

# Temporal-Topological Properties of Higher-Order Evolving Networks

*Alberto Ceria, and Huijuan Wang*

**January, 11th 2024, Delft**

**Graphs & Data Seminar,  
TU Delft**



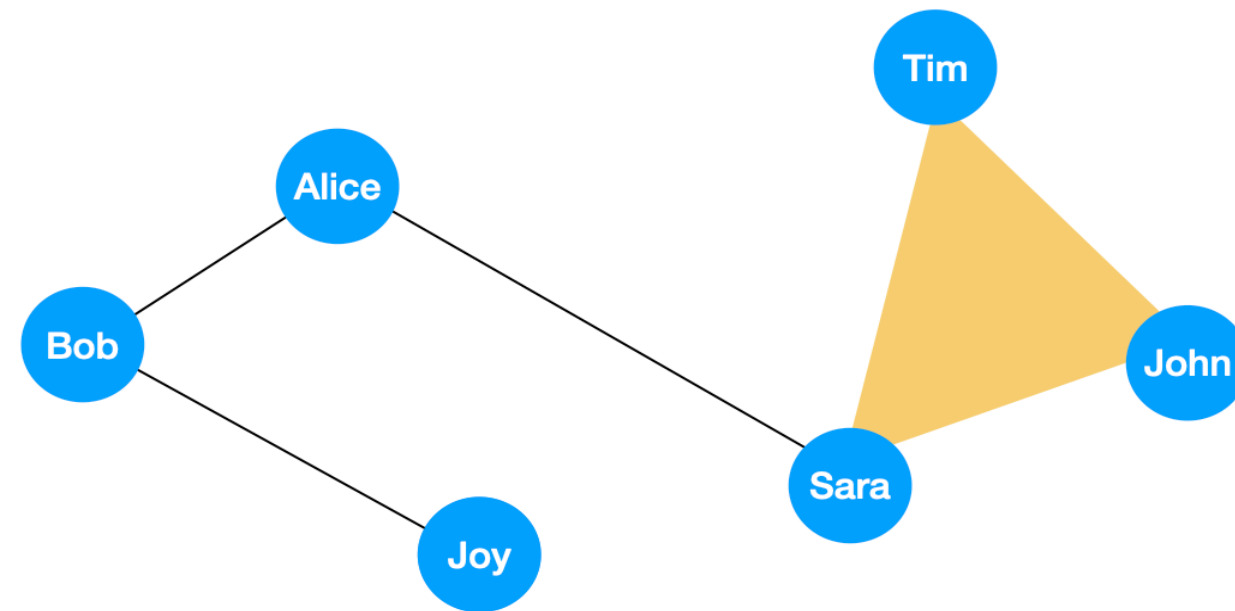
**Universiteit  
Leiden**



# Higher-order evolving networks?

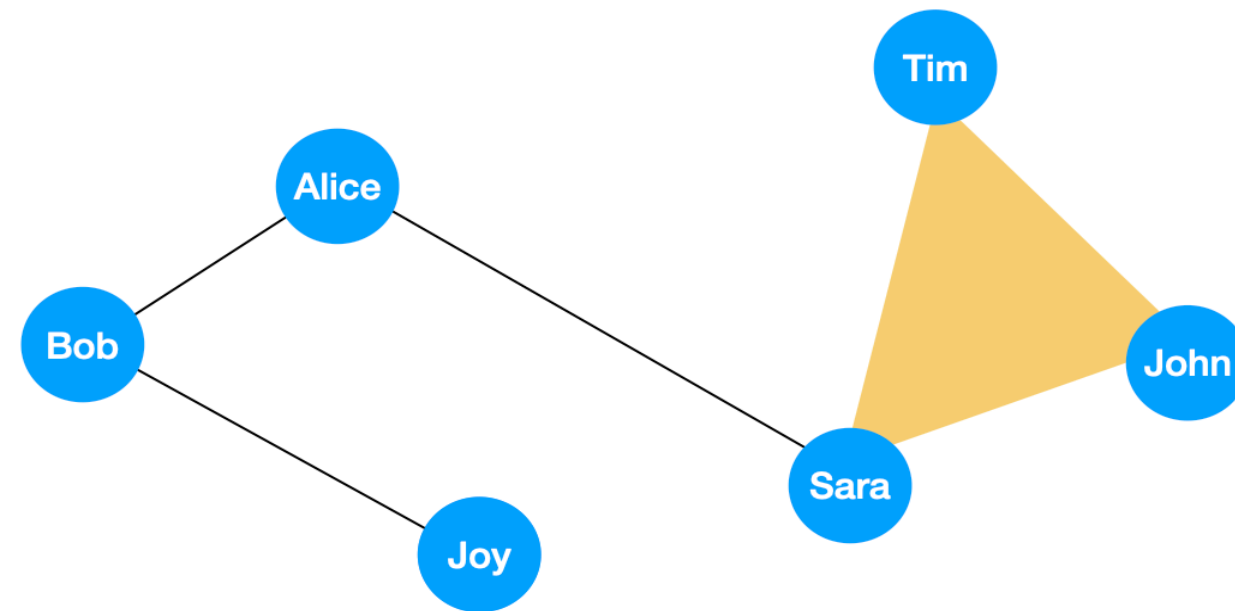
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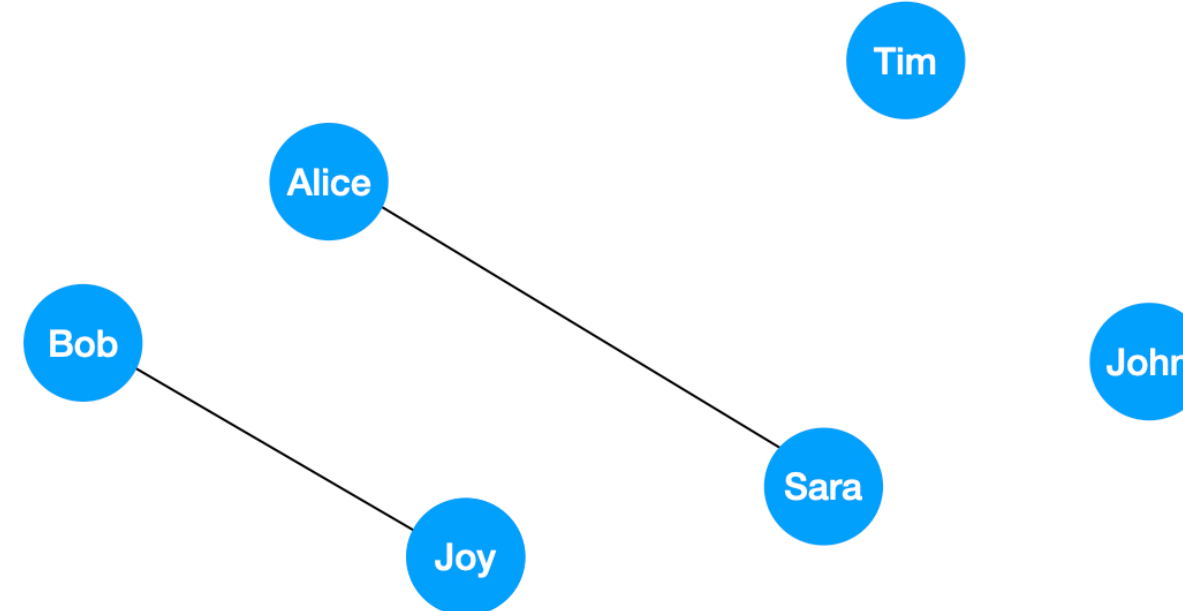


