SOME INTODUCTORY REMARKS ON TRANSITION PREDICTION BASED ON LINEAR STABILITY THEORY

by

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Note: The paper reviews the early developments of the eN method by the author and gives some comparisons with recent experimental verifications at the Low-Speed Aerodynamics Laboratory of the Faculty of Aerospace Engineering of Delft University

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Some Introductory Remarks on Transition Prediction Methods Based on Linear Stability Theory

Abstract

Nearly 40 years ago the e^n method was introduced independently by A.M.O. Smith and the present author. Further developments have been made by many researchers, extending it to higher speeds, three-dimensional flows and including the effects of suction and heat transfer. Various papers at the present Colloquium will discuss these developments.

That the method is still being used is on the one hand due to the inherent difficulties of transition prediction from first principles. On the other hand, the e^n method contains enough physics to allow it to "predict" the distance to transition with only a simple experimental calibration. It was realized from the beginning that not enough physics was included to predict the process of transition itself.

The paper reviews the early developments of the method by the author and gives some comparisons with recent experimental verifications at the Low-Speed Aerodynamics Laboratory of the Faculty of Aerospace Engineering of Delft University of Technology.

The birth of the e^n method

In 1956 the e^n method for transition prediction in 2D incompressible flow, using linear stability theory, was introduced simultaneously and independently by Smith & Gamberoni (1956a) and the present author (Van Ingen, 1956a,b). From the end of the 19th century to about 1940 linear stability theory had been developed by a large number of mathematicians and theoretical aerodynamicists. Only through the famous experiments by Schubauer and Skramstadt (1948) it was shown that the theory was indeed applicable to real flows (the experiments were done in the period 1940-1945, but due to the war conditions the results became only widely known in 1948).

Although Pretsch (1941, 1942) had already done some amplification calculations, it was only in the fifties that it was realized that linear stability theory might be used to bridge the sometimes large distance between the point of first instability and real transition.

Liepmann (1945) had postulated that at transition the maximum eddy shear stress due to the laminar instability would be equal to the maximum laminar

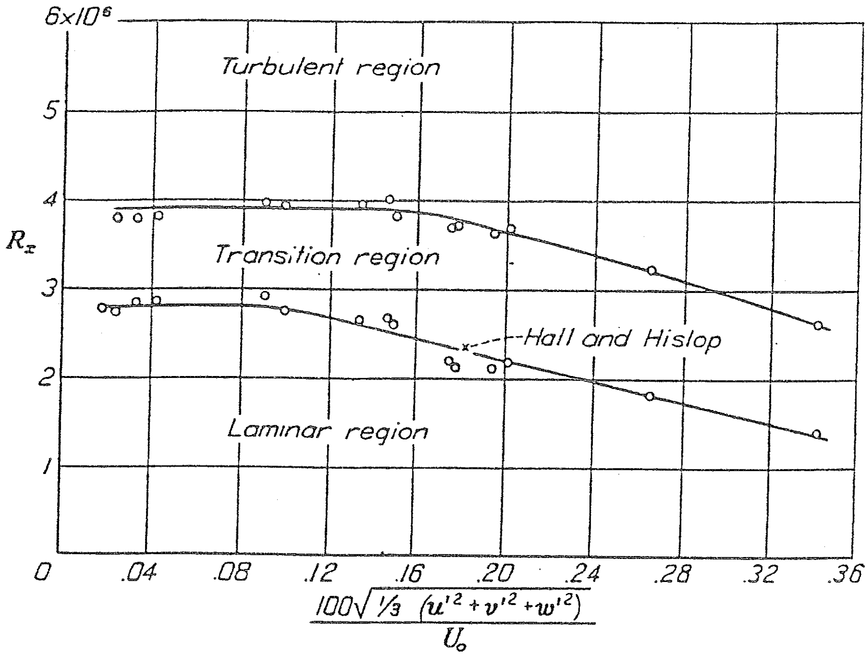

Figure 1: Transition Reynolds number for the flat plate according to Schubauer and Skramstadt (1948).

shear stress. This postulate was the starting point of the discussion by Smith and Gamberoni. Apparently Smith soon realized that it would be too ambitious to calculate the disturbance amplitude occurring in Liepmann's equation. Especially it was considered to be difficult - if not impossible - to specify the initial disturbances from which to start the amplification calculations. In fact up to the present time this remains a very difficult issue. How are disturbances generated inside the boundary layer? How are they related to outside disturbances like free stream turbulence, noise and vibration of the surface? At present this problem is denoted as "receptivity". Smith satisfied himself (and in fact had to be satisfied) with the calculation of the ratio between the amplitude of the most amplified disturbance according to linear theory at the experimental transition position and the original amplitude of this disturbance at its neutral position. From Smith's analysis it turned out that in many cases the same amplification ratio of about e^9 was found.

