ON THE DESIGN OF AIRFOIL SECTIONS UTILISING COMPUTER GRAPHICS

by

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Note: During the academic year 1966-1967 the author spent a sabbatical year at the Lockheed Georgia Research Laboratory. There pioneering research was being performed on "Computer Graphics". This new technique seemed to be extremely useful as a tool for airfoil design. During that year the foundation was laid for the author's later involvement with airfoil design. After returning to Delft this wonderful experience was transferred to staff and students. One of them (L.M.M. Boermans; at present associate professor at Delft and president of OSTIV) further developed this technique and became a well known sailplane designer. The author's cooperation with Prof. Boermans has lasted until the present day and in fact has been the stimulus to develop the new version of the method.

On the design of airfoil sections utilizing computer graphics [1])

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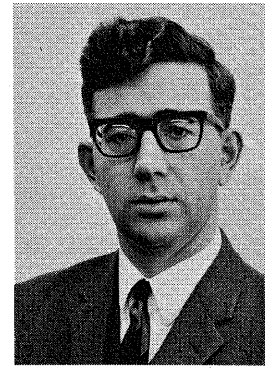

Summary: This paper will describe a computer programme which enables a designer not only to analyse the low-speed characteristics of an airfoil of arbitrary shape but also to obtain airfoils with improved pressure distributions. Use of the programme is facilitated by the application of computer graphics techniques. This implies that:

1. the results of calculations are presented by the computer in graphical form on a cathode-ray tube; and
2. the designer can communicate directly with the computer via the graphic display using a keyboard or a 'lightpen'.

1. Introduction

Designing a new airfoil for an advanced aircraft means finding a compromise among many conflicting requirements. Behaviour of a wing in cruising flight is vastly different from that in take-off and landing. It is often necessary, therefore, to perform a variety of theoretical and experimental investigations on many different airfoils before an optimum design can be selected.

[1]) Based on a lecture presented by the author to the Netherlands Association of Aeronautical Engineers on March 21, 1968.

During the past half century, a mass of knowledge has been accumulated about airfoils and wing designs. This knowledge is in the form of reference works such as the NACA catalogues of airfoils, and company-developed designs.

Up till now, the wind tunnel has been used extensively to derive the data required by the aeronautical engineer to complete his design. Wind tunnels will be used for this purpose for quite some time to come, but the state-of-the-art in aerospace sciences and modern computing equipment have advanced to the point where attempts may now be made to make a first selection of an airfoil on a theoretical basis. To be truly effective, these theoretical investigations have to be arranged so that results can be made available to the designer quickly and in such form that he can easily judge the merits of a certain design. It is precisely in this area that man-computer graphics can play an important role.

The foundations of the modern theories of airfoils in incompressible flow have been laid in the early 1900's. The classical potential flow theory in which viscous effects are neglected, failed to explain two important features of flow around airfoils that were observed in practice. The theory predicted a vanishing drag for an airfoil, in contrast to experimental results. In addition, the theory states that the lift of an airfoil increases as its angle of attack is increased up to a full 90°. This is only true for relatively small angles of attack. At high angles of attack an airfoil 'stalls', loosing lift rapidly. Both the drag and the occurrence of stall can be explained by the effects of the viscosity of air. Under normal flight conditions the direct effects of viscosity remain confined to a thin 'boundary layer' adjacent to the airfoil surface and a 'wake' downstream of the airfoil. The boundary-layer concept, as initiated by Prandtl in 1904, has to be taken into account when realistic predictions of airfoil characteristics are to be made.

Two aspects of boundary-layer flow are of utmost importance. The first is that a boundary layer may be either 'laminar' or 'turbulent'. While the boundary layer is neatly stratified in laminar flow, there is a strong mixing of air between different strata of the turbulent boundary layer which produces a high friction drag. Generally, on a smooth airfoil, the boundary layer is laminar over the front surface and turbulent over the rear areas. The point of *transition* between laminar and turbulent flow is determined by such factors as airfoil shape, angle of attack and flight speed. Due to the large difference in drag between laminar and turbulent boundary layer flows, it is important to know the point of transition and, where possible, to delay its occurrence by proper shaping of the airfoil. This will reduce drag significantly in many cases.

The second feature relates to *separation* of the boundary layer. This means that the flow no longer follows the airfoil surface, but breaks away from it. This separation process, which can occur for both laminar and turbulent flow, is not accounted for in the potential flow theory and is responsible for the stalling of aircraft wings.

The behaviour of the boundary layer is dictated by the streamwise pressure gradient. This gradient, in turn, depends on the shape of the airfoil and its angle of attack relative to the oncoming airflow.

Successful design of airfoils, therefore, requires combining the potential flow theory and boundary layer theory. One of the first successful applications of this principle was based on the discovery of the fact that transition of the boundary layer on a smooth airfoil is delayed when the pressure decreases in the streamwise direction. This is achieved for small values of the angle of attack, such as occur in cruising flight, by designing the airfoil to have its maximum thickness far backwards. These 'laminar-flow' airfoils have been used extensively since about 1940.

