Introduction to elementary particle physics (TN2811)

TU Delft: Bachelor program

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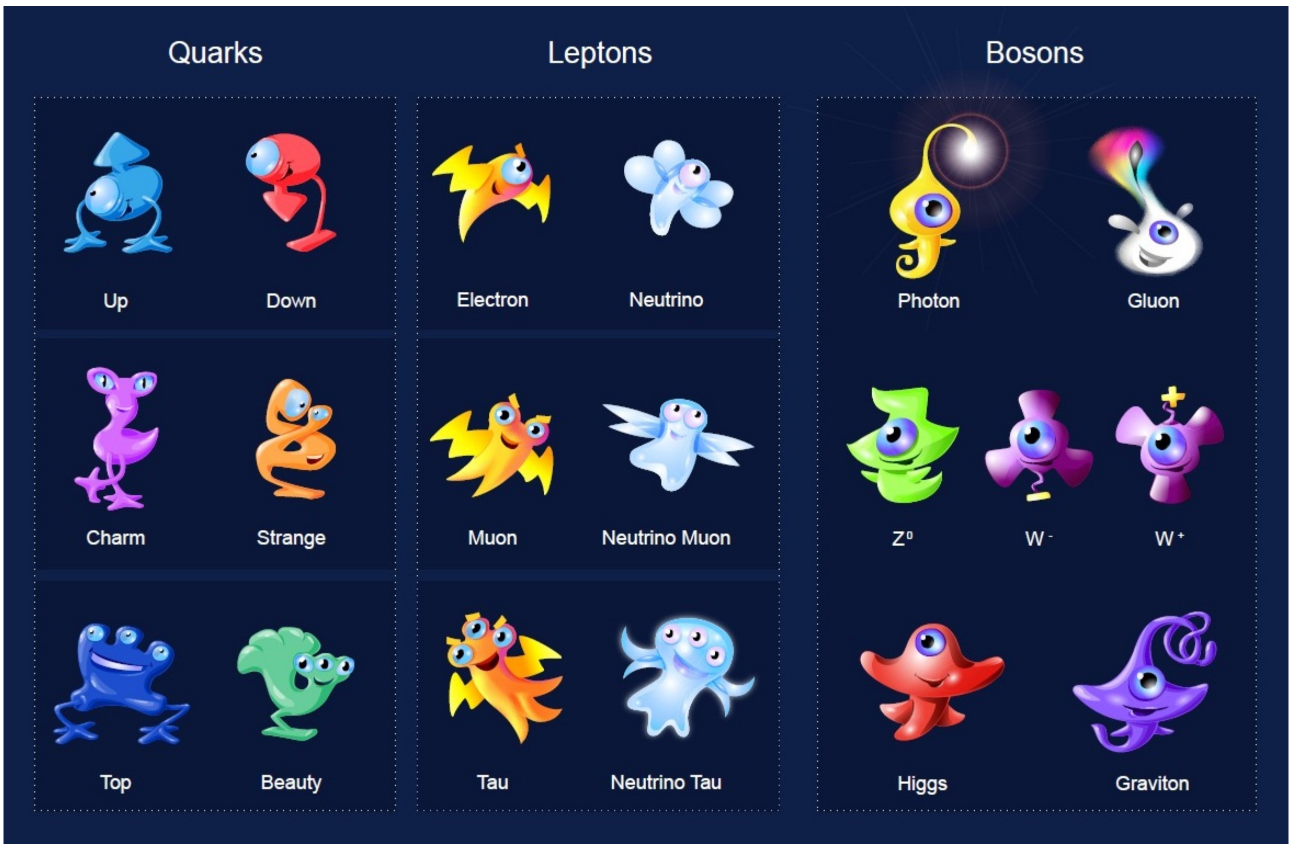

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Background material

It is important to note that the lecture notes and the slides are enough to complete the exercises and pass the course. The lectures are based on the following books:

- D. Griffiths, *Introduction to Elementary Particles*, Second Revised Edition, WILEY-VCH, (2008);
- F. Halzen and A.D. Martin, *Quarks and Leptons*, John Wiley & Sons, (1984).

The books above can be useful in case you want to read more on the topics.

How is the final grade computed

The grade of this course is made of two contributions:

- the final exam (or the retake),
- the weekly exercises.

The final grade of each student is a combination of the grade from the exam and the hand-in homework exercises. The relevant weight is 80 : 20 i.e. 80% of the grade comes from the exam and 20% from the hand-in exercises. However, it is important to note that homework can only work as a bonus for the student: if the grade a student gets after considering the homework and the exam is less than the relevant grade calculated just with the exam, then the latter is considered as the final grade. This might sound a bit complicated but I try to explain it below with some formulas and examples.

The formula based on which the grade is calculated is:

$$\text{grade} = MAX\Big(\big[0.8 \times (\text{final exam}) + 0.2 \times \frac{1}{N}\sum_{i=1}^{N}(\text{homework})_i\big], \big[(\text{final exam})\big]\Big)$$

Please note that no matter if you handed in exercises one time or as many times as the number of lectures, the average of the grade related to the exercises will be computed over the total number of lectures (i.e. you should hand-in as many sheets as the number of lectures to take advantage of the grading scheme). Furthermore, in order to have the possibility to use the bonus from homework you need to score at least 5.75. as a weighted average in the exams.In order to pass the course you need to get a grade larger than 5.75. This will be automatically translated into a 6, whereas anything below 5.75 will be downgraded to 5 (i.e. non-passing grade).

Both the final exam and the retake are “open book” so that you may consult the lecture notes and any book but not the worked-out exercises or any other material.

In all the examples the grade of the final exam can be replaced by the grade of the retake.

Some examples:

- **Example A:** A student hands in all or part of the exercise sheets and gets an average of 9.0 for this part. The same student scores 6.3 in the final exam. The final grade is then the maximum between $0.8 \times 6.3 + 0.2 \times 9.0 = 6.84 \to 7.0$ (including homework) and 6.3 (i.e., the exam grade). That means that in this case the final grade will be 7.
- **Example B:** A student hands in all or part of the exercise sheets and gets an average of 6.0 for this part. The same student scores 8.8 in the final exam. The final grade is then the maximum between $0.8 \times 8.8 + 0.2 \times 6.0 = 8.2 \to 8.0$ (including homework) and 8.8 (i.e., the exam grade). That means that in this case the final grade will be 8.8 that will be rounded to 9.
- **Example C:** A student hands in all or part of the exercise sheets and gets an average of 9.0 for this part. The same student scores 5 in the final exam. The exam grade is below 5.75 which makes 5 the final grade for the course i.e. the student did not pass the course. In this case the student won’t have the possibility to use the homework bonus which would have raised the grade to $0.8 \times 5 + 0.2 \times 9.0 = 5.8 \to 6.0$.

Contents

Chapter 1

Introduction

Already from early days, humans tried to understand the world that surround us i.e. how it is formed, which are the basic constituents and what are the fundamental laws that govern our Cosmos. Although there is evidence that the theory of the atom was also developed in India, I can not help but mentioning the developments that took place in ancient Greece.

Greek atomism, the term originating from the Greek word $\alpha\tau o\mu o$ i.e. uncuttable, was a response to problems in Greek philosophy. One aspect had to do with the problem of change – were we in a constant state of flux or was change illusory? How could one account for the seemingly endless multiplicity of Nature? Greek atomism was a means of answering philosophical difficulties and explaining the natural world. Around 440BC Leucippus of Miletus[1], in his lost book "The Greater World System," originated the atom concept. He and his pupil, Democritus[2] of Abdera, refined and extended it in future years. There are five major points to their atomic idea:

- All matter is composed of atoms, which are bits of matter too small to be seen. These atoms can not be further split into smaller portions.
- There is a void, which is empty space between atoms. In modern words, we would call this void *vacuum*.
- Atoms are completely solid objects. There can be no void inside the atom, otherwise it would be subject to changes from outside and would thus disintegrate i.e. it will no longer be an atom.
- Atoms are homogeneous, with no internal structure.
- Atoms are different in their sizes, their shapes and their weight, the latter property was added at a later stage by Epicurus[3].

Almost all of the original writings of Leucippus and Democritus are lost. About the only sources we have for their atomistic ideas are found in quotations of other writers. The idea of the atom was strongly opposed by Aristotle and others. Because of this, the atom receded into the background. Although there is a fairly continuous pattern of atomistic thought through the ages, only a relative few scholars gave it much thought. It was not until 1897 and J. J. Thomson's discovery of the electron that the atom was shown to have an internal structure.

1.1 The discovery of the electron

The previous century was the period during which the idea that all matter is composed of a few types of elementary particles, that can no be further subdivided, has been established. The list of elementary particles has changed over time, as new particles have been discovered and some old ones were proven to be composed of elementary constituents. Through all these changes, the one particle type that has always remained on the list was the electron.

The electron was the first elementary particle to be ever identified. It is also by far the lightest of the elementary particles that have charge and one of the few that does not decay into other particles. The discovery of the electron is credited

[1] Leucippus of Miletus: Greek philosopher that lived in the early 5th century BC in Miletus

[2] Democritus: Greek pre-Socratic philosopher from Abdera, Thrace (460-371 BC)

[3] Epicurus: Greek philosopher from Samos (341-270BC)

to J. J. Thomson[4]. Thomson was awarded the 1906 Nobel Prize in Physics for this discovery and for his work on the conduction of electricity in gases.

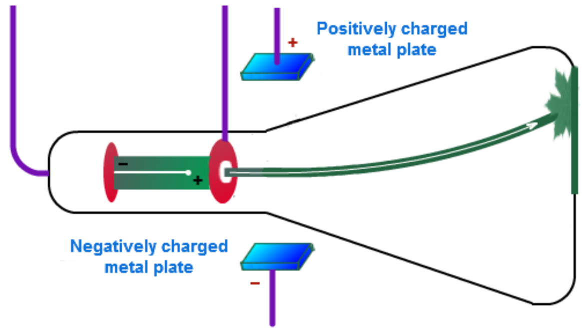


Fig. 1.1: One of the tubes that J. J. Thomson used to conduct his cathode-ray tubes experiments.

Thomson used Newton's second law of motion to obtain a general formula that would explain the measurements of the cathode-ray deflection, produced in his experiment by various electric or magnetic forces in terms of the properties of the cathode-ray particles. In the cathode-ray tube, shown in fig. 1.1, the particles pass through a region (i.e. the deflection region) in which they are subject to electric or magnetic forces acting at right angles to their original deflection and then through a force free region (i.e. the drift region) in which they travel freely until they hit the end of the tube. A glowing spot appears at the place where the ray particles hit the glass at the end of the tube. This made it easy for the observer to measure the displacement of the ray produced by the forces acting on it, by measuring the distance between the locations of the glowing spot when the forces were on and when they were switched off. The displacement could be calculated according to

$$d = \frac{F \times l_{deflection} \times l_{drift}}{m \times u^2}, \tag{1.1.1}$$

where d is the displacement of the ray-particles at the end of the tube, F is the force that acts upon the particles, $l_{deflection}$ and l_{drift} are the length that particles travel in the deflection and drift regions, respectively, m is the mass of the particles and u their velocity. The main idea behind the experiment is the way that the deflection is controlled via the external forces that are applied. These forces give an acceleration to the ray-particles with a component perpendicular to the original motion. In his experiment, Thomson measured the displacement produced by various electric and magnetic forces acting on the ray. Out of the quantities used in Eq. 1.1.1, the lengths of the deflection and drift regions are known quantities determined with high accuracy from the design of the cathode ray tube and the particle mass and velocity are the quantities that one would like to measure. The electric force acting on the particle is proportional to the charge of the particle itself. This makes the displacement measured with Eq. 1.1.1 a combination of different variables or parameters (i.e. mass and charge) of the ray-particles, but can not constrain them individually. To overcome this obstacle, Thomson measured the deflection induced by magnetic forces. A magnetic force is proportional to the the charge of a particle as well as, unlike electric forces, to its velocity. By measuring the deflections by both electric and magnetic forces, Thomson was able to determine at the same time the velocities and the ratio of electric charge to mass of the ray-particles.

The fact that the particles were 1000 times lighter than the known atoms suggested their elementary nature. Still, there were hints that led Thomson to further support his conclusion. On of the hints was given by the value of the charge to mass ratio that seemed not to depend on the circumstances under which the measurement was conducted (e.g. type of gas in the tube, type of cathode, even though the velocities were quite different). At first Thomson did not use any specific name for the particles that he discovered. The name was given by George Stoney[5] as the fundamental unit quantity of electricity.

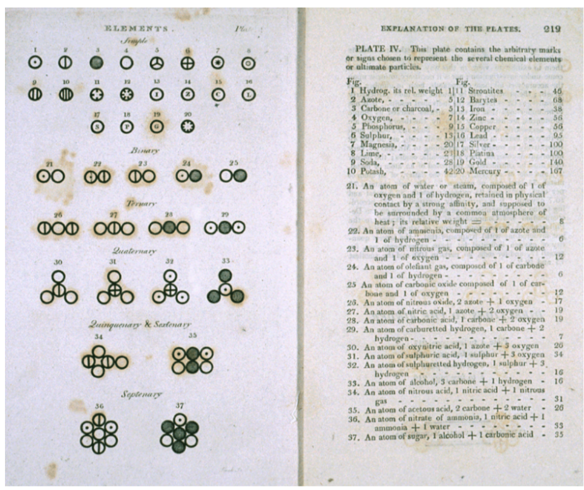
[4] Sir Joseph John Thomson, (18 December 1856 ? 30 August 1940) was an English physicist who became a senior professor at the University of Cambridge.

[5] George Johnstone Stoney (1826-1911) was an Anglo-Irish physicist.

1.2 The atomic scale

The concept of atoms was introduced in the early 1800s by John Dalton. He was the first to publish a table of relative atomic weights, containing six elements, namely: hydrogen, oxygen, nitrogen, carbon, sulphur, and phosphorus. The atom of hydrogen conventionally assumed to weight 1. A snapshot of one of Dalton's report where he was discussing his studies can be seen in 1.2. Dalton's theory of the atoms can be summarised by the following points:

- Elements are made of extremely small particles called atoms.
- Atoms of a given element are identical in size, mass, and other properties; atoms of different elements differ in size, mass, and other properties.
- Atoms cannot be subdivided, created, or destroyed.
- Atoms of different elements combine in simple whole-number ratios to form chemical compounds.
- In chemical reactions, atoms are combined, separated, or rearranged.



EXPLANATION OF THE PLATES. 219

PLATE IV. This plate contains the arbitrary marks or signs chosen to represent the several chemical elements or ultimate particles.

Fig.			Fig.		
1	Hydrog. its rel. weight	1	11	Strontites	46
2	Azote,	5	12	Barytes	68
3	Carbone or charcoal,	5	13	Iron	38
4	Oxygen,	7	14	Zinc	56
5	Phosphorus,	9	15	Copper	56
6	Sulphur,	13	16	Lead	95
7	Magnesia,	20	17	Silver	100
8	Lime,	23	18	Platina	100
9	Soda,	28	19	Gold	140
10	Potash,	42	20	Mercury	167

21. An atom of water or steam, composed of 1 of oxygen and 1 of hydrogen, retained in physical contact by a strong affinity, and supposed to be surrounded by a common atmosphere of heat; its relative weight = - - - - - - 8
22. An atom of ammonia, composed of 1 of azote and 1 of hydrogen - - - - - - 6
23. An atom of nitrous gas, composed of 1 of azote and 1 of oxygen - - - - - - 12
24. An atom of olefiant gas, composed of 1 of carbone and 1 of hydrogen - - - - - - 6
25 An atom of carbonic oxide composed of 1 of carbone and 1 of oxygen - - - - - - 12
26. An atom of nitrous oxide, 2 azote + 1 oxygen - 17
27. An atom of nitric acid, 1 azote + 2 oxygen - - 19
28. An atom of carbonic acid, 1 carbone + 2 oxygen 19
29. An atom of carburetted hydrogen, 1 carbone + 2 hydrogen - - - - - - 7
30. An atom of oxynitric acid, 1 azote + 3 oxygen 26
31. An atom of sulphuric acid, 1 sulphur + 3 oxygen 34
32. An atom of sulphuretted hydrogen, 1 sulphur + 3 hydrogen - - - - - - 16
33. An atom of alcohol, 3 carbone + 1 hydrogen - 16
34. An atom of nitrous acid, 1 nitric acid + 1 nitrous gas - - - - - - 31
35. An atom of acetous acid, 2 carbone + 2 water - 26
36. An atom of nitrate of ammonia, 1 nitric acid + 1 ammonia + 1 water - - - - - - 33
37. An atom of sugar, 1 alcohol + 1 carbonic acid - 35

Fig. 1.2: One of the pages in a report from J. Dalton describing his studies on the atoms.

At a later stage, when Thomson discovered electrons, he also built one of the first atomic models. Thomson knew that electrons had a negative charge and thought that matter must have a positive charge. His model looked like raisins stuck on the surface of a lump of pudding.

In 1900 Max Planck, a professor of theoretical physics in Berlin showed that when you vibrate atoms strong enough, such as when you heat an object until it glows, you can measure the energy only in discrete units. He called these energy packets, quanta.Physicists at the time thought that light consisted of waves but, according to Albert Einstein, the quanta behaved like discrete particles. Physicists call Einstein's discrete light particle, a *photon*. Atoms not only emit photons, but they can also absorb them. In 1905, Albert Einstein wrote a ground-breaking paper that explained that light absorption can release electrons from atoms, a phenomenon called the "photoelectric effect." Einstein received his only Nobel Prize for physics in 1921 for his work on the photoelectric effect.

In 1911, Ernest Rutherford via his experiments probed the internal structure of the atom by discovering, as we will see in the next section, the nucleus, the positively charged core of the atom. In 1912 a Danish physicist, Niels Bohr came up with a theory that said the electrons do not spiral into the nucleus and came up with some rules for what does happen. Bohr introduced the idea that electrons can orbit only at certain allowed distances from the nucleus. He also suggested that atoms radiate energy when an electron jumps from a higher-energy orbit to a lower-energy orbit. Furthermore, an atom could absorb energy when an electron gets boosted from a low-energy orbit to a high-energy orbit.

1.3 The nucleus

The atomic model introduced by Thomson was quite popular until around 1913, when Geiger and Marsden, under the supervision of Rutherford, performed the well known by now gold foil experiments. They pointed a beam of α-particles at a thin foil of metal and measured the scattering pattern by using a fluorescent screen. The setup used is schematically depicted in fig. 1.3. They spotted α-particles bouncing off the metal foil in all directions, some right back at the source. This should have been impossible according to Thomson's model: the α-particles should have all gone straight through. Obviously, those particles had encountered an electrostatic force far greater than Thomson's model suggested they would, which in turn implied that the atom's positive charge was concentrated in a much tinier volume than Thomson imagined.

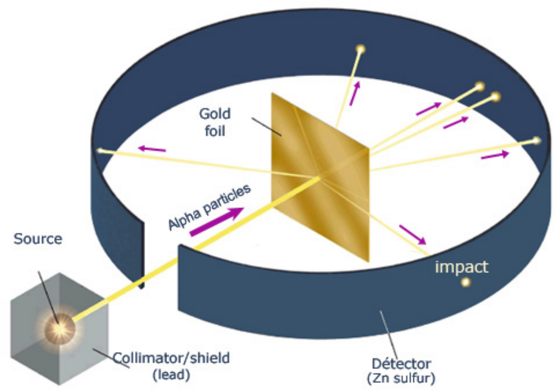


Fig. 1.3: A schematic representation of the experiment that Rutherford conducted.

Rutherford and his collaborator soon dismissed Thomson's model of the atom, and instead proposed a model where the atom consisted of mostly empty space, with all its positive charge concentrated in its centre in a very tiny volume, the nucleus, surrounded by a cloud of electrons.

The discovery of the neutron by Chadwick (1932) showed that atomic nuclei are made up of protons and neutrons. It was also clear that, in addition to gravitation and the electromagnetic force, there should exist two short-range forces in nature: a strong force which binds the nucleons together and a weak force which is responsible for radioactive β-decay. These forces had to be short-range because they were not felt at atomic scales.

1.4 The particles and forces of the Standard Model

The traditional goal of particle physics has been to identify what appears to be structureless units of matter and to understand the nature of the forces acting upon and between them. The connection between fundamental matter and forces was made clear by Thomson's discovery of the electron and Maxwell's theory of the electromagnetic field, which together by many are considered to mark the birth of modern particle physics. In the last part of the previous century and in the beginning of the current one, significant progress has been made not only in the development of theories that describe three of the fundamental forces i.e. the electromagnetic, the weak and the strong forces, but there were also compelling and convincing experimental evidence of their applicability. This collection of theories is usually called the Standard Model.

It turns out that the units of matter are fermions i.e. particles with half-integer spin ($\hbar/2$). There are two types of fermions, the leptons and the quarks, which are both considered elementary particles. The leptons are a generalisation of the electrons and interact electromagnetically (i.e. charged leptons) and weakly (i.e. neutral leptons). On the other hand the quarks, are considered to be the constituents of hadrons and interact via all interactions i.e. electromagnetically, weakly and strongly. The weak and electromagnetic interactions of particles are described by the electroweak theory developed by Glashow, Weinberg and Salam[6], which is in turns a generalisation of quantum electrodynamics or QED. The strong interactions

[6] Also known as GWS theory.

however are described by the Quantum ChromoDynamics (QCD). All three interactions are types of gauge theories, though realised in different ways.

Chapter 2
Special relativity

Let us consider one event A that happens in one coordinate system S at (t,x). Assume now that we have another system $S^{'}$ that moves with velocity u along the x-axis to the right. This is schematically depicted in fig. 2.1. For this new reference system, the coordinates of A are $(t^{'},x^{'})$. We assumed here that when the origin of the two systems coincided, the clocks synchronised at $t = t^{'} = 0$.

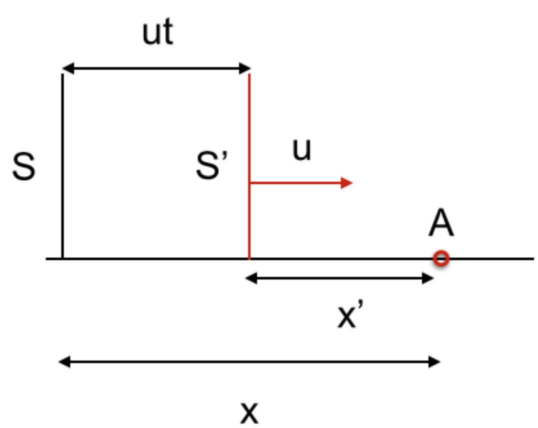


Fig. 2.1: A Galilean transformation between the two reference systems S and $S^{'}$.

To move from one system to the other, one simply has to perform the relevant transformation that takes the observer from one reference frame to the other. The connection between the two coordinate systems is given by:

$$t^{'} = t$$

$$x^{'} = x - ut$$

If the reference system $S^{'}$ travels with the same velocity as before u but in the opposite direction i.e. to the left, then the spacial coordinates are connected via:

$$x^{'} = x + ut$$

2.1 Special relativity

Let us see now how are these equations altered in the post-Einstein era!

For this we need to first agree that the speed of light is the same in both reference systems i.e. S and S'. That means that if we send a light signal, this will travel x (or x') for a time t (or t'), such that:

$$x = ct \tag{2.1.1}$$

$$x' = ct' \tag{2.1.2}$$

Now what used to work for the Galilean transformation $x' = x - ut$, should be altered in some way that we are going to try to extract below. Since the way this equation is altered is currently unknown, let us write it as:

$$x' = \gamma(x - ut), \tag{2.1.3}$$

where γ is an unknown fudge factor. The observer in the reference system S will see the object moving in a different way, with the relevant equation now being:

$$x = \gamma(x' + ut') \tag{2.1.4}$$

At this stage, it is important to note that:

- we assumed here that *gamma*, i.e. the unknown fudge factor, is the same in both systems. This is postulated by the fact that both observers are equivalent.
- The time elapsed from when we synchronised the two clocks and when the light pulse was sent can be different i.e. $t \neq t'$.

Multiplying the two sides of Eq. 2.1.3 and Eq. 2.1.4, we get:

$$xx' = \gamma^2(x - ut)(x' + ut')$$

If one also considers Eq. 2.1.1 and Eq. 2.1.2, the previous relation can be written:

$$c^2tt' = \gamma^2(xx' - utx' - ut'x - u^2tt') = \gamma^2(c^2tt' - \gamma utt' + \gamma utt' - u^2tt') = \gamma^2tt'(c^2 - u^2) \Leftrightarrow$$

$$\gamma^2 = \frac{c^2}{c^2 - u^2} \Leftrightarrow \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

The spatial coordinate thus transforms as:

$$x' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Let us now see how the time coordinate is modified:

$$t' = \frac{1}{u}\left(\frac{x}{\gamma} - x'\right) = \frac{1}{u}\left(\sqrt{1 - \frac{u^2}{c^2}}x - x'\right) = \frac{1}{u}\left(\sqrt{1 - \frac{u^2}{c^2}}x - \gamma x + \gamma ut\right) \Leftrightarrow$$

$$t' = \frac{1}{u}\left(\sqrt{1-\frac{u^2}{c^2}}x - \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}x + \frac{u}{\sqrt{1-\frac{u^2}{c^2}}}t\right) = \frac{1}{u\sqrt{1-\frac{u^2}{c^2}}}\left(x - \frac{u^2}{c^2}x - x + ut\right) \Leftrightarrow$$

$$t' = \frac{1}{u\sqrt{1-\frac{u^2}{c^2}}}\left(ut - \frac{u^2}{c^2}x\right) \Leftrightarrow t' = \frac{t-\frac{u}{c^2}x}{\sqrt{1-\frac{u^2}{c^2}}}$$

Note that similar equations can be written for differences between coordinates e.g. we have two events at A(t_1,x_1) and B(t_2,x_2):

$$\Delta x' = \frac{\Delta x - u\Delta t}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\Delta t' = \frac{\Delta t - \frac{u}{c^2}\Delta x}{\sqrt{1-\frac{u^2}{c^2}}}$$

2.1.1 Time dilation

Let us now consider a clock that in the reference frame S ticks every one second and imagine having two events that happened at the same special coordinate, whose time difference is Δt. The corresponding time difference as measured in a moving reference systems S', as we saw before, would be given by:

$$\Delta t' = \frac{\Delta t - \frac{u}{c^2}\Delta x}{\sqrt{1-\frac{u^2}{c^2}}}$$

However, for these two events $\Delta x = 0$, which means that:

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{u^2}{c^2}}}$$

What does this mean?