It is to be noted that under the many cases considered by Smith the Schubauer-Skramstadt flat plate experiment did not take a prominent place. The present author however started from this experiment (Fig. 1). At turbulence levels less than about 0.1% the transition region extends over a large distance, corresponding to Reynolds numbers Ux/ν from 2.8×10^6 to 3.9×10^6. In addition the present author considered some of his own transition experiments on an EC 1440 airfoil. Guided by the flat plate experiment this led to the conclusion that beginning and end of the transition region correspond to amplification ratios of $e^{7.8}$ and e^{10} respectively. On airfoils the transition region is in most cases only a few percent chord in length. Therefore it is not surprising that Smith, putting

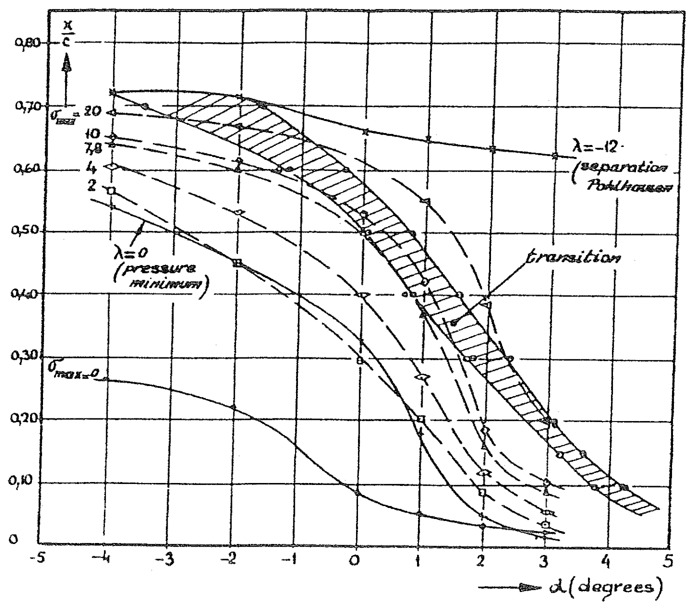

Figure 2: Calculated amplification factor and measured transition region for the EC 1440 airfoil section.

much emphasis on his results for airfoils, had concluded to a mean value of e^9. The exponent 9 was very close to the mean value indicated by van Ingen.

The present author, at that time not yet aware of Smith's results and not having access to as many transition experiments as Smith, had concluded that the prediction method looked promising but that more experiments would have to be evaluated. The reports by Smith and the present author were published only one month apart. A public paper on the method was given by Smith (1956b) in September 1956 at the 9th International Congress of Applied Mechanics at Brussels and by the present author (1956b) just one week later at the European Aeronautical Congress at Scheveningen.

It was at Scheveningen that Schlichting himself, having been present at both congresses, informed the present author about the presentation by Smith. Some of the results which the present author produced for the EC 1440 airfoil, making use of the Pretsch charts, are collected in Fig. 2. It should be noted that the factors 7.8 and 10 do not provide a very precise prediction of the transition region. This may have been caused by the fact that the laminar boundary layer was calculated by the Pohlhausen method which is known to be inaccurate near laminar separation. From Fig. 2 it follows that at the higher angles of attack transition occurs near or sometimes even downstream of laminar separation. Stability calculations were not available for separated flows (and hence Pretsch charts had to be extrapolated) and also the Pohlhausen method could not predict separated flows. It should also be realized that only later the possible existence of laminar separation bubbles was realized.

In 1956 both Smith and Van Ingen based their calculations on the temporal stability diagrams which had been calculated by Pretsch (1942) for some of the Falkner-Skan velocity profiles. Pretsch used an asymptotic method which was only applicable at very high Reynolds numbers. Therefore he had been unable to calculate the (very low) critical Reynolds number for the Falkner-Skan separation profile. To the present author the Pretsch diagrams were only available on the small scale presented in Pretsch (1942). Smith apparently had already available some larger scale diagrams from Pretsch (1945). Both authors had to do some tedious cross plotting from these charts. It should be noted that Pretsch was already aware of the fact that there may be a large distance between the position of the first instability and the actual transition position and that the then customary idea that transition location would be somewhere between the positions of instability and laminar separation, was not sufficiently precise. He even suggested that amplification calculations might give some more insight.

Since the Pretsch charts had been calculated for the temporal mode, a propagation speed of the disturbances had to be selected to calculate the streamwise development. The present author used the phase velocity in his first version of the method. Although Smith had realized that the group velocity should be taken, he used for convenience also the phase velocity. It should be noted that only in the sixties the importance of the group velocity was emphasized by Lighthill (1965) and especially by Gaster (1962).

To emphasize that in 1956 the available numerical results of stability theory were not very consistent, Table 1 gives the critical Reynolds number based on displacement thickness for the Blasius profile as calculated by different authors. A number of neutral curves for the flat plate boundary layer is shown in Fig. 3.

$\left(\frac{U\delta^*}{\nu}\right)_{crit}$	author
321	Timman (1956)
420	Tollmien (1929)
420	Lin (1945, 1946)
575	Ulrich (1944)
645	Schlichting-Ulrich (1942)
680	Pretsch (1941, 1942)

Table 1: Critical Reynolds numbers for the flat plate as calculated by various authors before 1956.

