TECHNOLOGICAL UNIVERSITY DELFT

DEPARTMENT OF AERONAUTICAL ENGINEERING

Report VTH-124

THEORETICAL AND EXPERIMENTAL INVESTIGATIONS OF INCOMPRESSIBLE LAMINAR BOUNDARY LAYERS WITH AND WITHOUT SUCTION

Ph.D THESIS

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the NETHERLANDS

OCTOBER, 1965

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Summary, contents and list of symbols

Summary.

In this dissertation the results are presented of some theoretical and experimental investigations of two-dimensional laminar boundary layers with and without suction. Throughout the work the velocities involved are assumed to be of such a small magnitude that the effects of compressibility can be neglected. The investigations were undertaken with the purpose to clarify some points concerned with maintaining laminar flow in a boundary layer by means of suction through a porous surface. In the course of this work several results were obtained which also may be of interest for laminar boundary layers without suction.
A first investigation is concerned with the calculation of laminar boundary layers by means of approximate methods of the type introduced by Pohlhausen. A new method is described which, by a special choice of the velocity profile, is capable of providing accurate results in those cases where the suction velocity is not too large.
The second theoretical investigation deals with a "phase plane" description of the laminar boundary layer flow between non-parallel plane walls. Here shear τ is plotted versus the velocity component u parallel to the wall.
This is analogous to the use of the phase plane method in the theory of non-linear oscillations with one degree of freedom where speed is plotted versus displacement. In the latter theory singular points in the phase-plane correspond to equilibrium positions of the oscillation. For the flow between non-parallel plane walls the singularities in the "phase plane" are shown to correspond to the edge of a boundary layer. It is shown that the occurrence of boundary layer type solutions depends on the character of the singularity which is determined by the amount of suction.
For the case of inflow between converging walls without suction τ^2 can be expressed as a polynomial in u. From this observation a new calculation method for laminar boundary layers evolves which is described in detail. The method assumes for τ^2 a polynomial expression in u with coefficients depending on the streamwise coordinate x. These coefficients are determined from compatibility conditions and from moments of a modified form of Crocco's boundary layer equation. In contrast to existing

approximate methods the new approach allows the degree N of the polynomial to be increased without unduly complicating the method. For increasing N the results of the approximate method seem to converge to the exact solution.
The experimental part of the work consists of measurements on two airfoil sections in a low speed wind tunnel. The first model is a 28°/o thick laminar flow airfoil section with an impermeable surface and a chord length of 1 meter. A detailed survey of the velocity profiles in the laminar boundary layer was made with hot wires; the measurements were extended so far downstream as to include the laminar separation point. Results of the measurements and a comparison with laminar boundary layer theory are presented.
The second model is a 15°/o thick, 1.35 meter chord, laminar flow wing section with porous upper- and lower surfaces between the 30°/o and 90°/o chord positions. The inside of the model is divided into 40 different compartments each with its own suction line, flow-regulating valve and -measuring device. Hence the chordwise suction distribution could be varied between wide limits. Wake drag and transition position were measured for several suction distributions; for some of these detailed boundary layer surveys were made. In one case the suction distribution was chosen in such a way that a separating laminar boundary layer was obtained.
From the transition measurements on the porous model a semi-empirical method is derived which permits the determination of the transition position for two-dimensional incompressible laminar boundary layers with arbitrary pressure- and suction distributions. This method is an extension of an existing method which was shown to be valid for the no-suction case both by Smith and Gamberoni [1,2] and the present author [3,4,5].
The boundary layer calculation methods and the transition criterion provide the means for a rational design of the suction distribution needed to maintain laminar flow for a given pressure distribution. For instance a suction distribution may be determined for which the total drag coefficient is as small as possible.

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List of symbols.