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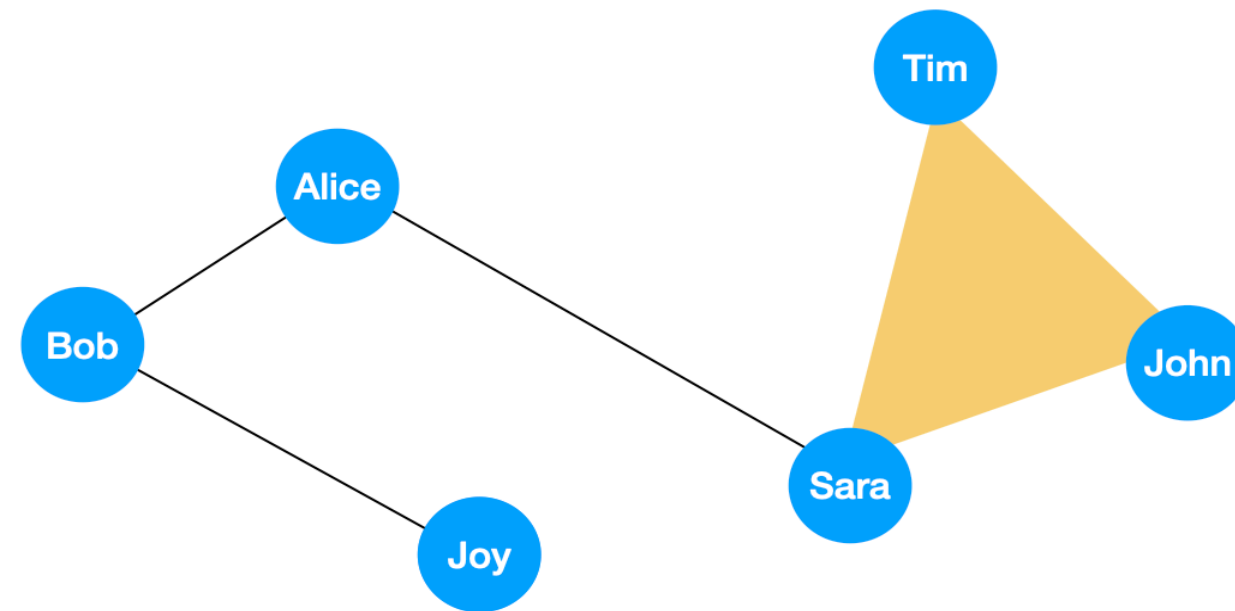
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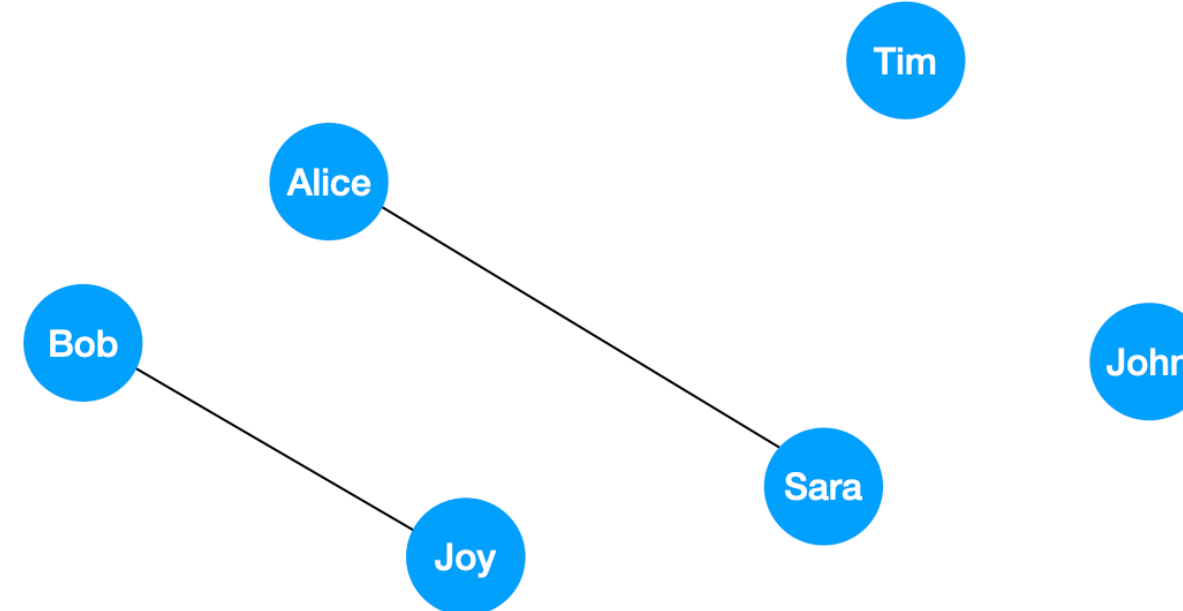


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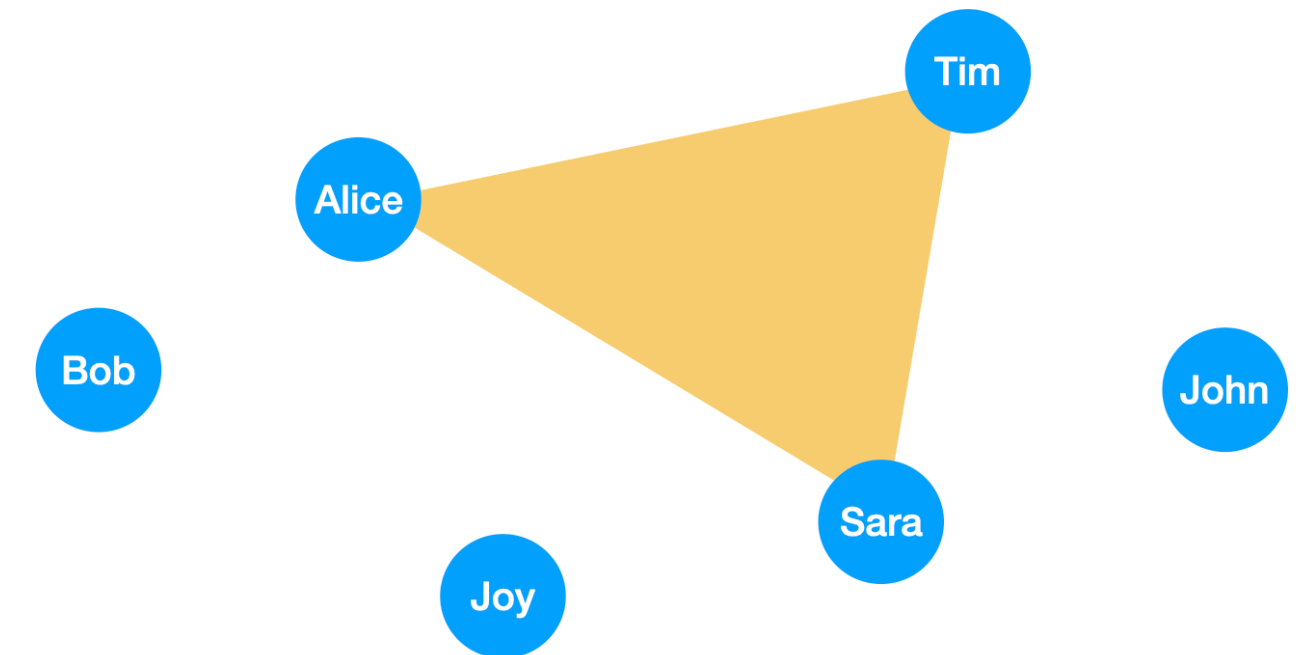
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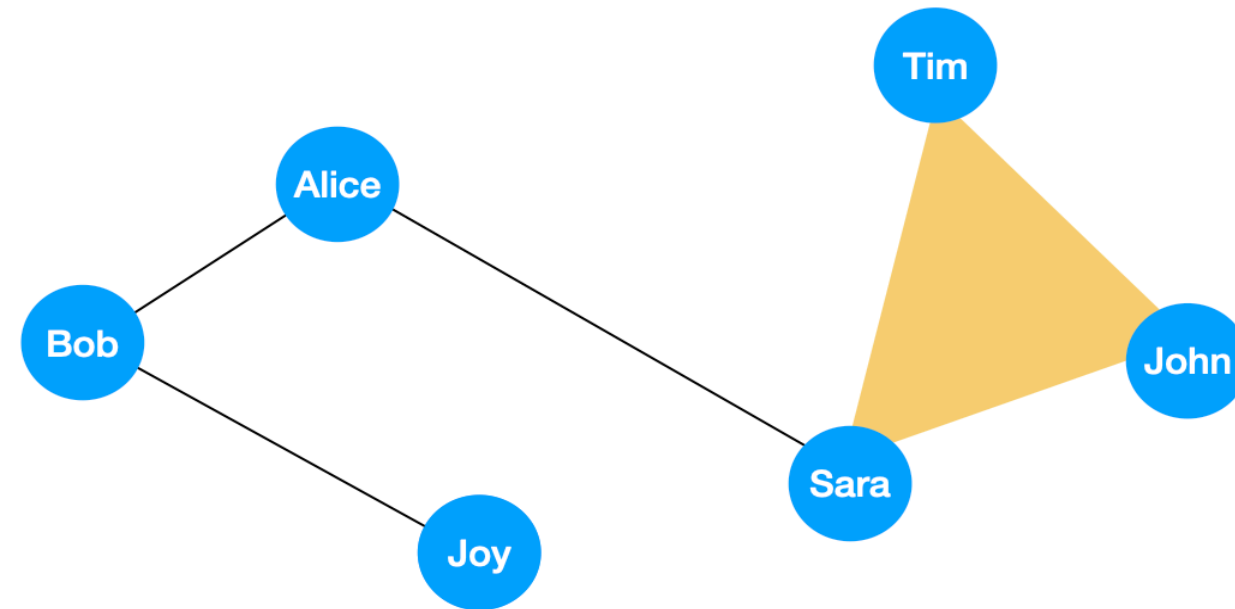