Along with the introduction of laminar-flow airfoils came the development of procedures to design airfoils having a required pressure distribution at a given angle of attack. Now, with the advent of modern high-speed computers, a closer and more complete integration of potential-flow and boundary-layer theories has become possible. For instance, methods have been developed whereby the shape of the airfoil is dictated in more detail by the pressure distribution required for a favourable development of the boundary layer than was the case for the older laminar-flow airfoils.

In this article a computer programme for airfoil-section design will be discussed in which the above-mentioned combination of potential-flow theory and boundary-layer theory has been accomplished. Use of the programme is facilitated by the application of computer graphics techniques. This means that the results of calculations are displayed by the computer in graphical form on a cathode-ray tube while the designer can interact with the programme via this display using a keyboard or a 'lightpen' (see section 2).

In the programme a conformal mapping method is used to obtain the potential flow around an airfoil from the known flow around a circular cylinder. The programme also contains a number of subroutines to enable calculations of the laminar boundary layer, the location of the transition point and the development of the turbulent boundary layer. These calculations require the pressure distribution as an input. On the other hand, the presence of the boundary layer changes the effective shape of the airfoil. This changes the pressure distribution and, in turn, the boundary-layer characteristics. The programme accounts for this mutual interaction. Finally, once the boundary layer has been calculated from the leading edge to the trailing edge of the airfoil, the drag coefficient is also known.

Space does not permit to describe these aerodynamic analysis subroutines in detail. In sections 3 and 4 a very brief review of them will be given. A more extensive discussion of the theory may be found in [1] and [13].

The airfoil analysis and synthesis programme, as described in this paper, is applicable only to two-dimensional incompressible flows. An extension to flows of higher speeds can be made by changing the aerodynamic subroutines involved, to account for the effects of compressibility.

The computer graphics applications, discussed in this paper, have been developed during the academic year 1966-1967 while the author was working with the Lockheed Georgia Company. The necessary graphics techniques have been developed by the Systems Sciences Group and the Department for Advanced Scientific Computing. In the early stages of the programme much work has been done by F. Akridge and J. Kennedy; later developments of the graphics part of the programme have been made by M. E. Haas. Some aerodynamic parts of the programme have been further developed in Delft; the author is indebted to ir. J. J. H. Blom for his help in this respect.

2. The principles of computer graphics

In many engineering problems it is essential that the results of computations are available – preferably in graphical form – before a decision can be made as to how the computation has to be continued. This is especially true in design problems were some alternative solutions may have to be analysed before one of them is selected for further refinement. In the conventional batch-type operation of computing centres the long turn-around time of programmes prevents an efficient use of modern computing methods in many areas of design. It is in this field that the application of man-computer graphics can improve the situation to a large extent. In this paper the term man-computer graphics will be used to refer to a process where:

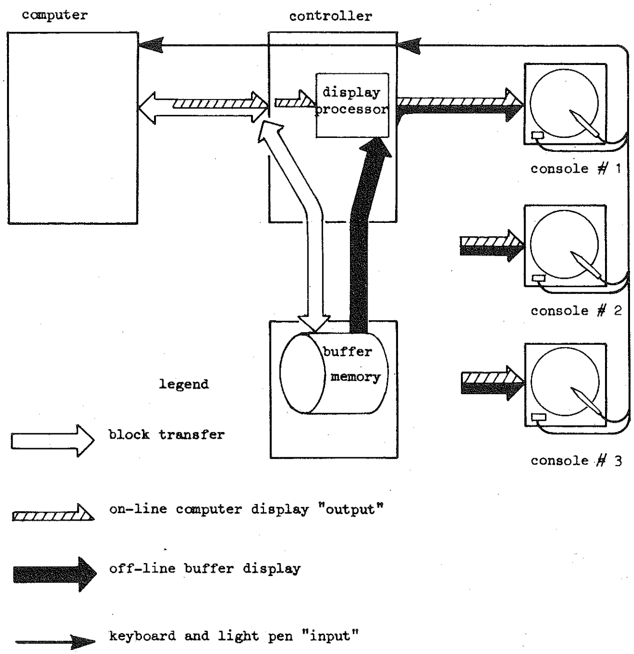

Fig. 1. Schematic diagram of Digigraphic system.

1. a computer displays results of calculations on a cathode-ray tube;
2. the user of the programme can communicate with the computer directly via the screen, using a keyboard or a lightpen.

In this way the experience and intuition of the human designer can be combined in an ideal way with the high-speed and large-memory capacity of the computer. The airfoil design programme, referred to in this paper, utilizing computer graphics has been run on Lockheed Georgia's Control Data Corporation (CDC) 3300 computer, coupled to the CDC Digigraphic system (Fig. 1). The display consoles each house a 20-inch diameter display tube and a fiber optics/photomultiplier lightpen.

Display commands to draw lines or figures are usually written into the drum memory and supplied by the drum to the console at a rate of 30 frames per second. These commands may also be routed directly to the display from the computer. The lightpen can be pointed at any item on the display to enable the Digigraphic system to identify that item to the computer.

