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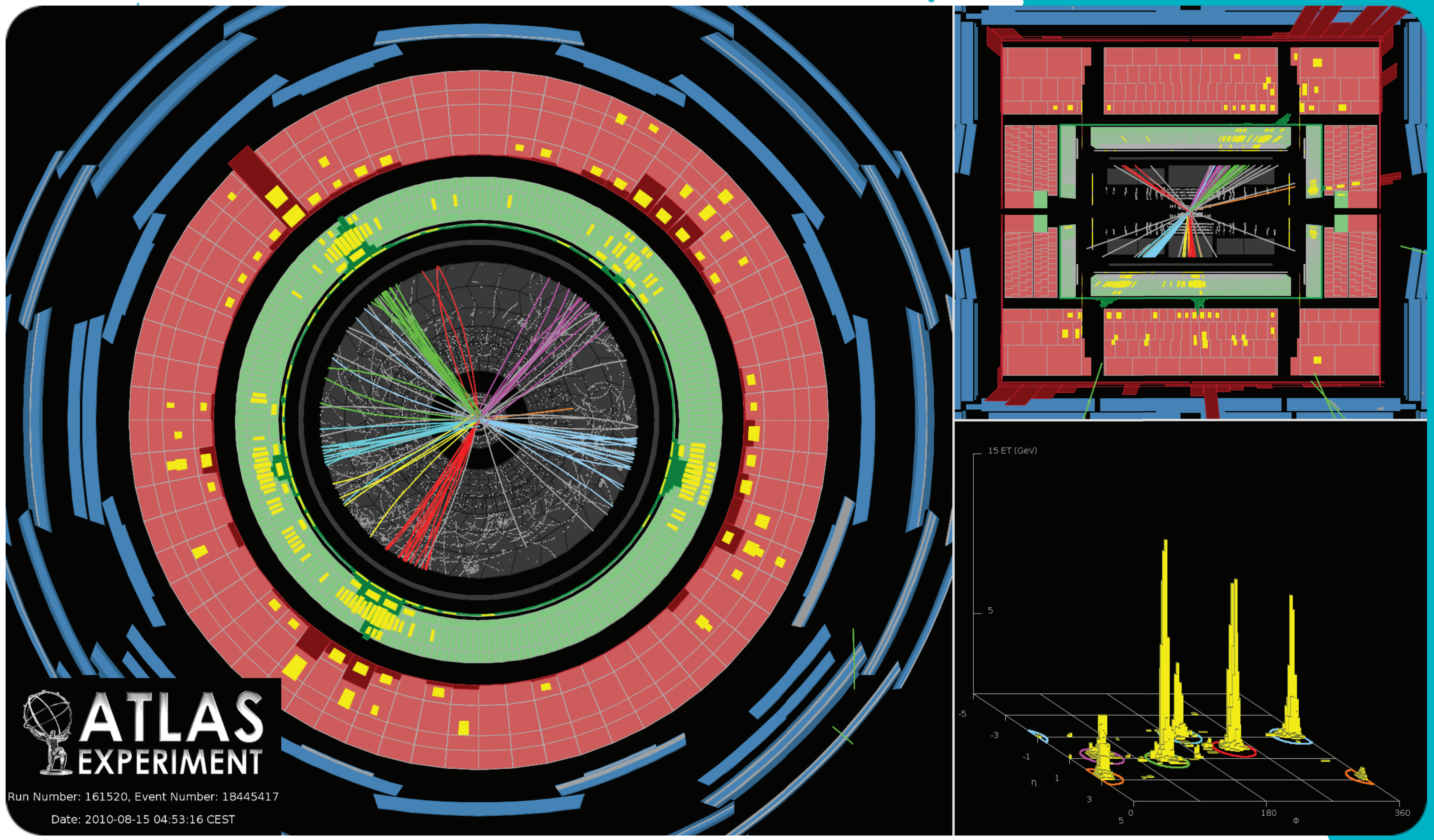
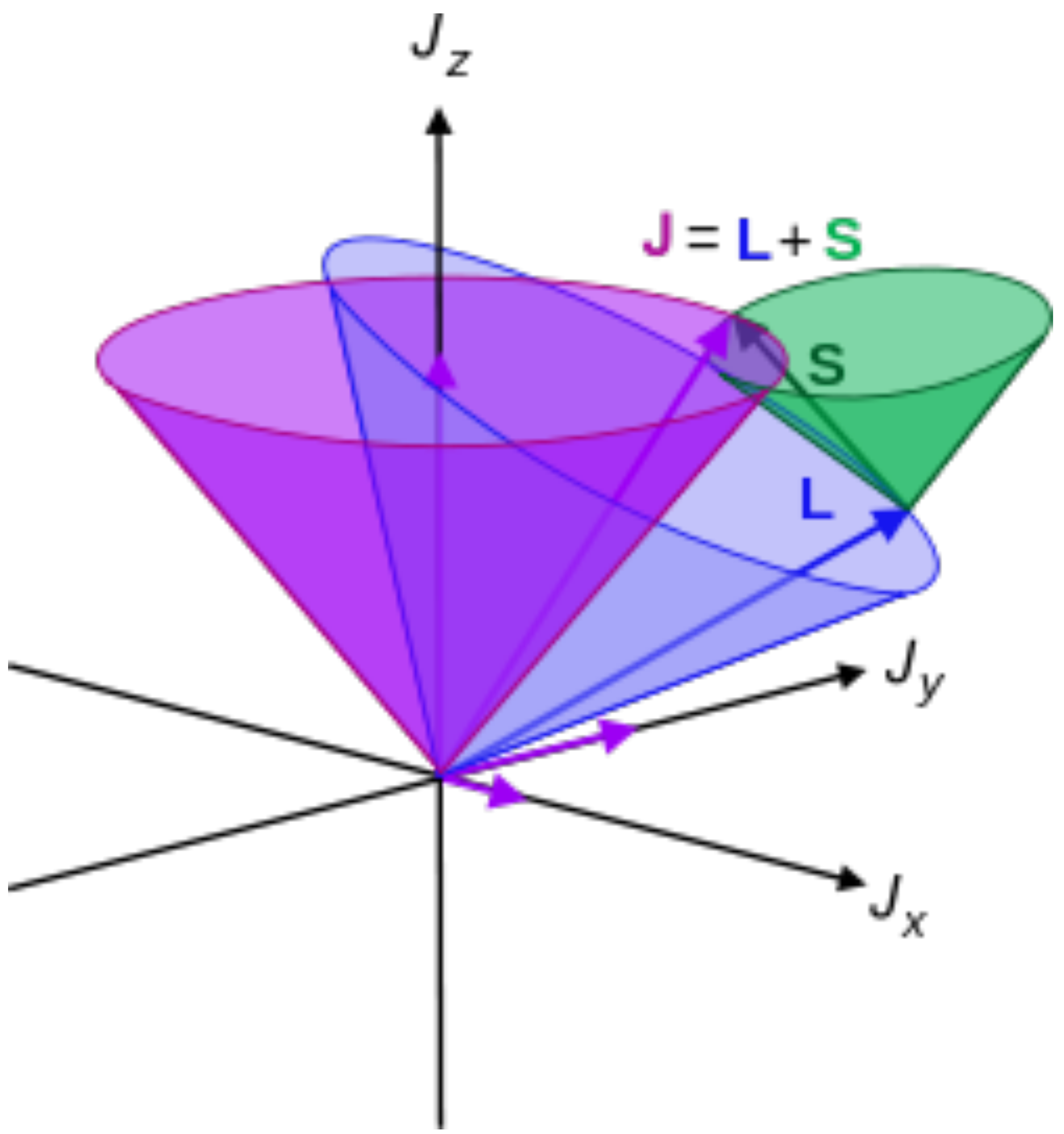


INTRODUCTION TO QUANTUM CHROMODYNAMICS

PARTICLE PHYSICS 2

Panos Christakoglou

SUMMARY



Today's lecture

- Elements of Quantum Mechanics
  - Orbital angular momentum
  - Spin
  - Clebsch-Gordan coefficients
- Categories of particles
  - Fermions vs bosons
  - Pauli principle
- Quark model
  - Mesons vs baryons
  - The need for a new QM i.e. colour
- Experimental evidence of colour

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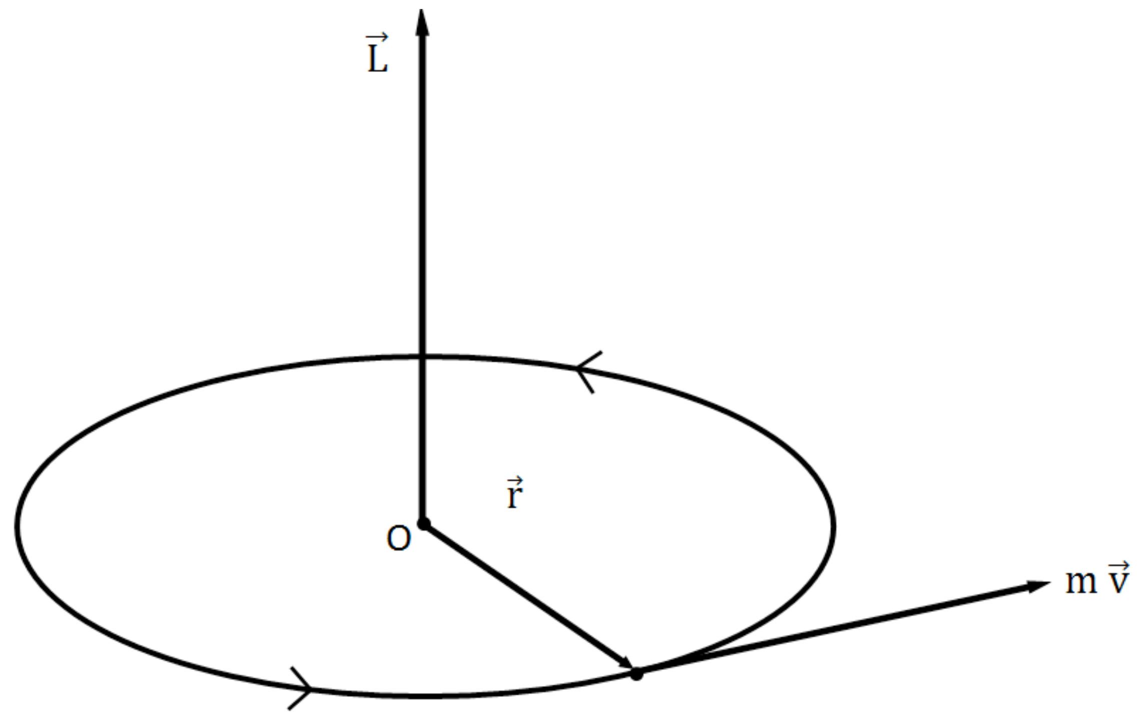
ELEMENTS OF QUANTUM MECHANICS

ORBITAL ANGULAR MOMENTUM

In classical mechanics

- a solid body rotating around one axis has associated **angular momentum→conserved** in the absence of external forces



- defined by the cross product of the momentum and position vectors

$$\vec{L} = \vec{r} \times \vec{P}$$

- can take any value

ORBITAL ANGULAR MOMENTUM

In quantum mechanics

- Angular momentum is represented by the corresponding operator
- can not take any value but it's quantised
- we cannot measure all components of angular momentum at the same time
  - We can measure its magnitude L^2 and its third component L_z

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{r} \rightarrow \hat{r} = (x\hat{x}, y\hat{y}, z\hat{z})$$

$$\vec{P} \rightarrow \hat{P} = (-i\hbar\frac{\partial}{\partial x}\hat{x}, -i\hbar\frac{\partial}{\partial}y\hat{y}, -i\hbar\frac{\partial}{\partial}z\hat{z})$$

$$\vec{L} \rightarrow \hat{L} = (L_x\hat{x}, L_y\hat{y}, L_z\hat{z})$$

$$\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$

$$\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)$$

$$\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

Show it!

ORBITAL ANGULAR MOMENTUM

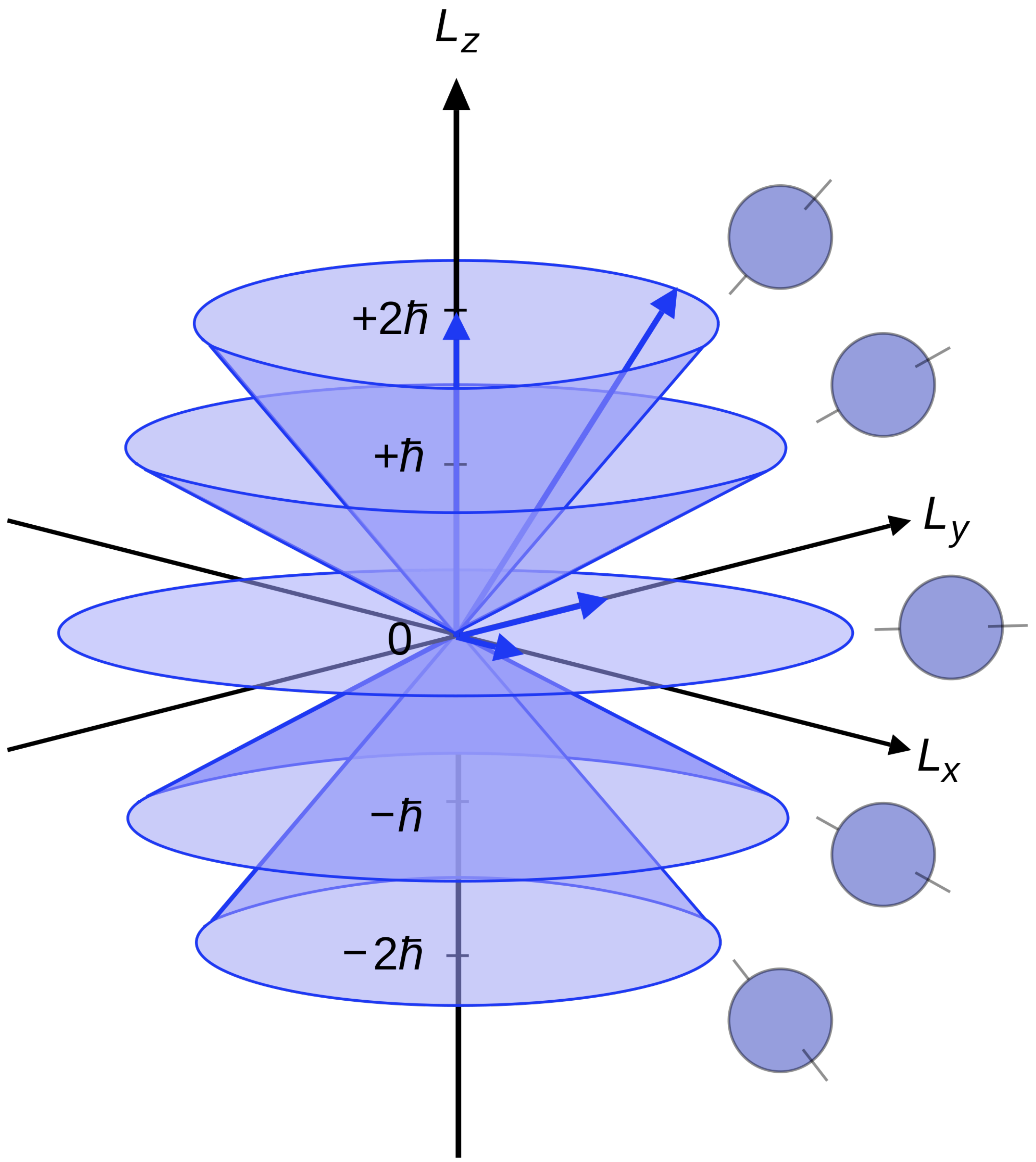
In quantum mechanics

- Assuming that the wave function of a particle is given by |ψ⟩ it can be chosen to be the eigenfunction of L^2 and L_z according to

$$\hat{L}^2|\Psi_{lm_l}\rangle = l\cdot(l+1)\hbar^2|\Psi_{lm_l}\rangle$$

$$\hat{L}_z|\Psi_{lm_l}\rangle = m_l\hbar|\Psi_{lm_l}\rangle$$

  - The quantum numbers l and m_l are integers and m_l can take any value from -l, -l+1,…,0,…,l-1,l (2l+1) values



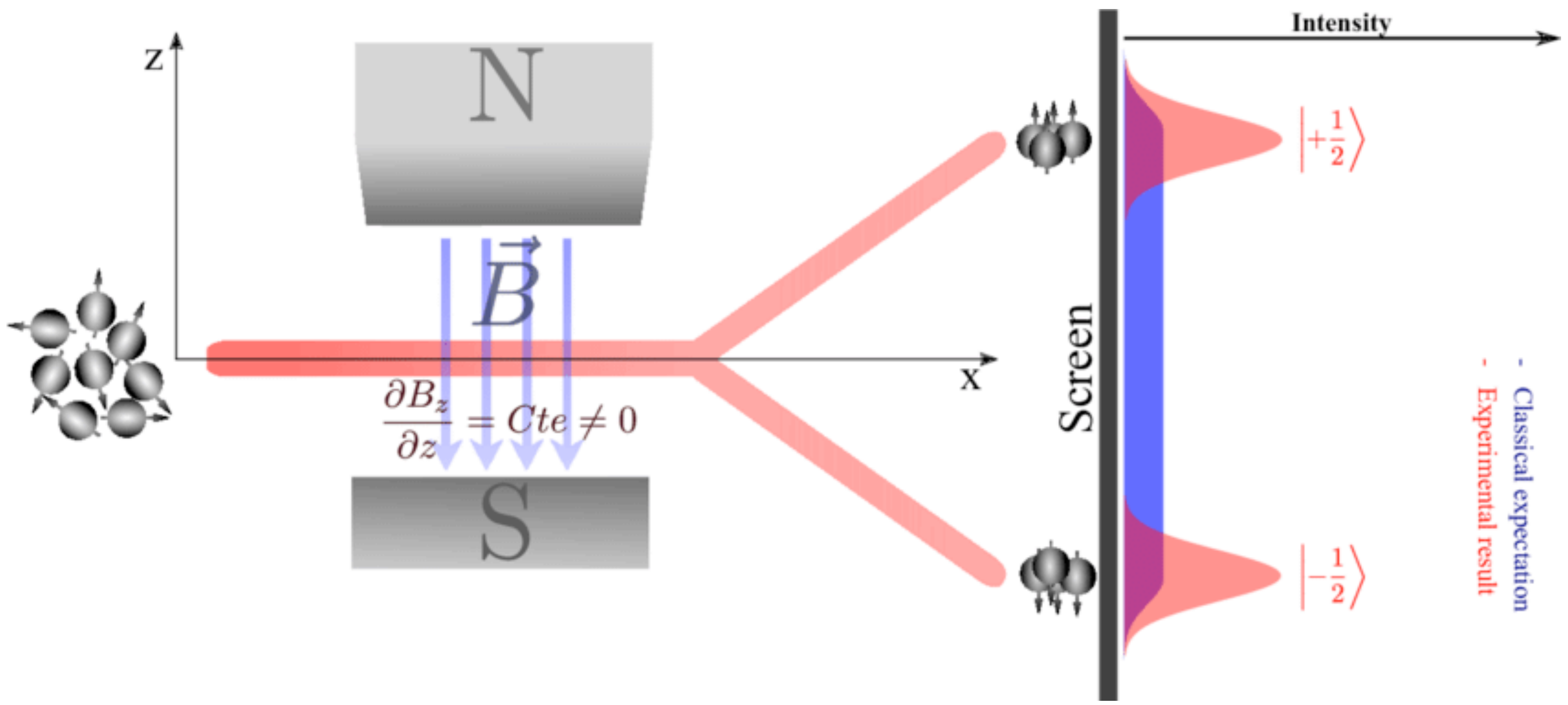
SPIN

The electron appeared (~1920) to have some **intrinsic angular momentum,** with only **two orientations** possible

$$|\hat{L}_z^e| = \frac{\hbar}{2}$$

$$l_e = \pm\frac{\hbar}{2}$$

Stern-Gerlach experiment



Is there a classical counterpart?

SPIN

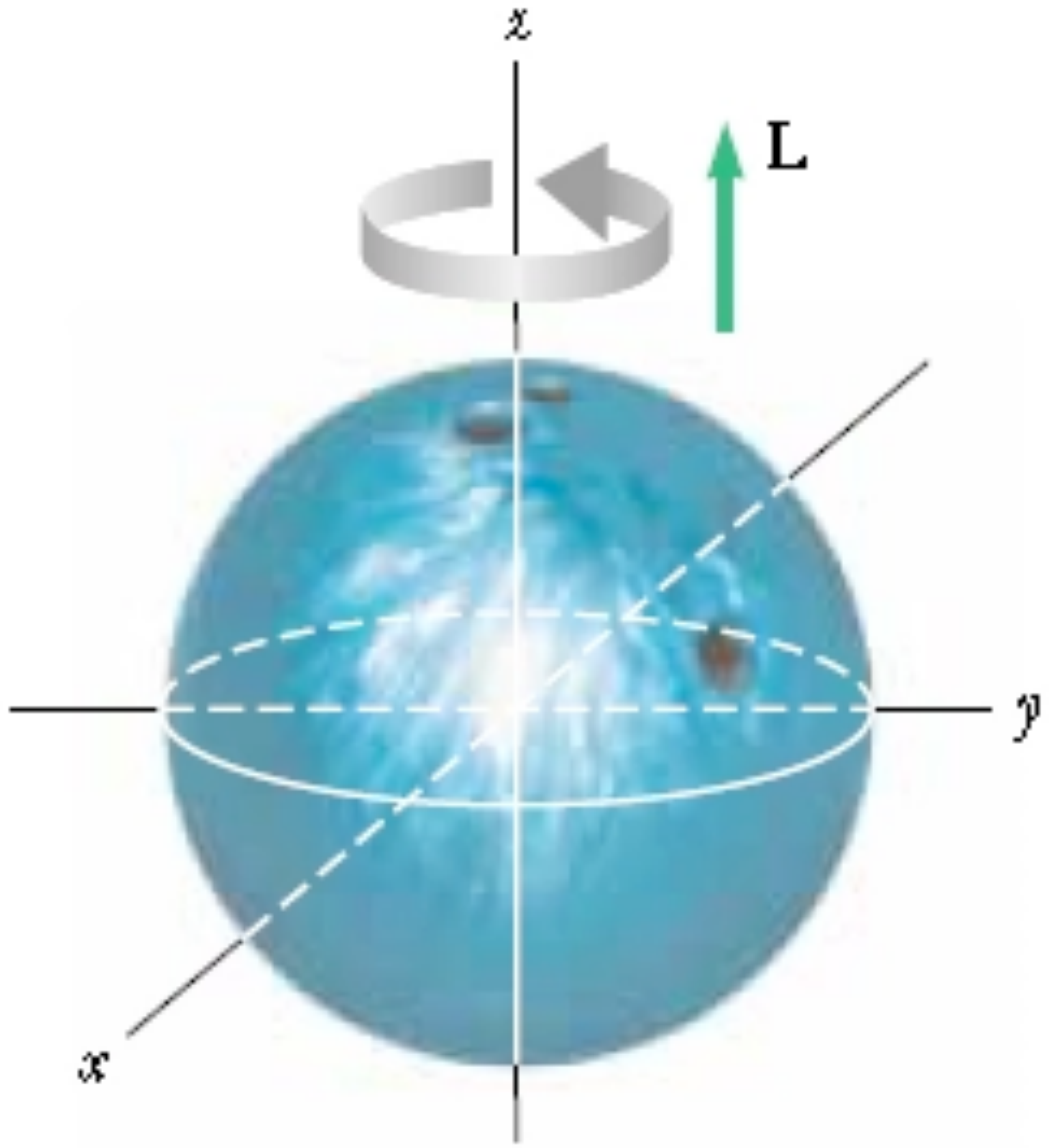
Can we understand intrinsic angular momentum of electron using **classical mechanics?**

$$|\vec{L}| = I \times \omega = \left(\frac{2}{5}MR^2\right) \cdot \omega$$

Moment of inertia   Angular velocity

If we assume the electron is a solid (classical) body, **at which speed** it needs to rotate?

$$v = \frac{5\hbar}{4m_e r_e} \approx 1.5 \times 10^{14} m/s$$



$$m_e = 9.11 \times 10^{-31} kg \qquad r_e \approx 10^{-18} m$$

$$\hbar = 1.05 \times 10^{-34} J \cdot s$$

$$|\vec{L}_e| = \left(\frac{2}{5}m_e r_e^2\right) \cdot \omega = \left(\frac{2}{5}m_e r_e^2\right) \cdot \frac{v}{r_e} = \frac{\hbar}{2}$$

No classical counterpart!!!