This shows that the time $\Delta t'$ between the two ticks as seen in the frame in which the clock is moving S', is longer than the time Δt between these ticks as measured in the rest frame of the clock S. In other words, moving clocks run slower.

2.1.2 Length contraction

Suppose we carry a rod of length L_0, aligned along the x-axis in the reference system S. To measure the length of this rod in the system S', in which the clock is moving, the two ends of the rod need to be measured at the same time in S'. This means that $\Delta t' = 0$. The relation between the lengths in both systems is given by

$$\Delta x = \frac{\Delta x' + u\Delta t'}{\sqrt{1-\frac{u^2}{c^2}}} \Leftrightarrow \Delta x' = L_0\sqrt{1-\frac{u^2}{c^2}}$$

Both time dilation and length contraction, as you can imagine, have quite some implications, the most characteristic of which is the example of the twins: at the age of 20, one of them leaves the earth for a round trip with a spacecraft that travels with a constant velocity $u = 0.5c$. The trip takes him $d = 10$ ly (i.e. light years) away and back to earth. For his brother who stayed back the time elapsed between the start and the end of the trip is:

$$\Delta t = \frac{2d}{u} = 40\text{years}$$

Let us now see what is the time elapsed for the astronaut. The astronaut travels at a constant speed but not for 10 ly. The distance is now:

$$\Delta x' = L_0\sqrt{1 - \frac{u^2}{c^2}} = 8.66\text{ly}$$

Hence, the trip lasts for $t = 34.6$years. The astronaut will return to the earth at the age of 54.6 and will meet his 60 year-old twin!

2.1.3 Causality

Assume that we have two events, one being the outcome and the other the cause. We say that these two events are causally related. It makes sense to say that no one should be able to see the outcome before the cause. On the other hand if the two events are not causally related, a reversed order will not lead to logical contradictions.

According to Lorentz transformations:

$$\Delta t' = \frac{\Delta t - \frac{u}{c^2}\Delta x}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Let's now assume that in S $\Delta t = t_2 - t_1 > 0$. To reverse the order of the two events in another, moving reference system S' we need to have $\Delta t' = t_2' - t_1' < 0$:

$$\frac{\Delta t - \frac{u}{c^2}\Delta x}{\sqrt{1 - \frac{u^2}{c^2}}} < 0 \Leftrightarrow \Delta t < \frac{u}{c^2}\Delta x \Leftrightarrow \frac{c\Delta t}{\Delta x} < \frac{u}{c}$$

- If we send a light pulse that travels for Δt, then if $c\Delta t > \Delta x$ (i.e. case A in fig. 2.2), the two events can be reversed in time if

  $$\frac{u}{c} > 1 \Leftrightarrow u > c$$

  If a light signal has enough time to travel between the two events (i.e. to cover their spatial separation) their order can not be reversed.

- On the other hand, if the light signal does not have enough time to travel the entire spatial separation of these two events (i.e. case B in fig. 2.2), then

  $$c\Delta t < \Delta x \Leftrightarrow u < c$$

  That means that we can find an observer for whom the ordering can be reversed. That is harmless though since if a light can not connect two events, nobody can!

In summary, if there is time for a light signal to connect two events which are causally connected, then there is no frame where the time ordering of these two events can be reversed.

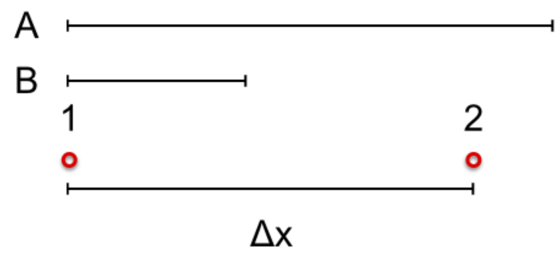


Fig. 2.2: Two events (1) and (2) that causally related in S occur at distance Δx. Depending on whether there is enough time for a light signal to travel (at least) the distance of the two events or not, the time ordering can not or can be reversed in another reference frame.

This now defines the so-called light cone seen in fig. 2.3. The two axis give the spatial (i.e. horizontal axis) and the time (i.e. vertical axis) coordinate. The light cone can be divided in three regions as follows

- The time-like region, with $c\Delta t > \Delta x$, also called as the absolute past (i.e. blue triangle) and absolute future (i.e. yellow triangle),
- the space-like region, with $c\Delta t < \Delta x$,
- and the lines along the diagonals, where $c\Delta t = \Delta x$, known as light-like.

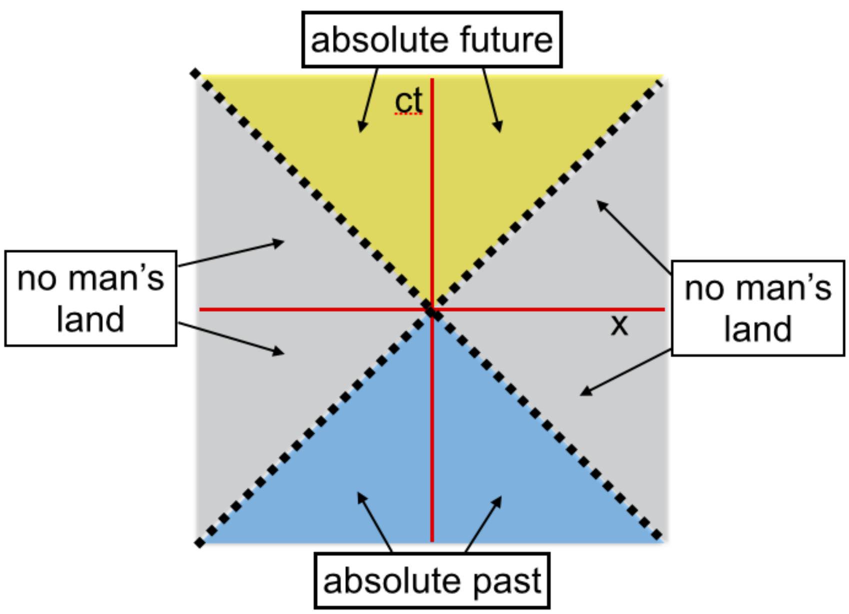


Fig. 2.3: The light cone in special relativity.

2.1.4 Velocity transformations

Let us now consider a particle that moves by Δx in a given time interval Δt measured in a reference system S. The same particle will be seen by an observer who travels along a moving reference system S' as if it traveled a distance $\Delta x'$ for an interval $\Delta t'$. We can now calculate the velocities in the two systems by taking the time derivative:

$$v = \frac{\Delta x}{\Delta t}$$

$$v^{'} = \frac{\Delta x^{'}}{\Delta t^{'}}$$

We will now derive the relation between the velocities in these two different systems:

$$v^{'} = \frac{\Delta x^{'}}{\Delta t^{'}} = \frac{\frac{\Delta x - u\Delta t}{\sqrt{1-\frac{u^2}{c^2}}}}{\frac{\Delta t - \frac{u}{c^2}\Delta x}{\sqrt{1-\frac{u^2}{c^2}}}}$$

$$\frac{\Delta x - u\Delta t}{\Delta t - \frac{u}{c^2}\Delta x} = \frac{\frac{\Delta x}{\Delta t} - u}{1 - \frac{u}{c^2}\frac{\Delta x}{\Delta t}} \Leftrightarrow$$

$$v^{'} = \frac{v - u}{1 - \frac{u}{c^2}v}$$

Going back from $S^{'}$ to S simply means that we have to follow:

$$v = \frac{v^{'} + u}{1 + \frac{u}{c^2}v^{'}}$$

It is interesting to see what kind of velocity one computes for a light pulse in the reference system $S^{'}$:

$$v^{'} = \frac{v-u}{1-\frac{u}{c^2}v} = \frac{c-u}{1-\frac{u}{c^2}c} = c\frac{c-u}{c-u} = c$$

We have now established a great part of special relativity, starting from the Lorentz transformations. At this stage, it is important to try to simplify a bit the notation by introducing the following variables:

- The velocity coefficient β:

$$\beta = \frac{u}{c}$$

- The Lorentz factor γ:

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}}$$

The Lorentz transformations can thus be written:

$$x^{'} = \frac{x - ut}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{x - \frac{uc}{c}t}{\sqrt{1-\frac{u^2}{c^2}}} \Leftrightarrow$$

$$x^{'} = \gamma\left[x - \beta(ct)\right]$$

$$t^{'} = \frac{t - \frac{u}{c^2}x}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{1}{c}\frac{ct - \frac{u}{c}x}{\sqrt{1-\frac{u^2}{c^2}}} \Leftrightarrow$$

$$(ct^{'}) = \gamma\left[(ct) - \beta x\right]$$

2.1.5 Invariant quantities

We now that although some coordinates change under a given transformation, certain combinations of these quantities remain invariant. A characteristic example is the rotation of the system S to $S^{'}$ by an angle θ. This transforms the vector $\vec{r} = x\hat{i} + y\hat{j}$ to $\vec{r}^{'} = x^{'}\hat{i} + y^{'}\hat{j}$. The connection between the coordinates in these two reference systems is given by:

$$x^{'} = x\cos\theta - y\sin\theta$$

$$y^{'} = x\sin\theta + y\cos\theta$$

If one takes the scalar (i.e. dot) product of each vector with itself, that is if we compute the length of the vector in both systems, then we will realise that it is invariant:

$$\vec{r}^{'}\vec{r}^{'} = (x^{'})^2 + (y^{'})^2 = (x\cos\theta - y\sin\theta)^2 + (x\sin\theta + y\cos\theta)^2 =$$

$$x^2\cos^2\theta + y^2\sin^2\theta - 2xy\cos\theta\sin\theta + x^2\sin^2\theta + y^2\cos^2\theta + 2xy\cos\theta\sin\theta =$$

$$x^2\left(\sin^2\theta + \cos^2\theta\right) + y^2\left(\sin^2\theta + \cos^2\theta\right) = x^2 + y^2 \Leftrightarrow$$

$$\vec{r}^{'}\vec{r}^{'} = \vec{r}\vec{r}$$

The same does not work in Minkowski space i.e. $(ct)^2 + x^2 \neq (ct)^{'2} + x^{'2}$. On the other hand, what seems to be invariant is the product:

$$s^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2$$

$$s^{2'} = x_0^{'2} - x_1^{'2} - x_2^{'2} - x_3^{'2}$$

Let us try to confirm this, by considering the Lorentz transformations in one direction e.g. on the x-axis, that is taking $x_2 = x_2^{'}$ and $x_3 = x_3^{'}$:

$$s^{2'} = x_0^{'2} - x_1^{'2} - x_2^{'2} - x_3^{'2} = \left[\gamma\left(x_0 - \beta x_1\right)\right]^2 - \left[\gamma\left(x_1 - \beta x_0\right)\right]^2 - x_2^2 - x_3^2$$

$$= \gamma^2 x_0^2 + \gamma^2\beta^2 x_1^2 - 2\beta\gamma^2 x_0 x_1 - \gamma^2 x_1^2 - \beta^2\gamma^2 x_0^2 + 2\beta\gamma^2 x_0 x_1 - x_2^2 - x_3^2$$

$$= \gamma^2(1-\beta^2)x_0^2 - \gamma^2(1-\beta^2)x_1^2 - x_2^2 - x_3^2$$

$$= \frac{1-\beta^2}{1-\beta^2}x_0^2 - \frac{1-\beta^2}{1-\beta^2}x_1^2 - x_2^2 - x_3^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2 \Leftrightarrow$$

$$s^{2'} = s^2$$

The quantity s^2 is invariant and it is called the space-time interval. Based on its value we can characterise it as:

- time-like if $s^2 > 0$,
- space-like if $s^2 < 0$,
- light-like if $s^2 = 0$,

2.1.6 Proper time

Let us now consider a particle that moves in the (x,t) plane with velocity v. The particle travels Δx at a time interval Δt. The space-time interval is:

$$(\Delta s)^2 = (\Delta x_0)^2 - (\Delta x_1)^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$= (c\Delta t)^2 \left[1 - \frac{1}{c^2}\left(\frac{\Delta x}{\Delta t}\right)^2\right] = (c\Delta t)^2 \left[1 - \frac{v^2}{c^2}\right] \Leftrightarrow$$

$$(\Delta s) = \frac{(c\Delta t)}{\gamma}$$

As we said before, the space-time interval (Δs) is invariant i.e. it does not change between different systems. Now let's calculate the same quantity as seen by the particle itself. For the reference frame that travels with the particle the time interval is $\Delta\tau$ but the space interval is $\Delta x^{'} = 0$ i.e. the space coordinate as seen by the particle did not change. That means that the space-time interval as seen by the particle is:

$$(\Delta s^{'})^2 = (c\Delta\tau)^2 - (\Delta x^{'})^2 = (c\Delta\tau)^2$$

Now from the invariance of (Δs) one gets:

$$\Delta\tau = \frac{\Delta t}{\gamma}$$

The parameter $\Delta\tau$ is also an invariant and is called **proper time**. One can further write the previous formula in terms of derivatives:

$$\frac{d\tau}{dt} = \frac{1}{\gamma} \Leftrightarrow \frac{dt}{d\tau} = \gamma$$

2.2 Energy-momentum

In Newtonian mechanics, we used to manufacture new vectors by taking derivatives e.g. $\vec{v} = d\vec{r}/dt$. Then momentum was defined as $\vec{P} = m\vec{v}$. Let us see how things change in the post-Einstein era. We can not take the time derivative of velocity

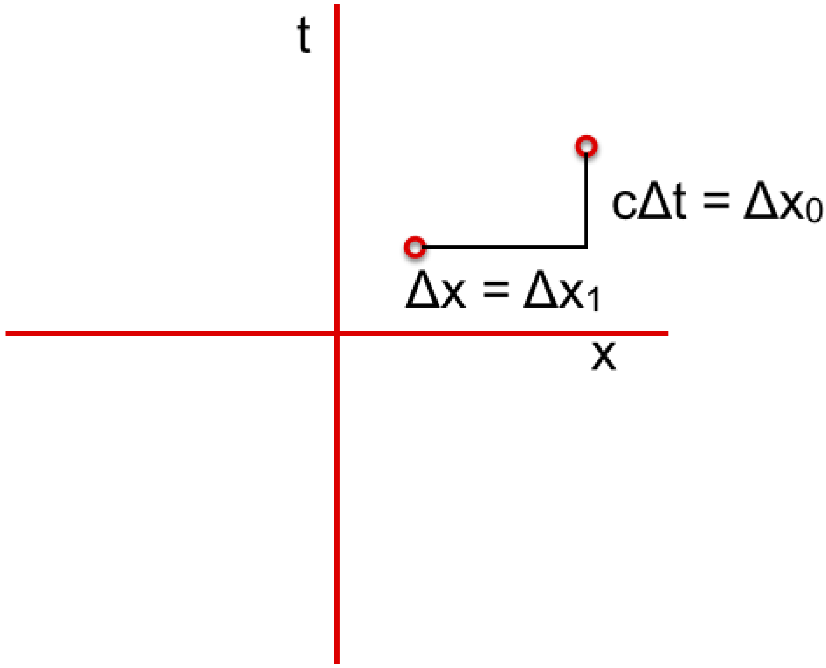


Fig. 2.4: A particle that moves in the (x,t) plane.

since now time is another coordinate and mixes with x. It is as if we take the derivative of x with respect to y in the vector notation i.e. it is meaningless! We need to take the derivative with respect to an invariant i.e. the proper time $\Delta\tau$:

$$v_i \equiv \frac{dx_i}{d\tau} = \frac{dx_i}{dt}\frac{dt}{d\tau} = \gamma\frac{dx_i}{dt}$$

$$\begin{pmatrix} v_0 = \gamma dx_0/dt = \gamma c \\ v_x = \gamma dx_1/dt = \gamma v_x \\ v_y = \gamma dx_2/dt = \gamma v_y \\ v_z = \gamma dx_3/dt = \gamma v_z \end{pmatrix}$$

The invariant quantity for the case of velocity is

$$(\gamma c)^2 - (\gamma \vec{v})^2 = \gamma^2 c^2 \big(1 - \frac{v^2}{c^2}\big) = c^2$$

Multiplying the previous with the mass, should give the relativistic momentum:

$$\begin{pmatrix} p_0 = m\gamma c \\ \vec{p} = m\gamma\vec{v} \end{pmatrix}$$

Let us look carefully at the two parts of the new four vector and let us start with the second part i.e. $m\gamma\vec{v}$.

$$m\gamma\vec{v} = \frac{m\vec{v}}{\sqrt{1-u^2/c^2}}$$

This seems as if it is the normal Newtonian mechanics momentum, with the addition of a correction due to velocity. In fact when $u \to 0$ then $m\gamma\vec{v} \to m\vec{v} = \vec{P}$. The momentum is an interesting quantity in relativity: although velocity seems to have an upper limit i.e. the speed of light, momentum does not. This is important for accelerators, where particles are moving very close to the speed of light e.g. $v = 0.9999c$. The next generation of accelerators can add another digit at the end, which makes little difference in the velocity but huge difference in the momentum.

Back to the components of the relativistic momentum now, with the first component which is of particular interest:

$$p_0 = m\gamma c = \frac{mc}{\sqrt{1-u^2/c^2}}$$

which when $u \to 0$, then $p_0 \to mc$ which does not remind us of something. This 0-th component can be written as:

$$p_0 = mc\big(1 - \frac{u^2}{c^2}\big)^{-1/2}$$

We should now remember the fact that

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots$$

for small values of x. This transforms p_0 as follows:

$$p_0 = mc\big(1 + \frac{1}{2}\frac{u^2}{c^2} + \frac{3}{8}\frac{u^4}{c^4} + \ldots\big)$$

$$= mc + \frac{1}{2}\frac{mu^2}{c} + \frac{3}{8}\frac{mu^4}{c^3} + \ldots$$

The first term still tells us nothing, while the second term starts to resemble the kinetic energy. To make the resemblance more clear one has to multiply p_0 by c:

$$cp_0 = mc^2 + \frac{1}{2}mu^2 + \ldots$$

Now the second term is definitely the old known kinetic energy. The first component is called the rest energy of the particle. It shows that even a particle at rest has energy!

Let us now define the relativistic energy

$$E = \gamma mc^2$$

which is connect to the 0-th component of the relativistic momentum as $p_0 = \gamma mc = E/c$. This means that the relativistic momentum can be written as:

$$\begin{pmatrix} p_0 = E/c \\ \vec{P} = m\gamma\vec{v} \end{pmatrix}$$

The invariant of the four momentum is:

$$\frac{E^2}{c^2} - \vec{P}^2 = (\gamma mc)^2 - (\gamma mv)^2$$

$$= m^2c^2\gamma^2\big(1 - \frac{v^2}{c^2}\big) \Leftrightarrow p^2 = m^2c^2$$

Being an invariant means that no matter in which frame we will calculate it, its value won't change. In these cases, it is always wise to choose the most convenient (e.g. in terms of complications related to calculations) reference system. In this case, we can try to calculate the invariant quantity in the frame that moves with the particle. In this frame, the particle is at rest and the relativistic momentum can be written as:

$$\begin{pmatrix} E/c \\ \vec{0} \end{pmatrix} = \begin{pmatrix} mc \\ \vec{0} \end{pmatrix}$$

It is obvious that the invariant quantity in this frame is simply:

$$m^2c^2$$

This invariant can be written as:

$$(\frac{E}{c})^2 - \vec{P}^2 = (mc)^2 \Leftrightarrow$$

$$E^2 = m^2c^4 + P^2c^2$$

This last formula clearly states that the energy of a particle with mass m consists of two parts, one related to its momentum and the other to its rest mass. What happens then for a massless particle e.g. the photon? For this $m = 0$ and $u = c$, and this means that the energy will be given by the magnitude of the momentum: $E = Pc$. In this case, the energy and momentum is determined by the frequency ν or the wave length λ of the photon:

$$E = h\nu = \frac{hc}{\lambda}$$

2.3 Relativistic collisions

In what follows, we use **natural units** i.e. $c = \hbar = 1$. In collisions between particles energy and momentum are always conserved:

$$E_1 + E_2 = E_3 + E_4 + ... + E_n$$

$$\vec{P}_1 + \vec{P}_2 = \vec{P}_3 + \vec{P}_4 + ... + \vec{P}_n$$

Note that by colliding two incoming particles, we can create at the final state more than two outgoing particles. This is also what is being achieved and studied at accelerator complexes where by colliding e.g. p on p, as done at the Large Hadron Collider (LHC), we can create 3, 4, 5,...,50 particles and more. The available energy for particle production is quantified by the centre-of-mass energy and is given by:

$$s = \sqrt{\Big(\sum_i E_i\Big)^2 - \Big(\sum_i p_i\Big)^2} \tag{2.3.1}$$

In a collider mode and when identical particles (i.e. same mass) are accelerated at the same momentum and thus energy, the centre-of-mass energy takes the form:

$$s = \sqrt{\Big(\sum_i E_i\Big)^2 - \Big(\sum_i p_i\Big)^2} = \sqrt{(E_1 + E_2)^2 - (\vec{P}_1 + \vec{P}_2)^2} = 2E \tag{2.3.2}$$

In a fixed target experiment, momentum conservation implies that the final state particles are always produced with significant kinetic energy and they are produced mainly in the so–called forward region. The corresponding centre–of–mass energy is given by:

$$s = \sqrt{\left(\sum_i E_i\right)^2 - \left(\sum_i p_i\right)^2} = \sqrt{(E_{\text{beam}} + M_{\text{target}})^2 - p_{\text{beam}}^2} = \sqrt{M_{beam}^2 + M_{\text{target}}^2 + 2M_{\text{target}}E_{\text{beam}}} \tag{2.3.3}$$

where E_{beam}, p_{beam} and M_{beam} are the energy, momentum and mass of the beam particles respectively, while M_{target} is the mass of the target particle. Colliding beam machines have the advantage of achieving much higher centre–of–mass energy than fixed target configurations.

Chapter 3
Elements of quantum mechanics

In this chapter we are first going to get familiar with the basic properties of particles. We are going to discuss about how particles are characterised based on the value of their orbital angular momentum, spin and total angular momentum but also isospin. At the end of the chapter we are going to introduce the concept of antimatter and review the elementary particles of the Standard Model.

3.1 From classical to quantum mechanics: representations

In quantum-mechanics free particles are described as waves that can be decomposed into a Fourier integral of plane waves according to

$$\Psi(\vec{x},t) \approx e^{i(\vec{k}\vec{x}-\omega t)} = \cos(\vec{k}\vec{x}-\omega t) + i\sin(\vec{k}\vec{x}-\omega t),$$

where $\vec{k} = \vec{p}/\hbar$ the wave vector connected with the wavelength λ via $\lambda = h/p$. The angular frequency gives the energy of the wave according to $E = \hbar\omega$. The particle in quantum mechanics is not localised and the probability to find a particle described by the wave function $\Psi(\vec{x},t)$ in a volume V is given by

$$P(\vec{x},t)dV = |\Psi(\vec{x},t)|^2 dV$$

The normalisation condition gives rise to

$$P = \int P(\vec{x},t)dV = \int_{\text{all-space}} |\Psi(\vec{x},t)|^2 dV = 1$$

All physical quantities and time-dependent variables are described by operators according to

$$\hat{A}\Psi = \alpha\Psi,$$

where $\hat{A}$ is the operator, α the eigenvalue that corresponds to the operator and Ψ is the eigenfunction. Table 3.1 gives some indicative examples for some of the basic “variables“ in their classical representation with their quantum-mechanical counterparts.

In quantum mechanics we thus have in one dimension:

$$\hat{P}_x\Psi(x,t) = -i\hbar\frac{\partial}{\partial x}(Ne^{i(kx-\omega t)}) = \hbar k N e^{i(kx-\omega t)} = p_x\Psi(x,t)$$

$$\hat{E}\Psi(x,t) = i\hbar\frac{\partial}{\partial t}(Ne^{i(kx-\omega t)}) = \hbar\omega N e^{i(kx-\omega t)} = E\Psi(x,t)$$

	Classical representation	QM representation
Scalar function	$f(x)$	$f(x)$
Momentum	$\vec{p}$	$-i\hbar\nabla$
Energy	E	$i\hbar\frac{\partial}{\partial t}$
Angular momentum (z-coordinate)	L_z	$-i\hbar\frac{\partial}{\partial\phi}$

Table 3.1: Association of some of basic "variables" in their classical representation with their quantum-mechanical counterparts.

