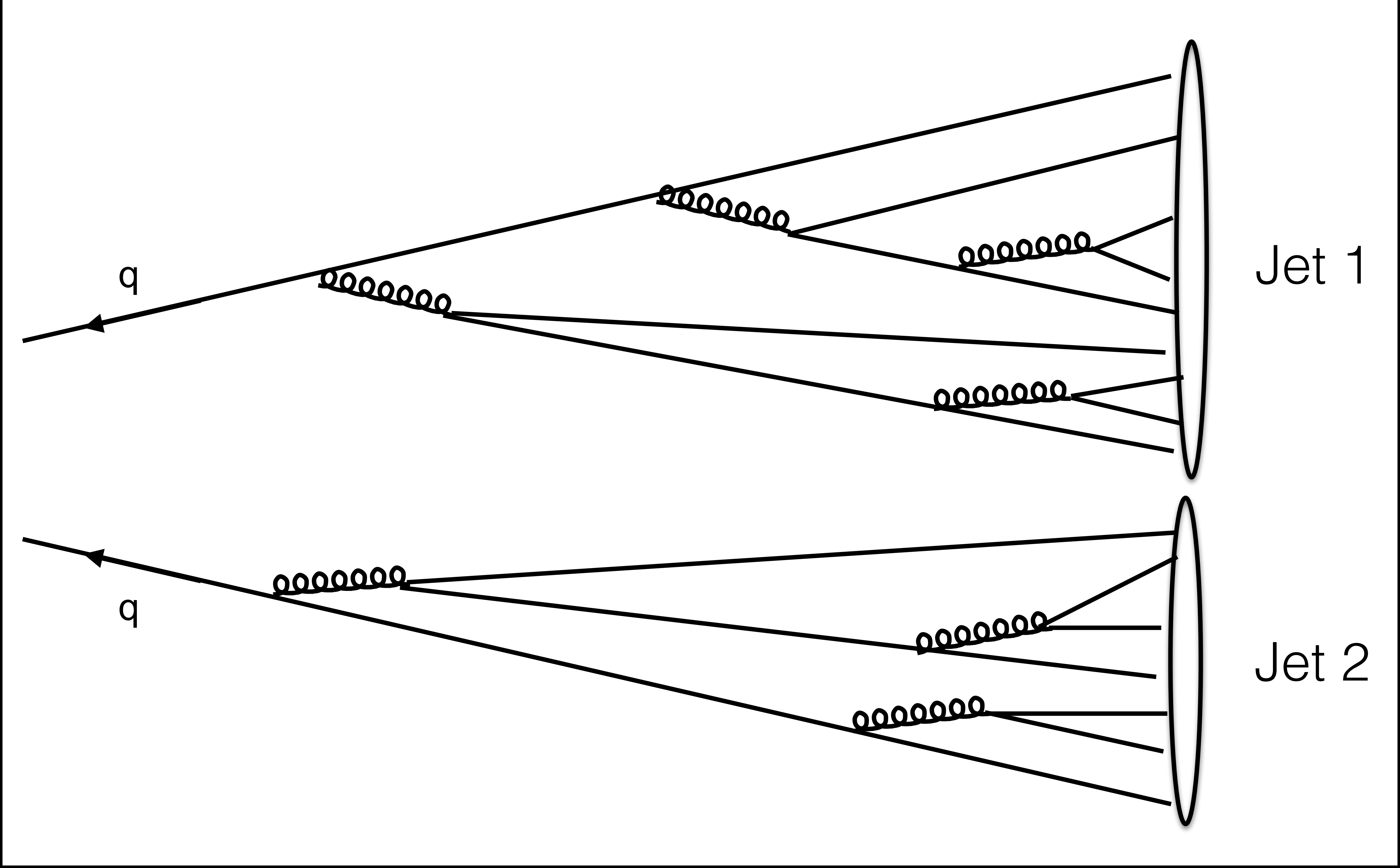
QCD process



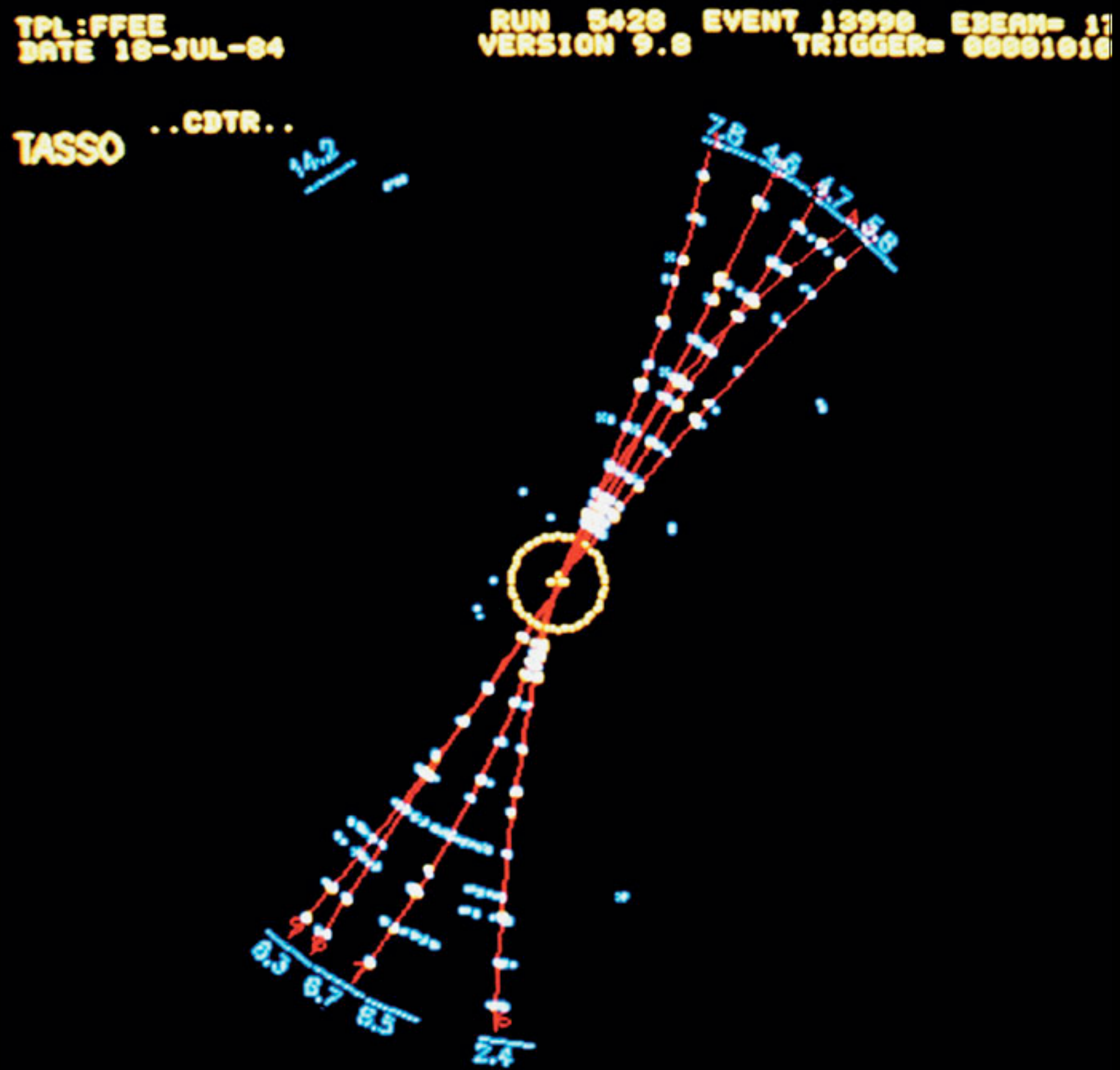
Evidence for colour (cont.)



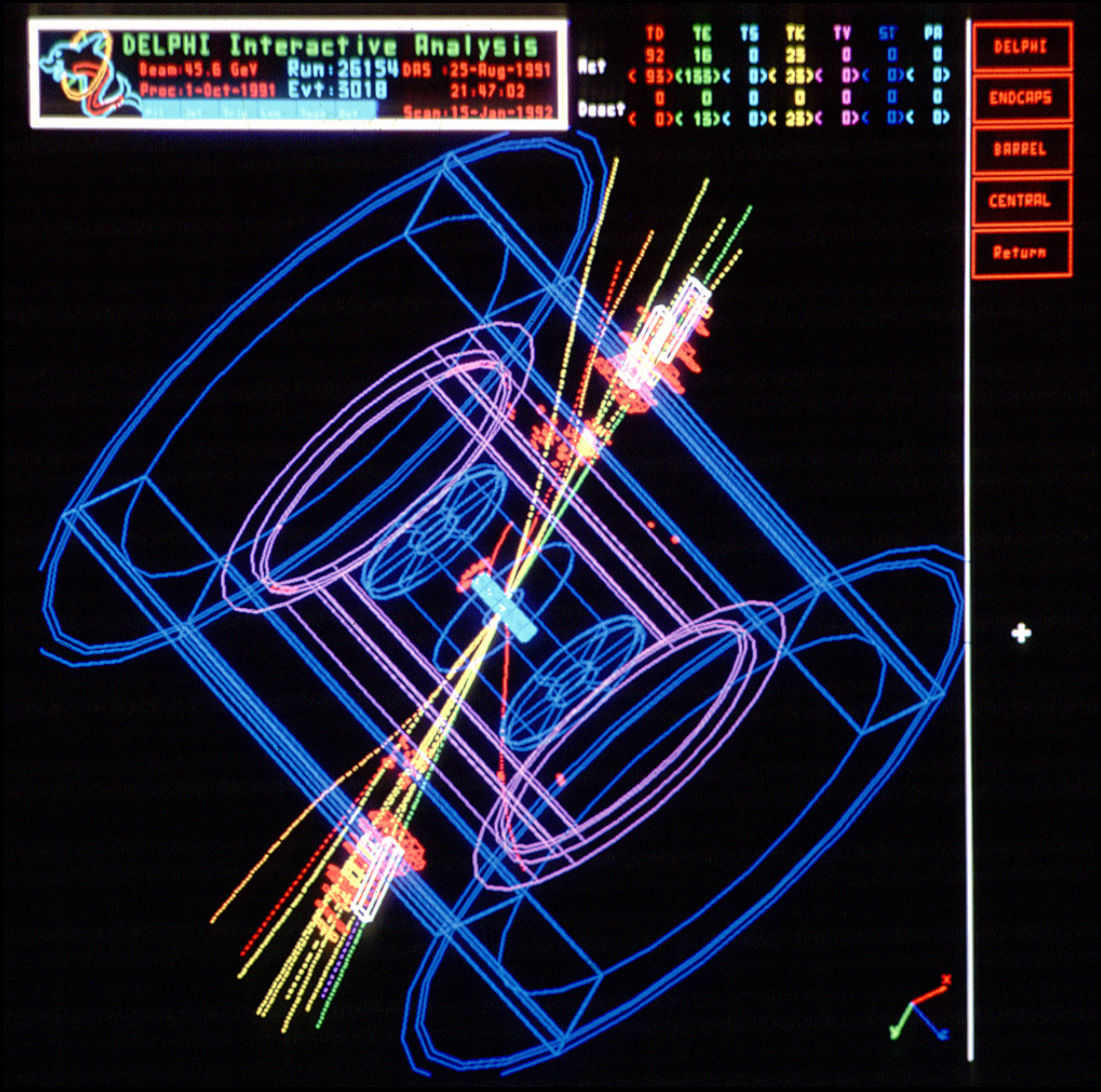
Fragmentation and jet production in the vacuum



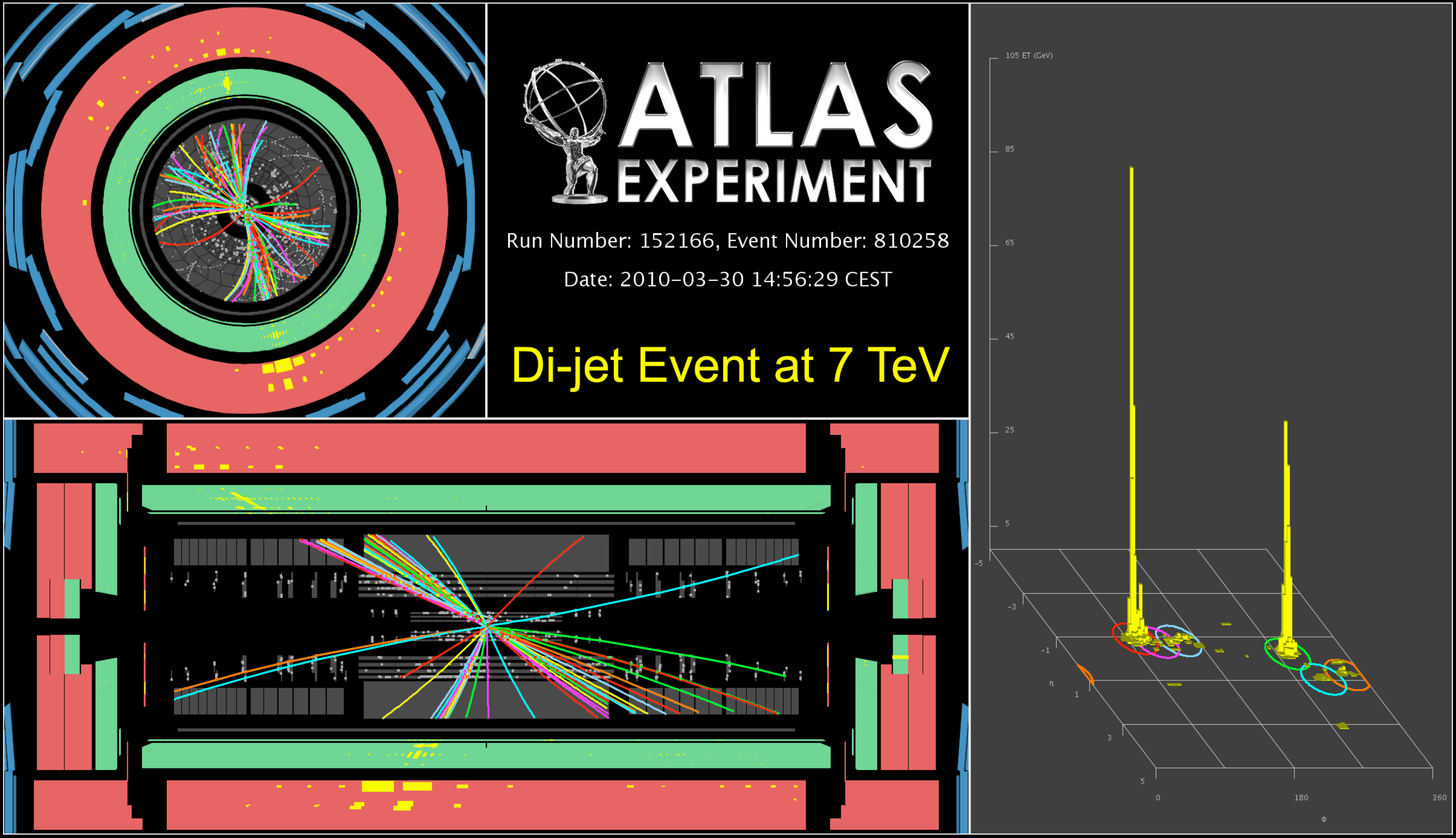
Two-jet events



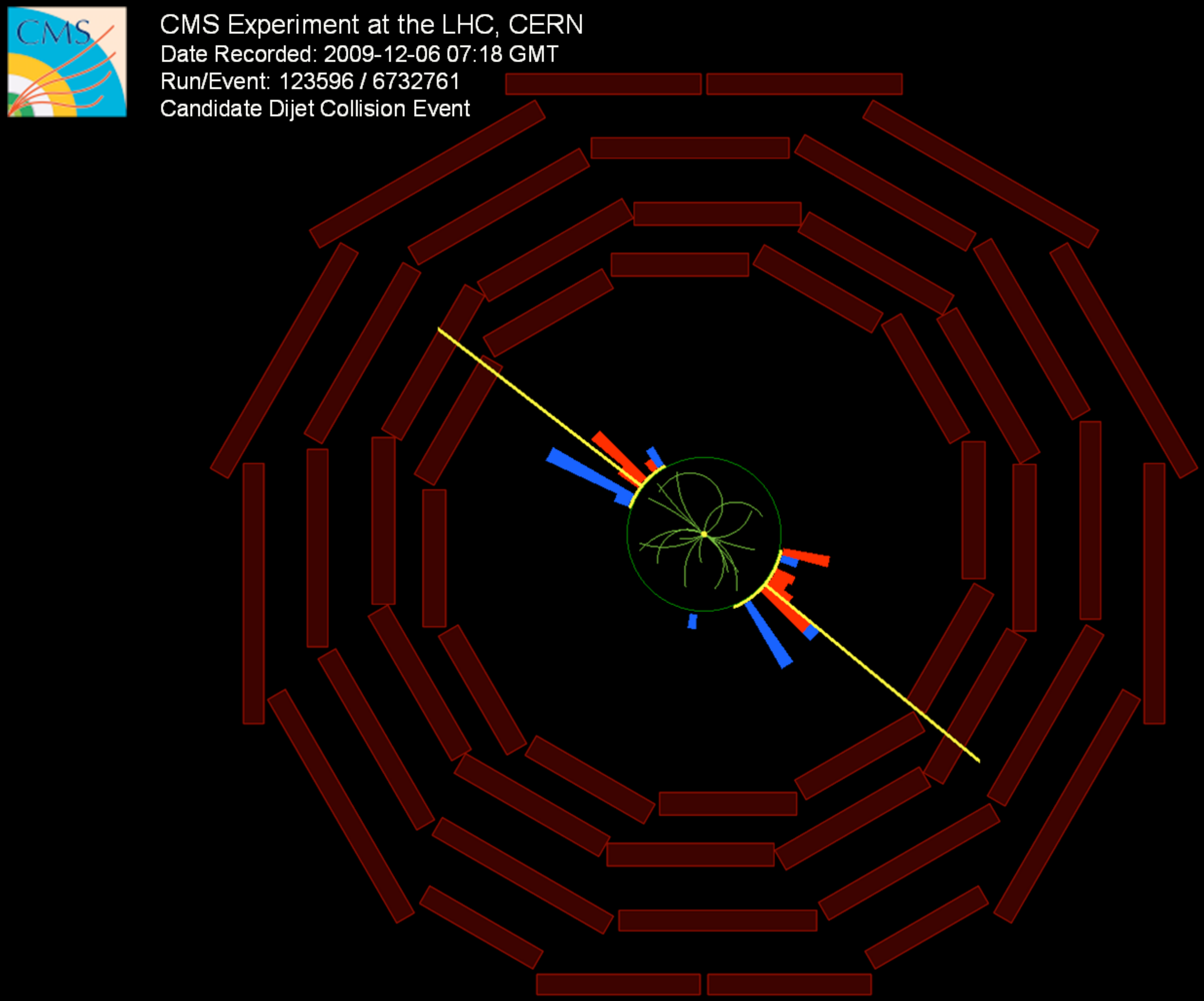
Two-jet events



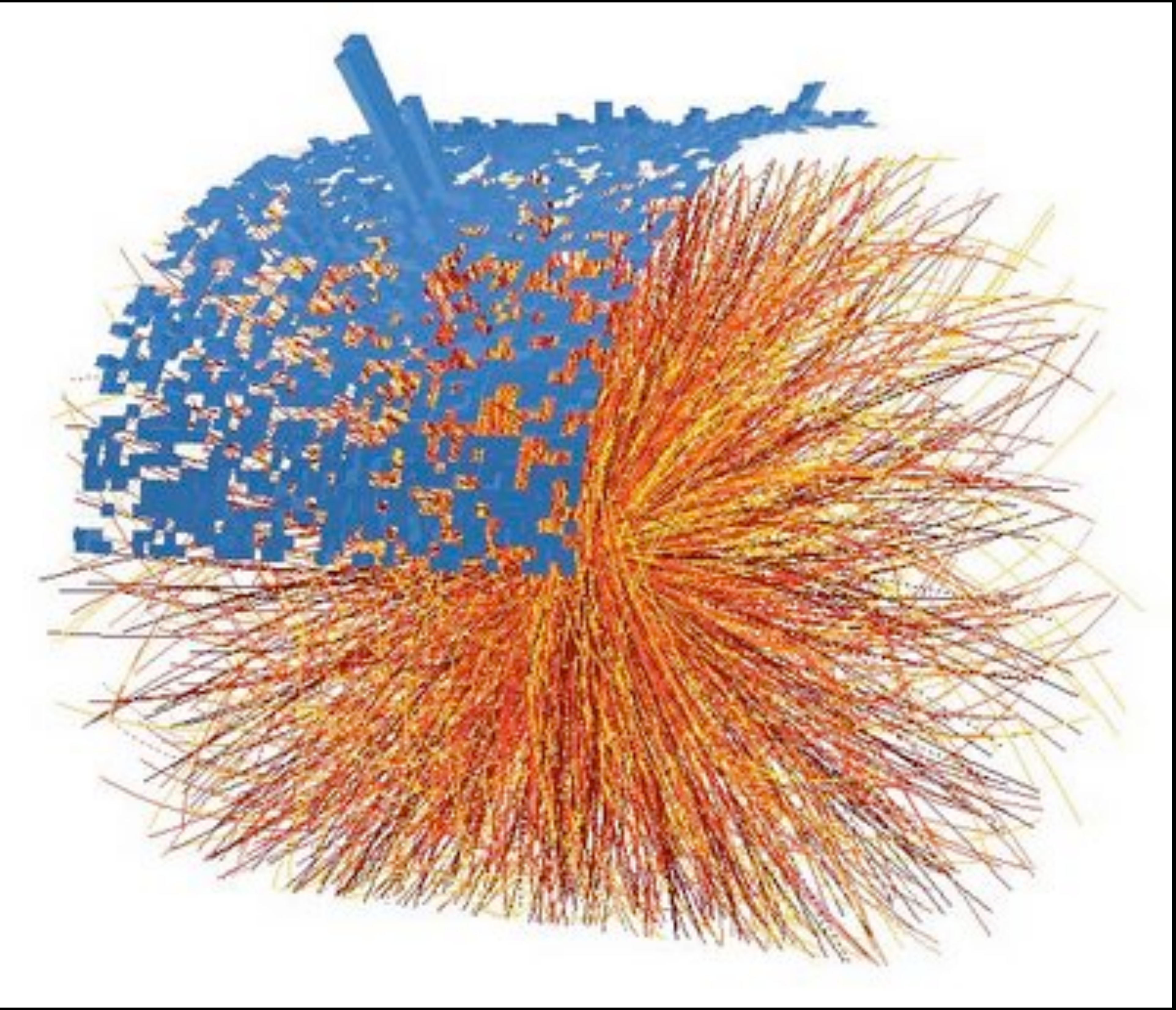
Two-jet events: ATLAS



Two-jet events: CMS

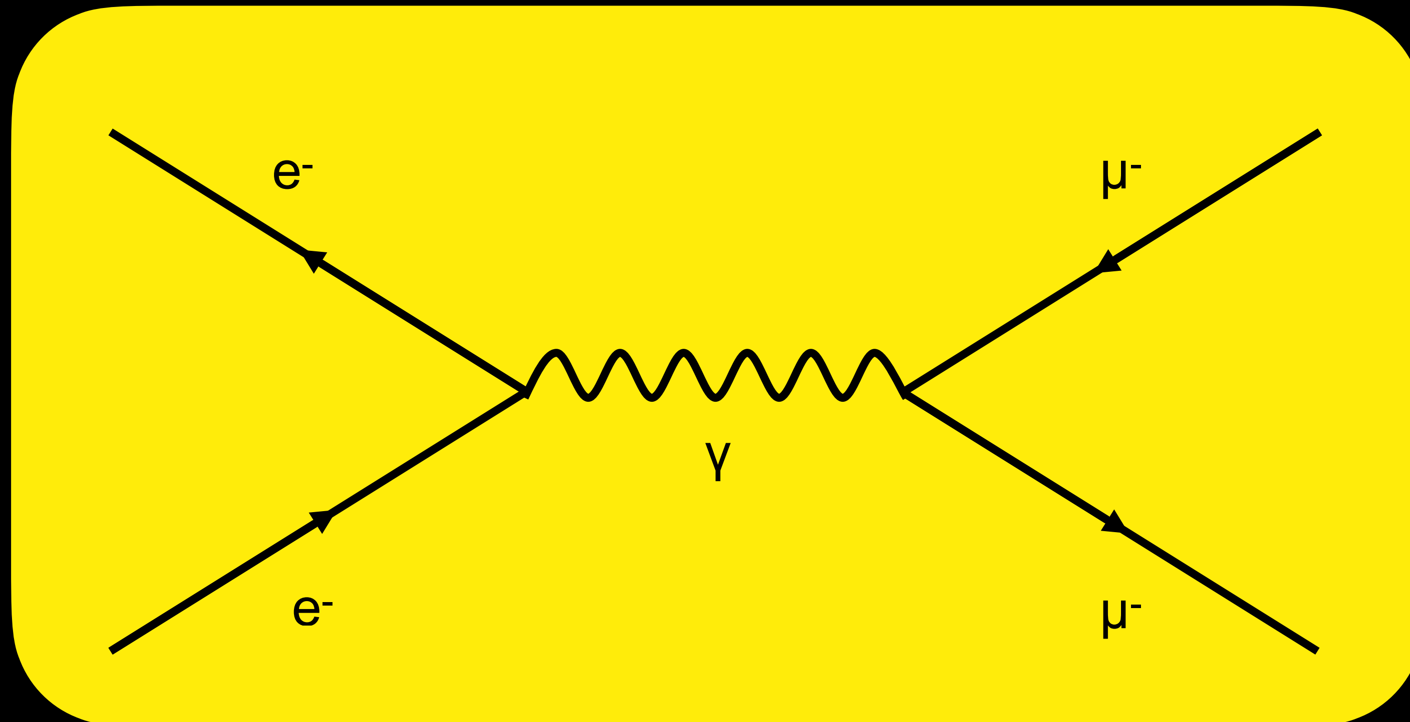


Jet events in heavy-ion collisions (ALICE)



Evidence for colour (cont.)

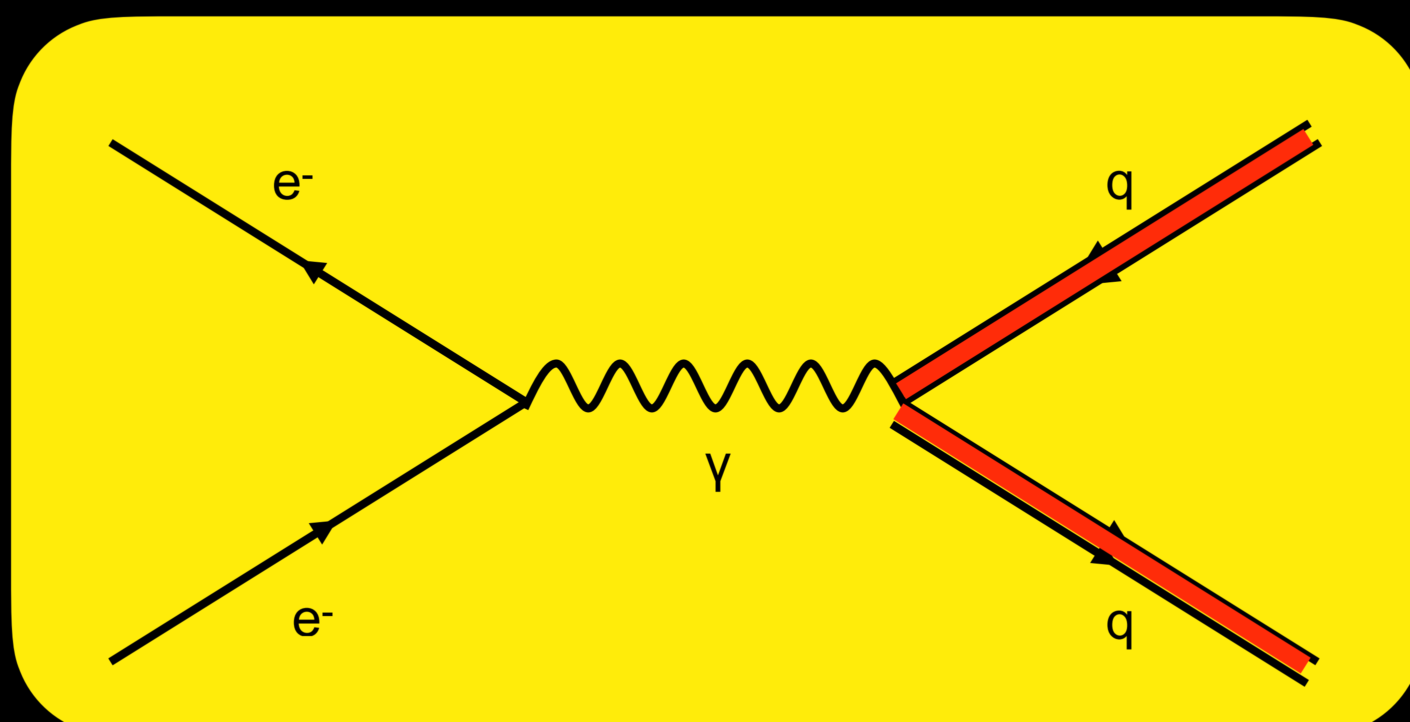
QED process



Final state
particles that can be seen in nature

$$\sigma \sim \left(\frac{a}{E}\right)^2$$

QED process



Final state
particles that do not fly free in nature

$$\sigma \sim \left(\frac{Qa}{E}\right)^2$$

Evidence for colour (cont.)

