University of Amsterdam
Amsterdam School of Economics

Machine Learning for Econometrics

Week 6: Causal Inference and Machine Learning

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Plan for Today

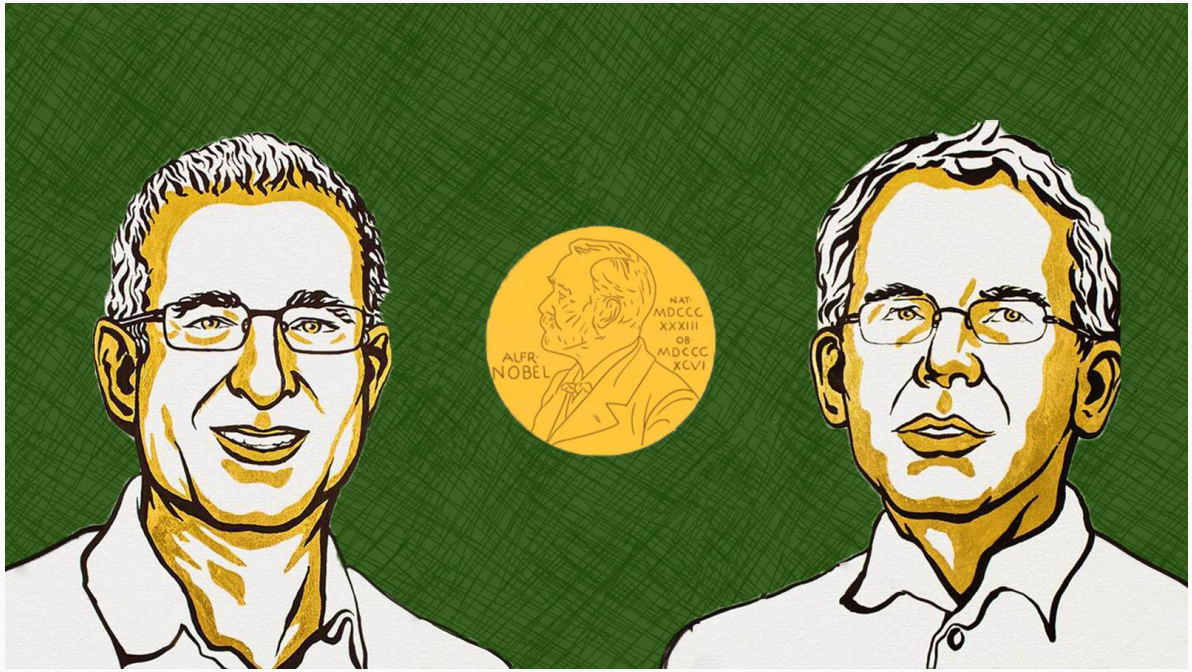
1. Introduction

2. Confounder Selection

3. Debiased Machine Learning

Introduction

Nobel Prize Economics 2021



Joshua Angrist and Guido Imbens [Dutch] share half the award

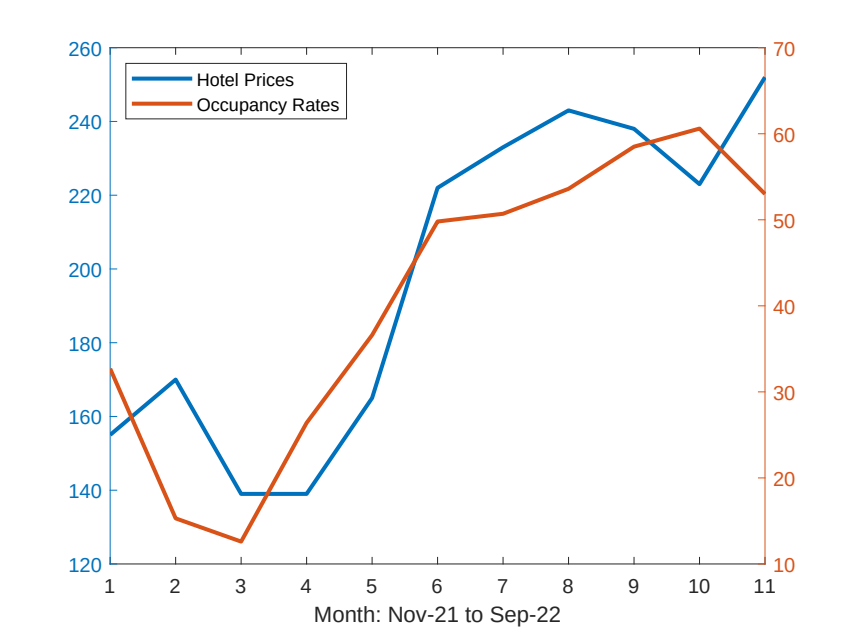
Achievement:

methodological contributions to the analysis of causal relationships

A Motivating Example

Imagine a hotel data set containing data about

- Occupancy rates
- Prices



Blue: the average rate per night for a standard double room in Amsterdam;

Red: hotel occupancy rate (in percentage) in the Netherlands

- Prices are easy to obtain through price comparison sites
- Occupancy rates are typically not made public by hotels
- Estimating the occupancy rates of *competitors*, based on publicly available prices, is a prediction problem:

  $$\text{Prices} \uparrow \text{ suggests occupancy rates} \uparrow$$

- Estimating how occupancy would change if the hotel raised prices across the board (e.g., $+5\%$ in prices in every state of the world) is a question of **causal inference**:

  $$\text{Prices} \uparrow \text{ causes occupancy rates} \uparrow ?$$

- Even though prices and occupancy are positively correlated in a typical dataset, we would not conclude that raising prices would increase occupancy

Potential Outcome Model

- Wish to find the causal effect of a treatment/policy variable D on the outcome variable Y
- Treatment ($D = 1$) or control ($D = 0$) groups
- $Y_i^{(d)}$ is the outcome for individual i when given treatment $d \in \{1, 0\}$:

$$Y_i = \begin{cases} Y_i^{(1)} & \text{if } D_i = 1, \\ Y_i^{(0)} & \text{if } D_i = 0, \end{cases} \quad \text{or } Y_i = D_i Y_i^{(1)} + (1 - D_i) Y_i^{(0)}.$$

- Binary treatment effect for individual i is given by

$$\delta_i = Y_i^{(1)} - Y_i^{(0)}$$

- *Counterfactual*: only observe $Y_i^{(D_i)} \in \{Y_i^{(1)}, Y_i^{(0)}\}$ but not both

The Fundamental Problem of Causal Inference

Table 2.1 in Morgan and Winship (2014)

Group	$Y^{(1)}$	$Y^{(0)}$
Treatment group ($D = 1$)	Observable as Y	Counterfactual
Control group ($D = 0$)	Counterfactual	Observable as Y

- Impossible to calculate individual-level treatment effects
- Estimate the *average treatment effect* (ATE)

$$\tau = \frac{1}{n}\sum_{i=1}^{n}\tau_i,\ \tau_i = \mathbb{E}\delta_i = \mathbb{E}\left[Y_i^{(1)} - Y_i^{(0)}\right] = \mathbb{E}\,Y_i^{(1)} - \mathbb{E}\,Y_i^{(0)}$$

where n denotes the total number of individuals

A Naive Estimator of the ATE

The difference in mean outcomes between

- the treatment group $\{i : D_i = 1\}$ with $n_1 = \sum_{i=1}^{n} D_i$ participants; and
- the control group $\{i : D_i = 0\}$ with $n_0 = n - n_1$ participants:

$$\begin{aligned}\widehat{\tau} =& \frac{1}{n_1} \sum_{D_i=1} Y_i - \frac{1}{n_0} \sum_{D_i=0} Y_i \\ =& \frac{1}{n_1} \sum_{D_i=1} Y_i^{(1)} - \frac{1}{n_0} \sum_{D_i=0} Y_i^{(0)}\end{aligned}$$

Is this a consistent estimator?