SPIN

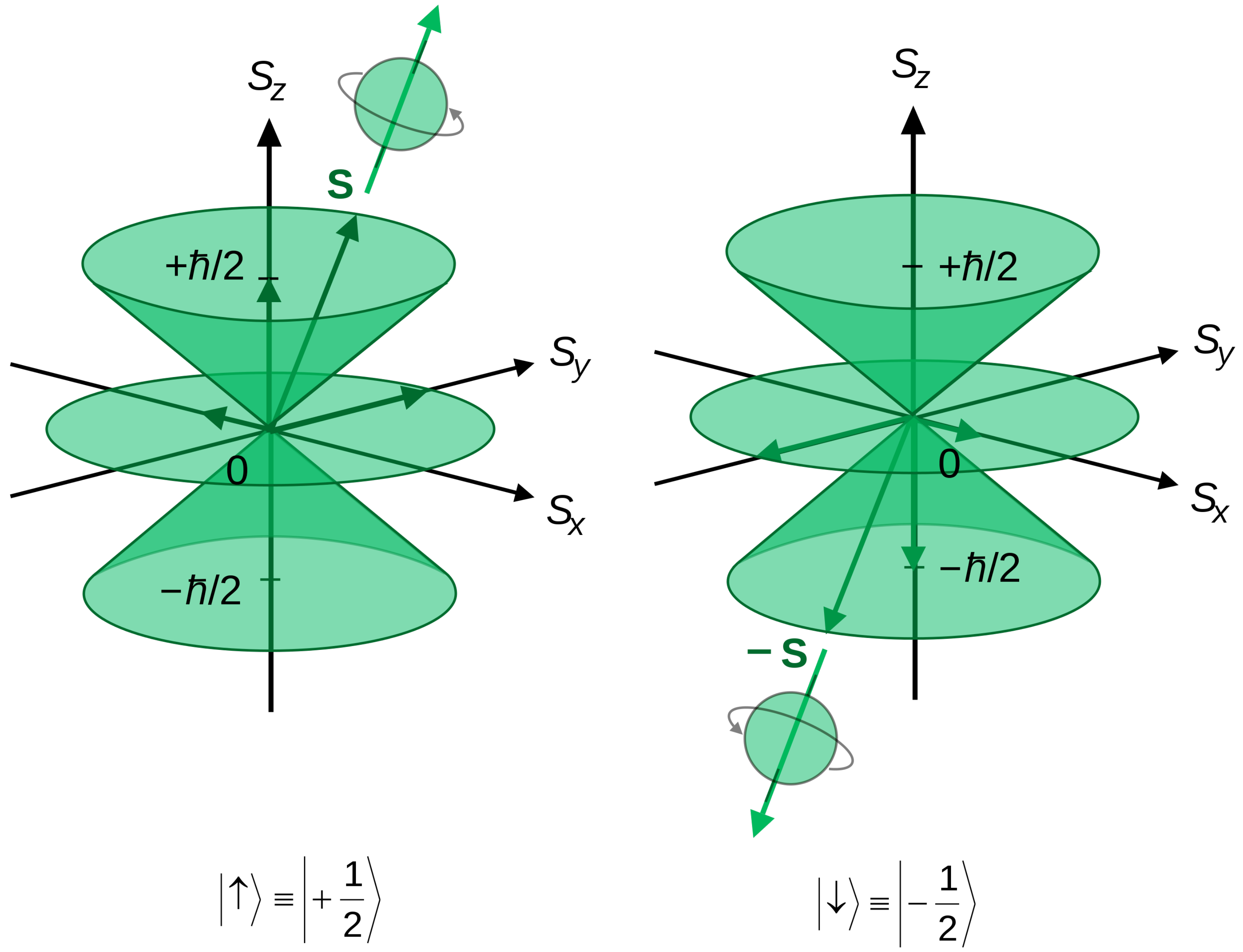
Spin: intrinsic angular momentum of elementary particles

- Even particles with zero rest mass have spin (e.g. γ)

The spin operators satisfy the following

$$\hat{S}^2|\Psi_{sm_s}\rangle = s\cdot(s+1)\hbar^2|\Psi_{sm_s}\rangle$$
$$\hat{S}_z|\Psi_{sm_s}\rangle = m_s\hbar|\Psi_{sm_s}\rangle$$

- The allowed values for s are not only integers but also half-integers: 0, 1/2, 1, 3/2, 2, 5/2…
- The allowed values for m_s are (2s+1): -s, -s+1,…0,…, s-1, s



TOTAL ANGULAR MOMENTUM

The eigenvalue of J_z adds up:

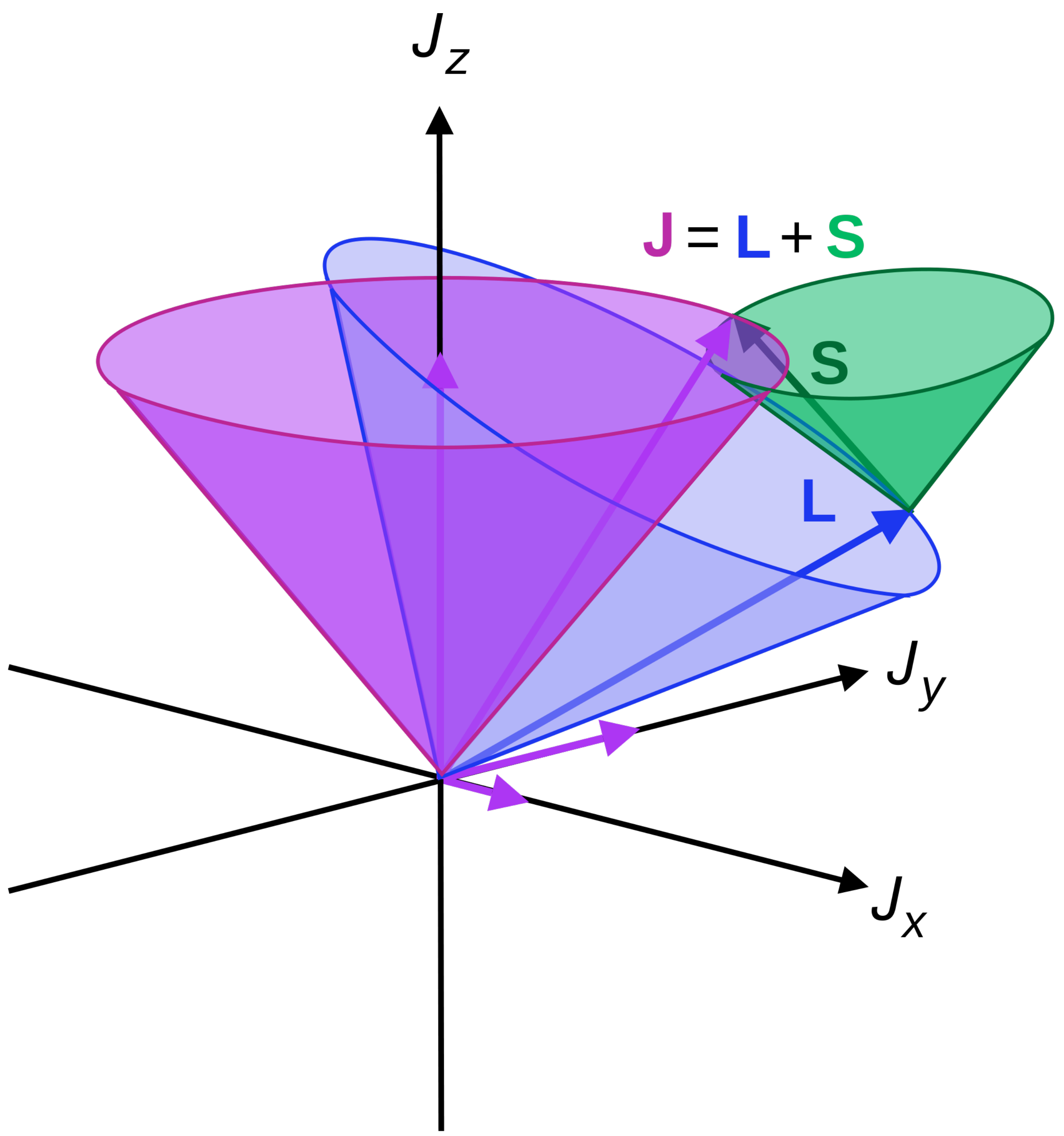
$m_j = m_l + m_s$

The eigenvalue of J can be anywhere between |l-s| and l+s

$$\vec{J} = \vec{L} + \vec{S}$$

$$\hat{J}^2 |\Psi_{jm_j}\rangle = j \cdot (j+1)\hbar^2 |\Psi_{jm_j}\rangle$$

$$\hat{J}_z |\Psi_{jm_j}\rangle = m_j \hbar |\Psi_{jm_j}\rangle$$



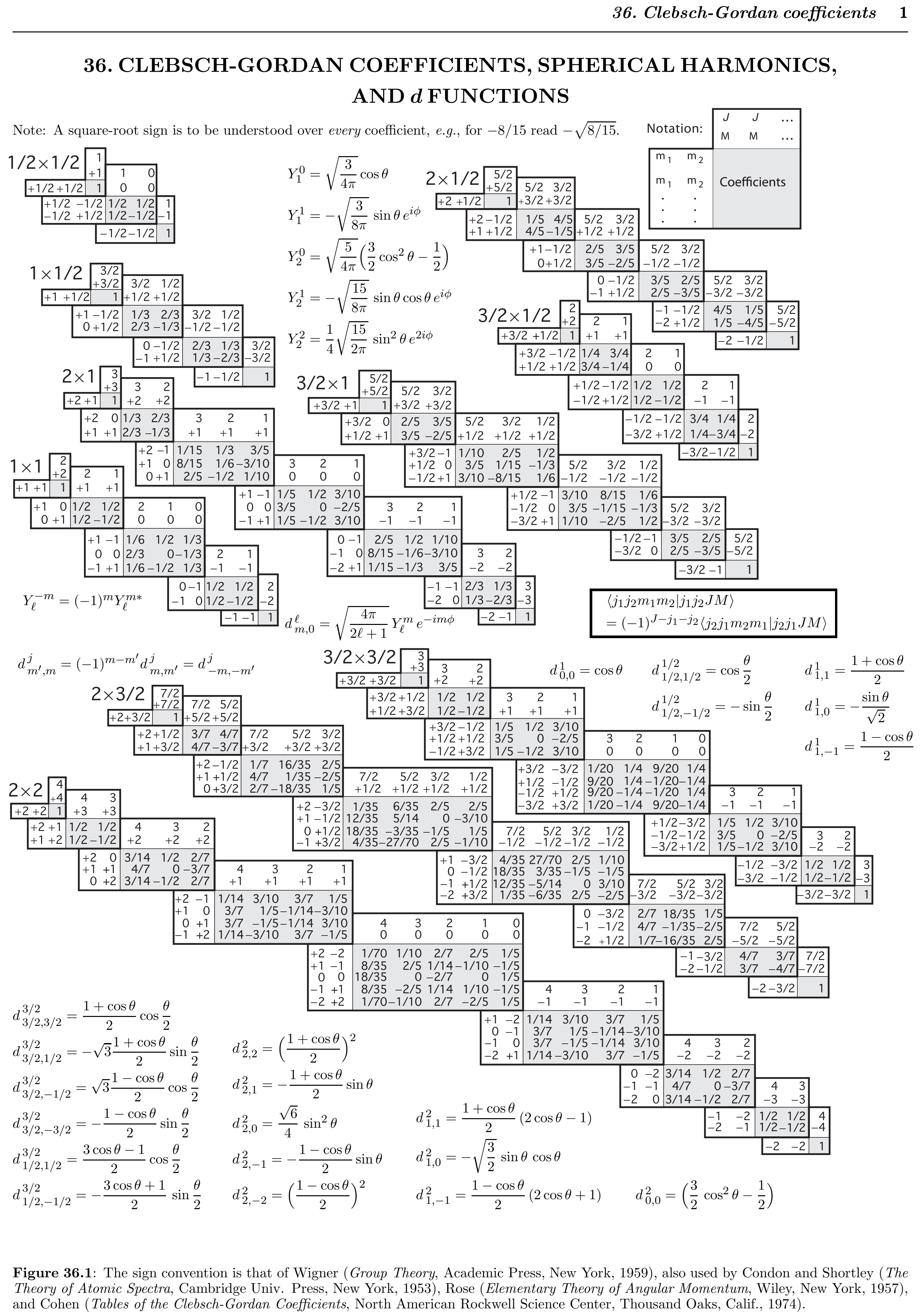
ANGULAR MOMENTUM AND SPIN OF A SYSTEM

The eigenvalue of s_z adds up: $s = s_1 + s_2$

The eigenvalue of s can be anywhere between $|s_1 - s_2|$ and $s_1 + s_2$

$$|s_1, m_{s1}\rangle \otimes |s_2, m_{s2}\rangle \rightarrow |s, m_s\rangle$$

$$|s, m_s\rangle = \sum_{m_s} C^{s, s_1, s_2}_{m_s, m_{s_1}, m_{s_2}} |s_1, m_{s_1}\rangle |s_2, m_{s_2}\rangle$$



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CATEGORIES OF PARTICLES

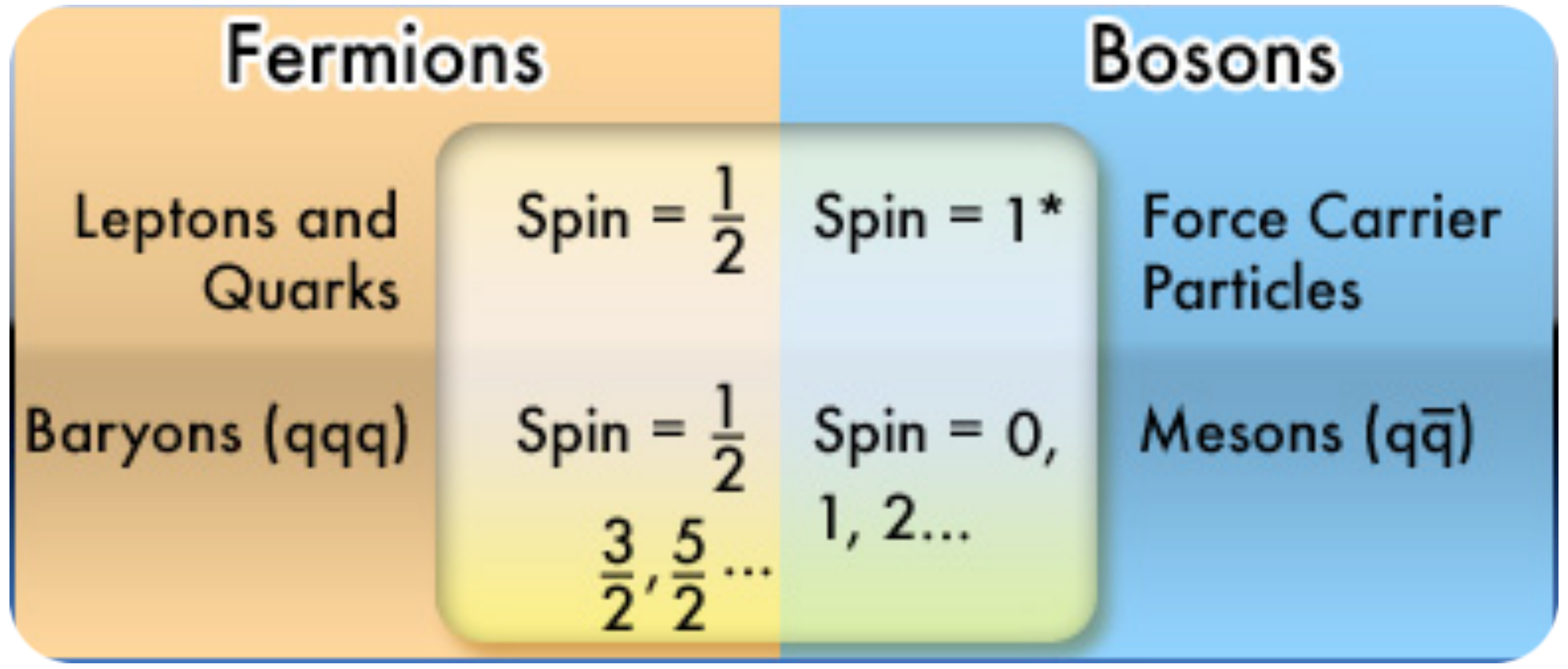
FERMIONS VS BOSONS

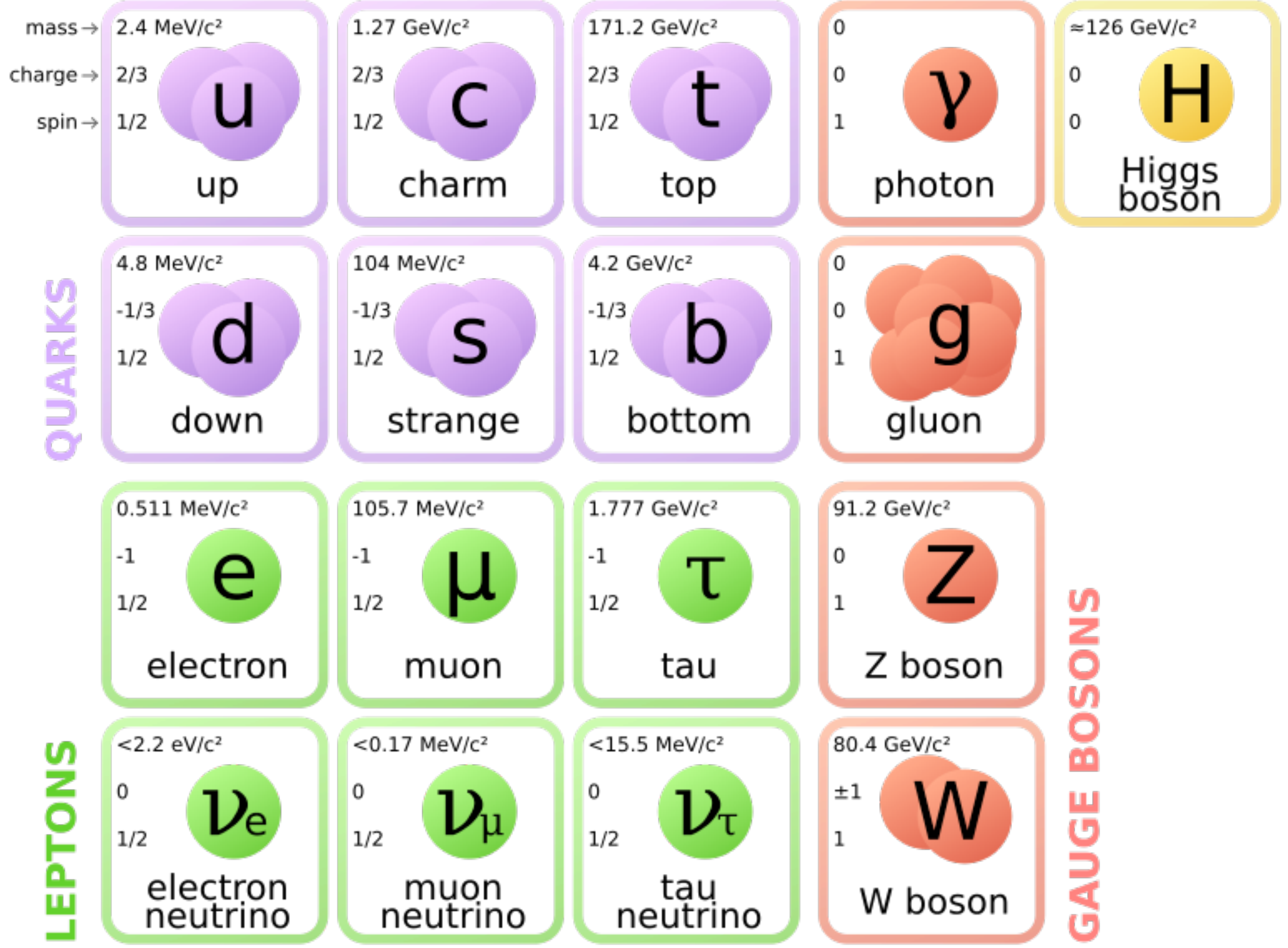
A fermion is any particle that has an odd half-integer (like 1/2, 3/2, and so forth) spin.

- Quarks and leptons are fermions with spin-1/2
- Baryons are composite particles, consisting of three quarks (anti-baryons consist of three anti-quarks) are fermions with spin 1/2, 3/2, 5/2,…

Bosons are those particles which have an integer spin (0, 1, 2...).

- All the force carrier particles are bosons with spin-1
- Mesons are composite particles consisting of a quark and an anti-quark are also mesons with spin 0, 1, 2,…





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FERMIONS VS BOSONS

Fermions and bosons do exhibit **vastly different properties** due to their different spins

Consider a quantum system composed by **two identical particles** with position $\mathbf{x}_1$ and $\mathbf{x}_2$

$$\psi_{tot.}(x_1, x_2) = \psi_1(x1)\psi_2(x2)$$

Now we can **exchange the position** of the two particles, and end up with:

$$\tilde{\psi}_{tot.}(x_1, x_2) = \psi_1(x2)\psi_2(x1)$$

FERMIONS VS BOSONS

Since the particles are identical, any physical measurements carried out in the system should yield exactly **the same result**

- In other words, the **probability** of finding the two particles at $\mathbf{x}_1$ and $\mathbf{x}_2$ should not change

So when we interchange the position of the two identical particles, the total wave function must be **unchanged up to a complex phase**

$$|\psi_{tot.}(x_1, x_2)|^2 = |\tilde{\psi}_{tot.}(x_1, x_2)|^2$$

$$|\psi_1(x1)\psi_2(x2)|^2 = |\psi_1(x2)\psi_2(x1)|^2$$

$$\downarrow$$

$$\psi_1(x1)\psi_2(x2) = e^{i\varphi}\psi_1(x2)\psi_2(x1)$$

FERMIONS VS BOSONS

What happens if we **exchange again** the position of the particles?

$$\psi_1(x1)\psi_2(x2) = e^{i\varphi}\psi_1(x2)\psi_2(x1) = e^{i\varphi}\left(e^{i\varphi}\psi_1(x1)\psi_2(x2)\right)$$

Which implies that the complex phase can only take **two values**

$$e^{i2\varphi} = 1 \rightarrow \varphi = 0, \pi \rightarrow e^{i\varphi} = 1, -1$$

Quantum mechanics tell us that there exist **two kinds of particles** depending on how they behave under exchanging them

Bosons: if we exchange two identical bosons, the wave function is **unchanged**

$$\psi_1(x_1)\psi_2(x_2) = \psi_1(x_2)\psi_2(x_1)$$

Fermions: if we exchange two identical fermions, the wave function **changes sign**

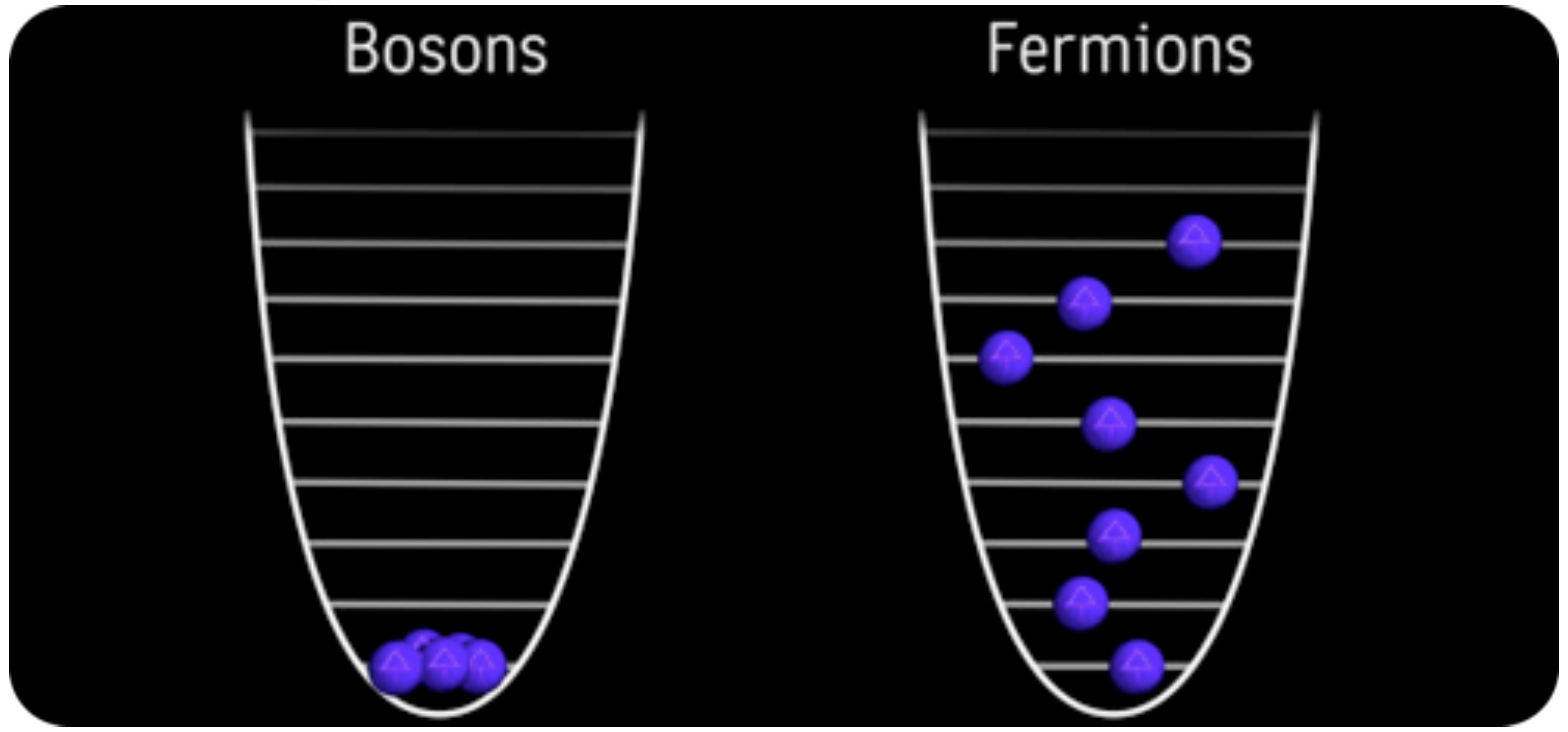
$$\psi_1(x_1)\psi_2(x_2) = -\psi_1(x_2)\psi_2(x_1)$$

FERMIONS VS BOSONS

What happens if two fermions occupy the same quantum state?

$$\psi_1(x_1)\psi_2(x_1) = -\psi_1(x_1)\psi_2(x_1)$$

$$\psi_{tot.}(x_1, x_1) = \frac{1}{\sqrt{2}}\Big(\psi_1(x1)\psi_2(x1) + \psi_1(x1)\psi_2(x1)\Big) = 0$$

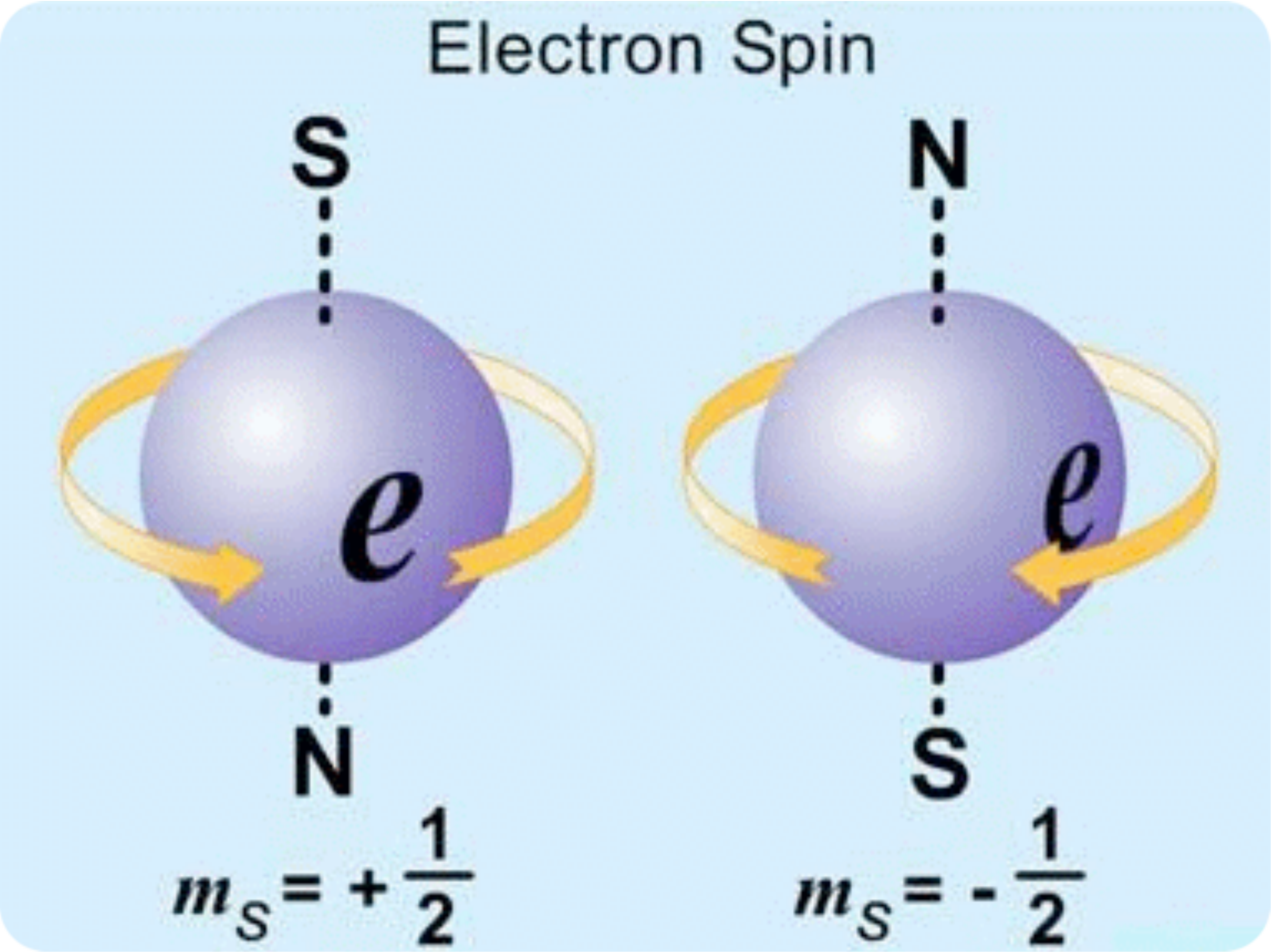


If two particles have the same quantum numbers, they are in the same state
If these two particles are fermions then the wave function vanishes

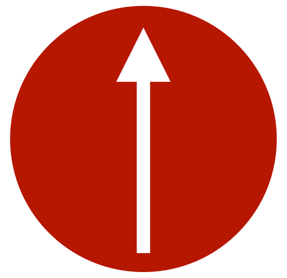
Pauli principle

A system cannot exist with two or more fermions in the same state

SPIN 1/2 PARTICLES



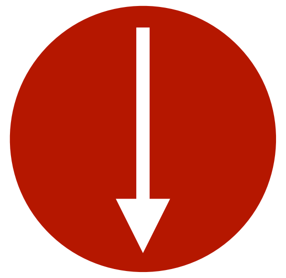
Spin-up



$$\begin{pmatrix}1\\0\end{pmatrix}$$

$$|\uparrow\rangle = |1/2, 1/2\rangle$$

Spin-down



$$\begin{pmatrix}0\\1\end{pmatrix}$$

$$|\downarrow\rangle = |1/2, -1/2\rangle$$

General representation

$$\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$$

$|\alpha|^2$: probability to have m_s=+1/2
$|\beta|^2$: probability to have m_s=-1/2

ISOSPIN

Neutrons and protons are quite similar apart from their charge

- Heisenberg proposed that they are regarded as the two states of the same particle
  - the nucleon
- Similar to the notation related to spin we can write p and n with a two component column matrix
- By direct analogy to spin we introduce isospin with coordinates in the isospin space:
  - I_1, I_2, I_3
- Strong interactions are invariant under rotations in isospin space
  - Isospin is conserved
  - Group theory wording:
    - Strong interactions are invariant under an internal symmetry of SU(2)