- Calculate the ratio of the cross-sections of the two processes

$$R=\frac{\sigma(e^-e^+\rightarrow q\overline{q}\rightarrow hadrons)}{\sigma(e^-e^+\rightarrow\mu^-\mu^+)}\sim\sum_{i=1}^{n}Q_i^2$$

- For three quark flavours (u,d,s) the ratio should give:

$$R=\left[\left(\frac{2}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{-1}{3}\right)^2\right]=\frac{2}{3}$$

- For four quark flavours (u,d,s,c) the ratio should give:

$$R=\left[\left(\frac{2}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{2}{3}\right)^2\right]=\frac{10}{9}$$

- For five quark flavours (u,d,s,c,b) the ratio should give:

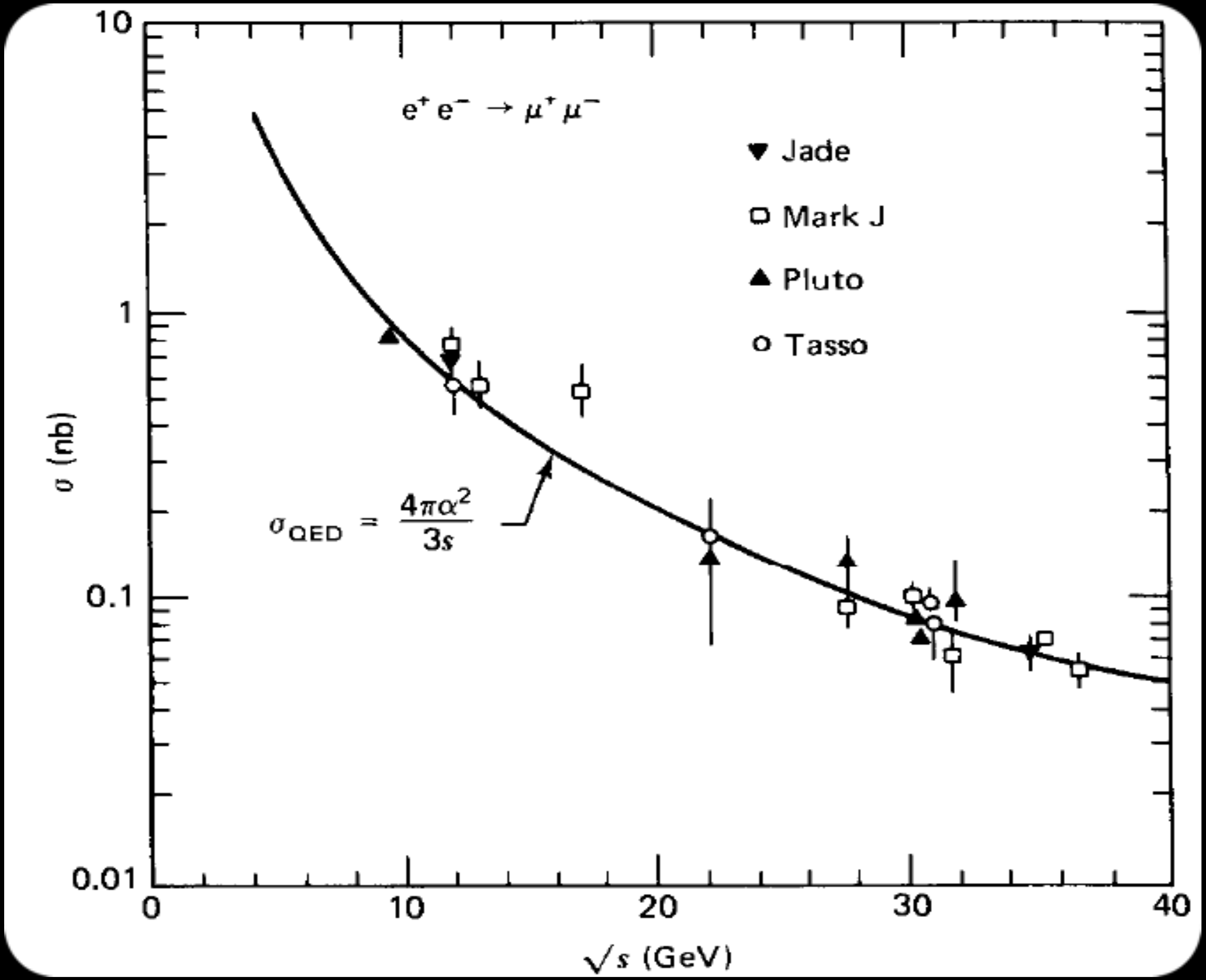
$$R=\left[\left(\frac{2}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{-1}{3}\right)^2\right]=\frac{11}{9}$$

- For all six quark flavours (u,d,s,c,b,t) the ratio should give:

$$R=\left[\left(\frac{2}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{2}{3}\right)^2\right]=\frac{15}{9}$$

Cross-section $e^-e^+ \rightarrow \mu^-\mu^+$

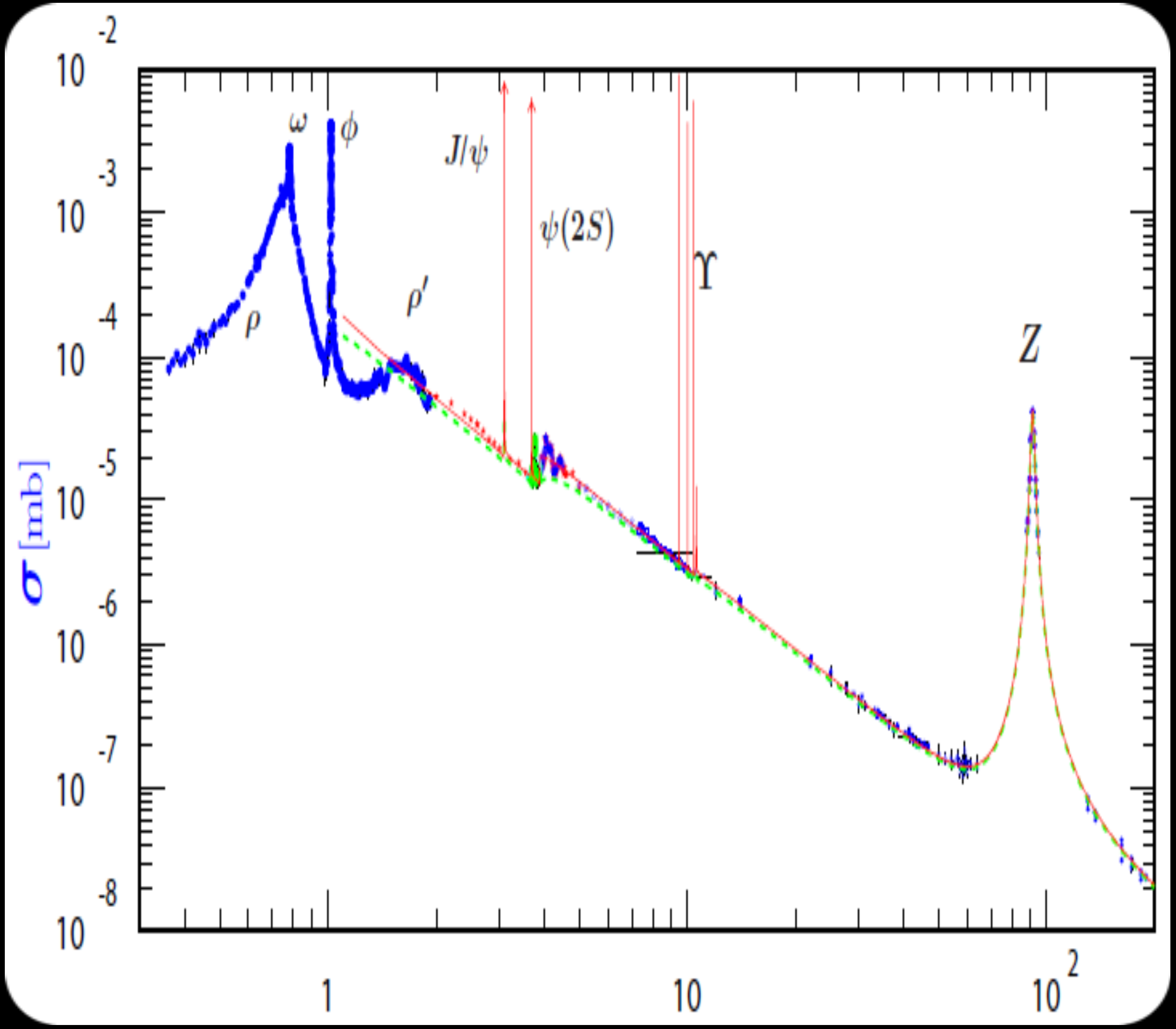
$$\sigma(e^-e^+ \rightarrow \mu^-\mu^+) \sim \left(\frac{a}{E}\right)^2$$



✓ At around $\sqrt{s_{NN}}$ = 10GeV → $\sigma(e^-e^+ \rightarrow \mu^-\mu^+) \sim 0.9$nb

Cross-section $e^-e^+ \rightarrow$ qqbar$\rightarrow$hadrons

$$\sigma(e^-e^+ \rightarrow q\bar{q} \rightarrow hadrons) \sim \left(\frac{Qa}{E}\right)^2$$



✓ At around $\sqrt{s_{NN}}$ = 10GeV → $\sigma(e^-e^+ \rightarrow$ qqbar$\rightarrow$hadrons) ~ 3.6nb

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Evidence for colour (cont.)

- At ~10GeV (beyond the threshold for the b-quark creation) the ratio should be

$$R=\left[\left(\frac{2}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{-1}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{-1}{3}\right)^2\right]=\frac{11}{9}$$

- But experimentally it turns out to be
  - $\sigma(e^-e^+\rightarrow\mu^-\mu^+)$ ~ 0.9nb
  - $\sigma(e^-e^+\rightarrow$qqbar$\rightarrow$hadrons) ~ 3.0nb
  - R~3

- The problem with the calculations assuming no additional quantum number persists for all energy ranges
- Solution: $$R=f\sum_{i=1}^{n}Q_i^2$$
  - where f is the number of colours
  - turns out to be 3 experimentally!!!

Evidence for colour (cont.)

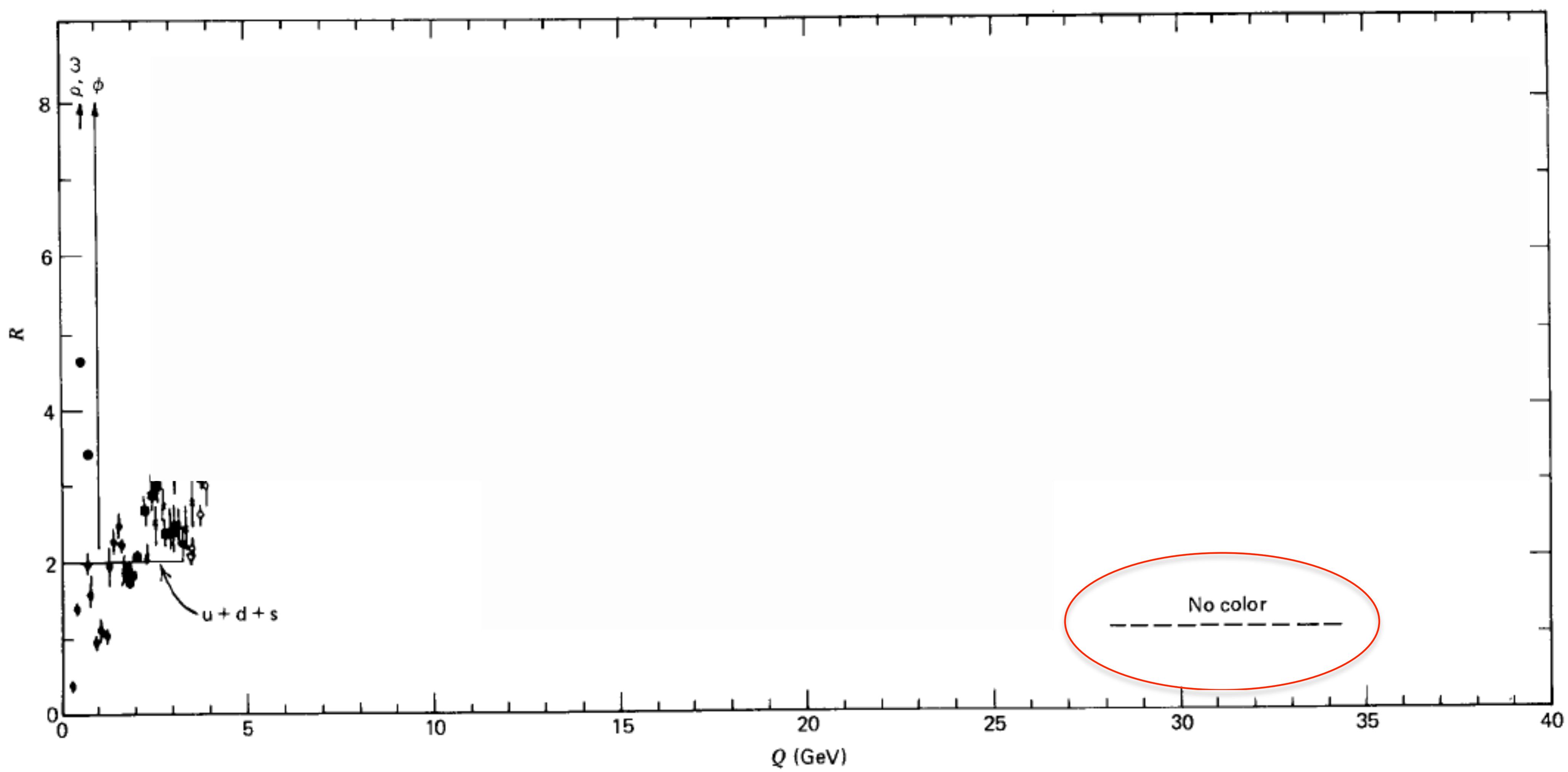
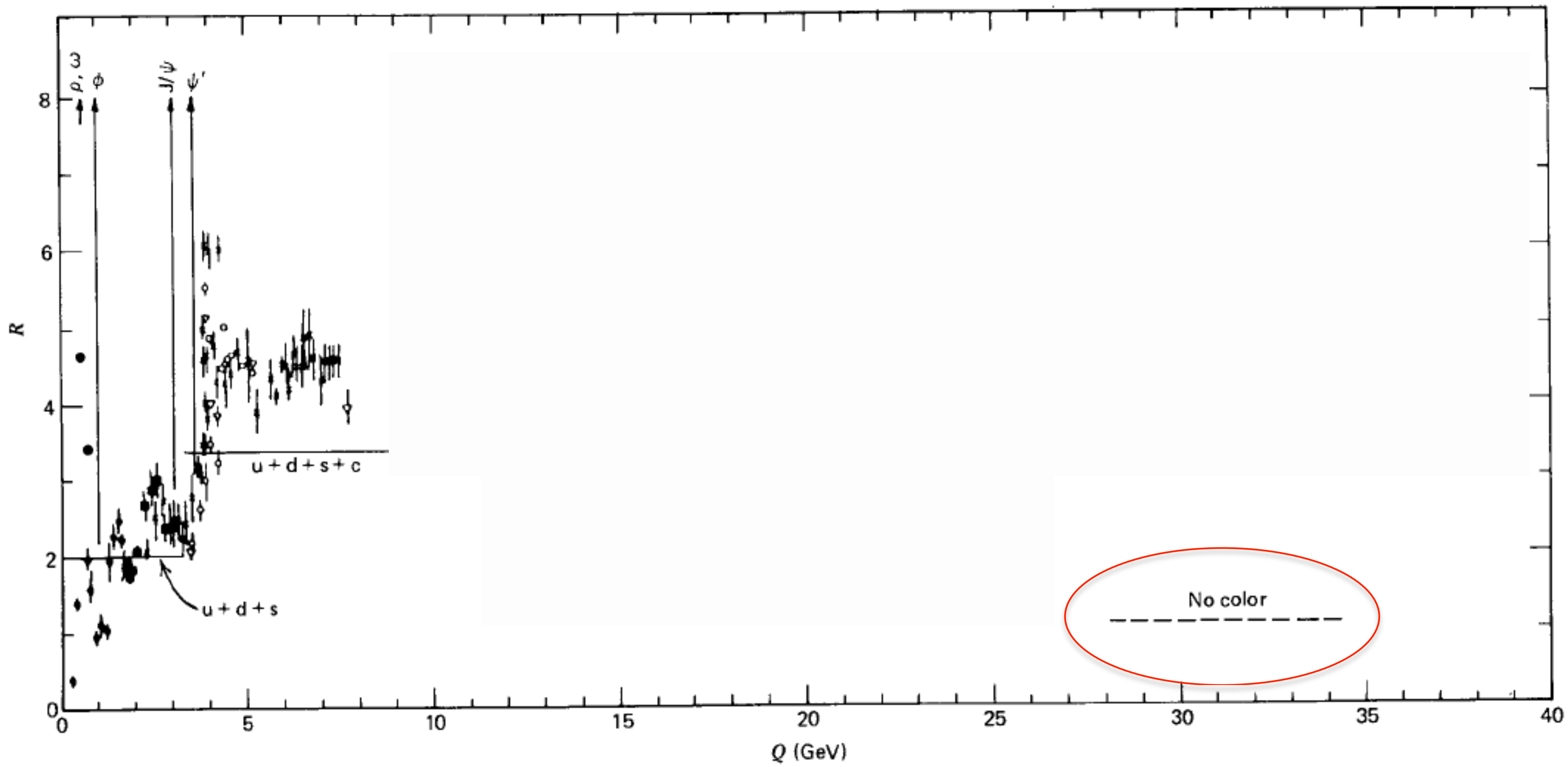


Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

Evidence for colour (cont.)



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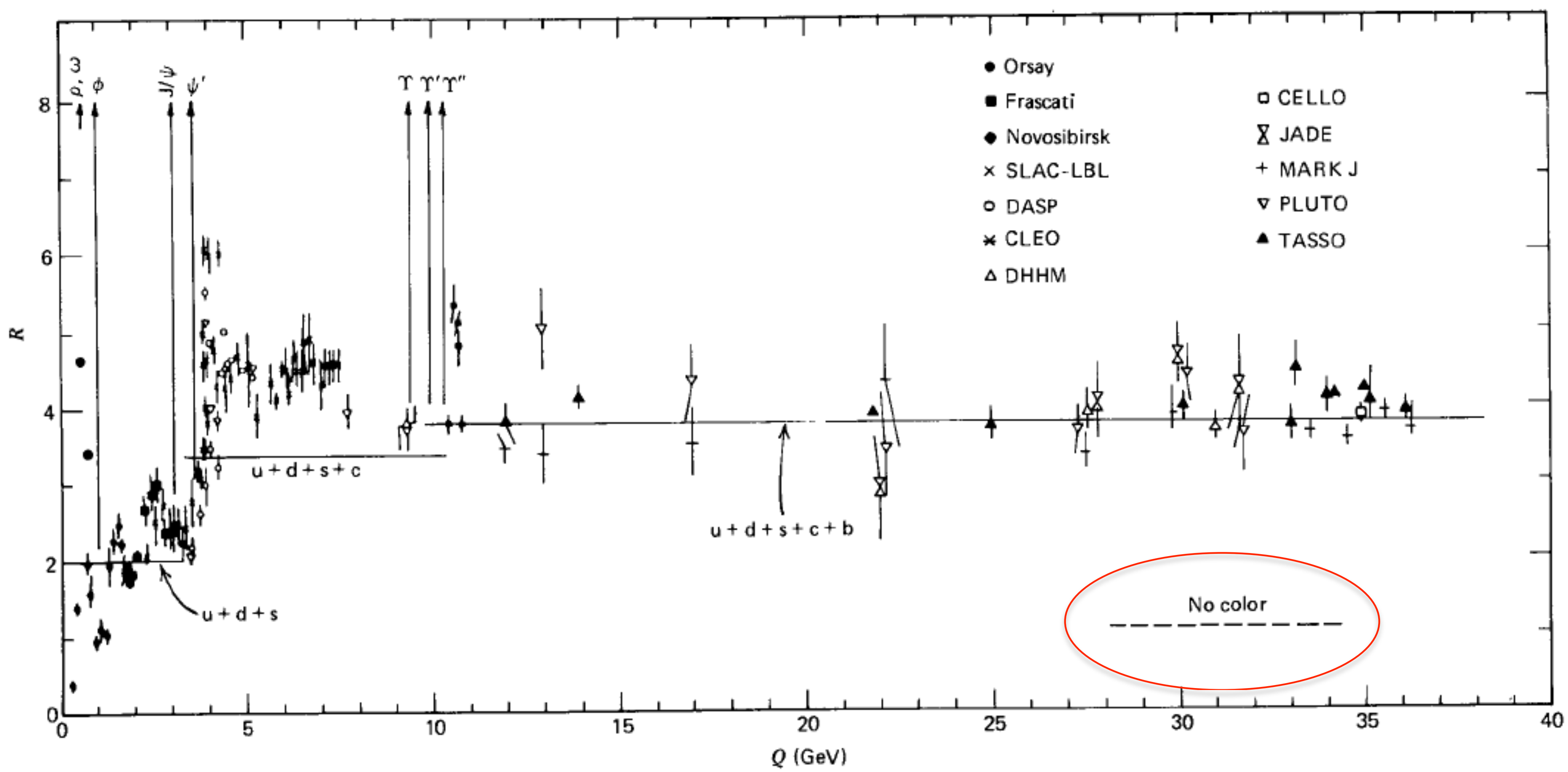
Evidence for colour (cont.)



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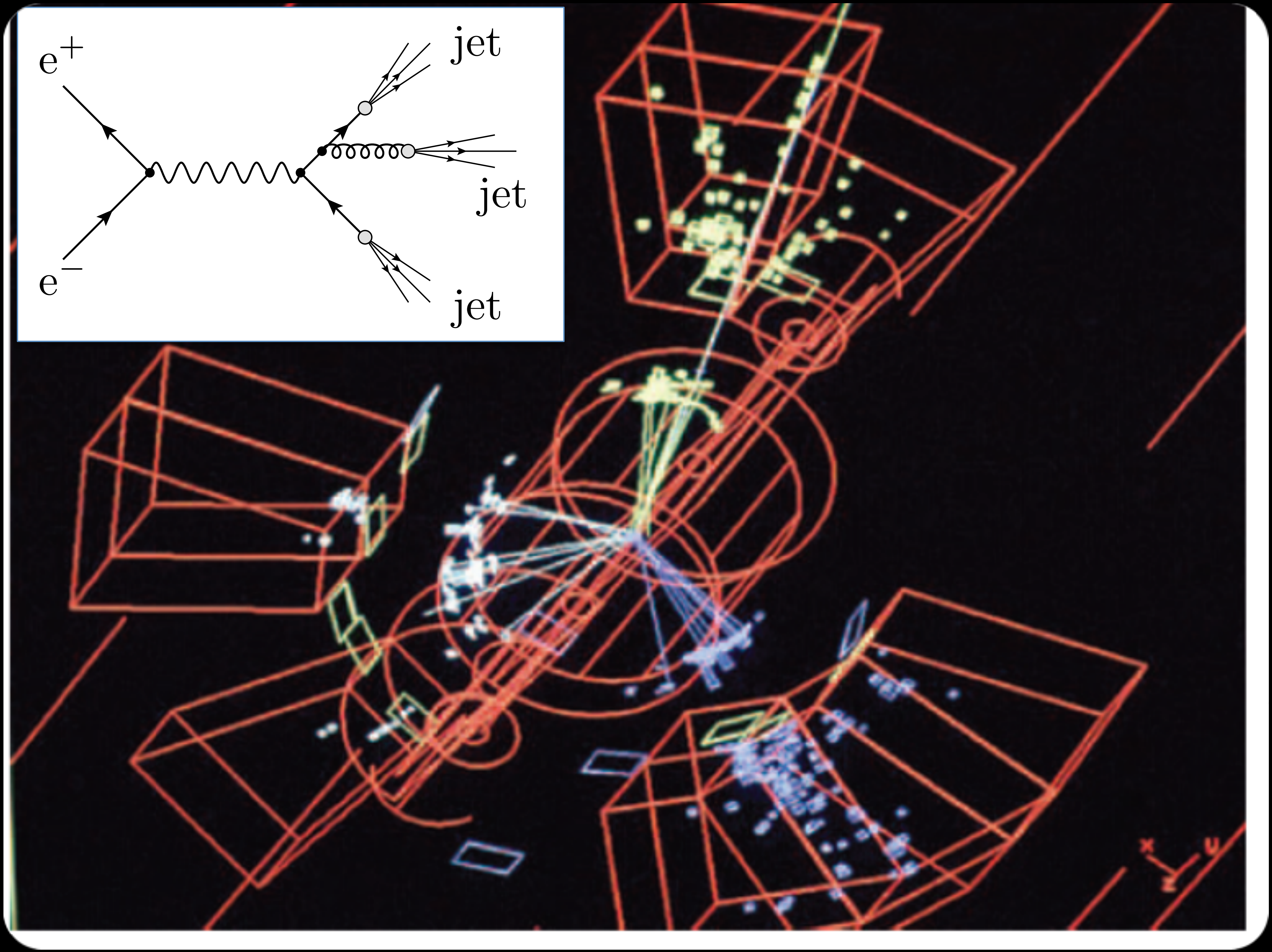
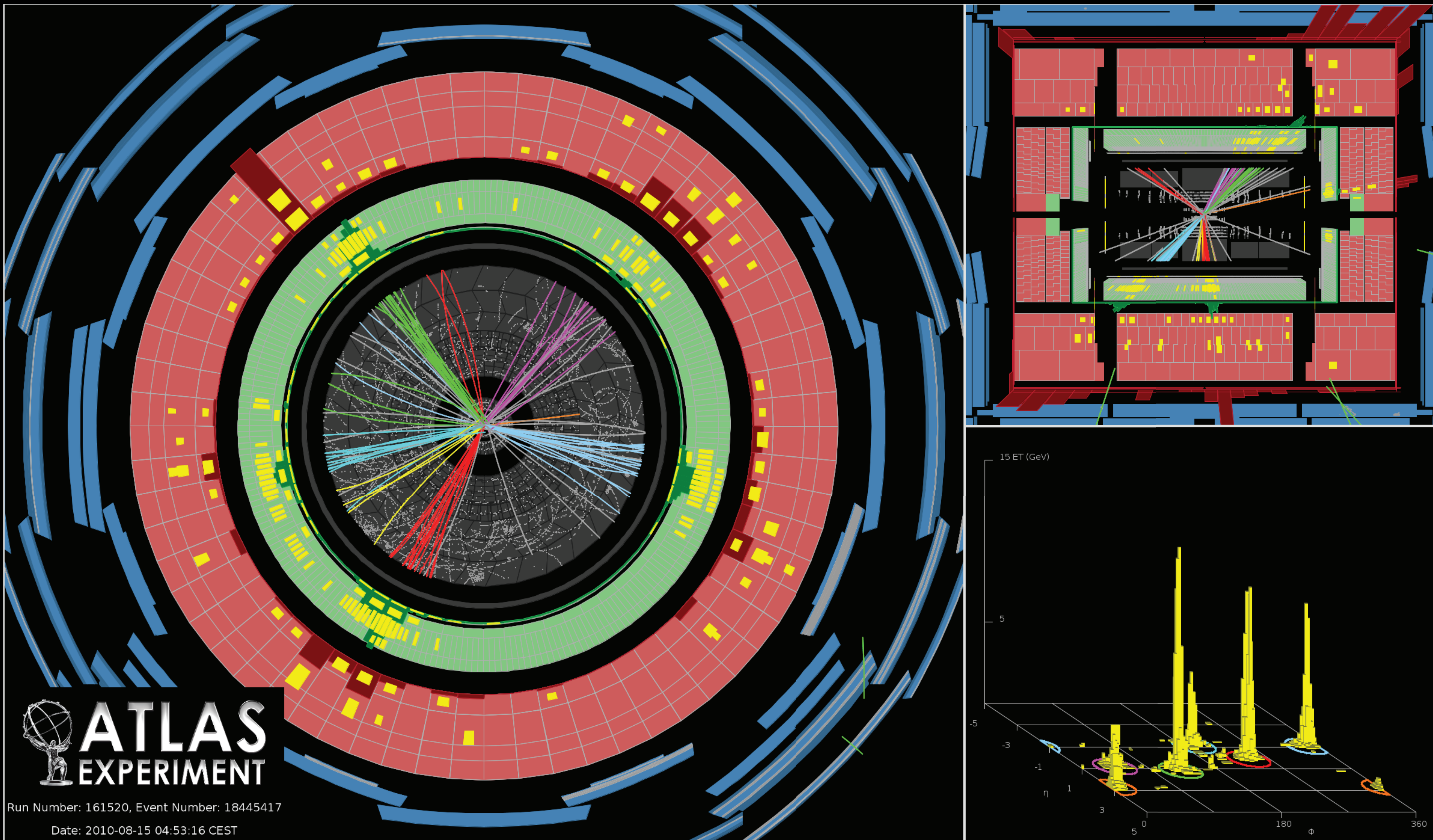
Existence of gluons: three-jet events

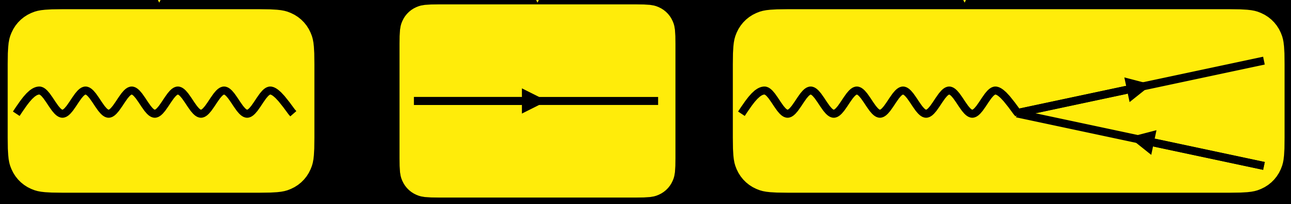


Existence of gluons: five-jet events



QCD Lagrangian formalism

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\,(i\gamma_\mu \partial^\mu - m)\psi - qA_\mu \bar{\psi}\,\gamma^\mu \psi$$



Free photon field A^μ

Fermion ψ with mass m

Interaction with coupling α

The birth of QCD

VOLUME 30, NUMBER 26 PHYSICAL REVIEW LETTERS 25 JUNE 1973

Ultraviolet Behavior of Non-Abelian Gauge Theories*

David J. Gross† and Frank Wilczek
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
(Received 27 April 1973)

It is shown that a wide class of non-Abelian gauge theories have, up to calculable logarithmic corrections, free-field–theory asymptotic behavior. It is suggested that Bjorken scaling may be obtained from strong-interaction dynamics based on non-Abelian gauge symmetry.

Non-Abelian gauge theories have received much attention recently as a means of constructing unified and renormalizable theories of the weak and electromagnetic interactions.[1] In this note we report on an investigation of the ultraviolet (UV) asymptotic behavior of such theories. We have found that they possess the remarkable feature, perhaps unique among renormalizable theories, of asymptotically approaching free-field theory. Such asymptotically free theories will exhibit, for matrix elements of currents between on-mass-shell states, Bjorken scaling. We therefore suggest that one should look to a non-Abelian gauge theory of the strong interactions to provide the explanation for Bjorken scaling, which has so far eluded field-theoretic understanding.

The UV behavior of renormalizable field theories can be discussed using the renormalization-group equations,[2,3] which for a theory involving one field (say $g\varphi^4$) are

$$[m\partial/\partial m + \beta(g)\partial/\partial g - n\gamma(g)]\Gamma_{asy}^{(n)}(g; P_1, \dots, P_n) = 0. \quad (1)$$

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[14] Y. Nambu and G. Jona-Lasino, Phys. Rev. 122, 345 (1961); S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[15] K. Symanzik (to be published) has recently suggested that one consider a $\lambda\varphi^4$ theory with a negative λ to achieve UV stability at $\lambda = 0$. However, one can show, using the renormalization-group equations, that in such theory the ground-state energy is unbounded from below (S. Coleman, private communication).

[16] W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, CERN Report No. CERN-TH-1538, 1972 (to be published).

[17] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972); S. Weinberg, Phys. Rev. D 5, 1962 (1972).

[18] For a review of this program, see S. L. Adler, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

Reliable Perturbative Results for Strong Interactions?*

H. David Politzer
Jefferson Physical Laboratories, Harvard University, Cambridge, Massachusetts 02138
(Received 3 May 1973)

An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek

The Nobel Prize in Physics 2004


David J. Gross


H. David Politzer


Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.

The birth of QCD

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The Nobel Prize in Physics 2004

David J. Gross — H. David Politzer — Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.

$$\mathcal{L}=\frac{1}{4g^2}G^a_{\mu\nu}G^a_{\mu\nu}+\sum_j\bar{q}_j(i\gamma^\mu D_\mu+m_j)q_j$$

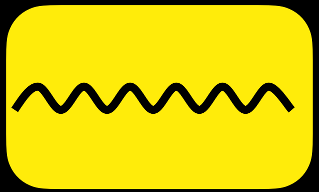
where $G^a_{\mu\nu}\equiv\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+if_{abc}A^b_\mu A^c_\nu$

and $D_\mu\equiv\partial_\mu+it^aA^a_\mu$

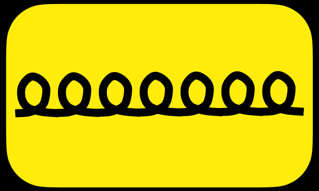
That's it!