Several separate display consoles may be operated simultaneously with the computer and each display may be used for a different computer-aided design or computer graphics application. Standard subroutines to display messages, draw axis lines, cross-plot arrays, position 'light buttons', control lightpen tracking, or to activate a type-in font are supplied by the system.

The 'light buttons' are display items used to activate subroutines when pointed to by the lightpen. Light buttons usually consist of words or short phrases. The type-in font is a displayed array of characters or numbers that, with the lightpen, serve, as a typewriter to input data to the computer. A 'tracking cross' displayed on the screen is programmed to move with the lightpen to indicate position.

Section 6 of this paper will describe how this computer-graphics equipment and related techniques can be applied to the airfoil design problem.

3. The theory of potential flow around airfoils

One of the ways in which the potential flow around an airfoil can be calculated is to transform the airfoil contour conformally into a circle. In the present programme a transformation method, due to Timman [1, 13] is employed. In the present section enough of this method will be discussed to enable the reader to understand the discussion of the programme in section 6.

Let the circle be in the ζ-plane and the airfoil, into which the circle is to be transformed, in the z-plane (Fig. 2) and furthermore let the transformation function be defined by

$$z = f(\zeta) \tag{1}$$

Two real functions σ and τ may now be defined by

$$\frac{\mathrm{d}z}{\mathrm{d}\zeta} = f'(\zeta) = \mathrm{e}^{\sigma + i\tau} \tag{2}$$

Making the complex potential Φ equal in corresponding points of both planes and using the relations:

$$U_c^* = \frac{\mathrm{d}\Phi}{\mathrm{d}\zeta}; \quad U_a^* = \frac{\mathrm{d}\Phi}{\mathrm{d}z} \tag{3}$$

we have

$$U_a^* = \frac{\mathrm{d}\Phi}{\mathrm{d}z} = \frac{\mathrm{d}\Phi}{\mathrm{d}\zeta} \bigg/ \frac{\mathrm{d}z}{\mathrm{d}\zeta} = U_c^* \, \mathrm{e}^{(\sigma + i\tau)} \tag{4}$$

In (3) and (4) U_c^* and U_a^* are the conjugates of the complex velocities in the planes of the circle and airfoil respectively. Taking absolute values in (4) gives

$$| U_a | = \mathrm{e}^{-\sigma} | U_c | \tag{5}$$

which shows that σ determines the ratio of $| U_a |$ to $| U_c |$.

Of course the pressure distribution around the airfoil can directly be determined from U_a using Bernoulli's law. The function τ gives the angle over which U_c has to be turned to obtain the direction of U_a (Fig. 2).

The value of U_c for any value of the angle of attack α follows from the well-known formula

$$U_c = 2U_\infty \left[\sin(\theta - \alpha) + \sin\alpha \right] \tag{6}$$

where the circulation has been given such a value that the rear stagnation point occurs at $\theta = 0$.

The geometry of the airfoil, including the radius of curvature of the contour, is easily obtained from an integration of (2) once σ and τ are known as functions of θ on the circle [1, 13].

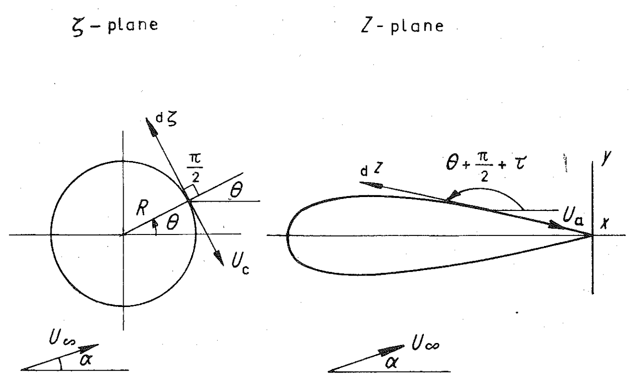

Fig. 2. Mapping of circle into airfoil.

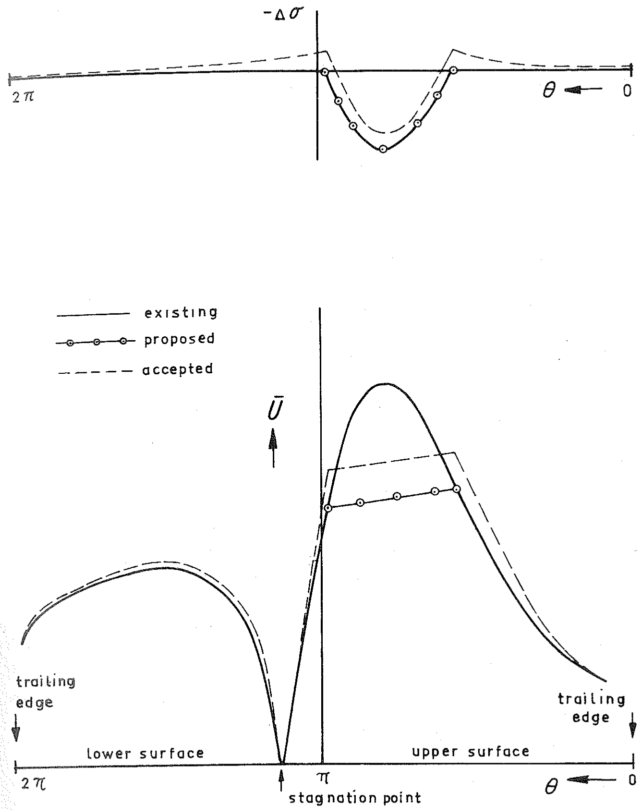

Fig. 3. Illustration of the procedure to find airfoils with improved pressure distributions.