Extension of the e^n method to suction (1965)

In his Ph.D. thesis, Van Ingen (1965) demonstrated that the e^9 method could also be used for the case of porous suction. An extensive series of wind tunnel measurements was done (using filtering paper as a porous surface). At that time larger scale stability diagrams were available to the present author (Pretsch, 1945). These charts had been reduced to a database containing about 100 numbers. In order to be able to analyse the suction experiments a two-parameter integral method for the calculation of the laminar boundary layer with suction was developed. Since in 1965 still only the Pretsch charts were available to the

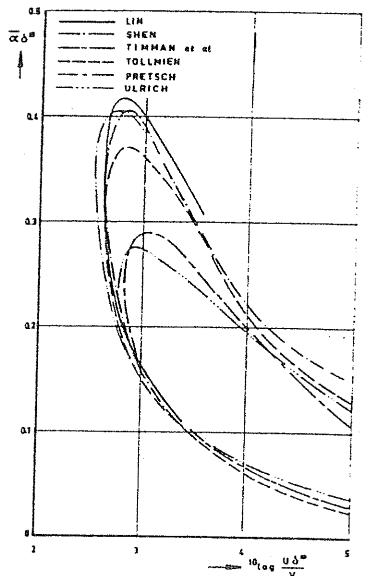

Figure 3: Neutral stability curves for e flat plate without suction from different sources.

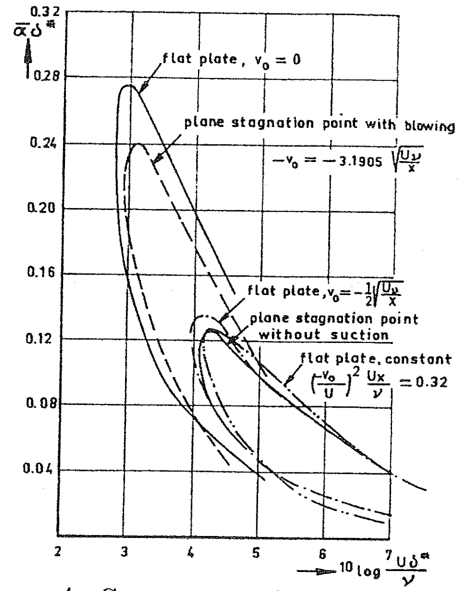

Figure 4: Some neutral stability curves for different boundary layers according to Ulrich.

present author, the database method mentioned above had to be made applicable to the suction case. This was done by assuming that:

- All possible stability diagrams form a one parameter family with the critical Reynolds number (based on momentum loss thickness) as the parameter;
- The critical Reynolds number for the velocity profiles used in the two-parameter method can be calculated from the approximate formula due to Lin (1955).

Fig. 4 gives a comparison of neutral curves for various flows with pressure gradient and/or suction or blowing. From such comparisons it was concluded in van Ingen (1965) that the above mentioned extension of the database method the suction case, where the effect of suction is replaced by an equal effect of the pressure gradient on the critical Reynolds number, might be a workable proposition.

It should be emphasized that each time one of the components in the whole e^n method is changed (new boundary layer calculation method, new database for the stability diagrams, possibly improved stability diagrams, new experiments in the same or a different wind tunnel or flight tests) the whole method will have to be re-calibrated. In this way the present author had come up in 1965 with n factors of 9.2 and 11.2 for the beginning and end of the transition region for the same EC 1440 results as in Fig. 2 (see Fig. 5).

It cannot be overemphasized that the n factor is not a magic number. It is just a convenient way to correlate into one single number a series of factors which are known from experiment to influence transition. The success of the method is due to the fact that an appreciable fraction of the distance between the point of instability and transition is covered by linear theory.

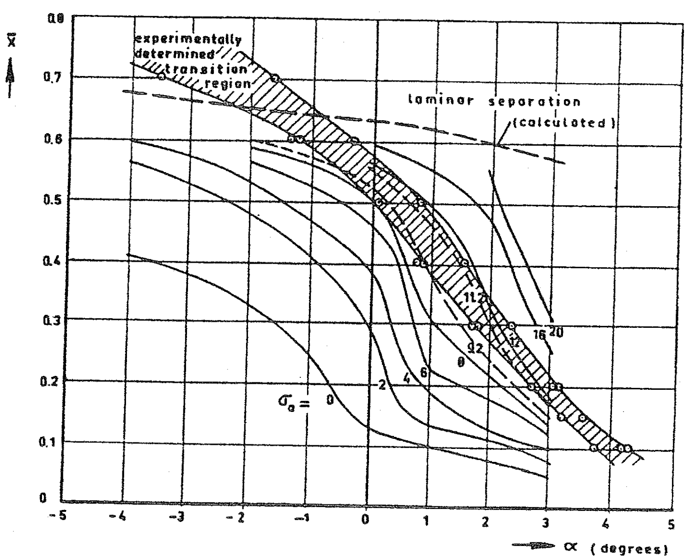

Figure 5: Calculated amplification factor and measured transition region for the EC 1440 airfoil section (Van Ingen, 1965).