Symmetries and gauge transformations (strong)

Conserved quantities	Gauge transformation	Symmetry group	Field
Charge: ψ	$\Psi \to \Psi' = e^{i\alpha}\Psi$	U(1) (QED)	A_μ (1 photon)
Colour	$\Psi \to \Psi' = e^{\frac{i}{2}\sum_{i=1}^{8}\theta_a\lambda_a}\Psi$	SU(3) (QCD)	A_μ (8 gluons g)



$$\Psi = \begin{pmatrix} R \\ B \\ G \end{pmatrix}$$



Feynman rules for QCD

- **Labeling:** We label every external line with the ingoing and outgoing momenta $\mathbf{P_1},\ldots,\mathbf{P_n}$, adding also an arrow indicating whether a particle is approaching or moving away from the vertex. If the diagram includes antiparticles, we still label them as particles but with the reverse direction of the arrow. We then label the 4-momenta for all internal lines $\mathbf{q_1},\ldots,\mathbf{q_j}$ and we give an arbitrary direction to the relevant arrow.
- **External lines:** Each external line contribute the following factors:

$$\text{Incoming quark} \rightarrow u^s \cdot c$$
$$\text{Outgoing quark} \rightarrow \overline{u}^s \cdot c^\dagger$$

$$\text{Incoming anti-quark} \rightarrow \overline{v}^s \cdot c^\dagger$$
$$\text{Outgoing anti-quark} \rightarrow v \cdot c$$

$$\text{Incoming gluon} \rightarrow \varepsilon_\mu \cdot a^\alpha$$
$$\text{Outgoing gluon} \rightarrow \varepsilon^*_\mu \cdot a^{\alpha *}$$

where u and v are the relevant Dirac spinors. In the previous c are the matrices that represent the colour:

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \text{ for R, } \begin{pmatrix}0\\1\\0\end{pmatrix} \text{ for G, } \begin{pmatrix}0\\0\\1\end{pmatrix} \text{ for B}$$

and a are the 8-element column matrices, one for each gluon state (i.e. α goes from 1 to 8):

$$|1\rangle \equiv \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}, |2\rangle \equiv \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix}, |3\rangle \equiv \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix}, |4\rangle \equiv \begin{pmatrix}0\\0\\0\\1\\0\\0\\0\\0\end{pmatrix}, |5\rangle \equiv \begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\end{pmatrix}, |6\rangle \equiv \begin{pmatrix}0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix}, |7\rangle \equiv \begin{pmatrix}0\\0\\0\\0\\0\\0\\1\\0\end{pmatrix}, |8\rangle \equiv \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix}$$

- **Vertices:** For each vertex we note down in the diagram the coupling constant factor $\approx g_s$. This factor is connected to the coupling constant via the equation

$$g_s = \sqrt{4\pi\alpha_s}$$

For a quark-gluon vertex (see fig. 3.4) the factor is of the form:

$$\frac{-ig_s}{2}\lambda^\alpha\gamma^\mu$$

where the parameters λ^α are the Gell-Man λ-matrices of SU(3).

For a 3-gluon vertex (see fig. 3.4) the factor is of the form:

$$-g_s f^{\alpha\beta\gamma}\left[g_{\mu\nu}(\mathbf{k_1}-\mathbf{k_2})_\rho + g_{\nu\rho}(\mathbf{k_2}-\mathbf{k_3})_\mu + g_{\rho\mu}(\mathbf{k_3}-\mathbf{k_1})_\nu\right]$$

,

where the factors $f^{\alpha\beta\gamma}$ are the structure constants of SU(3) and $\mathbf{k}_i$ are the 4-momenta of each internal line (with $i = 1,2,3$).

Finally, for a 4-gluon vertex (see fig. 3.4) the factor is of the form:

$$-ig_s^2\left[f^{\alpha\beta\eta}f^{\gamma\delta\eta}(g_{\mu\sigma}g_{\nu\rho}-g_{\mu\rho}g_{\nu\sigma}) + f^{\alpha\delta\eta}f^{\beta\gamma\eta}(g_{\mu\nu}g_{\sigma\rho}-g_{\mu\sigma}g_{\nu\rho}) + f^{\alpha\gamma\eta}f^{\delta\beta\eta}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\nu}g_{\sigma\rho})\right]$$

- **Propagators:** For each internal line, we give a factor of

$$q-\overline{q} : \frac{i(\not{q}+m)}{\mathbf{q}^2-m^2}$$

$$\text{gluon} : -ig_{\mu\nu}\delta^{\alpha\beta}$$

where $\not{q} \equiv \gamma_\nu q^\nu$.

- **δ-functions and integration:** The remaining steps are identical as in the general rules described before.

Figure 3.4 presents the lines for the basic particles and anti-particles but also the propagators for the strong interactions.

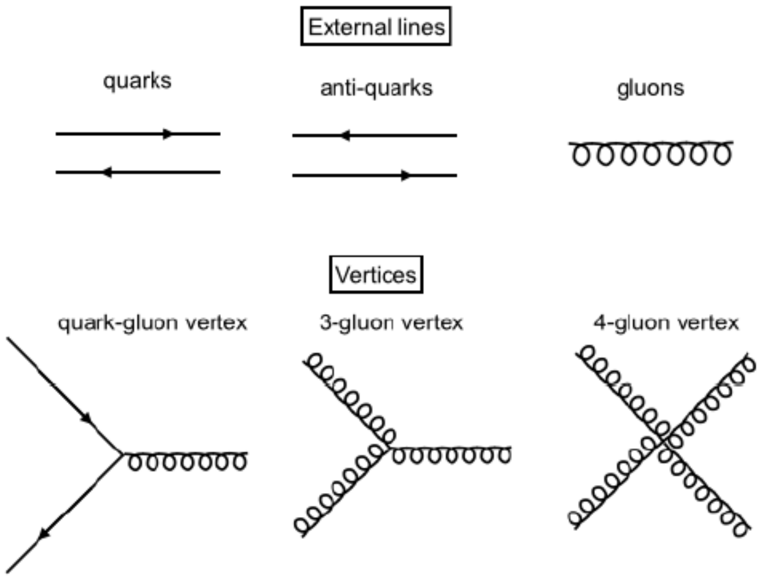
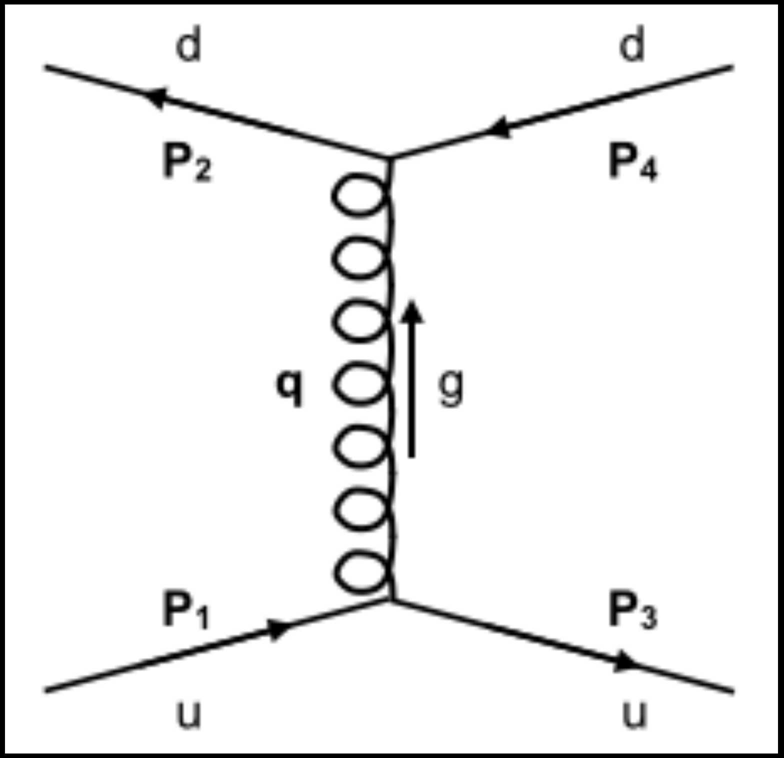


Fig. 3.4: The most characteristic lines for the Feynman diagrams in strong interactions.

Feynman rules for QCD (cont.)



$$\int [\overline{u}(3)c_3^\dagger]\cdot[\frac{-ig_s}{2}\lambda^\alpha\gamma^\mu]\cdot[u(1)c_1]\cdot[\frac{-ig_{\mu\nu}\delta^{\alpha\beta}}{\mathbf{q}^2}]\cdot[\overline{v}(2)c_2^\dagger]\cdot[\frac{-ig_s}{2}\lambda^\beta\gamma^\nu]\cdot[v(4)c_4]\cdot$$

$$[(2\pi)^4\delta^4(\mathbf{P_1}-\mathbf{P_3}-\mathbf{q})]\cdot[(2\pi)^4\delta^4(\mathbf{P_2}-\mathbf{P_4}+\mathbf{q})]\cdot\frac{d^4q}{(2\pi)^4}=$$

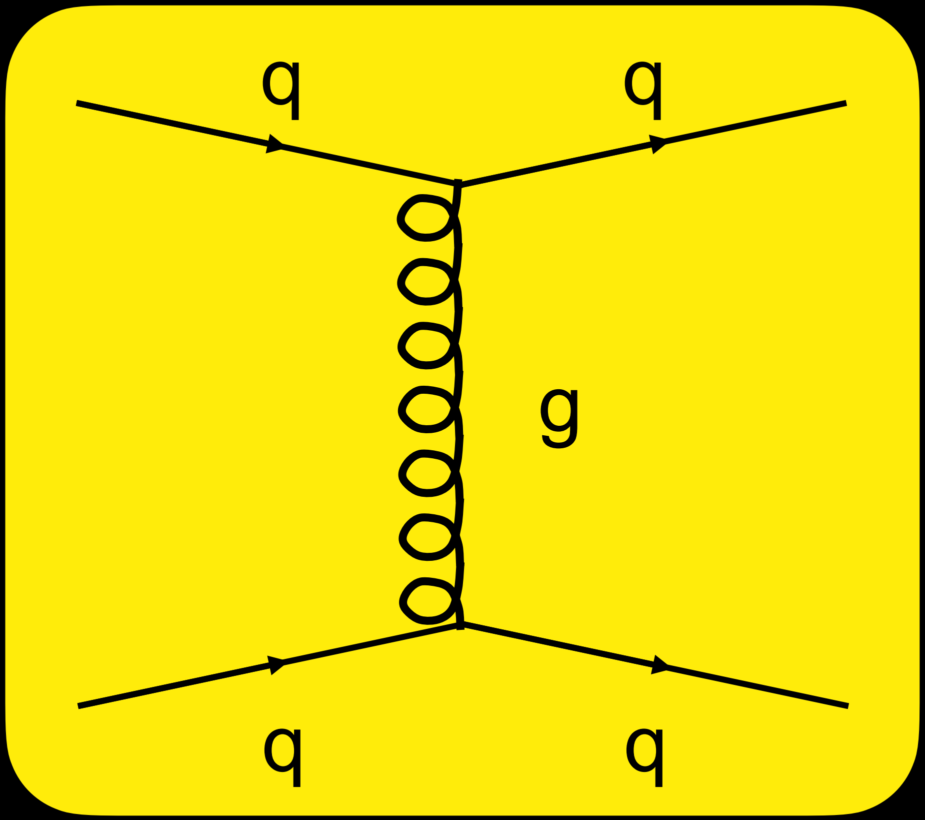
$$\frac{ig_s^2(2\pi)^4}{4}\cdot[\overline{u}(3)c_3^\dagger]\cdot[\lambda^\alpha\gamma^\mu]\cdot[u(1)c_1]\cdot[\frac{g_{\mu\nu}\delta^{\alpha\beta}}{\mathbf{q}^2}]\cdot[\overline{v}(2)c_2^\dagger]\cdot[\lambda^\beta\gamma^\nu]\cdot[v(4)c_4]\cdot\delta^4(\mathbf{P_1}+\mathbf{P_2}-\mathbf{P_3}-\mathbf{P_4})\Leftrightarrow$$

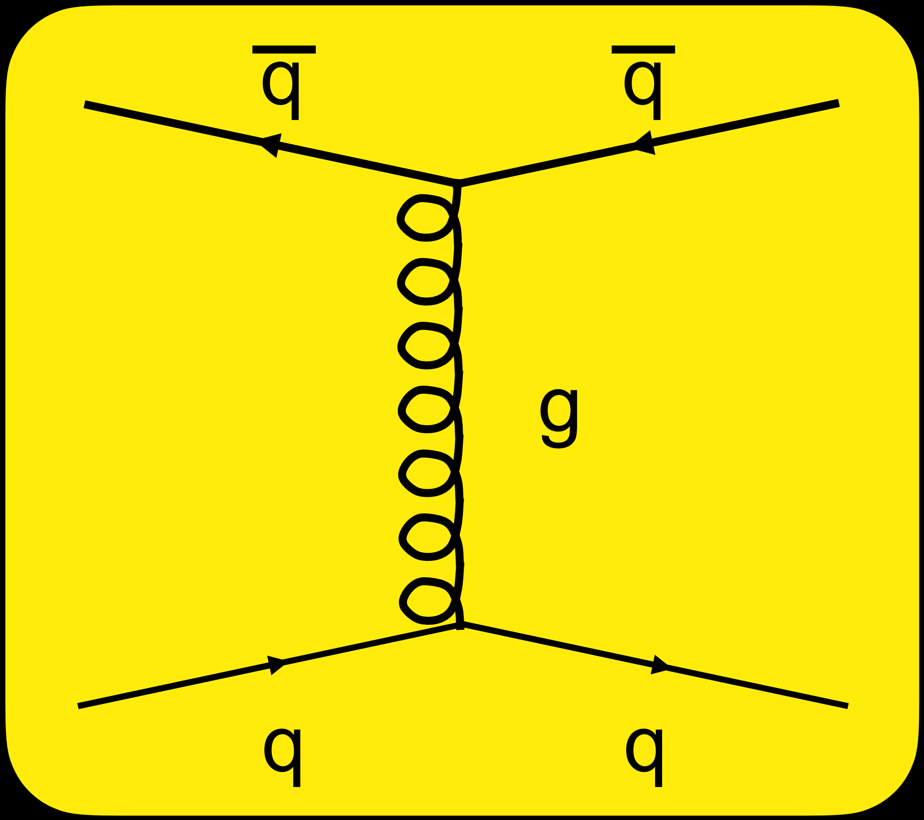
$$M_{if}=\frac{g_s^2}{4\mathbf{q}^2}\cdot[\overline{u}(3)c_3^\dagger]\cdot[\lambda^\alpha\gamma^\mu]\cdot[u(1)c_1]\cdot[g_{\mu\nu}\delta^{\alpha\beta}]\cdot[\overline{v}(2)c_2^\dagger]\cdot[\lambda^\beta\gamma^\nu]\cdot[v(4)c_4]\Leftrightarrow$$

$$M_{if}=\frac{g_s^2}{4\mathbf{q}^2}\cdot[\overline{u}(3)\gamma^\mu u(1)]\cdot[\overline{v}(2)\gamma_\mu v(4)]\cdot\left[[c_3^\dagger\lambda^\alpha c_1]\cdot(\delta^{\alpha\beta})\cdot[c_2^\dagger\lambda^\beta c_4]\right]\Leftrightarrow$$

$$M_{if}=\frac{g_s^2}{\mathbf{q}^2}\cdot(\overline{u}(3)\gamma^\mu u(1)\overline{v}(2)\gamma_\mu v(4))\cdot\left[\frac{1}{4}\cdot(c_3^\dagger\lambda^\alpha c_1)\cdot(c_2^\dagger\lambda^\alpha c_4)\right]$$

QCD processes

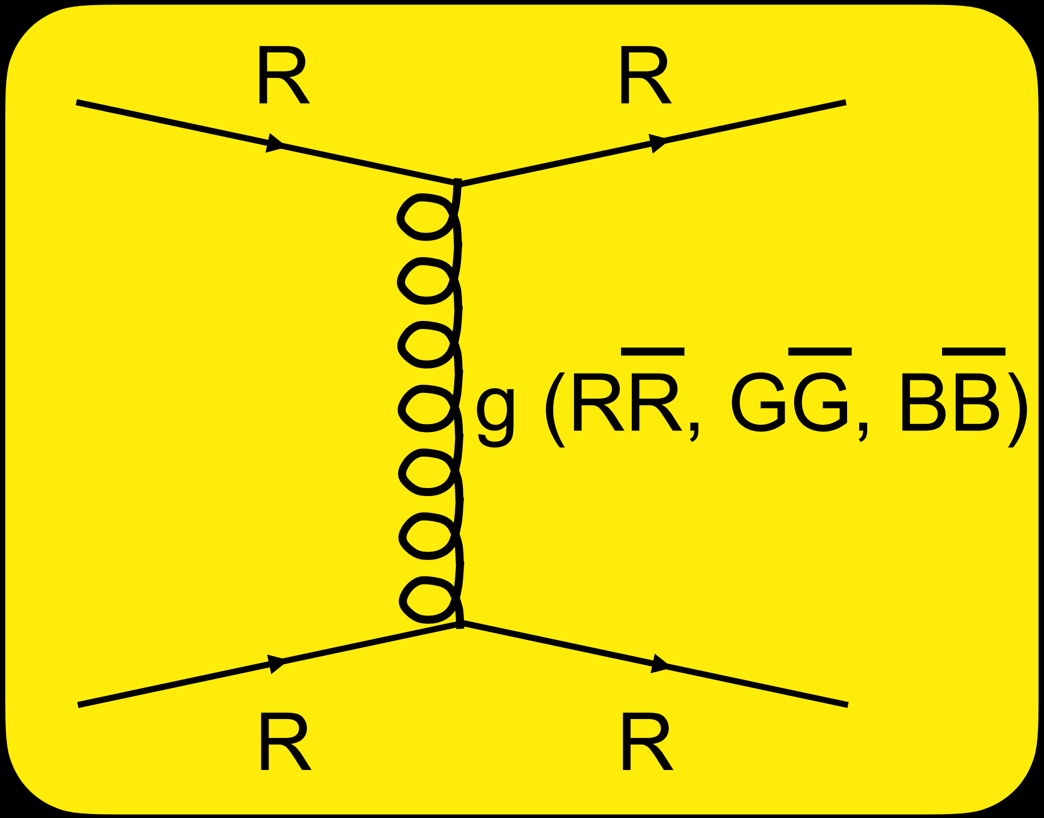


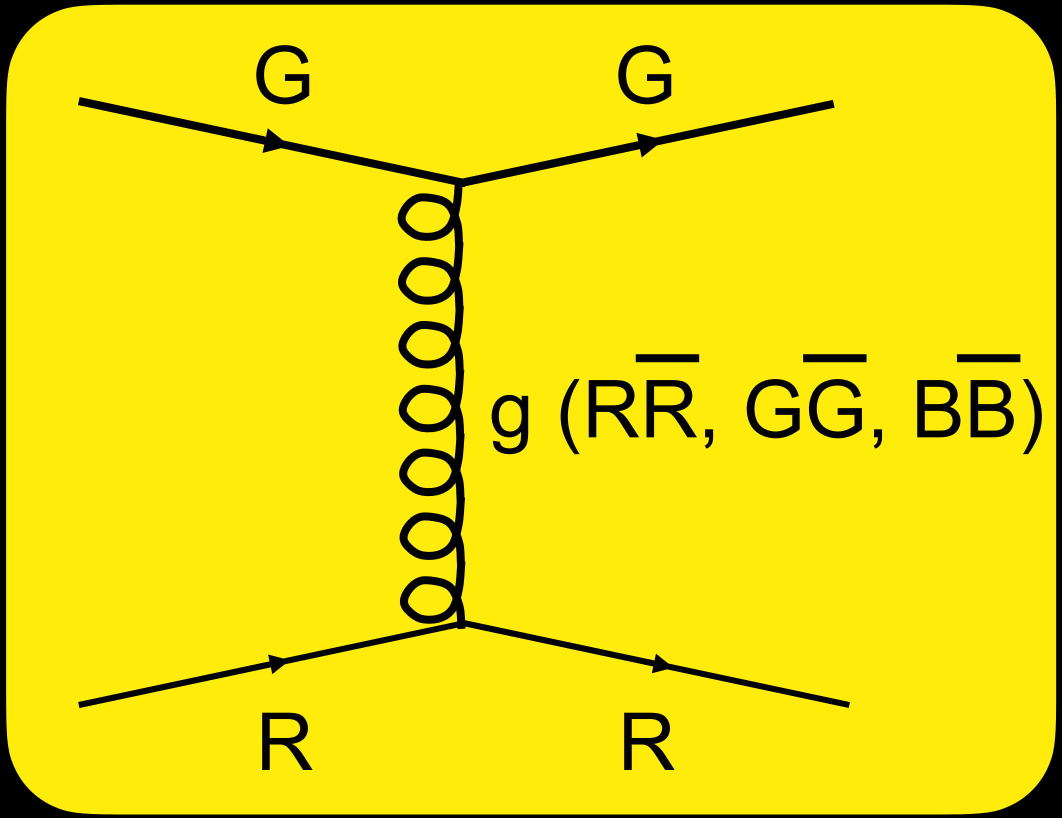


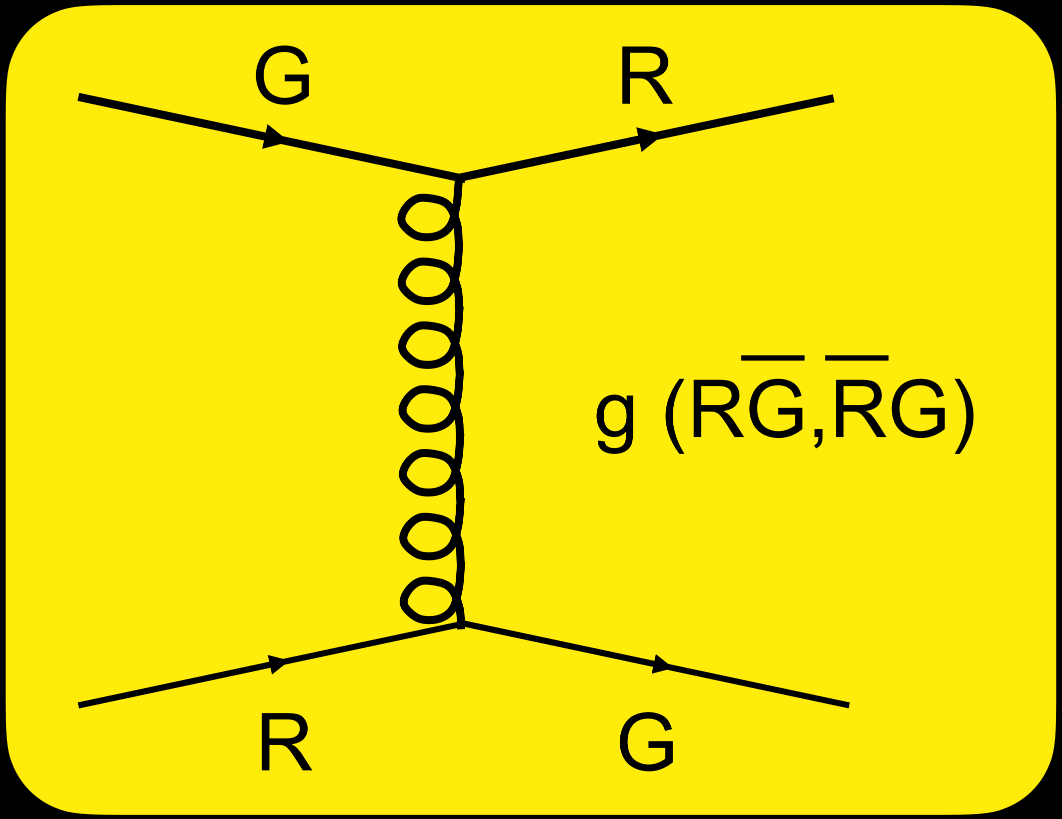
Note that such processes happen but are not directly seen in experiments (only indirectly via e.g. jet production)

- ✓ (anti)quarks have a(n) (anti)colour
- ✓ Gluons have a combination of colour and anti colour
- ✓ Colour needs to be conserved at every vertex

QCD processes (qq interactions)





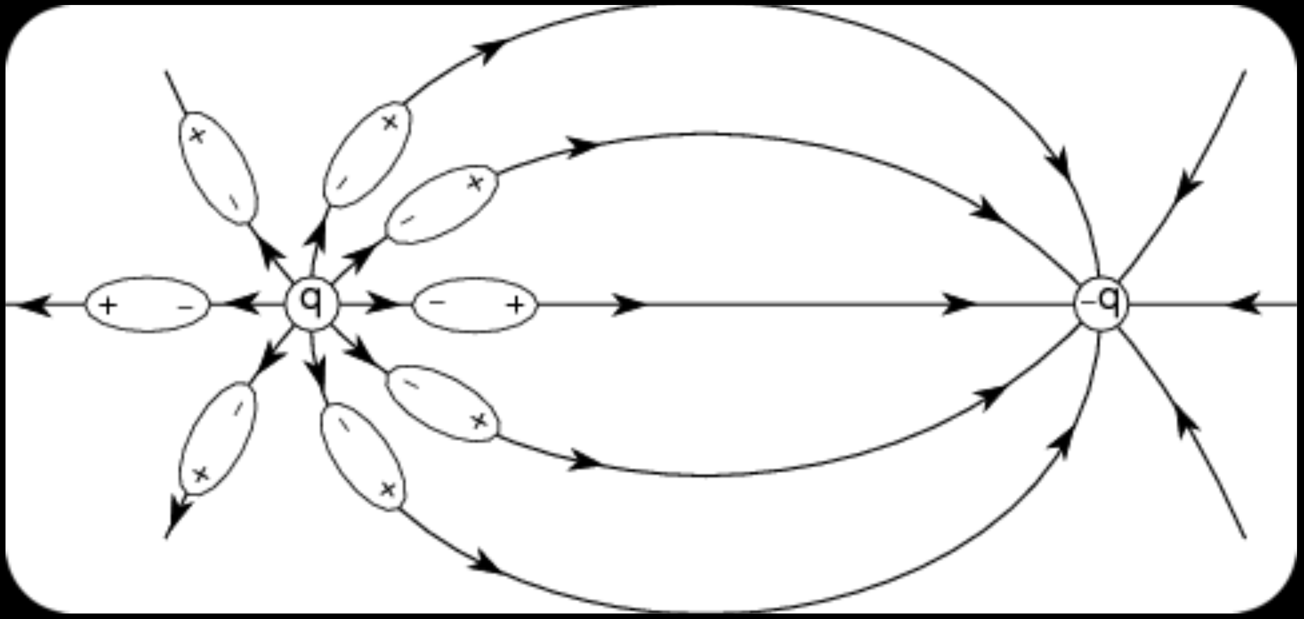


Strong coupling strength

QCD vs QED

- Quantum chromodynamics or else QCD is the theory that describes the strong interaction.
- Quite similar formulation between QED and QCD but also a number of fundamental differences
  - One of the fundamental differences:
    - in QED there is one charge (i.e. electric charge)
    - The equivalent in QCD is the colour
      - There are three kind of colours in QCD,: red (R), green (G) and blue (B).
    - Gluons have two colours, carrying one unit of color and one of anticolor.
      - There are 3x3 = 9 possibilities for the gluons but as we will see later there are only 8.
  - Since the gluons carry color, they can also couple directly to other gluons making the existence of gluon-gluon vertices possible.
  - Another difference comes from the fact that particles decaying strongly have typical lifetimes of 10^{-23}sec.

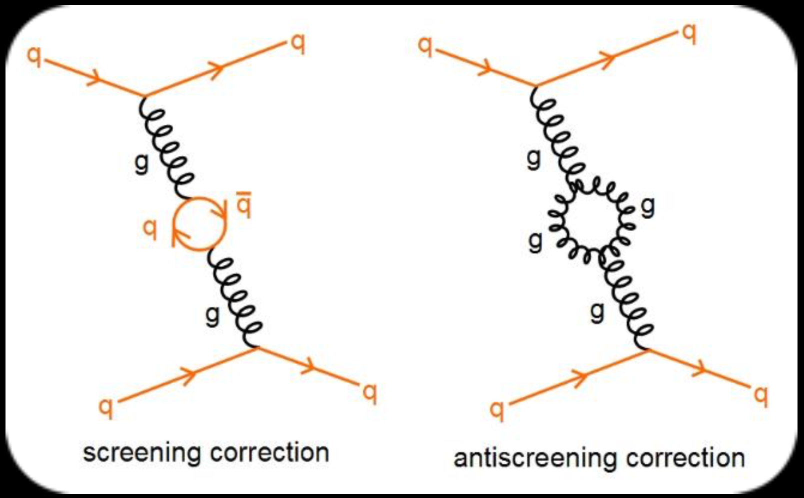
Charge screening in QED



- In QED a charged particle is surrounded by a cloud of virtual photons and electron-positron pairs continuously pop in and out of existence
- Because of attraction and repulsion in case of an electron, the positrons of the pairs tend to be closer and "screen" its charge
  - This is called vacuum polarisation and is analogous to the polarisation of a dielectric medium
- This gives rise to the notion of an effective charge that becomes smaller at large distances
- One defines the β-function that is positive in QED (as we will see later)

$$\beta = -\frac{de(r)}{d(\ln r)}$$

Charge antiscreening in QCD



- Likewise in QCD a quark is surrounded by quark-antiquark pairs and if this would be the only thing then the effect would be similar to the one we see in QED
  - the strong coupling constant would be small at large distances
- However, due to the gluon self-coupling the vacuum is also filled by gluon pairs
- Due to the colour charge that quarks and gluons carry the effective charge becomes larger with increasing energy
  - the β-function is negative in QCD
  - The effect is called anti-screening
- The negative contribution from the colour charge wins over the positive contribution of the normal charge and the quark colour charge
  - the strong coupling constant becomes small and short distances

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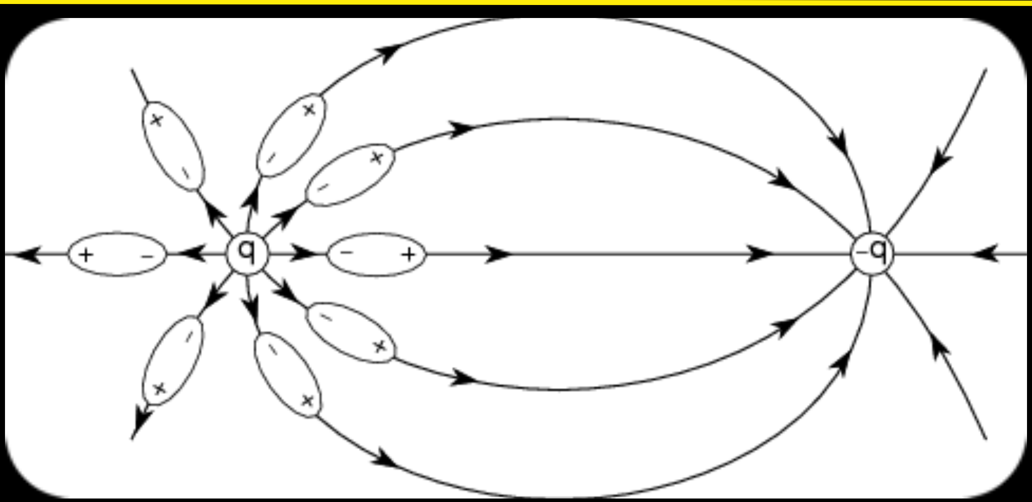
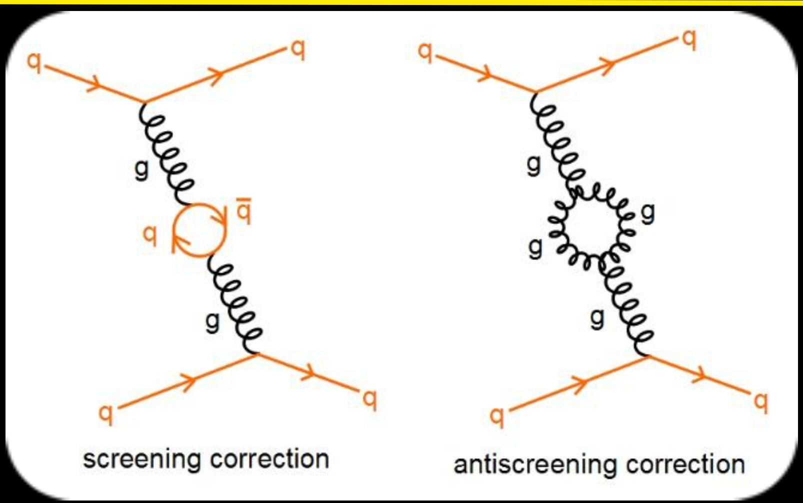
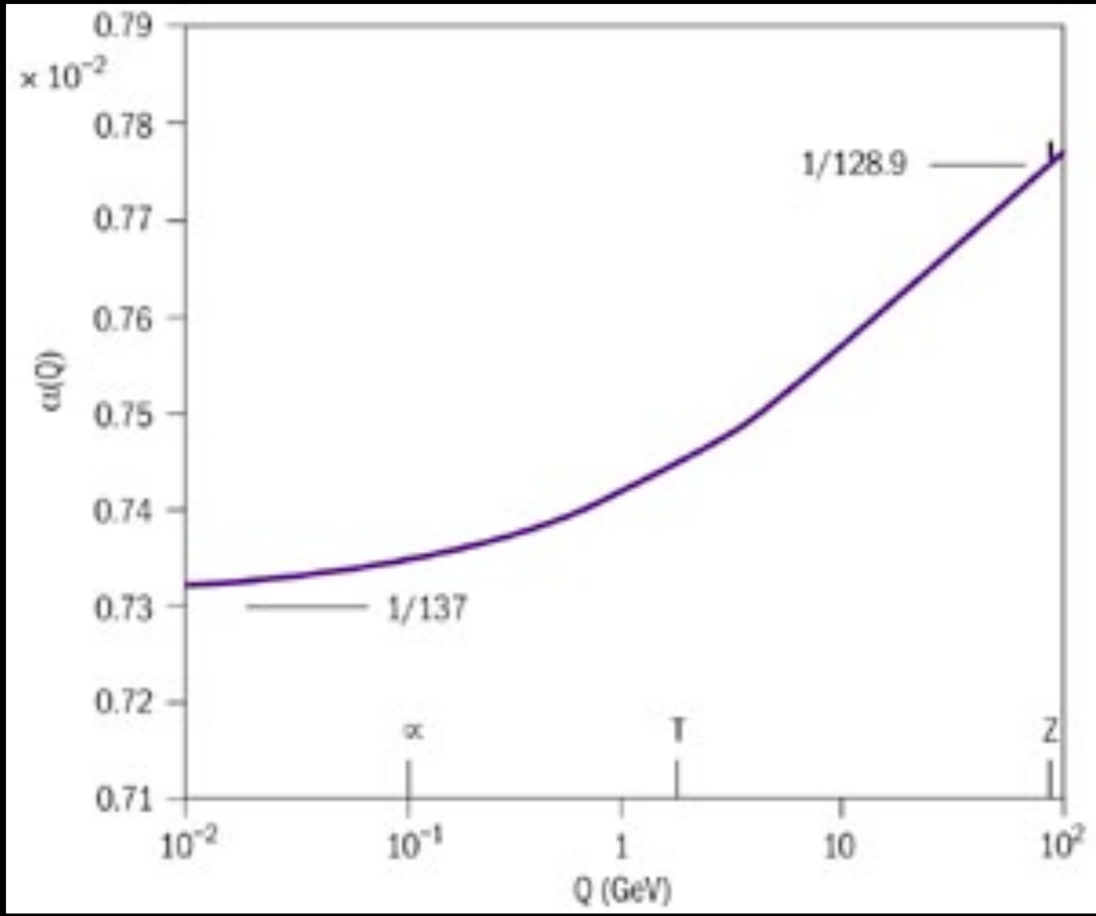
QCD vs QED (cont.)

