LECTURE 3:
* B-PHYSICS EXPERIMENTS
* TIME-DEPENDENT CP-VIOLATION

Essentials for “quark flavour” physics

- ”decay vertex resolution”

- “momentum resolution”

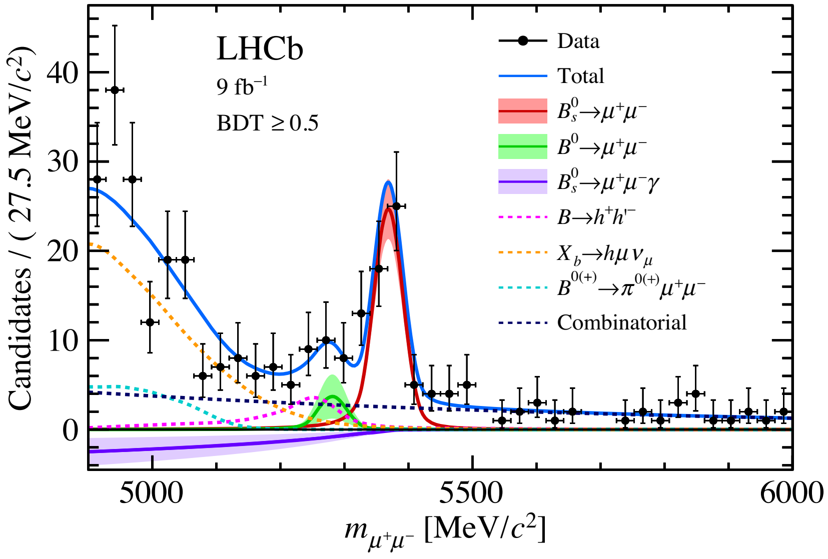
- “particle ID”

Essentials for “quark flavour” physics

- ”decay vertex resolution”
  - to identify weak decays
  - to measure decaytime
- “momentum resolution”
  - to identify different final states (‘mass peak’)
  - to measure decay time and kinematics of decay (‘decay angles’)
- “particle ID”
  - separate final state pions, kaons, protons, electrons, muons, photons
  - also needed for “flavour tagging”

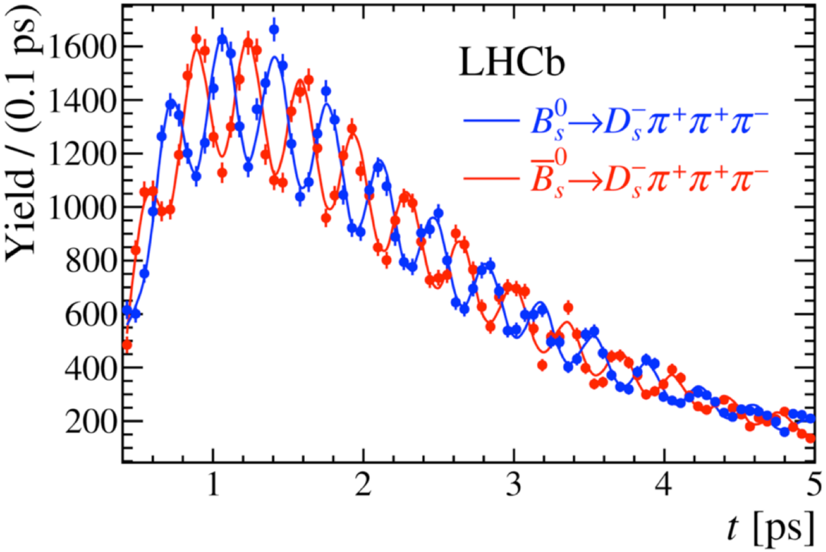
Why it matters

example: B mass resolution



Phys. Rev. Lett. 128, 041801

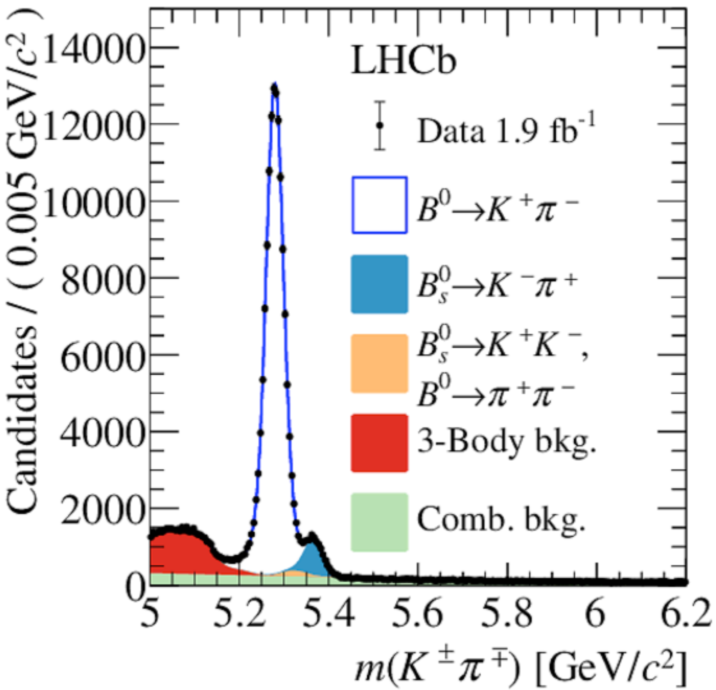
example: decay time resolution



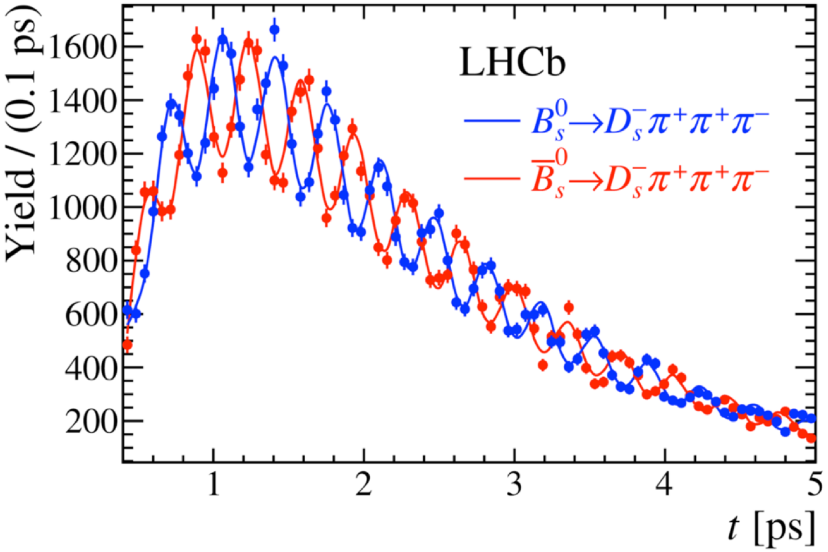
JHEP **2021**, 137 (2021)

Why it matters

example: K/π separation
(and momentum resolution)



example: decay time resolution



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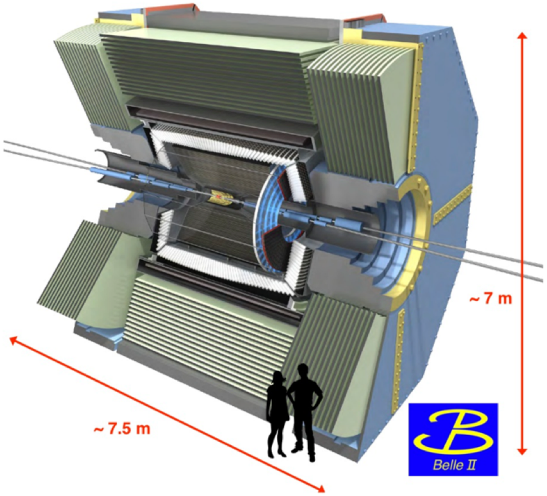
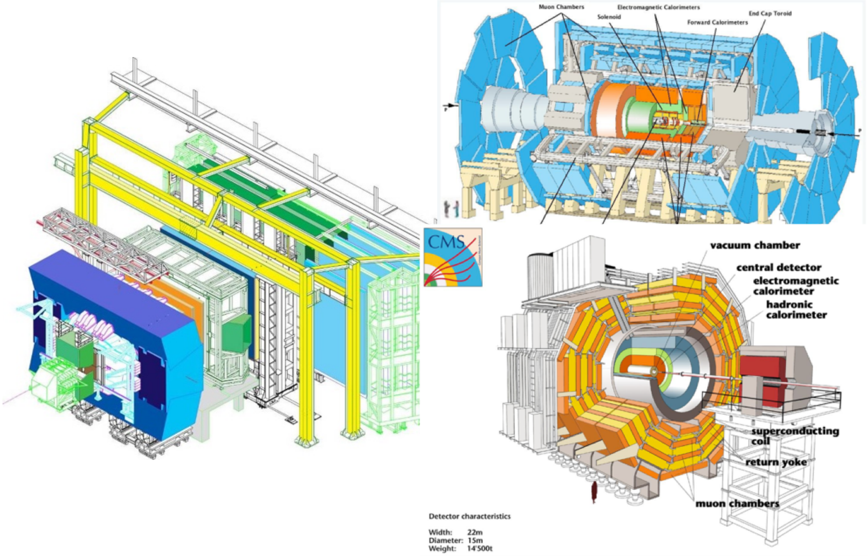
“B-physics” experiments