Some results of the suction experiments and the stability calculations are shown in Fig. 6. It should be noted that also in the suction case separating laminar boundary layers were encountered for which neither the two-parameter integral method nor the Pretsch charts were applicable (some results have been obtained through extrapolation). In view of all the simplifications which had to be made, the conclusion at that time was that the e^n method could be applied with some confidence to the suction case.

Extension of the method to laminar separation bubbles

In 1966 the present author started to be involved in the design of airfoil sections for 2D incompressible flows. The foundation of this work was laid while spending a sabbatical year at the Lockheed Georgia Research Laboratory. The then available numerical methods for conformal transformation, laminar and turbulent boundary layer calculation and the e^n transition prediction method were used (Van Ingen, 1970). Later in Delft these design methods were continuously improved, based on comparisons between calculations and wind tunnel tests. A large number of airfoil designs were made (especially by Boermans *et al.* 1976, 1982, 1988, 1989, 1994) and applied in many different sailplanes. It was soon realized that at the chord Reynolds numbers applicable to sailplanes (and also wind turbines) the occurrence of laminar separation bubbles was very important and warranted extensive research.

The e^n method could be extended to separated flows because stability diagrams had been made available by Taghavi and Wazzan (1974) for the Stewartson reversed flow solutions of the Falkner-Skan equation. Moreover improved stability calculations for the Falkner-Skan velocity profiles had been published

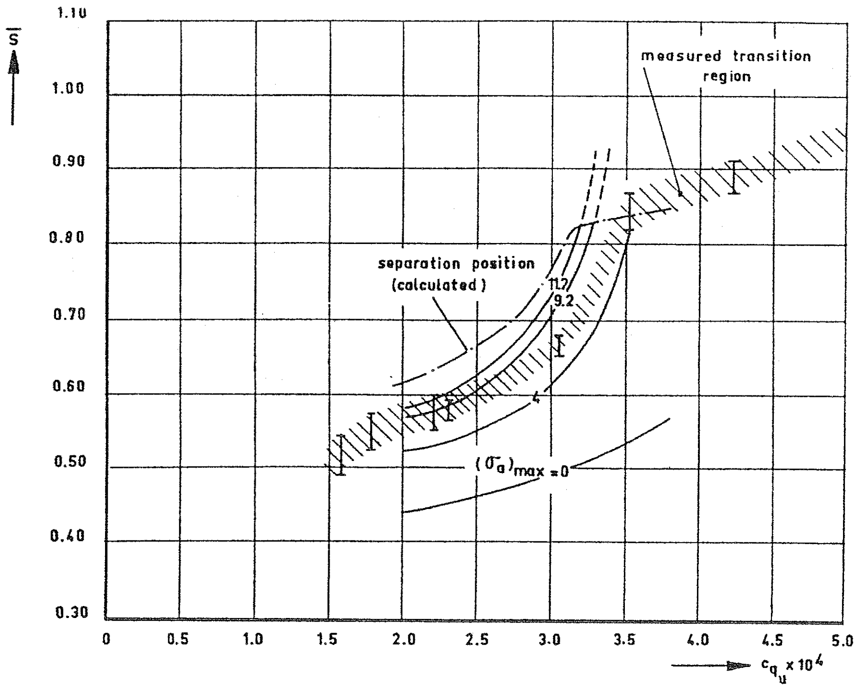

Figure 6: Measured transition region and calculated amplification factor for the upper surface of a suction airfoil.

by Wazzan, Okamura and Smith (1968) and by Kümmerer (1973). The present author had supplemented these results with solutions of the Rayleigh equation for the inviscid instability of the inflexional Falkner-Skan profiles for attached and reversed flow.

Using all the above mentioned results for the spatial mode, a new data base method was developed. The method is based on the observation that the stability diagrams show quite some similarity when as independent variable is used $^{10}\log(U\theta/\nu) - ^{10}\log(U\theta/\nu)_{crit}$ and the amplification rates are scaled with the maximum value for each diagram. The database consists of a table of about nly 300 numbers. Fig. 7 gives an application of the e^n method to laminar separated flow on a Wortmann airfoil.