In classical mechanics the total energy of a non-relativistic particle is the sum of the kinetic (T) and the potential (U) energy, given by

$$E = H = T + U = \frac{p^2}{2m} + U$$

If one writes down the quantum-mechanical counterparts of the previous equation then we end up with

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + \hat{U}\Psi(x,t)$$

The previous equation describes a particle moving in one dimension, along the x-axis. The generalisation in three dimensions gives rise to the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{x},t) + \hat{U}\Psi(\vec{x},t) \tag{3.1.1}$$

The physical interpretation of the wave function is that the product $\Psi^*\Psi$ calculated over a volume element d^3x gives the probability of finding a particle in the volume element.

3.2 Orbital angular momentum

Particles are also characterised by their angular momentum. The orbital angular momentum of a particle with momentum $\vec{p}$ is given by

$$\vec{L} = \vec{r} \times \vec{p}, \tag{3.2.1}$$

where $\vec{r}$ is the radius vector that connects the centre of mass of the particle to the point to which $\vec{L}$ refers to. Classically $\vec{L}$ can take any value. However in quantum mechanics $\hat{L}$, the quantum mechanical operator of the orbital angular momentum, has quantised eigenvalues. As we know, $\vec{p}$ is replaced in quantum mechanics by the operator:

$$\hat{p} = -i\hbar\vec{\nabla} = -i\hbar(\partial/\partial x, \partial/\partial y, \partial/\partial z)$$

Consequently, the orbital angular momentum is also expressed in terms of its quantum mechanical operator $\hat{L}$ expressed as:

$$\hat{L} = \hat{r} \times \hat{p} = (x\hat{x} + y\hat{y} + z\hat{z}) \times \Big[-i\hbar\Big(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\Big)\Big] =$$

$$-i\hbar x\frac{\partial}{\partial x}(\hat{x}\times\hat{x}) - i\hbar y\frac{\partial}{\partial x}(\hat{y}\times\hat{x}) - i\hbar z\frac{\partial}{\partial x}(\hat{z}\times\hat{x})$$

$$-i\hbar x\frac{\partial}{\partial y}(\hat{x}\times\hat{y})-i\hbar y\frac{\partial}{\partial y}(\hat{y}\times\hat{y})-i\hbar z\frac{\partial}{\partial y}(\hat{z}\times\hat{y})$$

$$-i\hbar x\frac{\partial}{\partial z}(\hat{x}\times\hat{z})-i\hbar y\frac{\partial}{\partial z}(\hat{y}\times\hat{z})-i\hbar z\frac{\partial}{\partial z}(\hat{z}\times\hat{z})=$$

$$i\hbar y\frac{\partial}{\partial x}\hat{z}-i\hbar z\frac{\partial}{\partial x}\hat{y}-i\hbar x\frac{\partial}{\partial y}\hat{z}+i\hbar z\frac{\partial}{\partial y}\hat{x}+i\hbar x\frac{\partial}{\partial z}\hat{y}-i\hbar y\frac{\partial}{\partial z}\hat{x}=$$

$$-i\hbar\Big(y\frac{\partial}{\partial z}-z\frac{\partial}{\partial y}\Big)\hat{x}-i\hbar\Big(z\frac{\partial}{\partial x}-x\frac{\partial}{\partial z}\Big)\hat{y}-i\hbar\Big(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\Big)\hat{z}\Rightarrow$$

$$\hat{L}_x=-i\hbar\Big(y\frac{\partial}{\partial z}-z\frac{\partial}{\partial y}\Big)$$

$$\hat{L}_y=-i\hbar\Big(z\frac{\partial}{\partial x}-x\frac{\partial}{\partial z}\Big)$$

$$\hat{L}_z=-i\hbar\Big(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\Big)$$

The different components of the orbital angular momentum satisfy the algebra of Lie:

$$[\hat{L}_i,\hat{L}_j]=i\hbar\varepsilon_{ij}^k\hat{L}_k,$$

where ε_{ij}^k is the Levi-Civita symbol that corresponds to 1 for different i,j,k indices that are properly ordered, -1 for different i,j,k indices that are not properly ordered and 0 in case any of the two indices are the same.

The wave function of a particle with a definite orbital angular momentum is an eigenfunction of L^2 and L_z such that

$$L^2|\psi_{lm}\rangle=l(l+1)\hbar^2|\psi_{lm}\rangle$$

$$L_z|\psi_{lm}\rangle=m_l\hbar|\psi_{lm}\rangle$$

The magnitude of L^2 can only take the values $l(l+1)\hbar^2$, where l is an integer number. The magnitude of L_z is $m_l\hbar$ and can take any integer value in the range: $-l,-l+1,...,0,...,l-1,l$. That means that there can be $(2l+1)$ values.

Figure 3.1 presents the possible orientations of the angular momentum vector for $l=2$.

3.3 Spin

In classical mechanics, objects spinning around a given rotation axis can have associated a a physical quantity called angular momentum, which is the analog of linear momentum in the case of rotational motion. For instance, in the case of a sphere rotating around an axis that goes through its center, we have that its angular momentum L will be given by

$$L=\Big(\frac{2}{5}MR^2\Big)\omega,$$

where M is its mass, R its radius, and ω the angular velocity of the sphere's rotation. So we see that the angular momentum is bigger the bigger the mass, radius, and rotation velocity of the sphere. Angular momentum is an important quantity since

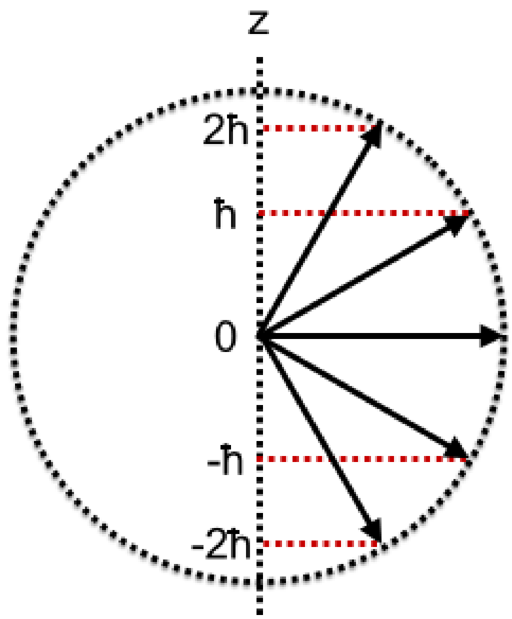


Fig. 3.1: Possible orientations of the angular momentum vector for $l = 2$.

under some circumstances, specifically in a configuration with rotational symmetry, it will be conserved, just like linear momentum of a system is conserved in the absence of external forces.

Another of the remarkable consequences of quantum theory is that even fundamental particles (that is, point particles without internal structure, meaning vanishing radius) do have associated angular momentum. This intrinsic angular momentum, which does not have a classical counterpart, is known as spin.

For the spin of a particle, one can measure its magnitude S^2 and the third component S_z. The value of S^2 can be of the form:

$$S^2 \to s(s+1)\hbar$$

In the case of the spin though, the quantum number s can take half-integer values as well as integer ones i.e. s : $0, 1/2, 1, 3/2, 2, 5/2, 3, 7/2, \ldots$. For a given value of s, its third component S_z can have values of the form $m_s\hbar$, where m_s is an integer or half integer in the range of $[-s, s]$ i.e. $m_s : -s, -s+1, \ldots, 0, \ldots, s-1, s$. Both L_z and S_z can take $2k+1$ values, where k is either l or s, respectively. In other words, just as several other properties of the quantum world, the spin of a fundamental particle is quantized: it can only take a finite set of values.

Every particle can have any value of angular momentum but their spin is fixed. We will see that we call particles with half integer spin fermions (e.g. leptons, quarks, baryons) and the ones with integer spin bosons (e.g. mesons and force mediators).

In the case of electrons, their total spin is $\hbar/2$, and therefore electrons exist in states with positive spin with respect a given axis, say the z axis, $m_s = +\hbar/2$ and in states with negative spin with respect to the same axis, $m_s = -\hbar/2$, see fig. 3.2. Interestingly, the superposition principle of quantum mechanics (see Sect. C) implies that an electron can also exist in a superposition of up and down spin states, for example with the wave function:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big(|\text{up}\rangle + |\text{down}\rangle\Big) = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + |\downarrow\rangle\Big)$$

Angular momentum and spin states are usually represented with a 'ket' i.e. $|l, m_l\rangle$ and $|s, m_s\rangle$. It can be that we are not interested in the value of the angular momentum or the spin separately but rather in the value of the total angular momentum $\vec{J} = \vec{L} + \vec{S}$. It can also be that we are interested in the relevant values of the total momentum of a particle,

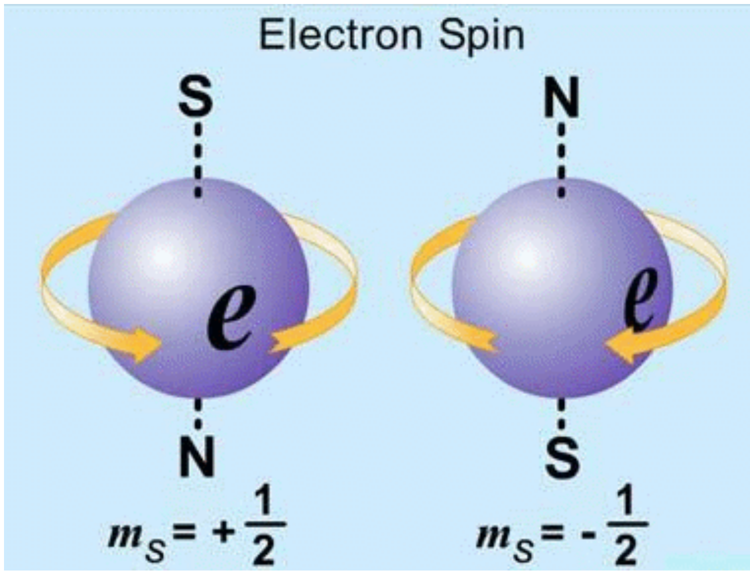


Fig. 3.2: Possible orientations of an electron in terms of its spin value.

composed by e.g. a quark and an anti-quark that, as we will see later, is called a meson. Combining angular momenta of two particles implies combining two states $|j_1, m_1\rangle$ and $|j_2, m_2\rangle$. Adding the third component is done naturally by

$$m = m_1 + m_2$$

But for the magnitude, it turns out that the quantum number j can take any value from $|j_1 - j_2|$ up to $|j_1 + j_2|$ in integer steps:

$$j = |j_1 - j_2|, |j_1 - j_2| + 1, ..., (j_1 + j_2) - 1, (j_1 + j_2)$$

3.4 Categories of particles: fermions vs bosons

In terms of their spin, all particles (either elementary, like quarks, or composite, like protons) can be divided into two main classes:

- Fermions: particles with half-integer spin: $\hbar/2$, $3\hbar/2$,...
- Bosons: particles with integer spin: 0, $\hbar$, $2\hbar$,...

For instance, all quarks and leptons are fermions, specifically with spin $\hbar/2$, while all force carries (photons, gluons, and W, Z bosons) are bosons with spin $\hbar$. On the other hand, the Higgs particle is the only known fundamental particle that is also a scalar, that is, a particle, with vanishing spin. Several composite particles that consist of quarks, called hadrons, have spin zero as well, such as the pion π and the kaon K.

This classification into fermions and bosons is summarised in fig. 3.3, which shows how the elementary particles and the associated force carriers can be classified into two main classes in terms of its spin.1 On one hand, all quarks and leptons have spin $\hbar/2$ and thus are classified as fermions. On the other hand, the Higgs boson and the force carries (photon, gluon, and W,Z particles) are all bosons, the first with spin 0 and the second with spin $\hbar$. The graviton, the hypothetical mediator of the quantum theory of gravity, is expected to have spin $2\hbar$.

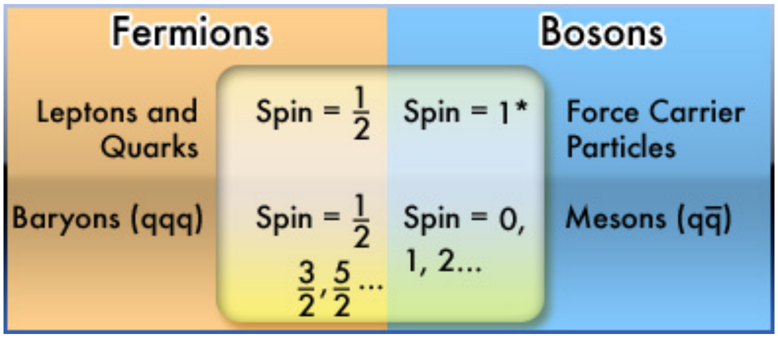


Fig. 3.3: The basic categories of both elementary and composite particles.

The main practical difference between fermions and bosons is related to the fact of whether or not two identical particles can occupy the same quantum state. To illustrate this property, let us consider a simple system composed by two identical particles, one at position x_1 and the other at position x_2. As discussed in Section 3.1, all the information about the system is encoded in its wave function. For this system composed by two particles, the wave function can be written as

$$\psi_{tot}(x_1,x_2) = \psi_1(x_1)\psi_2(x_2)$$

Now let us exchange the positions of the two particles $x_1 \to x_2$ and $x_2 \to x_1$. The new wave function of the system will be

$$\tilde{\psi}_{tot}(x_1,x_2) = \psi_1(x_2)\psi_2(x_1)$$

However, since the particles are identical, all physical information that I can extract from the system should be the same as before exchanging their positions. This requirement implies square of the wave function (which determines the probability of finding the system in a given quantum state) should be unchanged, that is,

$$|\psi_{tot}(x_1,x_2)|^2 = |\tilde{\psi}_{tot}(x_1,x_2)|^2,$$

which implies that the wave function itself can only vary up to a complex phase,

$$\tilde{\psi}_{tot}(x_1,x_2) = e^{i\phi}\psi_{tot}(x_1,x_2)$$

Moreover, if we exchange back again the positions of the two particles, one is back to the starting configuration $\psi_{tot}(x_1,x_2) = \psi_1(x_1)\psi_2(x_2)$, and then the wave function should be the same as before. This implies that

$$e^{2i\phi} = 1 \Rightarrow e^{i\phi} = \pm 1$$

This exercise tells us that identical quantum particles can behave only on two ways if they exchange positions (or more in general, if they exchange quantum states) within a system:

- either the wave function of the system remains unchanged,
- or else it get a minus sign.

We denote the first class of particles as bosons. Therefore, the wave function of a system composed by bosons satisfies upon the exchange of two particles:

$$\tilde{\psi}_{\text{bosons}}(x_1,x_2) = \psi_{\text{bosons}}(x_1,x_2),$$

while we denote the second class of particles as fermions, which behave as

$$\tilde{\psi}_{\text{fermions}}(x_1,x_2) = -\psi_{\text{fermions}}(x_1,x_2).$$

The latter implies that **two fermions cannot occupy simultaneously the same quantum state**. This is schematically illustrated in fig. 3.4.

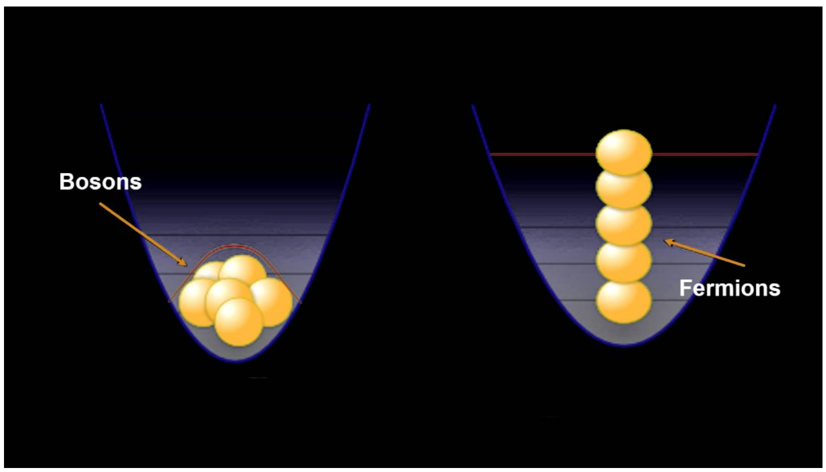


Fig. 3.4: The basic difference between bosons and fermions: while the former category of particles can be found in the same quantum state, this is not allowed for fermions.

The stability of everyday matter is indebted in great part to the Pauli exclusion principle, which prevents for instance electrons piling on top of each other within atoms. On the other hand, bosons can occupy the same quantum state without no limitations. You can either pile a macroscopic amount of bosons all in the same ground state of a given system, creating a new state of matter called a Bose-Einstein condensate. The fact that quarks and leptons are fermions has very important consequences for the theory of elementary particles, as we will illustrate in the following.

3.5 Elementary particles

In the Standard Model of particle physics, all matter and antimatter we know of is made of three categories of elementary particles:

- leptons, such as the electrons, muons, tau, the relevant neutrinos and their antiparticles,
- quarks, up, down, strange, charm, beauty, top and their antiquarks
- mediators, the massless photons and gluons and the massive $W^{\pm}$ and Z.

There are three generations of leptons: the one of electrons, positrons, the electron neutrino and antineutrino, the one of muons, the muon neutrino and antineutrino and the one of tau, the tau neutrino and the antineutrino. The basic properties

Lepton	Mass	Charge	L_e	L_μ	L_τ
e^-	0.511 MeV	-1(+1)	+1(-1)	0	0
ν_e	< 2 eV	0	+1(-1)	0	0
μ^-	105.7 MeV	-1(+1)	0	+1(-1)	0
ν_μ	< 0.17 MeV	0	0	+1(-1)	0
τ^-	1.777 GeV	-1(+1)	0	0	+1(-1)
ν_τ	< 15.5 MeV	0	0	0	+1(-1)

Table 3.2: The three generations of leptons, their masses, charges and their quantum numbers. In parenthesis, when different, the quantum numbers for antiparticles.

and quantum numbers of each lepton are summarised in Table 3.2. The table does not list antiparticles i.e. e^+, $\overline{\nu}_e$, μ^+, $\overline{\nu}_\mu$, τ^+ and $\overline{\nu}_\tau$, but note that their quantum numbers are reversed. That means that in total there are 12 leptons. Leptons fall in the category of spin-1/2 particles, as we will see towards the end of this chapter, and are thus fermions.

Quark	Mass	Charge	U	D	C	S	T	B
u	≈ 2.4 MeV	2/3(-2/3)	1(-1)	0	0	0	0	0
d	≈ 4.8 MeV	-1/3(1/3)	0	-1(1)	0	0	0	0
c	≈ 1.27 GeV	2/3(-2/3)	0	0	1(-1)	0	0	0
s	≈ 104 MeV	-1/3(1/3)	0	0	0	-1(1)	0	0
t	≈ 171 GeV	2/3(-2/3)	0	0	0	0	1(-1)	0
b	≈ 4.2 GeV	-1/3(1/3)	0	0	0	0	0	-1(1)

Table 3.3: The three generations of quarks, their masses, charges and their quantum numbers. In parenthesis, when different, the quantum numbers for antiquarks.

Similarly there are three generations of quarks, the building blocks that form all the composite particles in the Standard Model. Table 3.3 summarises their basic properties and quantum numbers. One distinct feature of quarks is that their charge is not integer multiple of the electron charge (q_e) but fractional: u, c and t all have $(2/3)\cdot q_e$, while d, s and b all have $-(1/3)\cdot q_e$. As in the case of leptons, the antiquarks all have reversed charge relative to their quark partners. Another distinct feature is the existence of additional quantum numbers, effective and applicable for each individual quark: upness for u and $\overline{u}$, downness for d and $\overline{d}$, strangeness for s and $\overline{s}$, charm for c and $\overline{c}$, beauty for b and $\overline{b}$, top for t and $\overline{t}$. All signs for these quantum numbers are reversed between quarks and antiquarks. In the meantime, each quark or antiquark comes in three colours as we will see later in this chapter, so in total we have 36 particles. Quarks also fall in the category of spin-1/2 particles, as we will see towards the end of this chapter, and are thus fermions.

Mediator	Mass	Charge
γ (photon)	0	0
W^-	≈ 80.4 GeV	-1
W^+	≈ 80.4 GeV	+1
Z^0	≈ 91.2 GeV	0
g (gluon)	0	0

Table 3.4: The mediators of interactions in the Standard Model with their masses and charges.

Finally, the Standard Model has mediators for the three interactions: the photon (γ) for the electromagnetic interactions, the $W^\pm$ and Z^0 for the weak interactions and the gluons (g) for the strong interactions. There are eight gluons that contain a combination of colour and anticolour. In total, the Standard Model has 12 mediators. These mediators fall in the category of spin-1 particles, as we will see towards the end of this chapter, and are thus bosons.

The last piece of the puzzle is the Higgs boson, discovered at the Large Hadron Collider (LHC) at CERN in Geneva in 2012. Its mass is around 125 GeV, with 0 charge and spin, hence a boson. We will briefly discuss about the Higgs boson in one of the last chapters.

Overall the Standard Model is built around 61 elementary particles.

3.6 Antimatter

In Chapter 2 we have seen how the energy of a particle with mass m consists of two parts, one related to its momentum and the other to its rest mass. The relation between these three quantities is given by the following formula:

$$E^2 = m^2c^4 + P^2c^2$$

The energy of a particle is always a positive number. However, the relationship above clearly allows the existence of negative values for E. What are these negative values? Are they unphysical or do they represent physical cases? In QM, we can't just discard a perfectly valid solution just because in classical mechanics it was not reflecting a physical system.

Paul Dirac while attempting to find a quantum field theory to describe elementary particles reached this question. He realised that, through a transformation, a particle moving forward in time with negative energy is equivalent with the picture where the particle has a positive energy but moves back in time. In addition to this, some of the quantum numbers of the particle in this second picture had to be reversed as well e.g. the electric charge. He interpreted these solutions as anti-particles that hold opposite quantum numbers to the "usual particles".

For instance, the electron e^- has associated an antimatter particle called the positron e^+ (i.e. positive electron). The two particles have identical mass $m_{e^-} = m_{e^+}$ but opposite charges, $Q_{e^+} = -Q_{e^-}$. Likewise, the antimatter partner of the proton p is the anti-proton $\bar{\mathrm{p}}$, with identical mass but opposite charge, $Q_{\bar{\mathrm{p}}} = -Q_{\mathrm{p}}$. So the anti-proton is negatively charged, in other words, the sign of its electric charge is the same as for the electron. We emphasize that there is nothing mysterious in antimatter particles: they are just the same type of matter as everyday particles, just with the opposite charges. At the same time, the universe we know of is made predominantly of matter. The mechanism that led to this asymmetry of matter over antimatter is still a complete mystery.

If antimatter is just like normal matter but with opposite charges, why we don't see positively-charged positrons and negatively-charged protons around? The reason is that as soon as a matter particle finds its corresponding antimatter particle, they annihilate into energy (into photons, the quanta of light, for example). Figure 3.5 provides a schematic view of the annihilation process between an electron and a positron that leads to the emission of two photons. In this phenomenon, two matter particles annihilate with the subsequent emission of energy, reflected in the two photons i.e. the quanta of light.

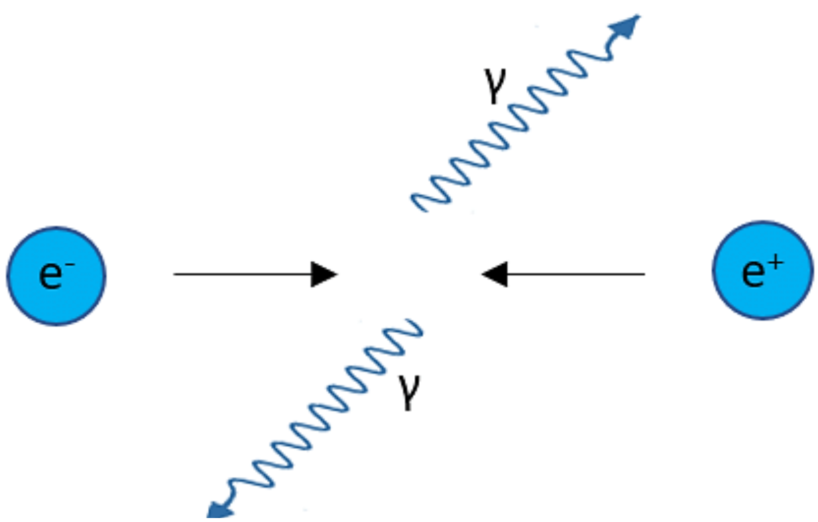


Fig. 3.5: The annihilation process between an electron and a positron that leads to the emission of two photons.

The inverse process is also possible, where a pair of photons produce a particle-anti-particle pair. In this respect, the fact that matter and antimatter particles have the opposite charges means that they can annihilate into energy without violating any conservation law, such as the conservation of the electric charge (more on this later).

Note that anti-particles experience the same types of interactions as the “regular” particles. In particular, it is easy to see that an anti-proton and a positron can combine to create an anti-hydrogen atom. The properties of anti-hydrogen are found indeed to be identical to those of normal hydrogen with very high precision. This same argument extends to other more complicated elements, for example anti-helium has also been created. Right now at CERN one can produce thousands of anti-hydrogen atoms per day, providing very stringent tests of the laws of nature.