This observation strongly recommends the use of σ and τ as primary functions describing the airfoil contour rather than using its co-ordinates x and y. It should be observed that σ and τ are the real and imaginary parts of an analytic function and hence are related by Poisson's integral [1, 13]. However, for convenience in the numerical calculations it is useful to record them separately.

To find the aerodynamic characteristics of an airfoil we need first to calculate the functions σ and τ. Only for very special classes of airfoils, such as the Joukowski and Kármán-Trefftz airfoils, the transformation function may be given in closed form. For arbitrary airfoils the functions σ and τ have to be tabulated for a large number of points on the circle. In all applications to be discussed in the present paper 100 points, evenly spaced around the circumference of the circle, have been used. The values of σ and τ in these points follow from an iteration process in which a Kármán-Trefftz airfoil, having the same trailing-edge angle as the given airfoil, is used as a first approximation [13].

Often the designer will not be satisfied with the characteristics of a given airfoil and he may want to change the functions σ and τ such that the pressure distribution changes in a required way. This can be done as follows. Let the pressure distribution for the existing airfoil be known at a certain value of the angle of attack and let it be plotted in the form of $\bar{U} = U_a/U_\infty$ versus θ (Fig. 3). The designer now may want to change this curve in a certain interval to the curve marked -o-o-o- in Fig. 3. Then the existing and proposed values of $\bar{U}$ define a correction $\Delta\sigma(\theta)$ to $\sigma(\theta)$ by the following equation, derived from (5):

$$\Delta\sigma(\theta) = -\ln \mid \bar{U}_{\text{proposed}}/\bar{U}_{\text{existing}} \mid \qquad (7)$$

It should be remarked that arbitrary modifications to σ are not allowed because these might produce airfoils which are not closed at the trailing edge [1, 13]. Therefore the computer programme has to check whether certain closure conditions are satisfied. If this is not the case certain restrictions on $\Delta\sigma(\theta)$ are imposed to ensure closure; this may lead to a permissible $\Delta\sigma(\theta)$ and hence a permissible pressure distribution which is slightly different from what the designer requested (Fig. 3). Once the correction $\Delta\sigma(\theta)$ is known, the related correction $\Delta\tau(\theta)$ can be calculated; then the new functions σ and τ define the new airfoil using (2).

4. The effects of viscosity

The present programme contains a number of subroutines which calculate the laminar boundary layer, the transition point location and the turbulent boundary layer. Many different methods are available for these calculations; in the present programme a certain choice between these methods has been made such that quickly a working overall-programme was obtained. However, the complete programme has been given a modular construction such that any subroutine can easily be replaced by a better one as soon as it becomes available.

In the present version the laminar boundary layer is calculated by a simple Pohlhausen-type method [2, chapter 5]. The transition location is calculated by an improved version [2, chapter 9] of a method developed independently by Smith and Gamberoni [3] and the present author [4]. The turbulent boundary layer is calculated by means of Head's entrainment method [5].

An important problem in the prediction of maximum lift is a knowledge of the behaviour of laminar separation bubbles. After the laminar boundary layer separates from the surface, in general, transition occurs rather rapidly because the separated layer is very unstable. Due to the turbulent mixing the separated flow may reattach to the surface, leaving a small region with back flow. This is a so-called short laminar separation bubble. The direct influence of such a bubble on the stalling characteristics is small and is neglected in the present programme.

Under certain circumstances, however, it is found that the separated flow, although it becomes turbulent, fails to reattach to the surface. The short bubble 'bursts' and a 'long bubble' is obtained. This bubble-bursting has a marked effect on the stalling characteristics resulting in a sudden loss of lift. It is clear that it is absolutely necessary to be able to ascertain whether a short or a long bubble will occur if a prediction of maximum lift of the airfoil is to be made.

A well-known criterion to discriminate between short and long bubbles has been advanced by Owen and Klanfer [6]. It says that the bubble will be short if the Reynolds number based on the momentum loss thickness of the boundary layer at separation is higher than 105-135; otherwise a long bubble will occur. This criterion is found to be rather crude, however. At the time of writing this paper some other criteria based on [7] and [8] were being tried out.

Once the boundary layer has been calculated, the profile drag coefficient can be determined from a well-known formula due to Squire and Young [10] which relates the drag coefficient to the boundary-layer momentum loss thickness at the trailing edge.