at the $e^+e^- \to \Upsilon(4S) \to b\bar{b}$ resonance



previously also: BaBar (at SLAC), CLEO (Cornell), ARGUS(DESY), …

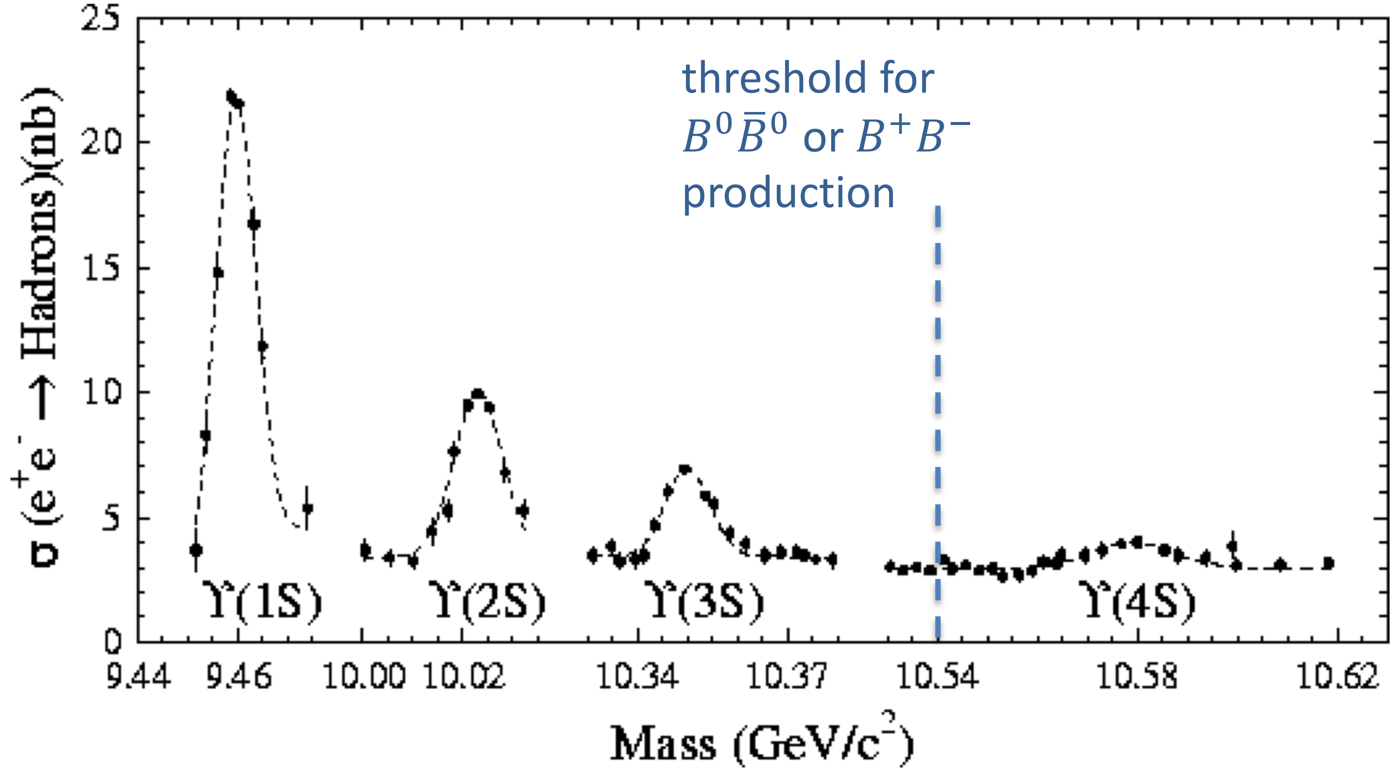
at high energy: $q\bar{q}/gg \to b\bar{b}$



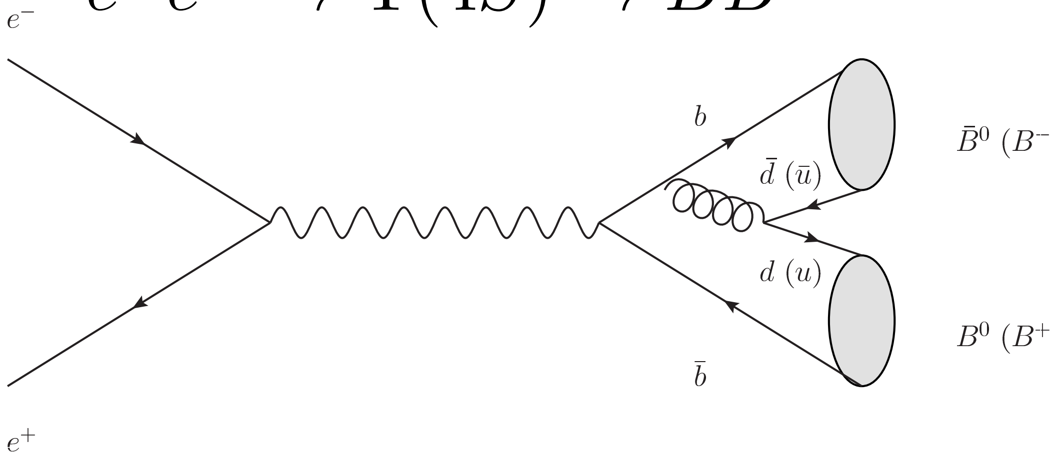
previously also: Tevatron, SLD, LEP, …

Υ resonances: $b\bar{b}$ bound states

cross section for e+ e- to hadrons



$$e^+e^- \to \Upsilon(4S) \to B\bar{B}$$



$e^+e^- \to$	Cross-section (nb)
$b\bar{b}$	1.05
$c\bar{c}$	1.30
$s\bar{s}$	0.35
$u\bar{u}$	1.39
$d\bar{d}$	0.35
$\tau^+\tau^-$	0.94
$\mu^+\mu^-$	1.16
e^+e^-	~ 40

at 10.48 GeV:

$$\sigma(B^0\bar{B}^0) : \sigma(B^+B^-) : \sigma(\text{everything else}) = 1 : 1 : \sim 6$$

- nice feature of Ups(4S) resonance
  - detector empty apart from BB decay products
  - kinematic constraint from energy of initial state

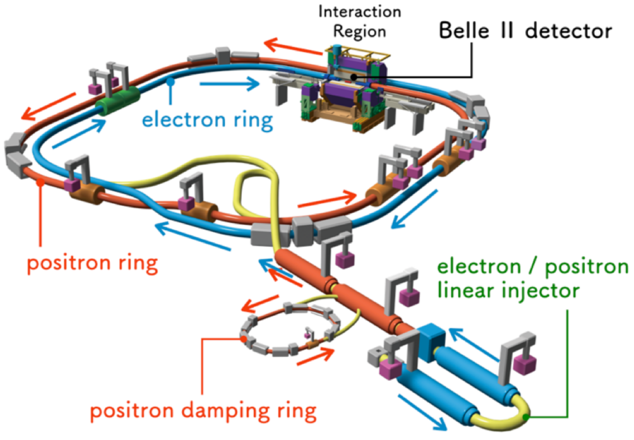
- great for
  - "flavour tagging"
  - 'inclusive' measurements such as V_{ub} with $B \to X_u \ell \nu$
  - other rare decays with neutrinos in final state

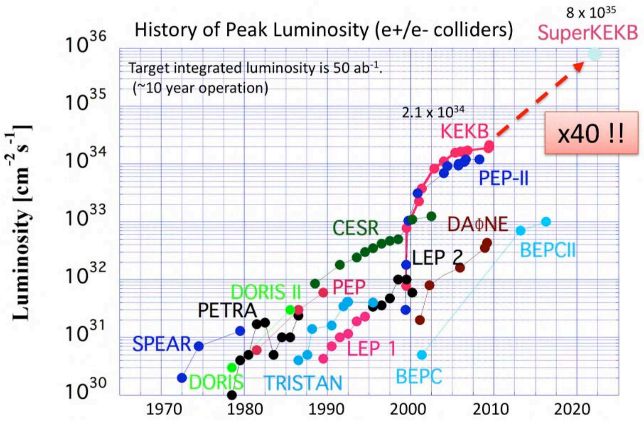
Belle-II at SuperKEKB

- "asymmetric-energy collider"
- to measure 'decaytime', B-mesons need to have non-zero velocity
- achieved by using **beams with different energy**

electrons: ~7 GeV
positrons: ~4 GeV

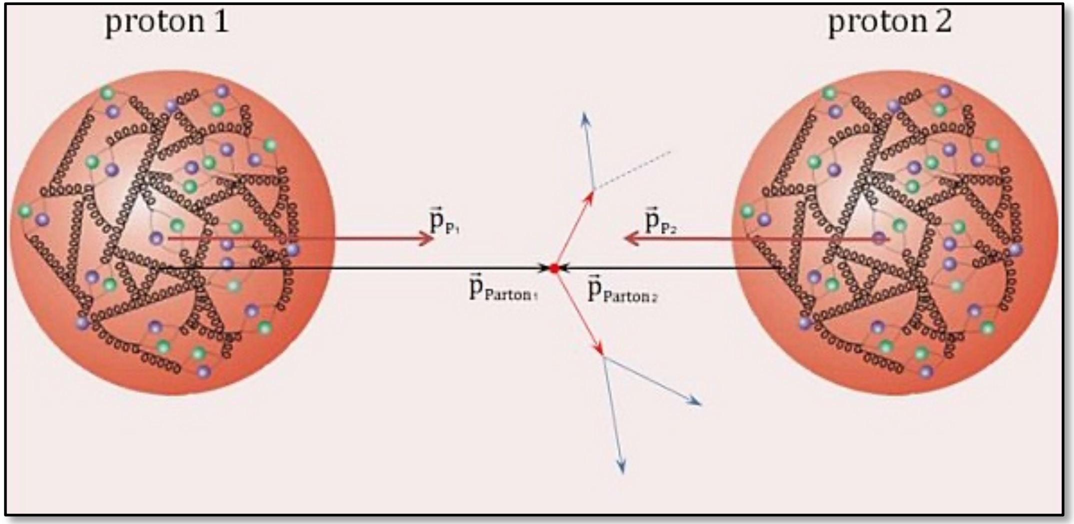
- main challenge: **collider luminosity**
  - aim: 10^{10} $B\bar{B}$ per year
  - LHCb: $>10^{11}$ $b\bar{b}$ per year





at the LHC: gluon-gluon fusion

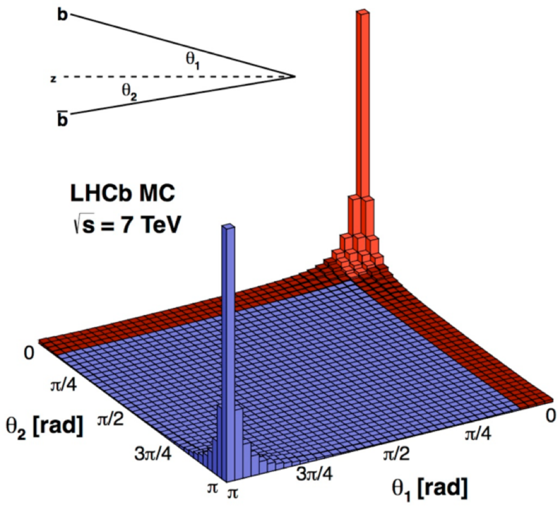
- dominant production mechanism is gluon-gluon fusion