The fact that matter and antimatter particles annihilate all their rest mass into energy suggests that they could represent the ultimately efficient fuel (or the ultimate weapon as well). But before this idea can be realised, first one needs to produce large amounts of antimatter and store it. This is easier said that done, since if antimatter touches a single matter particle it will annihilate into energy. Therefore, one needs to use techniques such as magnetic traps, which are able to store antimatter atoms preventing them of interacting (and thus annihilating) with normal matter.

Chapter 4

Fermi's golden rule

In this chapter we are going to describe the golden rule of Fermi for different processes. Before doing so, let's review a couple of important ingredients that we are going to encounter when dealing with particle interactions.

Let's imagine that we have N_0 particles that decay and after dt the relevant number becomes N. We define as decay rate Γ the probability that a particle will decay per unit time:

$$\frac{dN}{dt} = -\Gamma N$$

$$\frac{1}{N} dN = -\Gamma dt$$

$$\int_{N_0}^{N(t)} \frac{1}{N'} dN' = -\Gamma \int_{t_0}^{t} dt'$$

$$[lnN]_{N_0}^{N(t)} = -\Gamma (t - t_0)$$

$$lnN(t) - lnN_0 = -\Gamma \Delta t$$

$$ln \frac{N(t)}{N_0} = -\Gamma \Delta t$$

$$N(t) = N_0 e^{-\Gamma \Delta t}$$

We define as mean (or average) lifetime τ, the time it takes for a sample of particles to reach the value of $N(\tau) = N_0/e$. It turns out that:

$$\tau = \frac{1}{\Gamma}$$

If a particle has more than one decay mode, then

$$\Gamma_{tot} = \sum_{i=1}^{n} \Gamma_i$$

$$\tau = \frac{1}{\Gamma_{tot}}$$

In the case of a decay, an important property is the so-called branching ratio that defines the fraction of particles of a given type that decay by each mode. This is defined by

$$B.R. = \frac{\Gamma_i}{\Gamma_{tot}}$$

A different category of particle interactions is related to scattering. The relevant quantity that reflects the probability of such an interaction is the so-called cross-section. The cross-section depends on the nature of the interaction e.g. we can speak about elastic, inelastic, total or inclusive cross-section. This latter is calculated as the sum of every individual contribution, such that

$$\sigma_{tot} = \sum_{i=1}^{n} \sigma_i$$

Finally, let's define as luminosity the number of particles per unit time and per unit area, such that:

$$dN = L \cdot d\sigma$$

4.1 Golden rule for decays and scattering

To calculate the previous quantities one uses the golden rule for decays and scattering that states that a transition rate is given by the product of the amplitude or matrix element M_{if} squared and the phase space factor, the latter being purely kinematic:

$$\text{(transition rate)} = \frac{2\pi}{\hbar} |M_{if}|^2 \times \text{(phase space factor)}$$

4.2 General Feynman rules

We will first start by giving the general Feynman rules, no matter what the underlying theory is. In what follows we will be working in natural units i.e. $\hbar = c = 1$.

A characteristic diagram describing any kind of interaction of the type $A + B \to C + D$ is given in fig. 4.1.

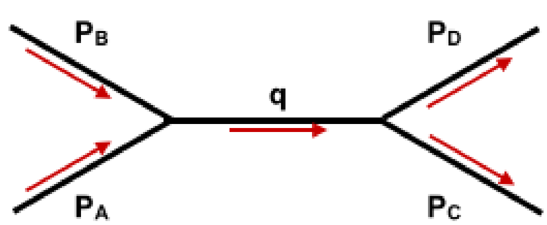


Fig. 4.1: A diagram describing the process $A + B \to C + D$.

The general rules that one needs to follow in order to calculate the matrix element M_{if} are given below:

- **Labeling:** We first label the incoming and outgoing energy and momenta $E_A, \vec{P}_A$, $E_B, \vec{P}_B$, $E_C, \vec{P}_C$ and $E_D, \vec{P}_D$. For each external line we add an arrow which indicates the positive (i.e. along the time axis) direction for particles. We then label all internal lines e.g. in fig. 4.1 we label the energy and momentum of the propagator as $E, \vec{q}$ and give an arbitrary direction to the relevant arrow.

- **Vertices:** For each vertex we note down in the diagram the coupling constant factor $-ig$ of each theory. Energy and momentum is conserved at every vertex. The same stands for all relevant quantum numbers that should be conserved for each theory.

Each of these points will be adjusted for the peculiarities of the three interactions of the Standard Model.

Chapter 5

The interactions of the Standard Model

According to the Standard Model of particle physics, there are four fundamental forces in nature: gravitational, weak, electromagnetic and strong. To each of these forces, a dedicated theory is developed and a relevant mediator, a particle carrier of the force is associated. The gravitational force is described by the general theory of relativity and its mediator which in many cases is referred to as the graviton. The weak force is described by the flavordynamic theory or better by the Glashow-Weinberg-Salam (GWS) theory. It accounts for the nuclear β decay, the decay of the pion, the muon and many strange particles. The mediators of the weak force are the $W^{\pm}$ and Z bosons. The electromagnetic interaction is described in terms of Quantum ElectroDynamics or QED. Its classical representation was developed by Maxwell, while it was refined by people like Tomonaga, Feynman and Schwinger in around 1940-1950. The mediator in QED is the photon. Finally, the strong force is mediated by the gluons and is described by the theory that emerged last among all, around 1970, the Quantum ChromoDynamics. Figure 5.1 reminds us of the three generations of matter in terms of leptons and quarks, the gauge bosons also known as force mediators and the Higgs boson. The latter is the representation of the Higgs field which is believed to explain why some fundamental particles have mass while the symmetries controlling their interactions should require them to be massless, and why the weak force has a much shorter range than the electromagnetic force.

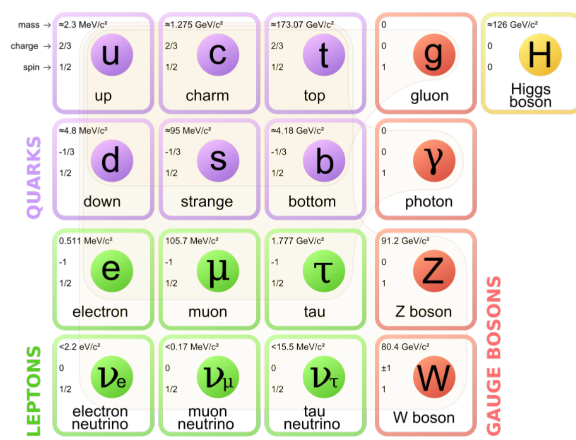


Fig. 5.1: The Standard Model of elementary particles, with the three generations of matter, gauge bosons in the fourth column, and the Higgs boson in the fifth.

5.1 Electromagnetic interactions

Quantum ElectroDynamics or QED is the simplest of the dynamic theories of particle physics. It describes all electromagnetic phenomena, using one basic diagram that presents an elementary process and is shown in fig. 5.2. In this Feynman diagram time flows from left to right, horizontally. It can be read as follows: an electron (a lepton or even a quark in general) enters, emits a photon and exits.

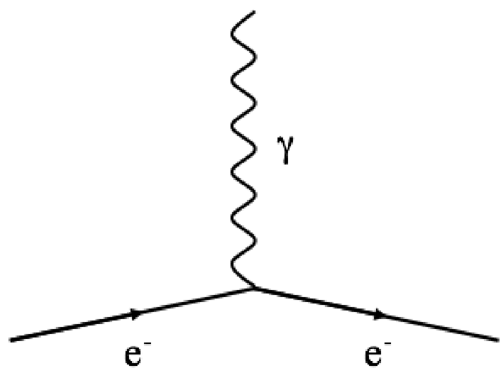


Fig. 5.2: The basic diagram that represents the most elementary process in QED.

Combining two of these elementary diagrams allows to describe more basic processes as the one presented in fig.5.3. This figure presents the scattering process between two electrons, also known as Møller scattering. It is seen that the process is mediated by a virtual photon and its cross-section is easy to be calculated using the Feynman rules.

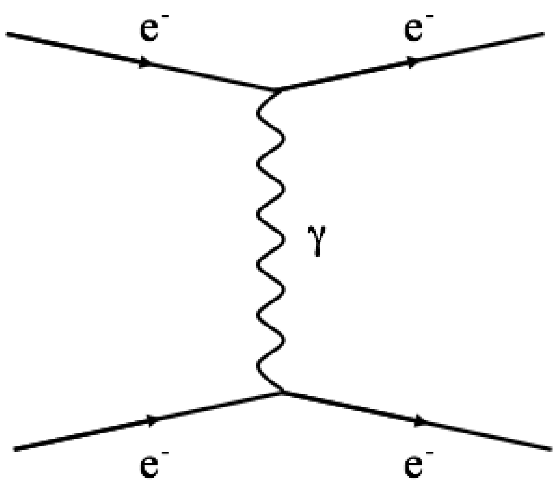


Fig. 5.3: The diagram describing the electron-electron scattering, known as Møller scattering.

Particles decaying electromagnetically have a typical lifetime of 10^{-16} sec. The strength of the electromagnetic interaction is characterised by the coupling constant, known as the fine structure constant α ($\alpha = e^2/\hbar c$). The value of α is estimated to be 1/137 with very high accuracy. It should be noted that it is believed that the fine structure constant is not a real constant but changes very slowly with energy, as we will see later in this chapter. Figure 5.4 presents the momentum transfer, Q, dependence of the fine structure constant. This momentum transfer reflects the softness or hardness of a given interaction or, equivalently, the energy with which it happens. Alternatively, Q can be considered as the inverse of the distance i.e. small values of Q correspond to large distances, while large values of Q imply small distances. It is seen that over many orders of magnitude in terms of Q, the value of α does not change drastically, justifying probably the usage of the term "constant" for α.

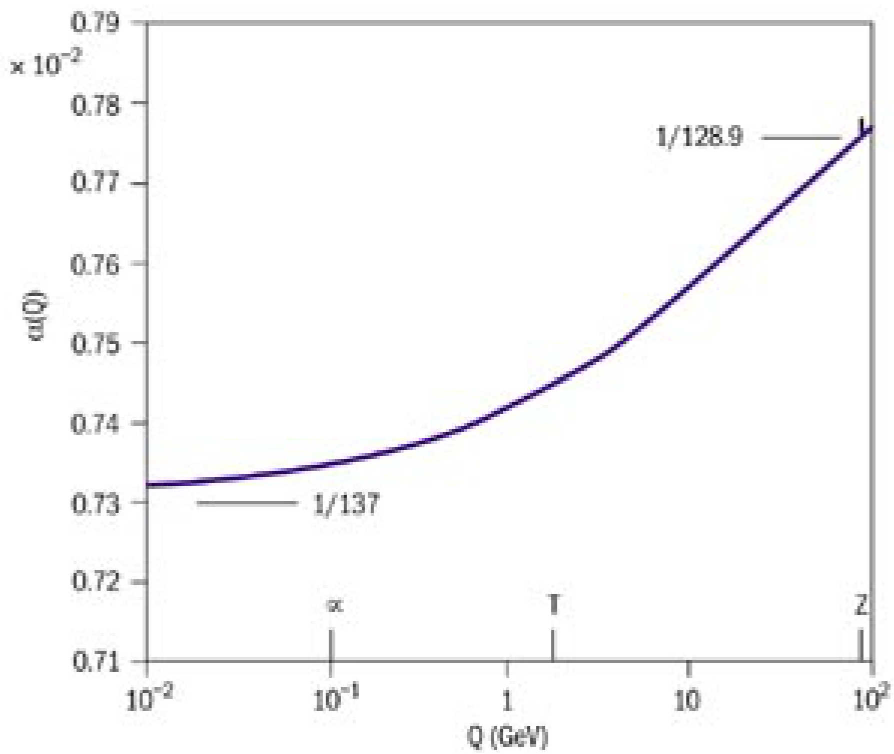


Fig. 5.4: The running of the QED coupling strength

5.2 Weak Interactions

Weak interactions are of fundamental significance in particle physics and are visible in processes involving leptons and quarks. The coupling constant is significantly smaller than the one of the electromagnetic force and it is of the order of 10^{-6}. Particles decaying weakly have a typical lifetime of $> 10^{-13}$ sec. There are two kinds of such interactions: the charged, mediated by the W-bosons, and the neutral, mediated by the Z-boson.

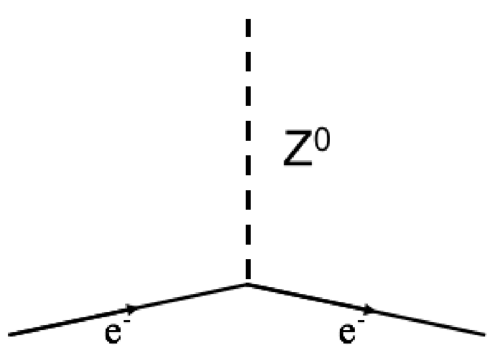


Fig. 5.5: The basic diagram that represents the most elementary neutral process in the GWS theory.

The diagram that is the basis of the neutral weak interactions is seen in fig. 5.5. It describes a Z-boson being emitted by an electron that enters and then exits. As in the case of QED, the combination of two of these diagrams can describe processes like the one that can be seen in fig. 5.6. The figure presents the scattering between an electron and a neutrino, which is mediated by there Z-boson.

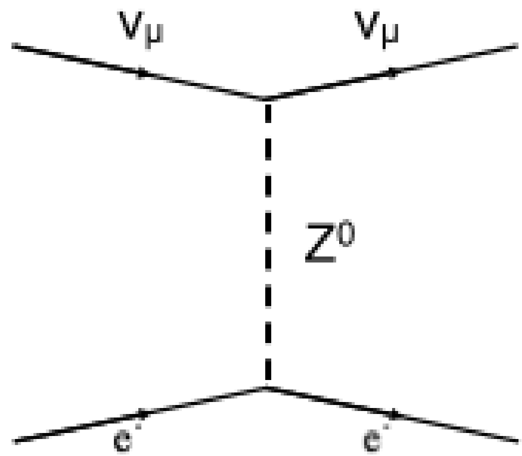


Fig. 5.6: The diagram describing the electron-neutrino scattering.

On the other hand, the charged weak decay is mediated by the charged W-bosons and its fundamental diagram can be seen in fig. 5.7.

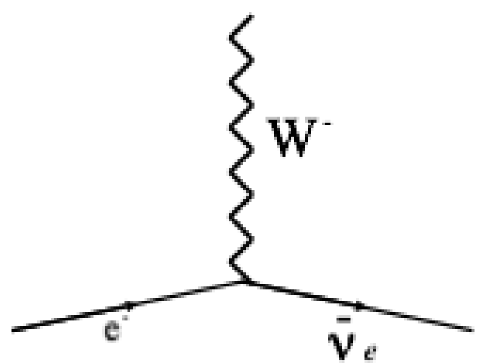


Fig. 5.7: The basic diagram that represents the most elementary charged process in the GWS theory.

Charged weak decays can occur in the presence of leptons as illustrated in fig. 5.8-left. A simple inversion of the neutrino line in the bottom part allows to describe the muon decay process ($\mu^- \to \nu_\mu + e^- + \overline{\nu}_e$), presented in fig. 5.8-right.

The charged weak decays are the only ones where the flavour of a quark can change during the process. A perfect example is the decay of the neutron ($n \to p + e^- + \overline{\nu}_e$) that is described by the diagram of fig. 5.9-left.

One interesting possibility though emerges by the observation of processes where strangeness changes, such as the decay of the Λ-baryon ($\Lambda \to p + \pi^-$). This process involves the conversion of a strange-quark to an up-quark, via the exchange of a W-boson as illustrated in fig. 5.9-right.

The solution to this was given by Cabbibo in 1963, and extended by Glashow, Iliopoulos and Maiani in 1970 and by Kobayashi and Maskawa in 1973. They suggested that the three quark generations have internal correlations and in particular that the d, s and b quarks are transformed to their respective prime particles that are linear combinations of the initial ones according to:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

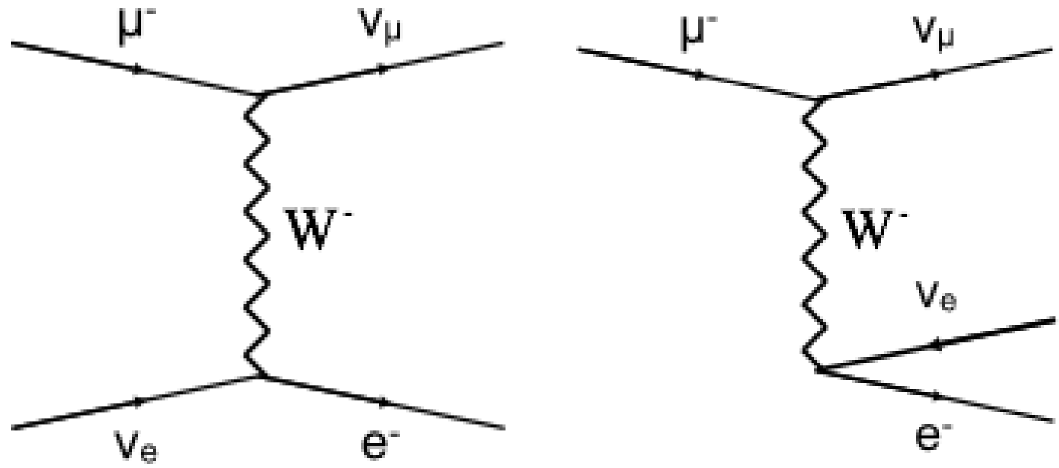


Fig. 5.8: The diagram describing the muon-neutrino charged scattering in the left plot. The right diagram results from the inversion of the line of the electron's neutrino and describes the muon decay.

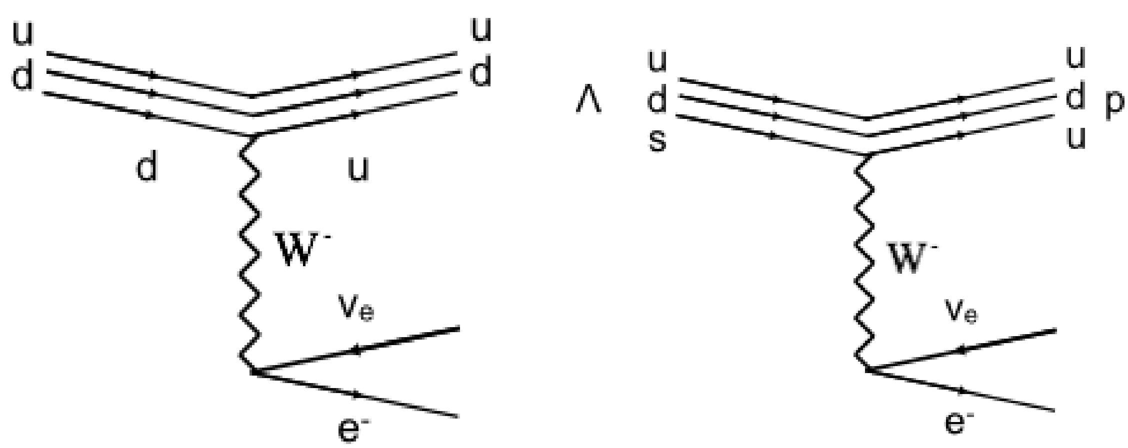


Fig. 5.9: The diagram describing the decay of the neutron (left) and of the Λ-baryon (right).

The 3×3 array is called the Kobayashi-Maskawa matrix and provides the coupling of u, c and t quarks to d, s and b.

5.3 Strong Interactions

Quantum chromodynamics or else QCD is the theory that describes the strong interaction. It describes all strong phenomena, using one basic diagram that presents an elementary process and is shown in fig. 5.10. The diagram can be read as follows: a quark enters, emits a gluon and exits.

QCD is quite similar in many ways to QED however, as we will see later, it has also distinct and fundamental differences. As in the case of QED, the combination of two similar diagrams as the one in fig. 5.10 can describe a known process i.e. the interactions between two quarks, as illustrated in fig. 5.11.

One of the fundamental differences of QCD with respect to QED is the fact that although in QED there is only one charge, in QCD the equivalent of the charge is the color. There are three kind of colours in QCD, that are conventionally called: red (R), green (G) and blue (B). Gluons have two colours, carrying one unit of color and one of anticolor. There are $3 \times 3 = 9$ possibilities for the gluons but there are only 8. Since the gluons carry color, they can also couple directly to

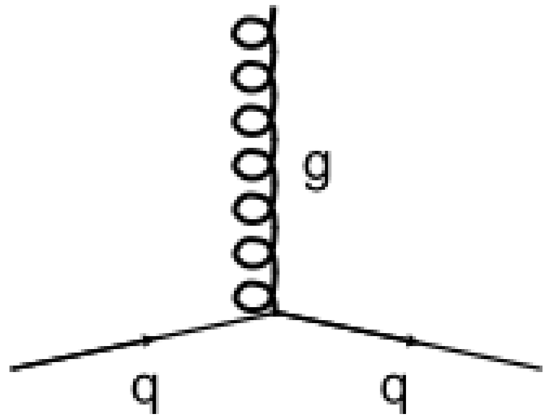


Fig. 5.10: The basic diagram that represents the most elementary process in QCD.

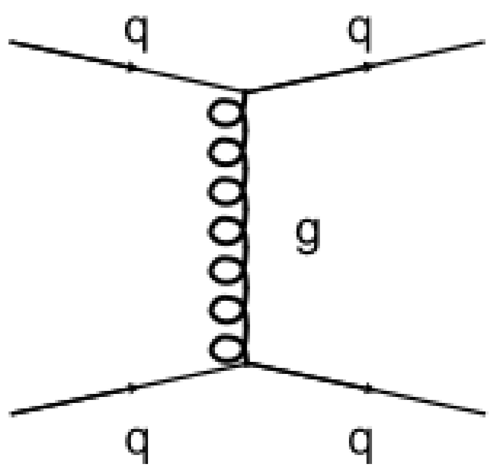


Fig. 5.11: The lower order diagram that describes the interaction between two quarks.

other gluons making the existence of gluon-gluon vertices possible. Another difference comes from the fact that particles decaying strongly have typical lifetimes of 10^{-23} sec.

The coupling constant is denoted by α_s and as in the case of QED is not a constant. In contrast to QED though, we will see that the strong coupling constant changes quite rapidly as a function of the distance between the interacting particles. In particular, at large distance α_s is big, making the observation of quarks and gluons move as free particles impossible. This phenomenon is called confinement. However, at short distances α_s becomes quite small, allowing the quarks e.g. within a proton to move freely without interacting much with their neighbouring quarks. This phenomenon is called asymptotic freedom and its existence was postulated in 1973 by Frank Wilczek, David Gross, and independently by David Politzer the same year. All three shared the Nobel Prize in physics in 2004.

In QED, a similar effect is observed which can be understood if one thinks that the vacuum itself behaves like a dielectric and creates electron-positron pairs as shown in the Feynman diagrams of fig. 5.12. The resulting vacuum polarisation screens the charge and reduces the field. This screening is reduced and eventually disappears if one approaches the charge.

An analogous picture emerges in QCD, however with the addition of the gluon-gluon vertices that we saw before. The main difference now is that in QCD the effect is opposite i.e. there is a competition between the quark polarisation diagram that tends to increase α_s and the corresponding gluon diagram that decreases the coupling constant. What the effect will be depends on the number if flavours and colours according to

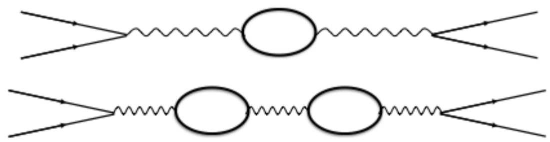


Fig. 5.12: The lower order diagrams that describe the vacuum polarisation in QED.

$$a = 2f - 11n, \tag{5.3.1}$$

where f is the number of flavours and n is the number of colours. It turns out that f = 6 and n = 3, resulting in a negative value of a of -21. This means that the QCD coupling constant decreases at small distances.

Finally it is important to note that the particles that stream freely in nature, are colourless. Quarks are confined in colourless configurations of three (baryons) or together with an antiquark (forming the mesons).

Similalry to the QED coupling strength, also in QCD we encounter the concept of a running coupling. In QED we have seen that the coupling becomes large at (very) short distance but its effect is small. In QCD, the strong coupling, α_s, has an opposite behaviour: α_s becomes small at short distance (large momentum transfer). This causes the quarks inside hadrons to behave more or less as free particles, when probed at large enough energies. This property of the strong interaction is called asymptotic freedom. Asymptotic freedom allows us to use perturbation theory, and by this arrive at quantitative predictions for hard scattering cross sections in hadronic interactions. On the other hand, at increasing distance the coupling becomes so strong that it is impossible to isolate a quark from a hadron (it becomes cheaper to create a quark-antiquark pair). This mechanism is called confinement. Confinement is verified in Lattice QCD calculations but, since it is non-perturbative, not mathematically proven from first principles.

The discovery of asymptotic freedom (1973) was a major breakthrough for QCD as the theory of the strong interaction, and was awarded the Nobel prize in 2004 to Gross, Politzer and Wilczek. [1]

Figure 5.13 presents the dependence of the strong coupling strength on the momentum transfer. The figure clearly demonstrates the drastic change in the value of α_s which becomes asymptotically small at short distances. On the other hand, at large distances the strong coupling becomes extremely large, at a momentum transfer range where confinement prevails.