Symbols which are used only locally are defined in the text; other symbols are listed below.

a	coefficient in eq. (4.1)
a	scaling factor for y; in chapter 5 only; numerical value 1.3
a_1	constant in equations (4.13) and (5.39)
a_n	coefficient in polynomial expression (7.29)
a_o	$\left(\frac{\tau_o x}{\mu U}\right)^2 \left(\frac{Ux}{\nu}\right)^{-1} = \bar{\tau}_o^{\,2}$
$a_{n,p}$	coefficients in series expansion (7.92) for a_n
Δa_n	increment of a_n; equation (7.70)
δa_n	increment of a_n; section 7.8
$\bar{a}_n$	starting value for a_n in sections 7.7 and 7.8
a_n^*	defined by equation (8.1)
A	integration constant in chapter 6
A_{ij}	elements of inverse matrix in chapter 7
b	coefficient in equation (4.1)
b	scaling factor for y; in chapter 5 only; numerical value 0.3
b	spanwise extent of porous surface in chapter 11 (0.805 m)
b_1	constant in equations (4.13) and (5.39)
c	coefficient in equation (4.1)
c	reference length, equal to chord length for airfoil sections
$\bar{c}$	$c_r + i\, c_i = \frac{\bar{\beta}}{\bar{\alpha}} = \frac{\beta_r}{\bar{\alpha}} + i\,\frac{\beta_i}{\bar{\alpha}}$; equation (9.6)
c_{d_f}	friction drag coefficient of flat plate defined by equation (3.18)
c_{d_s}	suction drag coefficient
c_{d_t}	total drag coefficient
c_{d_w}	wake drag coefficient
c_p	pressure loss coefficient $\frac{\Delta p}{\frac{1}{2}\rho U_\infty^2}$

c_q suction flow coefficient $\frac{Q}{U_\infty c}$

c_{q_i} suction flow coefficient for compartment number i in chapter 11

c_{q_u} suction flow coefficient for the upper surface

c_{q_ℓ} suction flow coefficient for the lower surface

d coefficient in equation (4.1)

d length scale in Lin's formulae (9.19) and (9.20)

D dissipation integral defined by equation (2.23)

$2D^*$ $2\int_0^\infty \left(\frac{\partial u/U}{\partial y/\theta}\right)^2 d\,\frac{y}{\theta}$; nondimensional dissipation integral

e_{i_n} $\frac{\partial E_i}{\partial a_n}$ defined by equation (7.76)

E_i defined by equation (7.74)

$\bar{E}_i$ starting value for E_i in section (7.8)

f number denoting value of reduced frequency $\frac{\beta_r \nu}{U^2}$ in table 9.4

f exponent in equation (7.79); used in chapter 7 and 8 only

$f(\eta)$ nondimensional streamfunction: for chapter 3 defined by eq. (3.6)

$f(\eta)$ nondimensional streamfunction: for chapter 6 defined by eq. (6.8)

$f_1(\eta)$, $f_2(\eta)$, $f_3(\eta)$ defined by equations 5.6-5.9 in chapter 5

$f_1, f_3, f_5 \ldots$ defined by equation (3.25)

$F_{2n+1}(\eta)$ defined by eq. (3.24)

$F(\xi,\eta)$ nondimensional streamfunction in Görtler's series method; defined by eq. (3.29)

$F_o, F_{\frac{1}{2}}, F_1, \ldots$ Görtler's universal functions in section 3.2.4.

$F(\eta)$ defined by eq. (4.9) in chapter 4

$F(\Lambda_1)$ Pohlhausen's universal function defined by equation (4.12) in chapter 4

$F_1(\eta)$	
$F_2(\eta)$	defined by equations 5.3-5.9 in chapter 5
$F_3(\eta)$	
$G(\eta)$	defined by equation (4.10) in chapter 4
H	$\frac{\delta^*}{\Theta}$
$\bar{H}$	$\frac{\varepsilon}{\Theta}$
$j_{k,n}$	defined by eq. (7.55)
J_k	defined by eq. (7.52)
$J_{k,p}$	defined by eq. (7.98) and (7.99)
k	$0,1,2,\ldots K$
K	N-5 in chapter 7
K	shape factor in chapter 5
K_1, K_2	constants in equation (9.25)
ℓ	$\left(\frac{\partial u/U}{\partial y/\Theta}\right)_o = \frac{\tau_o \Theta}{\mu U}$
ℓ_1	$\frac{\sigma^2}{\nu}\frac{dU}{dx} = \bar{\sigma}^2 \frac{d\bar{U}}{d\bar{x}}$
ℓ_2	$\frac{-v_o \sigma}{\nu} = \bar{v}_o \, \bar{\sigma}$
L	shape factor in chapter 5
m	$\left(\frac{\partial^2 u/U}{\partial (y/\Theta)^2}\right)_o$
m_1	constant in equation 3.1
M	$2\ell - 2(2+H)\Lambda_1 - 2\Lambda_2$
M_k	defined by equation (7.51)
$M_{k,p}$	defined by equation (7.98)-(7.99)
N	degree of polynomial expression (7.29)
p	static pressure in chapter 2 and 9
p	order of series expansion in chapter 7 and 8
p'	fluctuation of static pressure in chapter 9
p^*	p + p' in chapter 9