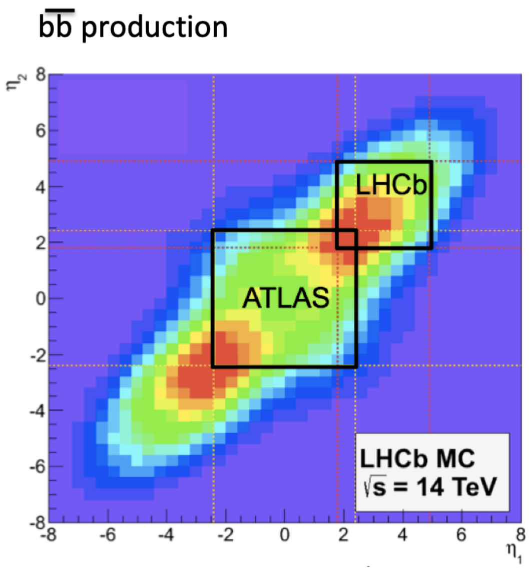


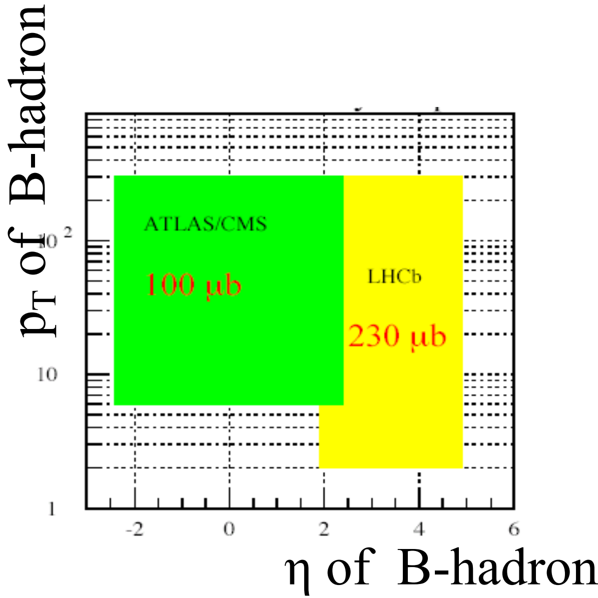
- about 1 $b\bar{b}$ pair in every 200 collisions (and 1 $c\bar{c}$ pair in every 20 collisions)
- large ‘background’ from underlying event

at the LHC: gluon-gluon fusion

- b-quarks are light compared to LHC energy: strongly ‘boosted’







- LHCb catches a larger fraction of B events
- ATLAS/CMS have (much) larger primary interaction rates

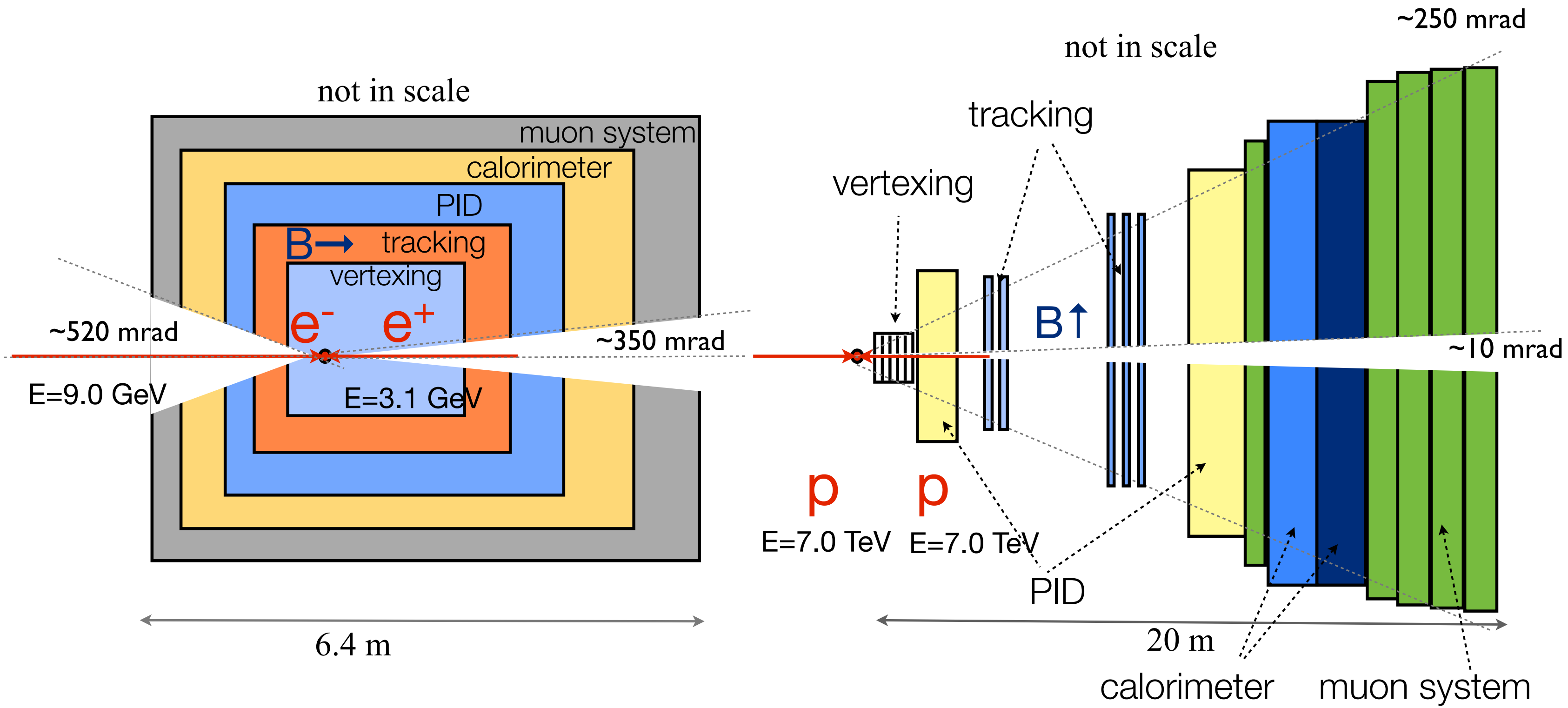
Comparison of current B-physics experiments

$e^+e^- \rightarrow \Upsilon(4S)$ | at high energy: $q\bar{q} \rightarrow b\bar{b}$

	Belle-II	LHCb	ATLAS/CMS
clean events	yes	no	no
which B hadrons?	B0/B+	all	all
decay time resolution	+++ (*)	+++	++
momentum resolution	+++	+++	++
electron/muon ID	+++	+++	+++
kaon/pion/proton ID	+++	+++	+

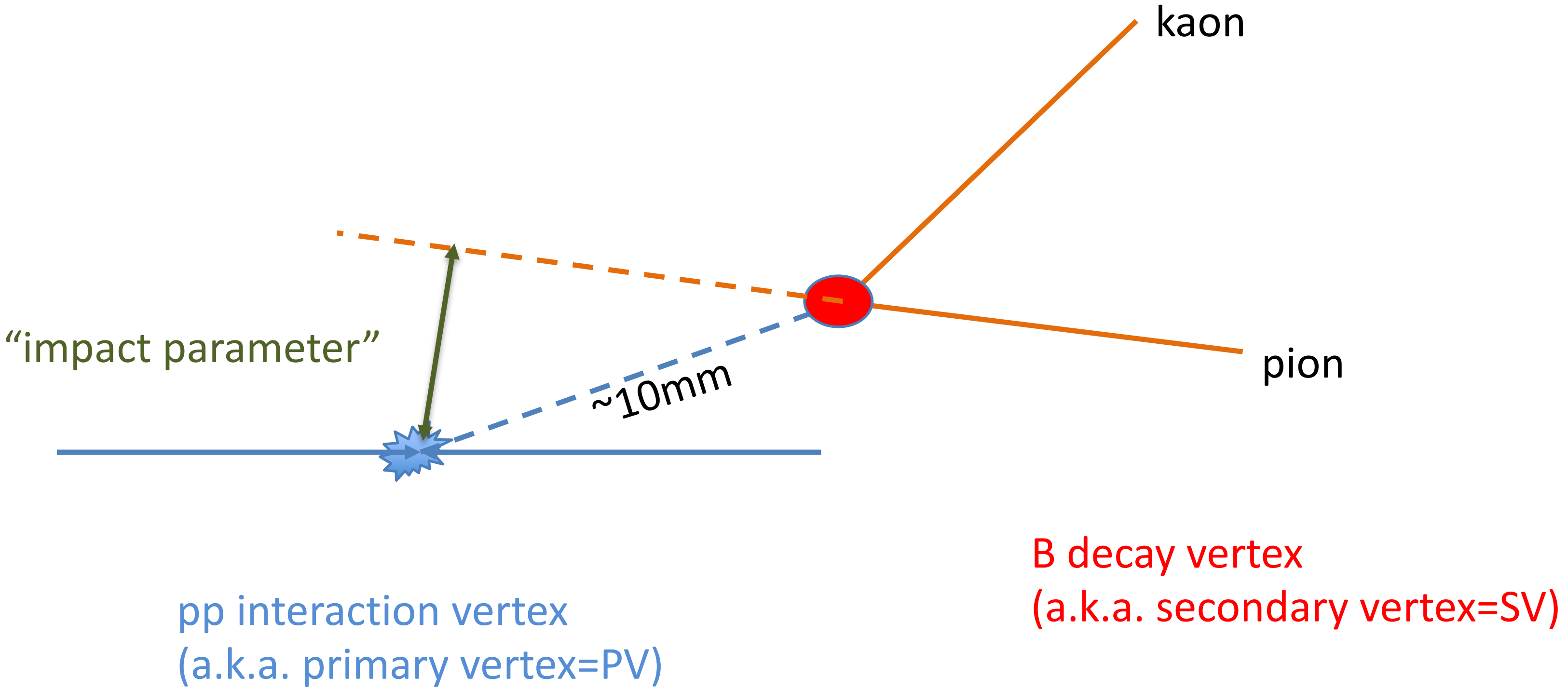
these detectors were designed for flavour physics