In first approximation the potential-flow pressure distribution is used as a basis for the boundary-layer calculation. Only near the trailing edge a correction is made to this pressure distribution in order to eliminate the unrealistic rear stagnation point. Once the boundary-layer displacement thickness δ_1 has been calculated the airfoil contour can be modified by adding δ_1 to it. This gives an airfoil with a tail extending to infinity in the wake. The influence of this correction is threefold:

- a change in angle of attack,
- a change in thickness, and
- a change in camber.

In order to eliminate the difficulty of having to deal with an airfoil which extends to infinity downstream, only the effects (1) and (3) have been incorporated in the present programme.

When it is assumed that the corrections are small it is justified to use thin airfoil theory to calculate the corresponding corrections to σ and τ. This allows us to correct the lift and pressure distribution for the presence of the boundary layer, the boundary layer can be recalculated, etc. This iteration process normally is found to converge after a few steps.

5. Some examples

The present section shows some results for the NACA 0012 airfoil which have been obtained with the latest version of the programme (without graphics capability) which is written in Fortran IV and run on the IBM 360-65 system at Delft University of Technology.[2]) Fig. 4 shows the result of the iteration procedure to obtain σ and τ for the given airfoil. Except for the trailing-edge angle, the approximate Kármán-Trefftz airfoil (no. 1 in Fig. 4) was chosen very different from the final airfoil. It is seen that the convergence to the final airfoil shape is very rapid (no. 1, 2, 3, 4 in Fig. 4.).

In general for this type of airfoil 5-figure accuracy is obtained after 5 iterations which in total takes about 5 seconds running time on IBM 360-65. Figs. 5 through 7 show the application of the procedure to obtain airfoils with improved pressure distributions to the NACA 0012 airfoil at zero angle of attack. A new function $\bar{U}$ was proposed which would modify the pressure distribution into one of the 'laminar-flow' type. The proposed and accepted functions $\bar{U}$ are shown in Fig. 5; the related functions $\Delta\sigma$ are given in Fig. 6. The existing and modified airfoils are shown in Fig. 7.

It follows that the proposed new pressure distribution has not been fully acceptable. The correction needed to make it acceptable has resulted in an airfoil which shows a pressure distribution which is less favourable to maintain laminar flow than was intended.

As another example, Figs. 8 and 9 show some results of applying the boundary-layer calculation procedures to the unmodified NACA 0012 airfoil. Fig. 8 shows the variation of the drag coefficient c_d with Reynolds number R_c at zero angle of attack. The lift curves for different values of the Reynolds number are shown in Fig. 9. Included is the correction for the presence of the boundary layer as discussed in section 4. It follows that the reduction in lift-curve slope is predicted very well at low angles of attack. At high angles of attack the present method is not yet satisfactory; also the drag coefficients (not shown) appear to be consistently too low at high angles of attack. Although it cannot be expected that a simple correction method as employed here would give very accurate results when appreciable separation occurs, it is felt that better results than shown here should be possible. The results shown in Fig. 9 have been obtained only recently; their improvement is the subject of current investigations.

6. Description of the complete programme utilizing computer graphics

The programme calculates all relevant quantities in N points which are evenly spaced around the circumference of the circle. Consequently, they are not evenly spaced around the airfoil contour but will have the closest spacing near the leading and trailing edges. In most applications $N = 100$ has been used. A schematic diagram of the programme is given in Fig. 10, where all the elements discussed in previous sections can be found.

Results of the programme may be obtained in the form of:

- printed output,
- graphical display on the scope, or
- graphical display on paper using a plotter.

After starting the computer and reading-in the programme together with certain fixed data, all further manipulations can

[2]) This computer system includes an IBM 2250 graphic-display unit; work is in progress to add the graphics capability to the programme which at present is used in Delft.

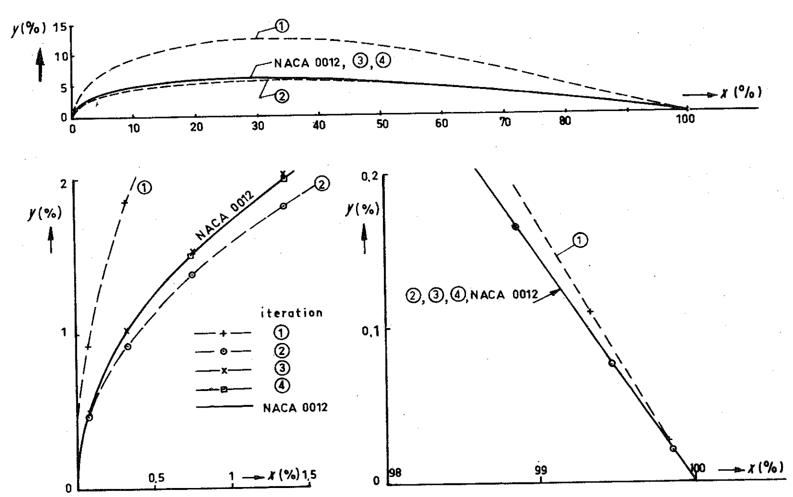

Fig. 4. Results of the iterative method to find the conformal transformation for the NACA 0012 airfoil.