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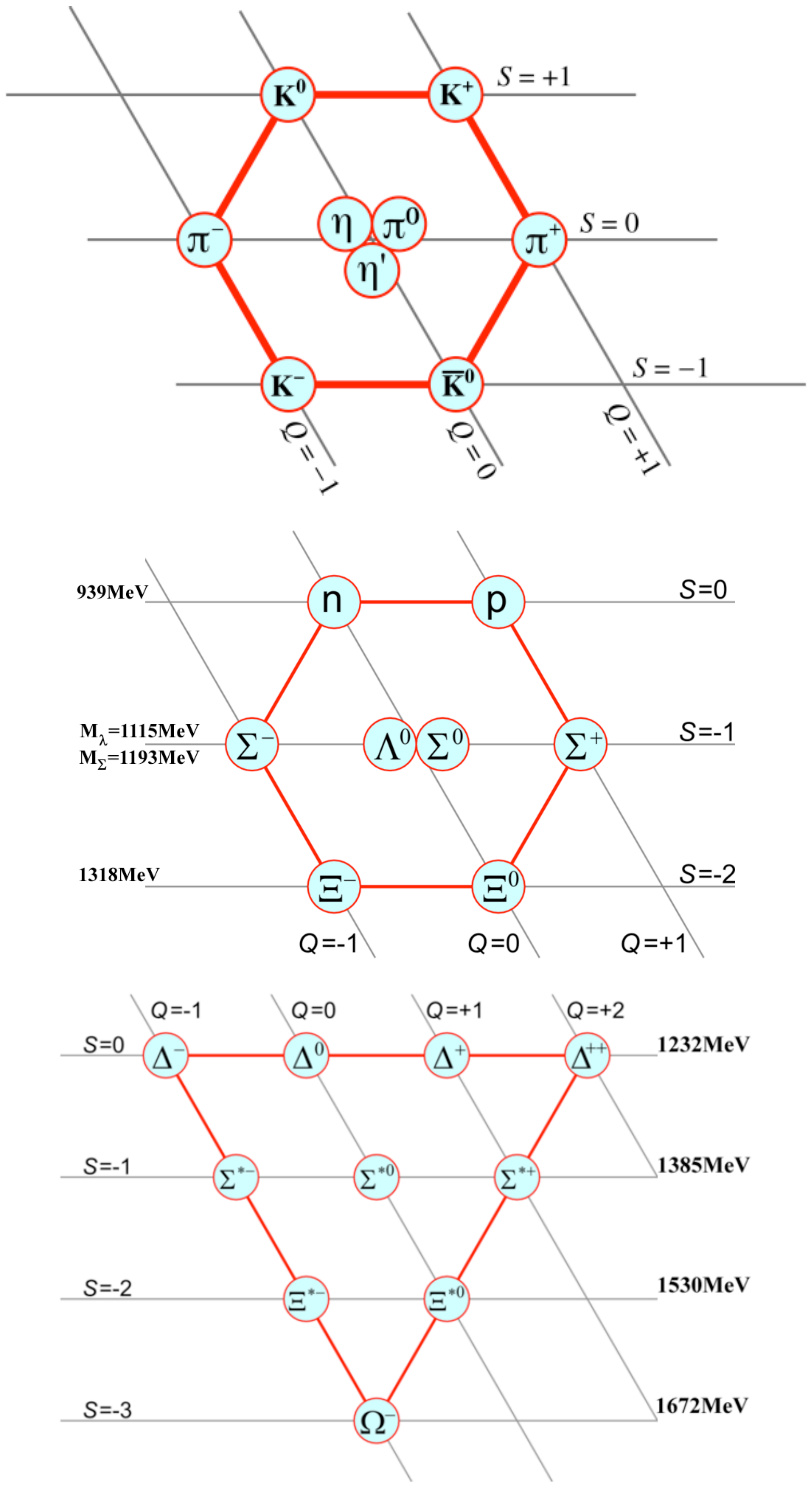
QUARK MODEL

QUARK MODEL

Introduced by Gell-Mann and Zweig (1964)

- All multiplets can be explained if you assume that hadrons are composite particles built from more elementary constituents: the quarks and antiquarks
  - Baryons are made of three quarks (Antibaryons are made of three antiquarks)
  - Mesons are made of a quark and an antiquark combination
- First quark model consisted of the three lightest quarks (and antiquarks)



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NOBEL PRIZE

The Nobel Prize in Physics 1969
Murray Gell-Mann

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The Nobel Prize in Physics 1969

Murray Gell-Mann
Prize share: 1/1

The Nobel Prize in Physics 1969 was awarded to Murray Gell-Mann *"for his contributions and discoveries concerning the classification of elementary particles and their interactions".*

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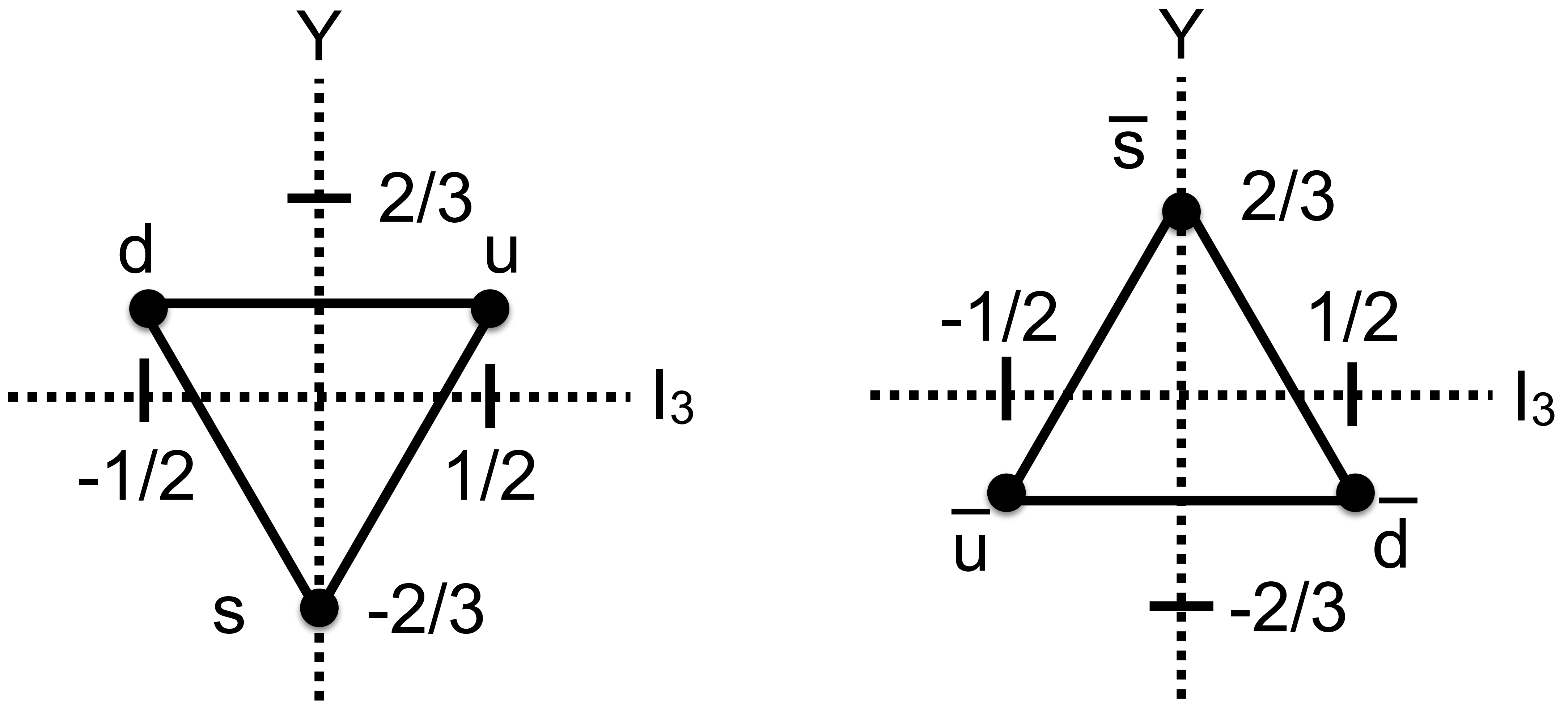
QUARK MODEL

Hypercharge Y = B+S



First quark model used the three lightest quarks

QUANTUM NUMBERS OF QUARKS

	u	d	s	c	b	t
Q - electric charge	+2/3	-1/3	-1/3	+2/3	-1/3	+2/3
I - isospin	1/2	1/2	0	0	0	0
I_3 - isospin z-component	1/2	-1/2	0	0	0	0
S - strangeness	0	0	-1	0	0	0
C - charm	0	0	0	1	0	0
B - beauty	0	0	0	0	-1	0
T - topness	0	0	0	0	0	1

MESONS

Mesons are part of the hadron family, together with the baryons

Mesons are particles composed of a combination of a quark and an antiquark

Since they consist of an even combination of subatomic particles with spin 1/2, mesons are bosons

Particle	Mass (MeV/c^2)	Charge (e)	Mean Life (sec)
π^0	135.0	0	0.84×10^{-16}
$\pi^\pm$	139.6	$+,-$	2.60×10^{-8}
$K^\pm$	493.7	$+,-$	1.24×10^{-8}
K^0	497.7	0	Complicated
η	547.8	0	5.1×10^{-19}
$D^\pm$	1869	$+,-$	1.0×10^{-12}
D^0	1865	0	4.1×10^{-13}
$B^\pm$	5279	$+,-$	$\sim 1.7 \times 10^{-12}$
B^0	5279	0	$\sim 1.5 \times 10^{-12}$

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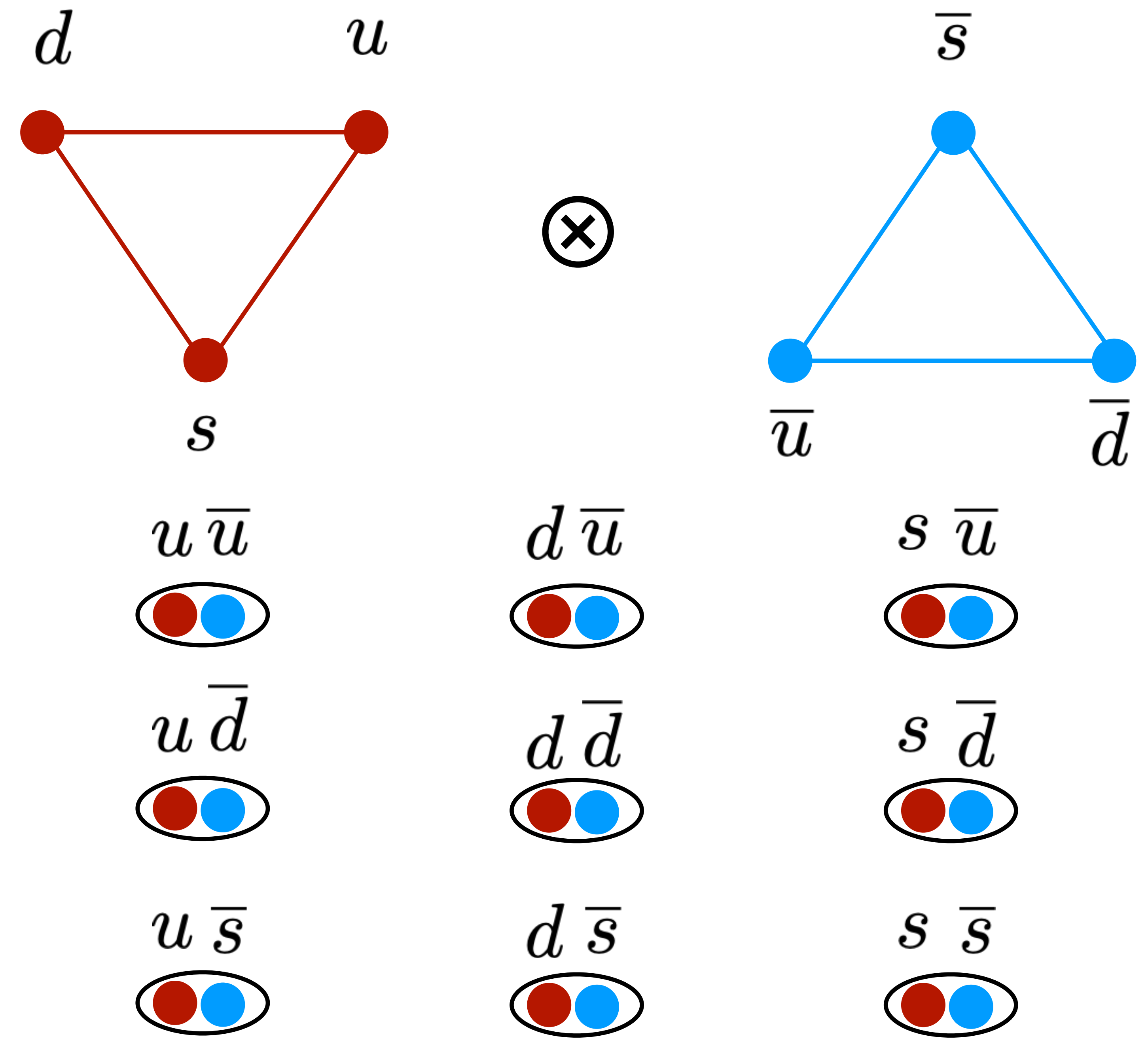
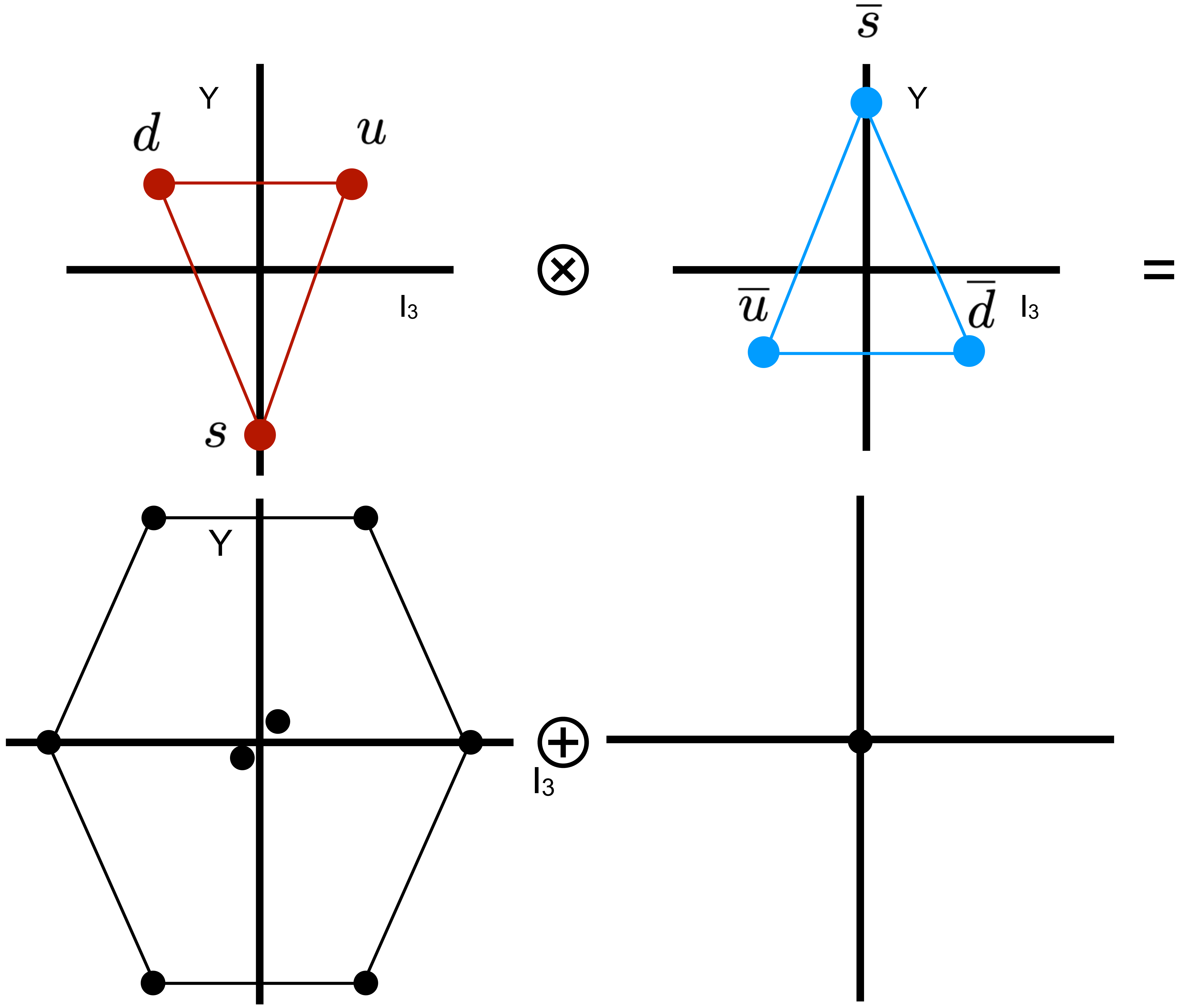
MESONS

Consider the three lightest (anti)quarks: u(bar), d(bar), s(bar)

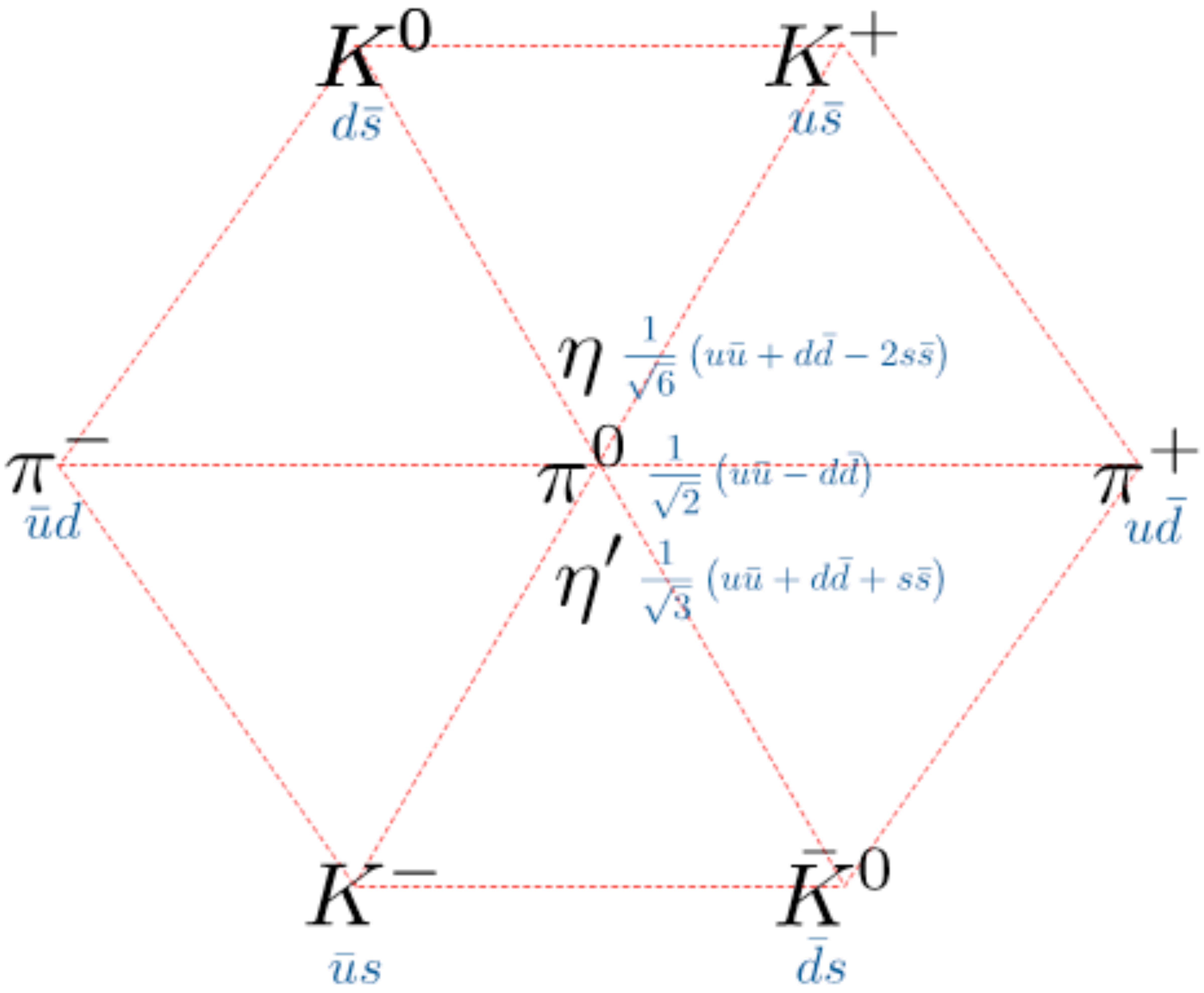
- Which mesons can one form?



MESONS



MESON OCTET



K^0
$d\bar{s}$
K^+
$u\bar{s}$
η $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$
π^-
$\bar{u}d$
π^0 $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$
π^+
$u\bar{d}$
η' $\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$
K^-
$\bar{u}s$
$\bar{K}^0$
$\bar{d}s$

SOME OF THE MESONS IN THE QUARK MODEL

Table 15.2: Suggested $q\overline{q}$ quark-model assignments for some of the observed light mesons. Mesons in bold face are included in the Meson Summary Table. The wave functions f and f' are given in the text. The singlet-octet mixing angles from the quadratic and linear mass formulae are also given for the well established nonets. The classification of the 0^{++} mesons is tentative: The light scalars $a_0(980)$, $f_0(980)$, and $f_0(500)$ are often considered as meson-meson resonances or four-quark states, and are omitted from the table. Not shown either is the $f_0(1500)$ which is hard to accommodate in the nonet. The isoscalar 0^{++} mesons are expected to mix. See the "Note on Scalar Mesons" in the Meson Listings for details and alternative schemes.

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\overline{d}$, $\overline{u}d$, $\frac{1}{\sqrt{2}}(d\overline{d}-u\overline{u})$	I = $\frac{1}{2}$ $u\overline{s}$, $d\overline{s}$; $\overline{d}s$, $-\overline{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	**π**	**K**	**η**	**$\eta'(958)$**	−11.4	−24.5
$1\,^3S_1$	1^{--}	**$\rho(770)$**	**$K^*(892)$**	**$\phi(1020)$**	**$\omega(782)$**	39.1	36.4
$1\,^1P_1$	1^{+-}	**$b_1(1235)$**	**K_{1B}**†	$h_1(1380)$	**$h_1(1170)$**		
$1\,^3P_0$	0^{++}	**$a_0(1450)$**	**$K_0^*(1430)$**	**$f_0(1710)$**	**$f_0(1370)$**		
$1\,^3P_1$	1^{++}	**$a_1(1260)$**	**K_{1A}**†	**$f_1(1420)$**	**$f_1(1285)$**		
$1\,^3P_2$	2^{++}	**$a_2(1320)$**	**$K_2^*(1430)$**	**$f_2'(1525)$**	**$f_2(1270)$**	32.1	30.5
$1\,^1D_2$	2^{-+}	**$\pi_2(1670)$**	**$K_2(1770)$**†	$\eta_2(1870)$	**$\eta_2(1645)$**		
$1\,^3D_1$	1^{--}	**$\rho(1700)$**	**$K^*(1680)$**		**$\omega(1650)$**		
$1\,^3D_2$	2^{--}		**$K_2(1820)$**				
$1\,^3D_3$	3^{--}	**$\rho_3(1690)$**	**$K_3^*(1780)$**	**$\phi_3(1850)$**	**$\omega_3(1670)$**	31.8	30.8
$1\,^3F_4$	4^{++}	**$a_4(2040)$**	**$K_4^*(2045)$**		**$f_4(2050)$**		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	**$\pi(1300)$**	$K(1460)$	**$\eta(1475)$**	**$\eta(1295)$**		
$2\,^3S_1$	1^{--}	**$\rho(1450)$**	**$K^*(1410)$**	**$\phi(1680)$**	**$\omega(1420)$**		

† The $1^{+\pm}$ and $2^{-\pm}$ isospin $\frac{1}{2}$ states mix. In particular, the K_{1A} and K_{1B} are nearly equal (45°) mixtures of the $K_1(1270)$ and $K_1(1400)$. The physical vector mesons listed under 1^3D_1 and 2^3S_1 may be mixtures of 1^3D_1 and 2^3S_1, or even have hybrid components.

SOME OF THE MESONS IN THE QUARK MODEL

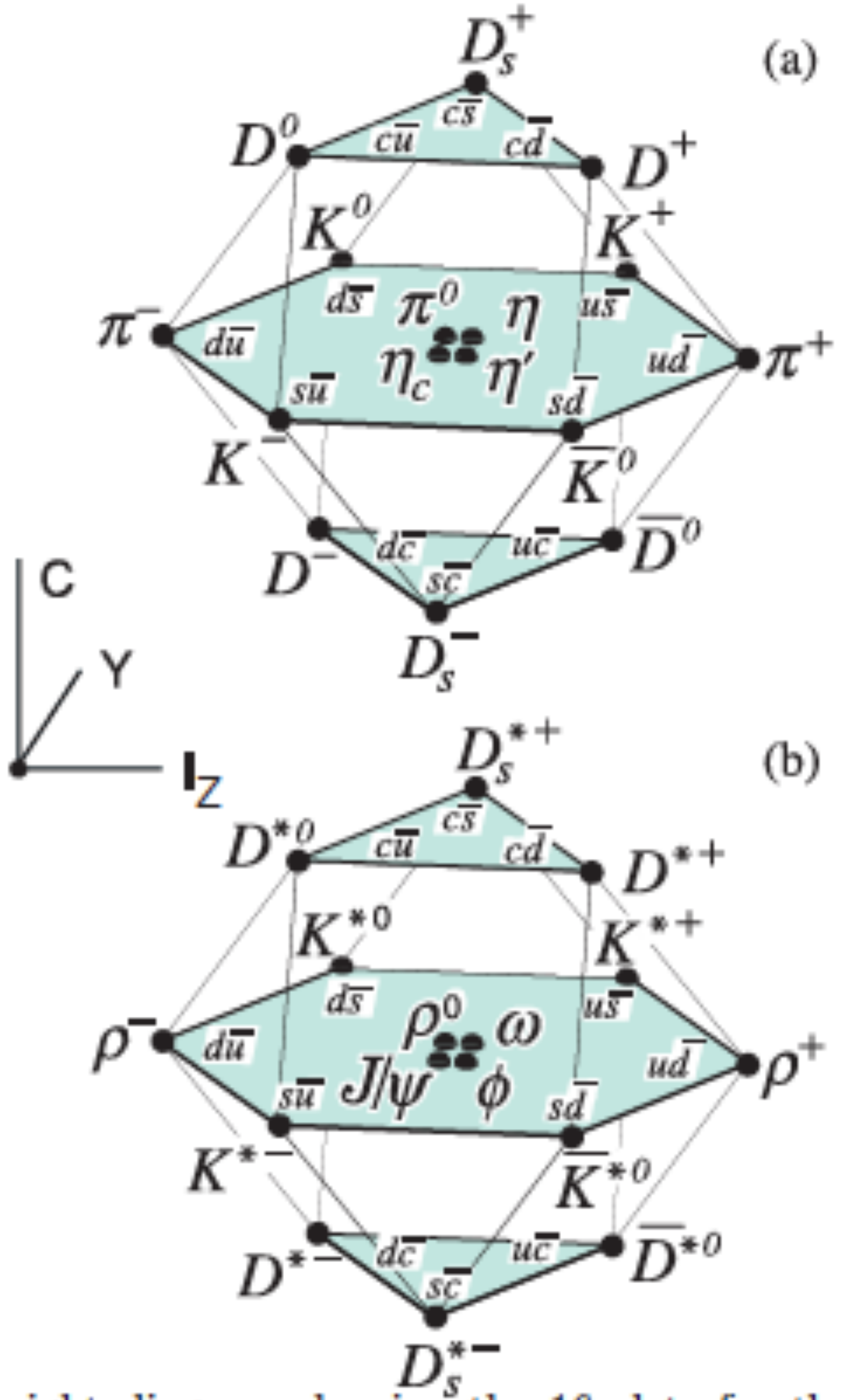


Figure 15.1: SU(4) weight diagram showing the 16-plets for the pseudoscalar (a) and vector mesons (b) made of the u, d, s, and c quarks as a function of isospin I_z, charm C, and hypercharge $Y = B + S - \frac{C}{3}$. The nonets of light mesons occupy the central planes to which the $c\bar{c}$ states have been added.