Different detectors



Courtesy: N.Neri, IDPASC school, May 2013

the “impact parameter”



IP/decay time resolution

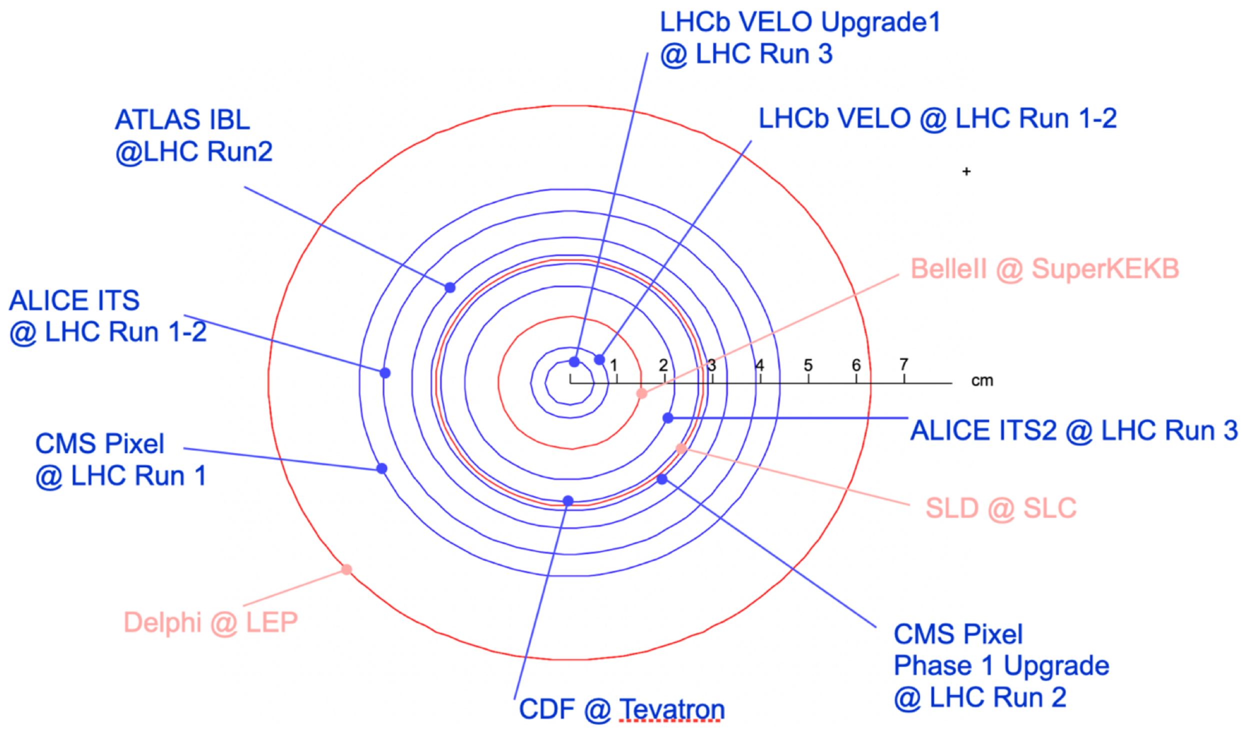
- typical requirement: IP resolution ~10µm
  - needed to separate B from (short-lived) background
  - needed to measure B oscillations (in particular Δm_s)

- resolution depends on 3 critical parameters:
  1. intrinsic **hit position resolution**

  2. **extrapolation distance** between hits and vertex

  3. **multiple scattering** between collision point and measured points from detector material

distance to vertex



resolution at LHCb:

- $\sigma(IP)$ ~10µm
- $\sigma(t)$ ~50fs

Fig.: Minimum radius of silicon vertex detectors at hadron and lepton colliders, up to start of LHC Run 3.

Courtesy: K.Akiba

Particle identification

- Many different B-decays!
  - "BtooKstarpipiDsgamma"
  - …

B^0 Decay Modes ▾ Collapse all

$\overline{B}^0$ modes are charge conjugates of the modes below. Reactions indicate the weak decay vertex and do not include mixing. Modes which do not identify the charge state of the B are listed in the $B^\pm / B^0$ ADMIXTURE section.

The branching fractions listed below assume 50% $B^0\overline{B}^0$ and 50% B^+B^- production at the $\Upsilon(4S)$. We have attempted to bring older measurements up to date by rescaling their assumed $\Upsilon(4S)$ production ratio to 50:50 and their assumed D, D_s, D^*, and ψ branching ratios to current values whenever this would affect our averages and best limits significantly.

Indentation is used to indicate a subchannel of a previous reaction. All resonant subchannels have been corrected for resonance branching fractions to the final state so the sum of the subchannel branching fractions can exceed that of the final state.

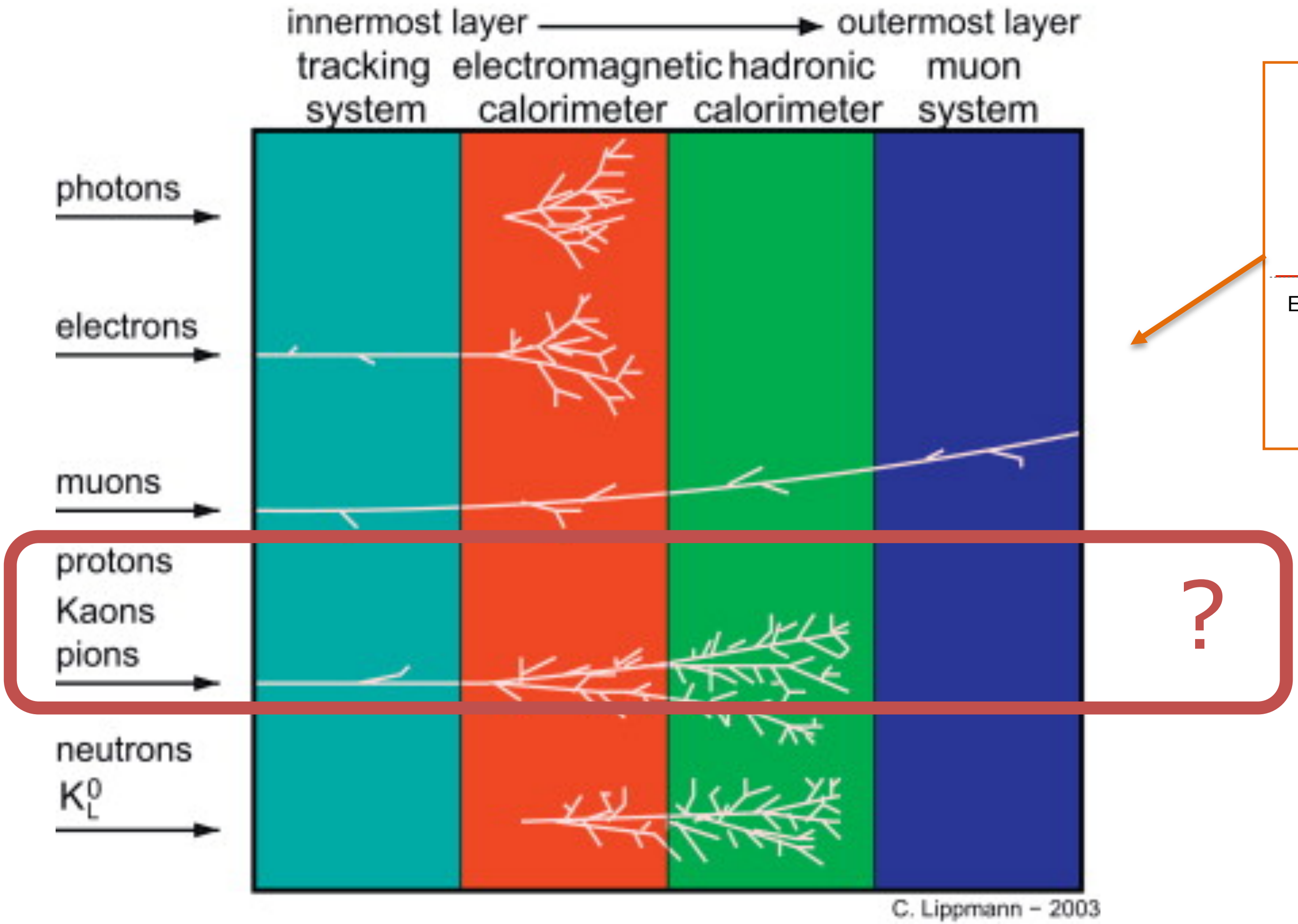
For inclusive branching fractions, $e.g.$, $B \to D^\pm X$, the values usually are multiplicities, not branching fractions. They can be greater than one.