5.4 Conservation laws

We already discussed about the lifetime of particles that decay electromagnetically, weakly or strongly. It is now time to look at some basic conservation laws. The first part of these laws is related to kinematics:

- Conservation of energy e.g. a particle can not decay spontaneously into particles heavier than itself
- Conservation of momentum
- Conservation of angular momentum

In addition to the previous constrains, there are also laws related to quantum numbers:

- All three interactions conserve the charge.
- Strong interactions conserve color.
- All three interactions conserve the baryon number.
- Lepton number is conserved in particle physics. Until recently there was no indication of a cross-talk or mixing between different leptons. However, neutrino oscillations make the previous statement not precise enough.

[1] The Nobel lecture of Frank Wilczek can be downloaded from `http://www.nobelprize.org` and makes highly recommended reading, both as an exposé of the basic ideas, and as a record of the hard struggle.

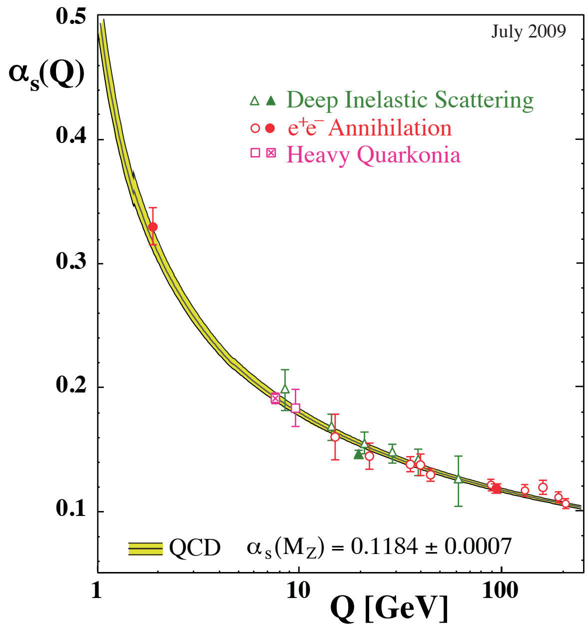


Fig. 5.13: The Q dependence of the strong coupling constant α_s.

- Quark flavour is conserved in the strong and electromagnetic interactions.

5.5 The Higgs boson

With the various elementary particles, and the corresponding force carriers, that we have presented so far we have almost completed the Standard Model of elementary particles, the unified theory of fundamental particles and their interactions. But we are still missing a very important part: the Higgs boson, which was only recently discovered at the LHC experiments at CERN. The Higgs is important because the symmetries of the Standard Model require that not only all particles are massless, but that also the W and Z bosons are massless. But we know that this is very far from the truth! The Higgs mechanism allows fundamental particles to acquire a mass without breaking its symmetries.

Indeed, there are two main reasons for which the picture of elementary particles and their interactions introduced up to now is incomplete:

- To begin with, the calculation of certain processes in electroweak theory leads to a violation of unitarity, which in a nutshell implies that the probability of this process becomes larger than one, which is certainly unphysical. As shown in Fig. 39, one needs to introduce a further particle, in this case a fundamental scalar (of spin zero, thus a boson) to obtain a sensible behaviour of the scattering amplitudes.
- In the quantum theory of the weak interactions, the W and Z should in principle be massless in order to respect some of the symmetries of the theory. Moreover, this also holds true for other particles that experience the weak interaction, such as the quarks and the charged leptons: they should have $m = 0$. The underlying reason is that the weak interaction has a built-in asymmetry between left and right spatial directions, which requires particles involved to be massless.

The way to bypass these two limitations is to break the relevant symmetry spontaneously, rather than directly, by introducing the Higgs boson by means of the Higgs mechanism. Schematically, the Higgs mechanism works as follows. In addition to the various quarks, leptons, and force carriers that the Standard Model contains, we introduce a new particle, called the Higgs boson, denoted by ϕ. This particle is different from all other SM particles in that it is a scalar, that is, its spin is zero, and thus is a boson. Recall that we have seen other spin zero particles before, such as the pions $\pi^{\pm}$ and π^0, but those particles are not fundamental but rather composite ones.

This new particle, the Higgs boson, is subject to a potential energy of the following form:

$$V(\phi^2) \equiv \mu^2\phi^2 + \lambda\phi^4,$$

with μ^2 and λ being in principle free parameters of the model. In Fig. 310 we represent the Higgs potential for two different possibilities for the sign of μ^2, either positive or negative. We see that the shape of the potential is quite different depending on the sign, in particular for $\mu^2 < 0$ the classical minimum of the potential corresponds to a non-zero value of the scalar field ϕ. This particular feature of the classical potential is crucial in the Higgs mechanism: the state with lowest energy of the theory is such that the Higgs particle is non-zero. On the other hand, for $\mu^2 \geq 0$ the minimum value of the potential (which corresponds to the vacuum state of the theory) is the one where the scalar field ϕ vanishes.

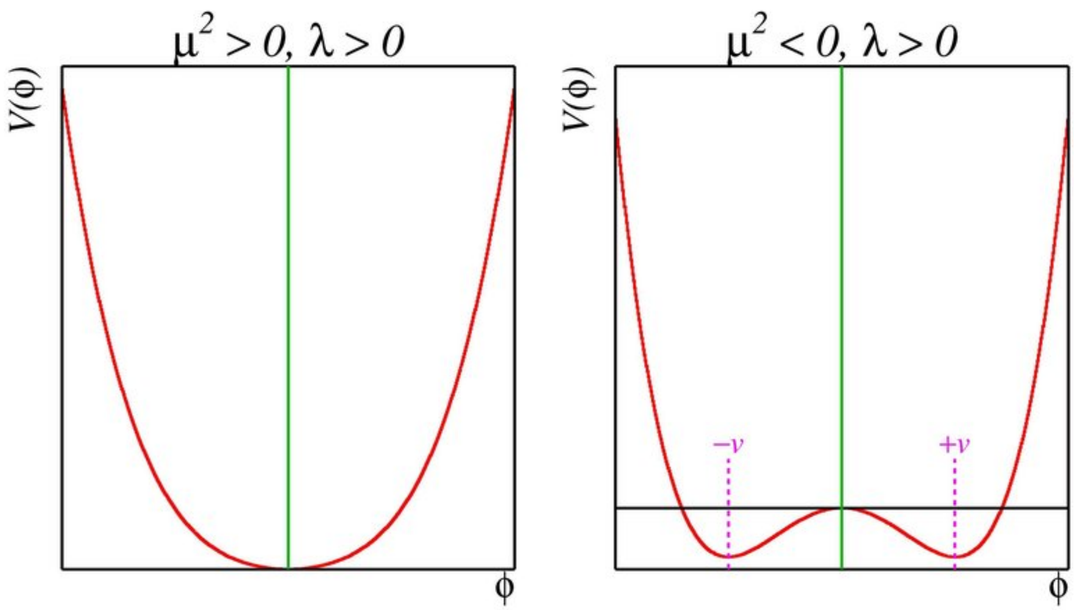


Fig. 5.14: The Higgs potential for two different sets of values for μ^2 and λ.

Classically, we know that the vacuum of the theory (that is, the state with the smallest total energy) will be the one for which the potential $V(\phi)$ has a minimum. Then, imposing this condition

$$\frac{\partial V(\phi)}{\partial \phi} = 0,$$

we find two possibilities for the vacuum state of our theory:

- For $\mu^2 \geq 0$, we find that the state of minimum energy of the theory is that where the field ϕ vanishes, i.e. $\langle\phi\rangle = 0$. In this case, the resulting theory is the standard classical electrodynamics with a massless photon coupled to a charged scalar particle.
- For $\mu^2 < 0$ instead, the state with minimum energy is such that $\langle\phi\rangle \neq 0$, and the scalar field will acquire a vacuum expectation value (VEV) $\langle\phi\rangle = \sqrt{\mu^4/2\lambda} \equiv v/\sqrt{2}$. In this case gauge symmetry will be spontaneously broken, due to the fact that the vacuum (preferred configuration) is not invariant under a gauge transformation. To see this, note that either $\langle\phi\rangle = +\sqrt{\mu^4/2\lambda}$ or $\langle\phi\rangle = -\sqrt{\mu^4/2\lambda}$ are equally good solutions (as follows from the U(1) rotational invariant

of the theory), however only one of the two options can actually be implemented in nature, breaking thus the original gauge invariance.

Therefore, we see that the key ingredient of the Higgs mechanism is that the state with lowest energy of the theory is not the state where the Higgs field takes a zero value, but rather than the Higgs field is non-vanishing. In other words, the Higgs field permeates all space, so that particles moving in this field acquire mass by coupling to it. In this context, the Higgs particle represents the quantum excitations of the Higgs field. One of the direct consequences of the Higgs mechanism is that the mass of each particle should then be proportional to the strength of its interaction with the Higgs boson. This fundamental prediction has been verified measurements of the Higgs properties by the ATLAS and CMS experiments at the CERN's Large Hadron Collider, as shown in the fig. 5.15.

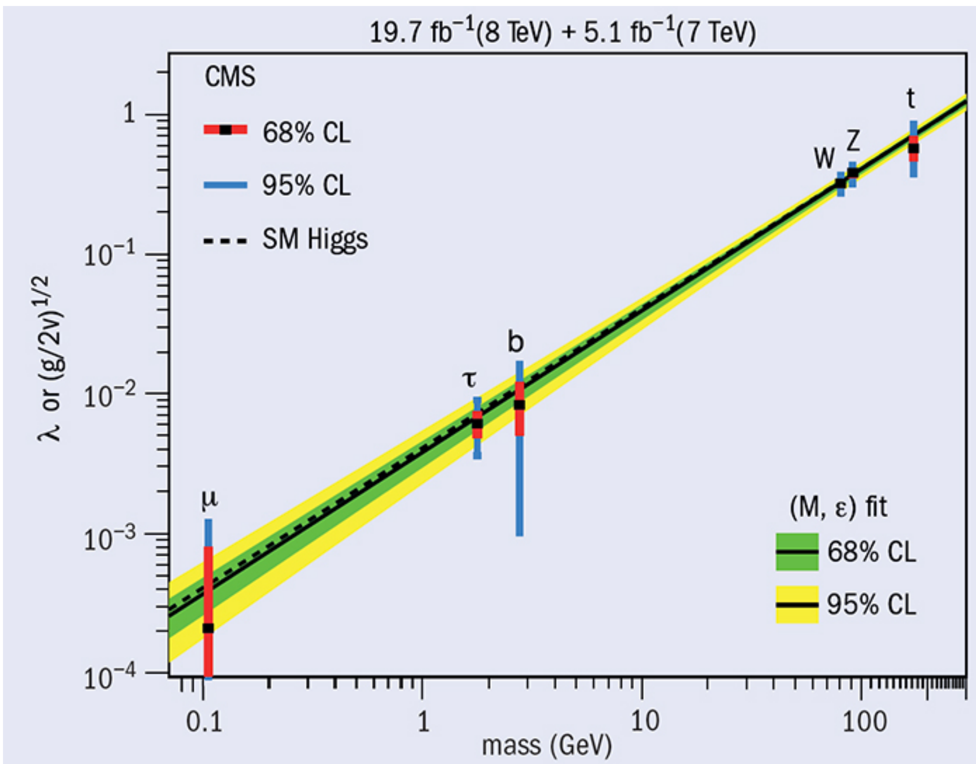


Fig. 5.15: The Higgs is responsible for giving mass to all elementary particles, including the W,Z bosons. The mass of each particle is then proportional to the strength of its interaction with the Higgs boson, as demonstrated by the measurements of the Higgs properties by the ATLAS and CMS experiments at the CERN's Large Hadron Collider.

Chapter 6

The particle zoo

In this chapter we are first going to get familiar with the basic properties of particles. We are going to discuss about mass and how one can calculate it, electric charge and magnetic dipole moment. We will also see how particles are characterised based on the value of their orbital angular momentum, spin and total angular momentum but also isospin. At the end of the chapter we are going to categorise particles and introduce the leptons, mesons, baryons and gauge bosons of the Standard Model. We will conclude by reviewing the basic conservation laws that different interactions of the Standard Model respect.

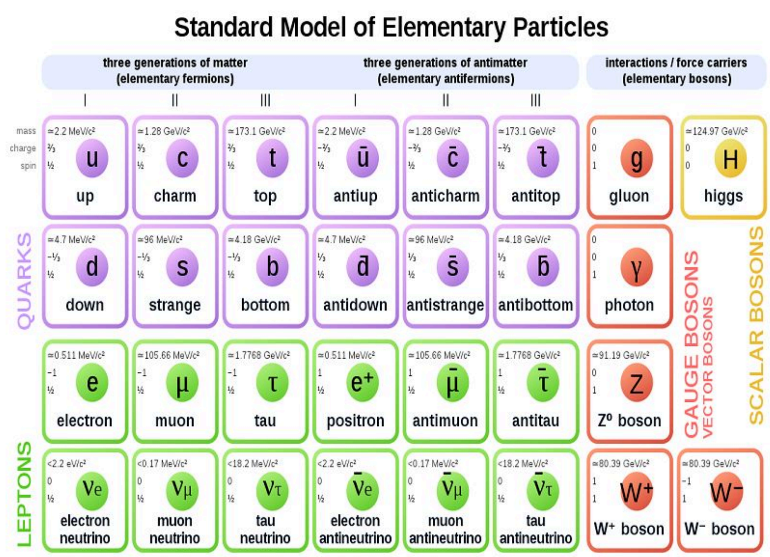


Fig. 6.1: The periodic table of elementary particles in the Standard Model.

As a reminder, fig. 6.1 presents the elementary particles of the Standard Model together with their basic properties. One sees the three generations of quarks and leptons, the gauge bosons that act as force carriers and the Higgs boson.

6.1 Particle mass

One of the obvious ways that a particle can be identified is via its mass. From the law of Newton, if a particle of mass m is positioned in a force field that acts a force $\vec{F}$ on it, then the particle accelerates with $\vec{a}$ according to

$$\vec{F} = m\vec{A}$$

The equation above does not hold at the relativistic limit but in this case we talk about the rest mass of a particle. Masses of particles can vary significantly, from the massless photon (γ) or gluons (g) to the light neutrinos (e.g. $m_{\nu_e} \approx 1$ eV, the electron ($m_e \approx 0.511$ MeV) up to nuclei e.g. the proton (p, nucleus of the hydrogen atom) with $m_{\mathrm{p}} \approx 1$ GeV $\approx 2000 m_e$. Note that the unit used to measure masses in particle physics is not anymore Kg or even g but multiples of "electronvolt"[1].

The mass of a particle can be deduced by calculating at the same time its momentum $\vec{p}$ and its energy E or its velocity $\vec{u}$. The mass then can be calculated using the relativistic equations

$$E^2 = p^2c^2 + m^2c^4$$

and

$$\vec{p} = m\gamma\vec{u},$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ with $\beta = \frac{u}{c}$.

Particle detectors are successful in helping measure the mass of charged particles. These detectors are usually embedded inside a magnetic field which is used to bend the trajectory of the particle from where on calculates its momentum. A schematic view of such a simple setup is given in fig. 6.2

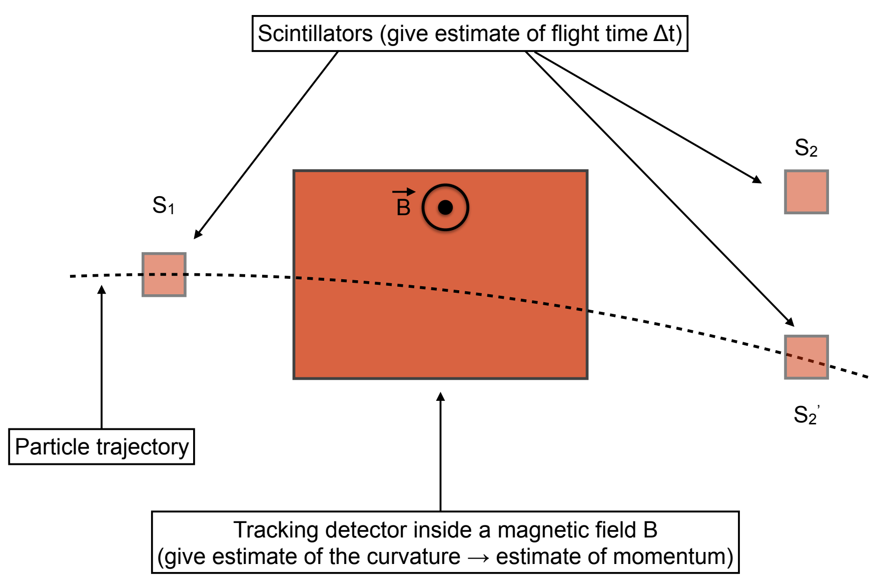


Fig. 6.2: A schematic view of a simple experimental setup that allows the measurement of particle masses.

This method obviously fails in the case of neutral particles or for particles whose lifetime is so sort that their momentum or energy can not be measured (i.e. resonances). In these cases the mass measurement is performed using the invariant mass. For this we will employ the knowledge we acquired in the Chapter 2. Let us suppose that we have a $\mathrm{K}_{\mathrm{s}}^0$–meson with a mass of around 498 MeV decaying into a pair of pions i.e. a positive π^+ and a negative one π^- with masses close to 139 MeV(do not worry about what these strange particles are at this stage; we will discuss about them in detail in the following sections of this chapter). Let's denote with $(E, \vec{P})$ the energy and momentum of the incoming particle and with $(E_1, \vec{P}_1)$ and $(E_2, \vec{P}_2)$ the energies and momenta of the outgoing pions. This decay is also schematically given

[1] The electronvolt (eV) is a typical unit of energy in particle physics which is equal to approximately 1.6×10^{-19} Joules. It is defined as the amount of energy gained or lost by a the charge of a single electron moving across an electric potential difference of one volt.

in fig. 6.3. The neutral particle leaves no trail in any particle tracking detector, even if the detector is embedded in an external magnetic field. Its trajectory is thus indicated with a dashed line in the figure. At some point it decays into a pair of charged pions that, in the presence of the magnetic field, have a curved trajectory from where information about their momenta can be deduced. There are also different detector techniques that can be employed that give us information about the identity of these particles and thus their masses. In addition, these two pions fly with an angle θ between them. Note that due to their decay topology that resembles a V–shape particles such as the K^0_s of this example or the Λ–baryon (decaying into a proton and a negative pion) are called generally V0s. Let us now try to get an estimate of the mass of this mysterious neutral particle from the information of its “daughters“. For this we are going to calculate the invariant quantities that we are very much aware of by now (working in natural units):

$$m^2_{K^0_s} = (E_1+E_2)^2 - (\vec{P}_1+\vec{P}_2)^2 \Rightarrow m^2_{K^0_s} = E_1^2+E_2^2+2E_1E_2-P_1^2-P_2^2+2P_1P_2\cos\theta \Rightarrow$$

$$m^2_{K^0_s} = m^2_{\pi^-}+m^2_{\pi^+}+2(E_1E_2+P_1P_2\cos\theta) \Rightarrow$$

$$m^2_{K^0_s} = 2m^2_{\pi}+2(E_1E_2+P_1P_2\cos\theta)$$

In particle physics experiments positioned in particle accelerators like the Large Hadron Collider (LHC), in a collision on let’s say proton on proton many particles are produced. That means that researchers do not have just the “real daughters“ at the final state to analyse. What they do is that they select a clean sample of pions with characteristic that fit the decay topology under discussion and they combine them in a statistical way to calculate the invariant mass making all possible combinations. As a result we get the right plot of fig. 6.3. This is a typical invariant mass plot, with the signal formed by combining pairs of pions that originate from decays of K^0_s lying on top of a combinatorial background (in this case really negligible) formed by pairs of pions not coming from a decay of a true K^0_s.

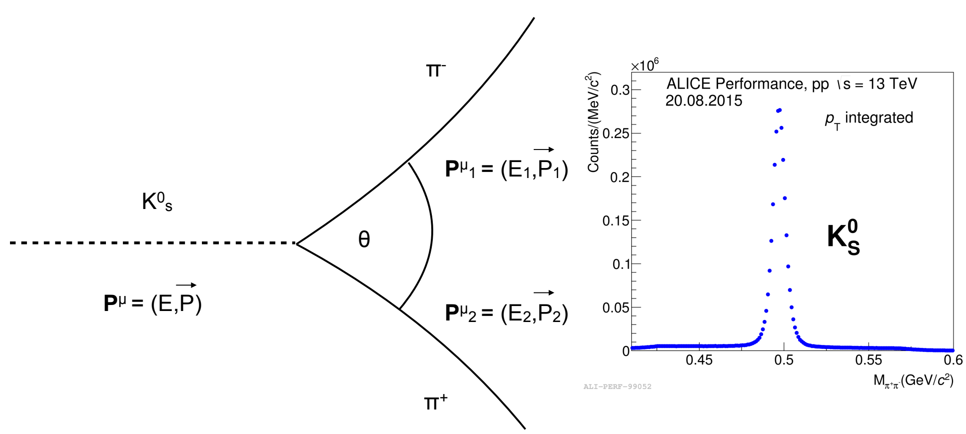


Fig. 6.3: A schematic view of the decay of a K^0_s–meson into a pair of pions. The 4–momenta of the incoming neutral and outgoing charged particles are also indicated in the plot.

6.2 Orbital angular momentum and spin

Particles are also characterised by their angular momentum and their spin as we have seen in Chapter 3. The wave function of a particle with a definite orbital angular momentum is an eigenfunction of L^2 and L_z such that

$$L^2|\psi_{lm}\rangle = l(l+1)\hbar^2|\psi_{lm}\rangle$$

$$L_z|\psi_{lm}\rangle = m_l\hbar|\psi_{lm}\rangle$$

The magnitude of L^2 can only take the values $l(l+1)\hbar^2$, where l is an integer number. The magnitude of L_z is $m_l\hbar$ and can take any integer value in the range: $-l, -l+1, ..., 0, ..., l-1, l$. That means that there can be $(2l+1)$ values.

Similarly for the spin of a particle, one can measure its magnitude S^2 and the third component S_z. The value of S^2 can be of the form:

$$S^2 \rightarrow s(s+1)\hbar$$

In the case of the spin though, the quantum number s can take half-integer values as well as integer ones i.e. s : $0, 1/2, 1, 3/2, 2, 5/2, 3, 7/2, ...$. For a given value of s, its third component S_z can have values of the form $m_s\hbar$, where m_s is an integer or half integer in the range of $[-s, s]$ i.e. $m_s : -s, -s+1, ..., 0, ..., s-1, s$. Both L_z and S_z can take $2k+1$ values, where k is either l or s, respectively.

Depending of the value of their spin, particles can be characterised as bosons i.e. integer spin, or fermions i.e. half-integer spin values.

6.3 Isospin

The original motivation to introduce this quantum number was that some hadrons seemed to be closely related between each other, in particular they had very similar masses. Such similarities often suggest that there is an underlying symmetry relating different hadrons. Two examples of groups of hadrons that seem to be related among them are:

- The proton and neutron have almost the same mass, $m_p = 938.27$ MeV and $m_n = 939.47$ MeV.
- The two charged and the neutral pions have also very similar masses, $m_{\pi^+} = m_{\pi^-} = 139.57$ MeV and $m_{\pi^0} = 134.98$ MeV.

These similarities can be accommodated by making the hypothesis that some hadrons have a property called isospin I that relates them. Following with the previous examples, we have that:

- The proton and neutron have both isospin $I = 1/2$. The difference is that one has its third component $I_3 = +1/2$ and other other $I_3 = -1/2$. This means that one can transform a proton into a neutron by a rotation in isospin space.
- The three pions, π^+, π^- and π^0 are all different realisations of the same underlying particle with isospin $I = 1$. Each of the three members of this multiplet has $I_3 = 1, 0, -1$. So we can rotate one pion into another by means of an isospin transformation.

Isospin symmetry can be explained by the fact that the u and d quark have very similar masses, as well as to the fact that from the point of view of the strong force the u and d quarks undergo the same interactions (electromagnetic effects can be neglected at this level). Therefore, isospin symmetry can be explained by the fact that from the strong force point of view, a transformation of the type $u \rightarrow d$ should leave the hadron properties unaffected. On the other hand $m_u \approx m_d$ but not quite identical, explaining why isospin is an approximate rather than an exact symmetry.

6.4 Electric charge

A particle with charge q in the presence of an external electromagnetic field feels a force given by

$$\vec{F} = q(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B})$$

The total charge of a particle is not entirely indicative of the particle itself or its internal structure (if any) since in nature only particles with integer multiples of the electron–charge exist. As an example the proton that consists of a combination of $2u$ and one d quarks has a charge

$$p(uud) \to q(p) = 2q(u) + q(d) = 2(\frac{2}{3}) + (\frac{-1}{3}) = 1 = +q(e)$$

but also the positron, which belongs to a completely different category of particles has the same charge as the proton.

6.5 Magnetic dipole moment

A classical particle with charge and spin contains currents and exhibits a magnetic dipole moment. If electric charge is distributed throughout a particle, then if the particle has spin it produces a magnetic dipole moment $\vec{\mu}$ with magnitude given by

$$|\vec{\mu}| = \frac{1}{c}(\text{current}) \times (\text{area})$$

The direction of $\vec{\mu}$ is perpendicular to the plane of the loop.