Even for the favorable case:

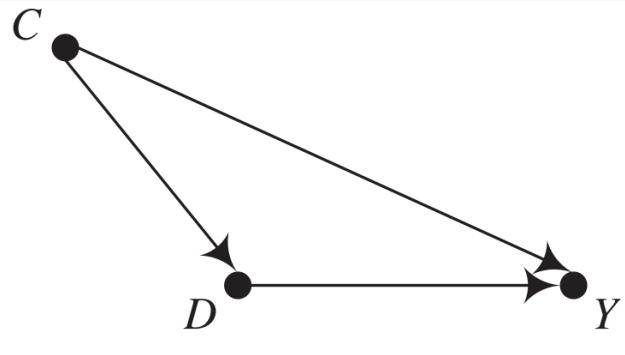
- $(Y_i^{(1)}, Y_i^{(0)}, D_i) \sim (Y^{(1)}, Y^{(0)}, D)$ are i.i.d.
- The treatment effect $\tau \equiv \tau_i \equiv \delta_i \equiv \delta$ is constant

$$
\begin{aligned}
\widehat{\tau} \xrightarrow{P} & \mathbb{E}[Y^{(1)} \mid D = 1] - \mathbb{E}[Y^{(0)} \mid D = 0] \\
= & \mathbb{E}[\delta + Y^{(0)} | D = 1] - \mathbb{E}[Y^{(0)} | D = 0] \\
= & \delta + \underbrace{\Big\{ \mathbb{E}[Y^{(0)} | D = 1] - \mathbb{E}[Y^{(0)} | D = 0] \Big\}}_{\text{bias}}
\end{aligned}
$$

- For $(Y^{(1)}, Y^{(0)})$ independent of D: $(Y^{(1)}, Y^{(0)}) \perp\!\!\!\perp D$
- ... the bias vanishes: $\widehat{\tau} \xrightarrow{P} \delta$

Unconfoundedness Assumption

- Confounding factors C influencing both D and Y:



- Example: college education and income both affected by parents' income & education, etc.
- Confounding implies that treatment assignment is not random but (self-)selected based on characteristics.
- Ideally confounders can be represented by features $X \in \mathbb{R}^d$.
- Unconfoundedness Assumption:

$$(Y^{(1)}, Y^{(0)}) \perp\!\!\!\perp D \mid X = x, \quad x \in \mathcal{X}$$

Confounder Selection

Structural Equations

If the treatment effect $\delta = Y_i^{(1)} - Y_i^{(0)}$ is fixed

$+$ linear model $Y_i^{(0)} = X_i^T \beta + \varepsilon_i = \sum_{j=1}^{d} X_{i,j}\beta_j + \varepsilon_i$

implies that

$$Y_i = \delta D_i + \sum_{j=1}^{d} X_{i,j}\beta_j + \varepsilon_i, \quad \underbrace{\mathbb{E}[\varepsilon_i | X_i, D_i] = 0}_{\text{Unconfoundedness}},$$

where we assume a zero intercept for simplicity.

Combining both information on policy Y and treatment D

$$\begin{cases} Y_i = \delta D_i + X_i^T \beta + \varepsilon_i, & \mathbb{E}[\varepsilon_i | X_i, v_i] = 0 \\ D_i = p(X_i) + v_i, & \mathbb{E}[v_i | X_i] = 0, \end{cases}$$

where $p(x)$ is called the *propensity score* function given by

$$p(x) = \mathbb{E}[D|X = x] = \mathbb{P}(D = 1|X = x)$$

Linear Model For Treatment Equation

For simplicity, consider an approximate linear probability model

$$p(X_i) = \sum_{j=1}^{d} X_{i,j}\gamma_j + r_i = X_i^T\gamma + r_i$$

where r_i should be negligible (stochastically).

Rewrite the model into a reduced form given by

$$\begin{cases} Y_i = X_i^T\bar{\beta} + \bar{\varepsilon}_i \\ D_i = X_i^T\gamma + \bar{v}_i, \end{cases}$$

where $\bar{\beta} = \delta\gamma + \beta$, $\bar{\varepsilon}_i = \delta\bar{v}_i + \varepsilon_i$ and $\bar{v}_i = r_i + v_i$.

Post-Selection Estimation

- When the dimension of confounders d is high relative to the sample size (with a large d/n), selecting a small set of the most useful confounders could be wise for variance reduction purpose.
- Run the least-squares regression on only the variables selected
- Post-selection estimator is not stable in general, but it can provide solid inference under sparsity assumption.

Sparsity Assumption

- If d is small, δ may be estimated by least-squares regression using the outcome equation:

$$Y_i = \delta D_i + X_i^T \beta + \varepsilon_i$$

- When d is large, assume that the true support set is however sparse:

$$\mathcal{J} = \{1 \leq j \leq d : \beta_j \neq 0\} \text{ has size } |\mathcal{J}| \ll n$$

- Only a few confounders are relevant to the population models, but we do not know which.