CATEGORIES OF MESONS

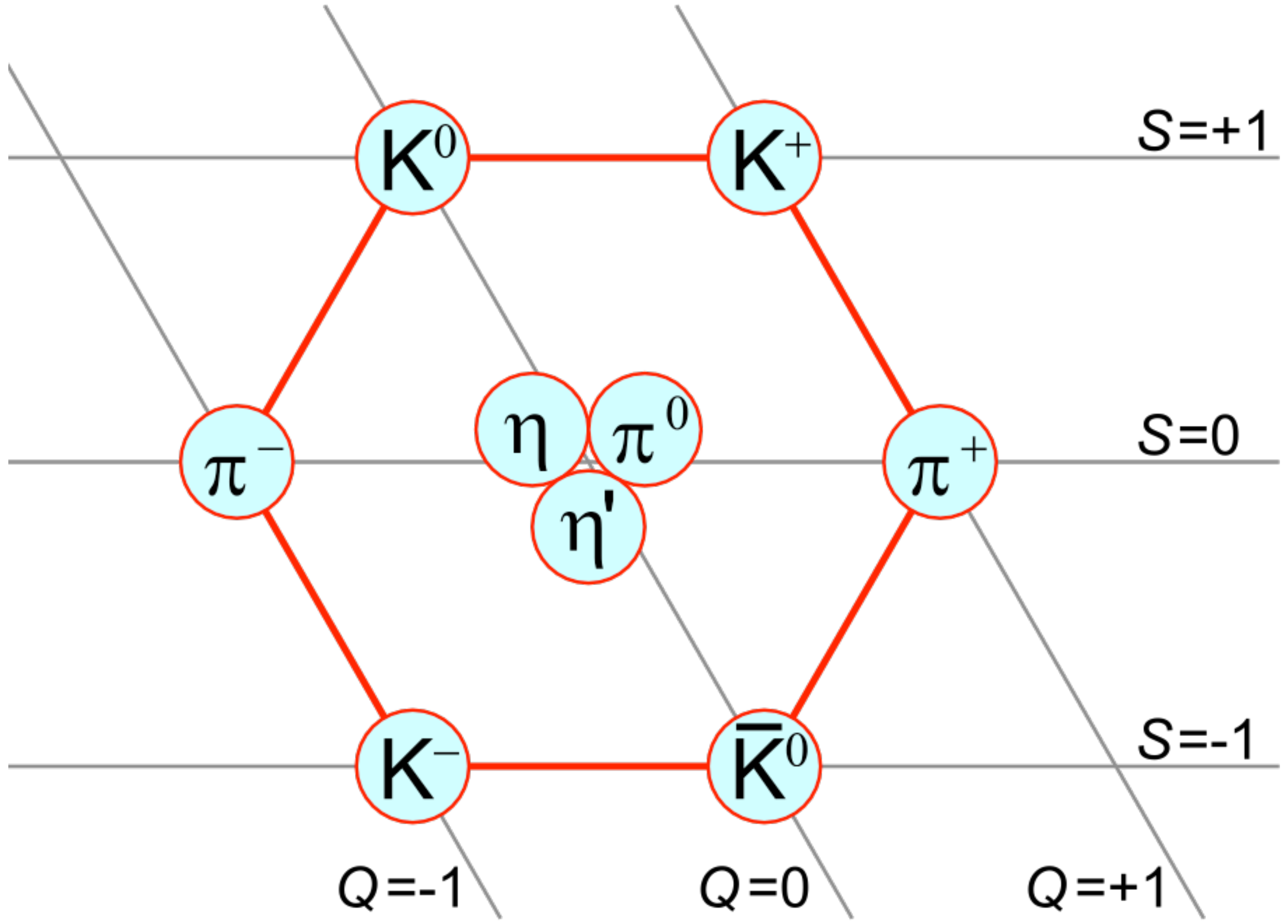
A meson contains a combination of a quark and an antiquark

- The spin of a meson can be either 0 or 1
- The angular momentum and thus the total momentum can take many different values

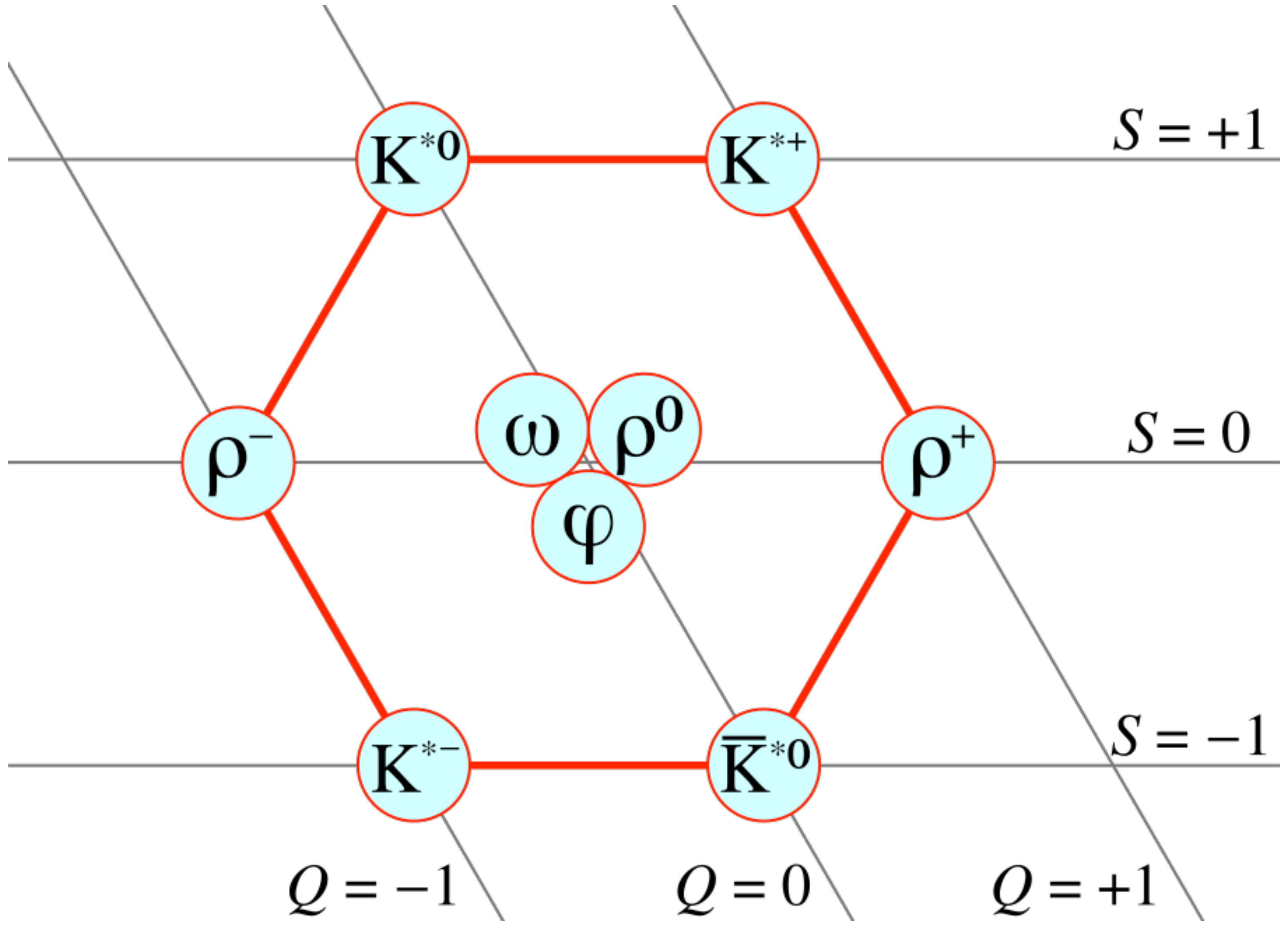
	S	L	J	$P \sim (-1)^{l+1}$	J^P
Pseudoscalar	0	0	0	-	0^-
Vector	1	0	1	-	1^-
Scalar	0	1	1	+	0^+
Pseudovector	1	1	1	+	1^+
Tensor	1	1	2	+	2^+

CATEGORIES OF MESONS

Pseudoscalar mesons:
s = 0 and l = 0



Vector mesons:
s = 1 and l = 0



for meson junkies:

http://pdg.lbl.gov/2014/tables/rpp2014-qtab-mesons.pdf

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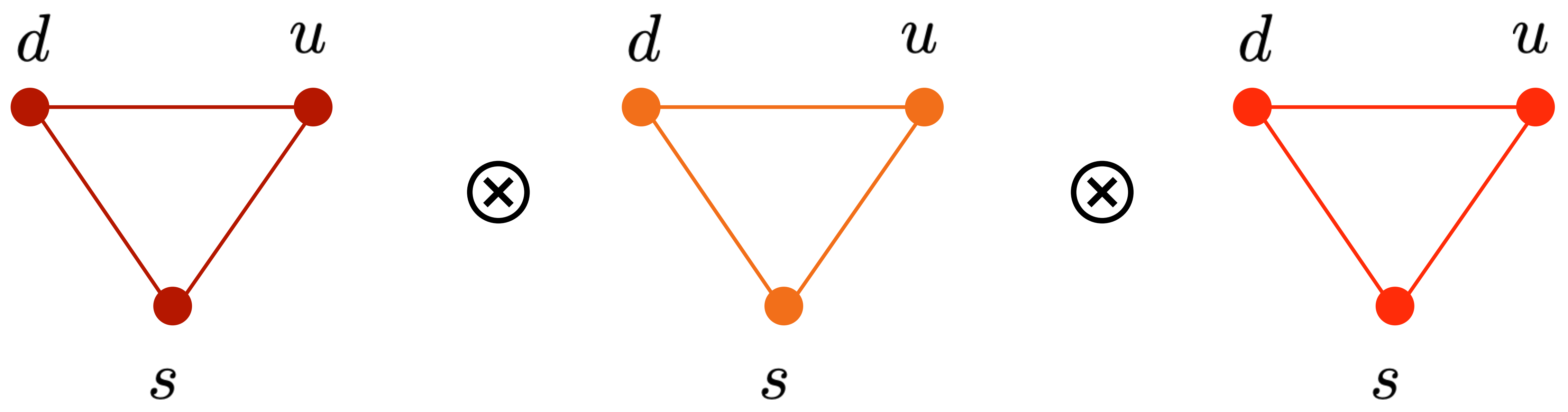
BARYONS

(Anti)Baryons are particles composed of a combination of three (anti)quarks

Since they consist of an odd combination of subatomic particles with spin 1/2, baryons are fermions



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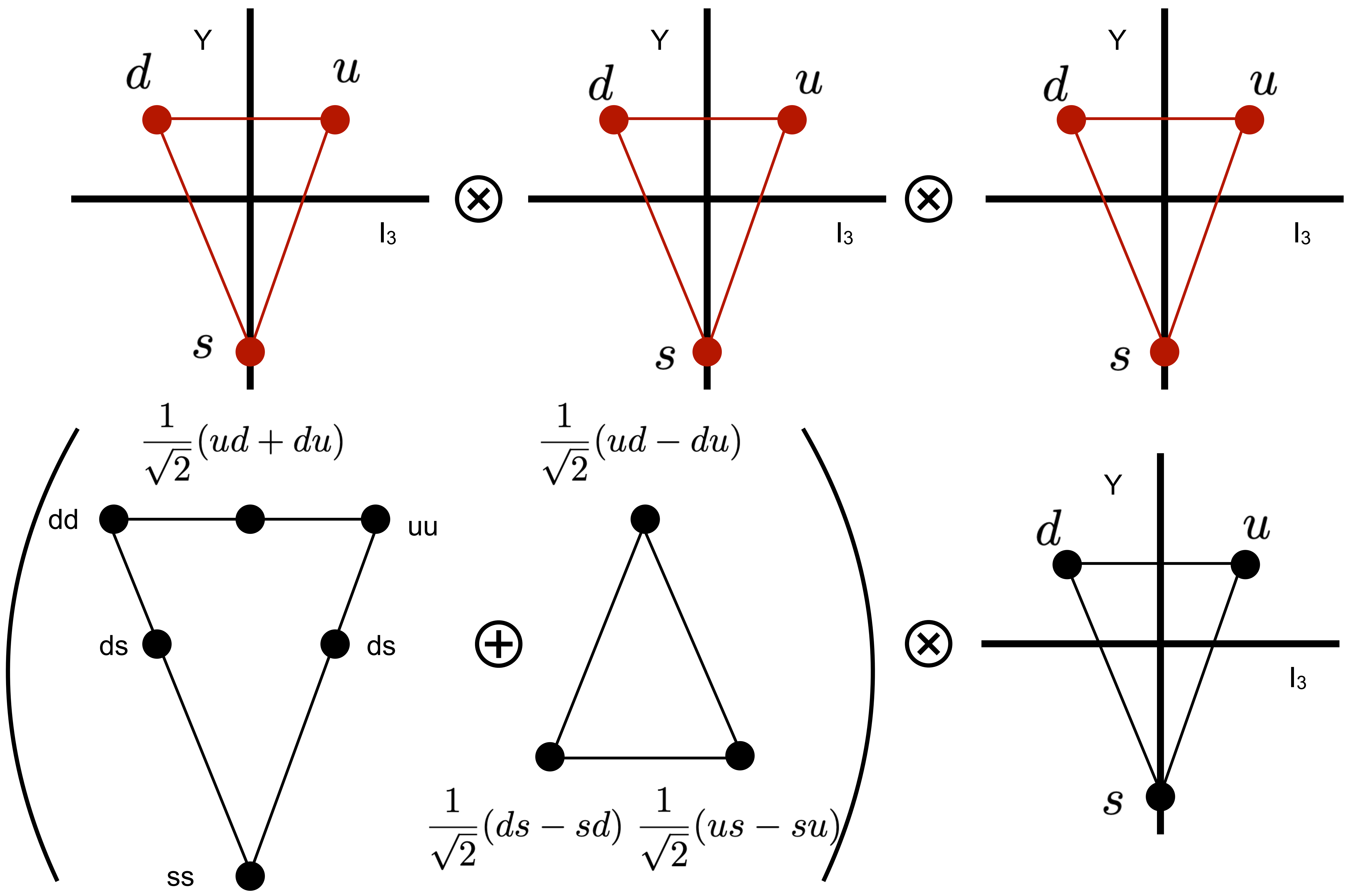
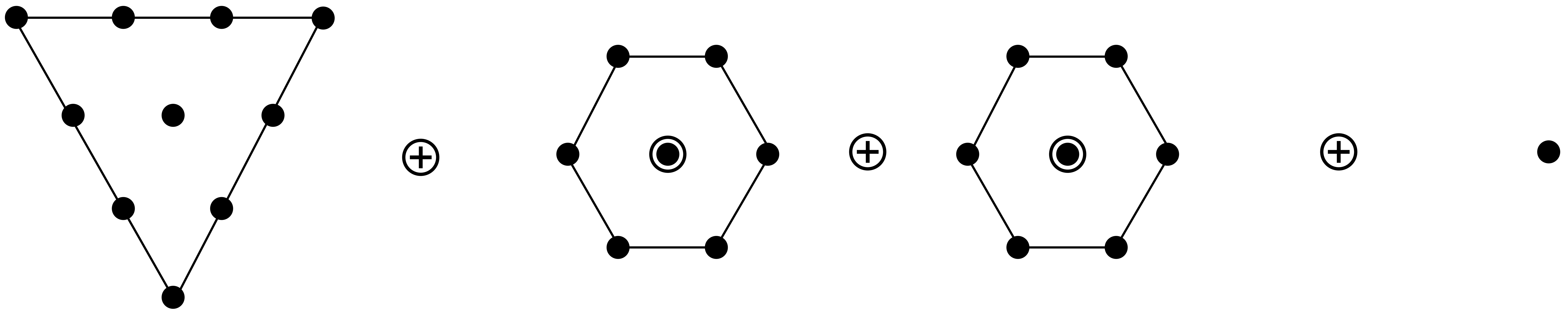
BARYONS



Y
d
u
I3
s
$\frac{1}{\sqrt{2}}(ud+du)$
$\frac{1}{\sqrt{2}}(ud-du)$
dd
uu
ds
ds
ss
$\frac{1}{\sqrt{2}}(ds-sd)$
$\frac{1}{\sqrt{2}}(us-su)$

BARYONS

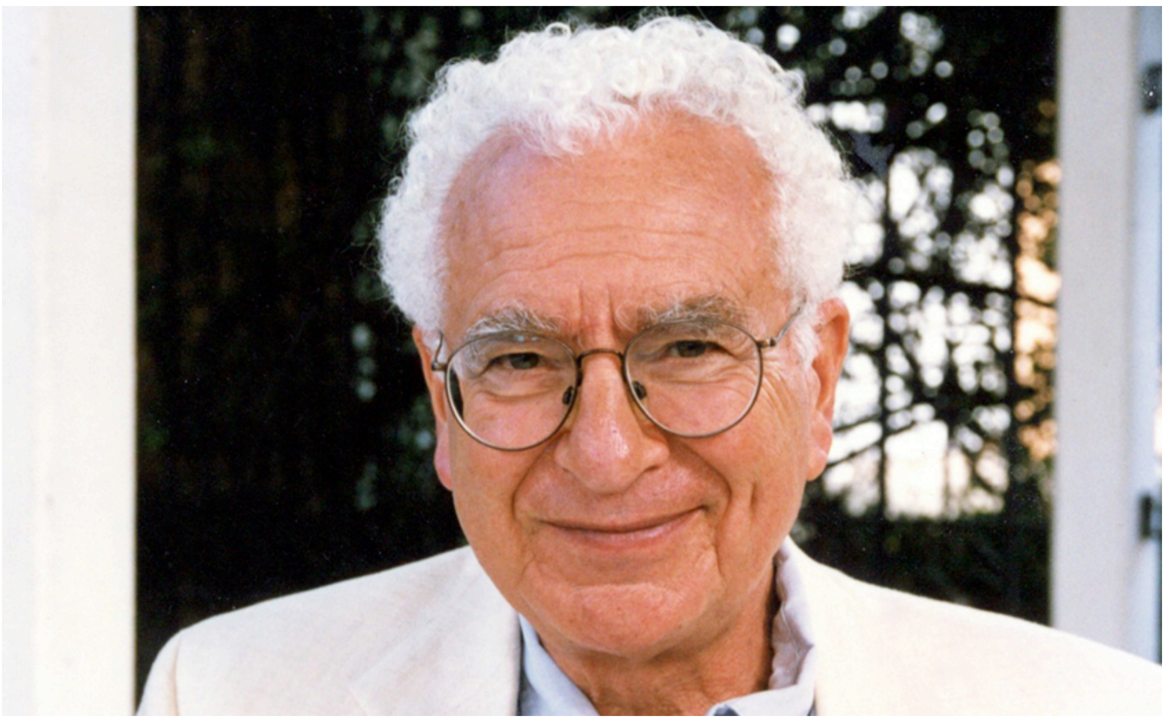
Symmetric states | Mixed states | Mixed states | Antisymmetric state



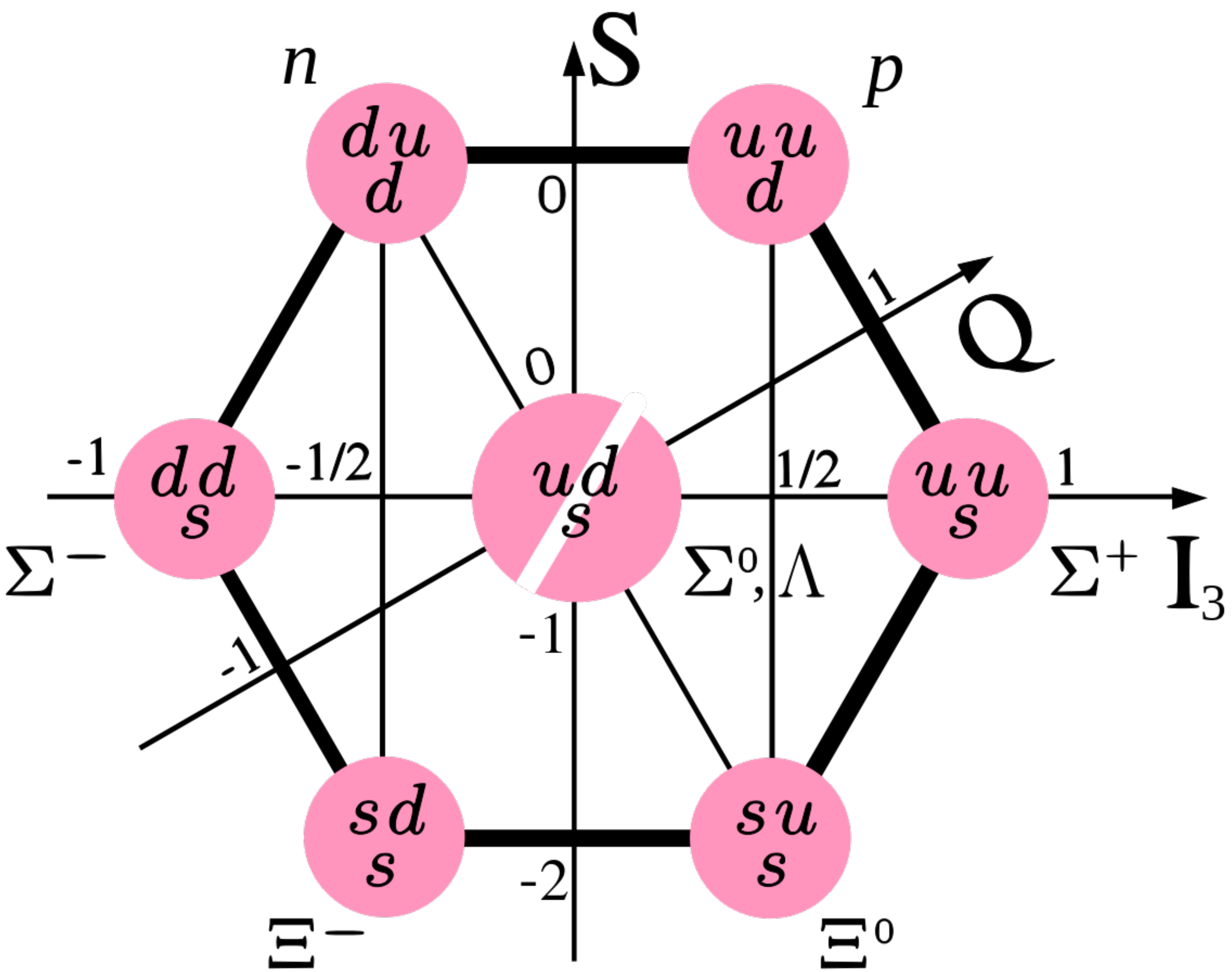
BARYON OCTET

Murray Gell-Mann “The eight-fold way” in 1961

Particle	Mass (MeV)	Strangeness
p	938.3	0
n	939.6	0
Λ	1115.6	-1
Σ^+	1189.4	-1
Σ^0	1192.6	-1
Σ^-	1197.4	-1
Ξ^0	1314.9	-2
Ξ^-	1321.3	-2



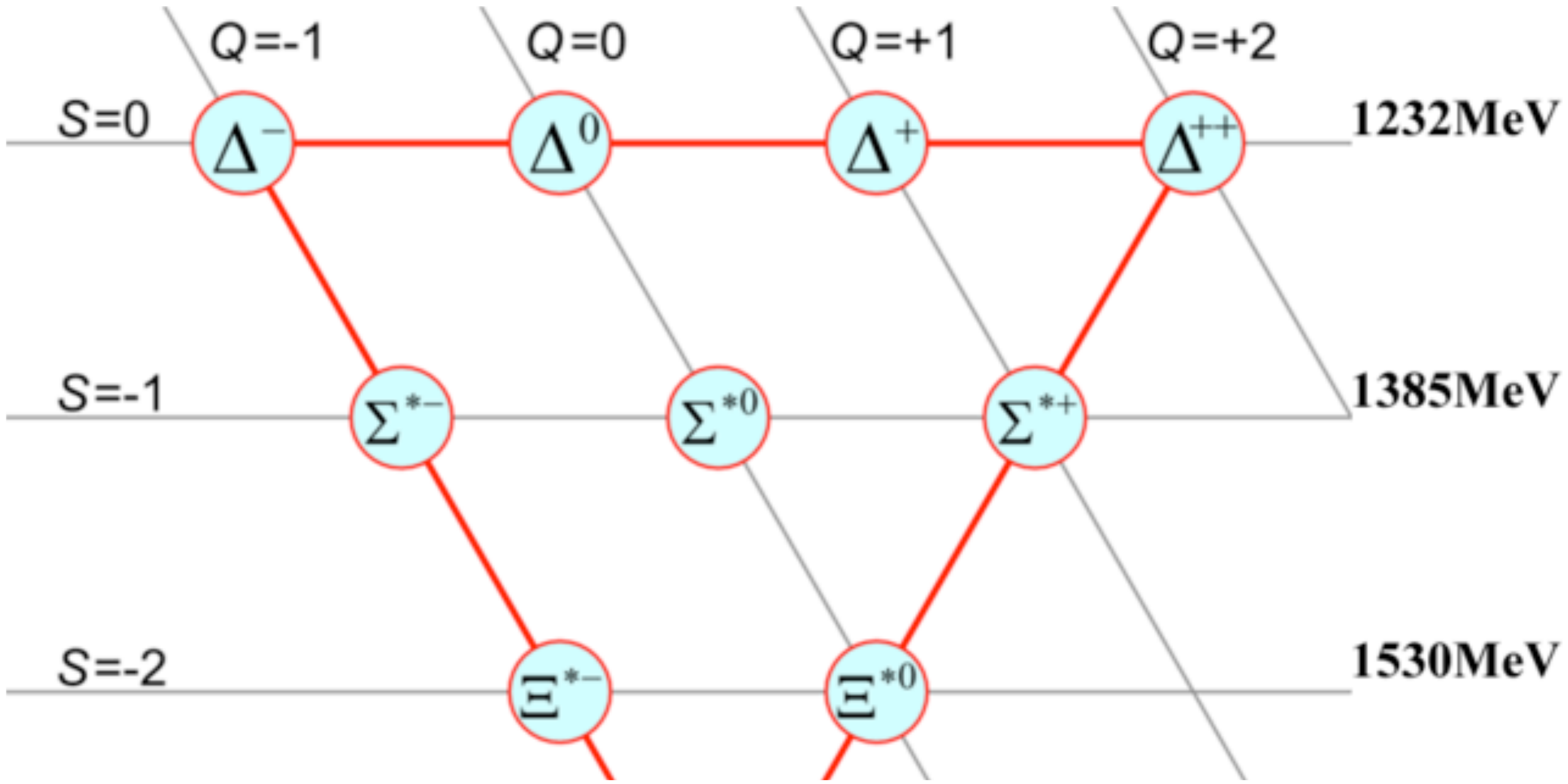
BARYON OCTET



Nikhef

BARYON DECOUPLET

Not all multiplets were complete…



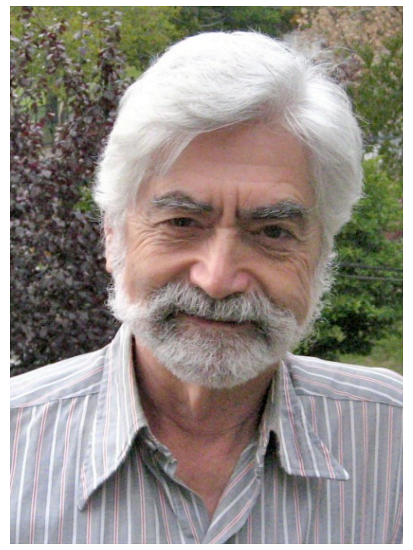
?