The influence of free stream turbulence on the n factor

At the time the above mentioned database was developed, it had been realized already for quite some time that a constant n factor could no longer be used. That for so long a constant n factor (with the value 9) had been useful, may have been due to the fact that most modern low speed, low turbulence wind tunnels had been built according to the same recipe, aiming at a turbulence level of just below 0.1% as had been suggested to be sufficiently low according to the Schubauer and Skramstad experiment. (Fig. 1). From this experiment it had been concluded that reducing the turbulence level Tu below 0.1% had no use because "transition would not be influenced by a reduction of Tu below

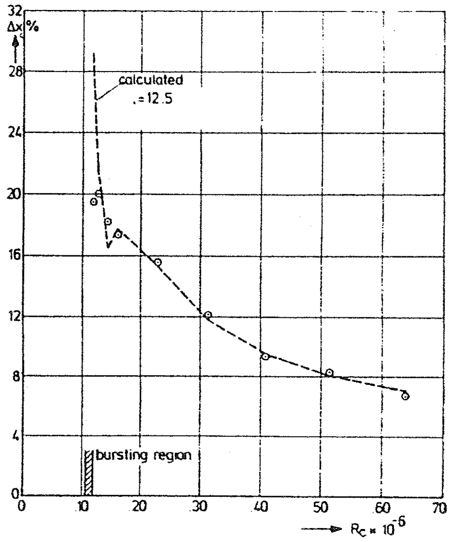

Figure 7: Distance between separation and transition on Wortmann airfoil in a small "noisy" wind tunnel.

0.1%". Since a further reduction of Tu requires a larger contraction ratio or more screens (and hence more money) most modern low speed wind tunnel designs have aimed at $Tu = 0.1\%$. The n factor needed to predict flat plate transition as a function of turbulence level Tu follows from an evaluation of various transition experiments on flat plates at various turbulence levels and from the calculation of the n factor as a function of Ux/ν (Fig. 8).

It was shown by Wells (1967) and by Spangler and Wells (1968) that transition Reynolds numbers larger than the Schubauer and Skramstadt values could be obtained by further reducing the turbulence level and the acoustic disturbances (apparently the acoustic disturbances rather than turbulence have caused transition in the Schubauer and Skramstadt experiments for $Tu < 0.1\%$). From Fig. 8. the present author concluded that beginning and end of transition could be predicted by n factors n_1 and n_2 respectively according to

$$n_1 = 2.13 - 6.18^{10}\log Tu$$

$$n_2 = 5 - 6.18^{10}\log Tu$$

(where Tu is the turbulence level in %). Mack (1975) has given independently a similar formula for n_1.

It should be clear that the free-stream turbulence level alone is not sufficient to describe the disturbance environment. Information about the distribution across the frequency spectrum should also be available and in addition to turbulence the acoustic disturbances are important. Of course the most important issue is "receptivity": how are the initial disturbances within the boundary layer related to the outside disturbances. Therefore we can only use Fig. 8 and the equations for n_1 and n_2 to specify the n factor if an "effective Tu" is known. This effective turbulence level can only be defined through a comparison of measured

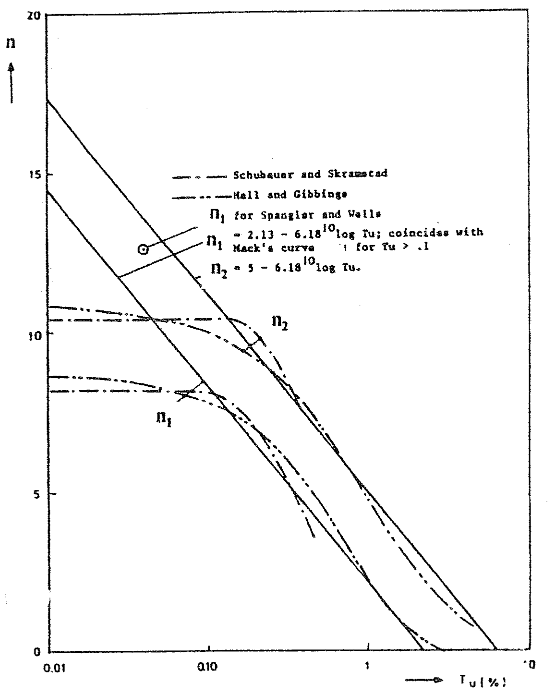

Figure 8: Relation between n_1, n_2 and Tu for the flat plate.

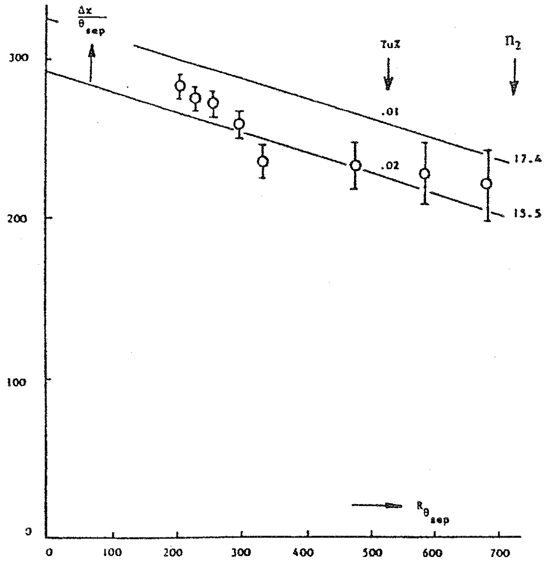

Figure 9: Length of the laminar part of the bubble on a Wortmann airfoil; bars indicate errors in Δx of + or -0.5% chord.

transition position with calculated amplification ratios. In fact it has become customary to define the quality of a wind tunnel by stating its "critical n factor".