Single Selection Using The Outcome Equation

- Use the LASSO regression minimizing

$$\frac{1}{2}\sum_{i=1}^{n}\left(Y_i - \delta D_i - \sum_{j=1}^{d} X_{i,j}\beta_j\right)^2 + \lambda\sum_{j=1}^{d}|\beta_j|$$

  where treatment is *not* penalized
- The confounders should be normalized.
- LASSO performs model selection: for non-trivial $\lambda > 0$

$$\widehat{\mathcal{J}} = \{1 \leq j \leq d : \widehat{\beta}_j \neq 0\} \ll n$$

Why Post-Selection Estimation?

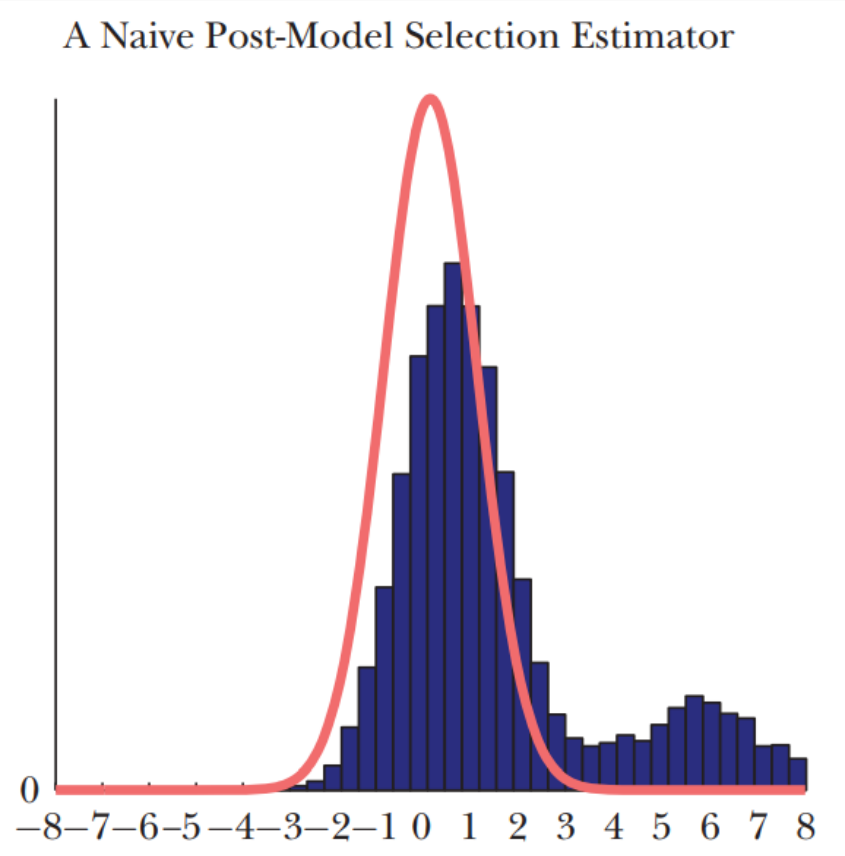
- Under more regularity conditions and an appropriate choice of penalty coefficient λ, Belloni and Chernozhukov (2013) show that with high probability and some constant C

$$\mathcal{J} \subset \widehat{\mathcal{J}}, \text{ and } \left|\widehat{\mathcal{J}}\right| \leq C|\mathcal{J}| \ll n$$

- The former implies a correct model selection, and the latter says our selected model is small (enough)
- LASSO estimator $\widehat{\delta}_{\text{LASSO}}$ often has a non-trivial bias: see, e.g., Belloni et al. (2013) for iterative algorithms for choosing λ
- Post-Lasso estimation tends to reduce bias: least-squares estimator after selection.

Naive Post Selection Estimator

- Single selection could miss the variables with true coefficients "moderately close to zero" with a non-trivial chance
- Select more variables: how?