~150 MeV

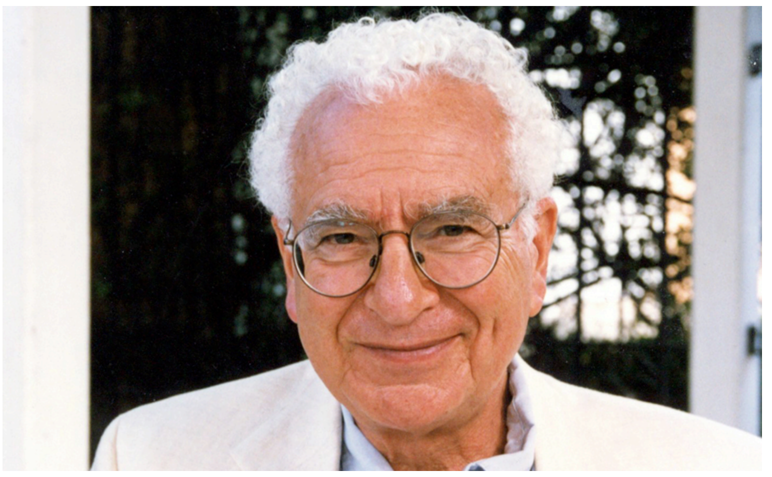
~150 MeV

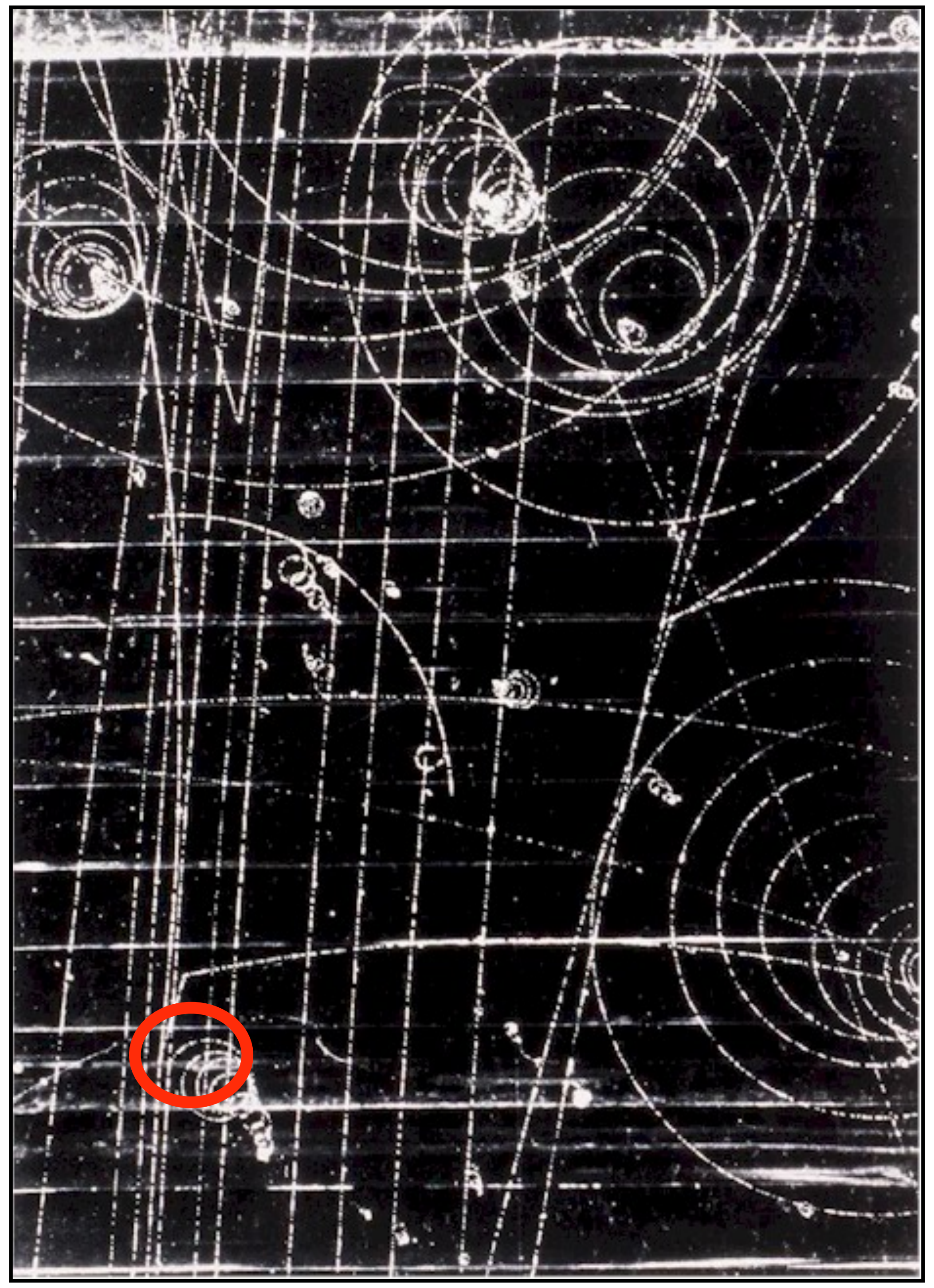
~150 MeV

George Zweig



Murray Gell-Mann

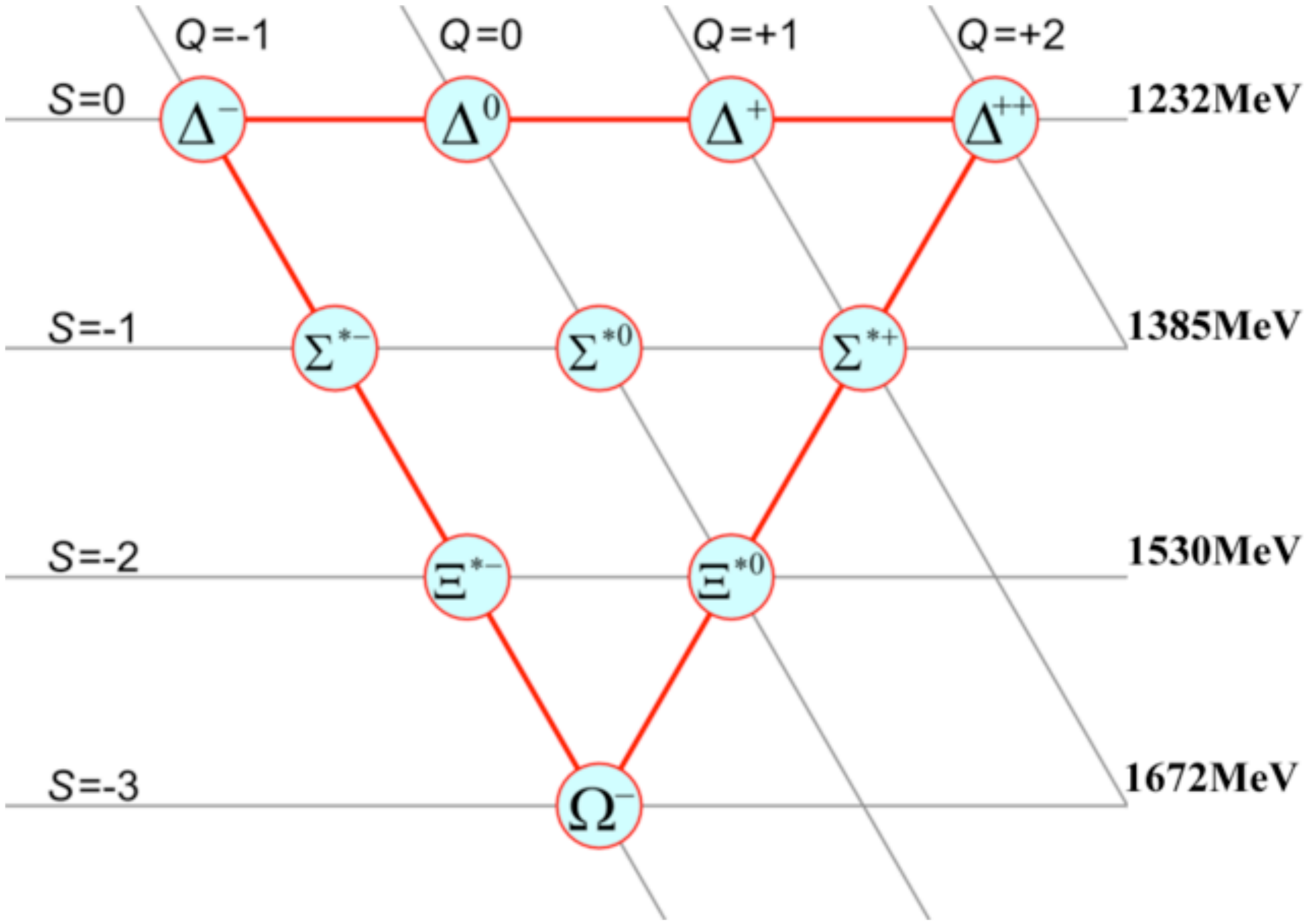




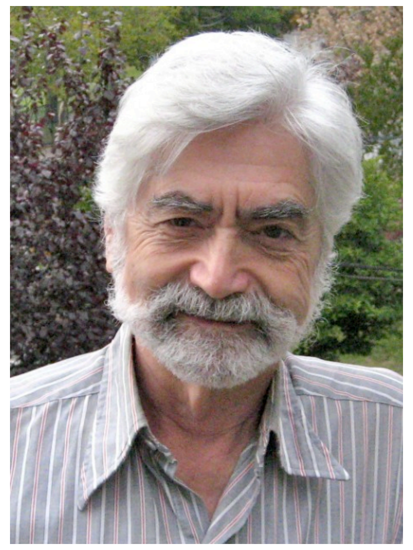
Nikhef

BARYON DECOUPLET

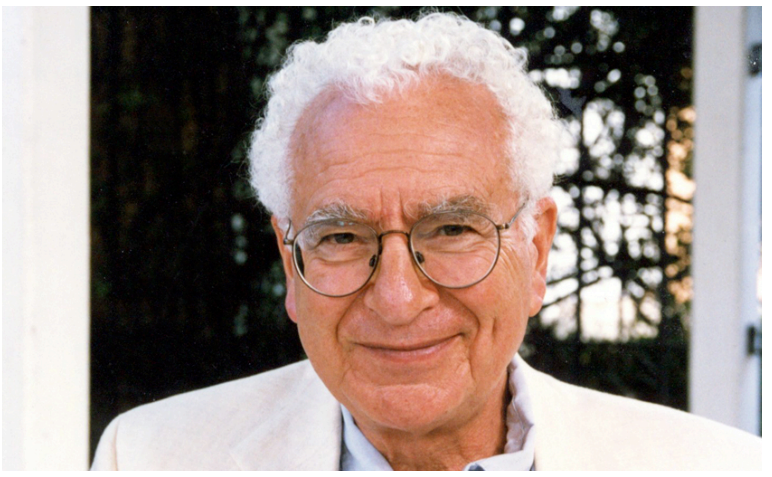
Not all multiplets were complete…

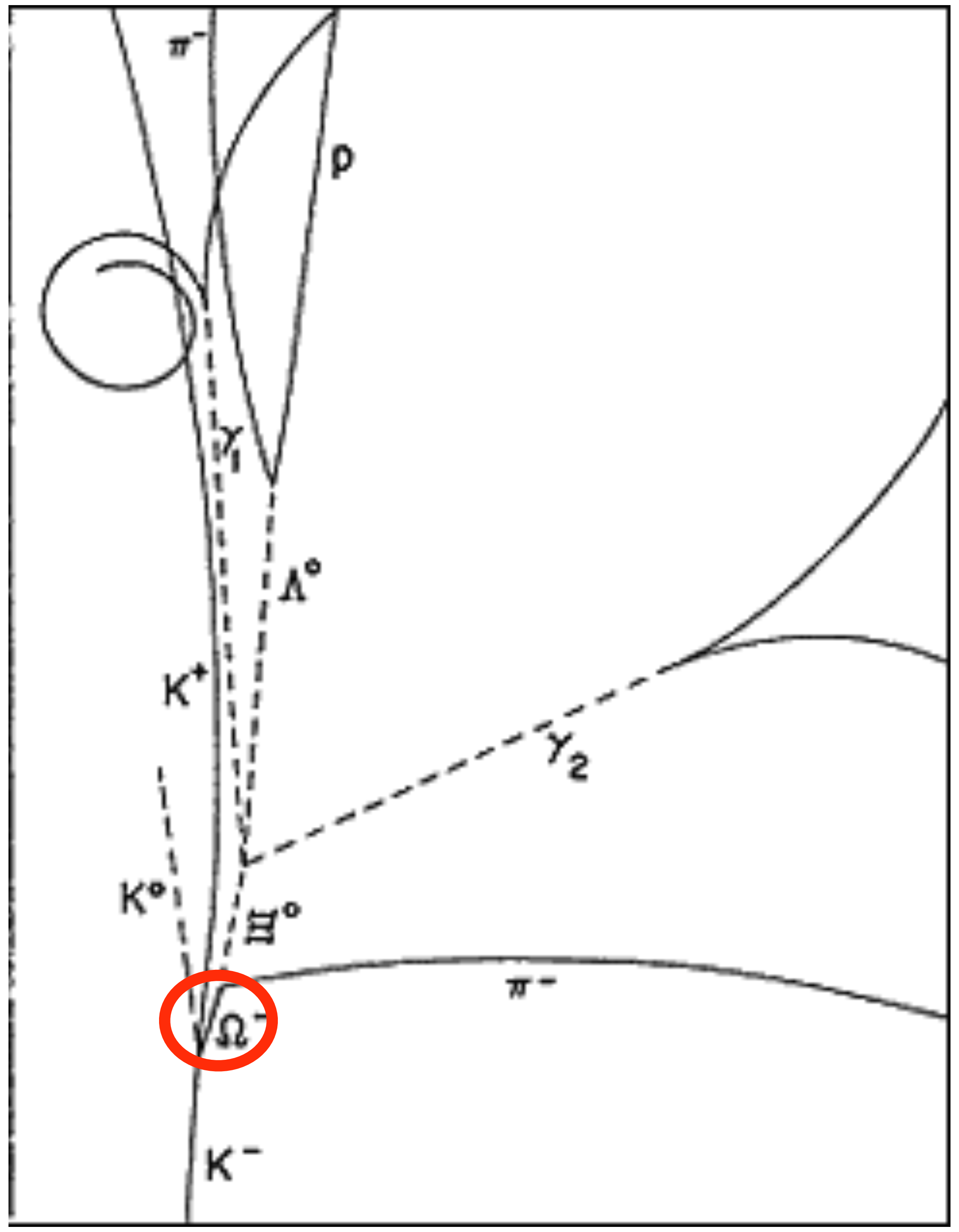


George Zweig

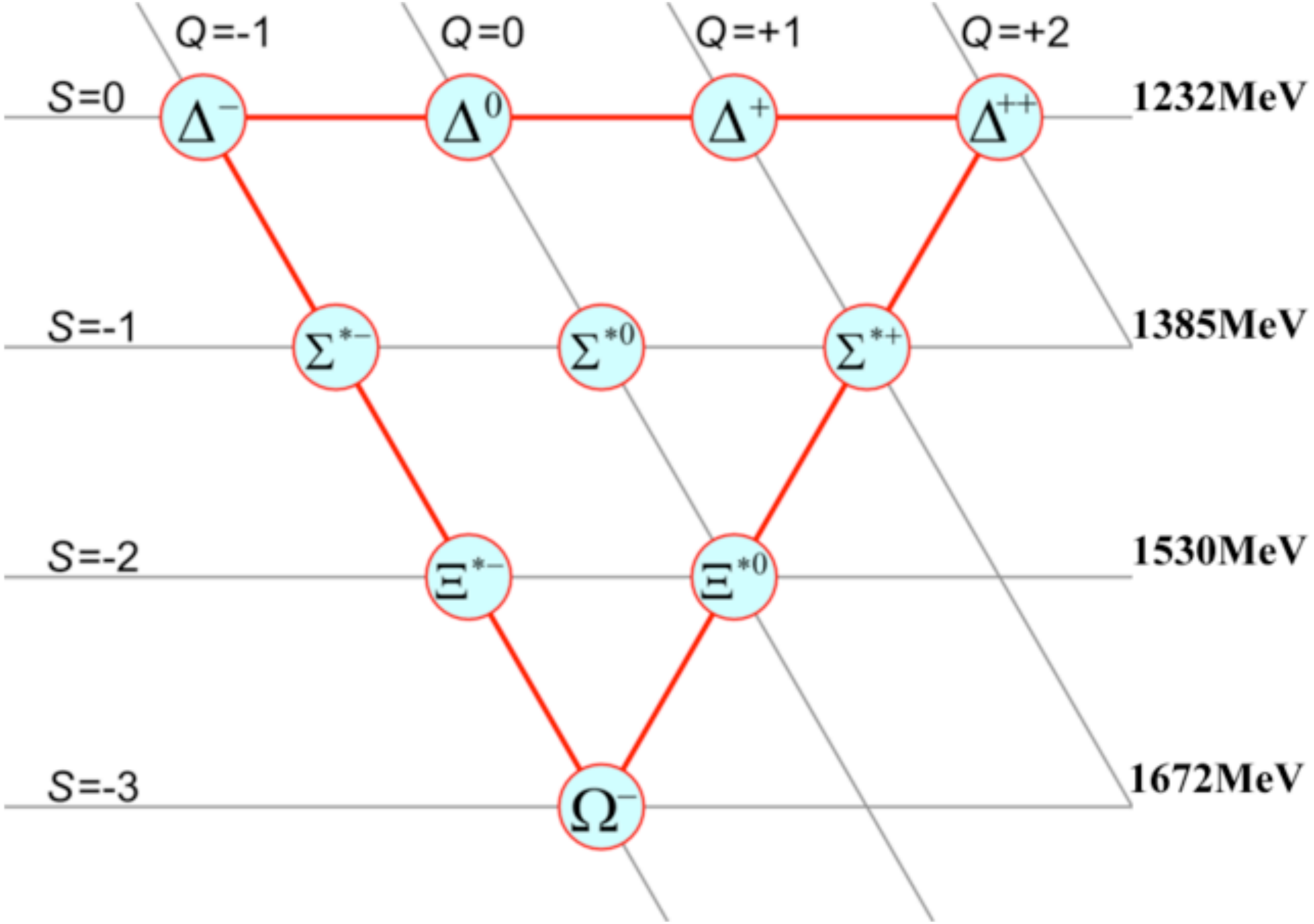


Murray Gell-Mann





BARYON DECOUPLET



SOME OF THE BARYONS OF THE QUARK MODEL

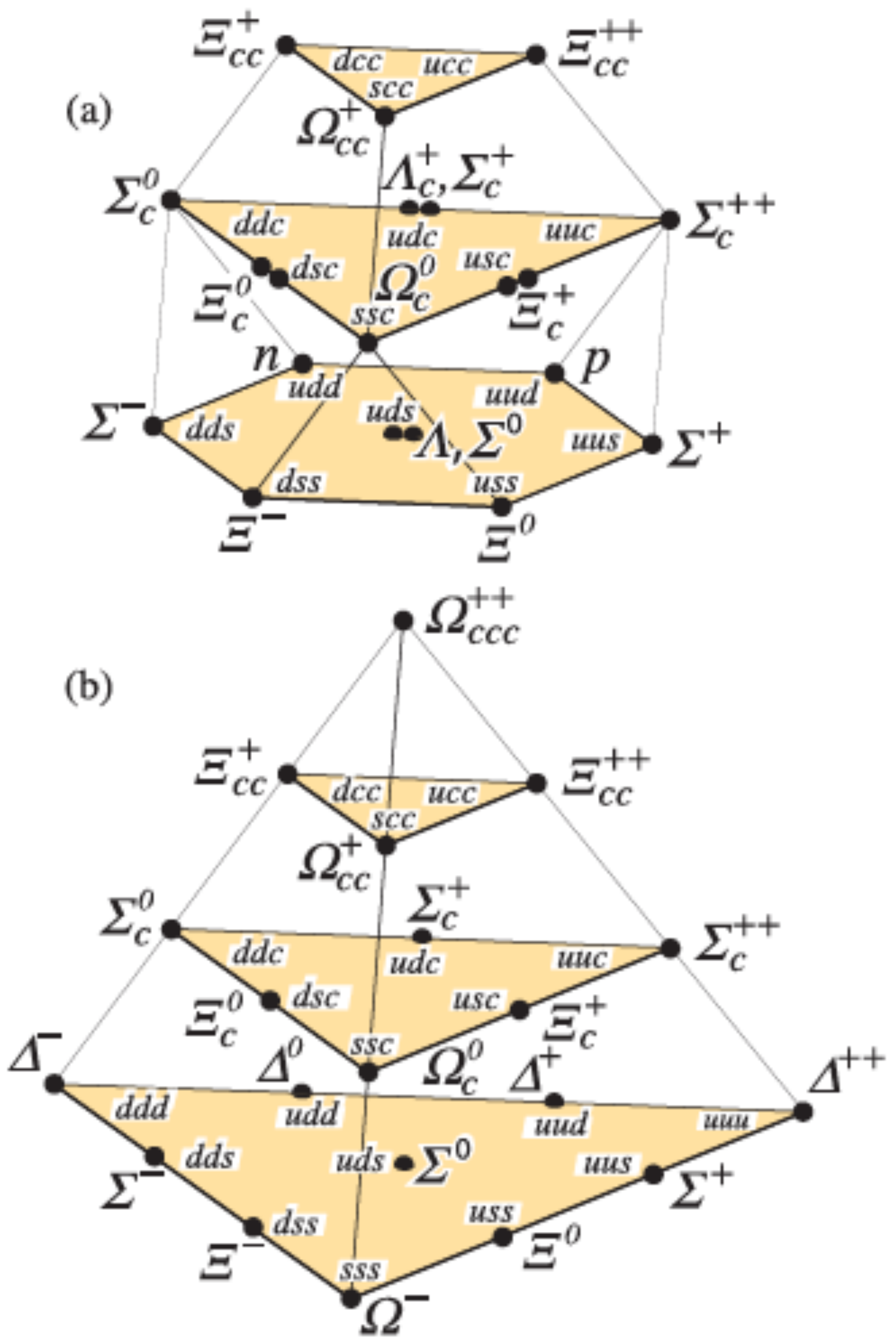
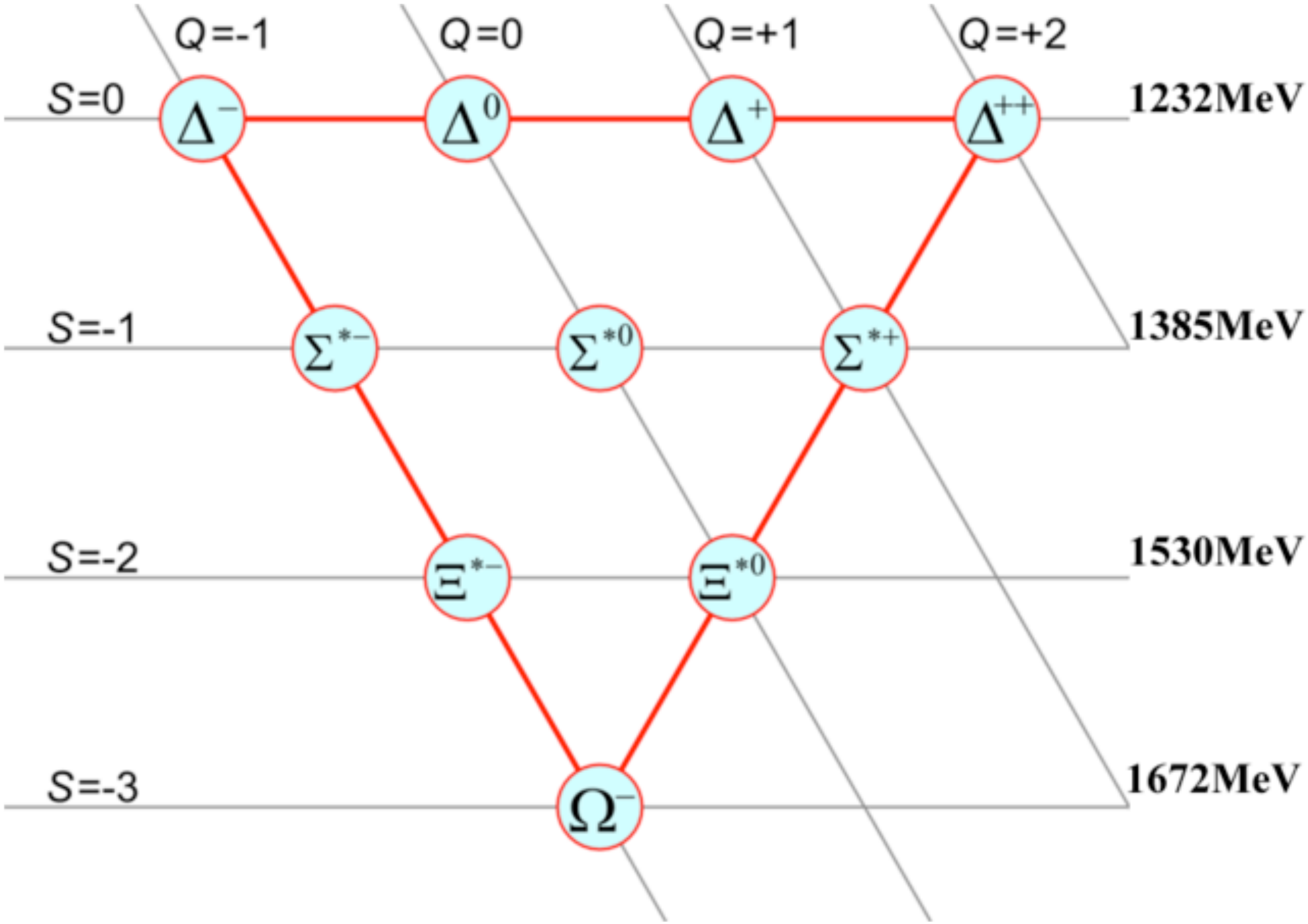


Figure 15.4: SU(4) multiplets of baryons made of u, d, s, and c quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

BARYONS



Q=-1
Q=0
Q=+1
Q=+2
S=0
S=-1
S=-2
S=-3
Δ⁻
Δ⁰
Δ⁺
Δ⁺⁺
Σ*⁻
Σ*⁰
Σ*⁺
Ξ*⁻
Ξ*⁰
Ω⁻
1232MeV
1385MeV
1530MeV
1672MeV

REMINDER: FERMIONS VS BOSONS

What happens if two fermions occupy the same quantum state?

$$\psi_1(x_1)\psi_2(x_1) = -\psi_1(x_1)\psi_2(x_1)$$

$$\psi_{tot.}(x_1, x_1) = \frac{1}{\sqrt{2}}\Big(\psi_1(x1)\psi_2(x1) + \psi_1(x1)\psi_2(x1)\Big) = 0$$

Bosons: if we exchange two identical bosons, the wave function is **unchanged**

$$\psi_1(x_1)\psi_2(x_2) = \psi_1(x_2)\psi_2(x_1)$$

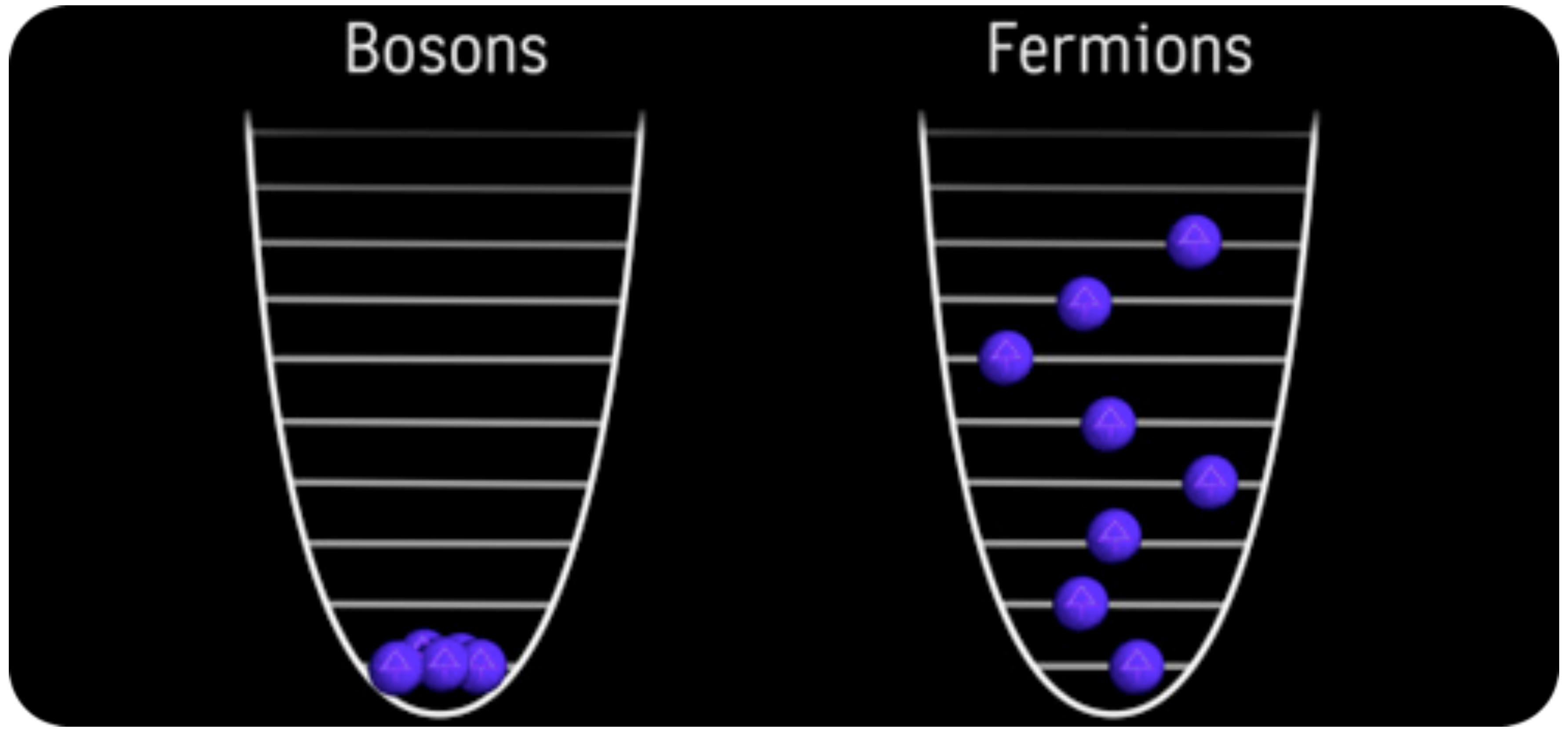
Fermions: if we exchange two identical fermions, the wave function **changes sign**

$$\psi_1(x_1)\psi_2(x_2) = -\psi_1(x_2)\psi_2(x_1)$$

Pauli principle

- ☑ **If two particles have the same quantum numbers, they are in the same state**
- ☑ **If these two particles are fermions then the wave function vanishes**

A system cannot exist with two or more fermions in the same state



PROBLEM OF THE QUARK MODEL

How can a baryon like the Δ^{++} (uuu), Δ^{-} (ddd) or the Ω^{-} (sss) given the Pauli principle?