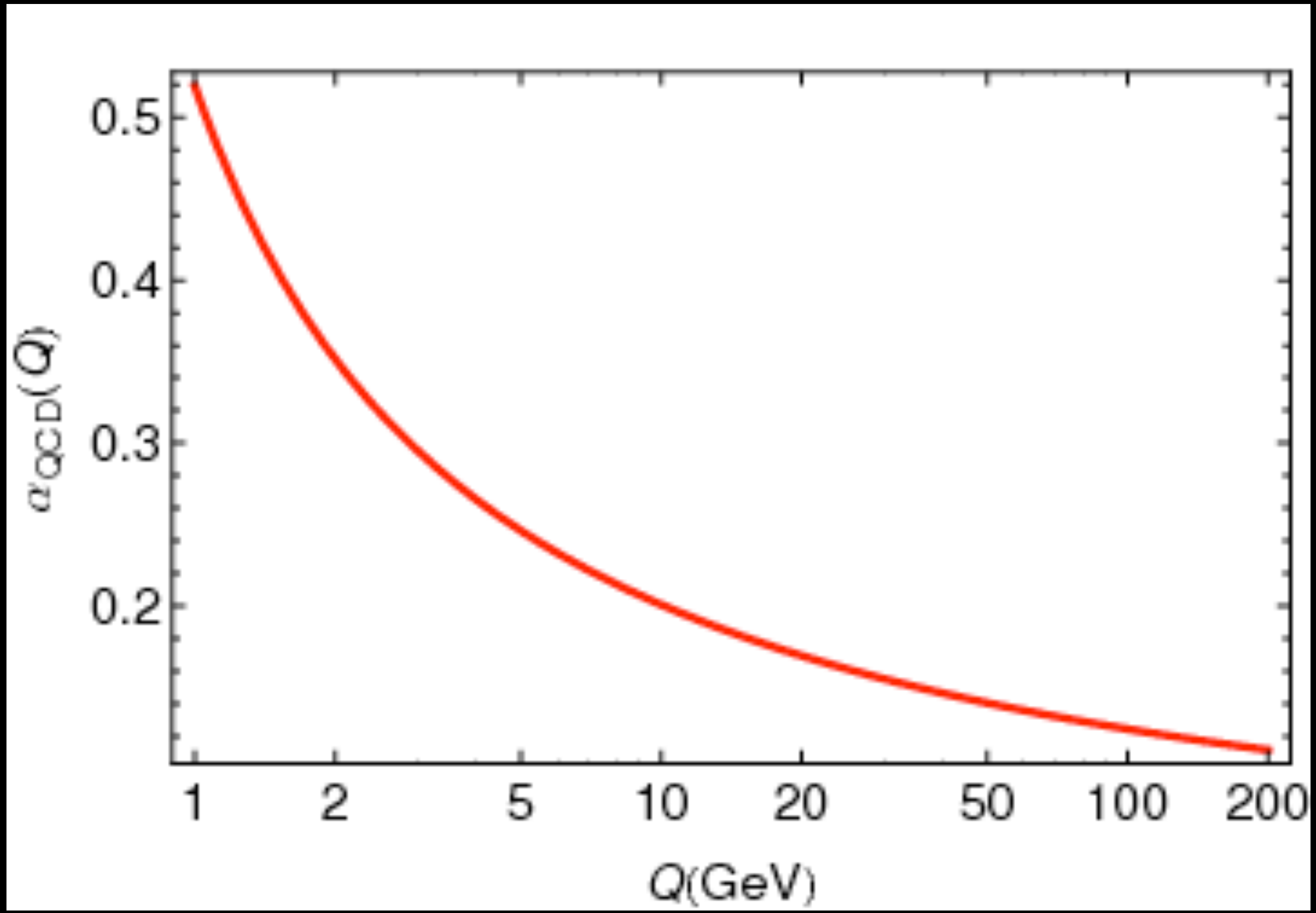




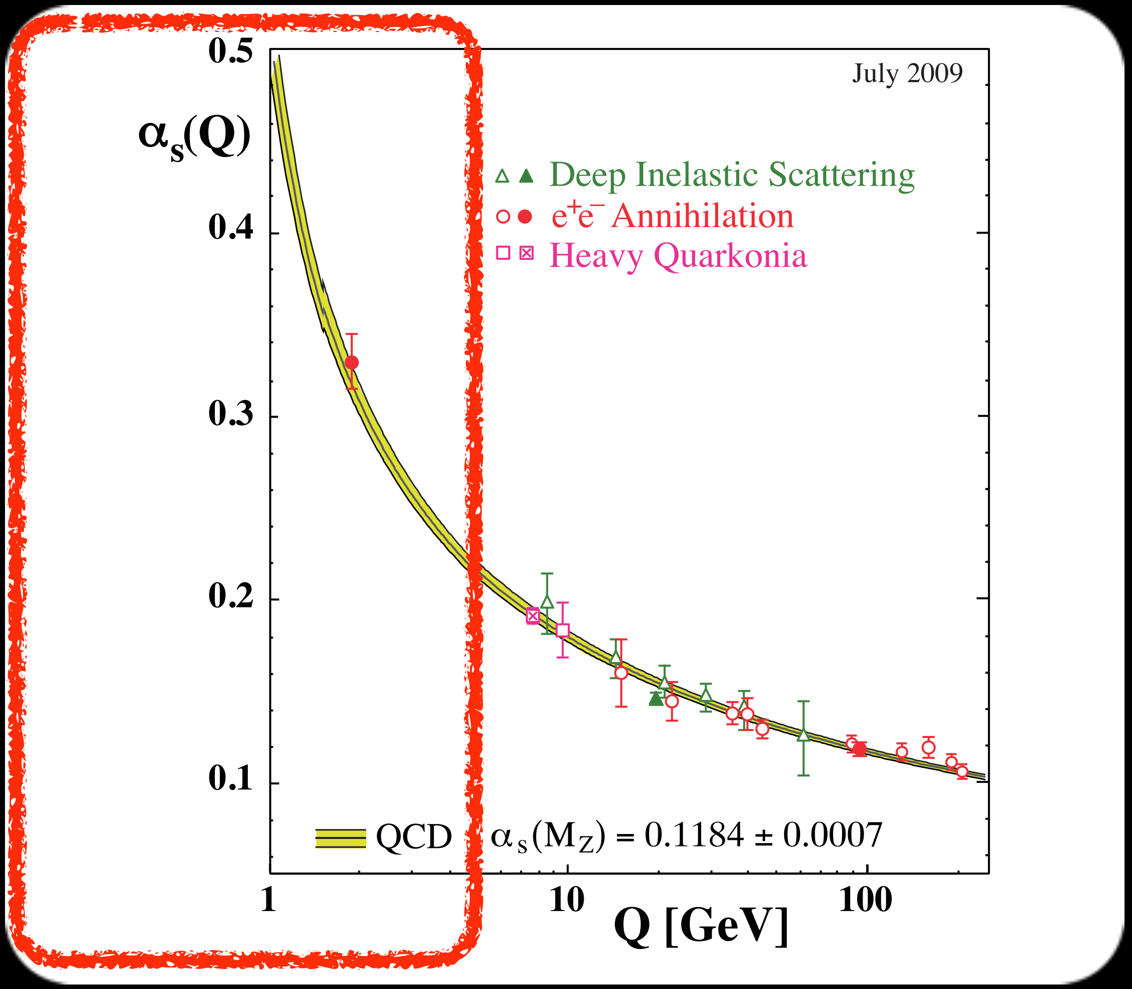
$$a(Q^2) = \frac{a(\mu^2)}{1 - \frac{a(\mu^2)}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)}$$

$$a_s(Q^2) = \frac{a_s(\mu^2)}{1 + \frac{a_s(\mu^2)}{12\pi}(11N_c - 2n_f)\ln\left(\frac{Q^2}{\mu^2}\right)}$$





Confinement

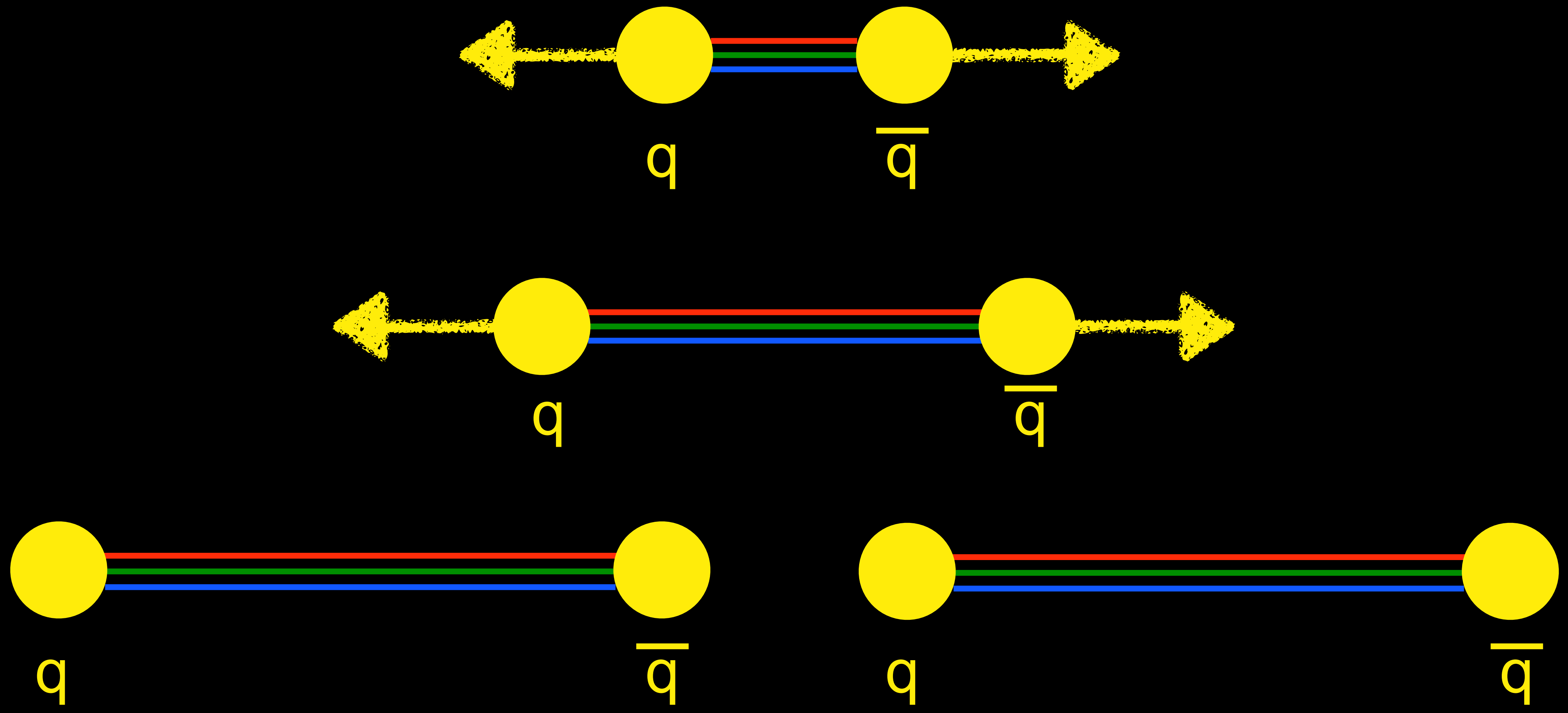


* Remember the uncertainty principle

$$\Delta P \Delta x \geq \hbar$$

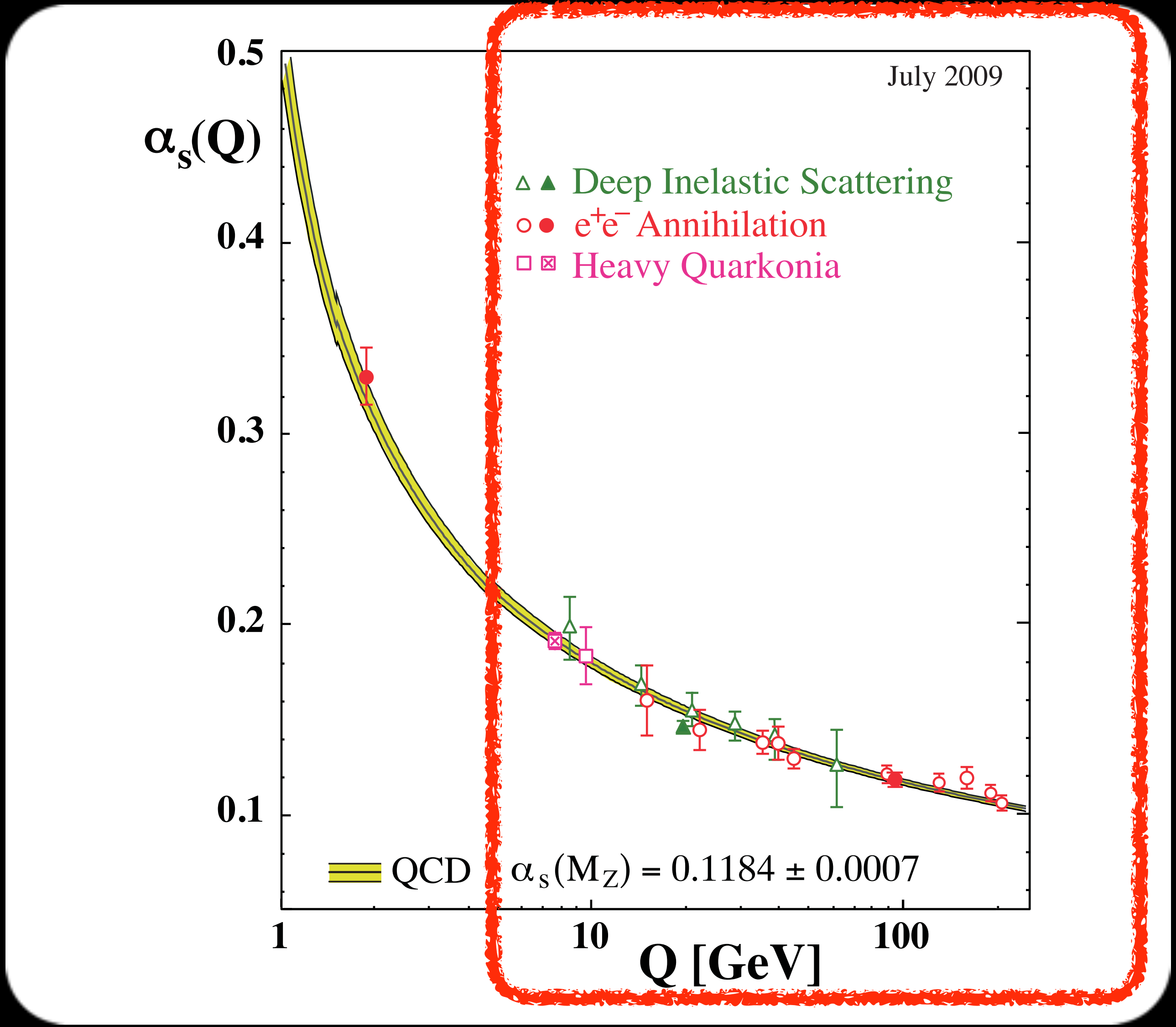
- For small values of momentum transfer, large spatial range*, the strong interactions are exceptionally strong
  - a_s >>
  - Quarks are bound together within hadrons
    - We can not find quarks and gluons moving freely in nature

Confinement (cont.)



- Breaking the colour string that connects a pair of quark and antiquark is not easy
- At some point, it is energetically more favourable to create a new pair of quarks and antiquarks
  - Quarks and gluons can not move freely

Asymptotic freedom



- For large value of momentum transfer, small spatial ranges, a_s becomes asymptotically small
  - Perturbation techniques can be used to perform high precision QCD calculations
  - Quarks and gluons can be considered as quasi-free particles

Asymptotic freedom (cont.)

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek

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The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.

Probing the strong coupling constant

$$R = \frac{\sigma(e^- + e^+ \to \text{hadrons})}{\sigma(e^- + e^+ \to \mu^- + \mu^+)} = R_{EW}(Q)\left[1 + \delta_{QCD}(Q)\right]$$

where $R_{EW}(Q)$ is the purely electroweak prediction for the ratio and $\delta_{QCD}(Q)$ is the correction due to QCD effects. To keep the discussion simple, we can restrict our attention to energies $Q << M_Z$, where the process is dominated by photon exchange ($R_{EW} = 3\sum_q e_q^2$, neglecting finite-quark-mass corrections, where the e_q are the electric charges of the quarks):

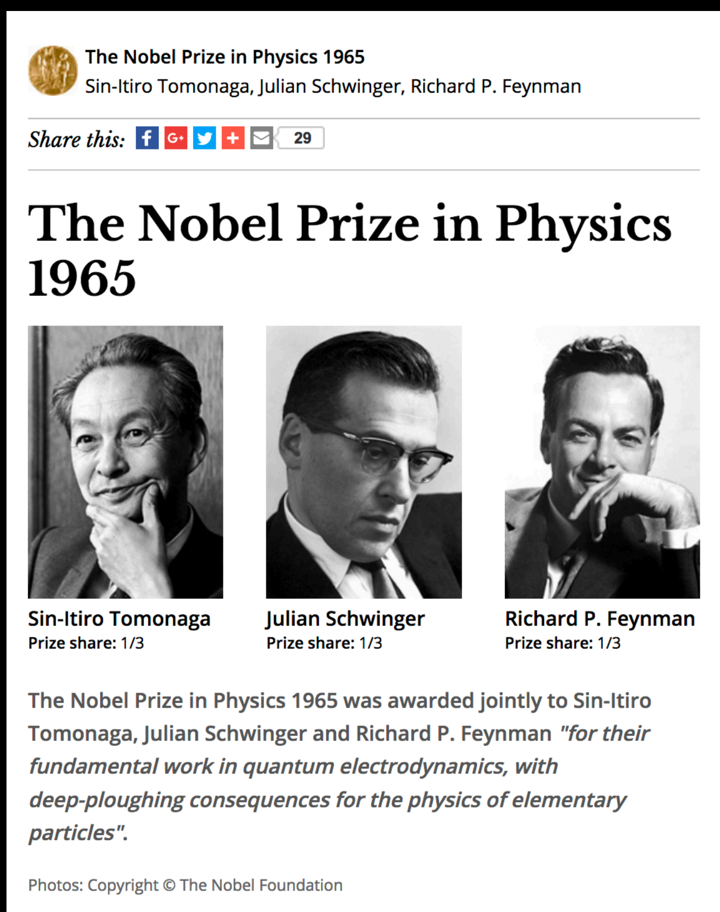
$$\delta_{QCD}(Q) = \sum_{n=1}^{\infty} c_n \left[\frac{\alpha_s(Q^2)}{\pi}\right]^n + \mathrm{O}\left(\frac{\Lambda^4}{Q^4}\right)$$

A new global analysis using all available precision data of deep inelastic and related hard scattering processes includes recent measurements of structure functions from HERA and of the inclusive jet cross sections at the Tevatron. After analysis of experimental and theoretical uncertainties the authors obtain

$$\alpha_s(M_{Z^0}) = 0.1165 \pm 0.002 \text{ (exp.)} \pm 0.003 \text{ (theo.)}, \quad (7.2.4)$$

The Standard Model (theoretical framework)

Quantum ElectroDynamics (QED)


The Nobel Prize in Physics 1965
Sin-Itiro Tomonaga, Julian Schwinger, Richard P. Feynman

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The Nobel Prize in Physics 1965

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Julian Schwinger
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Richard P. Feynman
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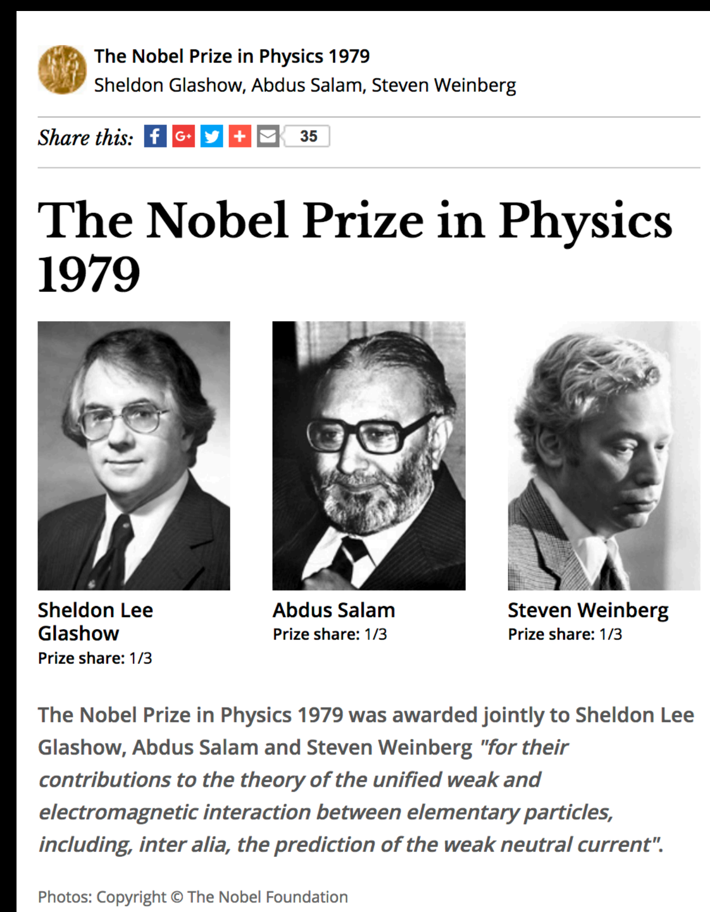
The Nobel Prize in Physics 1965 was awarded jointly to Sin-Itiro Tomonaga, Julian Schwinger and Richard P. Feynman *"for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"*.

Electroweak Unification (GSW)


The Nobel Prize in Physics 1979
Sheldon Glashow, Abdus Salam, Steven Weinberg

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Steven Weinberg
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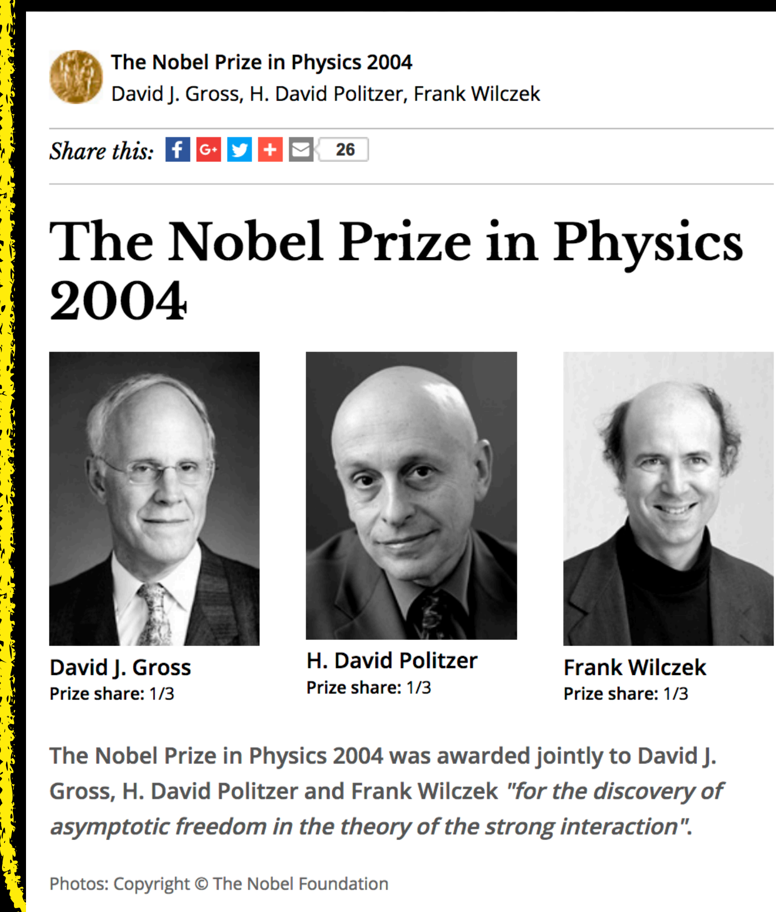
The Nobel Prize in Physics 1979 was awarded jointly to Sheldon Lee Glashow, Abdus Salam and Steven Weinberg *"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"*.