Let us now consider a particle of charge q, moving with velocity $\vec{v}$ in a circular orbit with radius r. The particle revolves with a period $T = 2\pi r/v$ and produces a current $I = qv/2\pi r$. The magnetic dipole moment will be then given by:

$$\vec{\mu} = \frac{1}{c}\vec{I} \times \vec{a} = \frac{1}{c}\frac{2}{2\pi r}|\vec{v}|\hat{v} \times \pi r^2 \hat{r} = \frac{q}{2c} vr(\hat{v} \times \hat{r})$$

But $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$ so that the previous equation takes the form:

$$\vec{\mu} = \frac{q}{2mc}\vec{L}$$

This tells us that the direction of the magnetic dipole moment is the one of the orbital angular momentum and that the ratio of the magnitudes μ/L is characteristic of the particle since

$$\frac{|\vec{\mu}|}{|\vec{L}|} = \frac{q}{2mc}$$

The operator of $\vec{\mu}$ is related to the total angular momentum via $\vec{\mu} = (\text{const.})\vec{J}$, where $(\text{const.}) = g(e/2mc)$:

$$\vec{\mu} = g\frac{e}{2mc}\vec{J}$$

The variable g is a dimensionless constant that measures the deviation of the magnetic moment from the simple value of $(e/2mc)$:

$$\vec{\mu} = g\frac{e\hbar}{2mc}\frac{\vec{J}}{\hbar} \Rightarrow \vec{\mu} = g\mu_0\frac{\vec{J}}{\hbar}$$

where $\mu_0 = e\hbar/2mc$ the unit of magnetic moment called magneton.

In atomic physics and for electrons $m = m_e$ we define the Bohr magneton according to

$$\mu = \mu_\mathrm{B} = \frac{e\hbar}{2m_e c}$$

In subatomic physics we define the nuclear magneton according to

$$\mu = \mu_\mathrm{N} = \frac{e\hbar}{2m_p c}$$

which implies that $\mu_\mathrm{N} \approx 10^{-3}\mu_\mathrm{B}$.

6.6 The leptons of the Standard Model

Forty years after the discovery of the electron by J. J. Thomson, the first member of another generation of leptons, the muon μ, was found independently by Street and Stevenson, and by Anderson and Neddermeyer. Following the convention of the electron, μ^- is the particle and μ^+ is the antiparticle. In 1975 Perl *et al.* discovered yet another replica of the electron. This particle was significantly heavier ($m \approx 1.78$ GeV/c^2) than the electron and was the tau τ^- with its antiparticle τ^+. Both particles behave quite similarly as the electrons and interact electromagnetically and weakly. These particles have half-integer spin of $1/2$.

An interesting observation about the behaviour of leptons is related to the fact that their decay into lighter leptons and a photon (e.g. $\tau^- \to \mu^- + \gamma$) is not observed. This indicated the necessity for a new conservation law which came in the form of a new additive quantum number called *lepton flavour*. We thus have the electron flavour number i.e. $L_e(e^-) = 1$ and $L_e(e^+) = -1$, the muon flavour number i.e. $L_\mu(\mu^-) = 1$ and $L_\mu(\mu^+) = -1$, and the tau flavour number i.e. $L_\tau(\tau^-) = 1$ and $L_\tau(\tau^+) = -1$. Each number is postulated to be conserved in all leptonic processes.

The electromagnetic interactions of these particles are the same, no matter what the flavour is. For the weak interactions, leptons are accompanied by their neutral partner, the neutrinos. The one emitted in the β decay of electrons was introduced by Pauli in 1930 as a last resort to restore the conservation laws of energy, momentum and angular momentum. These neutrinos are:

- ν_e with $L_e = 1$ and the antineutrino $\overline{\nu}_e$ with $L_e = -1$,
- ν_μ with $L_\mu = 1$ and the antineutrino $\overline{\nu}_\mu$ with $L_\mu = -1$,
- ν_τ with $L_\tau = 1$ and the antineutrino $\overline{\nu}_\tau$ with $L_\tau = -1$,

The realisation of the nature of antineutrinos came in 1956 by Cowan *et al.* by observing that they produce positrons via the inverse β decay:

$$\overline{\nu}_e + p \to n + e^+ \tag{6.6.1}$$

In addition, it was realised that the different neutrino flavours correspond indeed to different particles i.e. $\nu_e \neq \nu_\mu$ by observing the reaction $\overline{\nu}_\mu + p \to \mu^+ + n$ while at the same time the reaction $\overline{\nu}_\mu + p \to e^+ + n$ was not observed (i.e. an upper limit on the cross-section was given).

In summary there are three generations of leptons, grouped by flavour in charged and neutral: (ν_e, e^-, ν_μ, μ^- and ν_τ, τ^-. Some of the properties of these three generations are given in Table 6.1.

Particle	Mass (MeV)	Q/e	L_e	L_μ	L_τ
ν_e	$\leq 2\times10^{-6}$	0	1	0	0
$\overline{\nu}_e$	$\leq 2\times10^{-6}$	0	-1	0	0
e^-	0.511	-1	1	0	0
e^+	0.511	1	-1	0	0
ν_μ	≤ 0.19	0	0	1	0
$\overline{\nu}_\mu$	≤ 0.19	0	0	-1	0
μ^-	105.66	-1	0	1	0
μ^+	105.66	1	0	-1	0
ν_τ	≤ 18.2	0	0	0	1
$\overline{\nu}_\tau$	≤ 18.2	0	0	0	-1
τ^-	1777	-1	0	0	1
τ^+	1777	1	0	0	-1

Table 6.1: The basic properties of the Standard Model leptons.

6.7 The quarks of the Standard Model

Quarks are the constituents of hadrons, in which they are bound by the strong force. The quarks are grouped in three to form a baryon (e.g. p containing uud) or combined with antiquarks to form mesons (e.g. π^- containing $\overline{u}d$).

Evidence for the composite nature of hadrons was provided in the 60s and 70s. Elastic scattering of electrons from protons indicated that the latter was not point-like but had an approximately exponential distribution of charge with a root mean square value of 0.8 fm. In addition, in the field of baryon and meson spectroscopy, several excited states were revealed. The emerging picture resembled strongly the one associated to the atomic and nuclear physics. A proposal by Gell-Man and Zweig came in 1964 suggested that baryons are formed by the combination of three spin-1/2 constituents called quarks, while mesons contained a combination of a quark and its antiparticle.

When this proposal was put forward, three types of quarks were enough to account for the known (at that time) hadrons. These quarks were: u-quark (up) with a charge of 2/3, d-quark (down) with a charge of -1/3 and the s-quark (strange) with a charge of -1/3. Soon later, and mainly based on arguments related to the quark-lepton symmetry, a fourth quark was proposed, which Glashow, Iliopoulos and Maiani in 1970 estimated to have a mass of about 3-4 GeV. At a later stage (i.e. in 1974) Gaillard and Lee performed more detailed calculations and predicted $m \approx 1.5$ GeV/c^2. The prediction was confirmed in November of the same year with the discovery of the J/Ψ which was soon identified as a compound state of $c\overline{c}$, with c being the fourth quark called charm. The discovery of the third lepton generation hinted for the existence of a relevant third generation of quarks. The discovery of the b-quark (i.e. beauty) came shortly after, in 1977, from the observation of the heavy mesonic states known as Υ, identified as a bound $b\overline{b}$ state. Finally, the three generations of quarks was completed with the discovery of the top quark by the CDF and D0 collaborations.

Particle	Mass (MeV)	Q/e	Strangeness	Charm	Beauty	Topness
u	2	2/3	0	0	0	0
$\overline{u}$	2	-2/3	0	0	0	0
d	5	-1/3	0	0	0	0
$\overline{d}$	5	1/3	0	0	0	0
c	1200	2/3	0	1	0	0
$\overline{c}$	1200	-2/3	0	-1	0	0
s	100	-1/3	-1	0	0	0
$\overline{s}$	100	1/3	1	0	0	0
t	174000	2/3	0	0	0	1
$\overline{t}$	174000	-2/3	0	0	0	-1
b	4200	-1/3	0	0	-1	0
$\overline{b}$	4200	1/3	0	0	1	0

Table 6.2: The basic properties of the quarks of the Standard Model.

6.8 Hadrons: baryons and mesons

After the discovery of the proton, many more baryons were discovered, some of them seemed to behave strangely. These particles were produced in big numbers very fast (i.e. on a time scale of 10^{-23} sec) but they decayed very slowly, typically on a time scale of about 10^{-10} sec. This suggested that their creation process is governed by a different procedure as compared to their decay. The strange particles, that happened to contain also a strange quark, are produced by the strong interaction while they decay weakly. In 1953 Gell-Man and Nishijima suggested to assign a new property, a new quantum number to every particle that Gell-Man called *strangeness*. This new quantum number is conserved in strong interactions, contrary to weak interaction where this number is not preserved.

Soon enough, in 1961, Gell-Man introduced what is know as the *Eightfold Way*. Based on this, the mesons and baryons were arranged, initially, based on their charge and strangeness. An example is given in fig. 6.4, where the members of the meson octet are presented. In both cases the diagonal lines present the members of the group with the same charge.

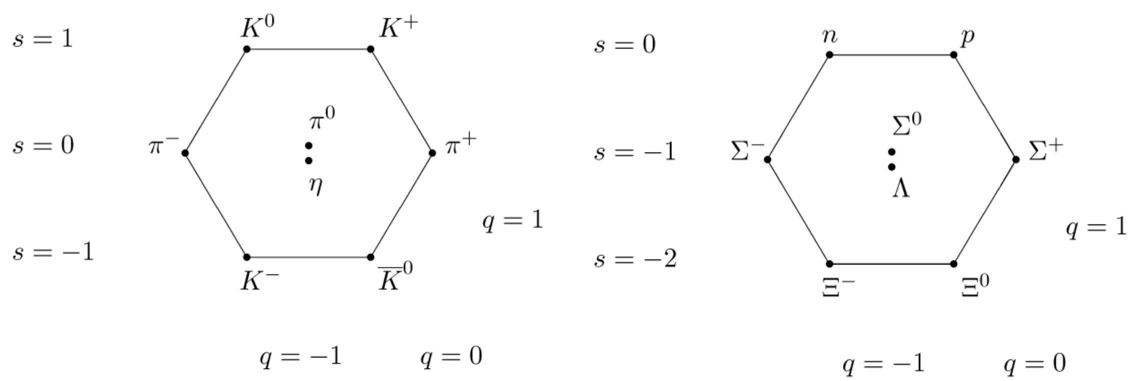


Fig. 6.4: The meson (left) and the baryon (right) octets.

Soon enough this classification illustrated its predicted power. Figure 6.5 presents the members of the baryon decuplet. At the time when this particles were positioned in the triangle, nine out of ten particles were already discovered. The remaining unknown particle had charge -1 and strangeness number -3 and it was expected to be positioned at the lower edge of the shape. Gell-Man went on to predict the existence of this multi-strange particle and suggested how this can be produced. Not long after, around 1964, the Ω^- was discovered.

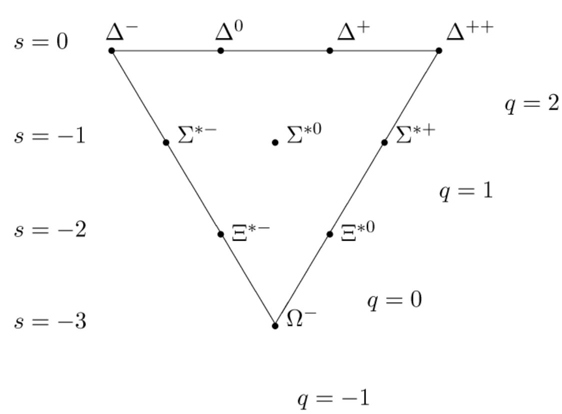


Fig. 6.5: The baryon octet (left) and decuplet (right).

Within the Standard Model we have:

- baryons, formed by a combination of three quarks or three antiquarks,
- mesons, that consist of a combination of a quark with an anti-quark.

Since baryons are formed by an odd combination of half-integer spin particles, they too have half integer spin and are thus fermions. On the other hand, the combination of a quark with an anti-quark leads to a quantum state with an integer spin value i.e. a boson.

A characteristic particle of the baryon family is the proton that consists of two up and one down quarks. The charge of the proton can be deduced from the charge of its constituents:

$$Q_p = 2 \times Q_u + Q_d = 2 \times \frac{2}{3}|e| - \frac{1}{3}|e| = +|e|$$

Similarly, the neutron, the neutral "cousin" of protons, consists of two down and one up quarks. That makes the charge of this particle $Q_n = 2 \times Q_d + Q_u = -2 \times \frac{1}{3}|e| + \frac{2}{3}|e| = 0$.

A characteristic example of the other category of hadrons, the mesons, is the pion that comes with three charges π^+, π^- and π^0. All pions contain different combinations of up and down (anti-quarks).

6.9 The gauge bosons of the Standard Model

The Standard Model of particle physics describes three of the four fundamental forces in nature: the weak, electromagnetic and strong forces; gravity is not part of the Standard Model. To each of these forces, a dedicated theory is developed and a relevant mediator, a particle carrier of the force is associated. The weak force is described by the flavordynamic theory or better by the Glashow-Weinberg-Salam (GWS) theory. It accounts for the nuclear β decay, the decay of the pion, the muon and many particles that contain strange quarks (and not only). The mediators of the weak force are the $W^\pm$ and Z bosons. The electromagnetic interaction is described in terms of Quantum ElectroDynamics or QED. Its classical representation was developed by Maxwell, while it was refined by people like Tomonaga, Feynman and Schwinger in around 1940-1950. The mediator in QED is the photon (γ). Finally, the strong force is mediated by the eight gluons (g) and is described by the theory that emerged last among all, around 1974, the Quantum ChromoDynamics.

6.10 A new quantum number: colour

One of the major achievements of quantum field theory was the proof of the connection between spin and statistics:

- Bosons, described by the Bose-Einstein statistics, have integer spin and have symmetric wave functions i.e. $\psi(1,2) = \psi(2,1)$. In this category fall
  - mediators (elementary particles) such as the photon, the gluons and the carriers of the weak force,
  - composite particles such as mesons, consisting of a combination of a quark and an antiquark
- Fermions, (described by the Fermi-Dirac statistics, have half-integer spin and have anti-symmetric wave functions i.e. $\psi(1,2) = -\psi(2,1)$. In this category fall
  - elementary particles such as quarks and leptons,
  - composite particles such as baryons, consisting of three quarks.

Suppose that we have two particles and one is in state ψ_α and the other in ψ_β. If these two particles are different (e.g. an electron and a muon), then one can discuss about which of the two is in which state. Depending on the answer, we can have $\psi(1,2) = \psi_\alpha(1)\psi_\beta(2)$ or $\psi(1,2) = \psi_\alpha(2)\psi_\beta(1)$. However, if the particles are identical, then if they are bosons then the wave function is a symmetric combination of ψ_α and ψ_β:

$$\psi(1,2) = \frac{1}{\sqrt{2}}[\psi_\alpha(1)\psi_\beta(2) + \psi_\alpha(2)\psi_\beta(1)]$$

On the other hand, if the two particles are identical fermions then the wave function is anti-symmetric:

$$\psi(1,2) = \frac{1}{\sqrt{2}}[\psi_\alpha(1)\psi_\beta(2) - \psi_\alpha(2)\psi_\beta(1)]$$

That implies that if one tries to put two identical fermions in the same state then the final wave function collapses to 0 i.e. 0 probability for this state to exist. This is another manifestation of the Pauli exclusion principle.

The wave functions of particles up to that moment consisted of the part that describe space and time ($\psi(\vec{r},t)$), the part that describe the spin ($\psi(s)$) and the part that describe the flavour ($\psi(flavour)$), such that ($\psi = \psi(\vec{r},t)\cdot\psi(s)\cdot\psi(flavour)$). This representation seems to be problematic if one considers e.g. the *uuu* baryon state (the so-called Δ^{++}). This baryon that consists of three identical fermions, should have an anti-symmetric wave function, however the function at this stage seems to be symmetric. To cure this, a new quantum number is introduced, the one of colour, with three possible values: red (R), green (G) and blue (B). The combination of quarks with different colours is done in such a way so that all hadrons are colourless particles. Thus, the wave function now reads

$$\psi = \psi(\vec{r},t)\cdot\psi(s)\cdot\psi(flavour)\cdot\psi(colour)$$

The quarks interact with each other, with the possibility to change the relevant colour. This is usually done at the quark-gluon vertex as we will see in the next chapters. This interaction is mediated by the gluons that carry the supplementary colour (and anti-colour). There are nine gluon species which are described by the SU(3) color symmetry:

$$3 \otimes \overline{3} = 8 \oplus \overline{1}$$

The octet consists of the following states:

$$|1\rangle = \frac{1}{\sqrt{2}}(R\overline{B} + B\overline{R})$$

$$|2\rangle = \frac{-i}{\sqrt{2}}(R\overline{B} - B\overline{R})$$

$$|3\rangle = \frac{1}{\sqrt{2}}(R\overline{R} - B\overline{B})$$

$$|4\rangle = \frac{1}{\sqrt{2}}(R\overline{G} + G\overline{R})$$

$$|5\rangle = \frac{-i}{\sqrt{2}}(R\overline{G} - G\overline{R})$$

$$|6\rangle = \frac{1}{\sqrt{2}}(B\overline{G} + G\overline{B})$$

$$|7\rangle = \frac{-i}{\sqrt{2}}(B\overline{G} - G\overline{B})$$

$$|8\rangle = \frac{1}{\sqrt{6}}(R\overline{R} + B\overline{B} - 2G\overline{G})$$

The colour singlet is formed by $|9\rangle = \frac{1}{\sqrt{3}}(R\overline{R} + B\overline{B} + G\overline{G})$. This last configuration is not physical. As we will see in a later chapter, confinement requires that all naturally occurring particles are colour singlet states. This explains why the colour octet states can not be seen in nature. But $|9\rangle$ is also a colour singlet, and thus would have been easy to detect it in nature, as a free particle. In addition, being a state of a mediator it could be exchanged by e.g. a proton and a neutron (i.e. colour singlet states) which would imply that the strong force is long range, while we know that it is short range. All these indicate that the colour singlet $|9\rangle$ does not occur in our world.

6.11 Conservation laws

After reviewing all the previous properties of the various particles of the Standard Model (i.e. elementary and composite), it is time to briefly introduce the basic interactions and the corresponding conservation laws. The Standard Model describes three types of interactions:

- Weak interactions, mediated by the massive $W^{\pm}$ and Z^0 bosons. The weak force acts on leptons but also quarks (and thus hadrons) and can change the flavour of quarks. The decay of particles through the weak force takes anything between 10^{-13} sec and several minutes.
- Electromagnetic interactions, mediated by the massless photon (γ). The electromagnetic force acts on leptons and quarks (and thus hadrons). A typical decay lifetime is about 10^{-16} sec.
- Strong interactions, mediated by the eight gluons (g). It is the strongest interaction in the standard model. The strong force acts on gluons and quarks (and thus hadrons) and is responsible for binding composite particles made of quarks and gluons (e.g. protons). A typical decay lifetime is about 10^{-23} sec.

Deciding which interaction is responsible about a given decay of the form $A \to B + C$ among the three available is not straightforward unless we know the lifetime of the decay. However, there are some ”standard candles” that help us decide which force is involved:

- If an interaction involves neutrinos, then it’s the weak force that is responsible.
- If an interaction involves photons, then the responsible force is the electromagnetic.
- If there is any (quark)flavour changing process, then it’s the weak force that is responsible.

We now review the basic conservation laws in the list below.

- **Kinematic constrains**: Conservation of energy, momentum and angular momentum. As an example a particle cannot decay spontaneously into particles heavier than itself.
- **Conservation of electric charge**: All three interactions conserve electric charge.
- **Conservation of colour**: Both the weak and the electromagnetic forces do not feel the colour. It’s only the strong interactions that can affect it and in these interactions colour is always conserved.
- **Conservation of baryon number**: In any interaction the baryon number (i.e. +1 for baryons and -1 for anti-baryons) is conserved.
- **Conservation of lepton number**: Leptons do not feel the strong force so the lepton number is conserved by construction. In both the weak and the electromagnetic interactions, the individual lepton numbers i.e. electron, muon and tau lepton numbers are conserved.

Chapter 7
Physics beyond the Standard Model

7.1 The shortcomings of the Standard Model

Up to this point we have presented the Standard Model as if it was a fully complete theory of the fundamental interactions between elementary particles. And though certainly the Standard Model is extremely successful in describing a wide array of different phenomena with astonishing precision (one could argue that the SM is the most successful physical theory ever constructed) there are a number of important questions that are left unanswered. In this lecture we discuss the drawbacks and limitations of the Standard Model, emphasizing that we still don't know how to address these limitations with an even better (and deeper) theory. What are some of these open questions?

- Why do we have three copies (families/generations) of quark and leptons, identical in all their properties except to their masses?
- Is there an underlying principle that determines the mass patterns of quarks and leptons? Why the difference between the lightest quark (up) and heaviest (top) quark is as large as a factor of $m_t/m_u \approx 10^6$?
- Are quarks and leptons really elementary particles?
- Why is the Universe made only of matter and how is this asymmetry created?
- Neutrinos are massless in the Standard Model, but recent measurements have demonstrated that they have a small yet finite mass, $m_\nu \neq 0$. This opens a number of interesting questions, such as if the Higgs mechanism also gives mass to neutrinos, or there is some altogether different mechanism. In the following we discuss these open questions in turn, and highlight briefly some of the possible avenues to overcome this limitations and develop and improved theory of the fundamental forces and interactions.
- The Universe contains five times more dark matter (matter that does not interact electromagnetically) than normal visible matter. The Standard Model does not contain any suitable candidate to act as this dark matter.
- The Universe is undergoing a period of accelerated expansion driven by some form of dark energy. Again, in the Standard Model there is no mechanism that explains this dark energy.
- Gravity is not part of the Standard Model. How does it fit into all this?

In what follows, we will briefly touch upon some of these topics and highlight potential avenues that can be followed.

7.2 Unification of forces

In previous chapters we have discussed the properties of the three fundamental interactions that are relevant to describe the world of elementary particles: electromagnetism, the weak and the strong forces. An obvious question is, why these three forces, and not say two or ten? Perhaps there is a unique underlying interaction, of which these three forces (and perhaps even also gravity) are different manifestations? The idea has certain appeal if we recall that the history of physics has many instances of such unification of seemingly unrelated phenomena, such as celestial and earth mechanics with Newton and electricity and magnetism with Maxwell. Actually, we already know that electromagnetism and the weak interactions are actually two aspects of the same underlying phenomenon, the electroweak interaction. The electromagnetic and weak interactions appear very differently at low energies because the masses of their corresponding force carries (the photon and the W, Z bosons) are very different, and thus the range of these interactions is also very different (very long and very short range interactions, respectively). But if we go to very high energies, in the sense that $E \gg m_Z$, then we observe the electromagnetic and weak interactions unify into a single electroweak interaction. So actually the Standard Model contains only two fundamental interactions: the strong and the electroweak forces.

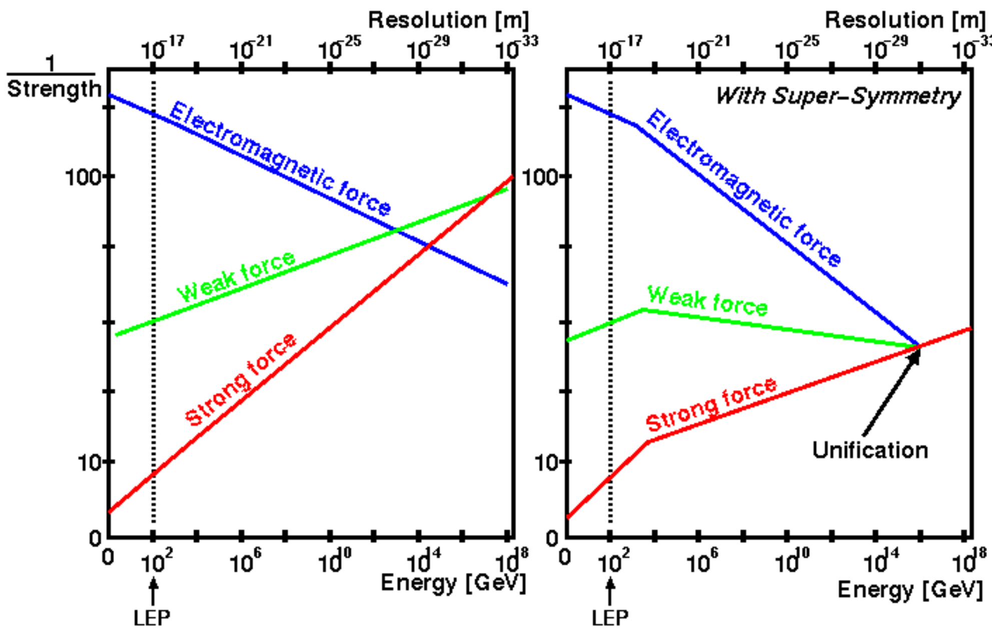


Fig. 7.1: The inverse of the strength of each interaction of the Standard Model as a function of energy. The three forces could be unified in one at higher energies within theories that extend the Standard Model.

Gravity, electricity, and magnetism were known since ancient times, but only around the 17th century a mathematical framework to describe these interactions that was able to describe phenomena was developed. Electricity and magnetism where unified into the theory of electromagnetism at the end of the 19th century following the work of Maxwell and several others. In the beginning of the 20th century, the strong and weak nuclear forces were discovered. The combination of quantum mechanics and special relativity lead to the formulation of Quantum Chromodynamics and of the Electroweak Theory for the strong, weak, and electromagnetic interactions respectively. The strong and electroweak interactions constitute the Standard Model of particle physics, which is as of today, as discussed in the previous section, our best theory for the world of elementary particles.