Δ^{++} (uuu) — u↑ u↑ u↑ — Intrinsic spin: symmetric — $|\frac{3}{2}, +\frac{3}{2}\rangle = \uparrow\uparrow\uparrow$

Quarks: symmetric — $|uuu\rangle$

Ω^{-}(sss) — s↑ s↑ s↑ — Intrinsic spin: symmetric — $|\frac{3}{2}, +\frac{3}{2}\rangle = \uparrow\uparrow\uparrow$

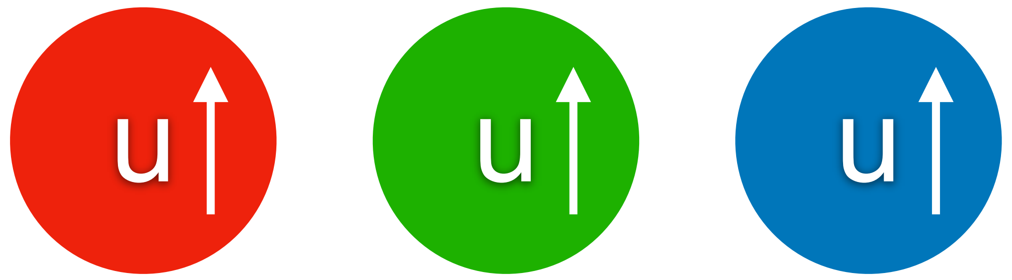
Quarks: symmetric — $|sss\rangle$

Half-integer spin particle ⇒ fermion that obeys the Fermi-Dirac statistics: anti-symmetric wave-functions

PROBLEM OF THE QUARK MODEL

How can a baryon like the Δ^{++} (uuu), Δ^{-} (ddd) or the Ω^{-} (sss) given the Pauli principle?

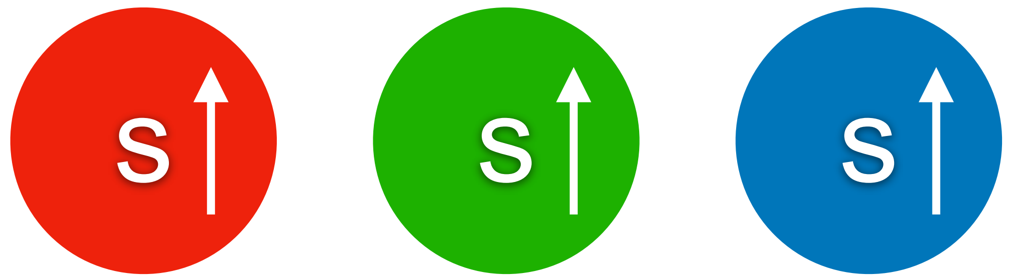
Δ^{++} (uuu)



Intrinsic spin: symmetric

$$|\frac{3}{2}, +\frac{3}{2}\rangle = \uparrow\uparrow\uparrow$$

Ω^{-}(sss)



Intrinsic spin: symmetric

$$|\frac{3}{2}, +\frac{3}{2}\rangle = \uparrow\uparrow\uparrow$$

Solution: Introduce a new quantum number ⇒ **COLOUR**

- ☑ Comes with three flavours: RED, GREEN, BLUE
- ☑ Applicable for quarks not leptons!

PROBLEM OF THE QUARK MODEL

How can a baryon like the Δ^{++} (uuu), Δ^{-} (ddd) or the Ω^{-} (sss) given the Pauli principle?

All naturally occurring particles come in **colour singlet** states: **invariant under rotations in colour space**

Solution: Introduce a new quantum number ⇒ **COLOUR**

- ☑ Comes with three flavours: RED, GREEN, BLUE
- ☑ Applicable for quarks not leptons!

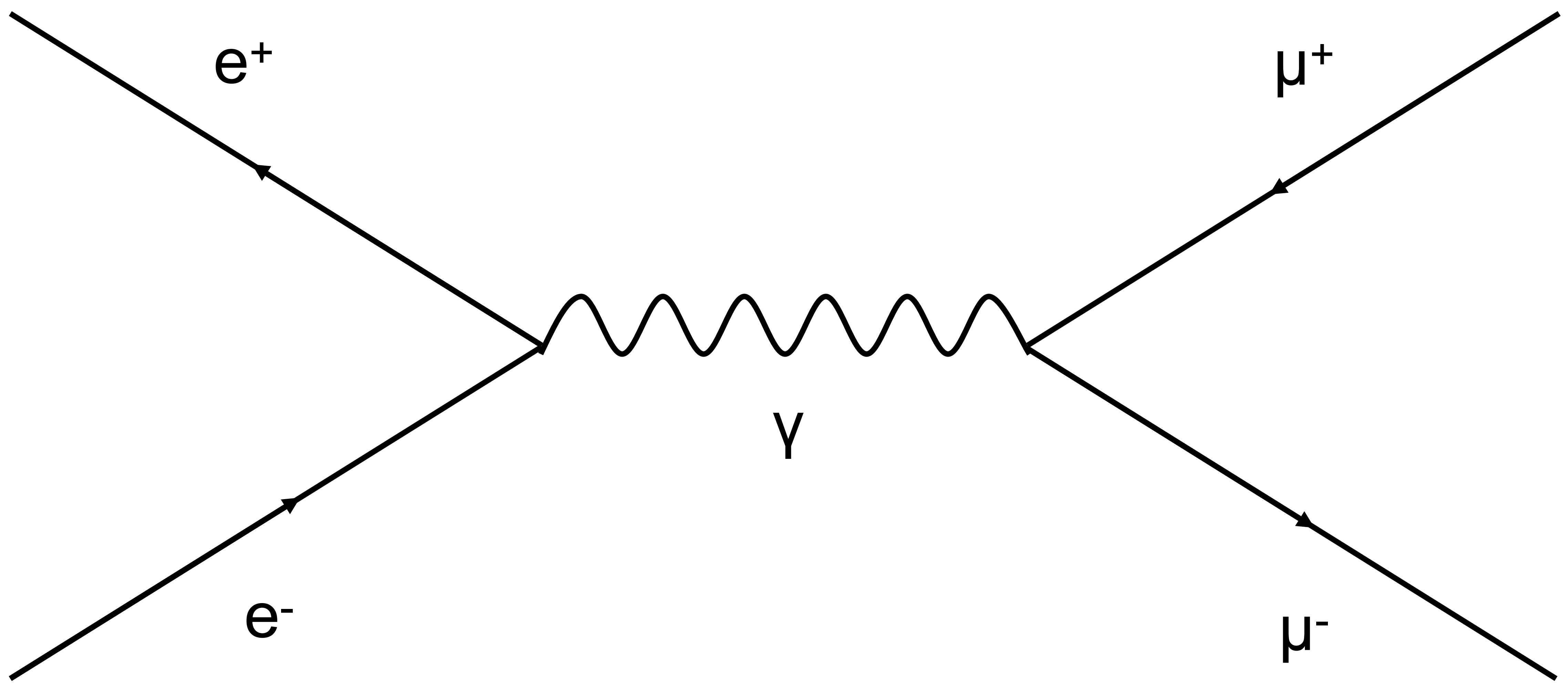
Nikhef

EXPERIMENTAL EVIDENCE OF COLOUR

EXPERIMENTAL EVIDENCE OF COLOUR

Evidence of colour via QED processes

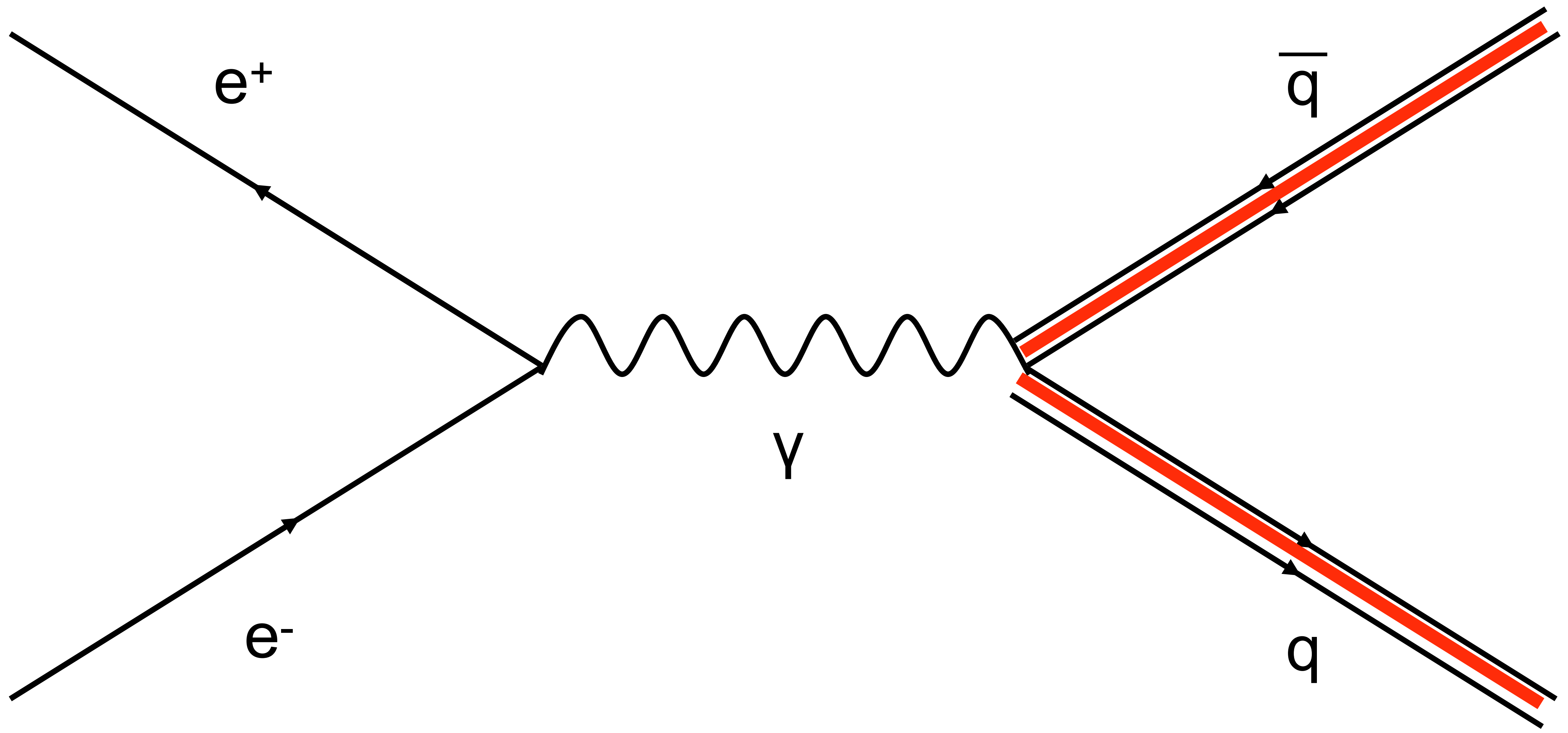
- The first one (i.e. muon production via electron-positron scattering) is well controlled experimental and is used as a reference



EXPERIMENTAL EVIDENCE OF COLOUR

How can we probe colour via a QED process?

- The second process (i.e. quark production via electron-positron scattering) cannot actually be observed directly in nature



EXPERIMENTAL EVIDENCE OF COLOUR

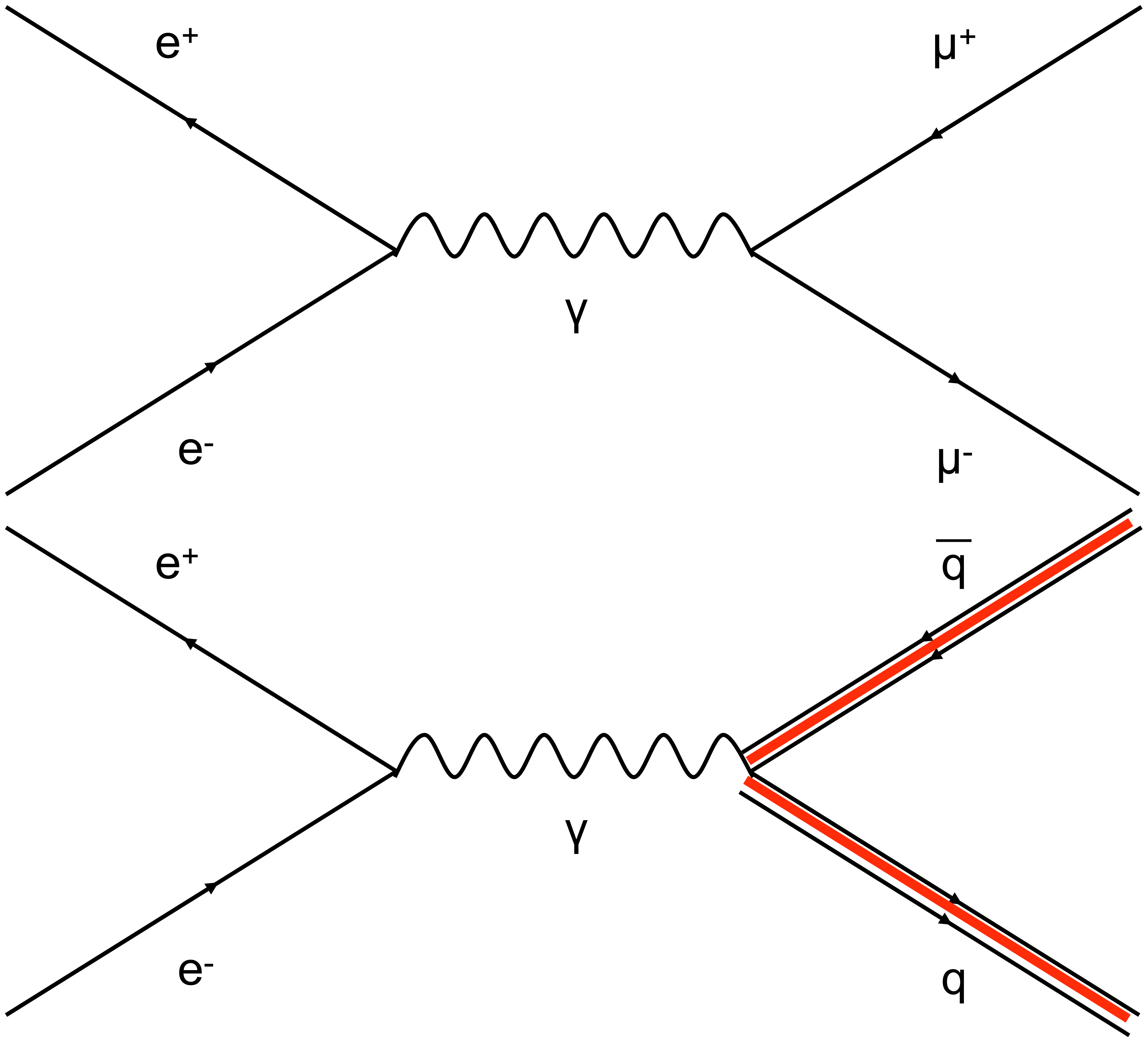
These quarks do not fly free for long (i.e. they can fly as “free” within the size of a hadron)

- They fragment producing additional q-qbar pairs that when combined form hadrons
  - This is a QCD-type of process
- The final state particles are detected as a collimated spray of hadrons ➡JETS
- Due to energy-momentum conservation, these jets emerge in a back-to-back topology

Any difference between the cross-sections of these two processes from a naive scaling with the square of the charge will signal the existence of additional factors

EXPERIMENTAL EVIDENCE OF COLOUR

QED process



<u>Final state</u> particles that can be seen in nature

QED process

<u>Final state</u> particles that do **not** fly free in nature

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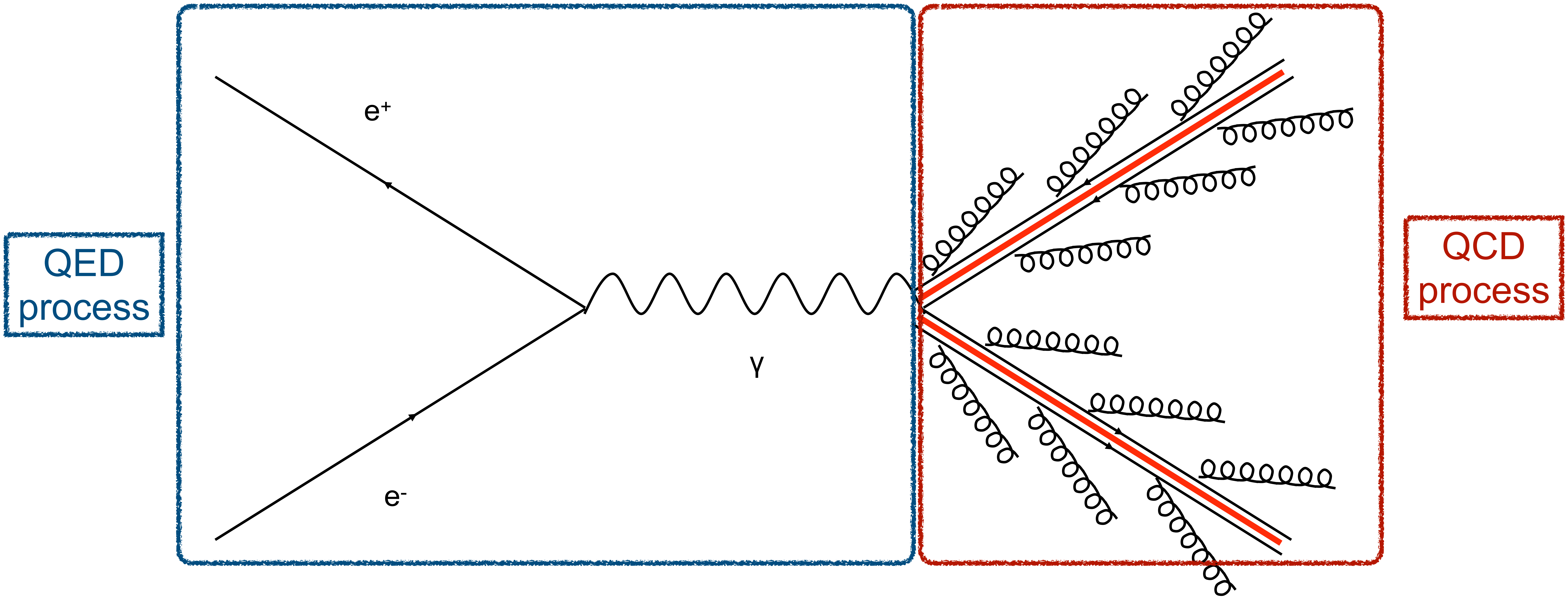
EXPERIMENTAL EVIDENCE OF COLOUR



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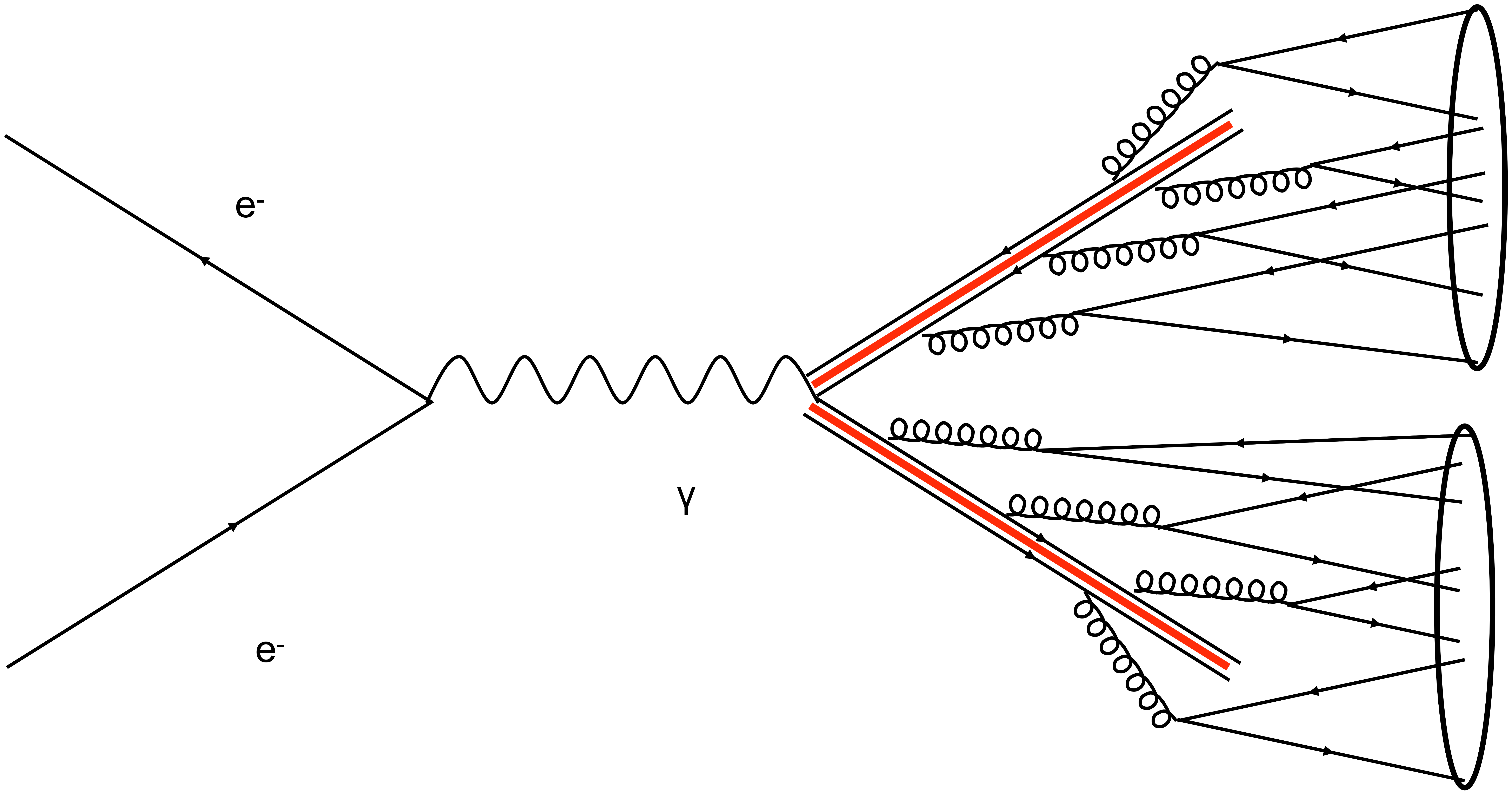
EXPERIMENTAL EVIDENCE OF COLOUR



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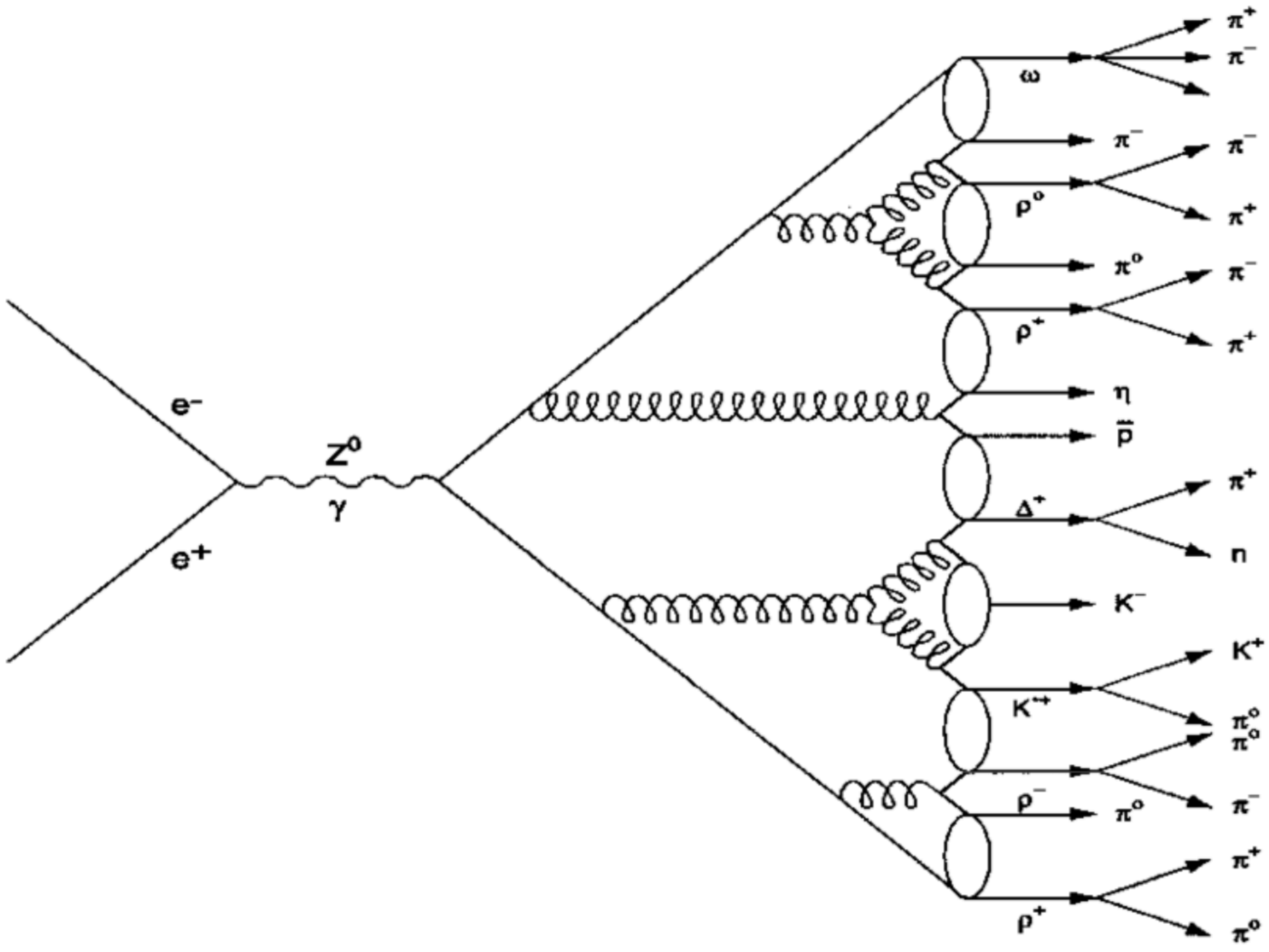
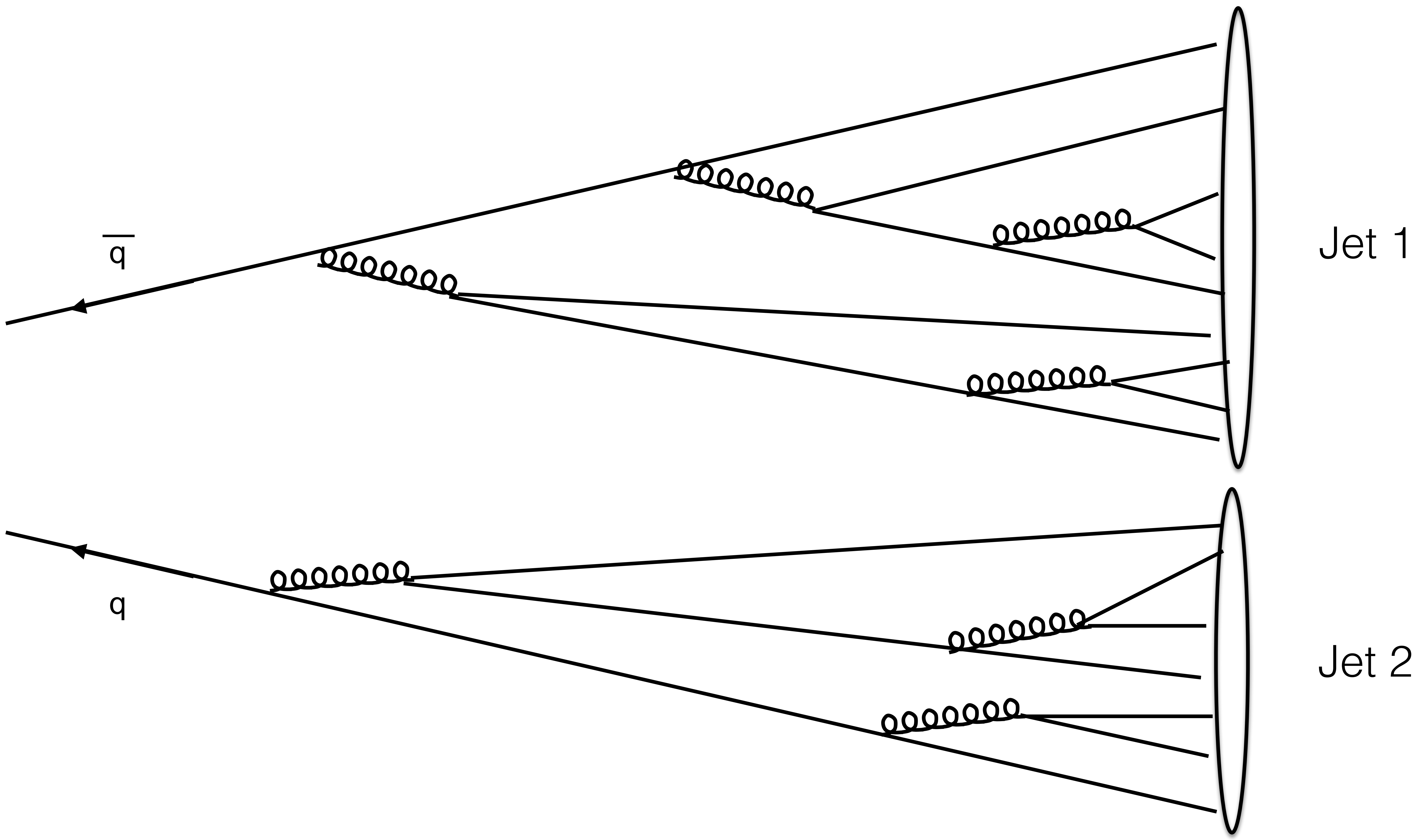
EXPERIMENTAL EVIDENCE OF COLOUR