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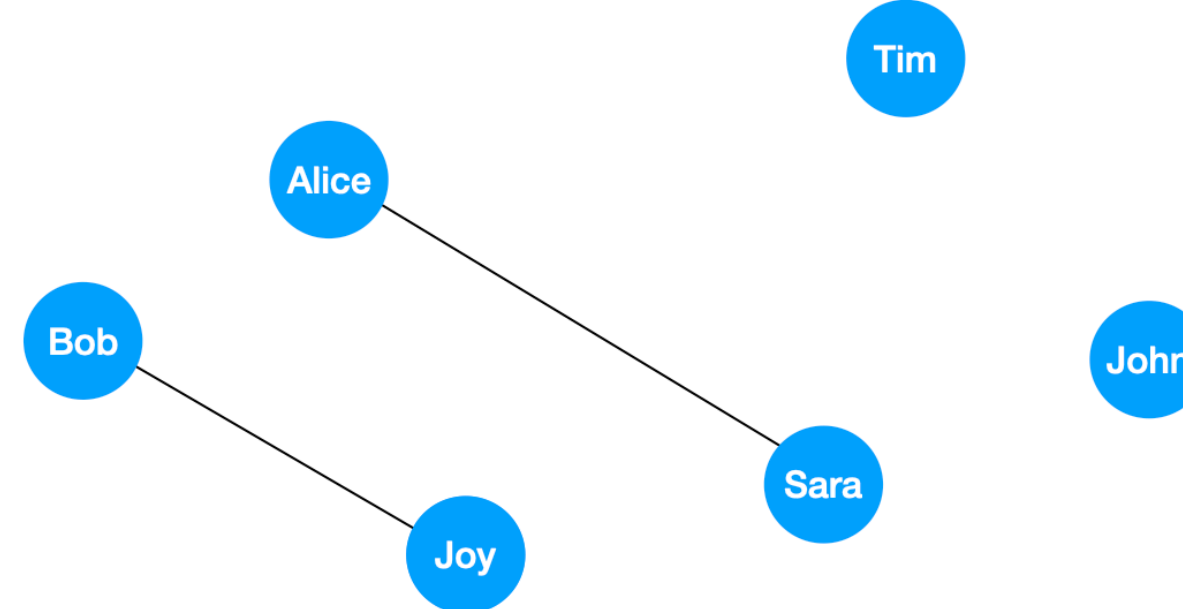


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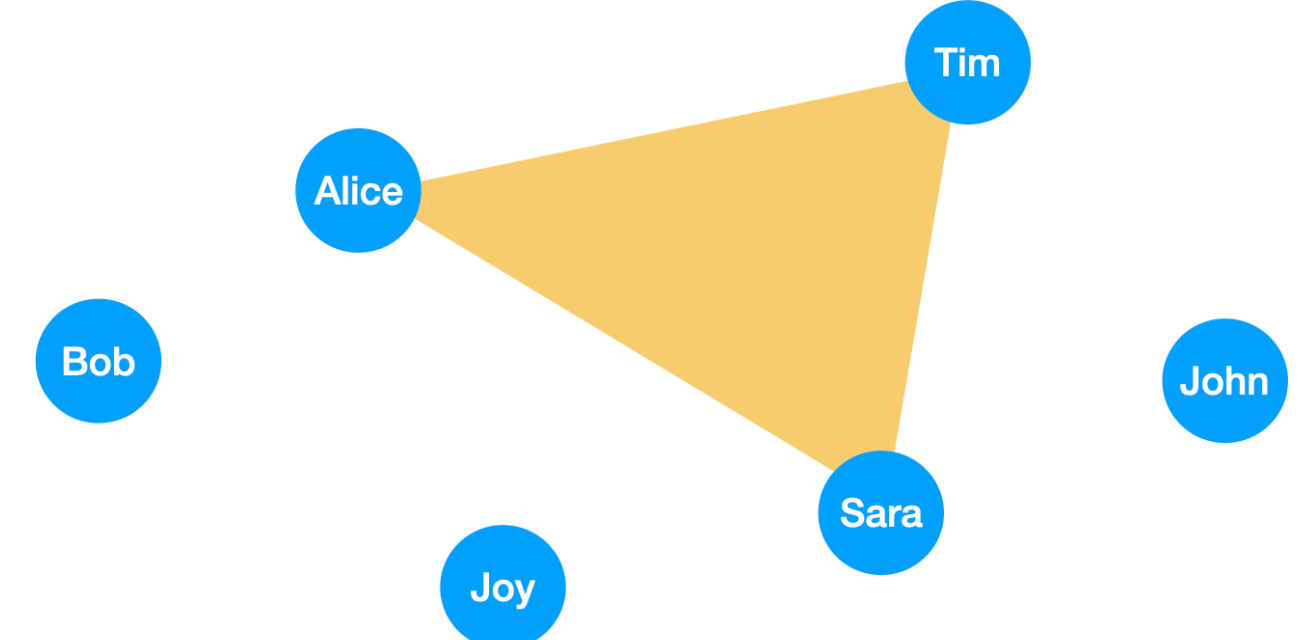
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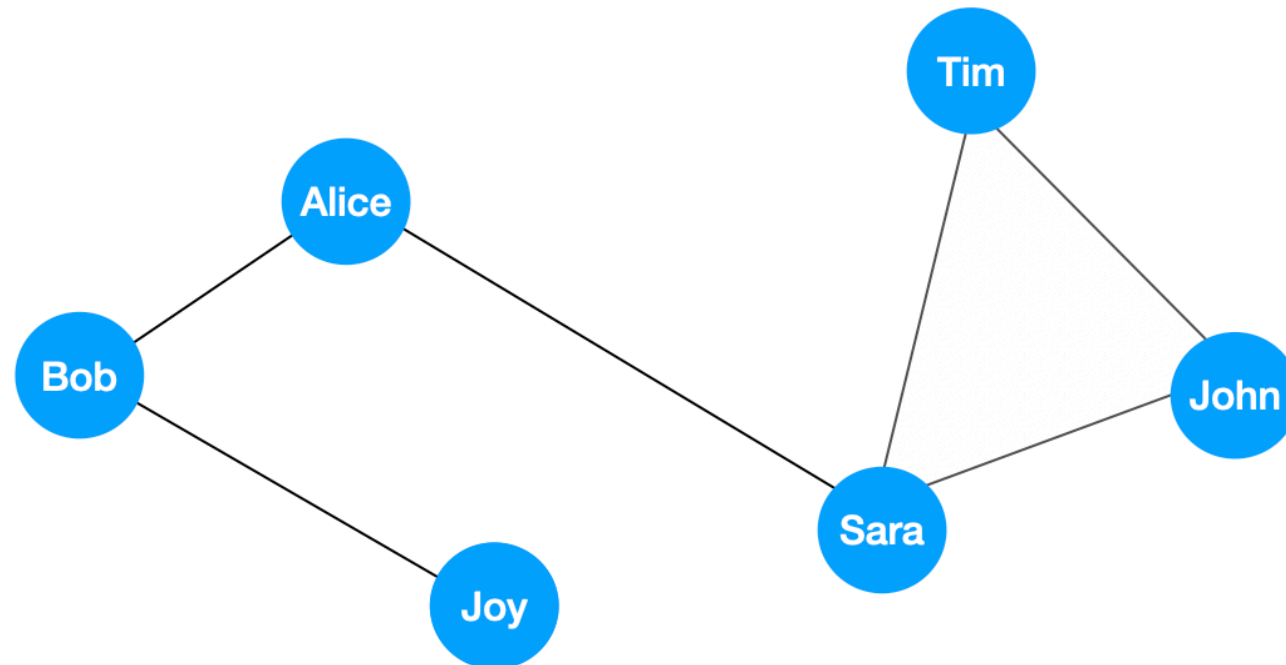