Through a combination of the e^n method with calculation methods for separated flows it has become possible to develop a suitable prediction method for laminar separation bubbles (van Ingen, 1975). Some examples, are given in Figs 9 and 10. Brief explanations are given in the captions to the figures. In a simplified version of the method, the length of the laminar part of the bubble is correlated with the effective turbulence level (and hence with the n factor).

At one time the author tried to do some additional calibrations of the e^n method for separation bubbles by trying to shorten the bubble by means of additional turbulence due to grids. Not much happened due to the fact that apparently turbulence was added by the grid in the wrong frequency band.

Some applications of the Delft e^n method to airfoil designs

It should be realized by now that the e^n method does not automatically lead to useful results. The airfoil designer should be aware of its shortcomings and should make a judicious choice of the n factors to be used. In general we use at Delft the following n factors and "effective turbulence level" for various circumstances.

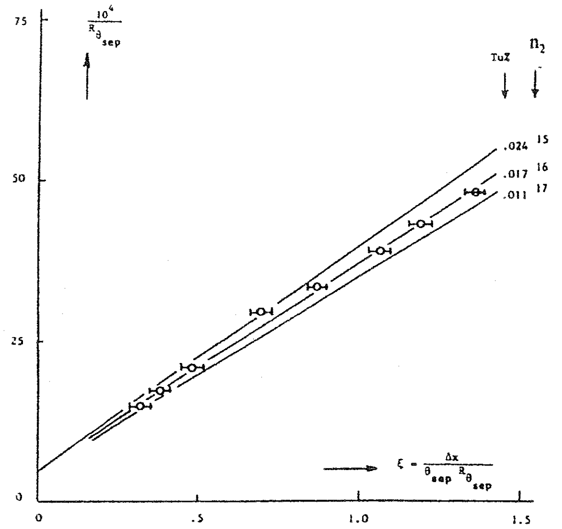

Figure 10: The same results as in Fig. 9, but plotted in an improved way.

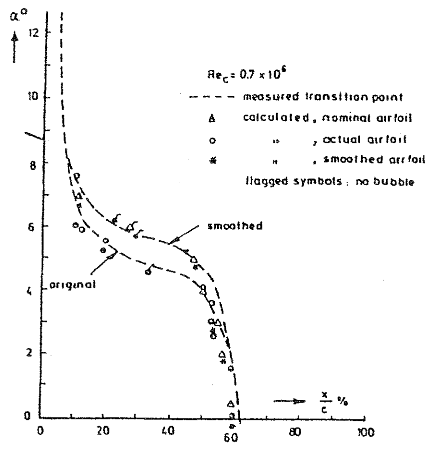

Figure 11: Position of transition on the upper surface of the M-300 airfoil (Boermans & Blom, 1976) according to viscous flow calculations and the stethoscope measurements.

environment	Tu %	n_1
NACA LTT	0.10	9.75
Delft Low Turbulence Tunnel	0.06	11.2
Free Flight of Gliders	0.014	15.0

In the author's group L.M.M. Boermans is the designer using the various calculation methods described above. He also has performed rather extensive wind tunnel measurements and has evaluated many flight tests. A number of examples can be found in the references to his work. As an example, Fig. 11, taken from Boermans & Blom (1976), shows results of computations with the e^n method for wavy and smooth versions of the same nominal airfoil. It follows that the e^n method is capable of predicting the shift in transition position due to waves in the surface. Fig. 12 gives the n factor for the beginning of transition on the airfoil DU89-122 tested in the low turbulence tunnel at Delft, using the infrared imaging technique (Boermans, private communication).

A possible explanation for the success of the e^n method for 2D incompressible flow at low Tu

In the past decades the e^n method has established itself as a useful method to predict the distance to transition in 2D incompressible flow. Apparently the linear stability theory has enough physics in it to account for the effects of