JET PRODUCTION IN VACUUM



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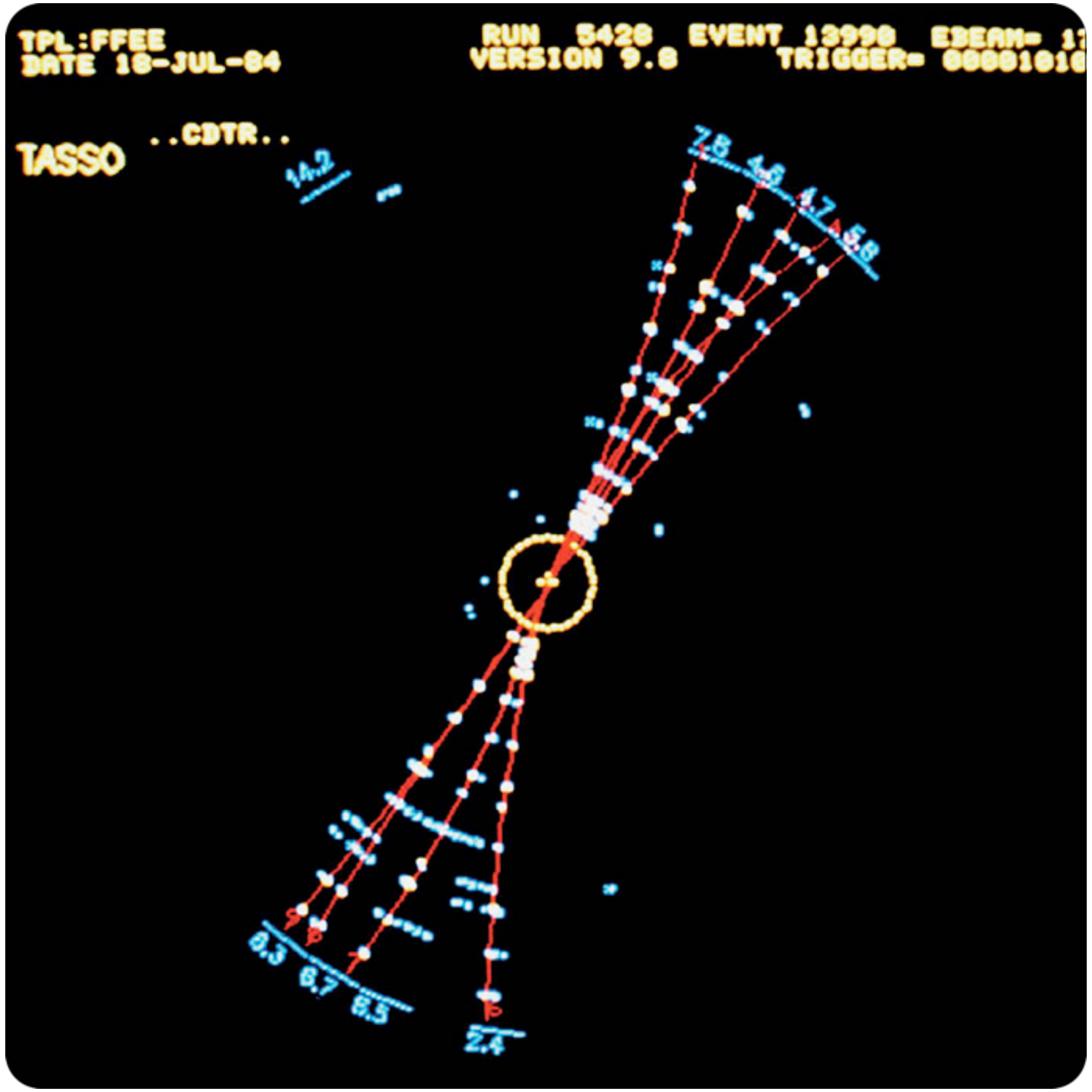
TWO-JET EVENT @ PETRA

Positron-Electron Tandem Ring Accelerator: electron-positron collisions between 1978 and 1986

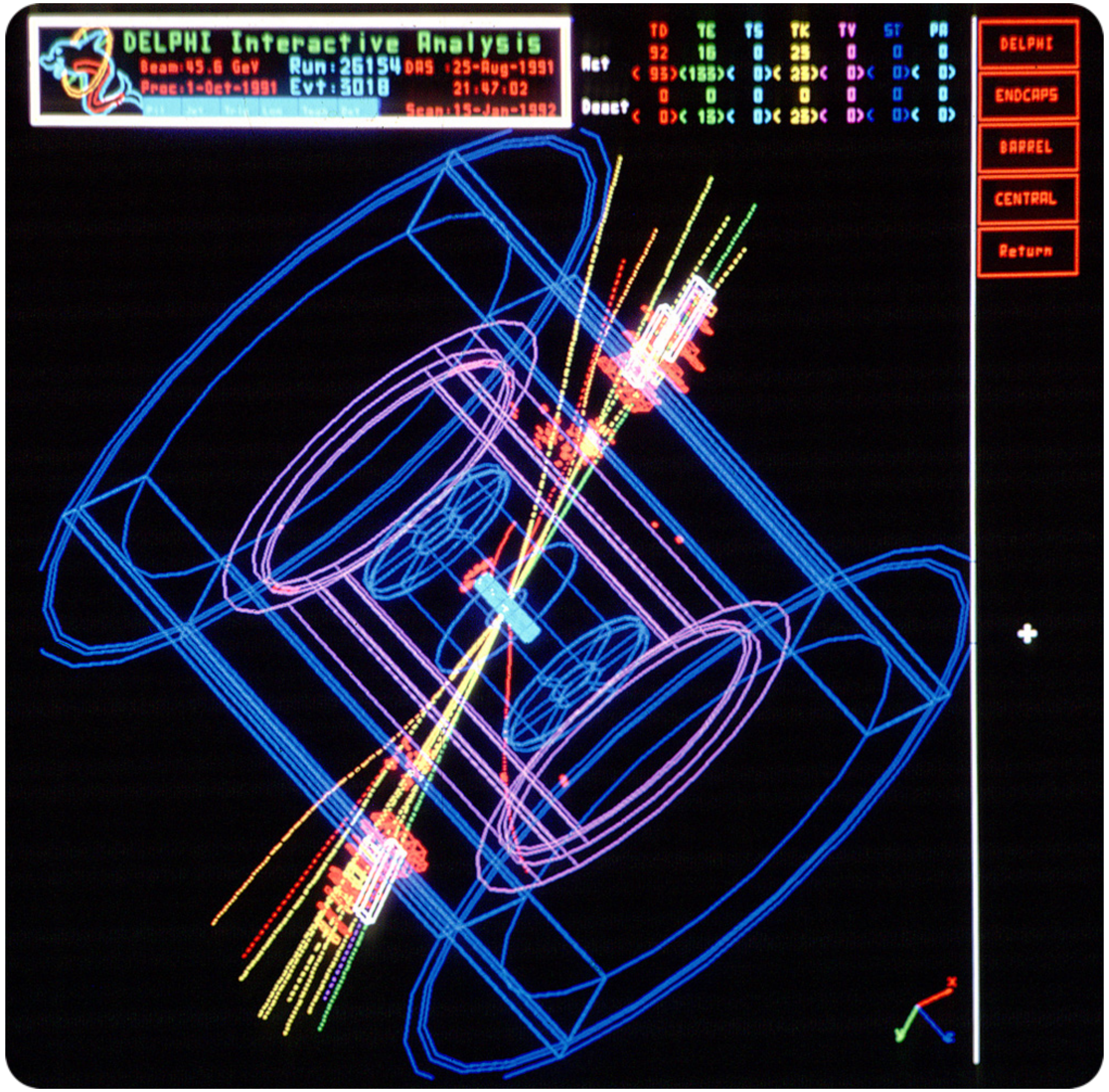


TASSO experiment @ PETRA @ DESY

e^-e^+ collisions @ $\sqrt{s}$ = 13-31 GeV

TWO-JET EVENT @ LEP

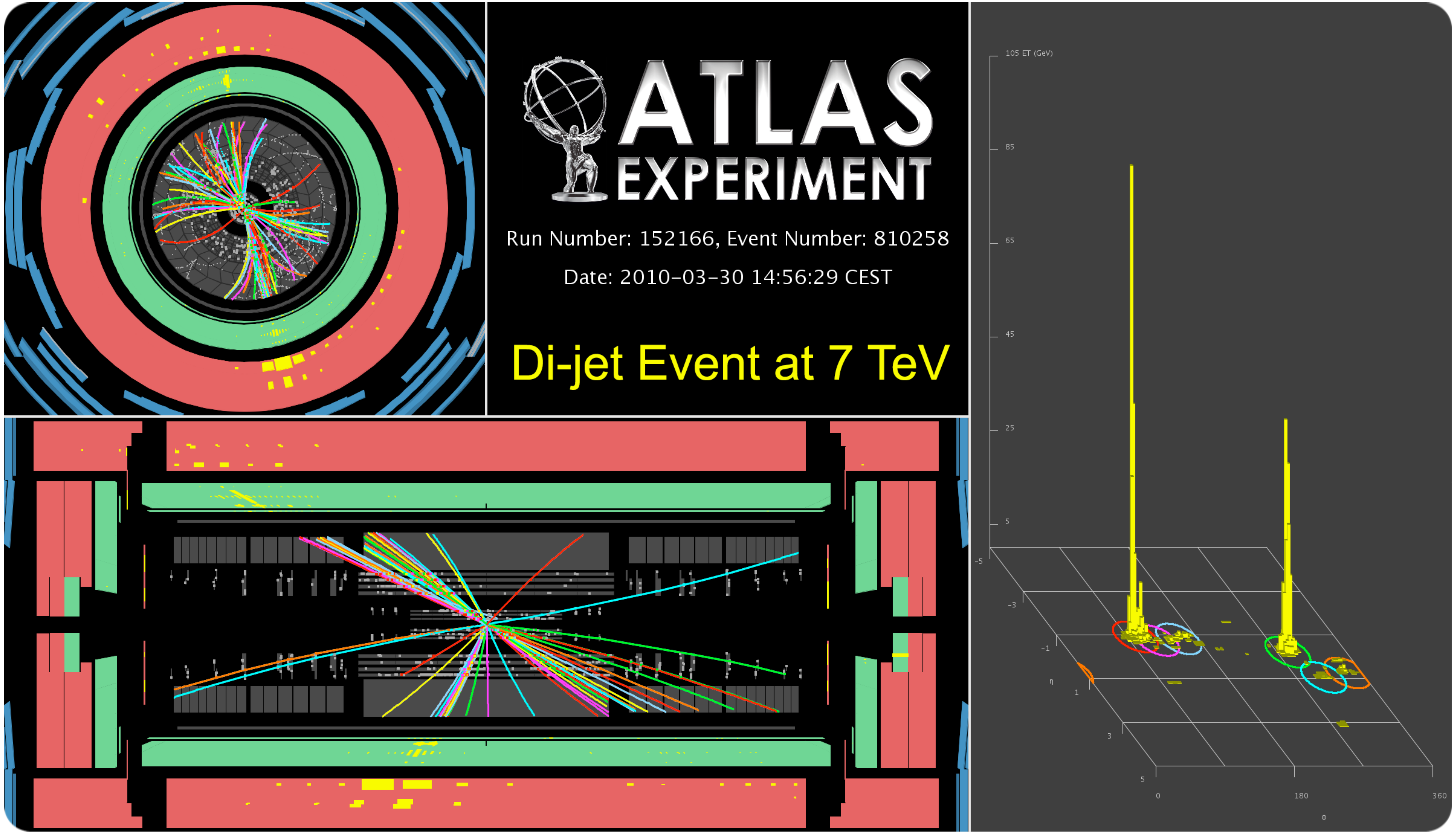
Large Electron-Positron collider @ CERN (predecessor of LHC): electron-positron collisions between 1989 and 2000



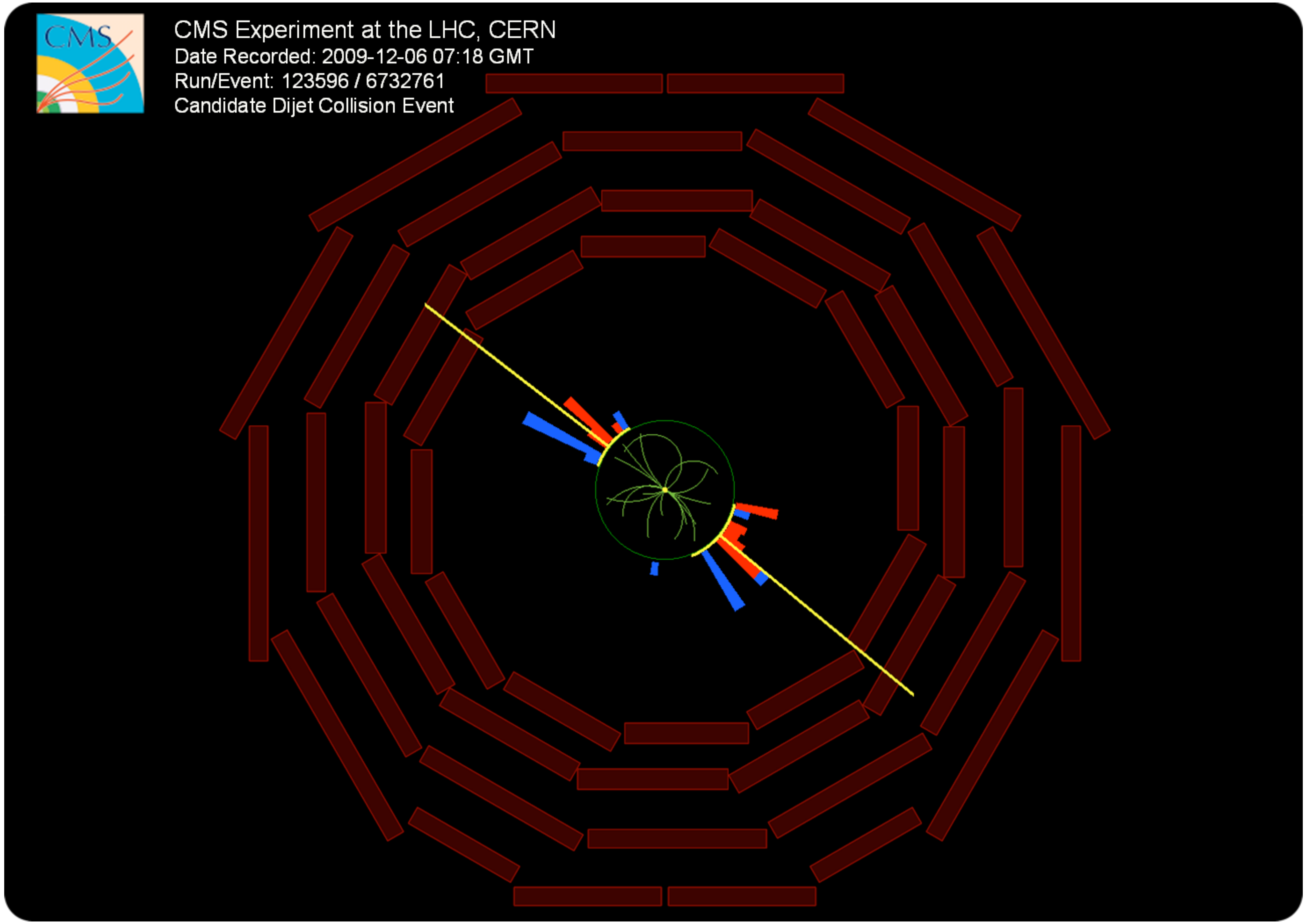
DELPHI experiment @ LEP @ CERN

$e^- e^+$ collisions @ $\sqrt{s}$ = 90-209 GeV

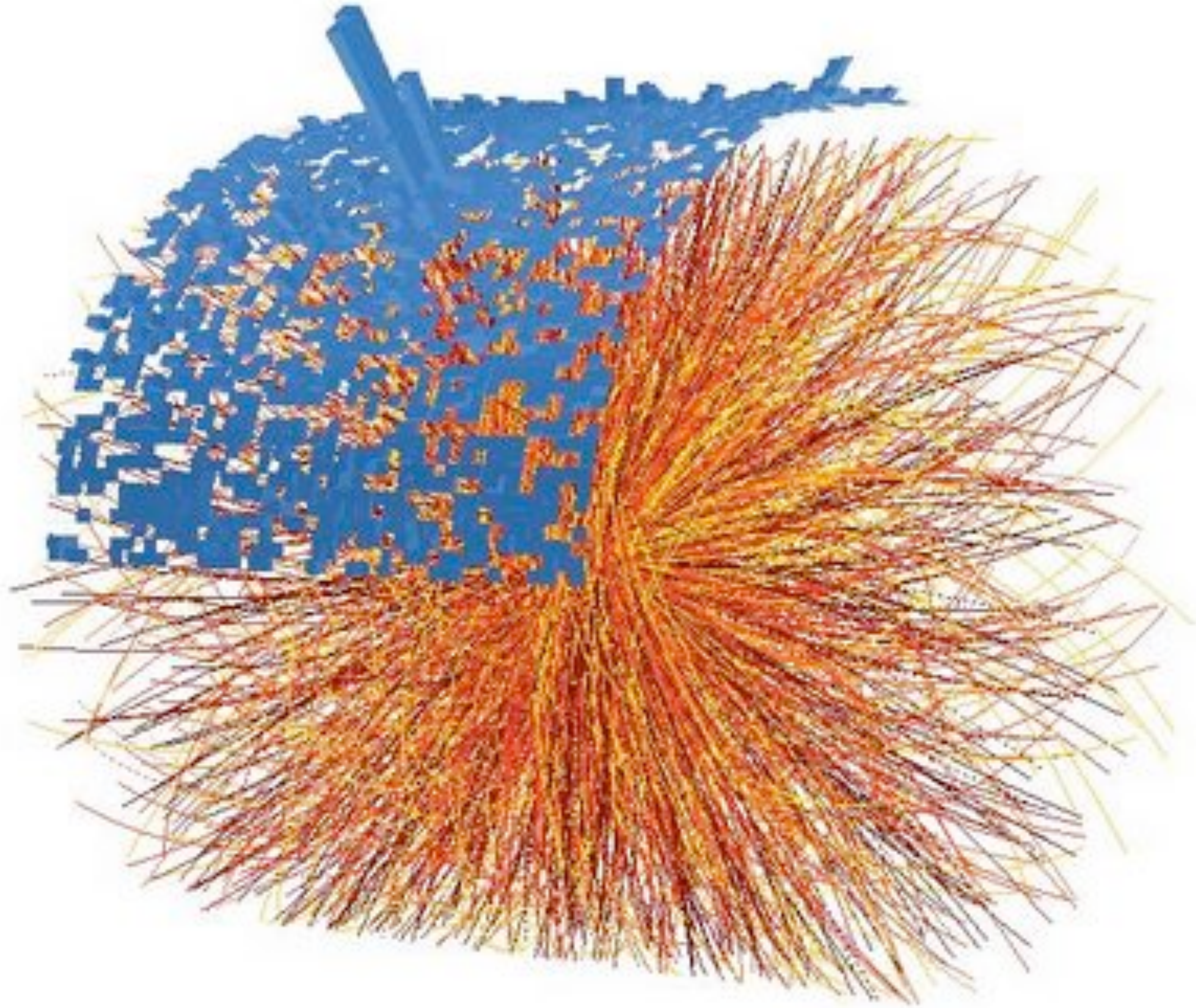
TWO JET EVENT @ LHC



TWO JET EVENT @ LHC



TWO JET EVENT @ LHC



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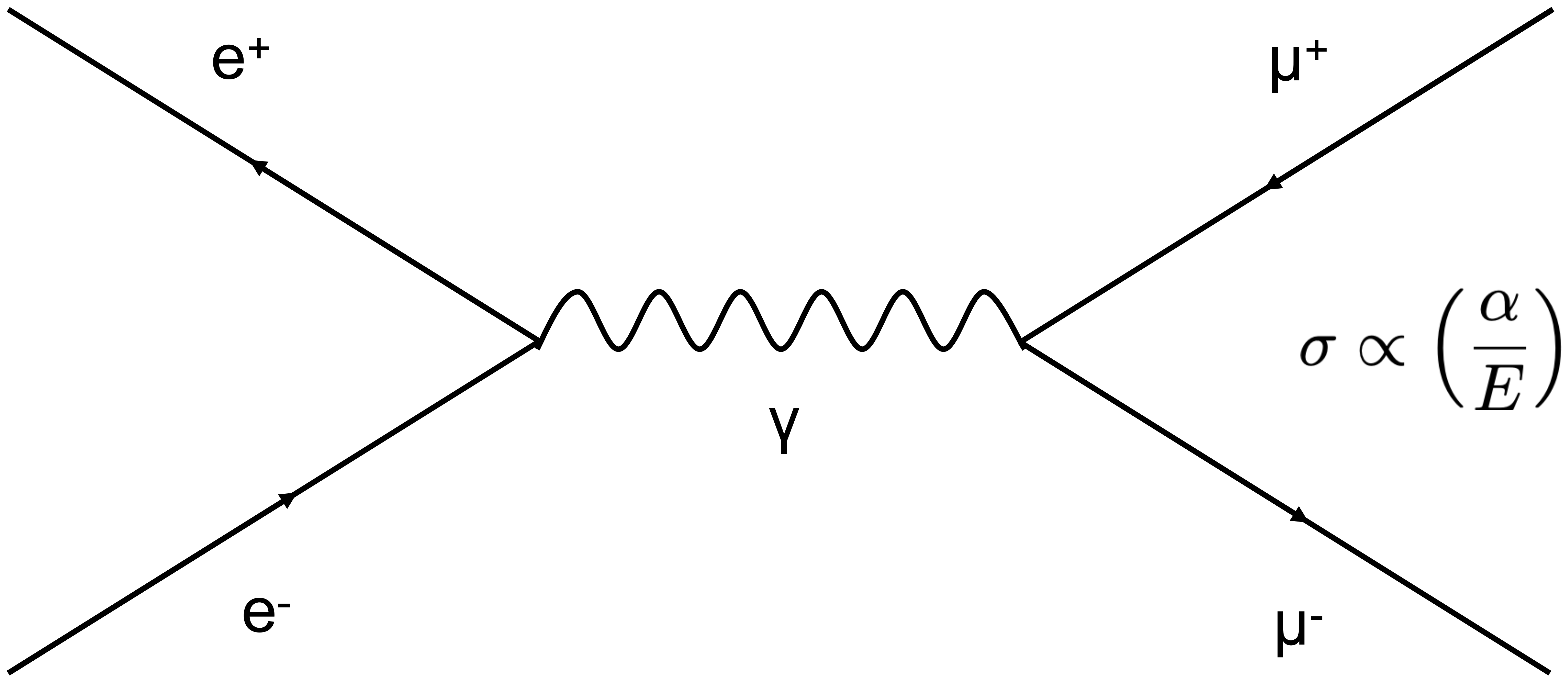
EXPERIMENTAL EVIDENCE OF COLOUR

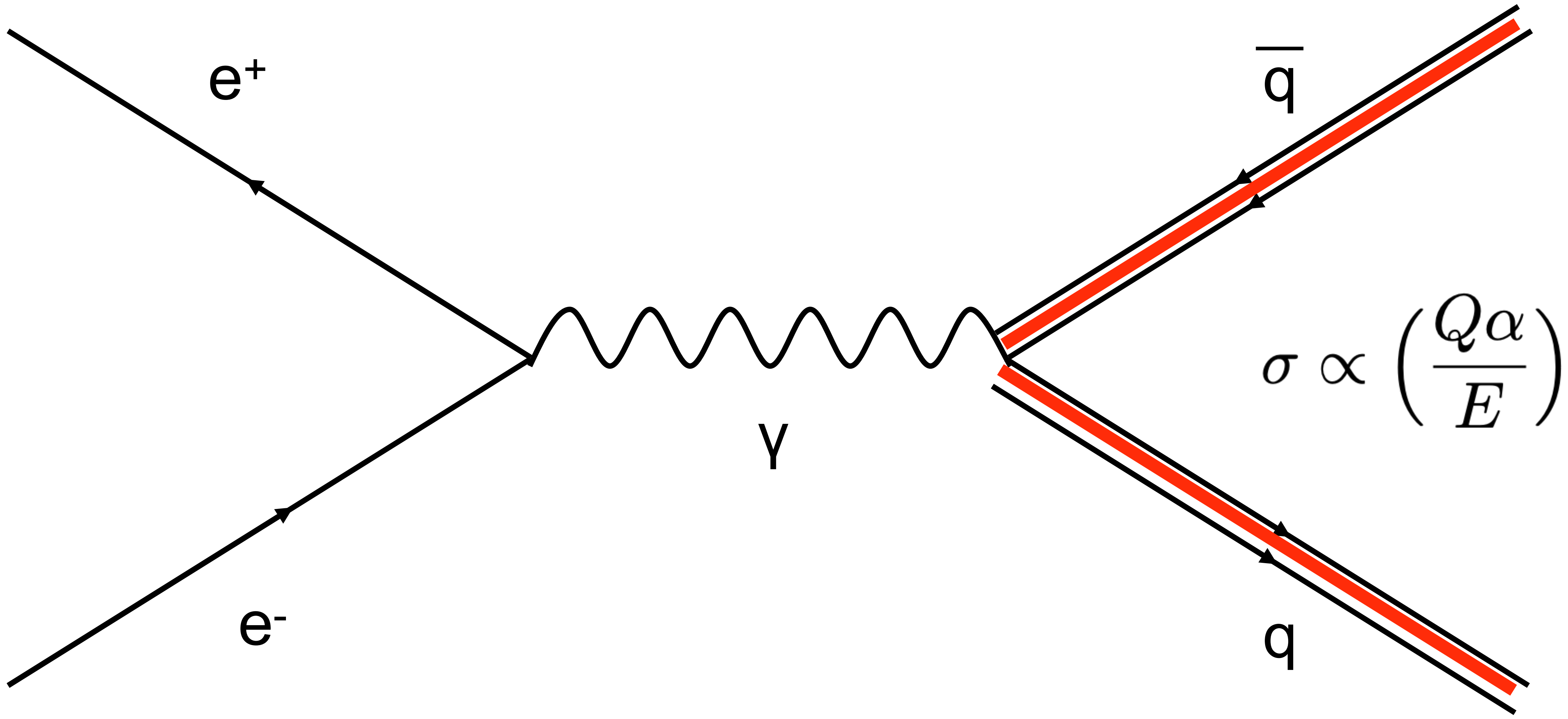
QED process



$\sigma \propto \left(\frac{\alpha}{E}\right)^2$

Final state particles that can be seen in nature

QED process



$\sigma \propto \left(\frac{Q\alpha}{E}\right)^2$

Final state particles that do **not** fly free in nature

EVIDENCE OF COLOUR

Calculate the ratio of the cross-sections of the two processes

$$R = \frac{\sigma(e^-e^+ \to q\bar{q} \to hadrons)}{\sigma(e^-e^+ \to \mu^-\mu^+)} \propto \sum_{i=1}^{n} Q_i^2$$

