TECHNOLOGICAL UNIVERSITY DELFT

DEPARTMENT OF AERONAUTICAL ENGINEERING

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THEORETICAL AND EXPERIMENTAL INVESTIGATIONS OF INCOMPRESSIBLE LAMINAR BOUNDARY LAYERS WITH AND WITHOUT SUCTION

Ph.D THESIS

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This PDF-file contains Appendix 1:

The drag of a flat plate with suction

15. Appendix 1: The drag of a flat plate with suction.

The drag of the flat plate with suction can easily be found from the momentum equation in the form (2.16)

$$\frac{U\theta}{\nu}\frac{d\theta}{dx} + (2+H)\frac{\theta^2}{\nu}\frac{dU}{dx} - \frac{v_o\theta}{\nu} = \frac{\tau_o\theta}{\mu U} \qquad (A.1)$$

For a flat plate U is constant and equal to the reference speed U_∞ so that (A.1) may be simplified to

$$2\frac{d\theta}{dx} - 2\frac{v_o}{U_\infty} = \frac{\tau_o}{\frac{1}{2}\rho U_\infty^2} \qquad (A.2)$$

In what follows a flat plate with unit span will be considered. Then defining the drag coefficient c_d and the suction flow coefficient c_q by

$$c_d = \frac{\int_o^c \tau_o \, dx}{\frac{1}{2}\rho U_\infty^2 c} \qquad (A.3)$$

and

$$c_q = \int_o^c \frac{-v_o}{U_\infty} \, d\frac{x}{c} \qquad (A.4)$$

equation (A.2) may be integrated to give:

$$c_d = 2\frac{\theta}{c} + 2c_q \qquad (A.5)$$

In (A.5) c is the length of the plate, c_d is the total drag coefficient experienced by the plate; $2\frac{\theta}{c}$ is the "wake drag" coefficient c_{d_w} which would be found from a wake survey method as described in section 11.3.4. The term $2c_q$ represents the so-called "sink drag" coefficient and expresses the fact that an amount of air of $c_q U_\infty c$ m^3/sec is brought to rest in the boundary layer causing a momentum loss which is experienced as drag. The sink drag can be disregarded if the air is expelled in downstream direction with the free stream speed, since in this case a thrust will be obtained which balances the sink drag. However, to overcome the pressure drops through the porous surface, suction ducts etc. a suction pump is required which consumes some power. If an amount of air of Q m^3/sec is sucked, which has to be given a pressure rise Δ p, then assuming incompressible flow this requires a pumping power of

$$P_p = \frac{Q\Delta p}{\eta_p} \tag{A.6}$$

where η_p is the efficiency of the pump.
If the suction power would have been used for propulsion of the plate a drag component Δ D could have been overcome which is given by

$$P_p = \frac{U_\infty \Delta D}{\eta_T} \tag{A.7}$$

In (A.7) η_T is the efficiency of the propulsion system; Δ D is called the "equivalent suction drag".
Usually a pressure loss coefficient c_p and a suction drag coefficient c_{d_s} are defined by

$$c_p = \frac{\Delta p}{\frac{1}{2}\rho U_\infty^2} \tag{A.8}$$

and

$$c_{d_s} = \frac{\Delta D}{\frac{1}{2}\rho U_\infty^2 c} \tag{A.9}$$

Equating the right hand sides of (A.6) and (A.7) and using (A.8) and (A.9) leads to the following expression for the suction drag coefficient

$$c_{d_s} = c_p c_q \frac{\eta_T}{\eta_p} \tag{A.10}$$

Assuming for convenience equal efficiencies η_T and η_p gives

$$c_{d_s} = c_p c_q \tag{A.11}$$

The total drag coefficient for the flat plate is now given by

$$c_{d_t} = c_{d_w} + c_p c_q \tag{A.12}$$

where $c_{d_w} = 2\,\frac{\Theta}{c}$.

Equation (A.12) may also be used to describe the total drag for an airfoil with suction if c_{d_w} represents the drag coefficient found from the wake traverse method.