Double Selection: Belloni et al. (2014)

Recall the reduced form

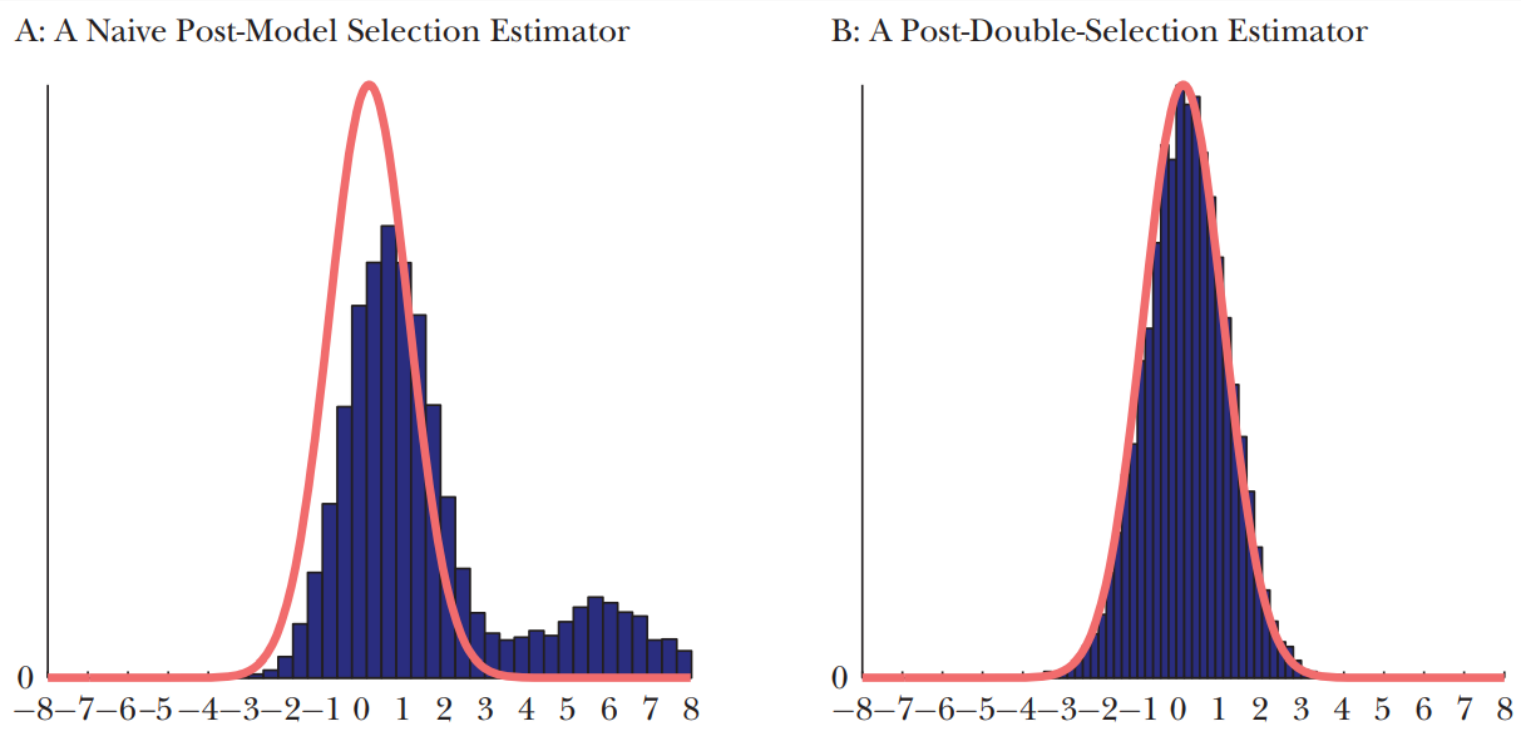
$$\begin{cases} Y_i = X_i^T \bar{\beta} + \bar{\varepsilon}_i \\ D_i = X_i^T \gamma + \bar{v}_i, \end{cases}$$

- Sparse index sets $\widehat{\mathcal{J}}_1, \widehat{\mathcal{J}}_2 \subset \{1, \ldots, d\}$ from **each** equation (using LASSO regression)
- Take a (small) set $\widehat{\mathcal{J}}_3$ that you think is important for ensuring robustness, if necessary
- The final index set is given by

$$\widehat{\mathcal{J}} = \widehat{\mathcal{J}}_1 \cup \widehat{\mathcal{J}}_2 \cup \widehat{\mathcal{J}}_3$$

- Least-squares regression after double selection.