p_t	free stream total pressure in chapters 10 and 11
$p_1, p_2, \ldots p_9$	coefficients in eq. (5.10)-(5.12)
p_x	static pressure on body surface at distance x from the stagnation point
$p_{k,n}$	defined by equation (7.55)
P_k	defined by equation (7.53)
$P_{k,p}$	defined by eqns. (7.98)-(7.99)
p_{w+}, p_{w-}	static pressures at side wall of wind tunnel (see fig.11.1)
Q	suction flow per unit span $m^3 sec^{-1} m^{-1}$
$q_{k,\ell,n}$	defined by equation (7.55)
Q_k	defined by equation (7.54)
$Q_{k,p}$	defined by equation (7.98)-(7.99)
r_p	defined by equations (7.94) and (7.96)
R	radius of cylinder in section 8.9
R_{δ^*}	$\frac{U\delta^*}{\nu}$
R_Θ	$\frac{U\Theta}{\nu}$
R_c	$\frac{U_\infty c}{\nu}$
S	$\bar{\tau}^2$
s	distance along contour of airfoil section, measured from leading edge (chapter 11 only)
$\bar{s}$	s/c
t	time
t_n	δa_n in section 7.8.
T	amplification rate of unstable disturbances defined by equation (9.17)
T_o	constant in eq. (9.25)
u	velocity component parallel to wall
u'	fluctuation of u
u^*	u+u' in chapter 9
$\bar{u}$	u/U in general; $u/\|U\|$ in chapter 6

$\bar{u}_p$	velocity measured by surface tubes; chapter 10
U	velocity component parallel to wall at edge of boundary layer
$\bar{U}$	U/U_∞
U_∞	reference speed, equal to free stream speed in chapters 10 and 11
v	velocity component normal to wall
v'	fluctuation of v in chapter 9
v^*	$v+v'$ in chapter 9
v_o	normal velocity at surface; negative for suction
$\bar{v}_o$	$\frac{-v_o}{U_\infty}\sqrt{\frac{U_\infty c}{\nu}}$
V	wind speed in section 8.9
x	distance along wall measured from stagnation point, (fig. 2.1)
x_p	distance along chord of airfoil section (fig. 2.1)
$\bar{x}$	x/c
y	distance normal to wall
y_p	ordinate of airfoil section
y_r	distance from reference position outside boundary layer (chapter 10)
$\bar{y}$	$\frac{y}{\delta} = \frac{y}{x}\sqrt{\frac{Ux}{\nu}}$
z	new variable, defined by eq. (7.79)
z_p	spanwise coordinate, chapter 10
α	angle of attack of airfoil section
α_1	$\bar{U}\,\bar{\delta}^2$ in chapter 7
$\bar{\alpha}$	$\frac{2\pi}{\lambda}$ in chapter 9 only
β	Hartree parameter defined by eq. (3.4)
$\beta_1, \beta_2, \beta_3, ..$	defined by eq. (7.68)
$\beta(\xi)$	Görtler's function defined by eq. (3.31)
$\bar{\beta}$	$\beta_r + i\beta_i$ in section 9.2.2.