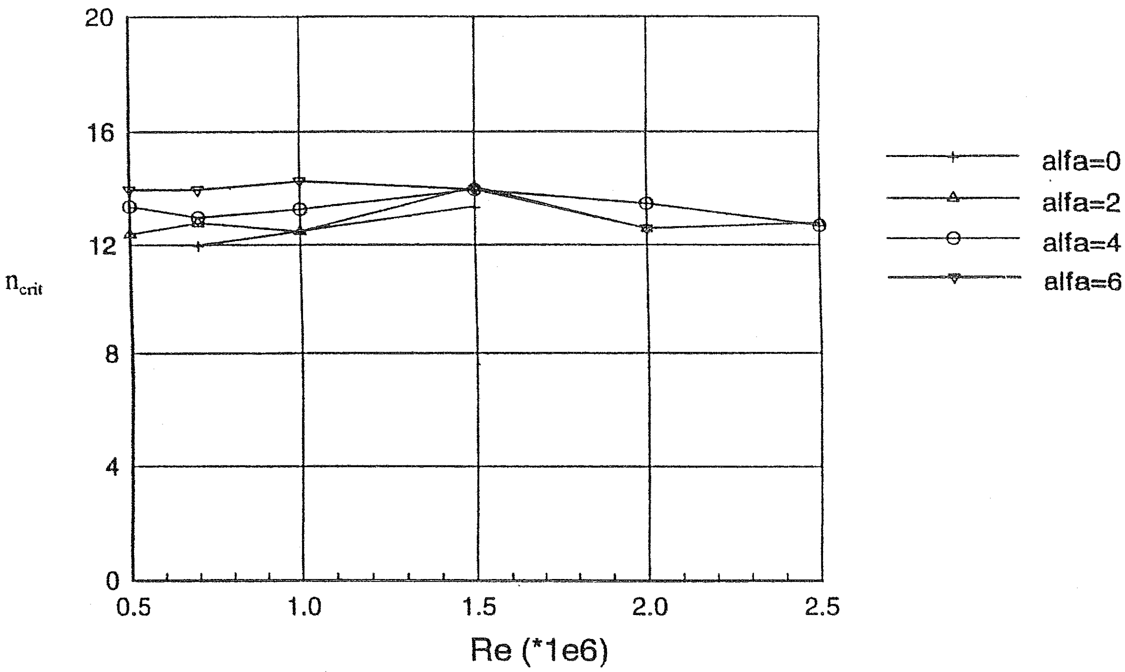

Figure 12: Critical n factors for the DU89-122 airfoil as a function of Reynolds number and angle of attack.

pressure gradient, suction, heating and cooling, etc. on transition. Hence it may be expected that under these circumstances the linear part of the amplification process covers a large percentage of the distance between first instability and transition. Obremski (1969) gives some background to this idea by quoting the following characteristic numbers.

In low speed, low turbulence wind tunnels (such as at Delft) the overall turbulence level at low speed is certainly less than 0.1%, even as low as 0.02%. Since only part of the spectrum contains the dangerous Tollmien-Schlichting frequencies, the amplitude of the neutral disturbances, being present inside the boundary layer and which somehow (through "receptivity") may be related to the external turbulence level, may be of the order of 0.001%. Linear theory is found to give a good description of the amplification process up till an amplitude of 1 to 1.5%. When transition is completed disturbance levels of the order of 10 to 20% are found. Hence the linear part extends to an n factor of 7. The nonlinear part only has to cover the range of n values between 7 and 10. Hence one should not be surprised about the relative success of the e^n method. A calibration by comparison with experiment should be adequate to "predict" the distance to transition. Of course the physical process of real transition is not described by linear theory.

An experimental illustration can be found in the work of Wubben *et al.* (1989). Transition experiments were done in a small boundary layer channel with a pressure distribution meant to represent a constant Hartree β-flow of -0.14 (Fig. 13). This tunnel has a rather large disturbance environment due to

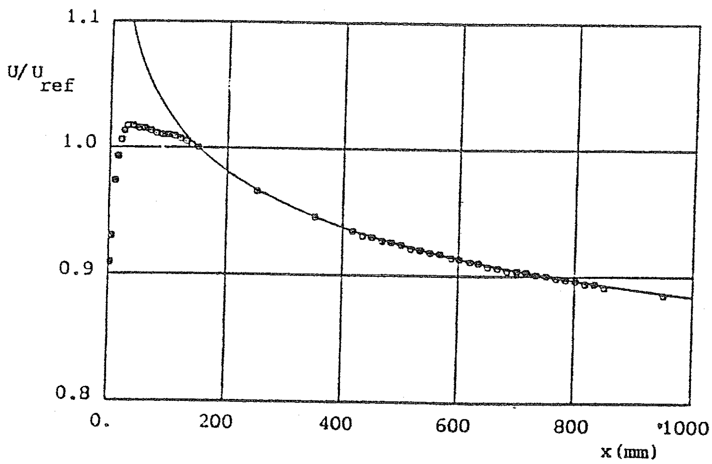

Figure 13: Experimental (⊙) and theoretical (-) velocity distribution, Hartree $\beta = -0.14$ ($U_{ref} = 10.96\ m/s$) (Wubben *et al.*, 1989).

turbulence, vibrations, blade passing frequency of the blower, etc. Therefore it is not easy to verify experimentally the initial amplification. Fig. 14 gives some velocity fluctuation spectra as measured with hot wires. To eliminate most of the noise and to determine the relative amplification the spectra at different x-values were compared to that at $x = 330$ mm by subtracting the latter from the others. Included in the figure is the amplification spectrum as calculated from our database method, also points calculated by the NLR version of the COSAL code are indicated.

Considering the various causes for inaccuracies it is seen that the linear calculations give a reasonable description of the amplification until $x = 616$ mm. Transition sets in at about $x = 790$ mm (to be concluded from the broadening of the spectrum). The relative n factor between $x = 330$ mm and $x = 518$ mm is equal to 4. As the calculated n factor at $x = 330$ mm equals 5, the results are comparable with the values mentioned by Obremski. A comparison between n factors obtained in flight and in the DNW low speed wind tunnel is given in Fig. 15 (Horstmann *et al.*, 1990).