Simulation Example: Belloni et al. (2014)



The post-double-selection estimator $\widehat{\delta}$ reduces bias, and its distribution gets closer to normal for sparse models.

Debiased Machine Learning

Auxiliary ML Estimator

Consider the partially linear model

$$Y_i = \delta D_i + \mu^{(0)}(X_i) + \varepsilon_i$$

where $\mu^{(0)}(x) = \mathbb{E}[Y^{(0)}|X = x]$ may be non-linear.

Suppose we have a machine learning estimator $\widehat{\mu}^{(0)}$ from another auxiliary sample independent of our training set with a learning bias

$$b(x) = \mathbb{E}\left[\widehat{\mu}^{(0)}(x)\right] - \mu^{(0)}(x) \not\equiv 0,$$

which is not identically zero.

The learning bias can emerge due to the bias-variance tradeoff in ML estimator.

Rewrite the model as

$$Y_i - \widehat{\mu}^{(0)}(X_i) = \delta D_i + \widetilde{\varepsilon}_i,\ \widetilde{\varepsilon}_i = \varepsilon_i + \mu^{(0)}(X_i) - \widehat{\mu}^{(0)}(X_i).$$

Regressing $Y_i - \widehat{\mu}^{(0)}(X_i)$ on D_i gives the least-squares estimator

$$\begin{aligned}
\widehat{\delta} &= \left(\sum D_i^2\right)^{-1}\left(\sum D_i(Y_i - \widehat{\mu}^{(0)}(X_i))\right) \\
&= \left(\sum D_i^2\right)^{-1}\left(\sum D_i(Y_i - \mu^{(0)}(X_i))\right) \\
&\quad + \left(\sum D_i^2\right)^{-1}\left(\sum D_i(\underbrace{\mu^{(0)}(X_i) - \widehat{\mu}^{(0)}(X_i)}_{\text{'prediction error' } e(X_i)})\right) \\
&= \delta + \underbrace{\left(\sum D_i^2\right)^{-1}\left(\sum D_i\varepsilon_i\right)}_{\text{variance component}} + \underbrace{\left(\sum D_i^2\right)^{-1}\left(\sum D_i e(X_i)\right)}_{\text{bias component}}
\end{aligned}$$

because

$$\mathbb{E}[D_i e(X_i)] = \mathbb{E}[D_i \cdot \mathbb{E}[e(X_i) \mid S]] = \mathbb{E}[\underbrace{D_i}_{p(X_i)+v_i} \cdot b(X_i)] \neq 0$$

Endogeneity and IV Regression

$$Y_i - \widehat{\mu}^{(0)}(X_i) = \delta D_i + \widetilde{\varepsilon}_i,\ \widetilde{\varepsilon}_i = \varepsilon_i + \mu^{(0)}(X_i) - \widehat{\mu}^{(0)}(X_i).$$

The learning bias issue is the endogeneity issue :

$$\mathbb{E}[D_i \widetilde{\varepsilon}_i] = \mathbb{E}[D_i b(X_i)] \neq 0.$$

because both D_i and $b(X_i)$ depend on X_i.

- Using the decomposition

$$D_i = p(X_i) + v_i, \quad \mathbb{E}[v_i | X_i] = 0$$

- If v_i were observable, it is a valid instrument for D_i as

$$\begin{cases} \mathbb{E}[v_i D_i] \neq 0 \\ \mathbb{E}[v_i \widetilde{\varepsilon}_i] = 0 \end{cases} \Rightarrow \delta = \frac{\mathbb{E}[v_i \left(Y_i - \widehat{\mu}^{(0)}(X_i)\right)]}{\mathbb{E}[v_i D_i]}$$

$$= \frac{\mathbb{E}[v_i \left(Y_i - \widehat{\mu}^{(0)}(X_i)\right)]}{\mathbb{E}[v_i^2]}$$

Debiased/Double ML: Chernozhukov et al. (2018)

One may estimate δ using the ratio of the sample averages

- Split into independent folds with index sets $\mathcal{I}$ and $\mathcal{I}^{\complement}$
- Learn both the regression function $\mu^{(0)}(\cdot)$ and propensity score functions $p(\cdot)$ on the auxiliary sample $\mathcal{I}^{\complement}$.
- Instrumental regression in sample $\mathcal{I}$:

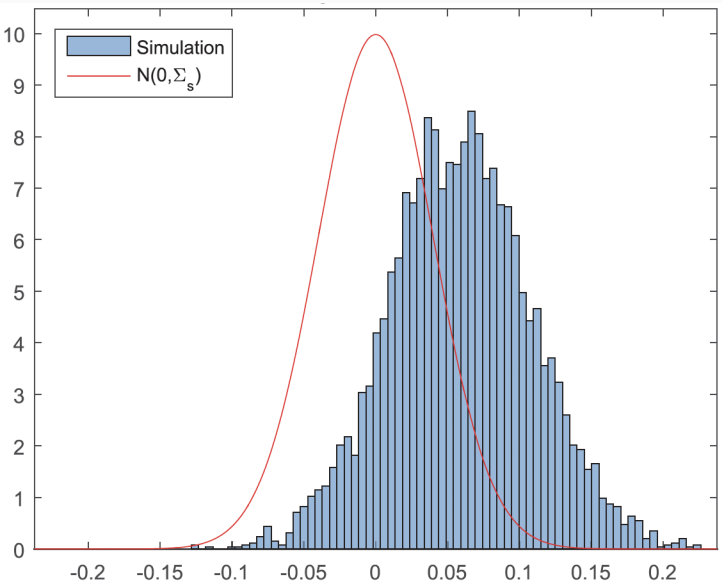
$$\widehat{\delta} = \left(\sum_{i \in \mathcal{I}} \widehat{v}_i D_i \right)^{-1} \sum_{i \in \mathcal{I}} \widehat{v}_i (Y_i - \widehat{\mu}^{(0)}(X_i)), \text{ where } \widehat{v}_i = D_i - \widehat{p}(X_i).$$

or

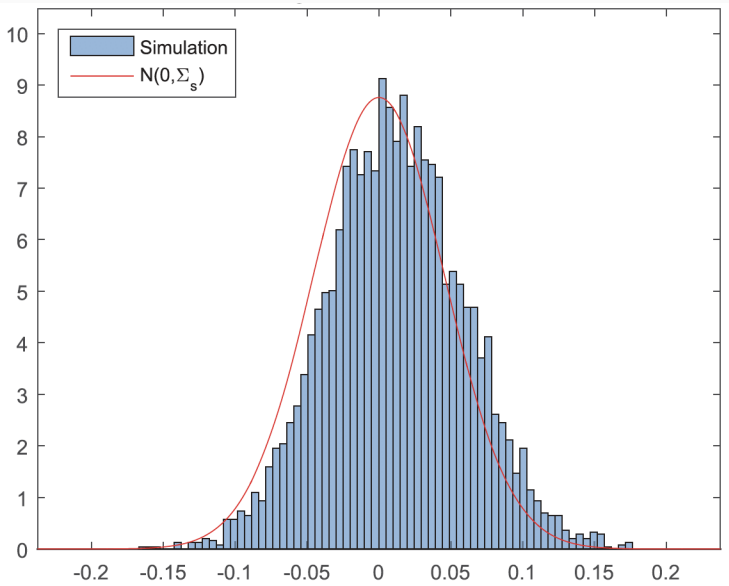
$$\widehat{\delta} = \left(\sum_{i \in \mathcal{I}} \widehat{v}_i^2 \right)^{-1} \sum_{i \in \mathcal{I}} \widehat{v}_i (Y_i - \widehat{\mu}^{(0)}(X_i))$$

- Optional: swap $\mathcal{I}$ and $\mathcal{I}^{\complement}$ and average the estimates

Simulation Example: Chernozhukov et al. (2018)



Conventional ML



Debiased ML

- $\mu^{(0)}(x)$ and $p(x)$ are learned by using random forests
- DML estimator has a smaller bias

Remark

- Due to the time constraint, today we cannot cover all machine learning methods in causal inference
- . . . such as the causal forest proposed Wager and Athey (2018) and the neural network approach in Farrell et al. (2021) based on influence functions
- You can find more explanations and details in our online reader.
- You will also learn more during the tutorial session.

References i

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