β_o	$\frac{2m_1}{m_1+1}$ (eq. 3.36)
γ	$1-3\lambda_1$
δ	measure for boundary layer thickness in general
δ	defined by $\frac{\delta}{x}\sqrt{\frac{Ux}{\nu}}=1$ in chapter 7 and further
$\bar{\delta}$	$\frac{\delta}{c}\sqrt{\frac{U_\infty c}{\nu}}$
δ^*	displacement thickness $\int_0^\infty (1-\bar{u})dy$
$\overline{\delta^*}$	$\frac{\delta^*}{c}\sqrt{\frac{U_\infty c}{\nu}}$
δ_{k+2}	defined by eq. (2.26)
ε	energy loss thickness $\int_0^\infty \bar{u}(1-\bar{u}^2)dy$
η	$y\sqrt{\frac{m_1+1}{2}\ \frac{U}{\nu x}}$ in section 3.12
η	$y\sqrt{\frac{u_1}{\nu c}}$ in section 3.2.2.
η	defined by eq. (3.28) in section (3.2.4)
η	$\frac{y}{\delta}$ in chapter 4
η	$\frac{y}{\sigma}$ in chapter 5
η	$\frac{y}{x}\sqrt{\frac{\lvert u_1\rvert}{\nu}} = \frac{y}{x}\sqrt{\frac{\lvert U\rvert x}{\nu}}$ in chapter 6
η_1	$-\lambda\eta$ in chapter 6
θ	momentum loss thickness $\int_0^\infty \bar{u}(1-\bar{u})dy$
$\bar{\theta}$	$\frac{\theta}{c}\sqrt{\frac{U_\infty c}{\nu}}$
λ	$\frac{v_o x}{\sqrt{\nu \lvert u_1\rvert}}$ in chapter 6
λ	$\frac{\delta^2}{\nu}\ \frac{dU}{dx}$ in chapter 4
λ	wave length of disturbance in chapter 9
λ_1	$\frac{x}{U}\frac{dU}{dx} = \frac{\bar{x}}{\bar{U}}\ \frac{d\bar{U}}{d\bar{x}} = \bar{\delta}^2\ \frac{d\bar{U}}{d\bar{x}}$

λ_2	$\frac{-v_o}{U}\sqrt{\frac{Ux}{\nu}} = \bar{v}_o . \bar{\delta} = \frac{-v_o \delta}{\nu}$
$\lambda_{1,p}$	coefficients in eq. (7.80)
$\lambda_{2,p}$	coefficients in eq. (7.81)
Λ_1	$\frac{\theta^2}{\nu}\frac{dU}{dx} = \bar{\theta}^2 \frac{d\bar{U}}{d\bar{x}} = \ell_1 (\frac{\theta}{\sigma})^2$
Λ_2	$\frac{-v_o \theta}{\nu} = \bar{v}_o \, \bar{\theta} = \ell_2 \frac{\theta}{\sigma}$
μ	coefficient of dynamic viscosity
ν	$\frac{\mu}{\rho}$ coefficient of kinematic viscosity
ξ	Görtler's variable defined by equation (3.27)
ρ	density
σ	measure for boundary layer thickness in chapter 5
$\bar{\sigma}$	$\frac{\sigma}{c}\sqrt{\frac{U_\infty c}{\nu}}$
σ_a	amplification factor defined by eq. (9.16)
$(\sigma_a)_{max}$	maximum value of σ_a at a certain position
σ_p	defined by equations (7.96) and (7.97)
τ	$\mu \frac{\partial u}{\partial y}$ shear stress
$\bar{\tau}$	$\frac{d\bar{u}}{d\eta}$ in chapter 6
$\bar{\bar{\tau}}$	$\frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\tau\delta}{\mu U}$ in chapter 7 and 8
τ_o	wall shear stress
$\bar{\tau}_o$	$\frac{\tau_o \delta}{\mu U} = \sqrt{a_o}$ in chapter 7 and further
$\varphi(y)$	$\varphi_r + i\varphi_i$, amplitude function in equation (9.4)
ψ	stream function in chapter 3 and 6
ψ	$\psi_r + i\psi_i$ stream function of disturbance defined by equation (9.4)

subscripts

i	at the instability point
o	at the surface
sep	at the separation point
st	at the stagnation point
tr	at transition
te	at trailing edge
le	at leading edge

primes denote differentiation with respect to η in chapter 3 and 6 and to $\overline{u}$ in chapter 7. They denote fluctuation components in chapter 9.