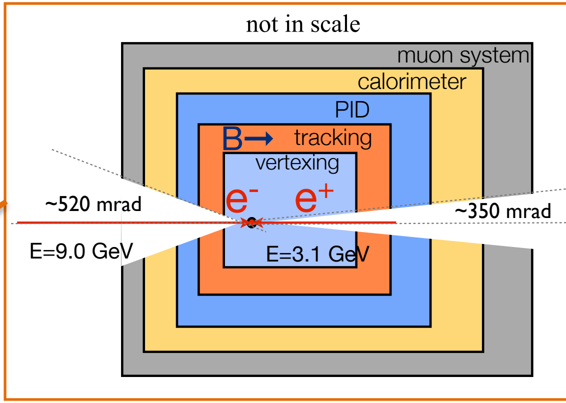
	Mode	Fraction (Γ_i / Γ)	Scale Factor/ Conf. Level	P(MeV/c)
Γ_1	$\ell^+\nu_\ell X$	$(10.33 \pm 0.28)\%$		
Γ_2	$e^+\nu_e X_c$	$(10.1 \pm 0.4)\%$		
Γ_3	$\ell^+\nu_\ell X_u$	$(1.51 \pm 0.19) \times 10^{-3}$		
Γ_4	$D\ell^+\nu_\ell X$	$(9.3 \pm 0.8)\%$		
Γ_5	$D^-\ell^+\nu_\ell$	$(2.24 \pm 0.09)\%$		2309
Γ_6	$D^-\tau^+\nu_\tau$	$(1.05 \pm 0.23)\%$		1909
Γ_7	$D^*(2010)^-\ell^+\nu_\ell$	$(4.97 \pm 0.12)\%$		2257

⋮

Γ_{535}	$\pi^0\nu\overline{\nu}$	$< 9 \times 10^{-6}$	CL=90%	2638
Γ_{536}	$K^0\ell^+\ell^-$	$(3.3 \pm 0.6) \times 10^{-7}$		2616
Γ_{537}	$K^0 e^+ e^-$	$(2.5^{+1.1}_{-0.9}) \times 10^{-7}$	S=1.3	2616
Γ_{538}	$K^0\mu^+\mu^-$	$(3.39 \pm 0.35) \times 10^{-7}$	S=1.1	2612
Γ_{539}	$K^0\nu\overline{\nu}$	$< 2.6 \times 10^{-5}$	CL=90%	2616
Γ_{540}	$\rho^0\nu\overline{\nu}$	$< 4.0 \times 10^{-5}$	CL=90%	2583
Γ_{541}	$K^*(892)^0\ell^+\ell^-$	$(9.9^{+1.2}_{-1.1}) \times 10^{-7}$		2565
Γ_{542}	$K^*(892)^0 e^+ e^-$	$(1.03^{+0.19}_{-0.17}) \times 10^{-6}$		2565
Γ_{543}	$K^*(892)^0\mu^+\mu^-$	$(9.4 \pm 0.5) \times 10^{-7}$		2560

- Need to distinguish:
  - e, μ, γ, π, K, p, …

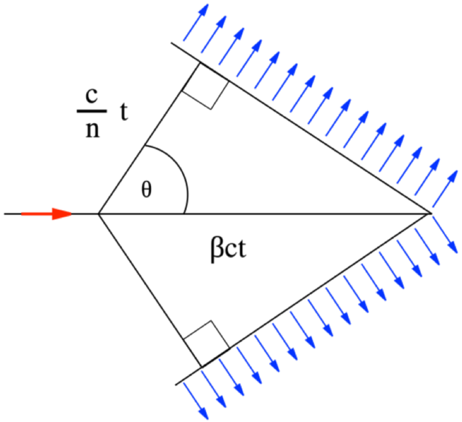

innermost layer
outermost layer
tracking system
electromagnetic calorimeter
hadronic calorimeter
muon system
photons
electrons
muons
protons
Kaons
pions
neutrons
K^0_L
?
C. Lippmann – 2003

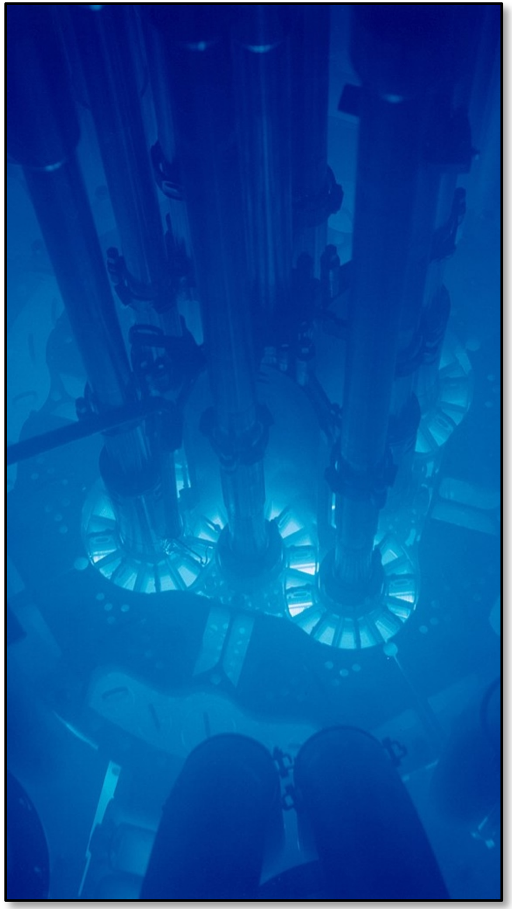

not in scale
muon system
calorimeter
PID
B→ tracking
vertexing
e^-
e^+
~520 mrad
~350 mrad
E=9.0 GeV
E=3.1 GeV

one solution: ring imaging cherenkov detector

- particles traversing medium with velocity $\beta > 1/n$, emit cherenkov radiation

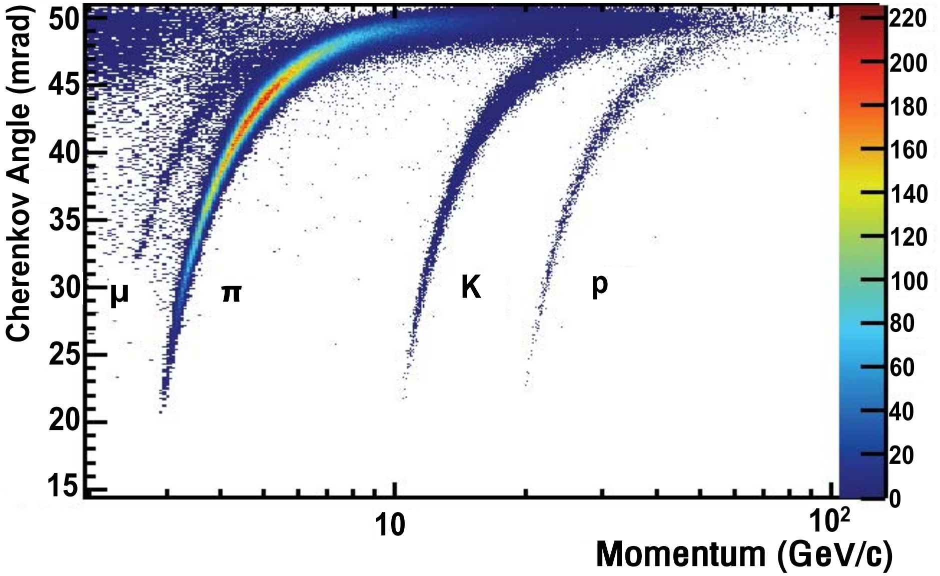
- angle of wave-front with direction is measure for velocity: $\cos\theta_c = 1/\beta n$





one solution: ring imaging cherenkov detector

- in combination with momentum measurement, $p = \gamma\beta m$, provides estimate of particle mass



- different radiators suitable for different momentum range:
    - Belle/Babar and LHCb make different choices

momentum resolution

- momentum measured by ‘deflection’ (or ‘curvature’ or ‘sagita’)

- detector resolution determined by
  - “B-ΔL” : strength of magnetic field times detector ‘length’ (arm)
  - hit resolution
  - multiple scattering

- different choices
  - LHCb is forward detector with very **long arm**
  - CMS/Belle-II have a **strong superconducting magnet**

- note: Belle-II works with much **lower momenta**: need less ‘sagitta’ resolution for same momentum resolution, but suffers more from multiple scattering

Time-dependent CP-violation

- will now discuss formalism of ‘time-dependent’ CP violation

- very interesting method to probe ‘phases’ in Vckm with relatively small ‘hadronic’ uncertainties

- will reuse mixing formalism discussed this morning, but also need to consider the ‘decay’ to the final state

Reminder: mixing formalism

- This is what we found this morning

$$\left|B^0(t)\right\rangle = g_+(t)\left|B^0\right\rangle + \frac{q}{p}\,g_-(t)\left|\bar{B}^0\right\rangle$$

$$\left|\bar{B}^0(t)\right\rangle = g_+(t)\left|\bar{B}^0\right\rangle + \frac{p}{q}\,g_-(t)\left|B^0\right\rangle$$

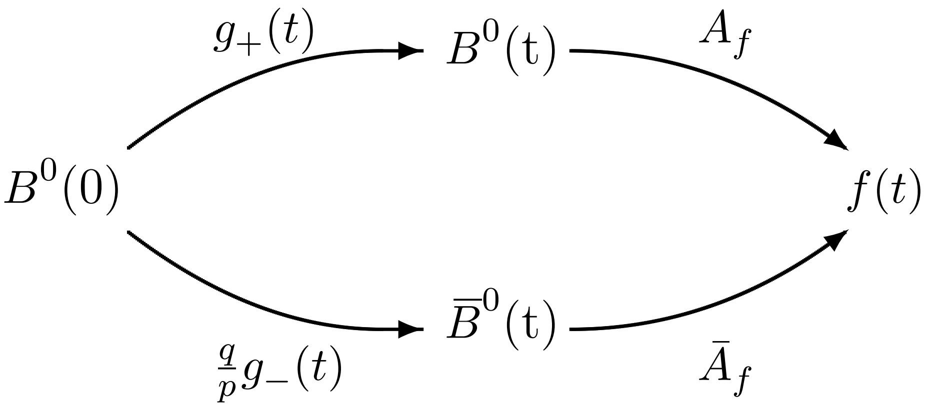
$$g_\pm(t) = \frac{1}{2}\left(e^{-i\,\mu_{\mathrm{L}}\,t} \pm e^{-i\,\mu_{\mathrm{H}}\,t}\right)$$

expressed in m, Γ, Δm, $\Delta\Gamma$

- next step: consider decay amplitude

Including decay amplitudes

- consider decay of B^0 to final state ‘f’ *accessible to both* B^0 *and anti-*B^0