This trend towards unification might suggest that by going to even higher energies the strong and electroweak interactions could unify into a single fundamental force. This seems difficult, since the relative strengths of these two interactions are rather different, so how can they arise from the same underlying interaction? Here a crucial consequence of quantum mechanics is that the strength of a specific interaction is not fixed, but actually depends on the characteristic energy scale of the interaction. For instance, in the specific case of the strong interaction, its coupling constant decreases as we increase the energy, while for the electroweak force, their couplings increase at higher energies.

To further illustrate this behaviour, in fig. 7.1 we show the inverse of the characteristic strength of the electromagnetic, weak and strong forces as a function of either the energy (in GeV) or the spatial resolution (in meters) of the process.

We show the results corresponding both to the Standard Model (left plot) and to the Standard Model extended to include low-scale supersymmetry (right plot), which will be discussed below. In the Standard Model, we find that the three couplings become similar (but not identical) at energies of around $E \approx 10^{14}$ GeV, corresponding to distances of the order of $r \approx 10^{-29}$ m. In the case of the SM extended with supersymmetry, the unification of the coupling constants of the fundamental interactions is much better, and takes place at an energy scale of $E \approx 10^{15}$ GeV, which is the so-called Grand Unification Scale.

So in principle it is possible that under specific conditions, as shown in fig. 7.1, the electromagnetic, weak, and strong interactions unify into a unique force, with a common value of the coupling constant. Note that however in this picture gravity is still excluded. On the other hand, so far we do not have experimental evidence of this unification, which might require probing energies much higher than those that are currently within our reach. All tests performed so far looking for a Grand Unified Theory have failed, such as the searches for the decays of the proton, which is predicted by such theories.

7.3 Dark matter

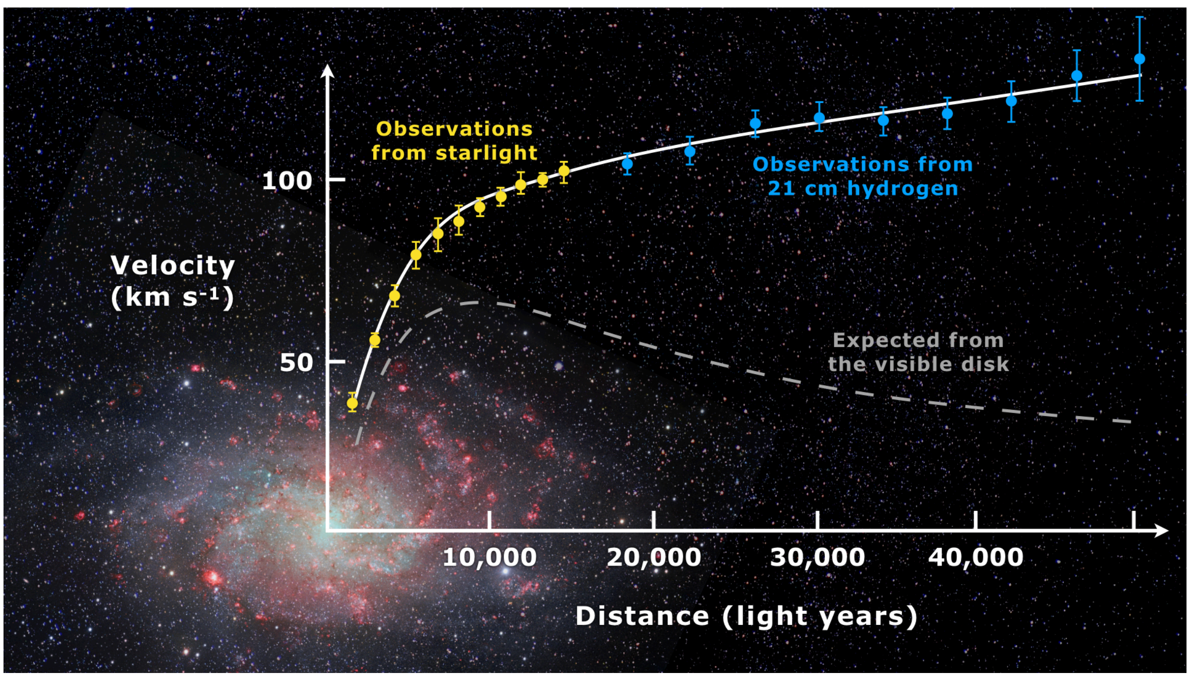


Fig. 7.2: Schematic representation of the galactic rotation curves, namely the dependence of the velocity v of its stars as a function of the distance R with respect to the galactic center. The dotted curve is the expectation from the visible mass of the galaxy, while the solid line is the best fit to the experimental data.

From a variety of astronomical and cosmological measurements we know that the visible mass of the universe (for example, the mass of visible stars) is only a small fraction of the total mass. We call Dark Matter this additional source of matter, which does not interact via electromagnetism (hence it is dark) and thus that we cannot see directly, but can only infer indirectly via its gravitational effects

As an example of this, in fig. 7.2 we show a schematic representation of the galactic rotation curves, namely the dependence of the velocity v(R) of its stars as a function of the distance R with respect to the galactic centre. The dotted curve is the expectation from the visible mass of the galaxy, while the solid line is the best fit to the experimental data. Clearly the observed velocity profile is very different as compared to the expectations based on visible matter, suggesting the presence of additional, dark, sources of matter in the galaxy, which increase the rotational velocity v(R) for large R.

Could this mysterious Dark Matter be composed by one or more of the Standard Model particles? Let us see if in the SM we have any particle that satisfies the properties of a suitable DM candidate:

- DM is electrically neutral, since it does not emit light. Therefore, a candidate for DM should have $Q = 0$. This eliminates all the quarks as well as the charged leptons, $e^{\pm}$, $\mu^{\pm}$ and $\tau^{\pm}$, and the charged weak bosons $W^{\pm}$.
- DM candidates should have a non-zero mass $m_{\rm DM} \neq 0$. This further eliminates the photon γ and the gluon g, both of which are massless.
- A suitable DM candidate should be stable on cosmological scales in order to seed galactic structure. This eliminates the Z^0 and h bosons, both of which have very short lifetimes.

The only SM particles that satisfies these three conditions (being electrically neutral, massive, and stable on cosmological time-scales) are the neutrinos ν. However, due to a number of reasons which we don't have the time to discuss here, the neutrinos cannot be the explanation for Dark Matter (at least for the greatest part of it). So the nature of DM is still basically unknown to us.

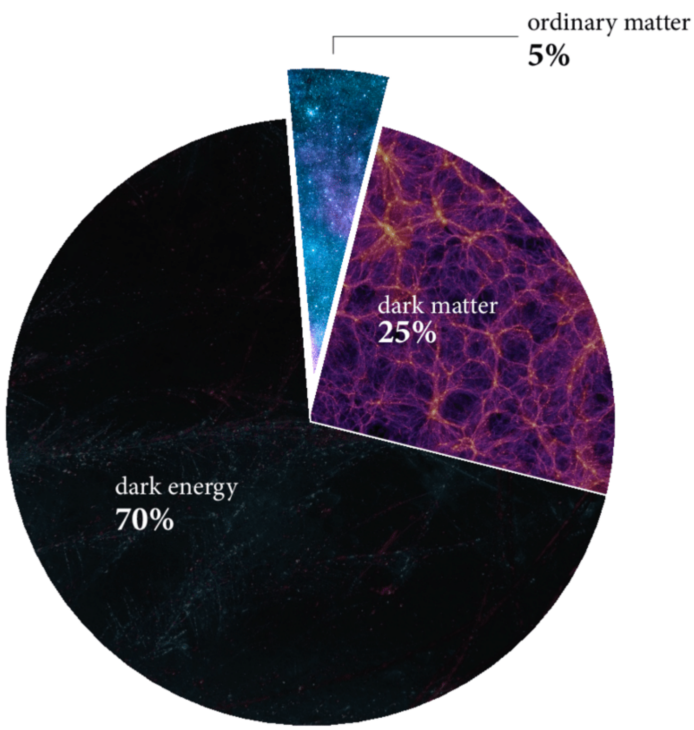


Fig. 7.3: Schematic schematic representation of our current understanding of the matter/energy content of the Universe. The known particles of the Standard Model add up to around 5% of this matter/energy content. There are around 5 times more dark matter than normal matter. Then dark energy, driving cosmic acceleration, makes up most of the energy content of the universe.

In fig. 7.3 we show a schematic representation of our current understanding of the matter/energy content of the Universe. The known particles of the Standard Model add up to around 5% of this matter/energy content. There is around 5 times more dark matter than normal matter: in other words, for each galaxy that we see in the Universe, there exists the equivalent of five galaxies composed of Dark Matter. The nature and properties of this mysterious dark matter, which certainly cannot be explained by the Standard Model of elementary particles, is now being actively searched for. Dark Matter for example could be produced at high-energy colliders such as the LHC. Direct searches for DM particles are being pursued in big underground experiences, shielded from all contamination from normal matter. Finally, hints for DM annihilation are also searched for in astrophysical measurements.

From the same figure we also see that the so-called Dark Energy constitutes the majority of the matter/energy content of the universe. The nature of this Dark Energy is also unknown: we can only infer its presence via its indirect effects, first and foremost driving the present phase of cosmic acceleration of the Universe. Information about dark matter and dark energy can be obtained from astronomical measurements such as those of the Cosmic Microwave Background, a kind of after glow of the Big Bang, that provides a snapshot of the very early Universe.

7.4 Gravity

As shown in fig. 7.1, while we now have a unified description of all known elementary particles and of their interactions by means of the strong, weak, and electromagnetic forces within the highly successful framework of the Standard Model, we still don't really know how to fit gravity in the picture. The main limitation is that we still have not managed to construct a quantum theory of gravity, or to be more precise, a quantum version of Einstein's theory of General Relativity.

The effects from gravitational interactions are really tiny in the world of elementary particles. This has one important corollary: testing a theory of quantum gravity is difficult since experimental measurements sensitive to quantum effects of the gravitational force are, for the time being, very far from our reach. This statement can be quantified as follows. The energy scale at which the quantum effects of gravity will become important is given by Planck's energy, a fundamental constant of Nature that is determined by Planck's constant h, Newton's gravitational constant c and the speed of light c as follows:

$$E_P = \sqrt{\frac{\hbar c^5}{G}} \approx 10^{19}\ \mathrm{GeV}$$

If we recall that the highest energies that we can achieve at man-made particle colliders is $E_{\mathrm{LHC}} \approx 10^4$ GeV, we see that we are still 15 orders of magnitude in energy below Planck's energy. Therefore, we are still a big way behind before we will be able to experimentally study quantum effects of gravity.

Planck's energy E_P has also distance and mass counterparts, which again reflect the typical distances and mass scales for which the effects of quantum gravity becomes relevant. In the case of distance scales, Planck's length L_P is given by

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.61 \times 10^{-35}\ \mathrm{m}$$

again much smaller than any distance that we can probe now or in the foreseeable future. Recall that the proton radius us about $R_p \approx 10^{-15}$ m, and therefore Planck's length is around 10^{-20} times smaller than the proton radius. With present and near-future particle colliders, we can at most probe distances as small as 10^{-20} meters, much larger than Planck's length, again preventing us from testing for the time being the quantum effects of gravity.

Despite the challenges is experimentally testing quantum gravity, this has not prevented theorists to developing new frameworks to unify the Standard Model with a quantum theory of gravity. There are a numbers of proposals that are being actively researched, such as string theory, loop quantum gravity, asymptotically free gravity, causal sets dynamical triangulation, and may others. In string theory for instance, elementary particles are replaced by finite-size one-dimensional strings, which avoids some of the problems affecting the quantum version of gravity. The different properties of each type of string then determine particle properties such as its mass and its spin. Beyond its potential to represent a quantum theory of gravity, string theory has found applications to many other fields from pure mathematics to condensed matter systems and heavy ion collisions.

7.5 Summary

At this point we have reached the end of our journey through the world of elementary particles. Let us know step back and recap what we have learned during these four weeks traveling around the particle zoo:

- A wide variety of extremely different phenomena can be explained in a rather economical way by just a handful of elementary particles and interactions: the Standard Model of particle physics.
- In addition to the well-known gravitational and electromagnetic forces, we have seen that two other interactions are present in the world of elementary particles: the strong nuclear force and the weak nuclear force.Their effects are visible in our everyday life: for instance, the Sun emits light thanks to nuclear reactions mediated by the weak force in its interior, and the fact that the protons inside atomic nuclei do not repeal themselves and that matter is thus stable is a direct consequence of the properties of the strong force.
- Even without entering into the details of how each of the three fundamental interactions work, we have seen that there exists a number of symmetries and conservation principles that determine very tightly whether or not a given particle reaction is physically allowed.
- The world of elementary particles is much richer than what normal matter, composed by only protons, neutrons, and electrons, would seem to imply. These additional particles have crucial implications even if their existence is fleeting. For instance, the Higgs boson is responsible to give mass to all elementary particles, and the fact that there are three generations of quark and leptons might be crucial to explain the asymmetry between matter and antimatter in the Universe, and so on.

Despite all the successes of the Standard Model, we have also seen in this last part of the course that this theory is ultimately incomplete, since it leaves open a number of crucial questions such as the nature of dark matter and dark energy, the origin of its flavour structure, the possible unification of the three fundamental forces among them and with gravity, and the explanation for the observed origin of the matter-antimatter asymmetry in the Universe. With this motivation, particle physicists are currently working very hard, both from the theoretical and experimental points of view, to unveil the properties of a better, more fundamental, theory of elementary particles beyond the Standard Model.

To conclude, let me emphasize that the field of elementary particles is a extremely alive and active area of research. There have been a number of breakthrough discoveries in the last 25 years, and only the future can tell which further discoveries lie ahead. Specifically, in the last 25 years the high-energy physics community has:

- Discovered the top quark, the heaviest of all known elementary particles.
- Discovered the Higgs boson, responsible for giving mass to all elementary particles and to the weak force carries $W^{\pm}$ and Z^0, which completes the Standard Model.
- Found evidence for neutrino oscillations, indicating that neutrinos have mass, that is, that neutrinos can change its flavour. Neutrino masses are one possible gateway to new physics beyond the SM, and imply that the individual leptonic quantum numbers are not conserved at large distances.
- Mapped the properties of Dark Matter to an unprecedented level of precision from a variety of experiments, and excluded a wide range of models for DM candidates.
- Identified the accelerated expansion of the universe, driven by the mysterious Dark Energy for which we do not have currently any explanation.
- Discovered gravitational waves, opening a new window to the universe, mapping extreme astrophysical events such as black hole and neutron star mergers.

Chapter 8

Experimental techniques and instrumentation

Particle physics experiments are designed to detect and identify particles produced in high-energy collisions. Of these particles, only the electrons, protons, photons and the notoriously hard to detect neutrinos are stable. Unstable particles travel a certain distance of the order of $\gamma v \tau$ before decaying, where here τ is the mean lifetime in the rest frame of the particle and $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor that accounts for the relativistic time dilation.

Relativistic particles with a lifetime greater than around 10^{-10} s will propagate over several meters when produced in high-energy particle collisions and thus can be detected directly. The long-lived particles include the muon ($\mu^{\pm}$), the charged pions ($\pi^{\pm}$) and charged kaons ($K^{\pm}$). Short-lived particles with lifetimes less than about 10^{-10} s will typically decay before they travel a significant distance from the point of production and only their decay products can be thus detected.

All these categories of particles, both stable and unstable, form the environment inside of which modern particle physics experiments operate. The goal of these experiments is to reconstruct and identify these particles. The techniques used to both detect and identify them differs and depends on the nature of their interaction with matter. Generally speaking, particle interactions can be divided into three categories:

- the interaction with charged particles,
- the electromagnetic interactions of electrons and photons,
- the strong interactions of charged and neutral hadrons.

8.1 Interaction of charged particles with matter

When a relativistic charged particle passes through a medium it interacts electromagnetically with the atomic electrons and loses energy through ionisation of the atoms. For a singly charged particle with velocity $v = \beta c$ traversing a medium with atomic number Z and number density n, the ionisation energy loss per unit length traversed is given by the Bethe-Bloch equation, according to

$$\frac{dE}{dx} \approx -4\pi\hbar^2 c^2 \alpha^2 \frac{nZ}{m_e v^2}\left[ln\left(\frac{2\beta^2\gamma^2 c^2 m_e}{I_e}\right) - \beta^2\right] \tag{8.1.1}$$

In this equation I_e is the effective ionisation potential of the material averaged over all atomic electrons, which is very approximately given by $I_e \approx 10Z$ eV. For a particular medium, the rate of the ionisation energy loss of a charged particle is a function of its velocity. Owing to its $1/v^2$ term in the Bethe-Bloch equation, dE/dx is greatest for low-velocity particles. Modern particle physics is mainly interested though in highly relativistic particles with $v \approx c$. In this case, for a given medium, dE/dx depends logarithmically on $(\beta\gamma)^2$ where

$$\beta\gamma = \frac{v/c}{\sqrt{1-(v/c)^2}} = \frac{p}{mc},$$

resulting into a slow relativistic rise of the rate of ionisation energy loss. The evolution of dE/dx as a function of $(\beta\gamma)$ is given in fig. 8.1. This also gives the stopping power. As can be seen from fig. 8.1, the $-\langle dE/dx\rangle$ defined in this way is about the same for most materials, decreasing slowly with Z. The rate of ionisation energy loss does not depend significantly on the material except through its density. This can be seen by expressing the number density of atoms as $n = \rho/(Am_u)$, where A is the atomic number and $m_u = 1.66 \times 10^{-27}$ kg is the unified atomic mass unit. The equation 8.1.1 can then be written as

$$\frac{1}{\rho}\frac{dE}{dx} \approx -\frac{4\pi\hbar^2 c^2\alpha^2}{m_e v^2 m_u}\frac{Z}{A}\Big[ln\Big(\frac{2\beta^2\gamma^2c^2m_e}{I_e}\Big) - \beta^2\Big], \tag{8.1.2}$$

where the proportionality of the energy loss to Z/A becomes evident. Since nuclei contain approximately similar number of protons and neutrons, the ratio Z/A is roughly constant and thus the rate of energy loss by ionisation is proportional to density and does not depend strongly on the material. This can be better seen in fig. 8.1 which shows the ionisation energy loss as a function of $\beta\gamma$ for a singly charged particle in various materials. It is seen that particles with $\beta\gamma \approx 3$ which corresponds to the minimum ionisation energy loss curve are referred to as minimum ionising particles.

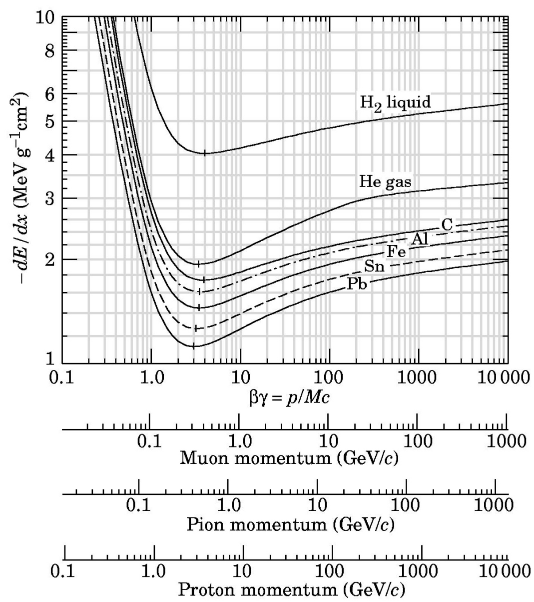


Fig. 8.1: Mean energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminium, iron, tin, and lead. Radiative effects, relevant for muons and pions, are not included. These become significant for muons in iron for $\beta\gamma > 1000$ and at lower momenta for muons in higher-Z absorbers

All charged particles lose energy through ionisation of the medium in which the are propagating. Depending on the particle type, other energy-loss mechanisms maybe present. For muons with energies below 100 GeV, ionisation is the dominant energy loss as can be seen in fig. 8.2. That means that a muon travels significant distances even in dense materials like iron. As an example, a muon with energy of around 10 GeV loses approximately 13 MeV cm^{-1} in iron and thus will be able to travel for several meters. As a result, the muons produced in collisions of particles in modern particle accelerators are very penetrating particles, they usually traverse the entire detector and leave a trail of ionisation. This is the feature that can be exploited to identify muons experimentally.

8.1.1 Tracking detectors

The detection and measurement of momenta of charged particles is an essential aspect of every large particle physics experiment. Regardless of the medium, through which a charged particle travels, it leaves a trail of ionised atoms and

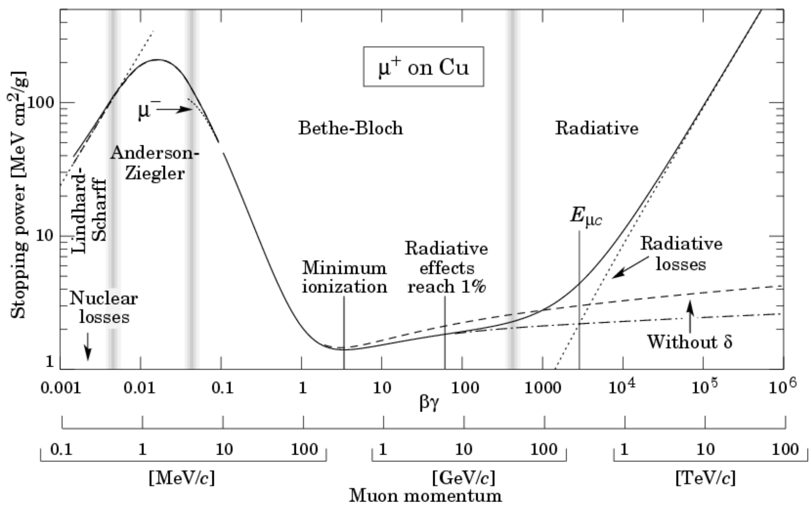


Fig. 8.2: Mass stopping power ($\approx -\langle dE/dx \rangle$) for positive muons in copper as a function of $\beta\gamma = p/Mc$ over nine orders of magnitude in momentum (12 orders of magnitude in kinetic energy).

liberated electrons. By detecting this ionisation it is possible to reconstruct the trajectory of a charged particle. There are two categories of tracking detectors used in modern experiments. Charged particles can be detected and their trajectories can be reconstructed in

- large gaseous detectors such as the Time Projection Chamber (TPC), where the liberated electrons are drifted in strong electric fields towards the sense wires where a signal is detected,
- detectors using semiconductor technology such as silicon pixels or strips.

8.1.1.1 Gaseous detectors

A detector that exploits this feature discussed above is the Time Projection Chamber (TPC). The TPC has been introduced in 1976 by D.R. Nygren. A TPC consists of a gas-filled detection volume in an electric field with a position-sensitive electron collection system. The design most commonly used is a cylindrical chamber with multi-wire proportional chambers (MWPC) as endplates. Along its length, the chamber is divided into halves by means of a central high-voltage electrode disc, which establishes an electric field between the centre and the end plates. Furthermore, a magnetic field is often applied along the length of the cylinder, parallel to the electric field, in order to minimise the diffusion of the electrons coming from the ionisation of the gas. On passing through the detector gas, a particle will produce primary ionisation along its track. The z coordinate (along the cylinder axis) is determined by measuring the drift time from the ionisation event to the MWPC at the end. This is done using the usual technique of a drift chamber. The MWPC at the end is arranged with the anode wires in the azimuthal direction, θ, which provides information on the radial coordinate, r. To obtain the azimuthal direction, each cathode plane is divided into strips along the radial direction.

The working principle of the TPC is sketched in fig. 8.3. A charged particle traversing the gas volume of the TPC will ionises the atoms of the gas mixture (usually around 90% noble gas and 10% quencher gas) along its trajectory. A high electric field is applied between the endplates of the chamber. The released electrons drift in this field towards the anode. To be able to measure the position of the particle trajectory as accurately as possible, the electric field has to be very homogeneous. This can be achieved by a field cage, which usually consists of conducting rings around the cylinder. These rings divide the potential from the cathode stepwise down to the anode. Additionally, a high magnetic field parallel to the electric field is used to "bend" the trajectory of the particle on a spiral track due to the Lorentz force. This gives the possibility to calculate the momentum of the particle from the knowledge of the curvature and the B-field.

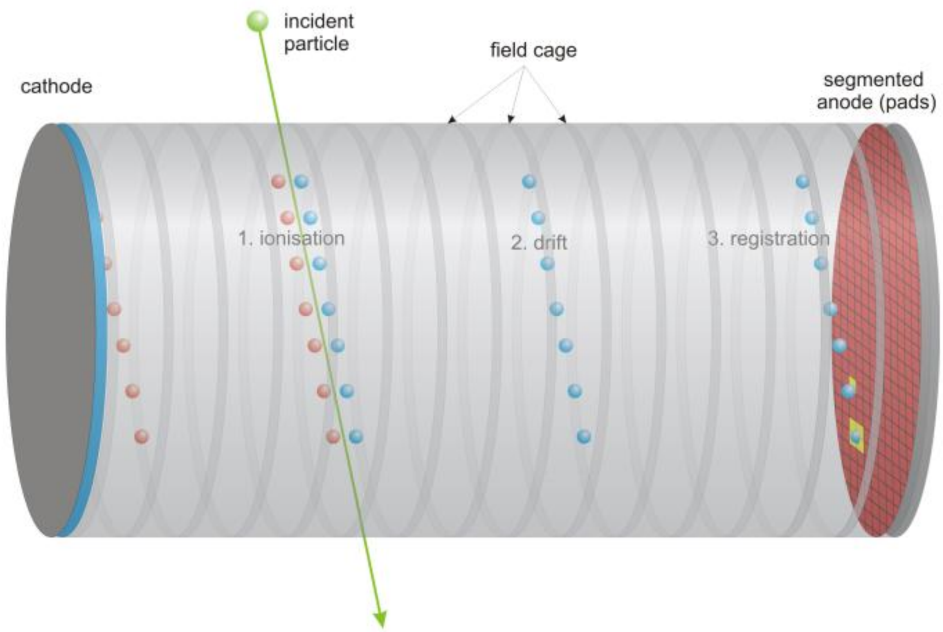


Fig. 8.3: The working principle of a Time Projection Chamber.