Quantum ChromoDynamics (QCD)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The Standard Model (theoretical framework)

The Higgs mechanism

The Nobel Prize in Physics 2013
François Englert, Peter Higgs

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The Nobel Prize in Physics 2013

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François Englert
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Photo: A. Mahmoud
Peter W. Higgs
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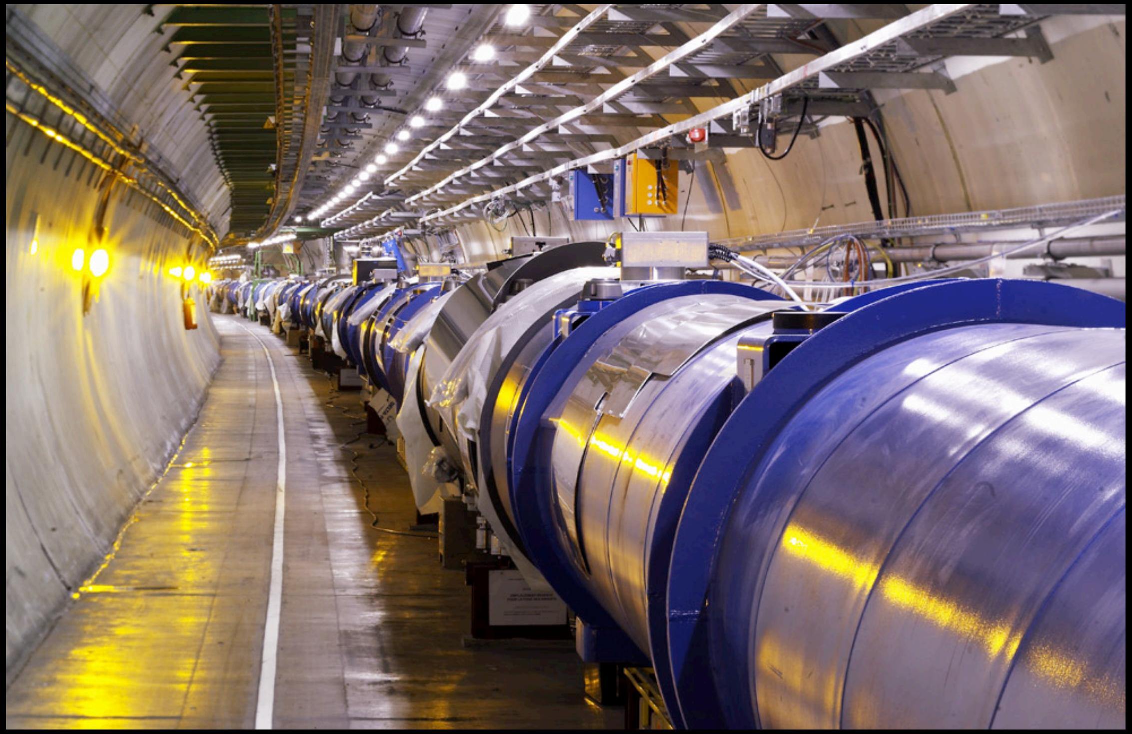
The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"*

Test of the Standard Model: Particle accelerators

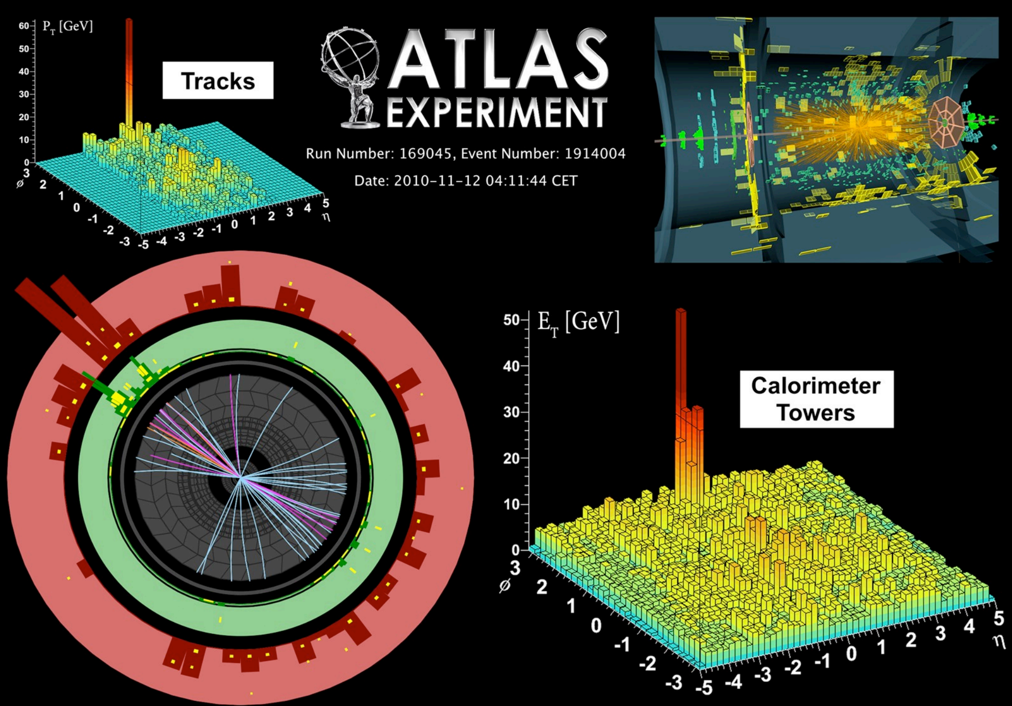
- The Large Hadron Collider (LHC)
  - The biggest and most powerful accelerator ever built
  - Used to accelerate protons but also heavy ions at ultra relativistic energies

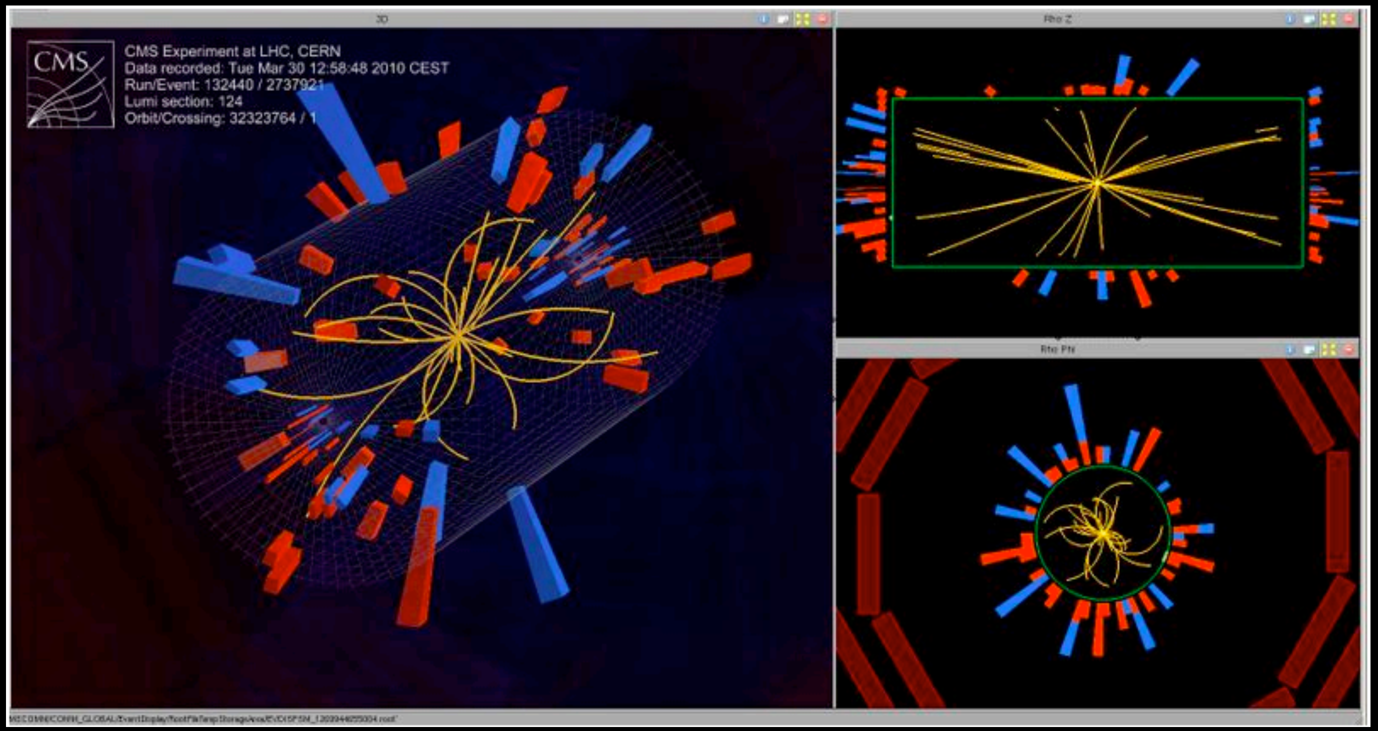
- LHCb (smallest) optimised to study pp collisions in forward regions
  - Study matter anti-matter asymmetry ⇒ weak interactions (main signal), QCD background
- ATLAS & CMS (largest) optimised to study pp collisions
  - Higgs and searches for physics beyond the Standard Model ⇒ QCD (huge) background
- ALICE (third largest) optimised to study heavy-ion collisions
  - Quark Gluon Plasma ⇒ QCD is the signal & the background

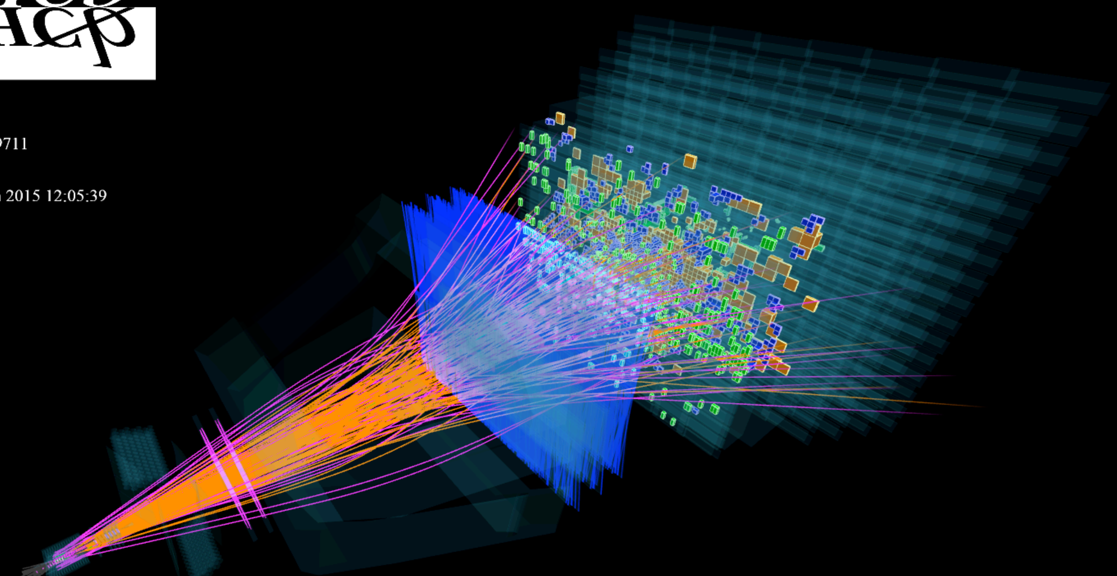
Understanding QCD is of great importance to interpret properly the LHC results!!!

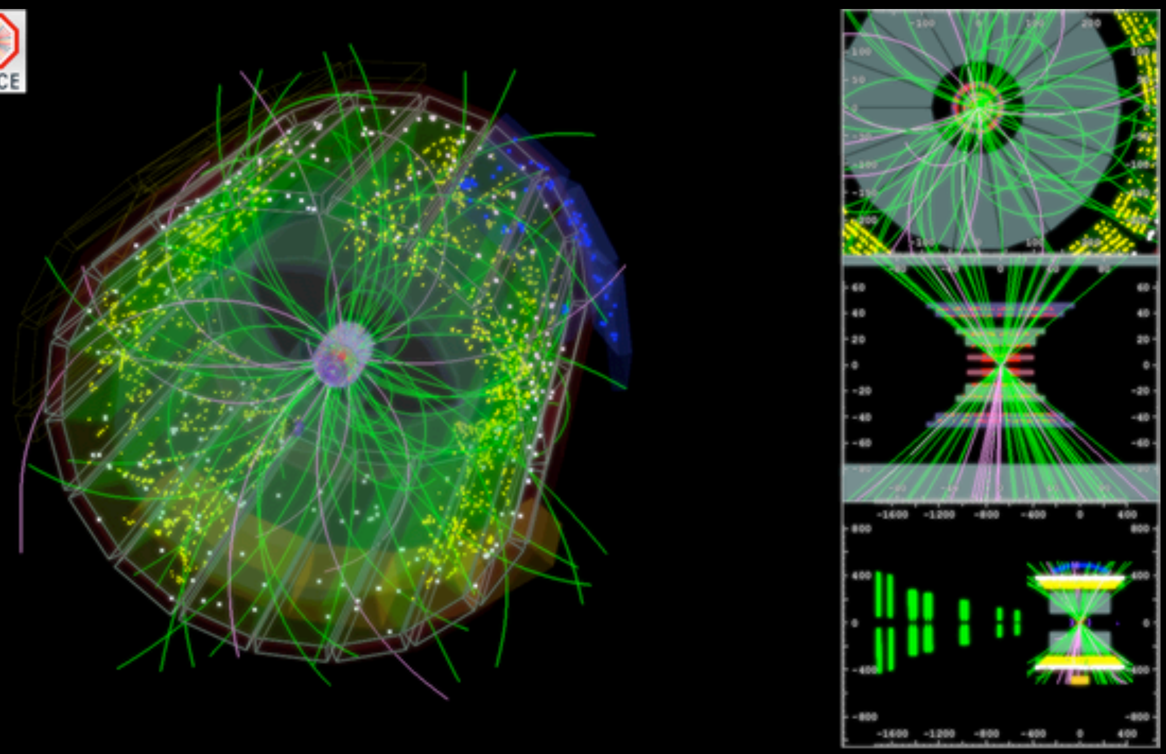


Particle detectors (pp collisions ➡ small number of particles)

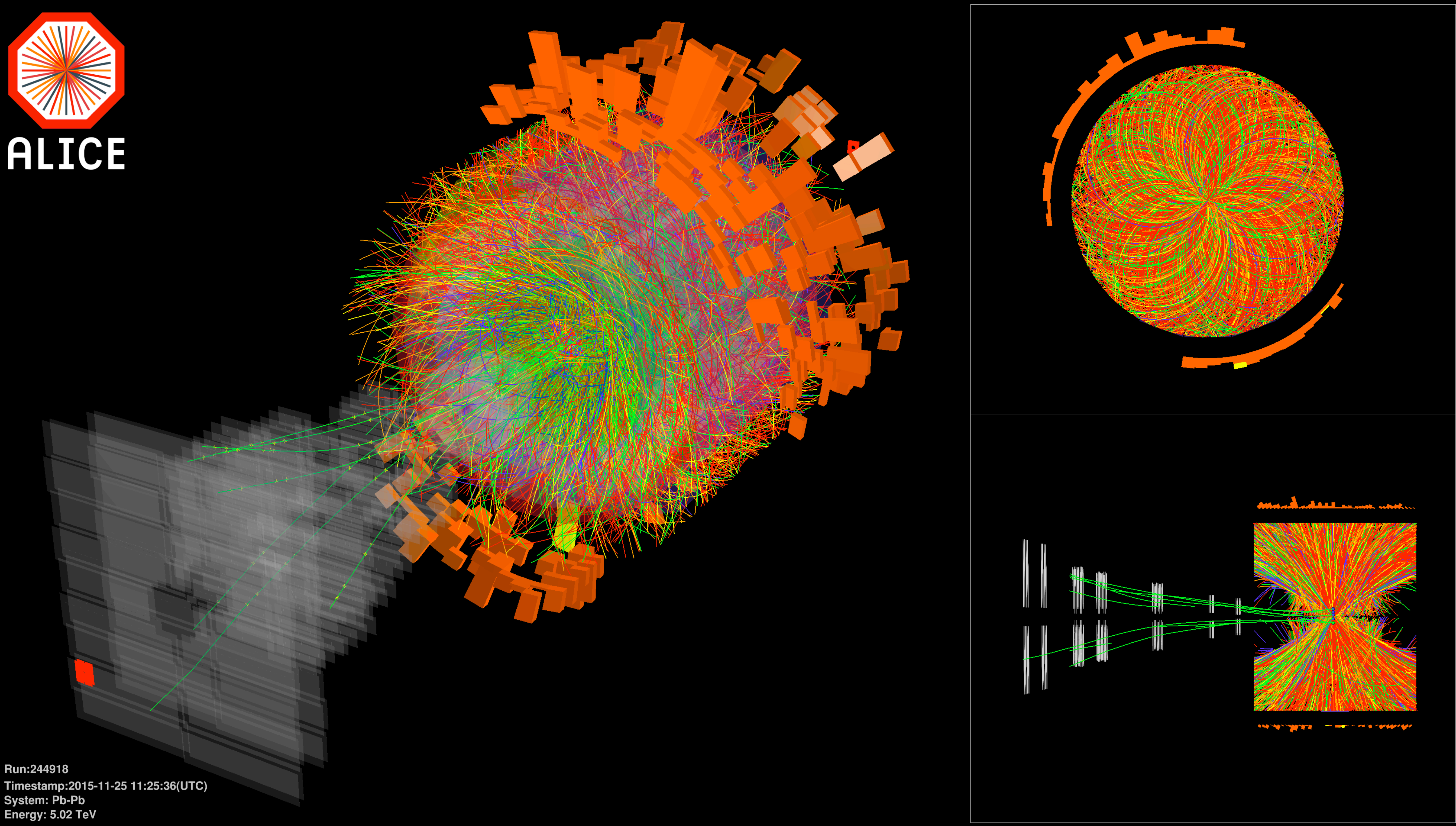








Particle detectors (Pb-Pb collisions ➡ large number of particles)



Deep Inelastic Scattering (DIS)

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Evidence of colour and different quark flavour

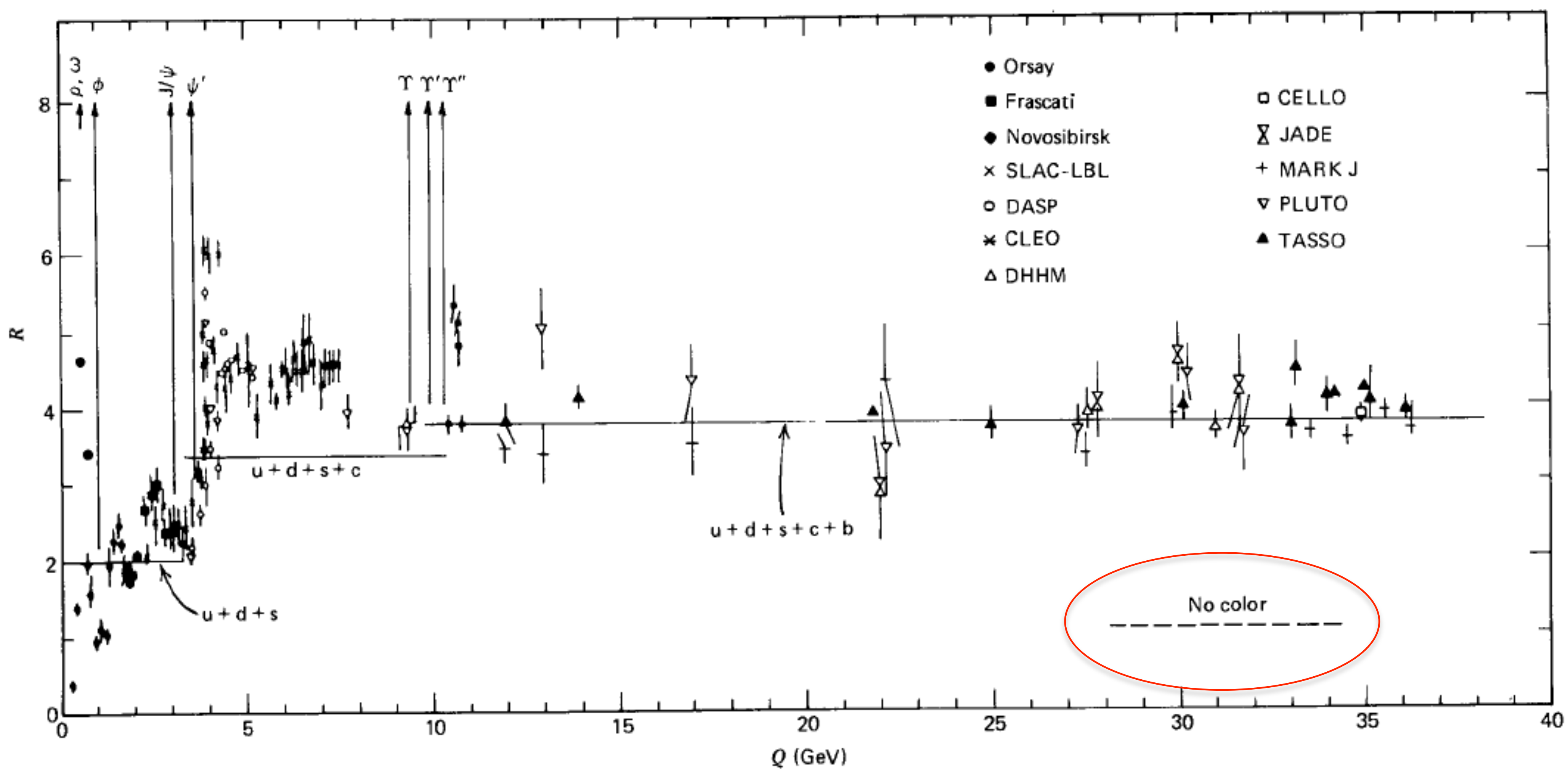
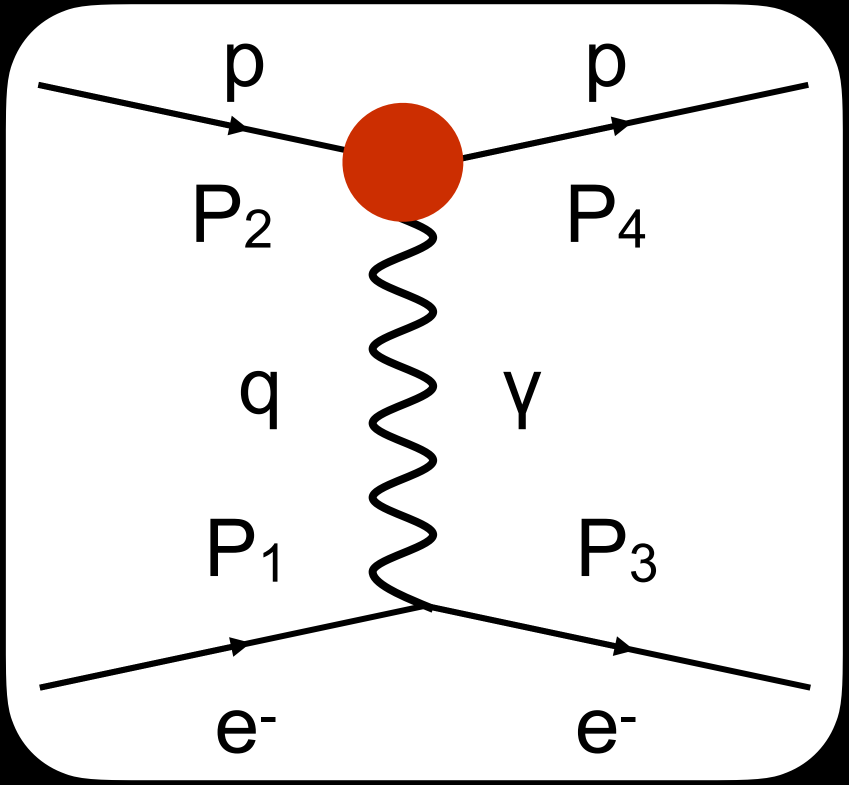
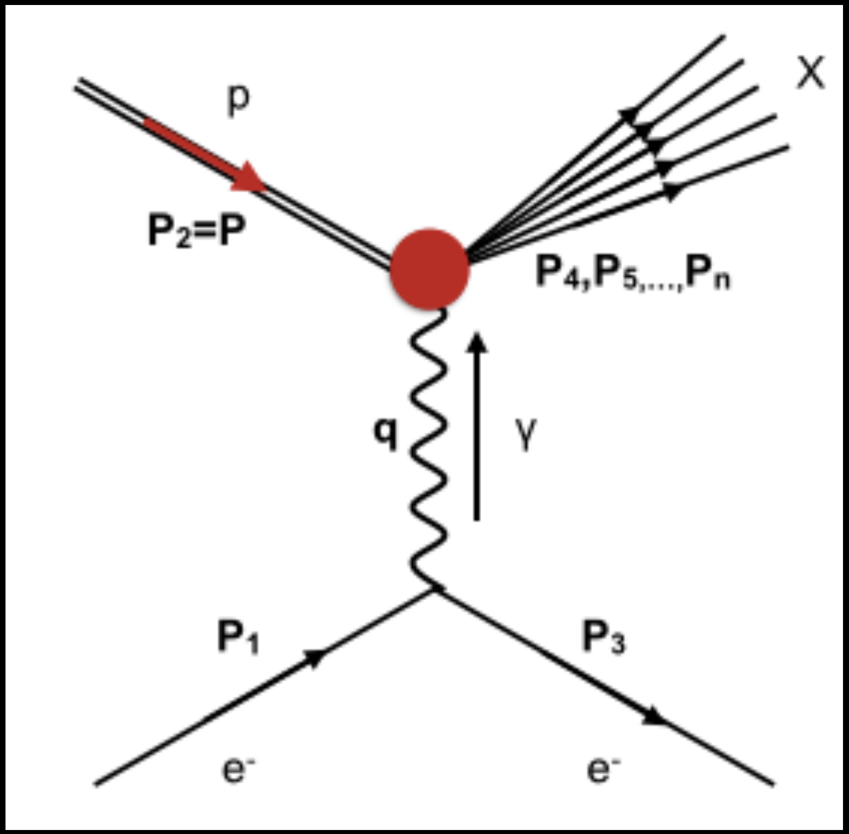


Fig. 11.3 Ratio R of (11.6) as a function of the total $e^- e^+$ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

Probing the structure of protons

- The precise nature of $e^- p \to e^- p$ scattering process depends on the wavelength of the virtual photon in comparison with the proton radius
  - at very low energies, where the electrons are non-relativistic and the wavelength of the virtual photon is large compared to the radius of the proton ($\lambda >> r_p$), the process is described by the elastic scattering of the electron in the static potential of an effective point-like proton
  - At higher electron energies, where $\lambda \sim r_p$, the scattering process is no longer purely electrostatic and the cross-section needs to account for the extended charge and magnetic moment distribution of protons
  - when the wavelength of the virtual photon becomes relatively small ($\lambda < r_p$), the contribution from the elastic process becomes also small. The dominant process is of inelastic nature, where the virtual photon interacts with the constituent quark of the proton and the proton breaks up
    - the inelastic electron-proton scattering can be considered an elastic electron-quark scattering process
  - at even higher energies, where the wavelength of the virtual photon is sufficiently short ($\lambda << r_p$) to resolve the detailed dynamic structure of the proton, the proton appears to consist of a sea of strongly interacting quarks and gluons





Sub-nuclear structure

- In order not to see the proton as one compound object but rather to probe its internal structure, one has to bombard it with highly energetic particles
  - Rutherford followed a similar trick by bombarding the thin gold foils with α-particles
- Experiments later used higher energy projectiles and revealed that the (up to that moment known as) point-like core had some internal structure
  - The scattering distributions were damped by the relevant form factors of the nucleus
- In 1932 Chadwick discovered the neutron and it became clear that the nucleus consisted of protons and neutrons

James Chadwick (1891 - 1974)

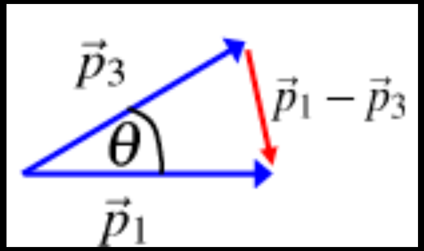
electron-proton scattering vs energy: Rutherford scattering

- ✓ The Rutherford scattering is the low-energy limits of e^-p scattering
- ✓ In this case the electron energy is sufficiently low that is considered as non-relativistic
- ✓ Also the kinetic energy of the recoiling proton is negligible compared to its rest mass
- ✓ In this case the proton can be considered as a fixed, point-like source of 1/r electrostatic potential
- ✓ The cross-section is calculated from scattering theory by using the first order terms in the perturbation expansion

electron-proton scattering vs energy: Rutherford scattering

- Rutherford scattering:
  - the proton recoil can be neglected and the electron is non-relativistic
  - The differential cross-section is given then by

$$\langle |M_{if}| \rangle^2 = \frac{16 M_p^2 m_e^2 e^4}{q^4}$$



$$q^2 = (p_1 - p_3)^2 = 4 |\vec{p}|^2 \sin^2(\theta/2)$$

$$\langle |M_{if}| \rangle^2 = \frac{M_p^2 m_e^2 e^4}{|\vec{p}|^4 \sin^4(\theta/2)}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left(\frac{1}{M_p + E_1 - E_1 \cos\theta} \right)^2 \langle |M_{if}| \rangle^2$$

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electron-proton scattering vs energy: Mott scattering

- The Mott scattering is the limit where the electron is relativistic but the proton recoil can still be negligible
- These conditions apply when $m_e << E << m_p$
- The matrix element is given this time by

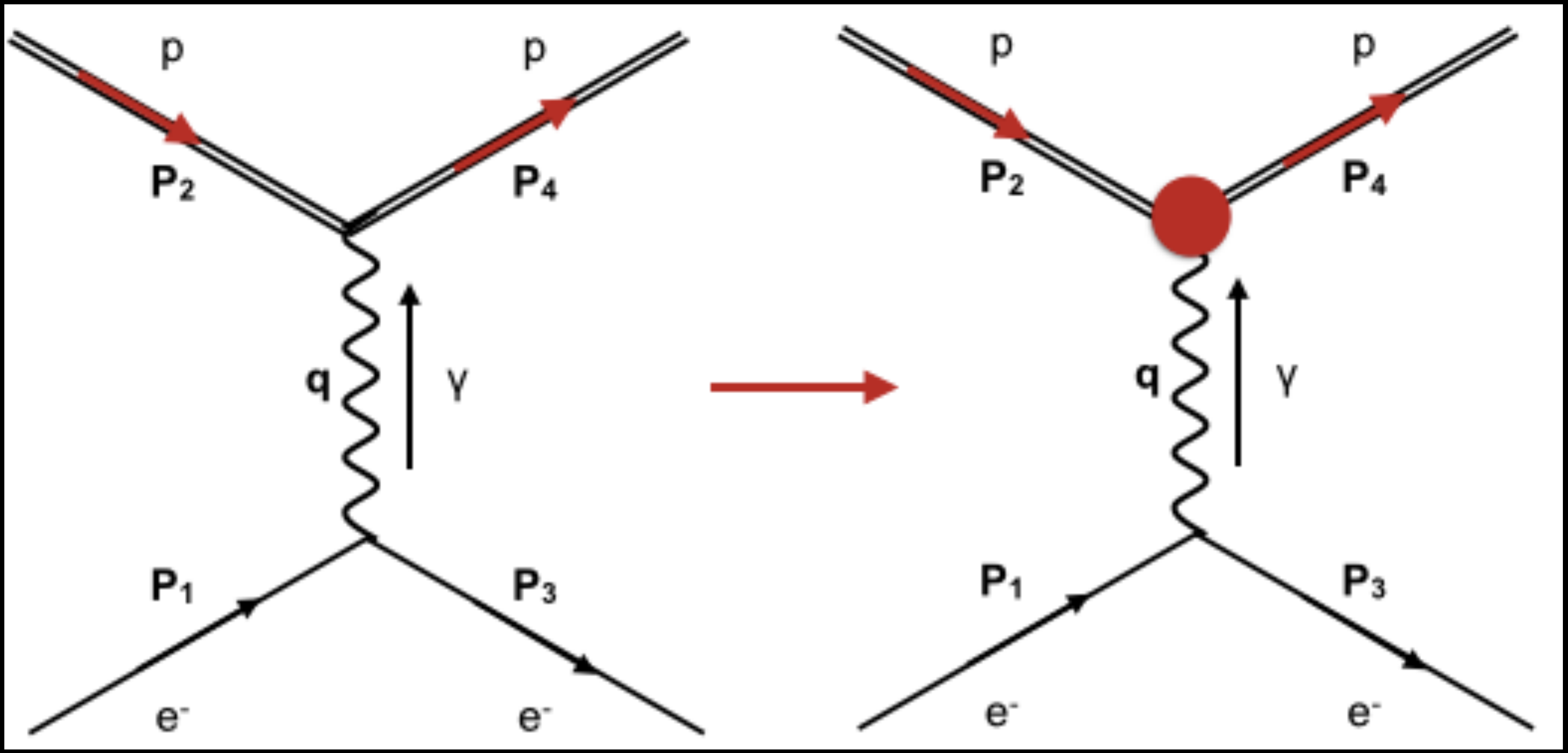
$$\langle |M_{if}|\rangle^2 \simeq \frac{e^4}{E^2 \sin^4(\theta/2)} \cos^2(\theta/2)$$

- while the differential cross-section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{a^2}{4E^2 \sin^4(\theta/2)} \cos^2(\theta/2)$$

Form factors

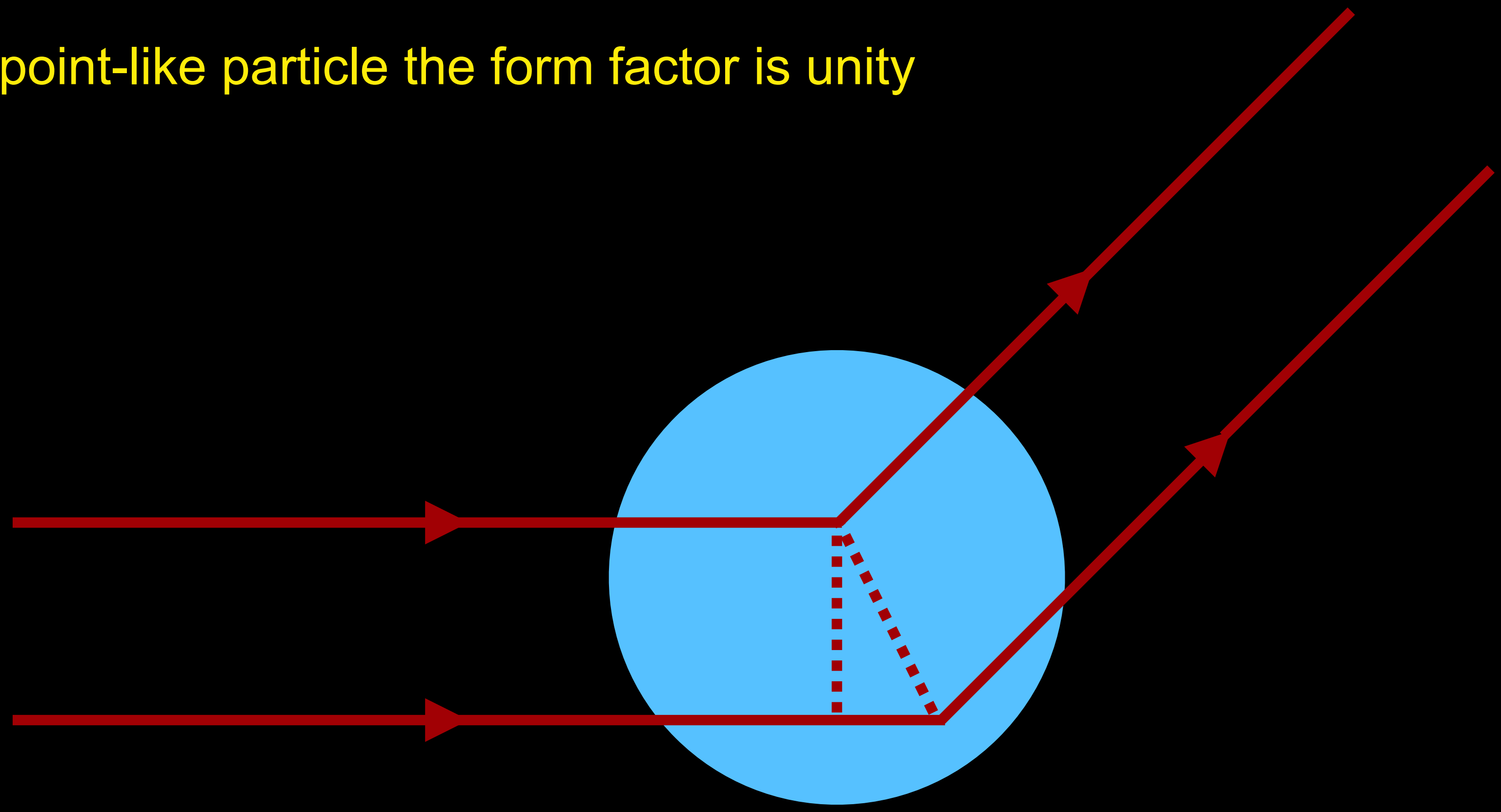
- ✓ Form factors are introduced to account for the fact that some particles are not elementary but have internal structure
- ✓ The electron and in general leptons are elementary particles and thus are ideal tools to probe the internal structure of particles
  - A typical example of such kind of interaction is the electron-proton elastic scattering



Form factors (cont.)