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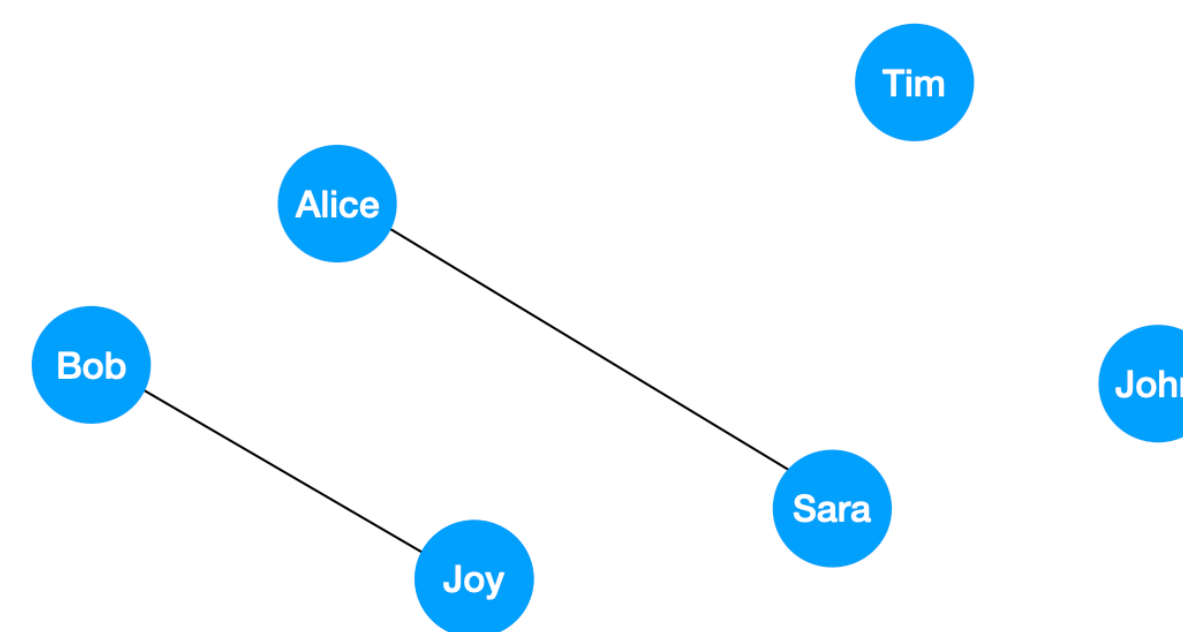


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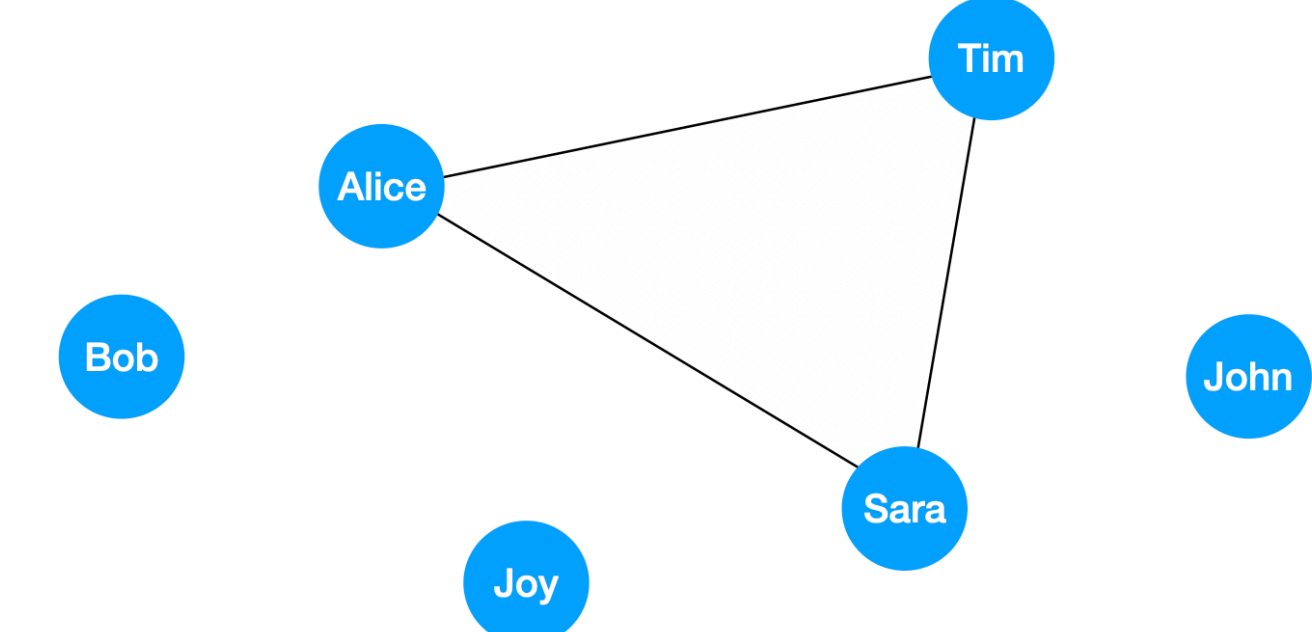
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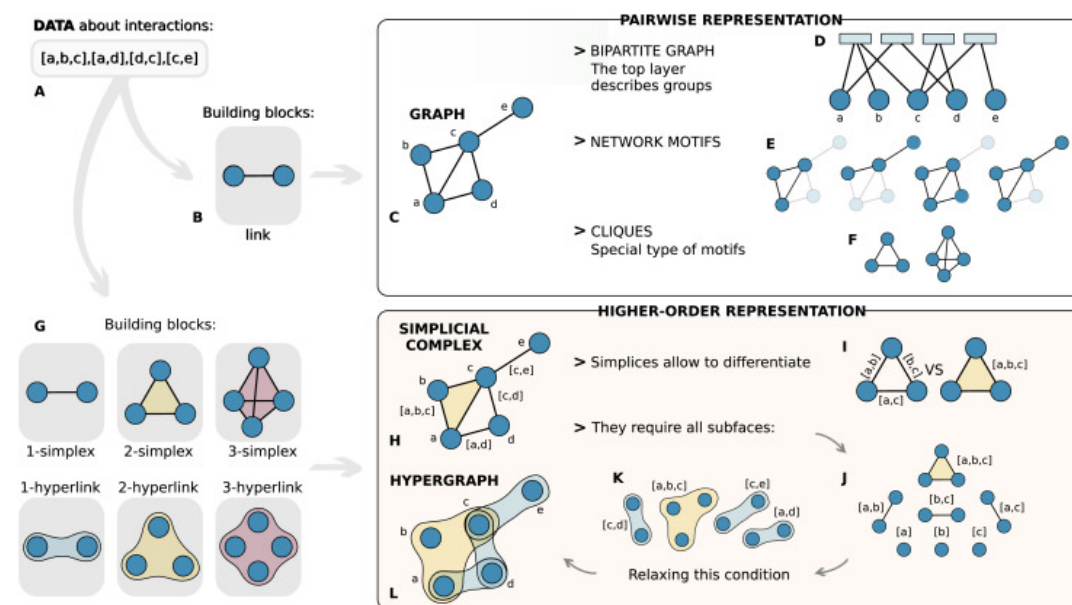
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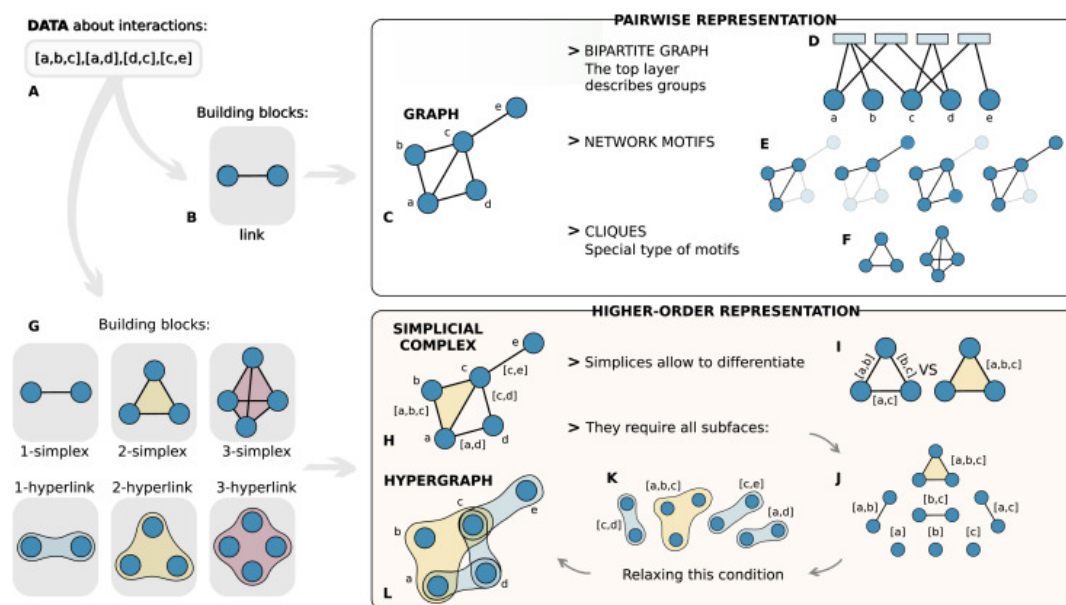
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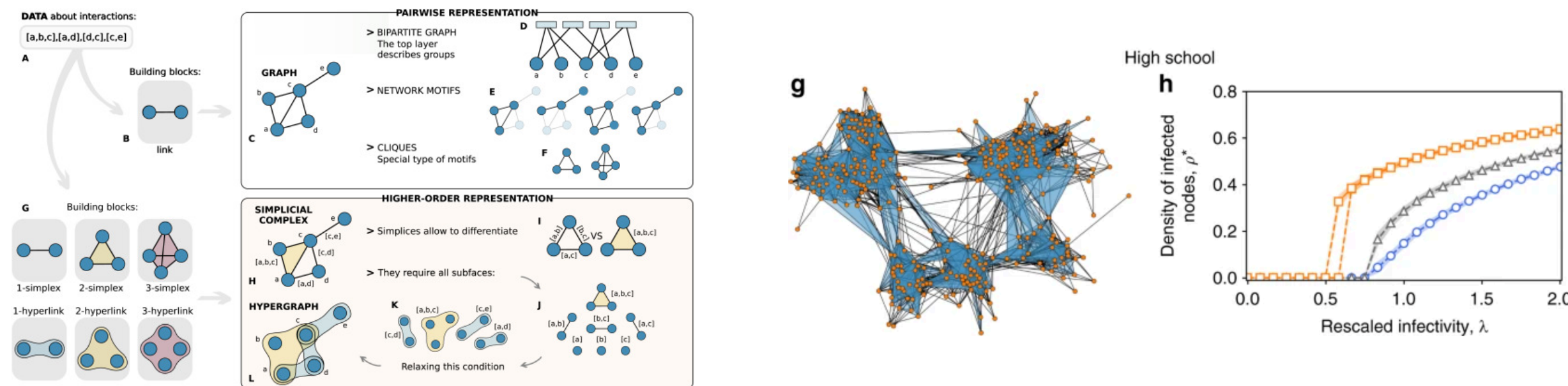
- **Comparing** different **higher-order evolving** networks
- Detecting properties that can influence **dynamic processes** unfolding on **higher-order evolving** networks



