CIEM1110-1: FEM, workshop 2.1

pyJive: Constrainer, ElasticModel

Frans van der Meer, Iuri Rocha

TU Delft

Dirichlet BCs

Up until now we have mostly dealt with zero prescribed displacements

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \Rightarrow \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \Rightarrow \begin{bmatrix} K_{11} & K_{13} \\ K_{31} & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_3 \end{bmatrix}$$

TU Delft

Dirichlet BCs

Up until now we have mostly dealt with zero prescribed displacements

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \Rightarrow \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \Rightarrow \begin{bmatrix} K_{11} & K_{13} \\ K_{31} & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_3 \end{bmatrix}$$

But what about non-zero Dirichlet boundary conditions? Two ways:

- Partitioning approach:

$$\begin{bmatrix} \mathbf{K}_{ff} & \mathbf{K}_{fc} \\ \mathbf{K}_{cf} & \mathbf{K}_{cc} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{a}_f \\ \mathbf{a}_c \end{bmatrix} = \begin{bmatrix} \mathbf{f}_f \\ \mathbf{f}_c \end{bmatrix} \Rightarrow \begin{cases} \mathbf{a}_f = \mathbf{K}_{ff}^{-1} \left(\mathbf{f}_f - \mathbf{K}_{fc} \mathbf{a}_c \right) \\ \mathbf{f}_c = \mathbf{K}_{cf} \mathbf{a}_f + \mathbf{K}_{cc} \mathbf{a}_c \end{cases}$$

Dirichlet BCs

Up until now we have mostly dealt with zero prescribed displacements

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \Rightarrow \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \Rightarrow \begin{bmatrix} K_{11} & K_{13} \\ K_{31} & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_3 \end{bmatrix}$$

But what about non-zero Dirichlet boundary conditions? Two ways:

- Partitioning approach:

$$\begin{bmatrix} \mathbf{K}_{ff} & \mathbf{K}_{fc} \\ \mathbf{K}_{cf} & \mathbf{K}_{cc} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{a}_f \\ \mathbf{a}_c \end{bmatrix} = \begin{bmatrix} \mathbf{f}_f \\ \mathbf{f}_c \end{bmatrix} \Rightarrow \begin{cases} \mathbf{a}_f = \mathbf{K}_{ff}^{-1} \left(\mathbf{f}_f - \mathbf{K}_{fc} \mathbf{a}_c \right) \\ \mathbf{f}_c = \mathbf{K}_{cf} \mathbf{a}_f + \mathbf{K}_{cc} \mathbf{a}_c \end{cases}$$

- Size-conserving approach:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \Rightarrow \begin{bmatrix} K_{11} & 0 & K_{13} \\ 0 & 1 & 0 \\ K_{31} & 0 & K_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 - K_{12}\overline{u} \\ \overline{u} \\ f_3 - K_{32}\overline{u} \end{bmatrix}$$