- For three quark flavours (u,d,s) the ratio should give:

$$R = \left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right] = \frac{2}{3}$$

- For four quark flavours (u,d,s,c) the ratio should give:

$$R = \left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right] = \frac{10}{9}$$

- For five quark flavours (u,d,s,c,b) the ratio should give:

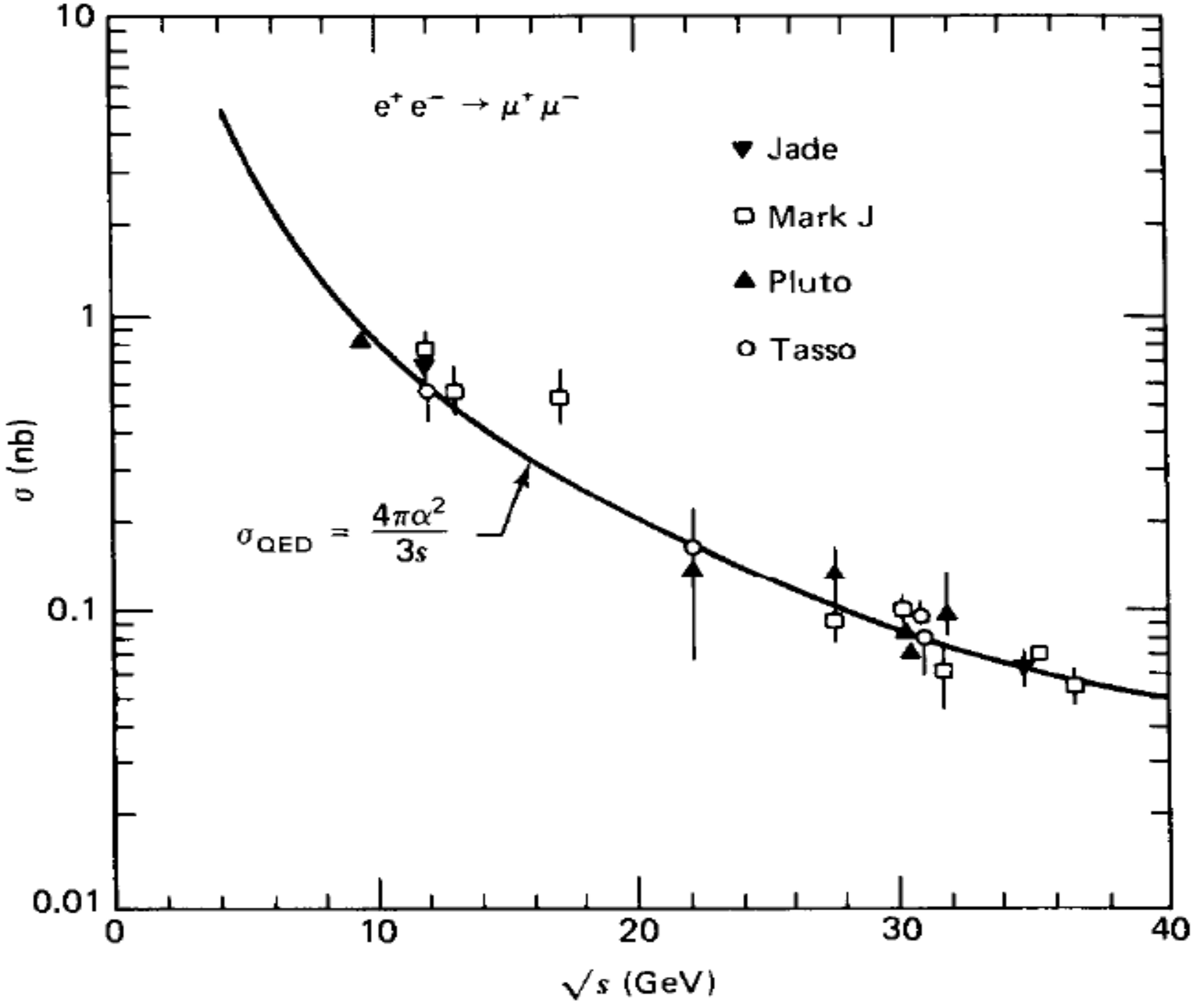
$$R = \left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right] = \frac{11}{9}$$

- For all six quark flavours (u,d,s,c,b,t) the ratio should give:

$$R = \left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right] = \frac{15}{9}$$

CALCULATING THE RATIO…

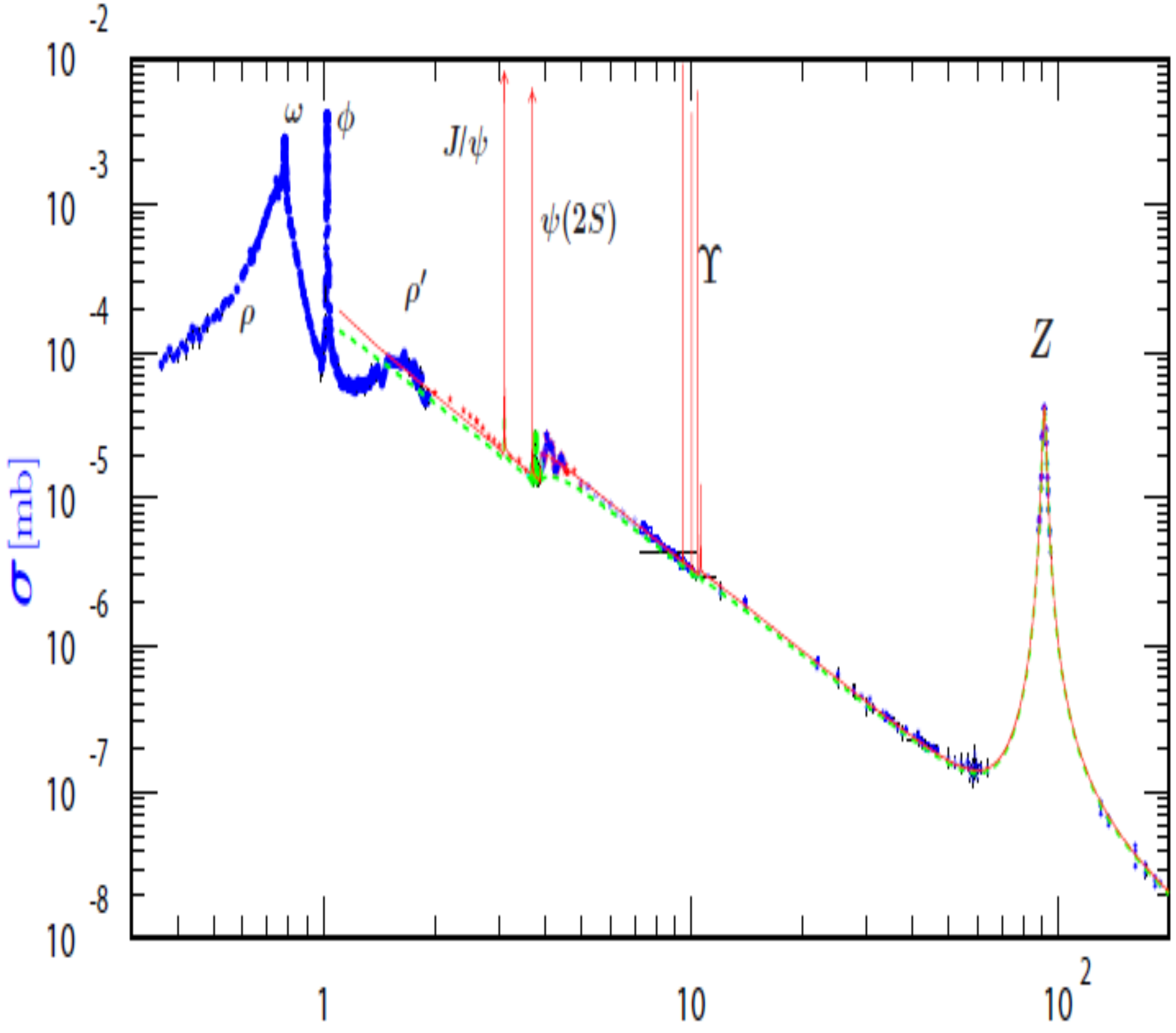
$$\sigma \propto \left(\frac{\alpha}{E}\right)^2$$



At around $\sqrt{s_{NN}}$ = 10GeV → $\sigma(e^-e^+ \to \mu^-\mu^+) \sim 0.8$nb

CALCULATING THE RATIO…

$$\sigma \propto \left(\frac{Q\alpha}{E}\right)^2$$



At around $\sqrt{s_{NN}}$ = 10GeV → σ(e^-e^+→qqbar→hadrons) ~ 3nb

CALCULATING THE RATIO…

At ~10GeV (beyond the threshold for the b-quark creation) the ratio should be

$$R = \left[\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right] = \frac{11}{9}$$

- But experimentally it turns out to be
  - $\sigma(e^-e^+ \to \mu^-\mu^+) \sim 0.8\text{nb}$
  - $\sigma(e^-e^+ \to \text{qqbar} \to \text{hadrons}) \sim 3.0\text{nb}$
  - R~3.7 instead of 11/9 → a factor of 3 missing!!!

- The problem with the calculations assuming no additional quantum number persists for all energy ranges

Nikhef

EVIDENCE OF COLOUR

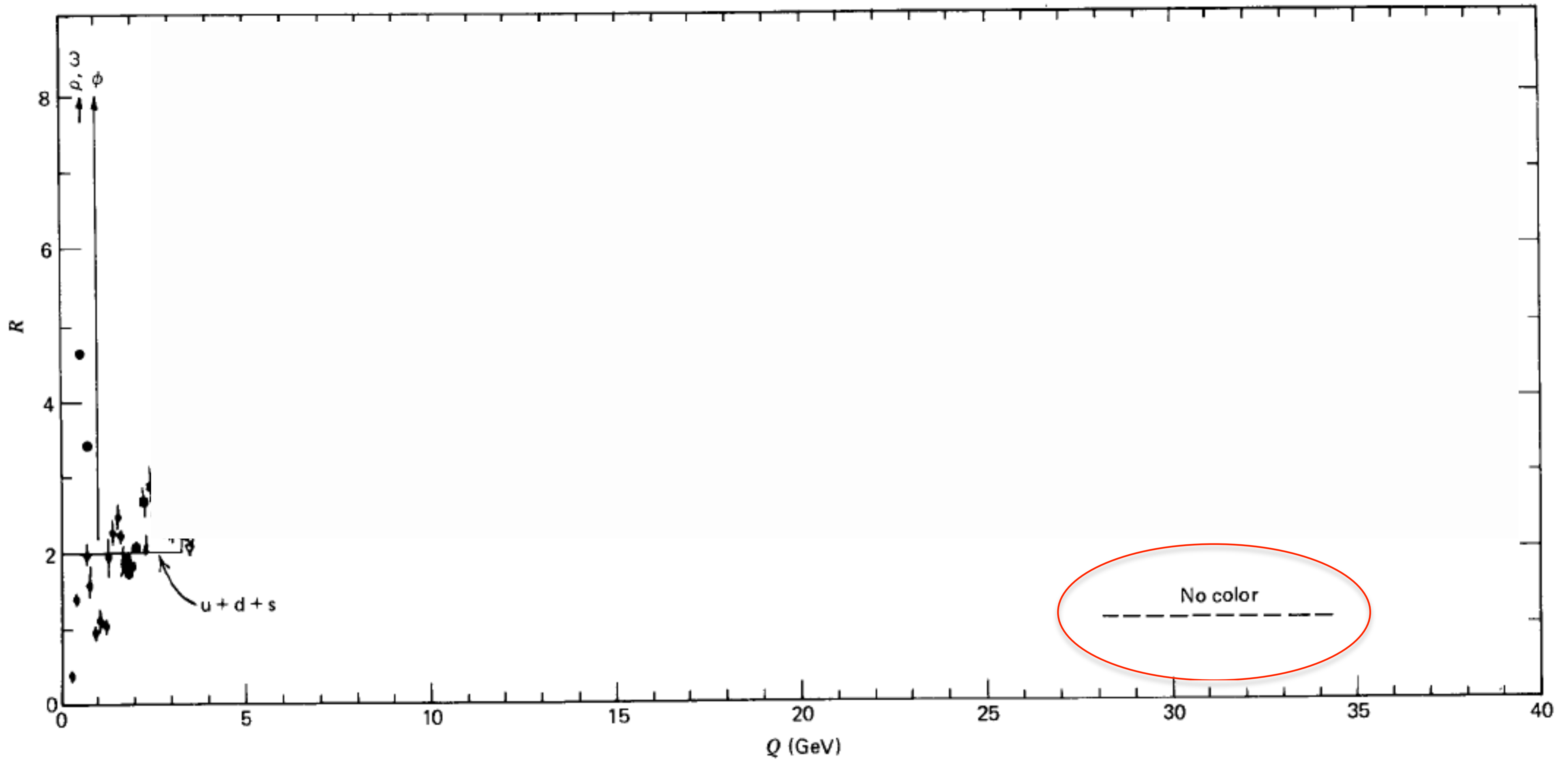


Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

EVIDENCE OF COLOUR

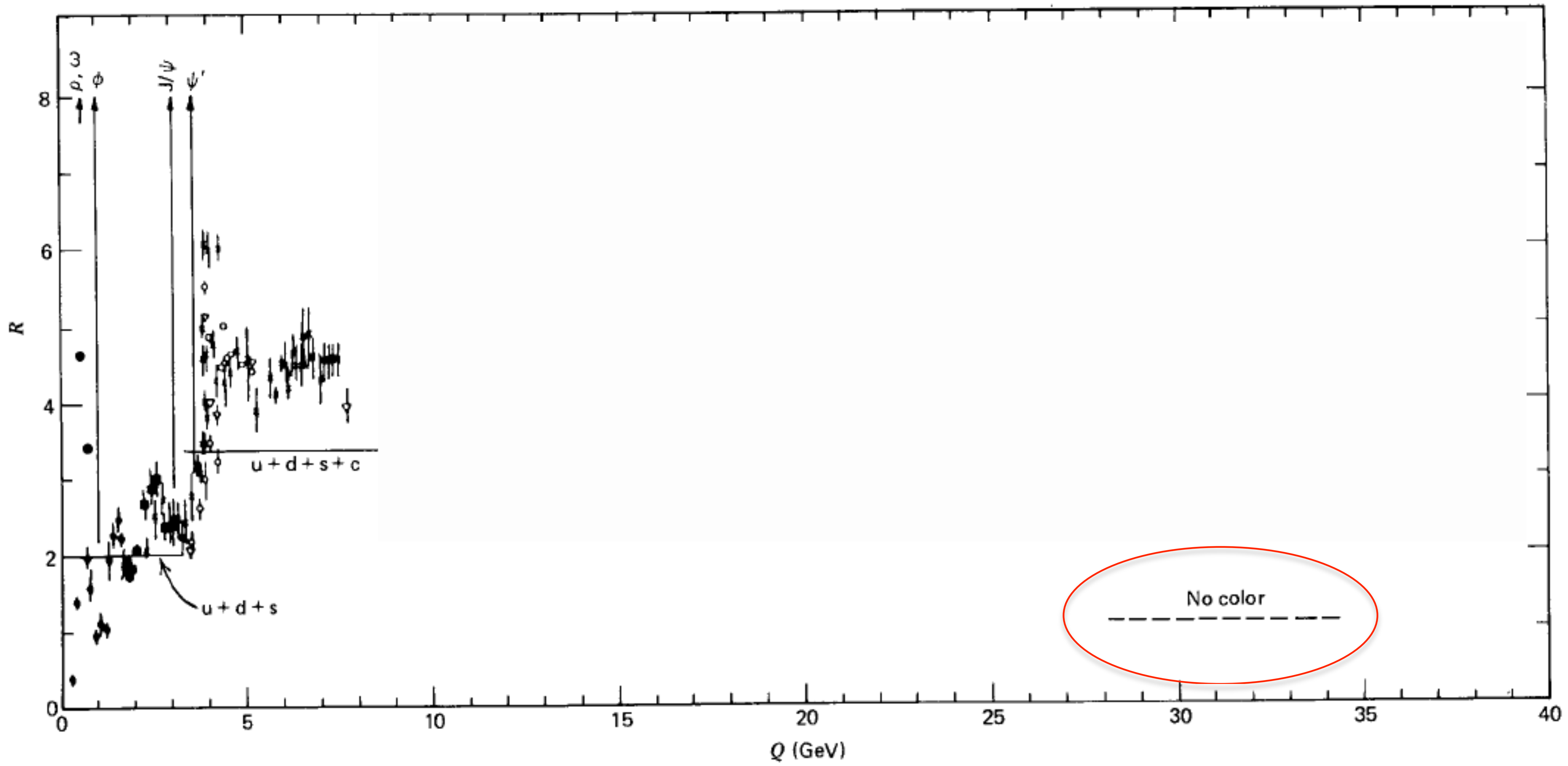


Fig. 11.3 Ratio *R* of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

EVIDENCE OF COLOUR

$$R = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c z_q^2$$

$$R = N_c z_q^2 \left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$$

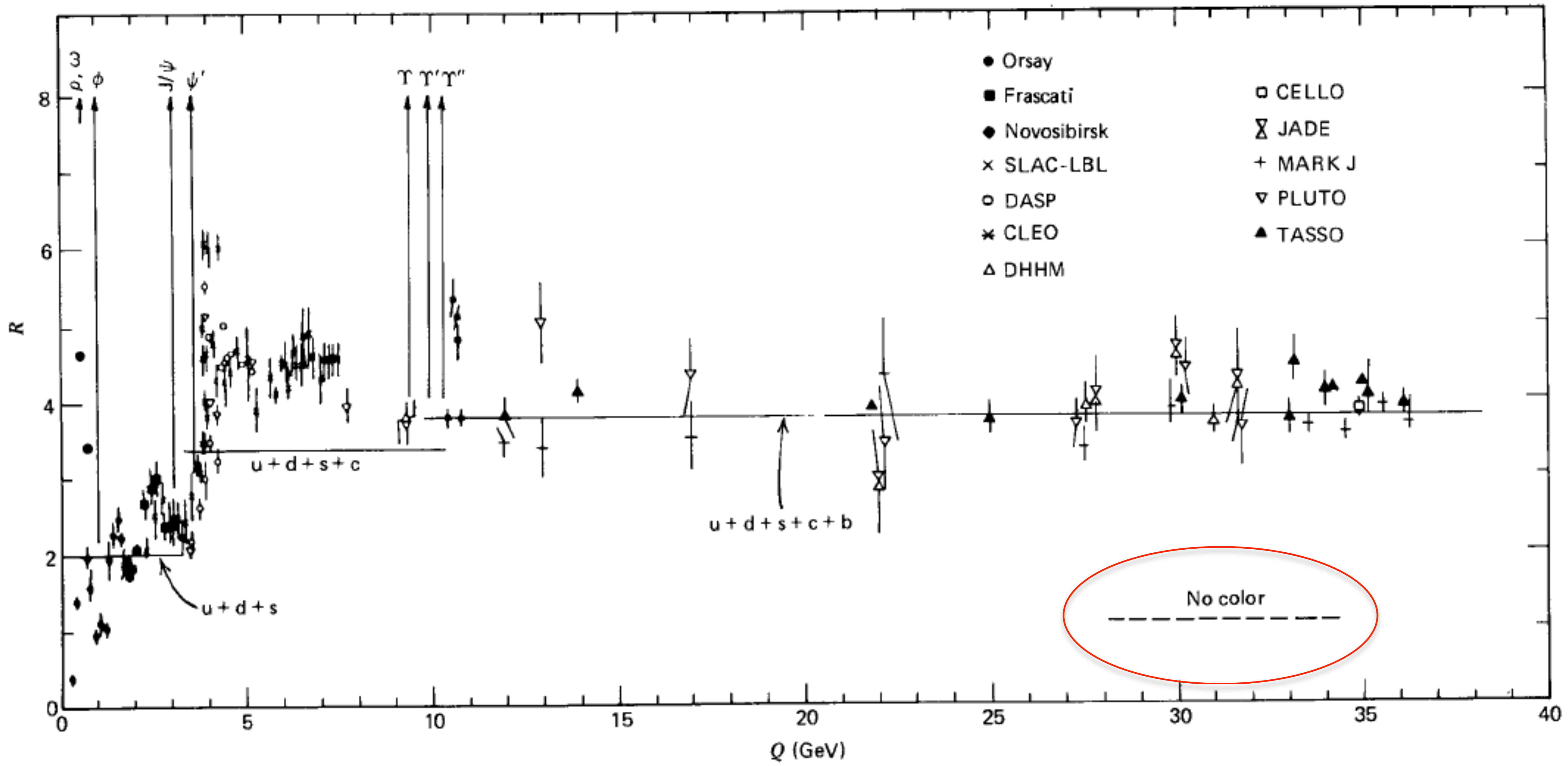
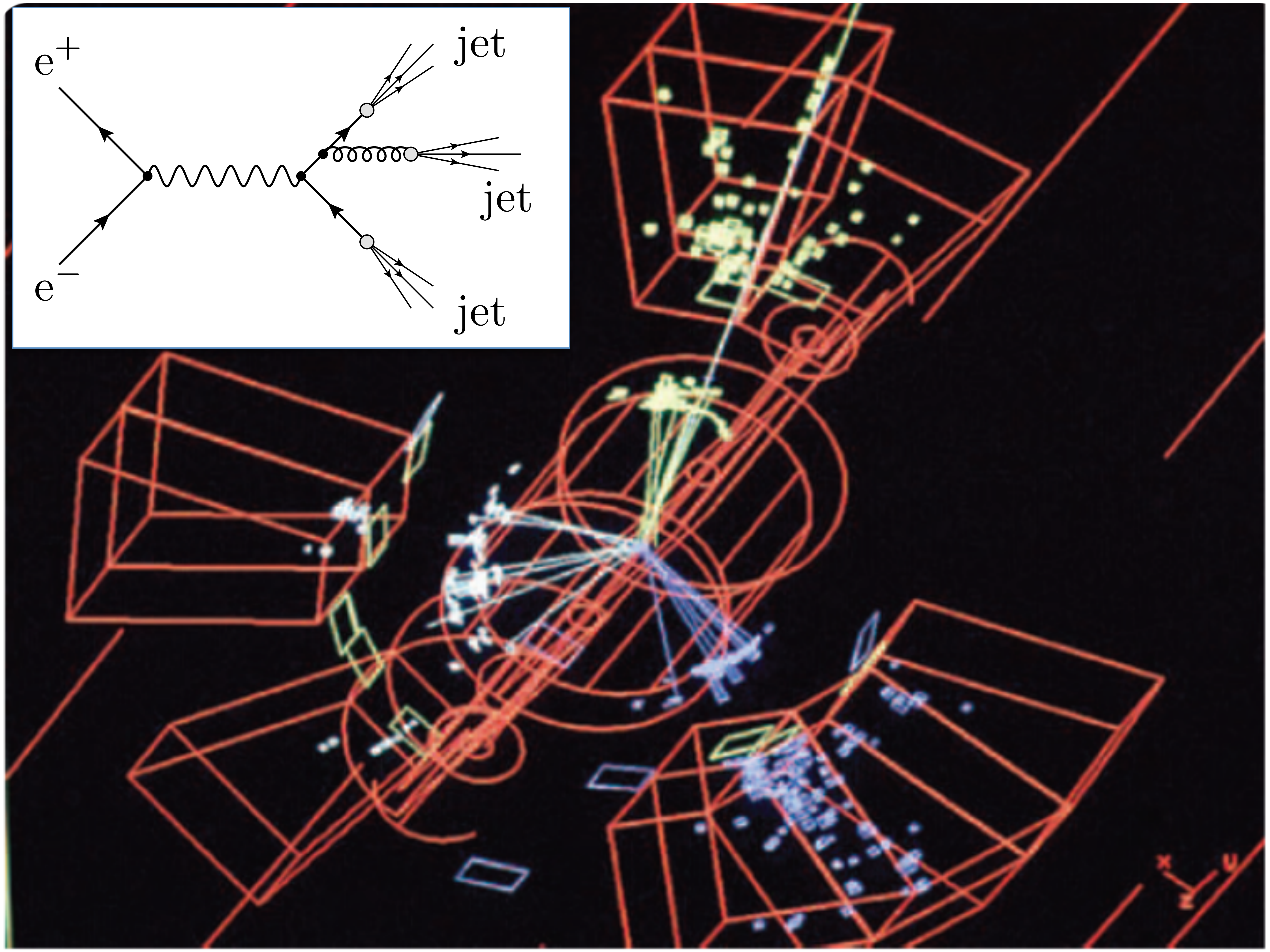


Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

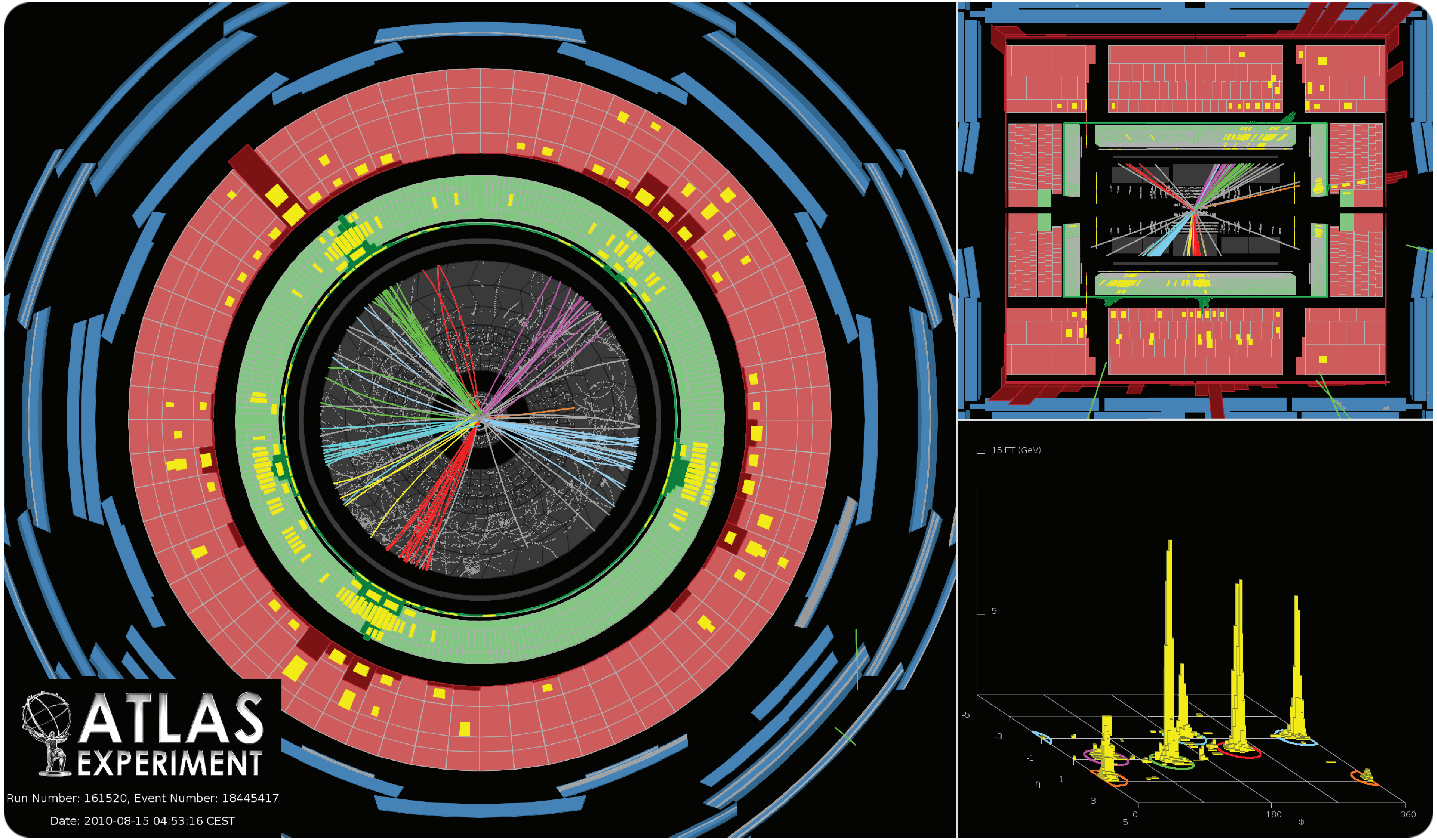
EXISTENCE OF GLUONS: 3-JET EVENTS

TASSO experiment @ PETRA @ DESY



e^-e^+ collisions @ $\sqrt{s}$ = 13-31 GeV

EXISTENCE OF GLUONS: 5-JET EVENTS



Thank you for
your attention!