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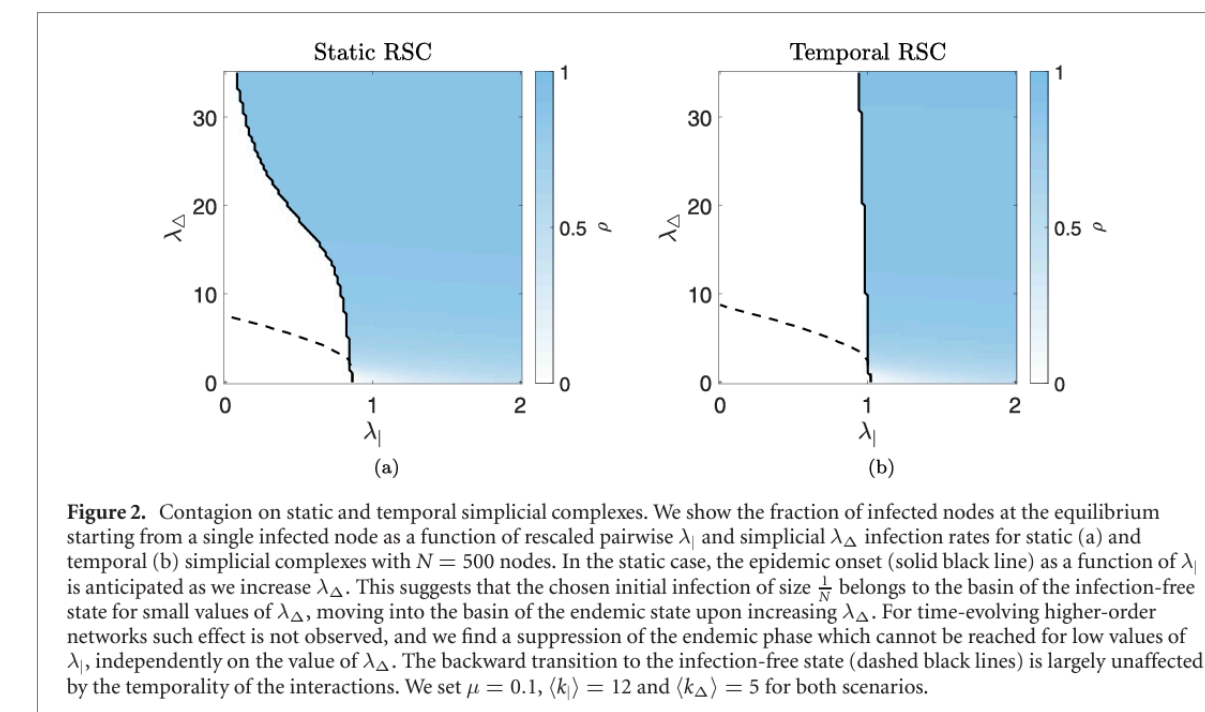
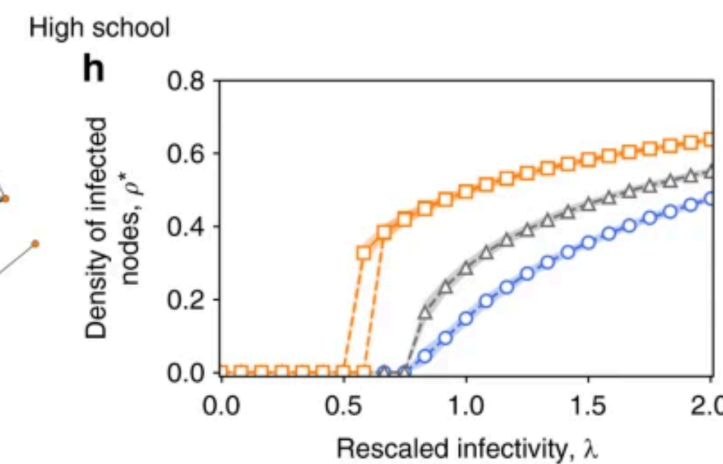
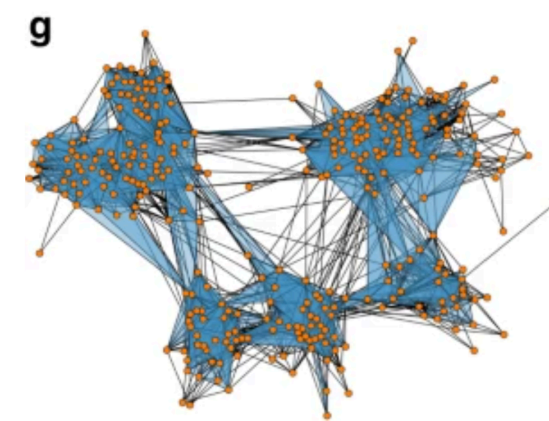
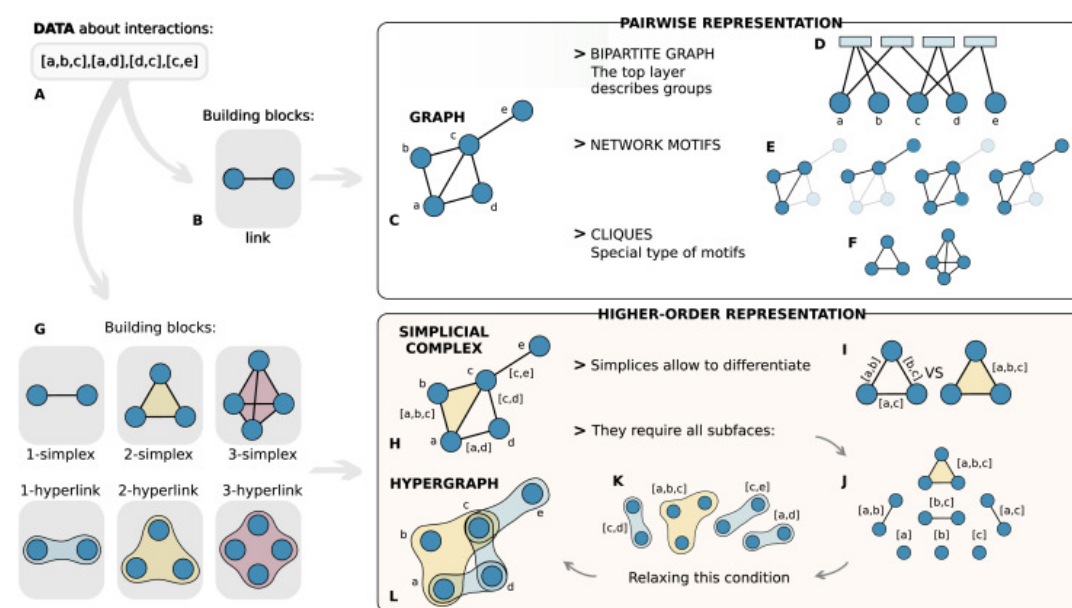
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# Why do we characterise higher-order evolving networks?