- computation needs to include the *decay amplitudes*

$$A_f = \mathcal{A}(B^0 \to f) = \langle f | H_{\text{weak}} | B^0 \rangle$$

$$\bar{A}_f = \mathcal{A}(\bar{B}^0 \to f) = \langle f | H_{\text{weak}} | \bar{B}^0 \rangle$$

Time-dependent amplitude

- if the meson started as a B^0, then time-dependent amplitude is

$$A_{B^0\to f}(t) \equiv \langle f|\, H_{\text{weak}}\, \left|B^0(t)\right\rangle \;=\; g_+(t)\, A_f \;+\; \frac{q}{p}\,\bar{A}_f\, g_-(t)$$

- two contributing amplitudes

- functions $g_\pm(t)$ are complex
  - relative size and phase depends on time
  - leads to “time-dependent CP violation”,
    provided A_f and $\bar{A}_f$ are approximately of equal size

Time-dependent rate

- next step: take the square of amplitude: $\Gamma_{B^0\to f}(t) \;=\; \left|A_{B^0\to f}(t)\right|^2$

- taking the square gives

$$\Gamma_{B^0\to f}(t) \;=\; |A_f|^2 \left[\; |g_+(t)|^2 \;+\; |\lambda_f|^2 \, |g_-(t)|^2 \;+\; 2\mathrm{Re}\left(\lambda_f \, g_-(t) \, g_+(t)^* \right) \; \right]$$

where we defined the "lambda" parameter: $\lambda_f \;=\; \dfrac{q}{p}\dfrac{\bar{A}_f}{A_f}$

Time-dependent rate

- next step: take the square of amplitude: $\Gamma_{B^0\to f}(t) \;=\; |A_{B^0\to f}(t)|^2$

- taking the square gives

$$\Gamma_{B^0\to f}(t) \;=\; |A_f|^2 \left[\; |g_+(t)|^2 \;+\; |\lambda_f|^2\,|g_-(t)|^2 \;+\; 2\mathrm{Re}\,(\,\lambda_f\; g_-(t)\; g_+(t)^*\,) \;\right]$$

$B^0 \to f$

$B^0 \to \overline{B^0} \to f$

interference

$$\lambda_f \;=\; \frac{q}{p}\,\frac{\overline{A}_f}{A_f}$$

Time-dependent rate

- now substitute the functions g
- *after some straightforward but time-consuming algebra*:

$$\Gamma_{B\to f}(t) \;=\; |A_f|^2 \left(1+|\lambda_f|^2\right) \frac{e^{-\Gamma t}}{2}$$
$$\left[\cosh\left(\tfrac{1}{2}\Delta\Gamma\, t\right) + D_f \sinh\left(\tfrac{1}{2}\Delta\Gamma\, t\right) + C_f \cos\left(\Delta m\, t\right) - S_f \sin\left(\Delta m\, t\right) \right]$$

interference terms

with
$$C_f \;\equiv\; \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \qquad S_f \;\equiv\; \frac{2\Im(\lambda_f)}{1+|\lambda_f|^2} \qquad D_f \;\equiv\; -\frac{2\Re(\lambda_f)}{1+|\lambda_f|^2}$$

Time-dependent rate for anti-B0 to f

- expression for anti-B0 to f follows with substitution $\bar{\lambda}_f = \frac{1}{\lambda_f}$

- together, expressions then look like

$$\Gamma_{P^0\to f}(t) = |A_f|^2 \quad (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t + D_f \sinh\frac{1}{2}\Delta\Gamma t + C_f\cos\Delta m t - S_f \sin\Delta m t\right)$$

$$\Gamma_{\bar{P}^0\to f}(t) = |A_f|^2\left|\frac{p}{q}\right|^2 (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t + D_f \sinh\frac{1}{2}\Delta\Gamma t - C_f\cos\Delta m t + S_f \sin\Delta m t\right)$$

these terms change sign

with $$C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \qquad S_f \equiv \frac{2\Im(\lambda_f)}{1+|\lambda_f|^2} \qquad D_f \equiv -\frac{2\Re(\lambda_f)}{1+|\lambda_f|^2}$$

Rates to CP-conjugate final state

- we are not quite there yet: also need to consider the rates to $\bar{f}$
- it is mostly a matter of notation:

$$
\begin{aligned}
A_f &= \mathcal{A}(B^0 \to f) &= \langle f | \, H_{\text{weak}} \, | B^0 \rangle \\
\bar{A}_f &= \mathcal{A}(\bar{B}^0 \to f) &= \langle f | \, H_{\text{weak}} \, | \bar{B}^0 \rangle \\
A_{\bar{f}} &= \mathcal{A}(B^0 \to \bar{f}) &= \langle \bar{f} | \, H_{\text{weak}} \, | B^0 \rangle \\
\bar{A}_{\bar{f}} &= \mathcal{A}(\bar{B}^0 \to \bar{f}) &= \langle \bar{f} | \, H_{\text{weak}} \, | \bar{B}^0 \rangle
\end{aligned}
$$

- note: if there is no CP violation in the decay, then

$$
\text{no CP-violation in decay} \quad \Longleftrightarrow \quad |A_f| = |\bar{A}_{\bar{f}}| \quad \left(\text{and } |\bar{A}_f| = |A_{\bar{f}}|\right)
$$

Two important cases

- case 1: f is <u>flavour-specific final state</u>

$$|A_{\bar{f}}| \ll |A_f| \quad \Longrightarrow \quad \lambda_f \approx \bar{\lambda}_{\bar{f}} \approx 0$$

→

$$A_{\text{mix}}(t) \;=\; \frac{1}{2}\left(\,1 + \cos(\Delta m\; t)\,\right)$$

  - example: B0 -> J/psi K*, but many others

- case 2: f is <u>CP eigenstate with only single contributing amplitude</u>

$$|A_f| = |\bar{A}_f| \quad \Longrightarrow \quad \lambda_f = \frac{q}{p}\eta_f e^{-i2\phi_D}$$

→

$$A_{CP}(t) \;=\; S\sin(\Delta m\, t)$$

  - example B0 -> J/psi Ks and Bs -> J/psi phi (“golden modes”)

Two important cases

- case 1: f is <u>flavour-specific final state</u>

$$|A_{\bar{f}}| \ll |A_f| \quad \Longrightarrow \quad \lambda_f \approx \bar{\lambda}_{\bar{f}} \approx 0$$

$$A_{\rm mix}(t) = \frac{N(B^0 \to f \text{ or } \bar{B}^0 \to \bar{f}) - N(B^0 \to \bar{f} \text{ or } \bar{B}^0 \to f)}{N(B^0 \to f \text{ or } \bar{B}^0 \to \bar{f}) + N(B^0 \to \bar{f} \text{ or } \bar{B}^0 \to f)}$$

$$= \frac{1}{2}\left(1 + \cos(\Delta m\, t) \right)$$

- this is how we measure mixing frequency
- example: B0 -> J/psi K*, but many others

Two important cases

- case 2: f is CP eigenstate with only single contributing amplitude

$$|A_f| = |\bar{A}_f| \quad \Longrightarrow \quad \lambda_f = \frac{q}{p}\,\eta_f\, e^{-i2\phi_D}$$

mixing phase

phase from decay: $\phi_D = \arg(A_f)$

$$A_{CP}(t) = \frac{N(\bar{B}^0 \to f) - N(B^0 \to f)}{N(\bar{B}^0 \to f) - N(B^0 \to f)} = S\sin(\Delta m\, t)$$

Two important cases

- case 2: f is <u>CP eigenstate with only single contributing amplitude</u>

$$|A_f| = |\bar{A}_f| \quad \Longrightarrow \quad \lambda_f = \frac{q}{p}\, \eta_f\, e^{-i2\phi_D}$$

|q/p|=1, ΔΓ=0

$$A_{CP}(t) = \frac{N(\bar{B}^0 \to f) - N(B^0 \to f)}{N(\bar{B}^0 \to f) - N(B^0 \to f)} = S \sin(\Delta m\, t)$$

- this measures CKM phases → "angles of unitarity triangle"
- example B0 -> J/psi Ks and Bs -> J/psi phi ("golden modes")

Three types of CP violation

1. ***"direct"*** CP violation

- Decay rates $\Gamma(K^0 \to \pi^+\pi^-) \neq \Gamma(\overline{K^0} \to \pi^+\pi^-)$
- Also called: ***CPV in decay***

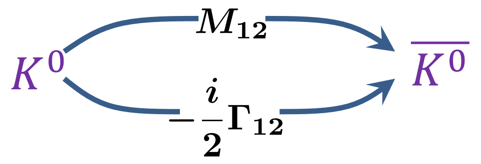
Interfere *decay amplitudes*:

$$A = a_{0(K\to\pi\pi)} + a_{2(K\to\pi\pi)}$$

2. ***"indirect"*** CP Violation: 1964 (CCFT)

- Prob$(K^0 \to \overline{K^0}) \neq$ Prob $(\overline{K^0} \to K^0)$
- Also called: ***CPV in mixing***

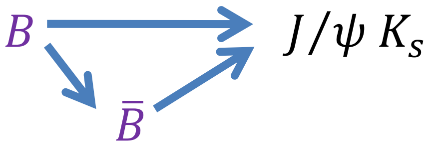
Interfere *dispersive* and *absorptive*:



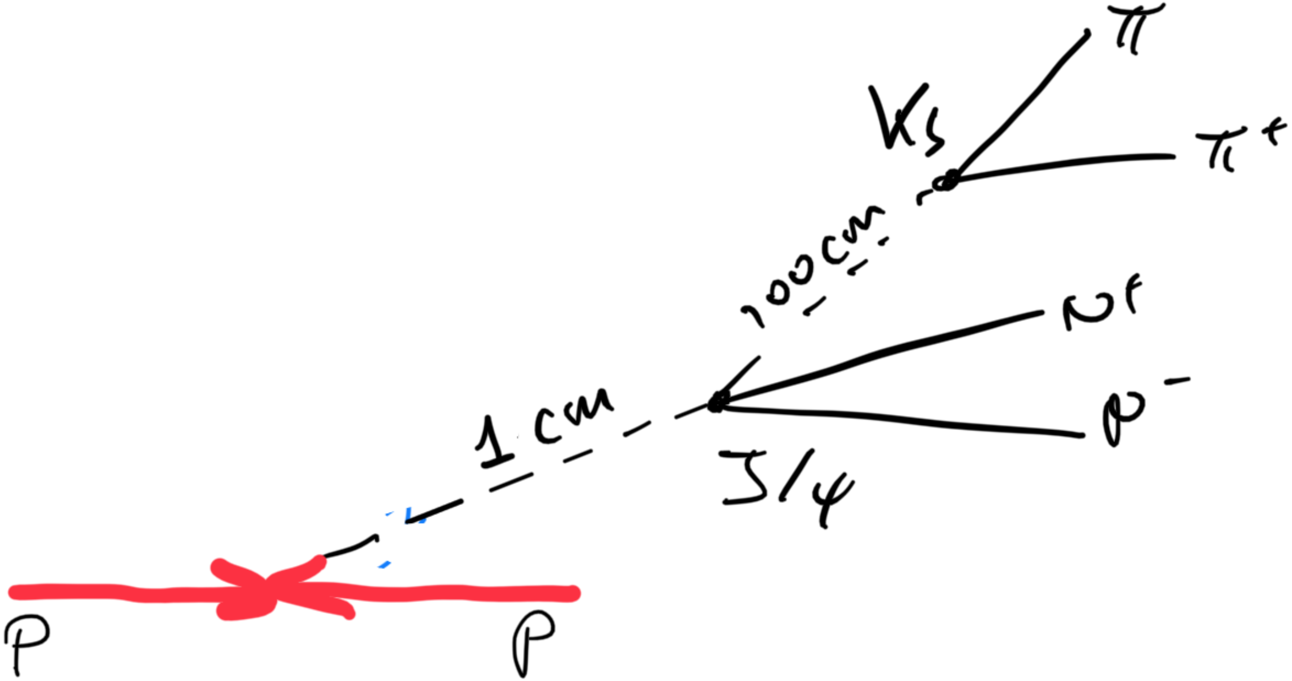
3. **"mixing induced"** CP violation: 2001 (Belle & Babar):

- Also: ***CPV in interference of mixing and decay***

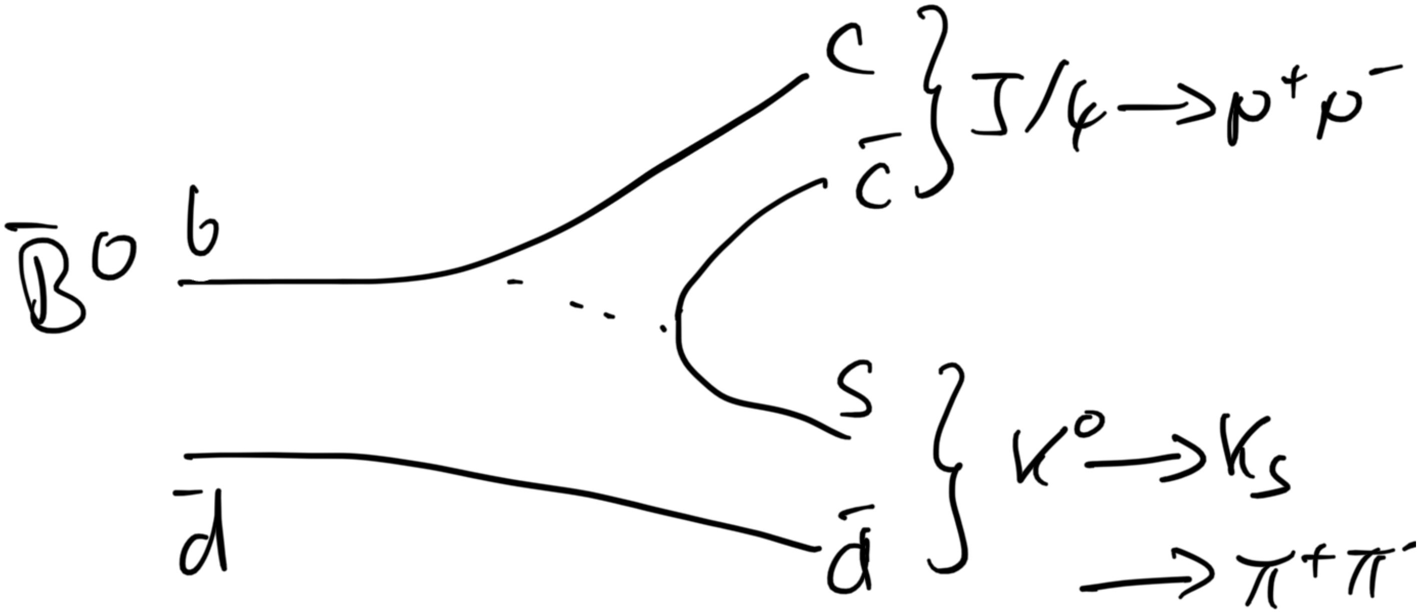
Interfere *direct and mixed*:



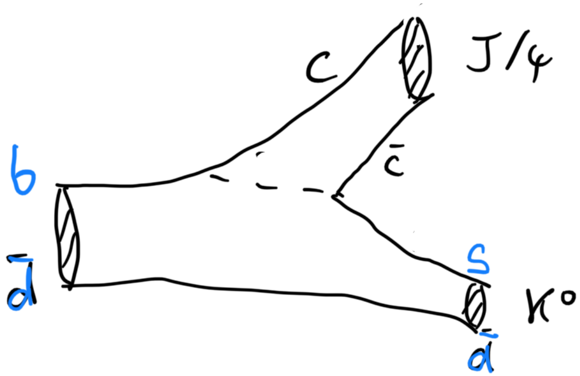
Example: $B^0 \to J/\psi\ K_S$

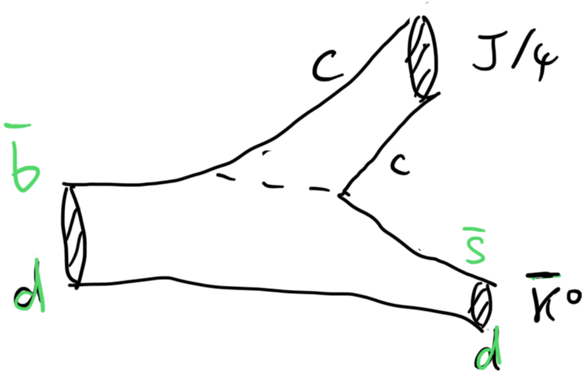


Example: $B^0 \to J/\psi\ K_S$

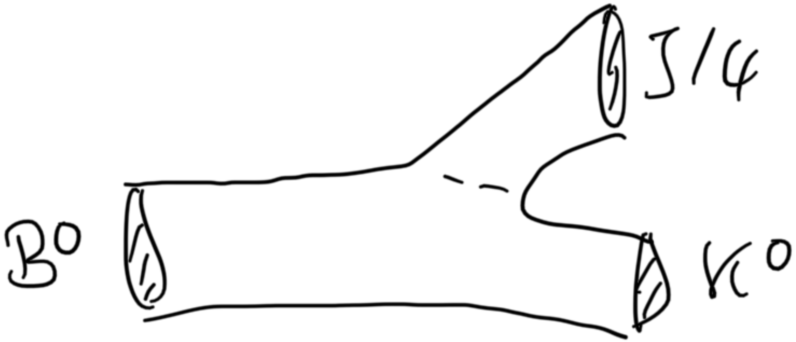


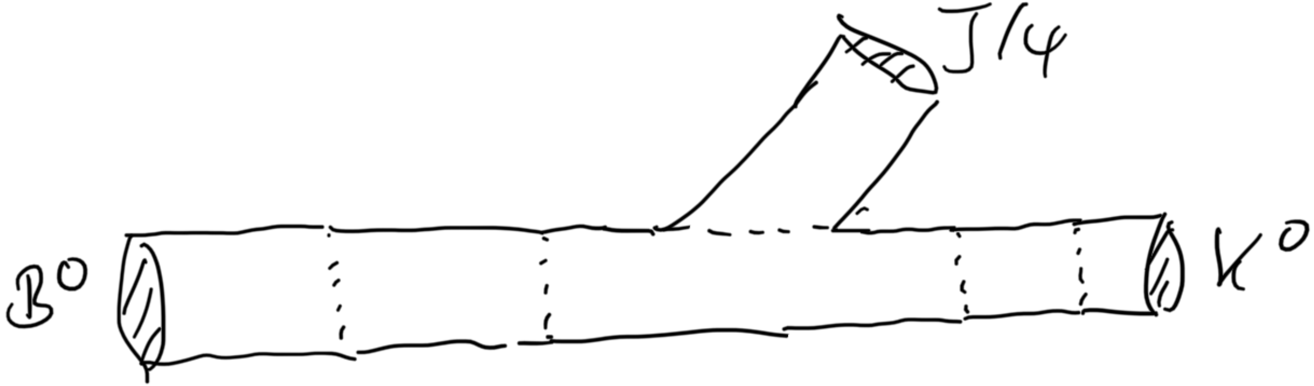
Example: $B^0 \to J/\psi\ K_S$





to get the interference, also need
to take into account K0-K0bar mixing

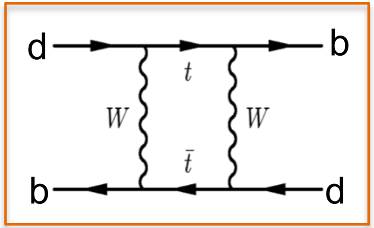

B^0
J/ψ
K^0

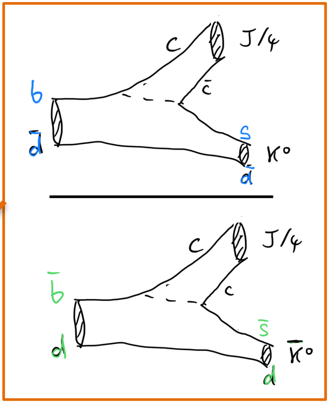

B^0
J/ψ
K^0

λ_f for $B^0 \to J/\psi K^0_s$

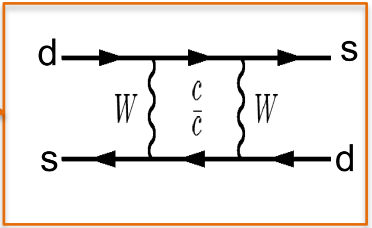
$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

$$\lambda_{J/\psi K_s} = \left(\frac{q}{p}\right)_{B^0} \left(\frac{A(B^0 \to J/\psi K^0)}{A(\bar{B}^0 \to J/\psi \bar{K}^0)}\right) \left(\frac{q}{p}\right)_{K^0}$$