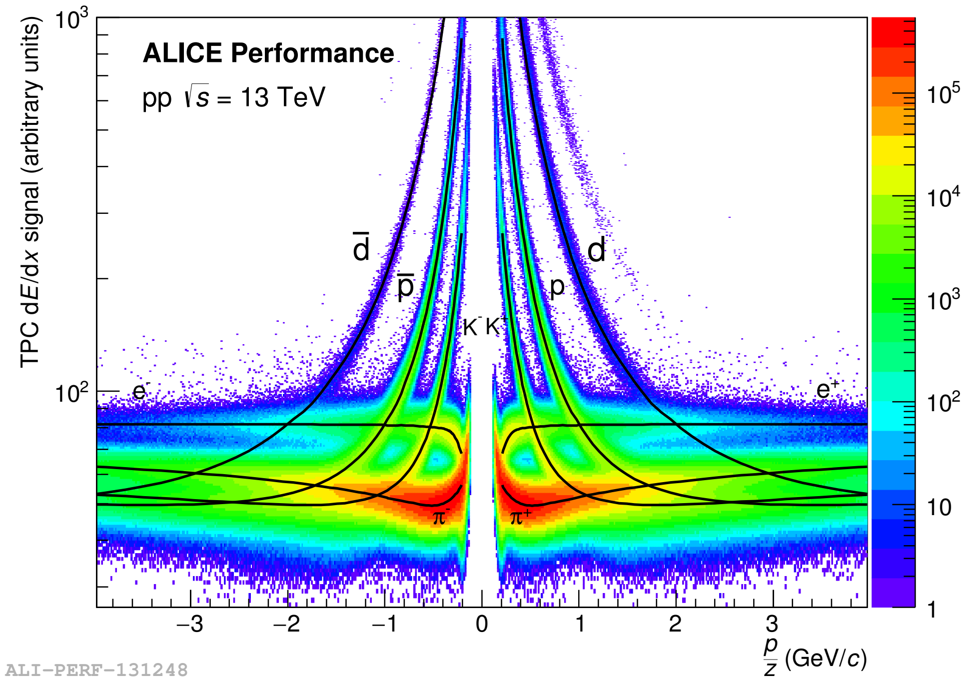


Fig. 8.4: The ionisation energy loss measured in the Time Projection Chamber of ALICE at the Large Hadron Collider (LHC) as a function of the rigidity of particles.

Figure 8.4 shows the ionisation energy loss measured by the biggest TPC ever built and functioned in a high energy experiment, the ALICE (A Large Ion Collider Experiment) at the Large Hadron Collider (LHC) at CERN. The value of dE/dx is measured as a function of the rigidity, defined by the momentum of each particle over the atomic number. One can clearly see the different bands corresponding to different particle species that interact with the gas of the TPC and are reconstructed and at a later stage identified by the TPC.

8.1.1.2 Solid state detectors

When a charged particle traverses an appropriately doped silicon wafer, electron-hole pairs are created by the ionisation process. If a potential difference is applied across the silicon, the holes will drift in the direction of the electric field where they can be collected by the p–n junctions. The sensors can be shaped into silicon strips, typically separated by $\approx 25\ \mu$m or into silicon pixels giving a precise two–dimensional space point.

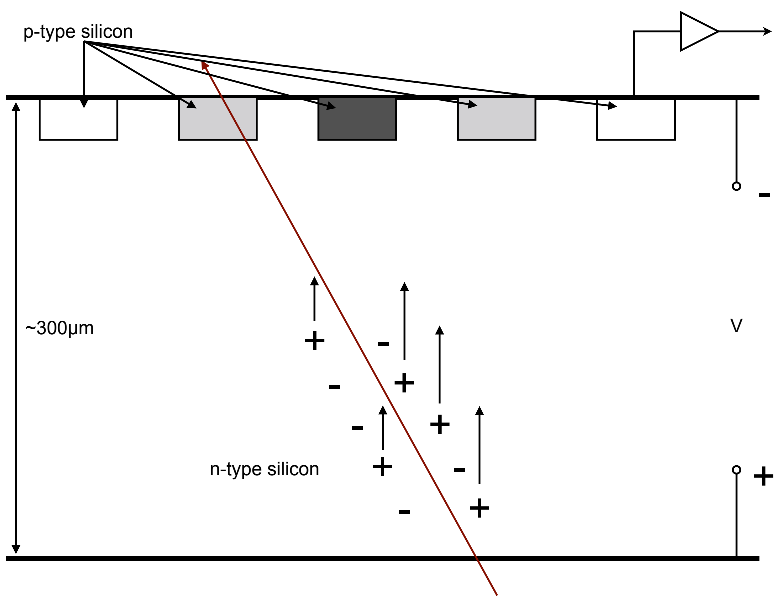


Fig. 8.5: The working principle of a silicon detector.

Silicon tracking detectors typically consist of several layers, cylindrical surfaces of silicon wafers. A schematic view of such configuration is given in fig. 8.6. A charged particle will leave a hit on a silicon sensor in each of the cylindrical layers from where the trajectory of the charged particle track can be reconstructed. The tracking system is usually placed inside a large solenoid producing approximately a uniform magnetic field in the direction of the colliding beams, which is taken to be the z–axis in fig. 8.6. Due to the Lorentz force the trajectory of the particle in the axial magnetic field is a helix with a radius of curvature R and a pitch angle λ. These two variables are connected with the particle's momentum according to

$$p\cos\lambda = 0.3BR,$$

where the momentum p is given in GeV/c, B is the magnetic flux density in T and R is given in meters. By determining the curvature and the pitch angle through the measurement of the helical trajectory, the momentum of the particle can be estimated. For high momentum particles the radius of the curvature can be large. As an example the radius of the curvature of a 100 GeV pion in a 4 T magnetic field as the one used by the Compact Muon Solenoid (CMS) detector at CERN is

$R \approx 100$ m. Even if such particle tracks seem quite straight this small deflection is efficiently detected and measured using precise space points from silicon strip detectors.

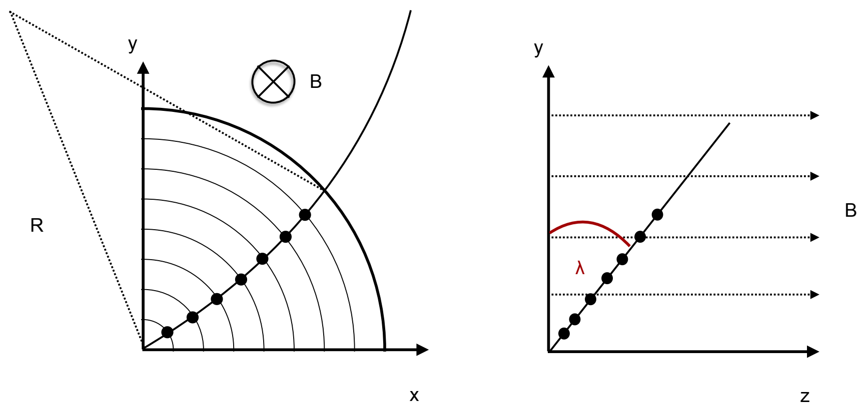


Fig. 8.6: Charged particle track reconstruction from the space points observed in a seven-layer tracking detector. The curvature in the x–y plane (left plot) determines the transverse momentum of the track.

8.2 Interactions and detection of electrons and photons

At low energies the energy loss of electrons is dominated by ionisation. However, for energies above a critical energy E_{cr}, the main energy loss mechanism is bremsstrahlung where the electron radiates a photon in the presence of the electrostatic field of a nucleus. The critical energy is related to the charge Z of the nucleus and is given by

$$E_{cr} \approx \frac{800}{Z} \text{ MeV}$$

The electrons of interest in the majority of the modern particle physics experiments have an energy at the multi–GeV scale, significantly larger than the critical energy. They therefore interact with matter primarily through bremsstrahlung. The bremsstrahlung effect can occur for all charged particles but the rate is inversely proportional to the square of the mass of the particle. Hence for muons the rate of energy loss through bremsstrahlung is suppressed compared to electrons by a factor of $(m_e/m_\mu)^2$. This explains why bremsstrahlung is the dominant energy loss mechanism for electrons while for muons ionisation energy loss is still the main energy loss effect (at least for $E_\mu < 100$ GeV).

Photons, at low energies, interact with matter primarily via the photoelectric effect. In this process a photon is absorbed by an atomic electron that is ejected from the atom. At higher energies, $E_\gamma \approx 1$ MeV, the Compton scattering process $\gamma e^- \to \gamma e^-$ becomes significant. For even higher energies, for $E_\gamma > 10$ MeV, the interactions of photons are dominated by electron–positron pair production in the presence of the field of a nucleus.

The electromagnetic interactions of high energy electrons and photons in matter is characterised by the radiation length X_0. The radiation length is the average distance over which the energy of an electron is reduced by bremsstrahlung by a factor of $1/e$. It is approximately $7/9$ of the mean free path of the e^+e^- pair production for a high–energy photon. The radiation length is related to the atomic number Z of the material and can be approximated by the expression

$$X_0 = \frac{1}{4\alpha n Z^2 r_e^2 ln(287/Z^{1/2})},$$

where n is the number density of nuclei and r_e is the classical radius of the electron, given by

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = 2.8 \times 10^{-15}\ \mathrm{m}$$

For high–Z materials the radiation length is relatively short. As an example $X_0(\mathrm{Fe}) = 1.76$ cm and $X_0(\mathrm{Pb}) = 0.56$ cm.

8.2.1 Electromagnetic showers

When a high–energy electron interacts with mater in a medium it radiates a bremsstrahlung photon which in turns produces an electron–positron pair. The process of bremsstrahlung and pair production continues to produce a cascade of photons, electrons and positrons. This whole process is referred to as electromagnetic shower and an example is given schematically in fig. 8.7. Similarly a high–energy photon will create an electron–positron pair that will in turns produce an electromagnetic shower.

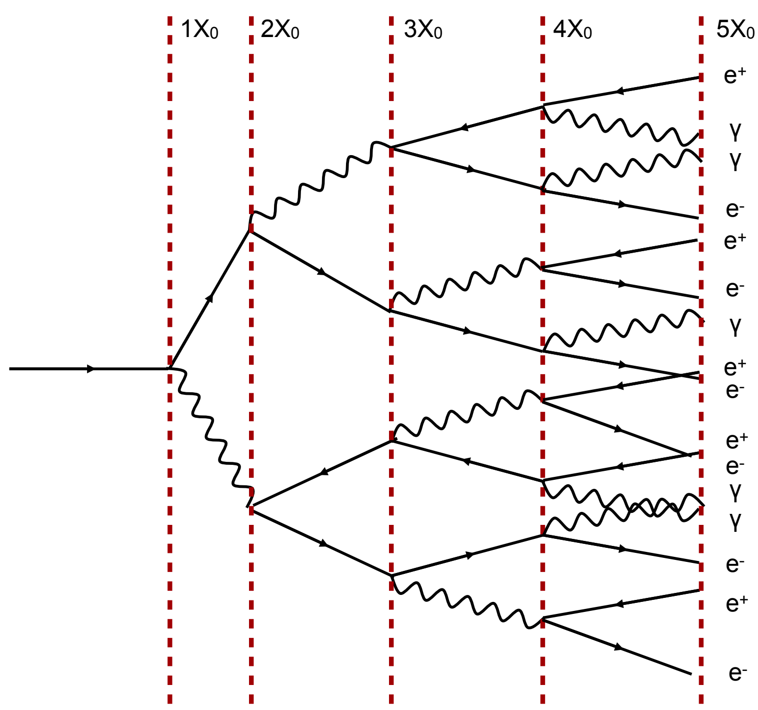


Fig. 8.7: Identifying particles in pp collisions at the LHC using a ring-imaging Cherenkov detector.

The number of particles in an electromagnetic shower approximately doubles after every radiation length of material traversed. Hence in an electromagnetic shower produced by an electron or photon of energy E, the average energy of particles after x radiation lengths is given by

$$\langle E \rangle = \frac{E}{2^x}$$

The shower continues to develop until the average energy of the particles falls below the critical energy E_{cr} at which point the electrons and positrons in the cascade loose energy primarily through ionisation. The electromagnetic shower has thus the maximum number of particles after X_{max} radiation lengths given by the condition $\langle E \rangle = E_{cr}$. This point is reached after

$$X_{max} = \frac{ln(E/E_{cr})}{ln2}$$

In a high–Z material such as lead with $E_{cr} \approx 10$ MeV, a 100 GeV electromagnetic shower reaches its maximum at $\approx 13X_0$. This corresponds to about 10 cm of lead. Consequently electromagnetic showers deposit most of their energy in a relatively small region of space.

8.2.2 Electromagnetic calorimeters

In high–energy particle physics experiments the energies of electrons and photons are measured using electromagnetic calorimeters constructed from high–Z material. A popular choice is the usage of lead tungstate ($PbWO_4$) crystals which is an inorganic scintillator. These crystals are both optically transparent and have a short radiation length of $X_0 = 0.83$ cm, allowing the electromagnetic shower to be contained in a compact region. The electrons in the electromagnetic shower produce scintillation light that can be collected and amplified by efficient photon detectors. The amount of scintillation light produced is proportional to the total energy of the original electron or photon. Alternatively, electromagnetic calorimeters can be constructed by alternating layers of high–Z material, such as lead, with an active layer in which the ionisation from the electrons in the electromagnetic shower can be measured. For the electromagnetic calorimeters in large, modern particle physics detectors, the energy resolution for electrons and photons is typically in the range

$$\frac{\sigma_E}{E} \approx \frac{3\% - 10\%}{\sqrt{E/\mathrm{GeV}}}$$

8.3 Interactions and detection of hadrons

Charged hadrons lose energy continuously through ionisation when they interact with matter. In addition, both charged and neutral hadrons can undergo strong interactions with the nuclei of the medium. Particle produced in this primary hadronic interaction will subsequently interact further downstream in the medium, giving rise to a cascade of particles of hadronic, this time, nature. The development of hadronic showers is characterised by the nuclear interaction length, λ_f, defined as the mean distance between hadronic interactions of relativistic hadrons. The nuclear interaction length is significantly larger than the radiation length. As an example, the interaction length for Fe is $\lambda_f(\mathrm{Fe}) \approx 17$ cm, compared to its radiation length of 1.8 cm.

Unlike electromagnetic showers which develop in a uniform manner, hadronic showers are more variable because of the many different final stets that can be produced in high–energy hadronic interactions. In addition, any π^0 produced in the hadronic shower decays almost instantaneously through the channel $\pi \to \gamma\gamma$, leading to an electromagnetic component of the shower. The fraction of the energy in this electromagnetic component depends on the number of π^0s produced and varies from shower to shower. Finally, not all of the energy in a hadronic shower is detectable. On average, around 30% of the incident energy is effectively lost in the form of nuclear excitation and break–up.

8.3.1 Hadronic calorimeters

In particle detector systems the energies of such hadronic showers are measured in a hadron calorimeter. Because of the relatively large distance between nuclear interactions, hadronic showers will occupy a significant volume in any detector.

For example, in a typical hadron calorimeter, the shower of a 100 GeV hadron has longitudinal and lateral extents of the order of 2 m and 0.5 m, respectively. A number of different technologies have been used to construct hadron calorimeters. A commonly used technique is to use a sandwich structure of thick layers of high–density absorber material, where the shower develops, and thin layers of active material where the energy deposition from the charged particle in the shower are sampled. Fluctuations in the electromagnetic fraction of the shower and the amount of energy lost in nuclear break–up limits the precision to which the energy can be measured. A typical value for the energy resolution of a hadronic calorimeter is given by

$$\frac{\sigma_E}{E} \geq \frac{50\%}{\sqrt{E/\mathrm{GeV}}}$$

8.4 Setup of modern particle physics experiments

Based on what was described before, a modern particle physics experiment should have the possibility to reconstruct and identify hadrons, measure the energy of highly energetic electrons and photons, estimate the energy deposited by highly energetic hadrons and reconstruct and identify muons. Following these requirements, the basic structure of a modern particle physics experiment is given in fig. 8.8. The experiment typically consists of a cylindrical barrel with its axis parallel to the beam pipe. This cylindrical structure is closed at the two ends with two flat end–caps, thus providing full solid angle coverage, almost down to the beam pipe. The inner region of the experiment is devoted to tracking and identifying charged particles with suitable tracking detectors e.g. based on semi-conductor technology (silicon detectors) or gaseous volumes (TPC). The tracking volume is usually surrounded by an electromagnetic calorimeter that allows to detect electrons and photons. This is followed by a large volume hadronic calorimeter, capable of detecting and measuring the energy of highly energetic hadrons. Finally, there are dedicated detectors positioned further outwards, devoted to the detection of muons that, as neutrinos, are able to go through the electromagnetic and hadronic calorimeters.

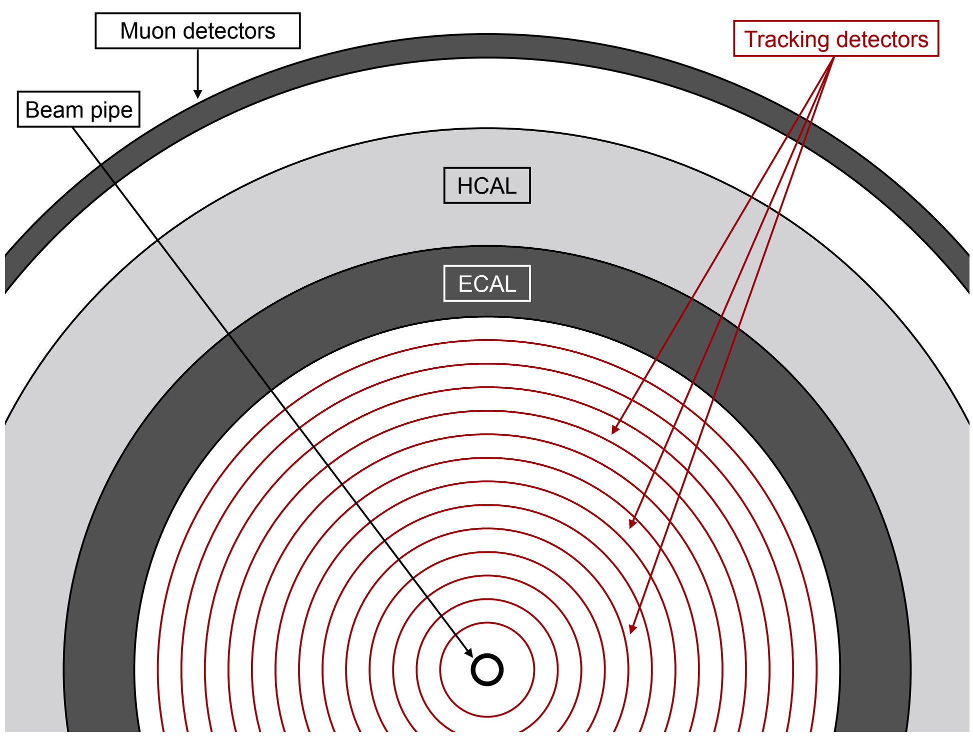


Fig. 8.8: A transverse slice of a typical modern particle physics experiment.

To measure the momentum of particles, as discussed before, a suitable (usually solenoid) magnetic fields is applied inside this barrel region. Different particle species can be detected and identified using one or a combination of techniques discussed in previous paragraphs:

- Information about momentum of charged particles is deduced from the measurement of the curvature of the particle track inside the magnetic field.
- Electrons are identified as charged–particle tracks that leave hits in the tracking detectors and subsequently initiate an electromagnetic shower in the electromagnetic calorimeter.
- Neutral particles are either reconstructed in the tracking detectors (e.g. decays) or their energy is measured in the calorimeters.

  - Neutral particles that decay are reconstructed via their decay topology in the tracking detectors in combination with information from the calorimeters about their decay products (depending on the nature of the decay)

  - Photons are identified in the electromagnetic calorimeter as sources of isolated showers.

  - On the other hand, neutral hadrons will interact with the material in the hadronic calorimeter and initiate an isolated hadronic shower.
- Charged hadrons will be reconstructed from their hits in the tracking detectors, followed by the combination of a small energy deposition via ionisation energy loss in the electromagnetic calorimeter and a large energy deposition in the hadronic calorimeter.
- Finally, special detectors outside the calorimeters are sensitive to the passage of muon tracks, in combination with hits in the tracking detectors and very small energy deposition in both the electromagnetic and the hadronic calorimeters.

One of the last pieces of the puzzle is the detection of neutrinos. Neutrinos, as we have seen, barely interact with matter. However they are carriers of important information and thus need to be accounted for. Their presence in modern particle physics experiments, whose purpose is not solely the detection of neutrinos, is through the presence of missing momentum, defined as:

$$\vec{p}_{missing} = -\sum_i \vec{p}_i,$$

where the sum extends over all measured momenta of all observed particles in all directions of an event. If all particles produced in the collision are detected, this sum should be zero provided that the collisions take place in the centre–of–mass frame. Any significant deviation from zero indicates the presence of energetic neutrinos in the event.

8.5 Measurements at particle accelerators

If one excludes the studies related to properties of neutrinos and the discovery of gravitational waves (not directly particle physics related topic), the major breakthroughs in particle physics have come from experiments at high–energy particle accelerators. Particle accelerators can be divided in two categories:

- colliding beams machines where two beams of particles are accelerated at high velocities, circulate in opposite directions and are brought to collision,
- fixed–target experiments where a single beam is accelerated at high velocities and collides into a stationary target.

The production and eventual discovery of massive particles, such as the carriers of the weak force Z^0 and $W^{\pm}$, or even the Higgs boson discovered in 2012 at CERN requires that the energy available in such collisions in the centre–of–mass frame is greater than the sum of masses of the final state particles. The centre–of–mass energy, as we saw in previous chapters, is a Lorentz invariant quantity and is given by

$$s = \sqrt{\vec{P}_{i\mu}\vec{P}_i^{\mu}} = \sqrt{\left(\sum_i E_i\right)^2 - \left(\sum_i p_i\right)^2} \tag{8.5.1}$$

In a fixed target experiment, momentum conservation implies that the final state particles are always produced with significant kinetic energy and they are produced mainly in the so–called forward region. The corresponding centre–of–mass energy is given by:

$$s = (E_{\text{beam}} + M_{\text{target}})^2 - p_{\text{beam}}^2 = M_{beam}^2 + M_{\text{target}}^2 + 2M_{\text{target}}E_{\text{beam}} \tag{8.5.2}$$

where E_{beam}, p_{beam} and M_{beam} are the energy, momentum and mass of the beam particles respectively, while M_{target} is the mass of the target particle. Colliding beam machines have the advantage of achieving much higher centre–of–mass energy than fixed target configurations.

Only stable charged particles can be accelerated to high energies, thus limiting the possibilities for colliding configurations to e^+e^- colliders, hadron colliders (e.g. pp, p$\bar{\text{p}}$, e^+p or e^-p colliders and finally heavy–ion colliders. Table 8.1 presents the basic parameters of recent particle accelerators.

Collider	Laboratory	Colliding system	Data of operation	$\sqrt{s}$/GeV	Luminosity/cm^{-2}s^{-1}
PEP–B	SLAC	e^+e^-	1999–2008	10.5	1.2×10^{34}
KEKB	KEK	e^+e^-	1999–2010	10.6	12.1×10^{34}
LEP	CERN	e^+e^-	1989–2000	90–209	10^{32}
HERA	DESY	e^+p/e^-p	1992–2007	320	8×10^{31}
Tevatron	Fermilab	p$\bar{\text{p}}$	1987–2002	1960	4×10^{32}
LHC	CERN	pp/pPb/PbPb	2009–today	14000/5000/5000	10^{34} (for pp)

Table 8.1: The basic parameters of the recent particle accelerators.

Two of the most important parameters of these accelerators are the centre–of–mass energy that gives an idea of the physics reach of each machine but also the luminosity L which determines the rate of events. For a given process the number of interactions is the product of the luminosity integrated over the lifetime of the operation of the machine and the cross-section σ of the process in question:

$$N = \sigma \int L(t)dt$$

The cross-section is a measure of the probability for a given interaction to occur. In order to convert the observed number of events of a particular type into the cross-section of a given process, the integrated luminosity needs to be estimated. Typically the particles in an accelerator are grouped in bunches that are brought into collisions at one or more interaction points. As an example, at the LHC the bunches are separated by 25 ns corresponding to a collision frequency of $f =$ 40 MHz. The instantaneous luminosity of the machine can be expressed in terms of the number of particles in the colliding bunches, n_1 and n_2, the frequency at which the bunches collide, and the root–mean–square (rms) of the horizontal and vertical beam sizes, σ_x and σ_y. Assuming that the beams have a Gaussian profile and collide head–on, the instantaneous luminosity is given by

$$L = f\frac{n_1 n_2}{4\pi\sigma_x\sigma_y} \tag{8.5.3}$$