- Comparing different **higher-order evolving** networks
- Detecting properties that can influence **dynamic processes** unfolding on **higher-order evolving** networks
- Guiding the development of **higher order evolving** network **models**

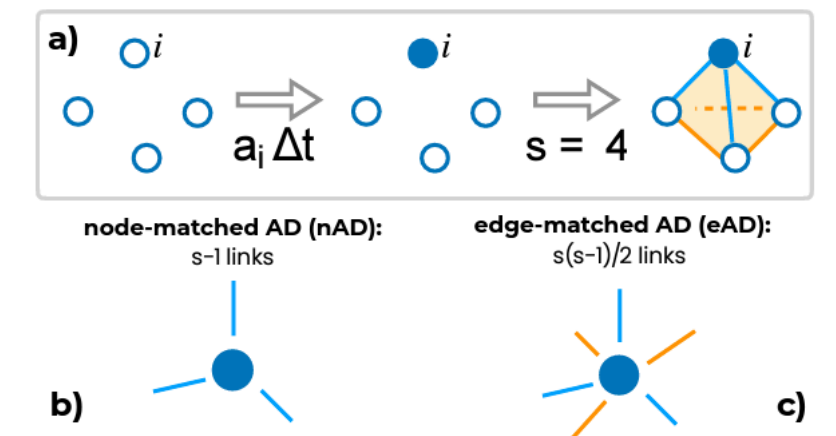
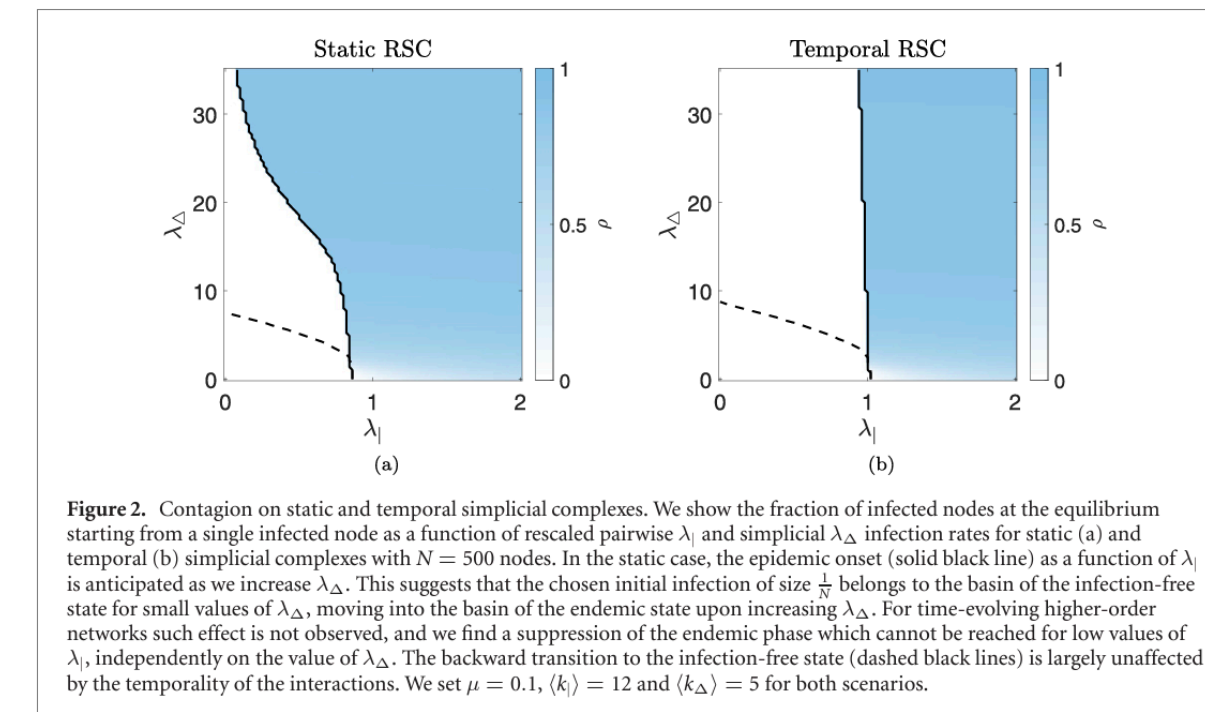
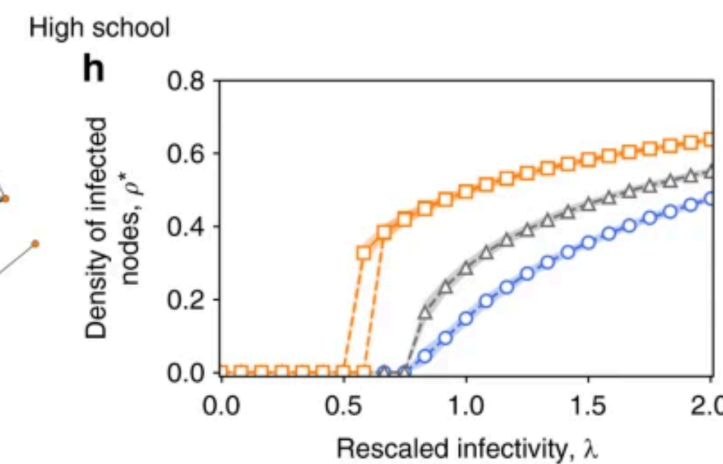
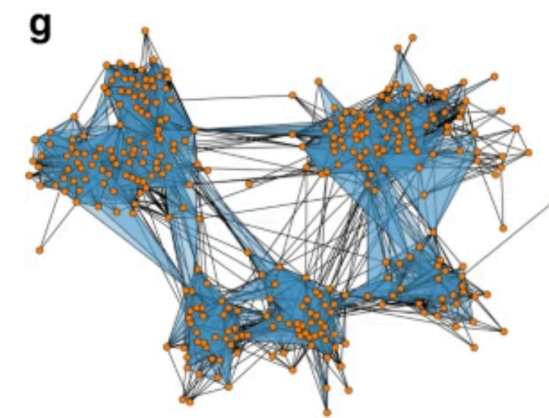
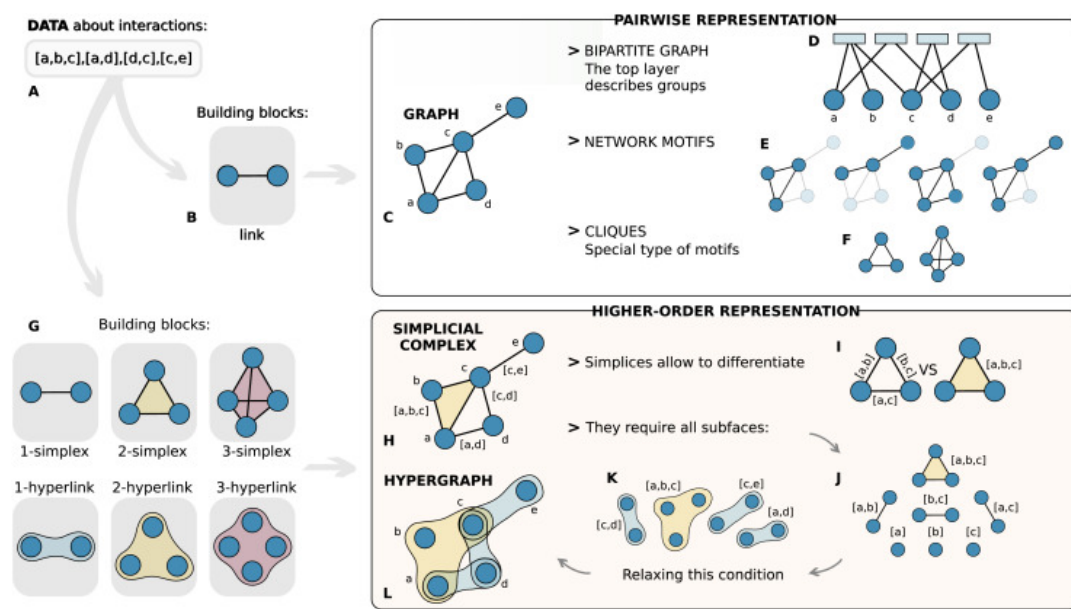