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} \rightarrow \frac{a^2}{4E^2 \sin^4(\theta/2)} \cos^2(\theta/2) \; |F(q^2)|^2$$

- Form factors act similar to diffraction of plane waves in optics
- The finite size of the scattering centre introduces a phase difference between plane waves “scattered from different points in space”
  - If the wavelength is long compared to the size of the object to be probed, all waves are in phase and $F(q^2) = 1$
- For a point-like particle the form factor is unity



Elastic form factors of nucleons

- Nucleons are not point like particles
  - They have an internal structure, containing a combination of uud (protons) and udd (neutrons)
- The best way to explore the charge and current distributions of nucleons is again via the elastic scattering with electron beams
- For protons, one can use a liquid hydrogen target in the path of an electron beam and determine the differential cross-section of the scattered electrons
- For neutrons things become more complicated as there is no neutron target
  - One relies on deuteron targets and subtract the effect of protons
  - This usually leads to significantly large uncertainties in the measurements
- The form factors for spineless particles is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} |F(q^2)|^2$$

  - This formula needs to be generalised for spin-1/2 particles with internal structure

Elastic form factors of nucleons (cont.)

- The form factor in the previous formula describes the electric charge distribution, and thus $F(q^2)$ is called the electric form factor
- A proton has also a magnetic moment with its "magnetisation" being distributed over the volume of the nucleon and is described by the magnetic form factor
- The generalisation of the previous formula is given by the Rosenbluth equation

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} = \frac{a^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + bG_E^2}{1+b} \cos^2(\theta/2) + 2bG_M^2 \sin^2(\theta/2) \right)$$

$$b = \frac{-q^2}{4M^2}$$

  - where G_E and G_M are the electric and magnetic form factors and they are both a function of q^2
  - M is the mass of the nucleon, θ is the scattering angle between the incoming and outgoing electrons with energies E_1 and E_3, respectively
  - q is the momentum transfer to the nucleon

Elastic form factors of nucleons (cont.)

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} = \frac{a^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + bG_E^2}{1+b} \cos^2(\theta/2) + 2bG_M^2 \sin^2(\theta/2) \right)$$

$$b = \frac{-q^2}{4M^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} = \frac{a^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \cos^2(\theta/2) \left(\frac{G_E^2 + bG_E^2}{1+b} + 2bG_M^2 \tan^2(\theta/2) \right)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(\frac{G_E^2 + bG_E^2}{1+b} + 2bG_M^2 \tan^2(\theta/2) \right)$$

- ✓ At low values of $q^2 \rightarrow b \sim 0$

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} / \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \simeq G_E^2(q^2)$$

- ✓ At high values of $q^2 \rightarrow b \gg 1$

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} / \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \simeq (1 + 2b \tan^2(\theta/2))\, G_M^2(q^2)$$

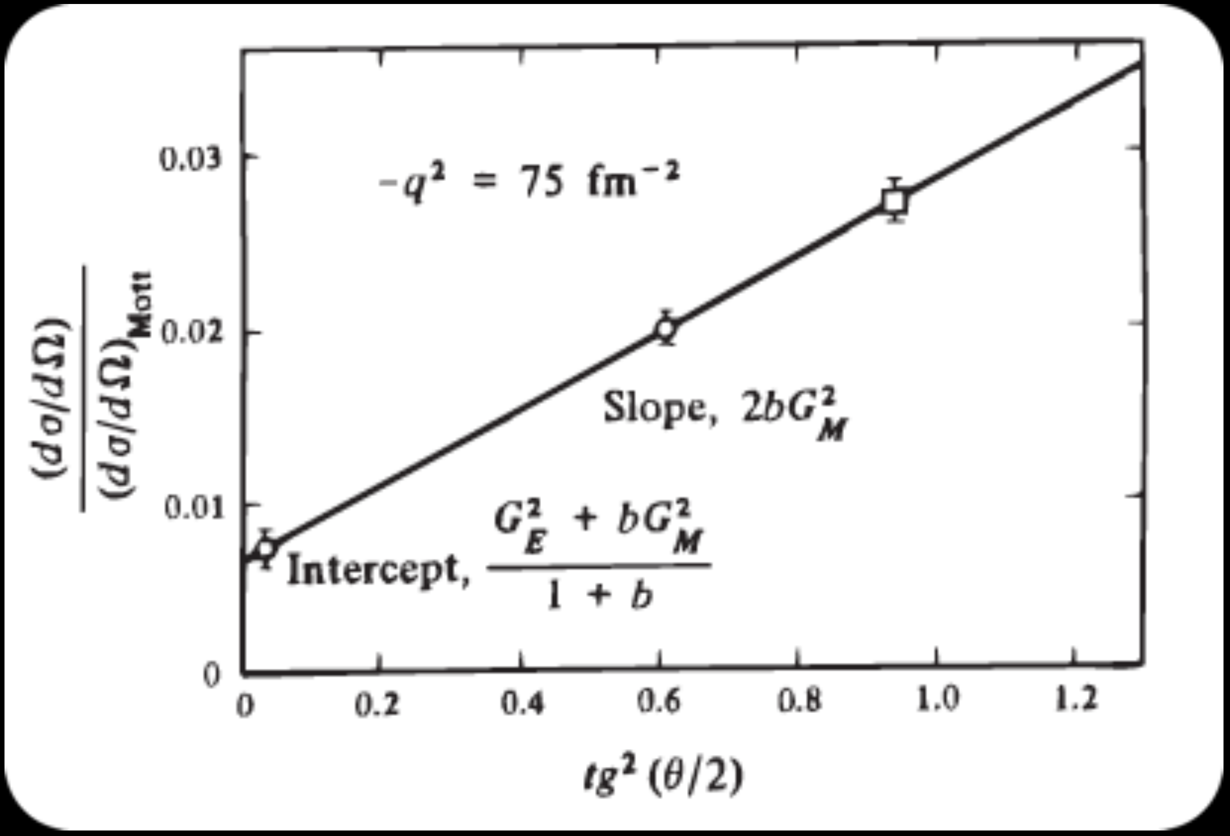
- ✓ In general we are sensitive to both form factors

Elastic form factors of nucleons (cont.)

- To extract the form factors, we use this formula

$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left(\frac{G_E^2 + bG_E^2}{1+b} + 2bG_M^2\tan^2(\theta/2)\right)$$

  - and plot the ratio of the measured cross-section for a given q^2 value over the Mott cross-section as a function of $\tan^2(\theta/2)$
  - The slope gives the factor multiplying $\tan^2(\theta/2)$ from where we extract G_M^2
  - The intercept on the y-axis gives the other factor from where we extract G_E^2



Elastic form factors of nucleons (cont.)

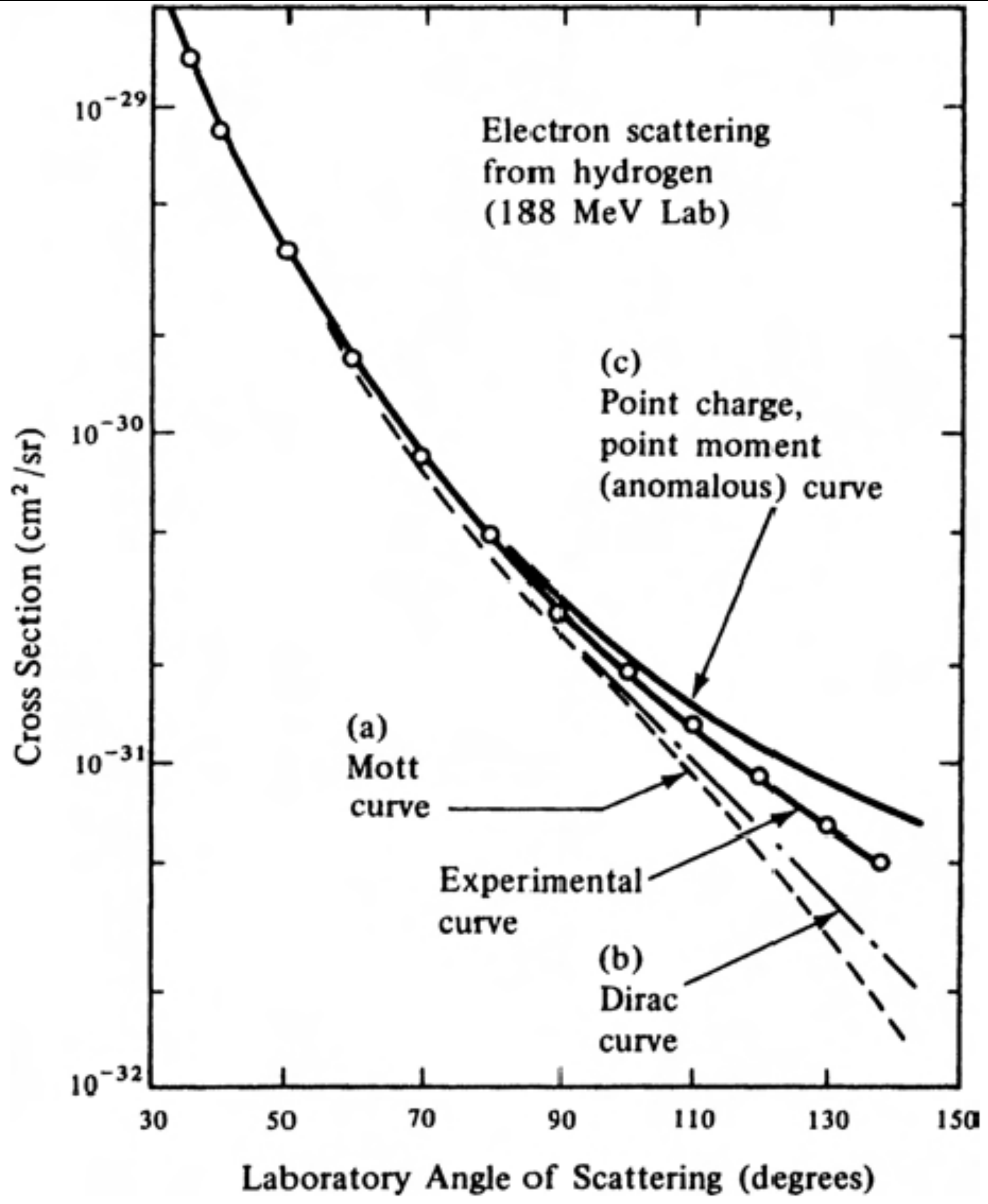
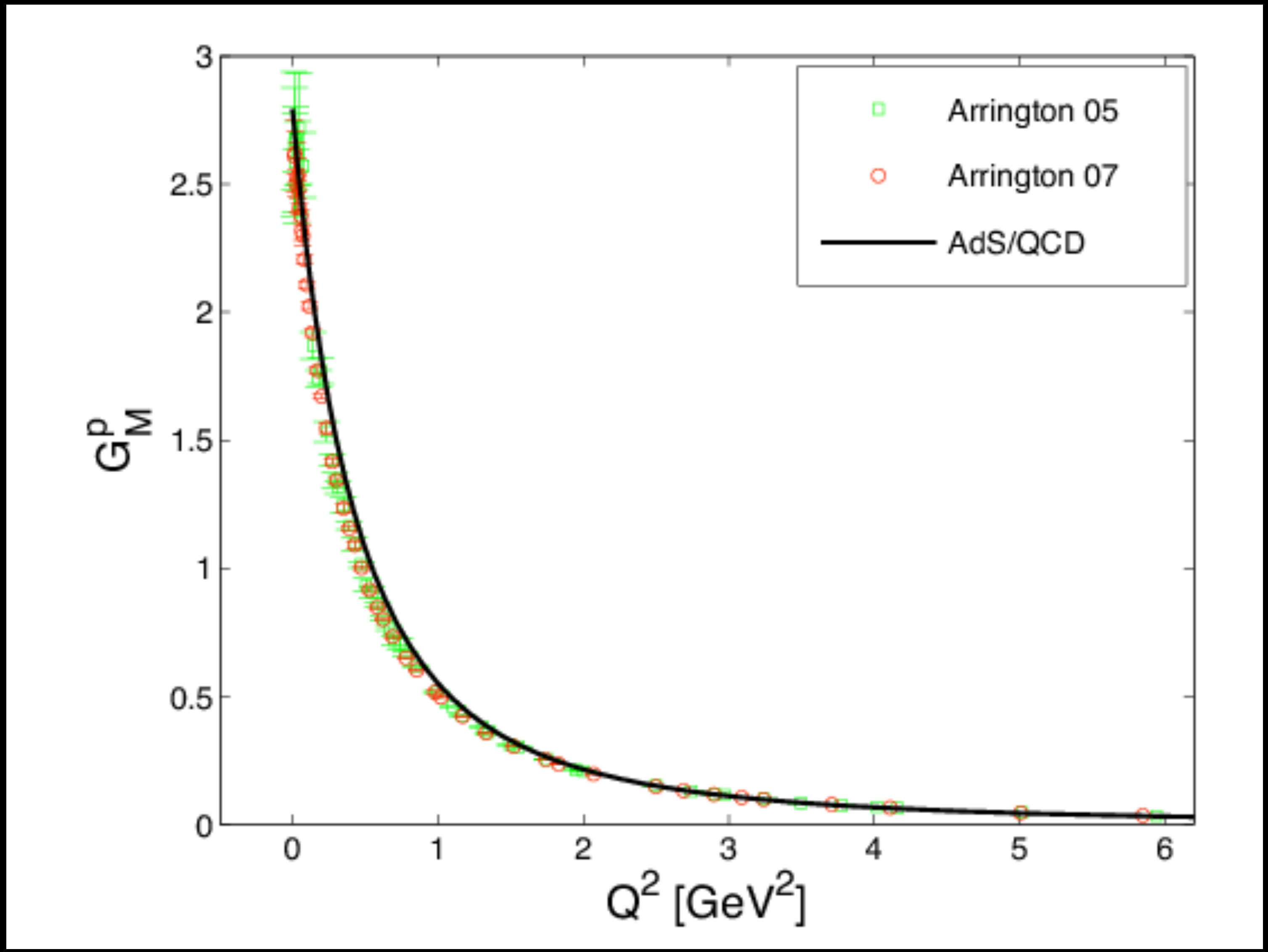


Figure 6.11: Electron–proton scattering with 188 MeV electrons. [R. W. McAllister and R. Hofstadter, *Phys. Rev.* **102**, 851 (1956).] The theoretical curves correspond to the following values of G_E and G_M: Mott (1;0), Dirac (1;1), anomalous (1;2.79).

Conclusion: Nucleons are not point like particles!

Proton form factors



$$G_E(q^2) = \left(1 - \frac{q^2}{0.71}\right)^{-2}$$

$$\langle r^2 \rangle = 6\left(\frac{dG_E(q^2)}{dq^2}\right)_{q^2=0} = (0.81 \cdot 10^{-13}\mathrm{cm})^2$$

Nobel prize in physics 1961

The Nobel Prize in Physics 1961
Robert Hofstadter, Rudolf Mössbauer

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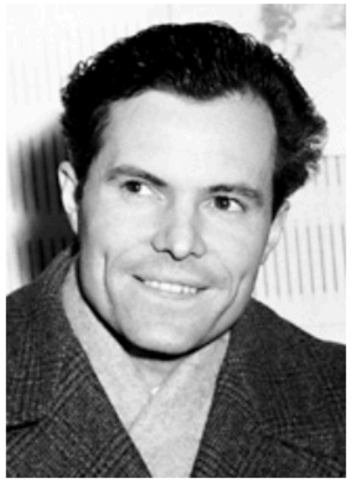
The Nobel Prize in Physics 1961

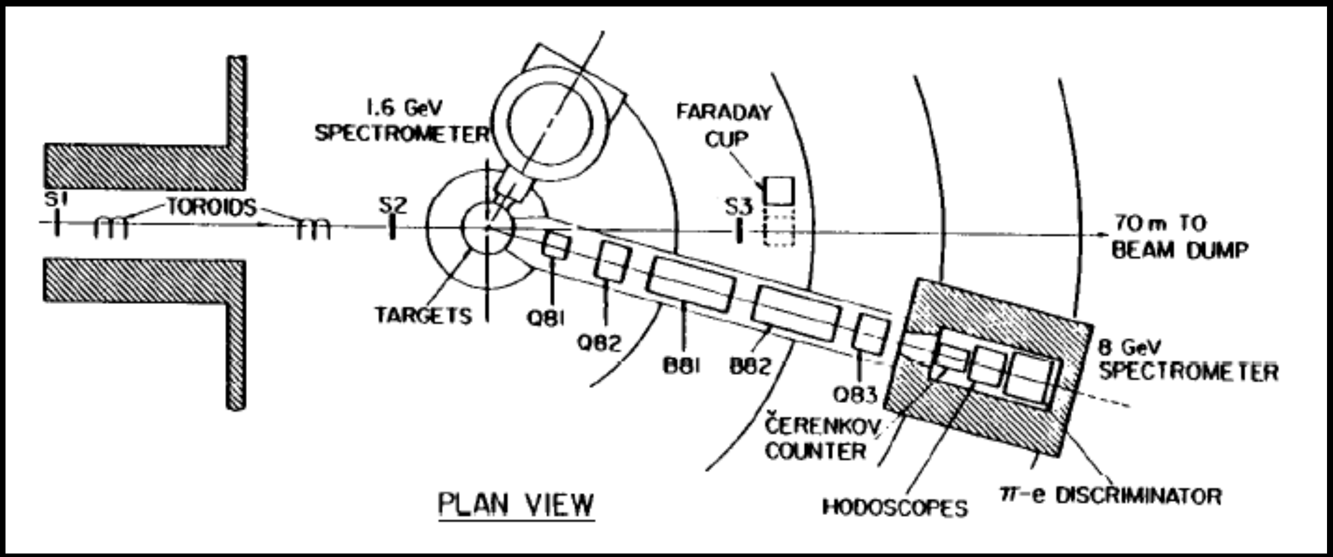
Robert Hofstadter
Prize share: 1/2



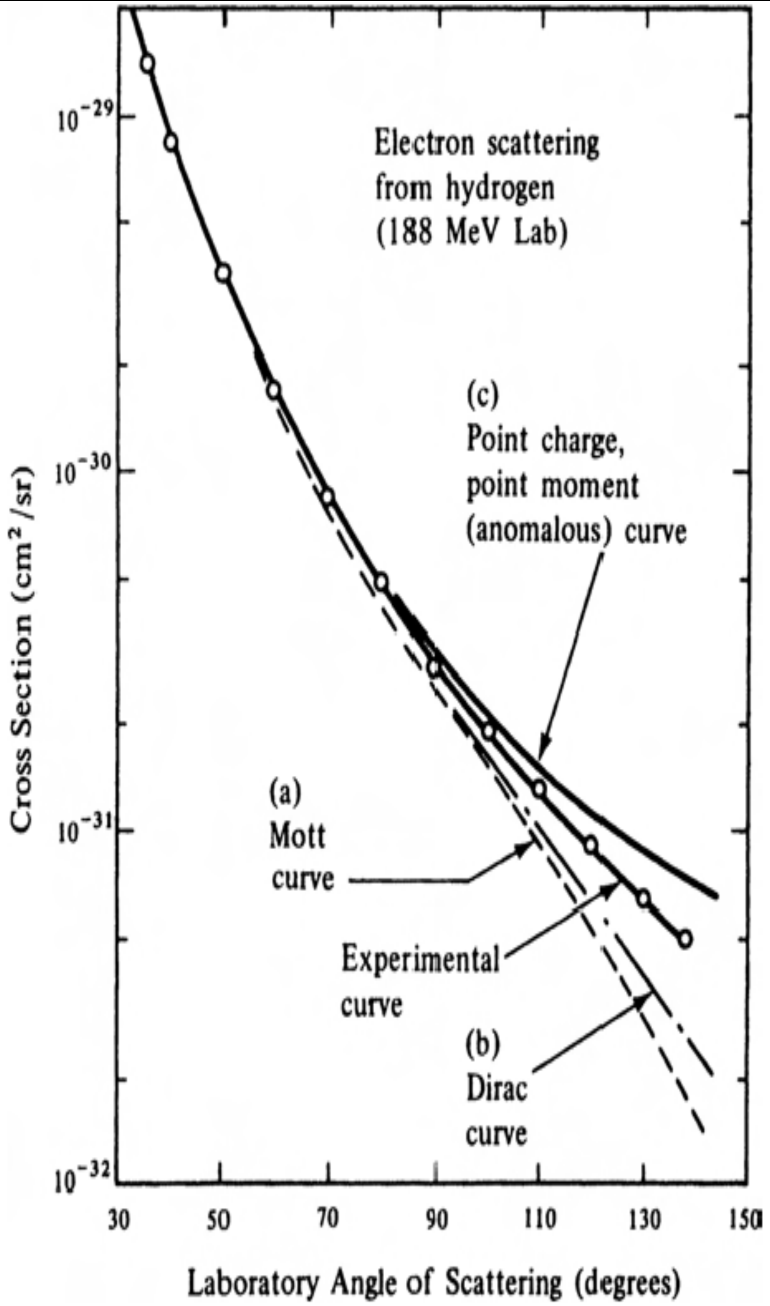
Rudolf Ludwig Mössbauer
Prize share: 1/2

The Nobel Prize in Physics 1961 was divided equally between Robert Hofstadter *"for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons"* and Rudolf Ludwig Mössbauer *"for his researches concerning the resonance absorption of gamma radiation and his discovery in this connection of the effect which bears his name"*.

Internal structure of protons



Robert Hofstadter



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Are nucleons elementary particles?

- ✓ In 1956 Stanford staff met in Prof. W. Panofsky's home to discuss Hofstadter's suggestion to build a linear accelerator that was at least 10 times as powerful as the previous one (called Mark III) to study the structure of sub-nuclear matter. This idea was called "The M(onster)-project" because the accelerator would need to be 2 miles long!
    - 1957 A detailed proposal was presented
    - 1959 Eisenhower said yes
    - 1961 Congress approved the project ($114M)
    - While excavating SLAC the workers discovered a nearly complete skeleton of a 10-foot mammal, Paleoparadoxia, which roamed earth 14 millions years ago…

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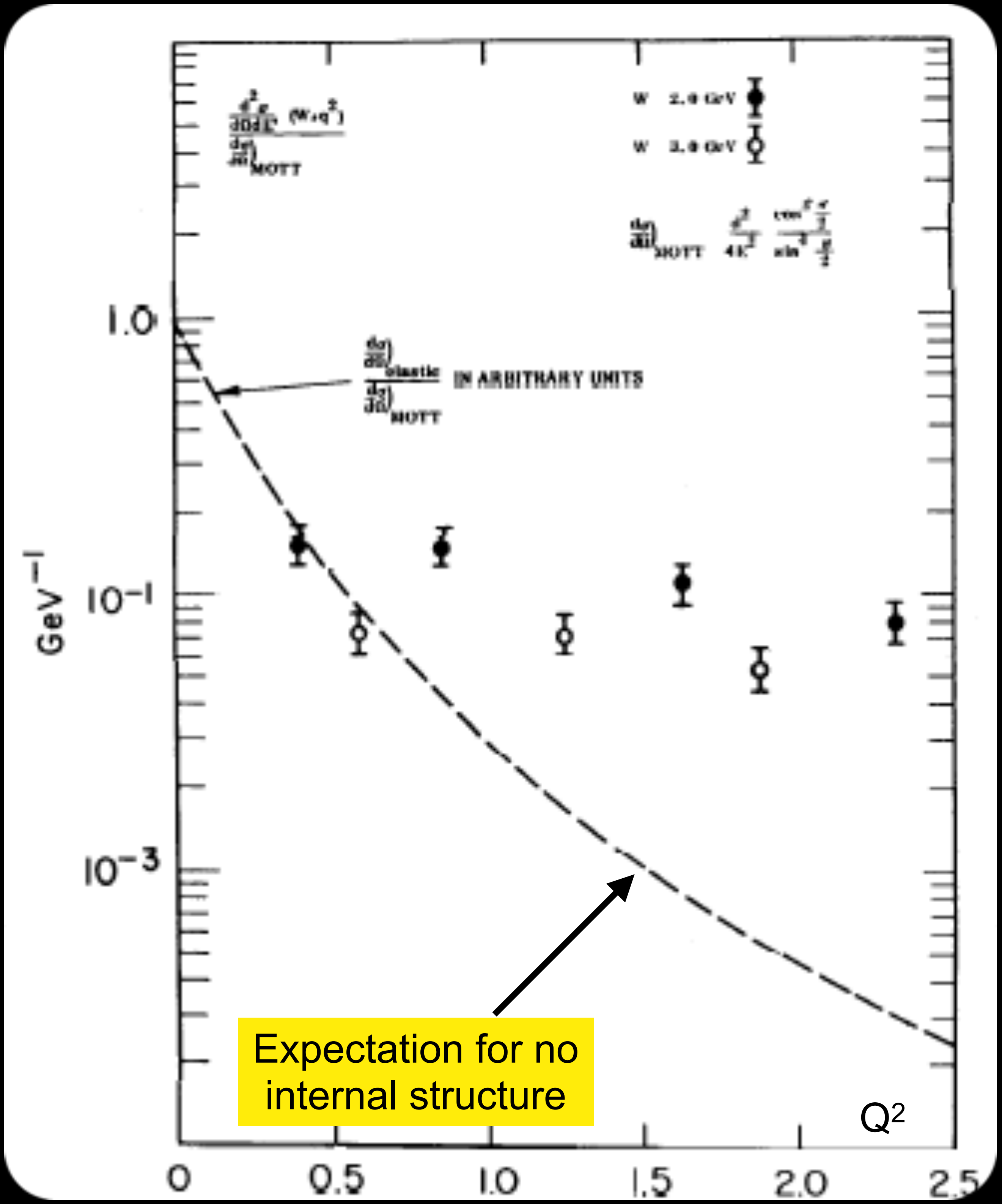
Deep inelastic scattering

- ✓ A real breakthrough in probing the internal structure of the proton came with a series of deep inelastic scattering experiments at SLAC - Stanford Linear Accelerator Centre
  - Electrons were scattered off quasi-free point-like constituents inside the protons i.e. the quarks!
  - Nobel prize in 1990 to Friedman, Kendall and Taylor
- ✓ These experiments were followed up by other experiments @ CERN and @ Fermilab using e, μ, v and anti-v beams as probes



Nobel prize in Physics 1990

The Nobel Prize in Physics 1990
Jerome I. Friedman, Henry W. Kendall, Richard E. Taylor

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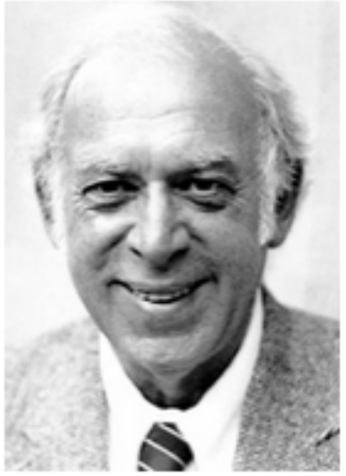
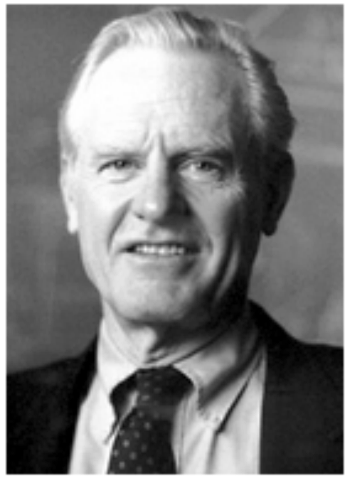
The Nobel Prize in Physics 1990



Jerome I. Friedman
Prize share: 1/3



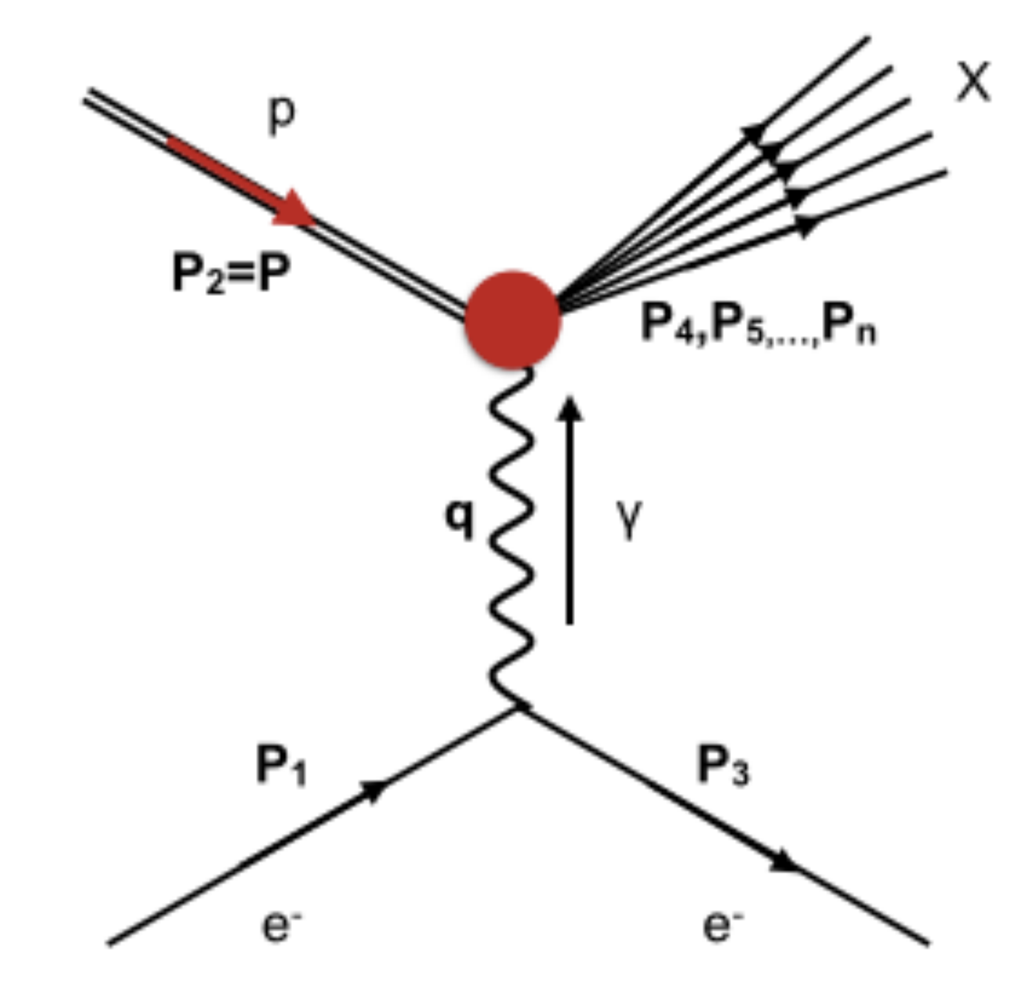
Henry W. Kendall
Prize share: 1/3

Photo: T. Nakashima
Richard E. Taylor
Prize share: 1/3

The Nobel Prize in Physics 1990 was awarded jointly to Jerome I. Friedman, Henry W. Kendall and Richard E. Taylor *"for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"*.