Concluding remarks

In the present paper some of the history of the e^n method for 2D incompressible flow has been highlighted. A discussion of further developments by other researchers would have been appropriate but proved to be impossible due to lack of space. Other papers in this volume may be consulted to learn about extensions to 3D flows and higher speeds. Not in all cases the extension is straightforward. An important issue is to define the proper "integration strategy" for 3D flows. Also an extensive discussion of simpler methods which also have some relation to the e^n method such as Michel's (1953) and Granville's (1953) methods would have been relevant. The Granville method uses a correlation between the difference between $U\theta/\nu$ at transition and at the instability point and the mean

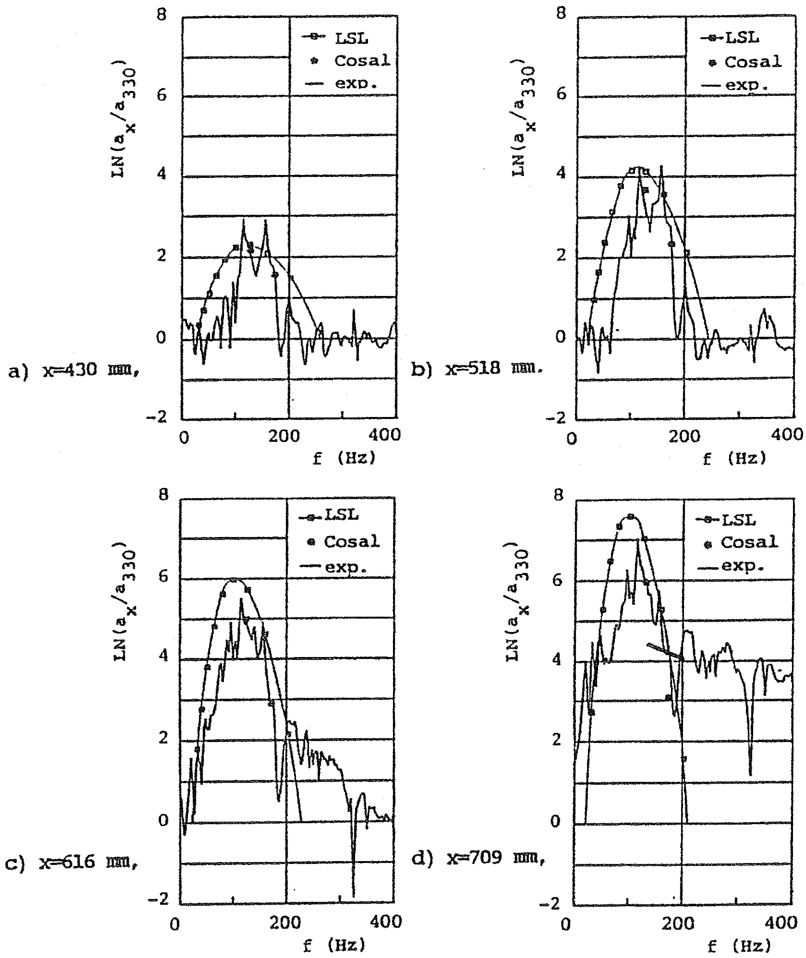

Figure 14: Amplification spectra. Amplification is with respect to the spectrum, measured at $x = 330$ mm; here a calculated 'n-factor' of 5 is found. Hence, figure 14b shows an n-factor of about 9.

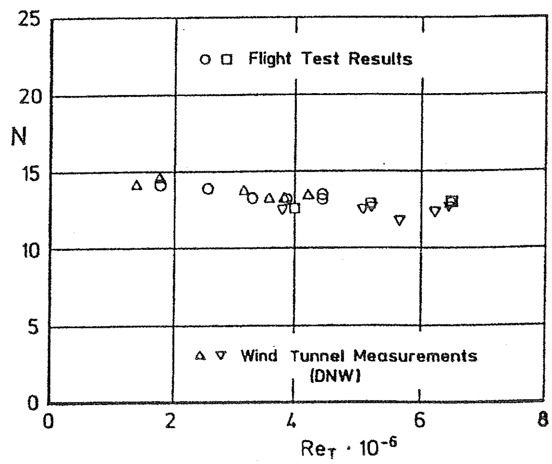

Figure 15: Calculated n-factors at transition based on flight and wind tunnel test (o Δ: 0 deg. flap; □ ∇: 3.5 deg. flap) (Horstmann *et al.*, 1990).

value of the pressure gradient parameter $K = \frac{\theta^2}{\nu}\frac{dU}{dx}$. Michel uses a correlation between $\frac{Ux}{\nu}$ and $\frac{U\theta}{\nu}$ at transition. It was shown by Smith that Michel's curve corresponds to the e^9 criterion for the similar Falkner-Skan boundary layers. The reader is referred to an extensive discussion of all these problems by Arnal (1993).

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