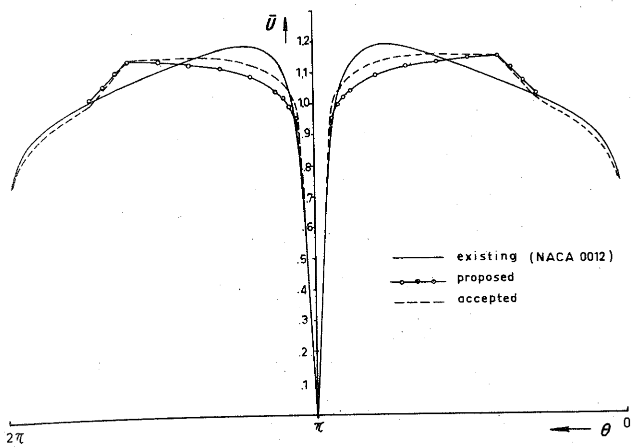

Fig. 5. Modification of pressure distribution of NACA 0012 into one of the 'laminar-flow' type ($\alpha = 0$).

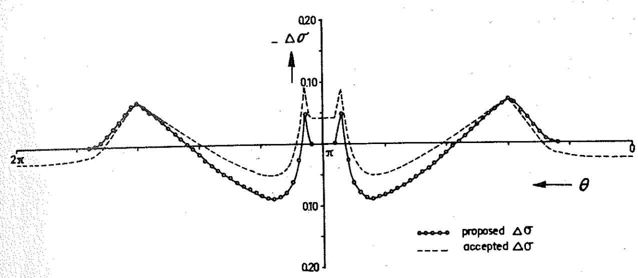

Fig. 6. The function $\Delta\sigma$ used to modify NACA 0012.

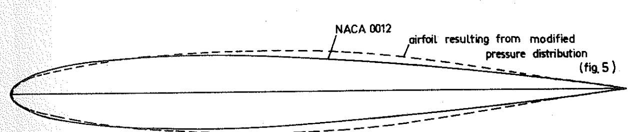

Fig. 7. NACA 0012 airfoil modified to 'laminar flow' type.

be performed on the scope. The diameter of the scope is 20 in, on which a work area of 10 in × 17-in has been defined. The 10 in height of this area corresponds to the 10 in width of the graph paper used on the plotter.

Figures which will appear successively in the same area of the scope will be plotted with a 17-in spacing in the horizontal direction on the plotter. In what follows only the scope display will be discussed using the photographs reproduced in Figs. 11.1 through 11.6. The early version of the programme which was used to generate the displays shown in these pictures allows the following operations to be performed:

1. Using the lightpen and typefont, an initial airfoil may be chosen which is either a Kármán-Trefftz airfoil or any airfoil generated previously by the programme (Fig. 11.1). The airfoil is displayed twice; first in terms of x/R and y/R and secondly in terms of x/c and y/c where R is the radius of the circle and c is the chord of the airfoil, defined as the straight line connecting the forward and rearward extremities of the airfoil (Fig. 11.2). The computer then asks 'Do you like airfoil?' With the lightpen either the answer 'yes' or 'no' may be indicated on the scope. When the answer is 'no' the parameters defining the initial airfoil may be varied until a satisfactory shaped airfoil has been obtained which can serve as a starting airfoil for further investigations.
2. As soon as the answer 'yes' is given the computer asks for the value of the Reynolds number for which subsequent calculations are to be made.
3. Next a choice can be made from the following 'menu' of subroutines (Fig. 11.3):

a. PRESSURE
b. BOUNDARY LAYER
c. CHANGE
d. NEW RC
e. DONE

After execution of any of the subroutines a. through d. the programme returns to 3; after executing DONE it returns to 1.

PRESSURE asks for a value of the angle of attack α and calculates the pressure distribution for that value of α. The result is displayed in the form $\bar{U} = U_a/U_\infty$ versus x/c in the lower left-hand corner of the work area (Fig. 11.3). This subroutine can be repeated for other values of α.

BOUNDARY LAYER asks for a value of α and then calculates for both surfaces of the airfoil:

- the development of the laminar boundary layer,
- the location of transition, and
- the development of the turbulent boundary layer.

The results are displayed in the right-hand side of the work area (Fig. 11.4). All relevant quantities are plotted versus the angle θ on the circle. The plot contains, for both the laminar and the turbulent boundary layer, values of the skin friction coefficient, shape factor and boundary layer thickness. In addition for the laminar boundary layer the amplification factor

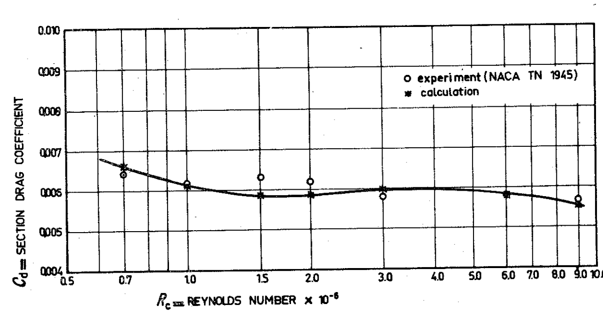

Fig. 8. Variation with Reynolds number of section drag coefficient at $\alpha = 0$ for NACA 0012.