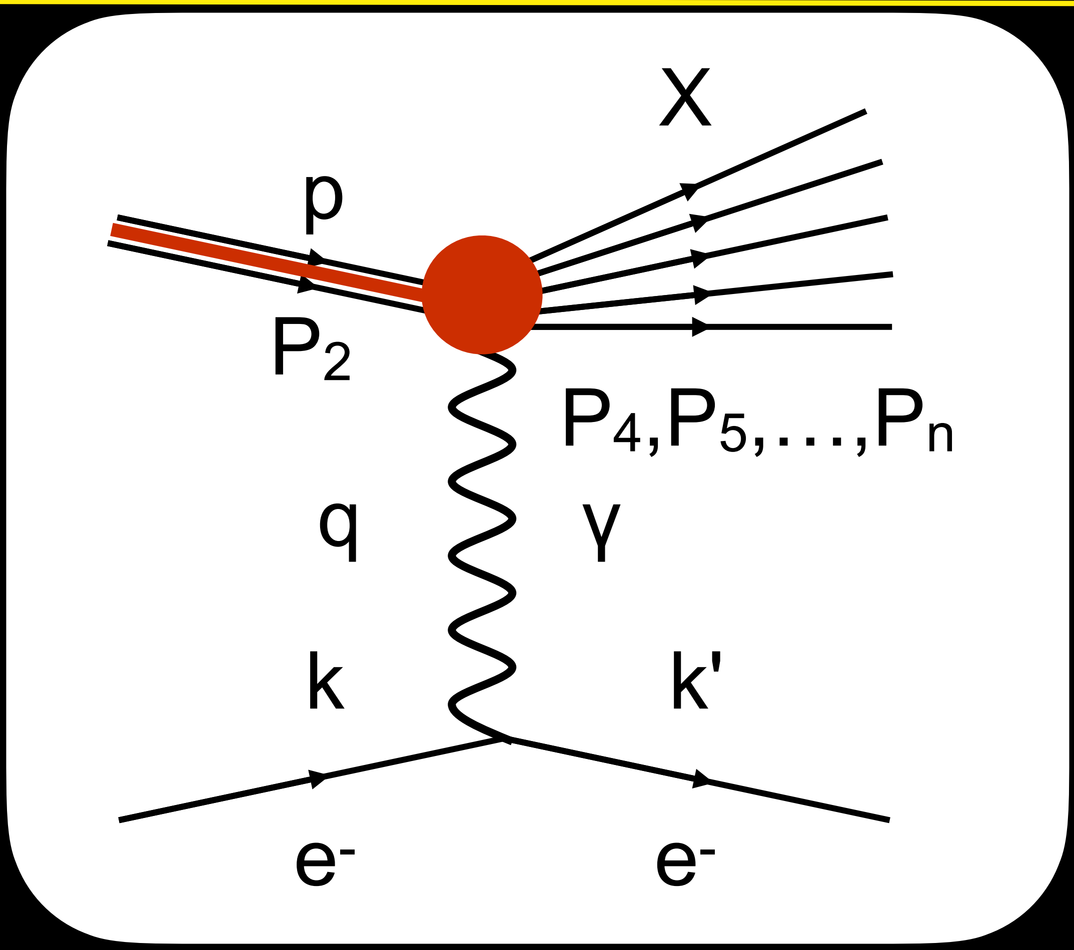
Inelastic electron-proton scattering

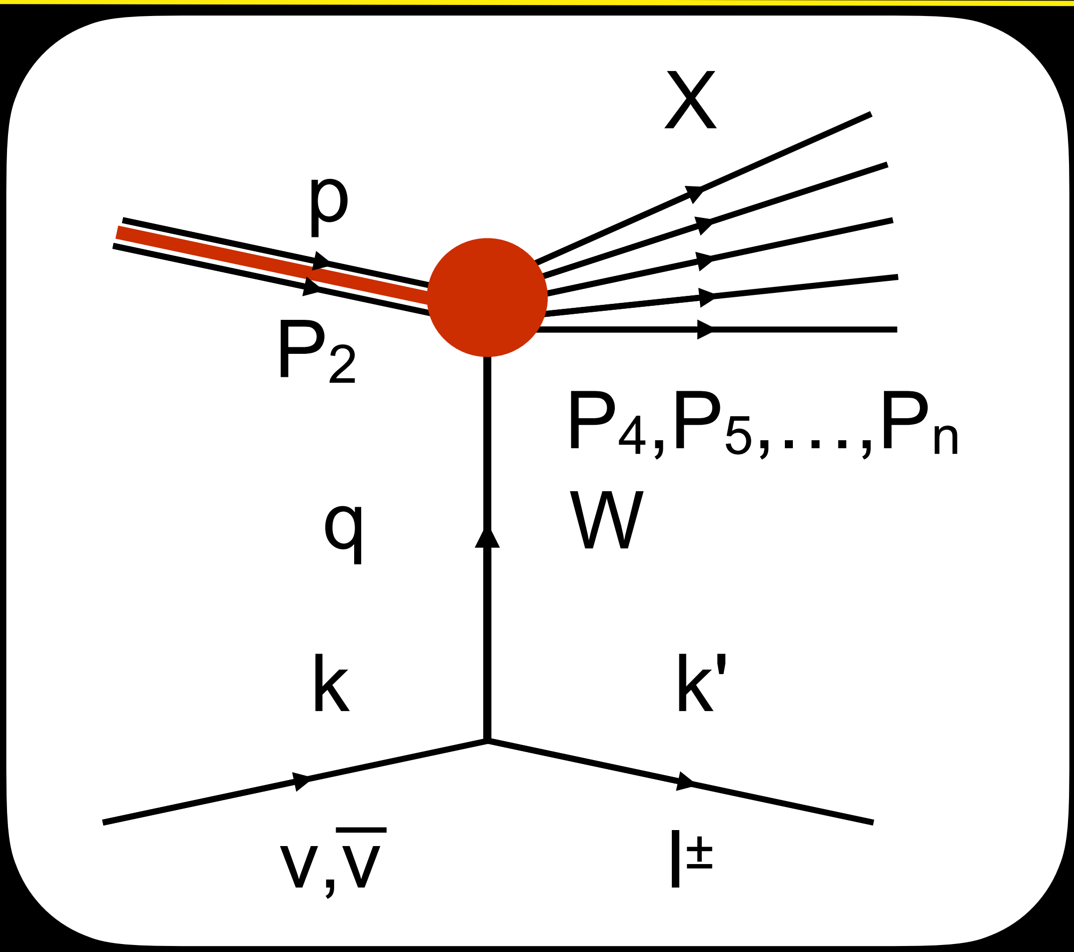
- Elastic e-p scattering dominates at lower energies
  - During this process an electron interacts with a proton which emerges intact
- Cranking up the energy of this process, the interaction becomes inelastic



- In experiments we record P_3 and what is usually measured is the inclusive cross-section in which all available final states are included

Kinematics of DIS





- ✓ Build invariant quantities to describe the interaction between the virtual photon or the W and the proton

Momentum transfer squared: $Q^2 = -q^2$

Invariant mass of X squared: $W^2 = (P+q)^2$

Bjorken-x (the fraction of the proton's momentum carried by the struck quark): $x = \frac{Q^2}{2Pq}$

Fractional energy transfer in the lab: $y = \frac{Pq}{Pk}$

Square of the proton mass: $M^2 = P_\mu P^\mu = P^2$

Energy transfer in the lab: $\nu = \frac{Pq}{M}$

Centre of mass energy squared: $s = (P+k)^2$

Bjorken scaling

- The differential cross section is now given by

$$\frac{d\sigma}{dE'\,d\Omega}=\left[\frac{\alpha}{2E\sin^2(\theta/2)}\right]^2\left[2W_1\sin^2(\theta/2)+W_2\cos^2(\theta/2)\right]$$

- The two structure functions W_1 and W_2 depend on q^2 and on qP

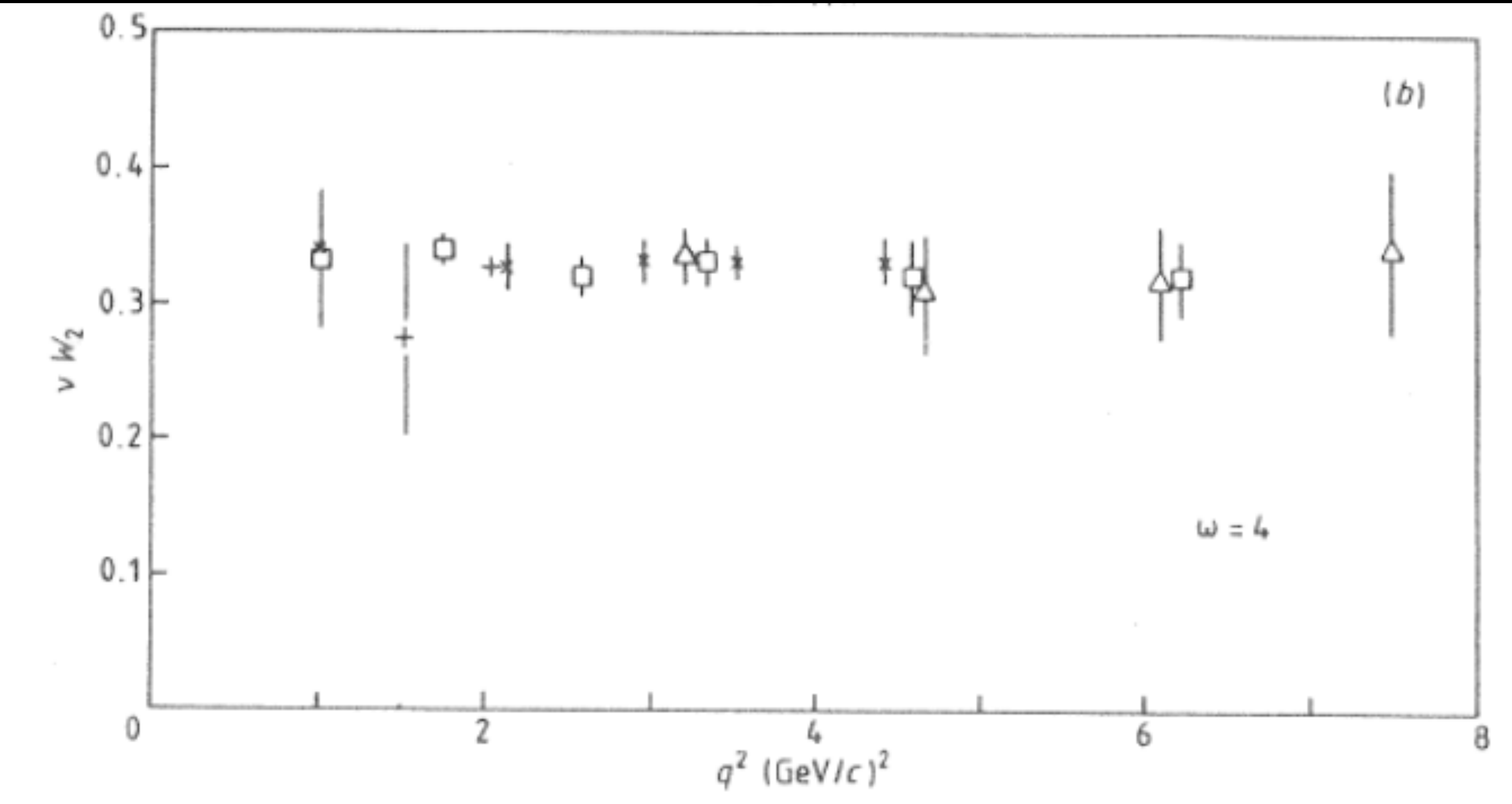
$$W_1=W_1(q^2,x) \qquad\qquad W_2=W_2(q^2,x)$$

- We can define the x-Bjorken
  - so that $x=\frac{q^2}{2qP}$
- At large values of momentum transfer Q, W_1 and W_2 do not depend on both q^2 and x but rather only on x
- This was predicted by Bjorken and can be explained as follows: $\lambda\simeq\frac{1}{Q}$
  - the wavelength of the virtual photon is inversely proportional to the momentum transfer Q and is connected to the resolution or better the resolving power of the internal structure
  - the higher the energy of the interaction between the electron and the proton, the higher the momentum transfer Q and the smaller the resolution

Bjorken scaling (cont.)

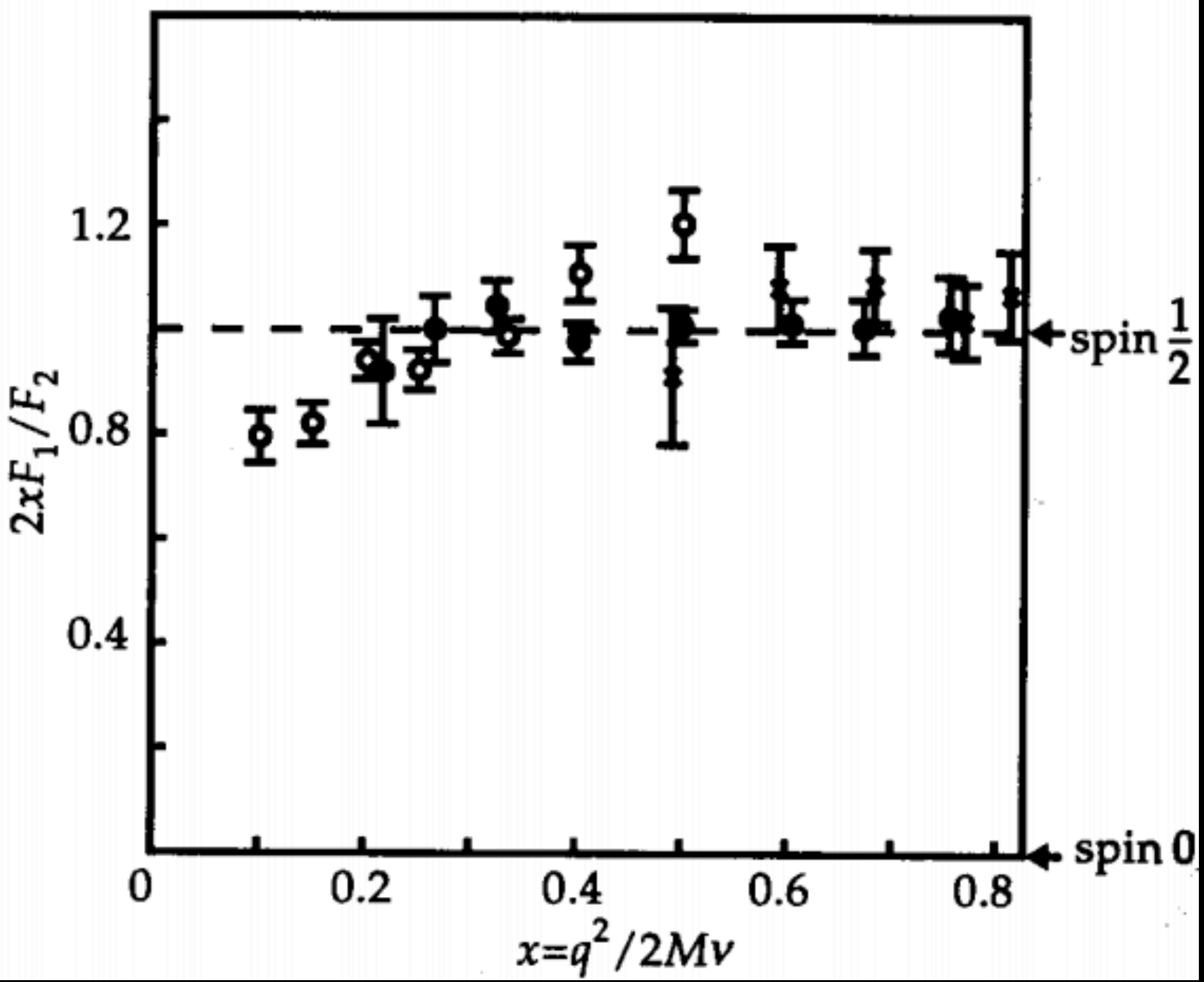
- By increasing the energy we start probing the internal structure of the proton
  - while at modest energies the structure functions have a dependence on both q^2 and x
  - at higher energies the virtual photon interacts with a point-like particle i.e. the parton (or better the quark) which has no internal structure
    - at least not visible at the current energy regime!

$$MW_1(Q^2,x) \rightarrow F_1(x) \qquad \frac{Q^2}{2Mx} W_2(Q^2,x) \rightarrow F_2(x)$$



- In addition, F_1 and F_2 are connected as suggested by Callan and Gross within the range that the scaling holds
  - Callan-Gross relation: $2xF_1(x) = F_2(x)$

Fig. 3.6. The ratio of the structure functions F_1 and F_2 within the scaling region provides a test of the so-called Callan–Gross relations (see (3.82))



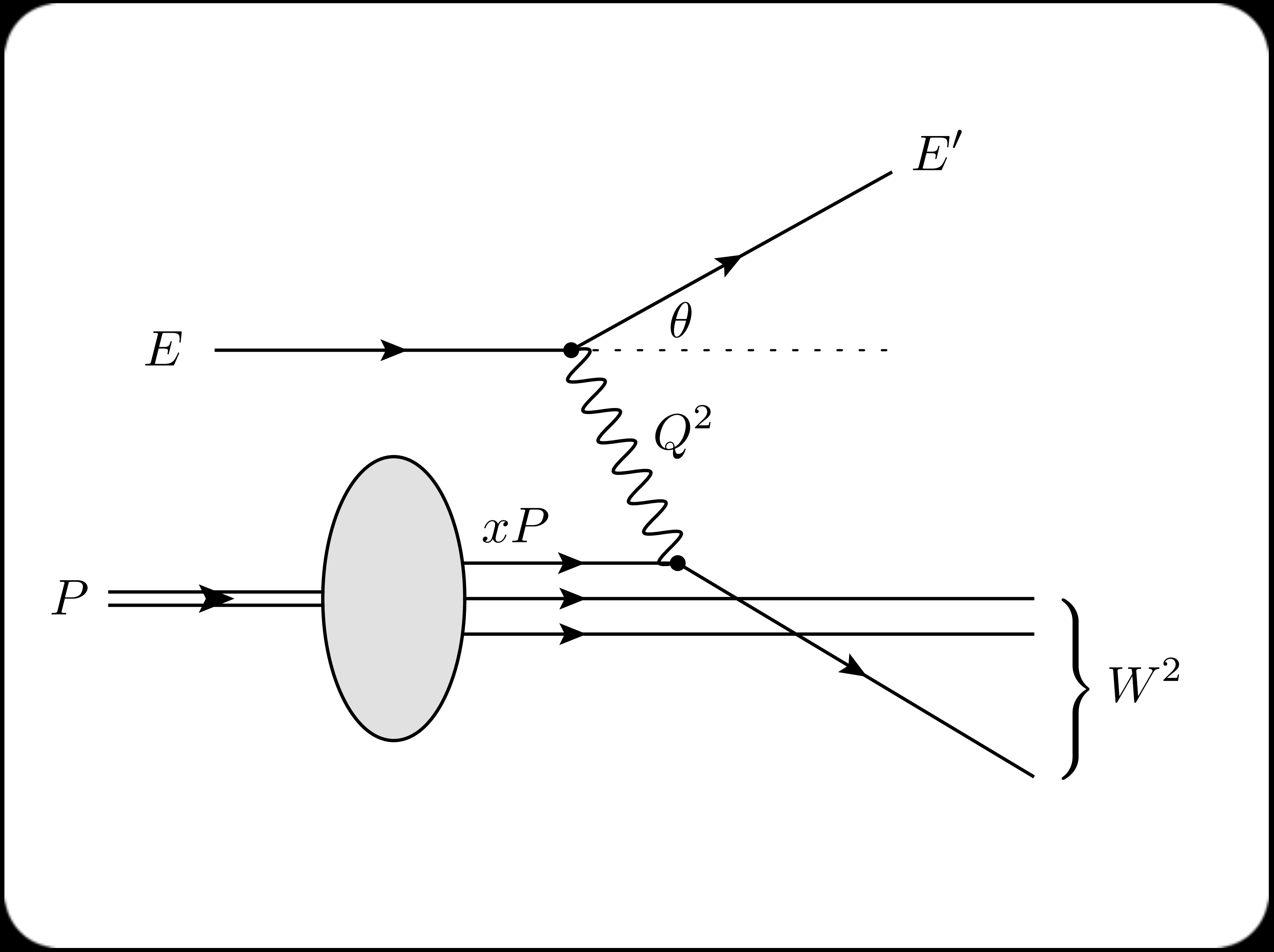
The parton model

- Bjorken's scaling hypothesis
  - if scattering is caused by point-like constituents, then the structure functions should be independent of Q^2

- Feynman's parton model
  - a proton consists of constituents
  - the term "parton" was used by Feynman at the early stages of his formulation and stands until our days
    - Physicists were reluctant to talk about quarks at that stage, let alone about gluons

The parton model (cont.)

- A virtual photon can penetrate a proton and can interact with its constituents
- We can thus probe the partons i.e. in this case the quarks
- There are more than one type of such particles in the proton and each one carries a different fraction of the proton's momentum and energy



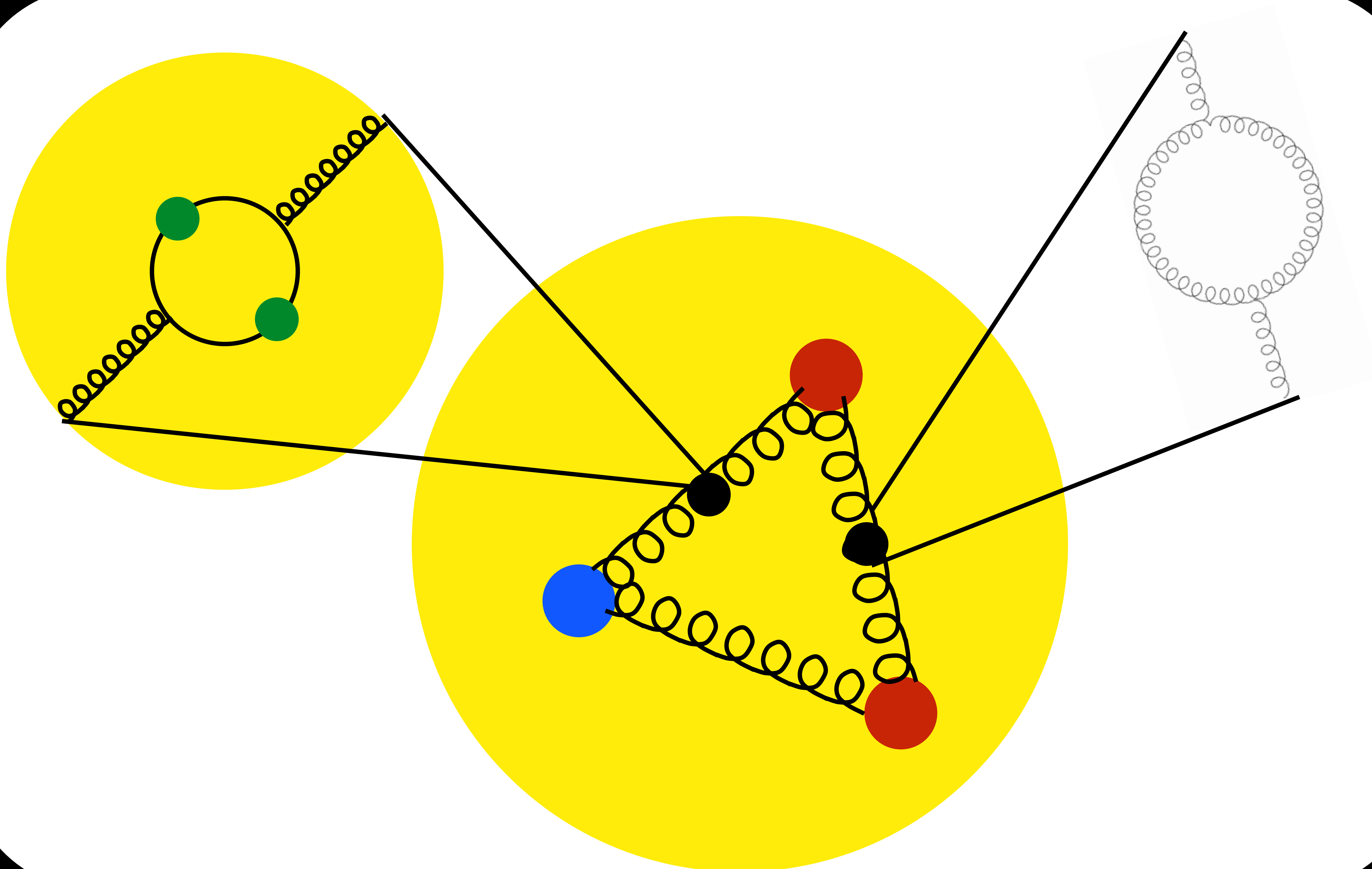
The parton model (cont.)

- We introduce the parton distribution function f(x):
  - the probability that the struck parton carries a fraction x of the proton's momentum
  - All fractions have to add up to unity such that

$$\sum_{i=1}^{N} \int x f_i(x)\, dx = 1$$

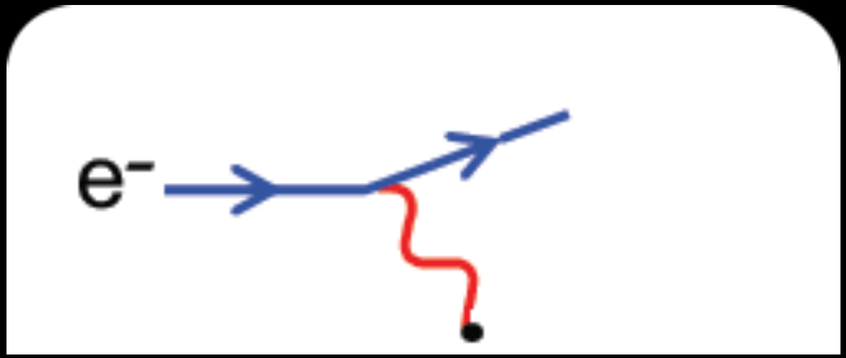
    - Where i is an index for all patrons that do not interact with the photon
- But inside the proton there are also other pardons (e.g. sea quarks and gluons)
- However there is a net excess of three quarks that carry the quantum numbers of the proton
  - these are the valence quarks
- These valence quarks are dressed with gluons and sea q-qbar pairs

The parton model (cont.)