FIG. 1. **SAD model.** a) At each timestep, a node  $i$  activates with probability  $a_i \Delta t$ . Upon activation it creates a coherent unit of  $s$  nodes (an  $(s-1)$ -simplex), with links between all pairs. b) In contrast, in the standard AD model (nAD) only the  $s-1$  edges stemming from the activated node are added. c) In the eAD model  $\binom{s}{2}$  links stem from the activated node, conserving at each interaction the number of links of the SAD model.

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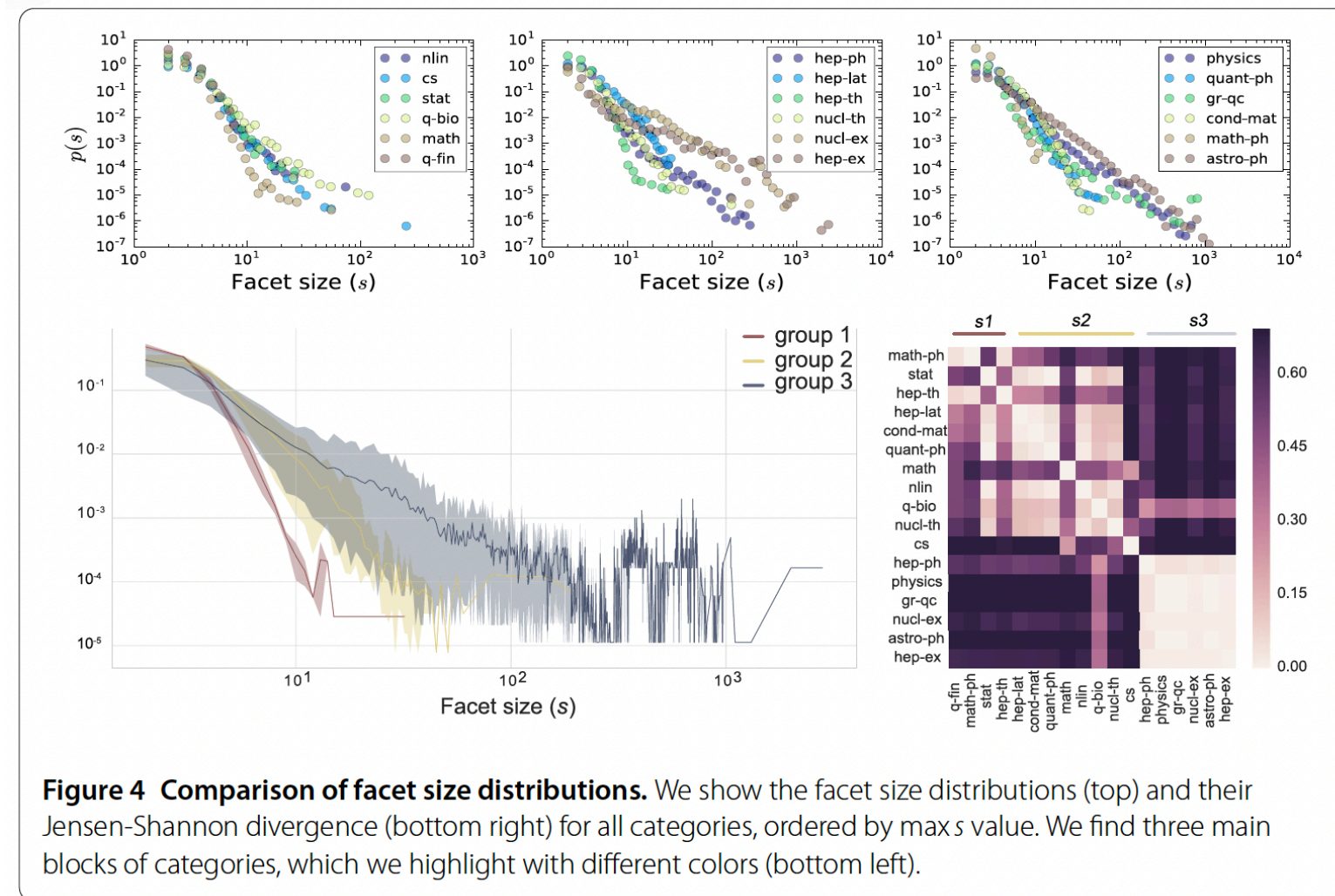
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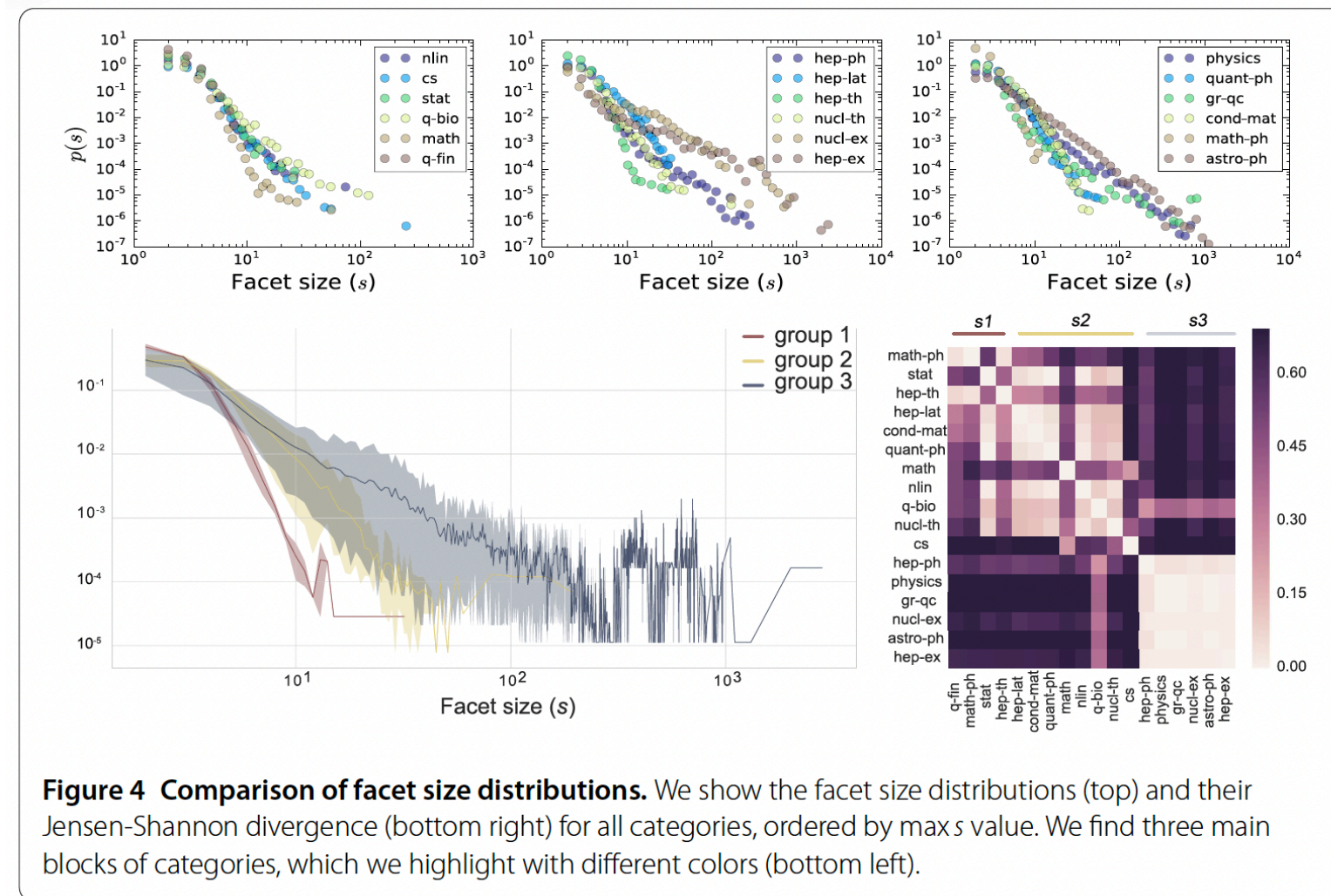