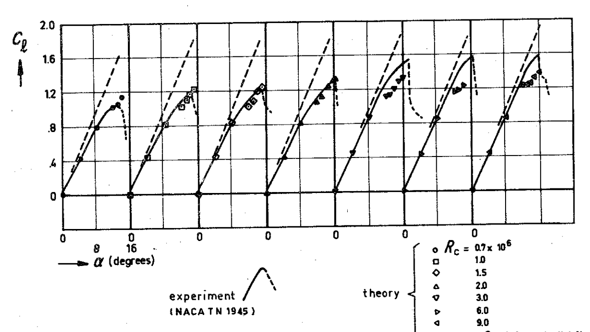

Fig. 9. Comparison of theoretical and experimental lift curves for the NACA 0012 airfoil at different values of R_c.

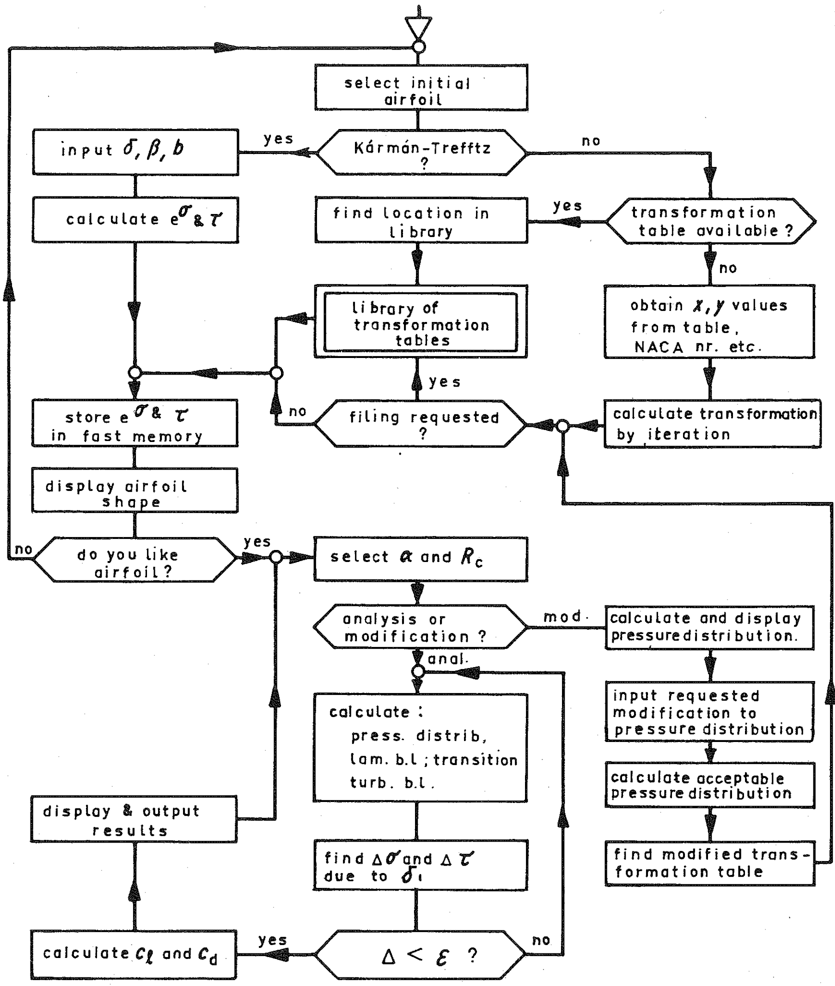

Fig. 10. Schematic diagram of complete programme.

of unstable disturbances is shown, indicating the danger of transition.

In the boundary layer plot and the pressure distribution plot the positions of some important points are indicated with different symbols for laminar separation (short/long bubble), transition point and turbulent separation.

The drag coefficient for the lower surface and both surfaces together as well as the lift coefficient are shown in the upper circular segment of the scope. When BOUNDARY LAYER is called again for a different angle of attack, the results of the preceding boundary-layer calculations will be erased from the right-hand side of the work area. The pressure plot with the indicated positions of separation and transition as well as the c_l— α and c_d— α plots remain displayed however. After executing PRESSURE and BOUNDARY LAYER for some values of α the scope will appear like indicated in Fig. 11.4. In general the lift and drag characteristics of the initial airfoil will not satisfy the design specifications. Hence we may try to change the airfoil in such a way that more favourable pressure distributions will be obtained. This may be achieved by calling the subroutine CHANGE. This routine asks for a value of α and displays the pressure distribution in the form $\bar{U}$ versus θ in the same area of the scope which was occupied earlier by the results of BOUNDARY LAYER (Fig. 11.5). In this plot a new pressure distribution may be proposed by manipulating with the lightpen; the computer then calculates the acceptable pressure distribution and the corresponding new airfoil shape. The results of these calculations appear as shown in Fig. 11.5. The new airfoil is shown superimposed on the old one so that the difference in shape may be seen on the scope. If needed the old airfoil may be erased; as soon as 4 airfoils are present on the scope the oldest one will be erased automatically when the next modification to the airfoil shape is made. It should be understood, however, that the characteristics of all airfoils are available in the form of the printed and plotted output.