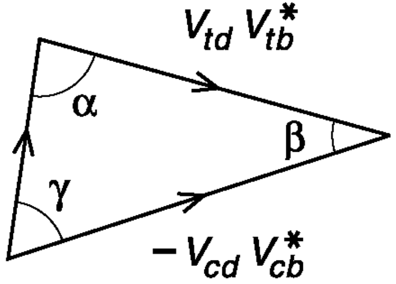
$$\lambda_{J/\psi K_s} = -\left(\frac{V_{tb}^{*}V_{td}}{V_{tb}V_{td}^{*}}\right)\left(\frac{V_{cb}V_{cs}^{*}}{V_{cb}^{*}V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^{*}}{V_{cs}^{*}V_{cd}}\right)$$



λ_f for $B^0 \rightarrow J/\psi K^0_S$

$$\lambda_{J/\psi K_s} = -\left(\frac{V_{tb}^{*}V_{td}}{V_{tb}V_{td}^{*}}\right)\left(\frac{V_{cb}V_{cs}^{*}}{V_{cb}^{*}V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^{*}}{V_{cs}^{*}V_{cd}}\right)$$

$$= -e^{-2i\beta}$$



Time-dependent CP asymmetry

$$A_{CP}(t) = -\sin 2\beta \ \sin(\Delta m t)$$

- Theoretically clean way to measure β
- Clean experimental signature
- Branching fraction: $O(10^{-4})$
  - "Large" compared to other CP modes!

Exercises

- see README.md file at

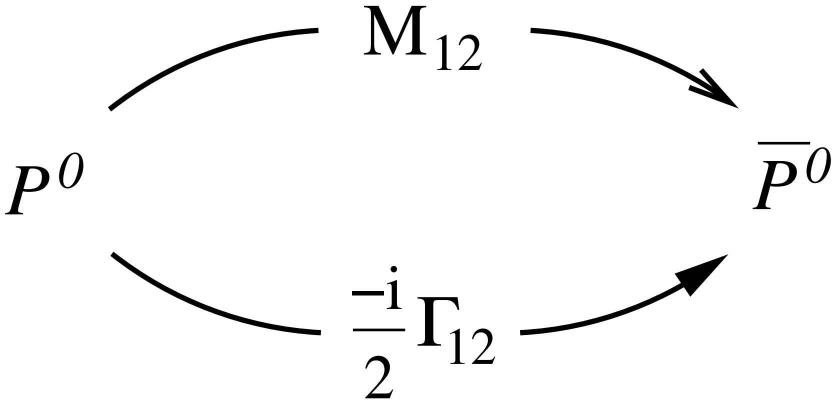
  https://github.com/wouterhuls/FlavourPhysicsBND2023/

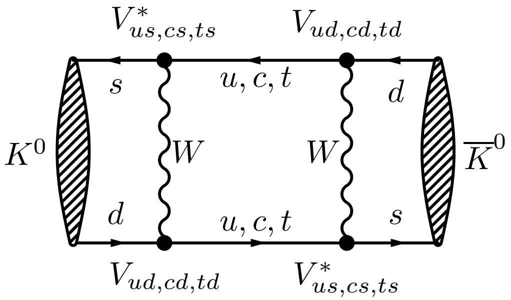
- **now: exercises 8-9**

- **(my apologies: still need to finish notebook for exercise 10)**

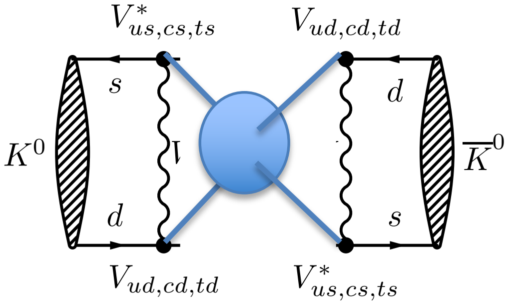
BACKUP

Computation of M_12 and G_12



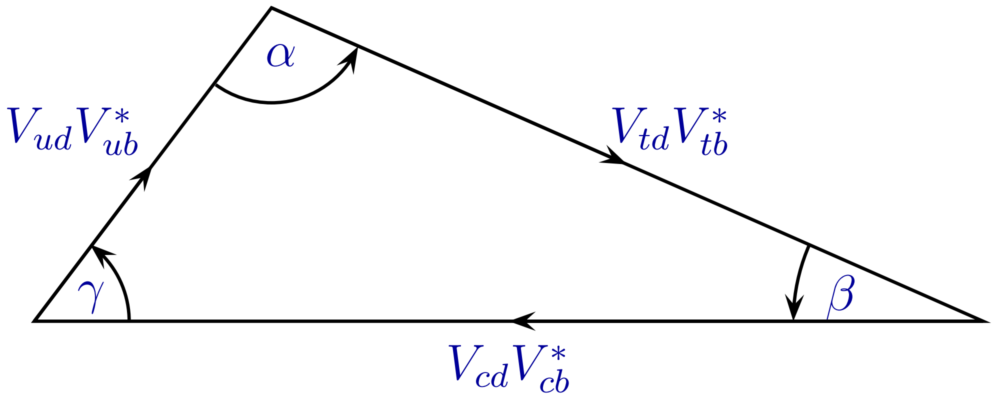


all 'virtual' quarks contribute



only states with sufficiently small mass contribute

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$



$$\alpha = \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right] \quad \beta = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] \quad \gamma = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

Illustration of computation of M_12

1. Amplitude for free quarks

$$\mathcal{M}_{uu} = i\left(\frac{-ig_w}{2\sqrt{2}}\right)^4 (V_{us}^* V_{ud} V_{us}^* V_{ud}) \int \frac{d^4k}{(2\pi)^4}\left(\frac{-ig^{\lambda\sigma} - k^\lambda k^\sigma/m_W^2}{k^2 - m_W^2}\right)\left(\frac{-ig^{\alpha\rho} - k^\alpha k^\rho/m_W^2}{k^2 - m_W^2}\right) \left[\bar{u}_s\gamma_\lambda(1-\gamma^5)\frac{\not{k} + m_u}{k^2 - m_u^2}\gamma_\rho(1-\gamma^5)u_d\right]\left[\bar{v}_s\gamma_\alpha(1-\gamma^5)\frac{\not{k} + m_u}{k^2 - m_u^2}\gamma_\sigma(1-\gamma^5)v_d\right]$$

- CKM factors
- quark masses

2. Hadronic corrections

$$M_{12} = \frac{G_F^2 m_W^2 m_{B_d}}{12\pi^2}\left(\sum_{\alpha\beta}\left(V_{\alpha d}^* V_{\beta b}\right)^2 S_0\left(\frac{m_\alpha m_\beta}{m_W^2}\right)\eta_B\right)\hat{B}_{B_d} f_{B_d}^2$$

Illustration of formula for M_12

$$M_{12} = \frac{G_F^2 m_W^2 m_{B_d}}{12\pi^2} \left(\sum_{\alpha\beta} V_{\alpha d}^* V_{\alpha b} V_{\beta d}^* V_{\beta b}\, S_0\left(\frac{m_\alpha m_\beta}{m_W^2}\right) \eta_B \right) \hat{B}_{B_d} f_{B_d}^2$$

1. Amplitude for free quarks

$$\begin{aligned}\mathcal{M}_{uu} &= i\left(\frac{-ig_w}{2\sqrt{2}}\right)^4 (V_{us}^* V_{ud} V_{us}^* V_{ud}) \\ &\int \frac{d^4k}{(2\pi)^4} \left(\frac{-ig^{\lambda\sigma} - k^\lambda k^\sigma/m_W^2}{k^2 - m_W^2}\right)\left(\frac{-ig^{\alpha\rho} - k^\alpha k^\rho/m_W^2}{k^2 - m_W^2}\right) \\ &\left[\bar{u}_s \gamma_\lambda (1-\gamma^5) \frac{\not{k} + m_u}{k^2 - m_u^2} \gamma_\rho (1-\gamma^5) u_d\right]\left[\bar{v}_s \gamma_\alpha (1-\gamma^5) \frac{\not{k} + m_u}{k^2 - m_u^2} \gamma_\sigma (1-\gamma^5) v_d\right]\end{aligned}$$

- CKM factors
- quark masses

2. Hadronic corrections

Illustration of formula for M_12

- for different mesons, different quarks 'dominate' inside the loop
- e.g. for B mesons, the top quark dominates:

$$M_{12}^{q} = \frac{G_F^2 m_W^2}{12\pi^2} \left(V_{tq}^* V_{tb}\right)^2 S_0\left(\frac{m_t^2}{m_W^2}\right) \eta_B \hat{B}_{B_q} f_{B_q}^2 m_{B_q}$$

CKM phase of M_12: we'll need this later on

Illustration of computation for G_12

- this if for B mesons as well:

top doesn't actually contribute:
this uses unitarity to replace phase of up quark contribution

$$\Gamma_{12}^{q} = -\frac{G_F^2 m_b^2}{8\pi^2}\left[(V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{cb}\,\mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) + (V_{cq}^* V_{cb})^2\,\mathcal{O}\left(\frac{m_c^2}{m_b^2}\right)\right] \eta_B' \, \hat{B}_{B_q} f_{B_q}^2 m_{B_q}$$

- to take away from this
  - computations are very hard work
  - only source of phases are CKM phases
  - uncertainty on size and phase of M_12/G_12 reasonably small