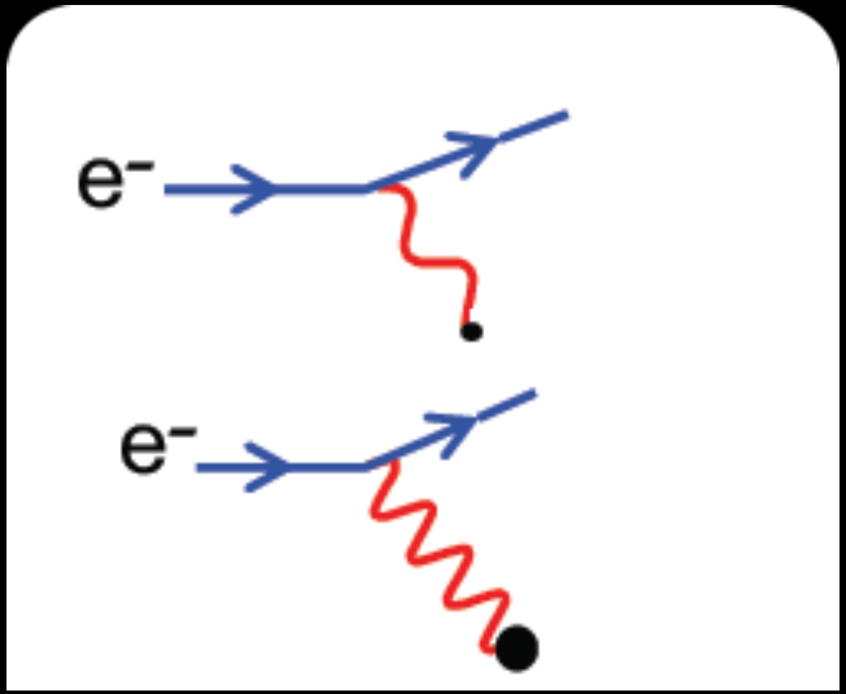
The internal structure of proton

- We started off with a "kid's microscope" with low resolution i.e. small momentum transfers $\Longrightarrow$ low energy
  - able to detect the existence of a static electric potential



The internal structure of proton

- We started off with a "kid's microscope" with low resolution i.e. small momentum transfers $\Longrightarrow$ low energy
  - able to detect the existence of a static electric potential
- Increasing the energy led to better resolution
  - our target has a sizeable charge distribution



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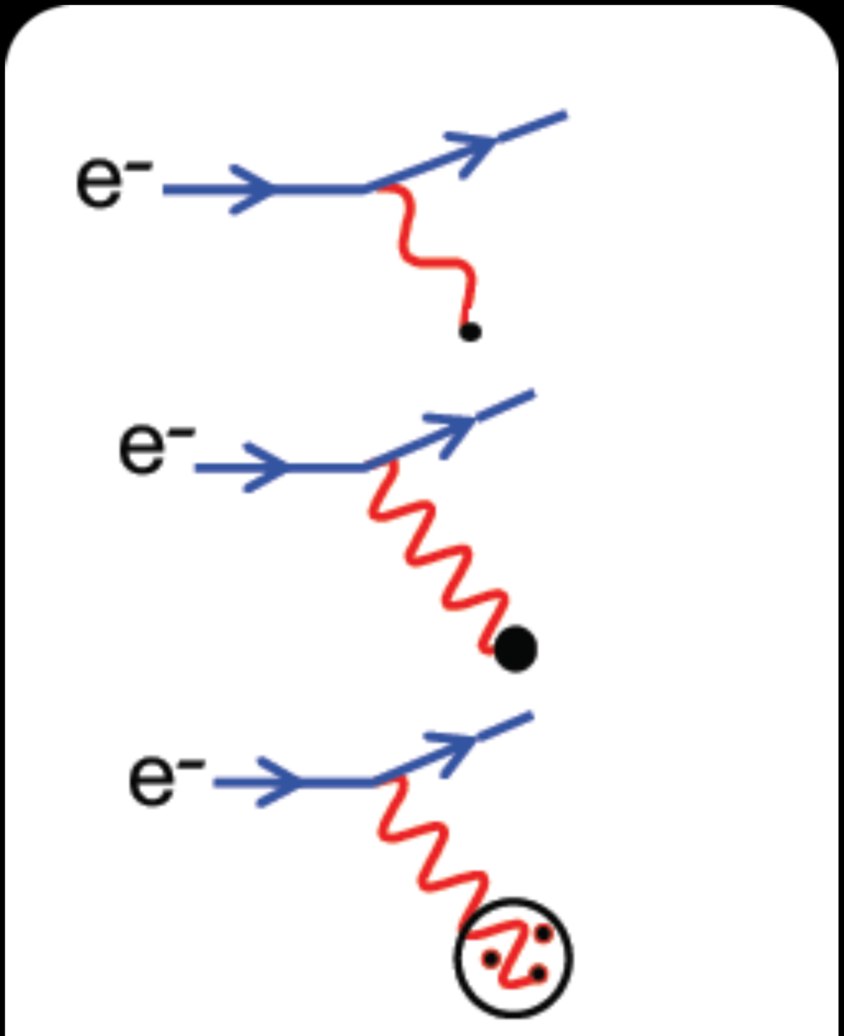
- Increasing the energy led to better resolution
  - our target has a sizeable charge distribution
- For large momentum transfers
  - our target has internal structure i.e. valence quarks
  - Electron-proton inelastic scattering is seen as an electron-quark elastic scattering process



The internal structure of proton

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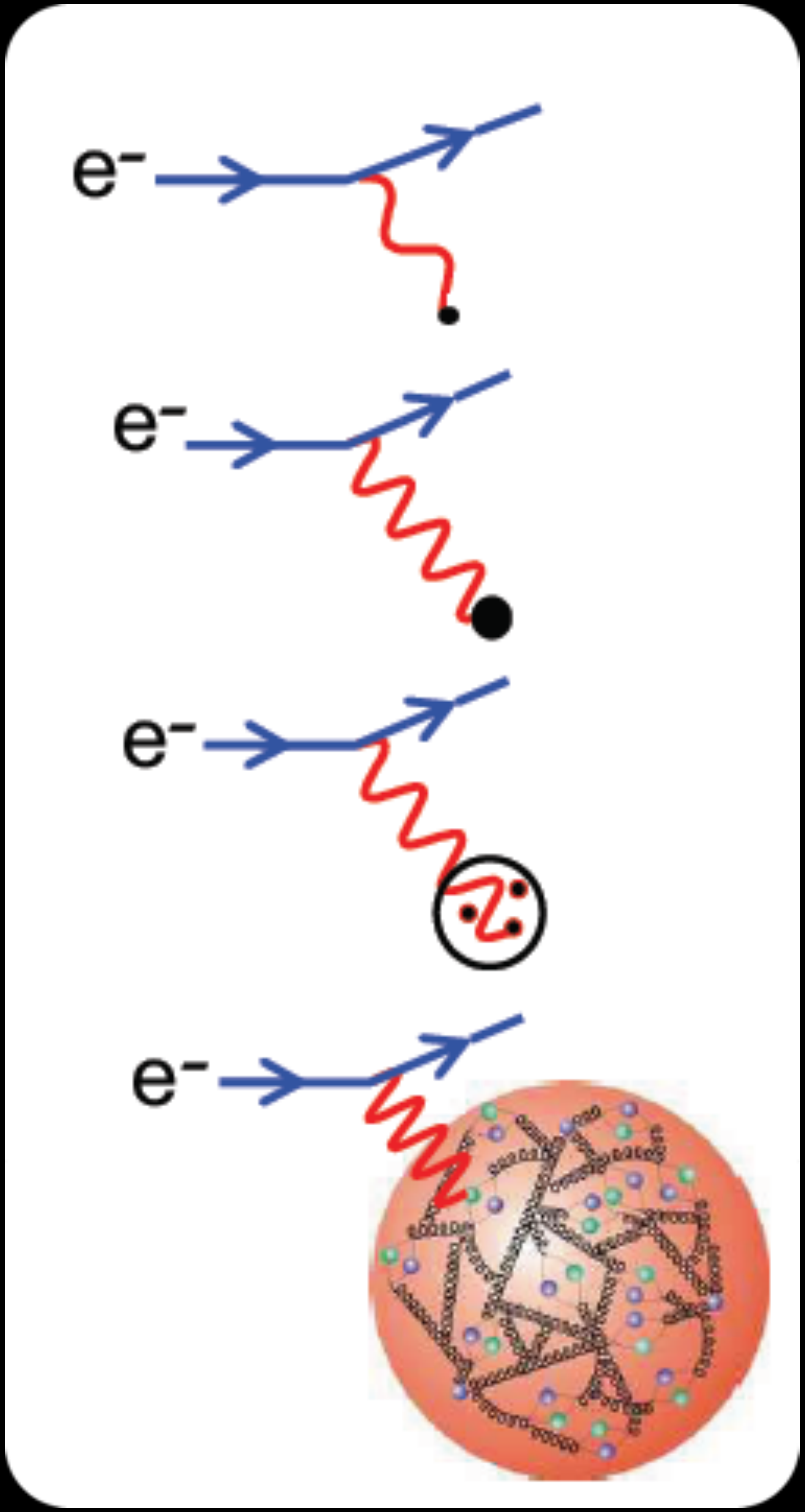
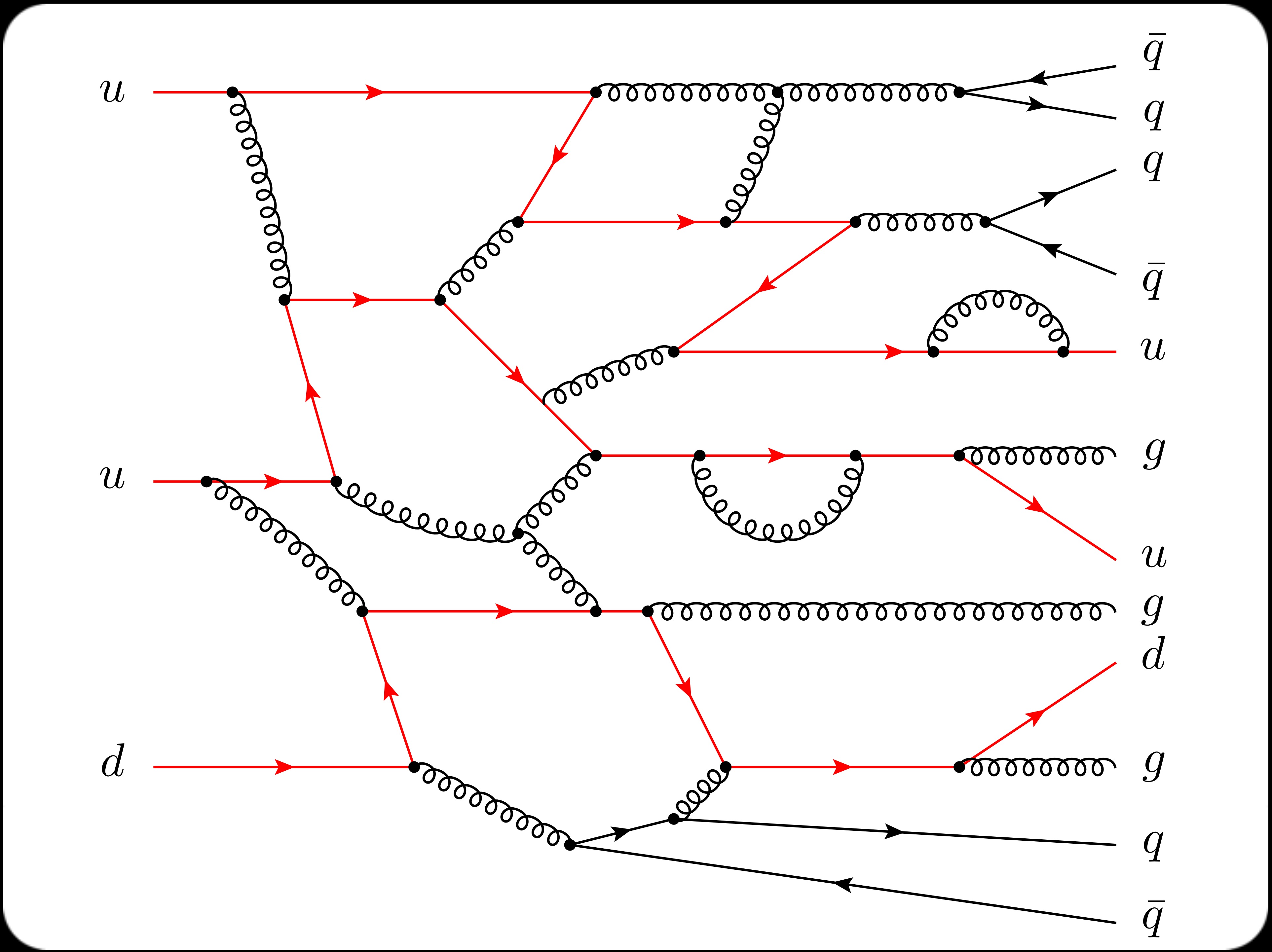
  - able to detect the existence of a static electric potential
- Increasing the energy led to better resolution
  - our target has a sizeable charge distribution
- For large momentum transfers
  - our target has internal structure i.e. valence quarks
  - Electron-proton inelastic scattering is seen as an electron-quark elastic scattering process
- For even larger momentum transfers
  - The internal structure of our target is even richer than we thought
  - not only valence but also sea quarks and gluons!
  - Introduce the parton distribution functions



A high resolution picture of the proton



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Quark and gluon distribution

- At high Q^2 we see a high resolution picture of the proton with not only the valence quarks but also the sear quarks and the gluons
- We focus on the three lightest quarks i.e. u, d, s, as the heavier ones are subject to threshold effects
- The structure function can be written as

$$\frac{1}{x}F_2^p(x)=\sum_{i=1}^{N}e_i^2 f_i^p(x)$$

$$=\left(\frac{2}{3}\right)^2\left[u^p(x)+\overline{u}^p(x)\right]+\left(\frac{1}{3}\right)^2\left[d^p(x)+\overline{d}^p(x)\right]+\left(\frac{1}{3}\right)^2\left[s^p(x)+\overline{s}^p(x)\right]$$

  - where u(x), d(x) and s(x) are the probability distributions of u, d and s quarks within the proton (similarly for antiquarks)
- $F_2(x)$ has six unknown quantities
  - To overcome this one first relies on the fact that protons and neutrons are members of the isospin doublet and thus their quark content is related

$$\frac{1}{x}F_2^n(x)=\left(\frac{2}{3}\right)^2\left[u^n(x)+\overline{u}^n(x)\right]+\left(\frac{1}{3}\right)^2\left[d^n(x)+\overline{d}^n(x)\right]+\left(\frac{1}{3}\right)^2\left[s^n(x)+\overline{s}^n(x)\right]$$

Quark and gluon distribution (cont.)

- The quark content of the two nucleons are connected if one exchanges u with d and vice-versa

$$\left\{\begin{matrix} u^p(x)=d^n(x)=u(x) \\ d^p(x)=u^n(x)=d(x) \\ s^p(x)=s^n(x)=s(x) \end{matrix}\right\}$$

- Another constrain comes from the fact that the quantum numbers are carried by the valence quarks
- One can also consider that the sea quarks occur to first order at the same rate and have similar momentum distributions
- We define the valence and the sea quarks as

$$\left\{\begin{matrix} u_v(x)=u(x)-\overline{u}(x) & d_v(x)=d(x)-\overline{d}(x) & s_v(x)=s(x)-\overline{s}(x)=0 \\ u_s(x)=2\overline{u}(x) & d_s(x)=2\overline{d}(x) & s_s(x)=2\overline{s}(x) \end{matrix}\right\}$$

  - So that $u(x)+\overline{u}(x)=u_v(x)+u_s(x)$

Quark and gluon distribution (cont.)

- Summing over all the pardons we should recover the quantum numbers of the proton

$$\int_0^1 u_v(x)\,dx = 2 \qquad \int_0^1 d_v(x)\,dx = 1$$

- As a result the proton and neutron structure functions are written as

$$\frac{1}{x}F_2^p(x) = \frac{1}{9}\left[4u_v(x) + d_v(x)\right] + \frac{4}{3}S(x) \qquad \frac{1}{x}F_2^n(x) = \frac{1}{9}\left[u_v(x) + 4d_v(x)\right] + \frac{4}{3}S(x)$$

  - where S(x) is the sea quark distribution

$$S(x) \equiv u_s(x) = \overline{u}_s(x) = d_s(x) = \overline{d}_s(x) = s_s(x) = \overline{s}_s(x)$$

- When studying the small momentum part of the proton i.e. x→0, one probes the low momentum sea quarks
- At high momenta i.e. x→1, the high momentum valence quarks leave little unoccupied room for the sea quarks and one probes mainly the valence quarks

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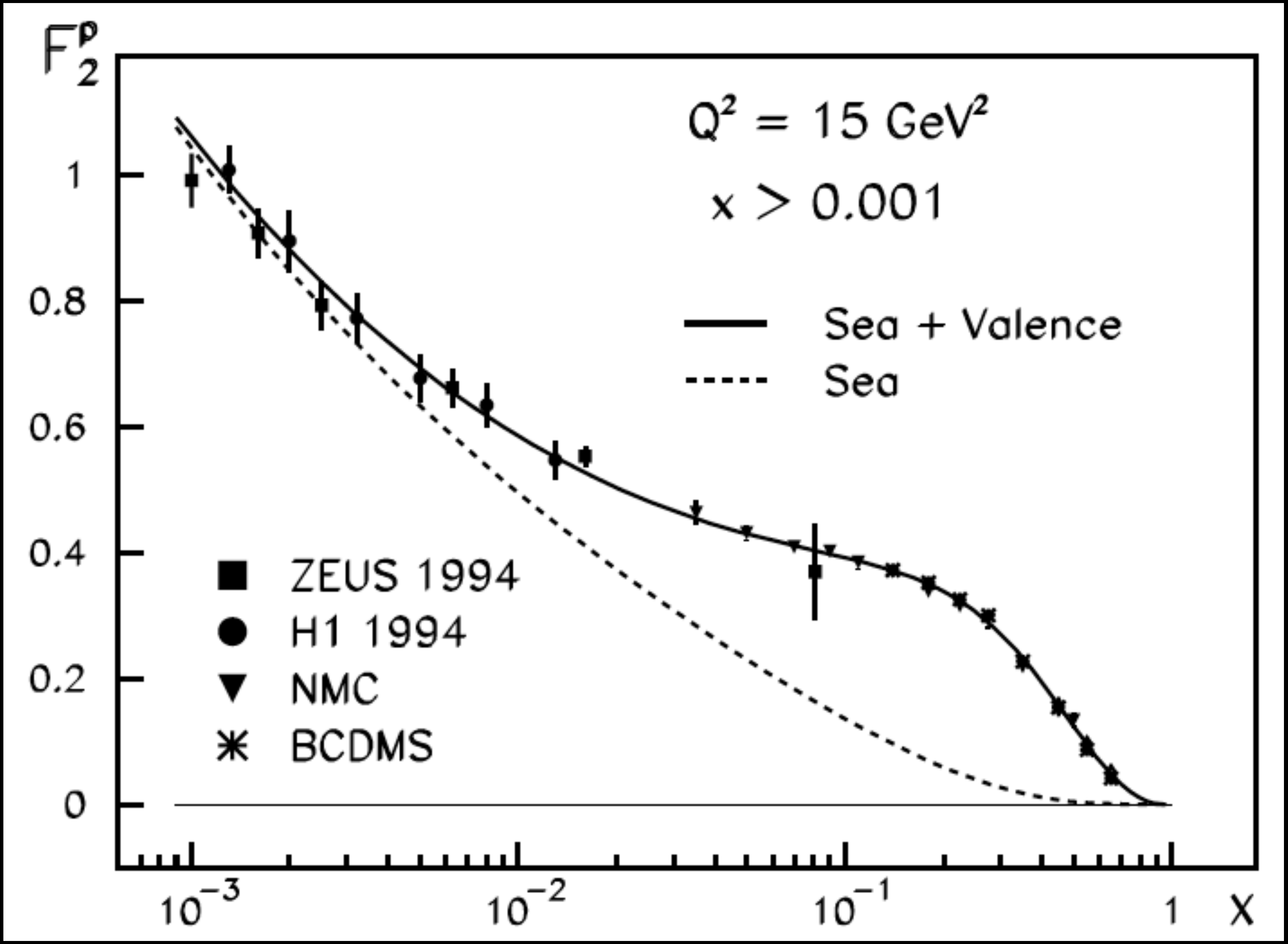
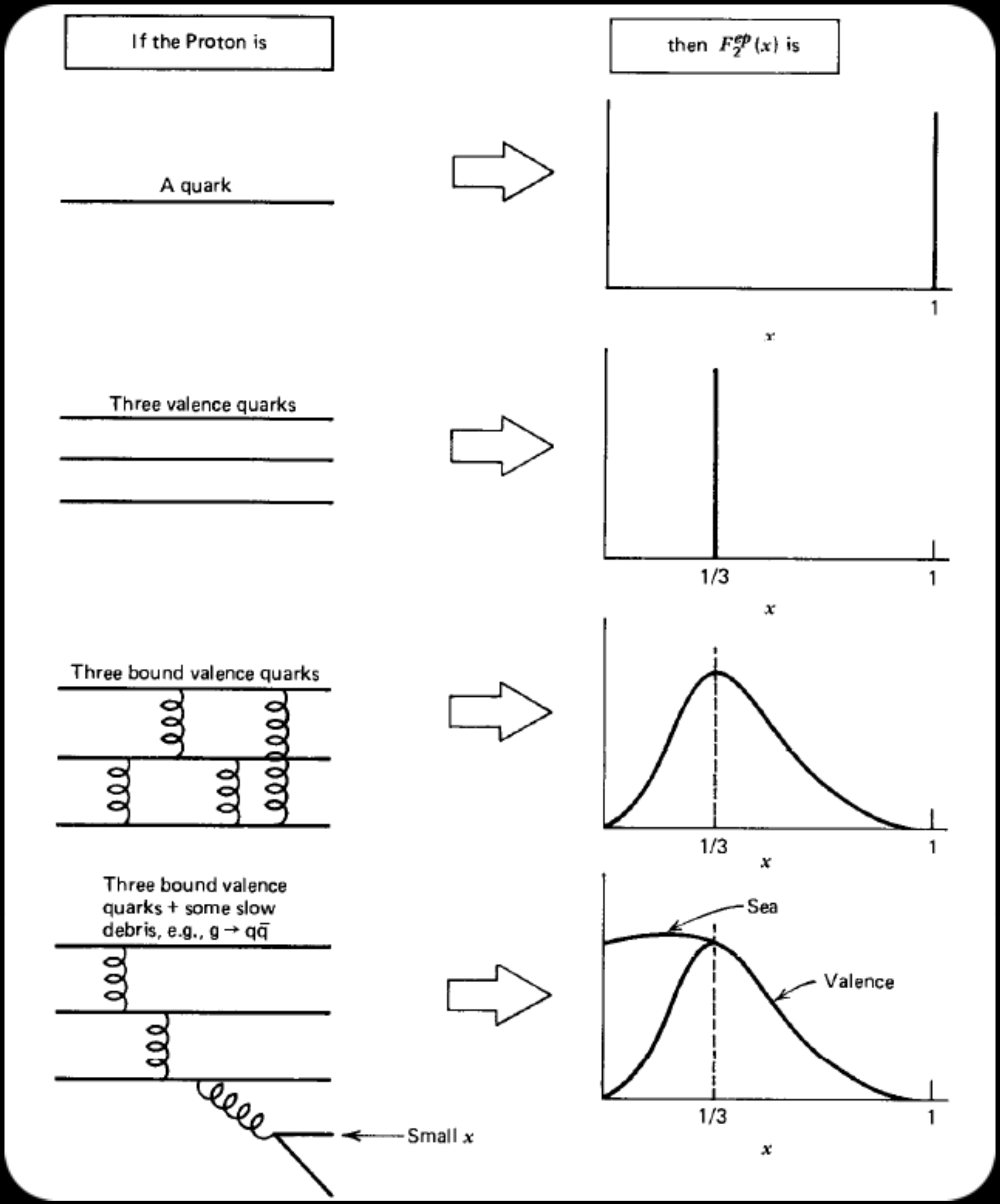
Sea and valence quark distributions



Probing the internal structure of protons



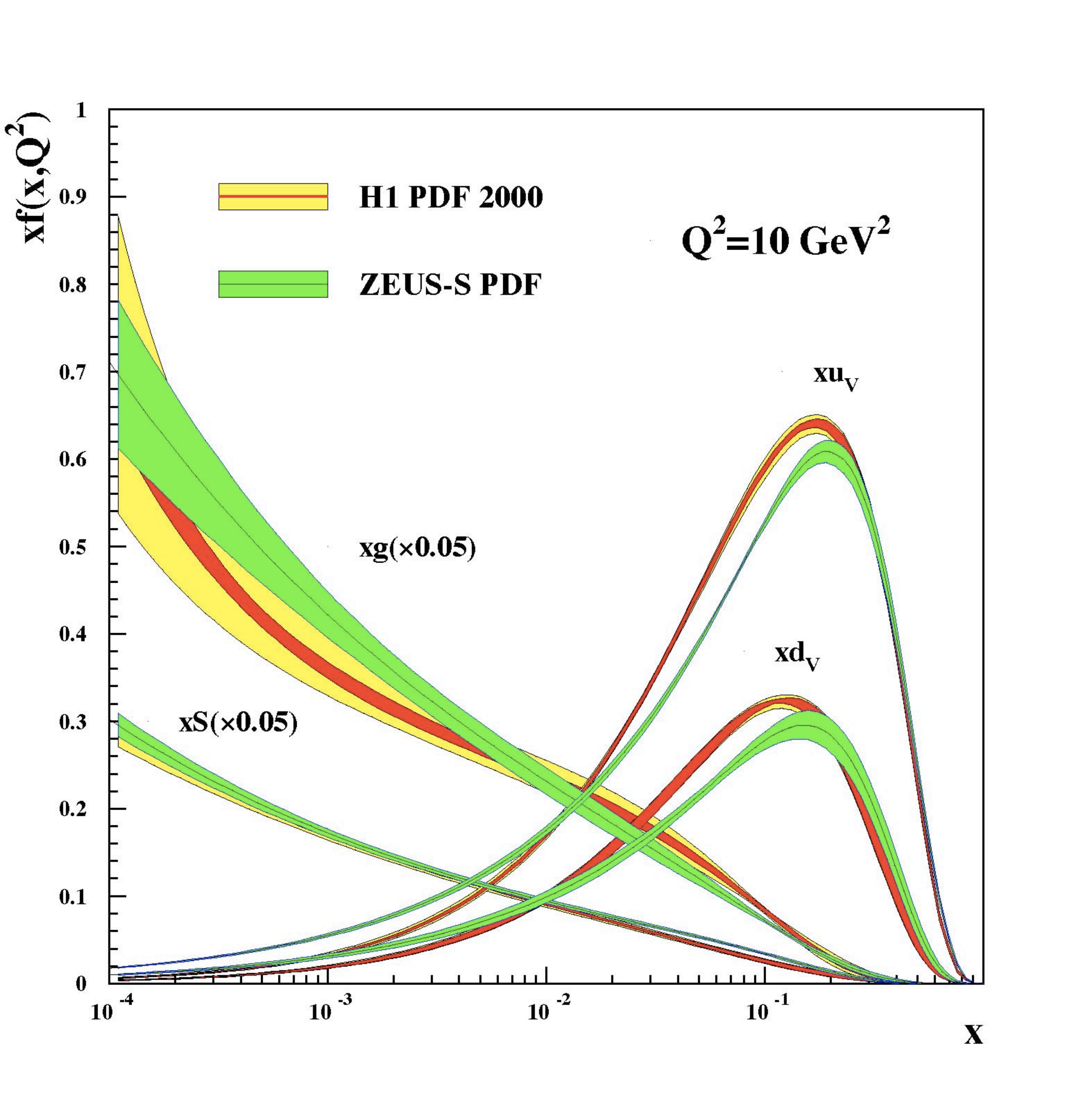
Gluon distribution

- Integrating the quark distributions obtained from DIS and neutrino scattering experiments gives

$$\sum_{i=1}^{N}\int_0^1 xf_i(x)dx \approx 0.5$$

- The missing momentum is carried by gluons!
- Introducing the gluon distribution function g(x), the correct sum rule reads

$$\sum_{i=1}^{N}\int_0^1 xf_i(x)dx + \int_0^1 xg(x)dx = 1$$



Nobel prize in physics 1990

The Nobel Prize in Physics 1990
Jerome I. Friedman, Henry W. Kendall, Richard E. Taylor

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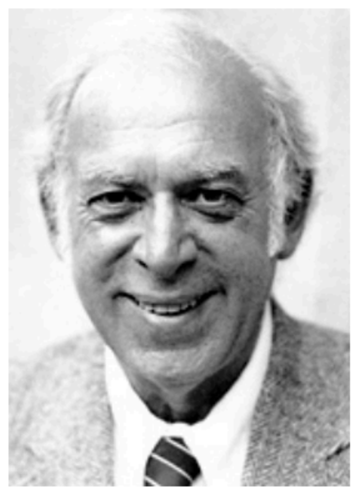
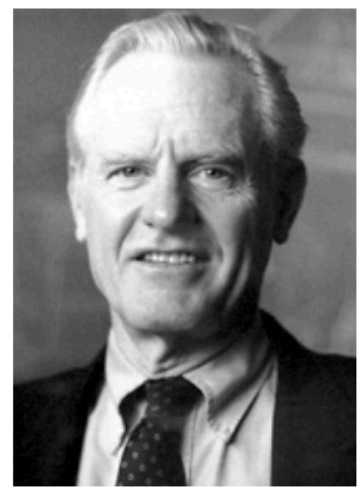
The Nobel Prize in Physics 1990



Jerome I. Friedman
Prize share: 1/3



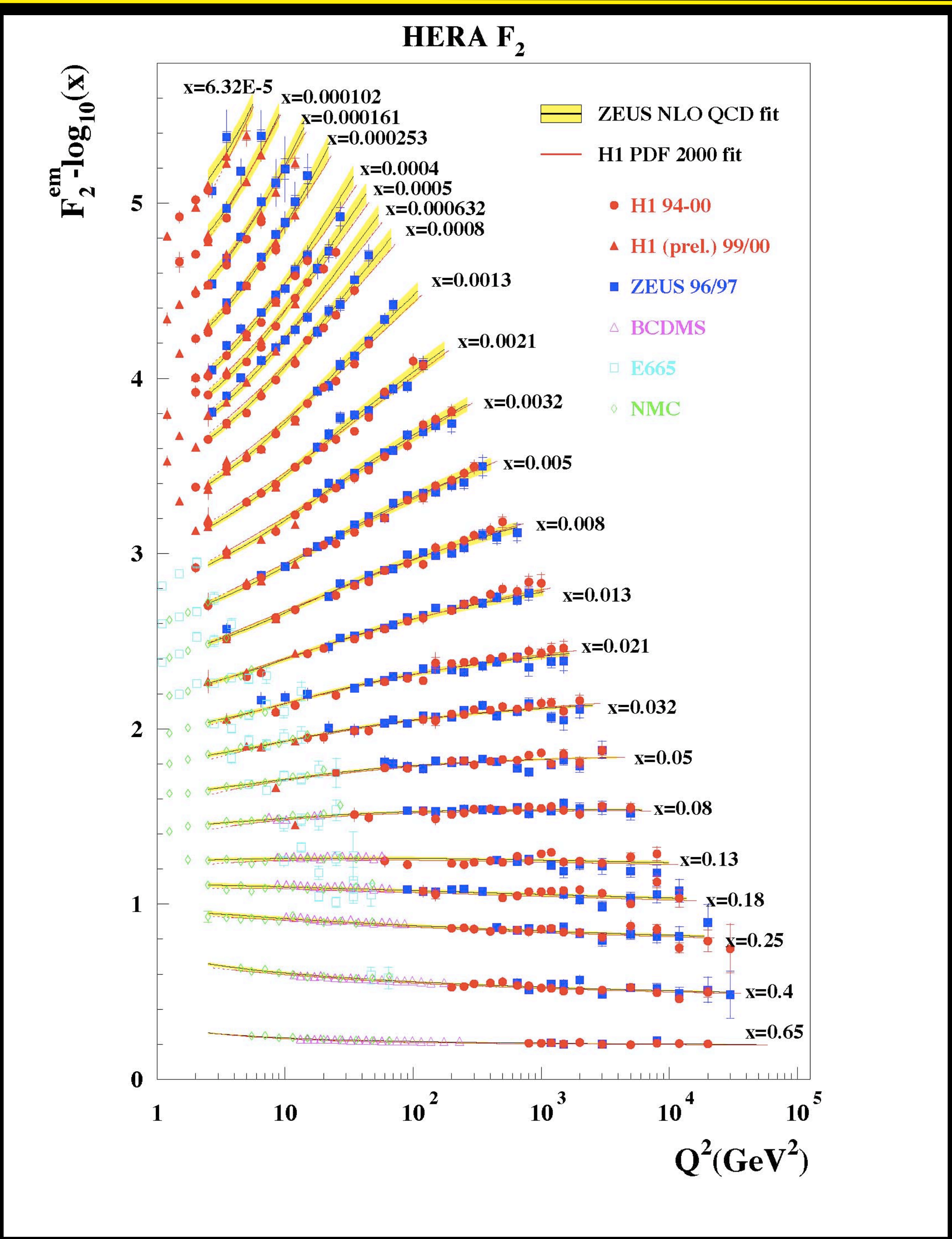
Henry W. Kendall
Prize share: 1/3

Photo: T. Nakashima
Richard E. Taylor
Prize share: 1/3

The Nobel Prize in Physics 1990 was awarded jointly to Jerome I. Friedman, Henry W. Kendall and Richard E. Taylor *"for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"*.

Violation of Bjorken scaling

- The plot shows the F_2 structure function of the proton as a function of Q^2 for different values of x, the Bjorken scaling variable.
- The data includes measurements from fixed target experiments as well as the HERA results.
- The measurements are impressive as they span four orders of magnitude in both x and Q^2.
- At high x values the structure function does not vary with Q^2
  - Bjorken scaling



Violation of Bjorken scaling

- As x decreases below ~0.1 this scaling fails, or is violated, and the structure function rises with Q^2.
- The great success of QCD is that this behavior is expected and can be calculated (using DGLAP evolution) given the structure function at some low Q^2 value usually around 4 GeV^2.
- At high x (x>0.1) the scattering is from a valence quark and is independent of momentum transfer
- As smaller x regions are studied the contribution from the gluons and sea quarks increase and these contributions are not constant but increase as you resolve smaller and smaller scales with increasing momentum transfer.