The new airfoil can now be analysed. Fig. 11.6 shows the new pressure distributions for instance.

NEW RC allows the introduction of a new value of the Reynolds number based on chord length c.

DONE returns the program to its starting configuration.

7. Concluding remarks

It has been shown that the application of man-computer graphics greatly facilitates the design of airfoil sections.

It should be emphasized that the results of the present programme will not be better than the accuracy of the underlying calculation methods allows. Therefore the capabilities and limitations of the programme should be carefully assessed by comparison with accurate experiments. The accuracy of the methods should be continuously reviewed against new experimental results and, where possible, be improved.

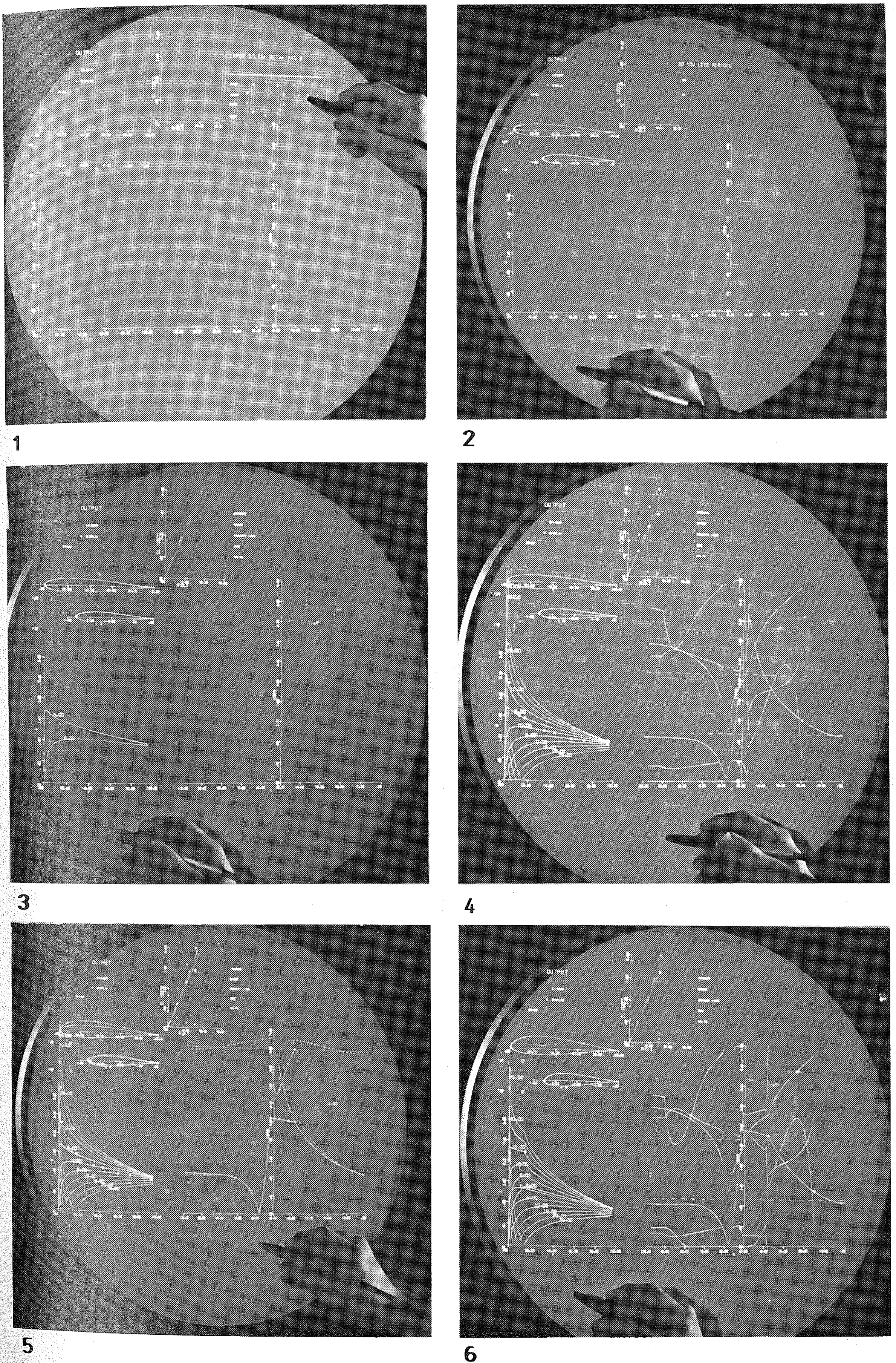

Fig. 11.1 ... 11.6. Successive scope displays during execution of airfoil design programme.

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