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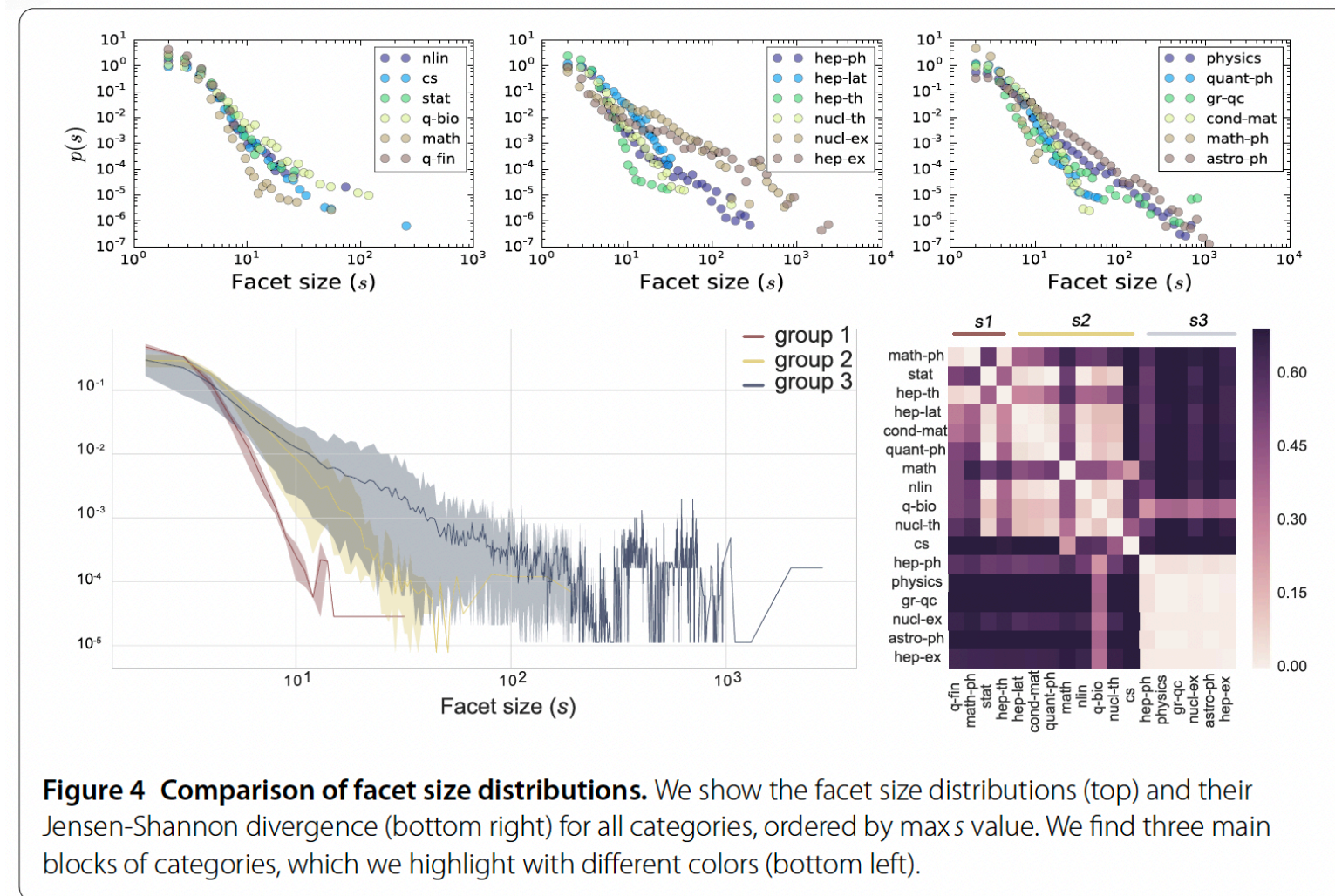


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- Characterization of properties of (higher-order) time-aggregated topology (**ignores time!!**)



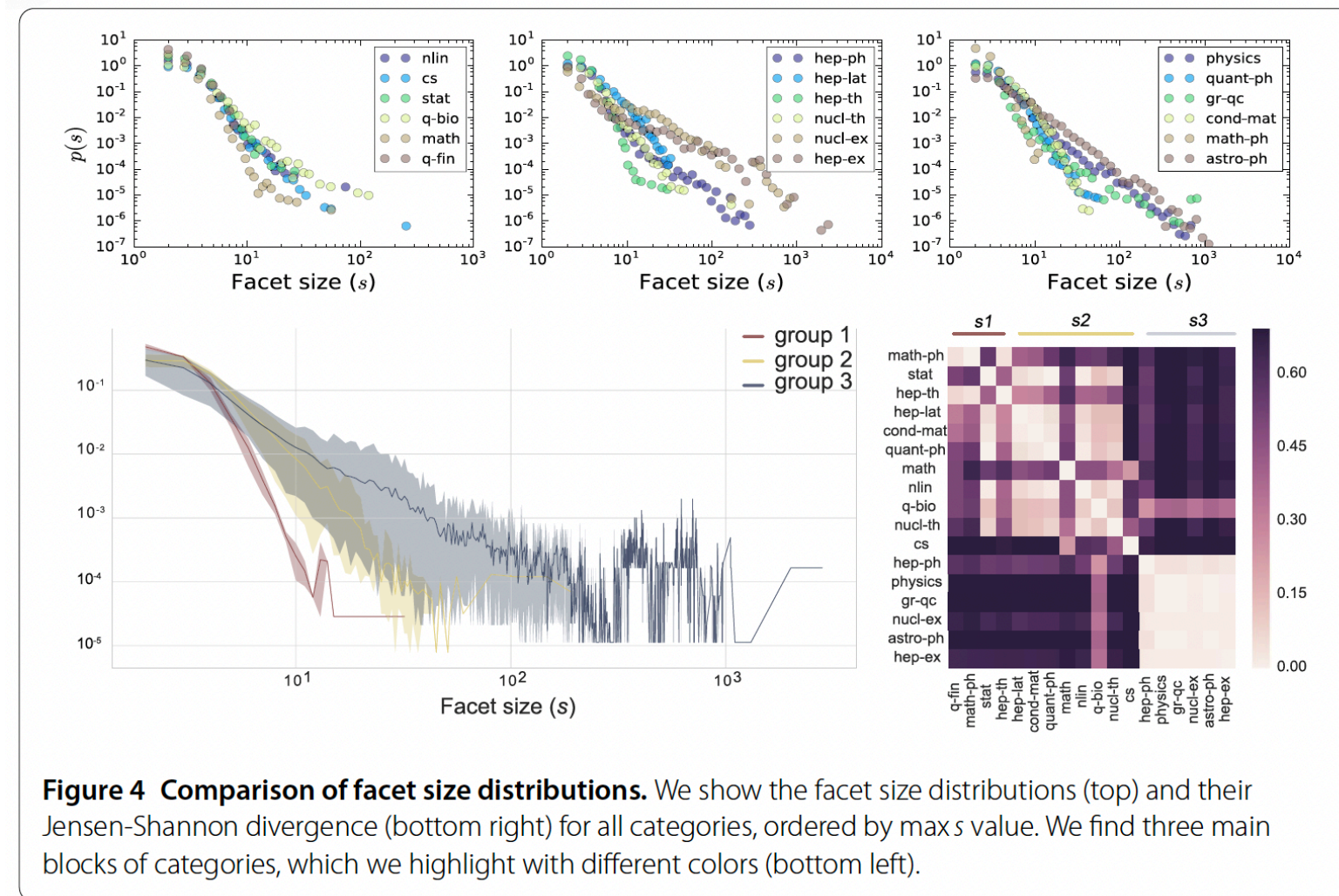
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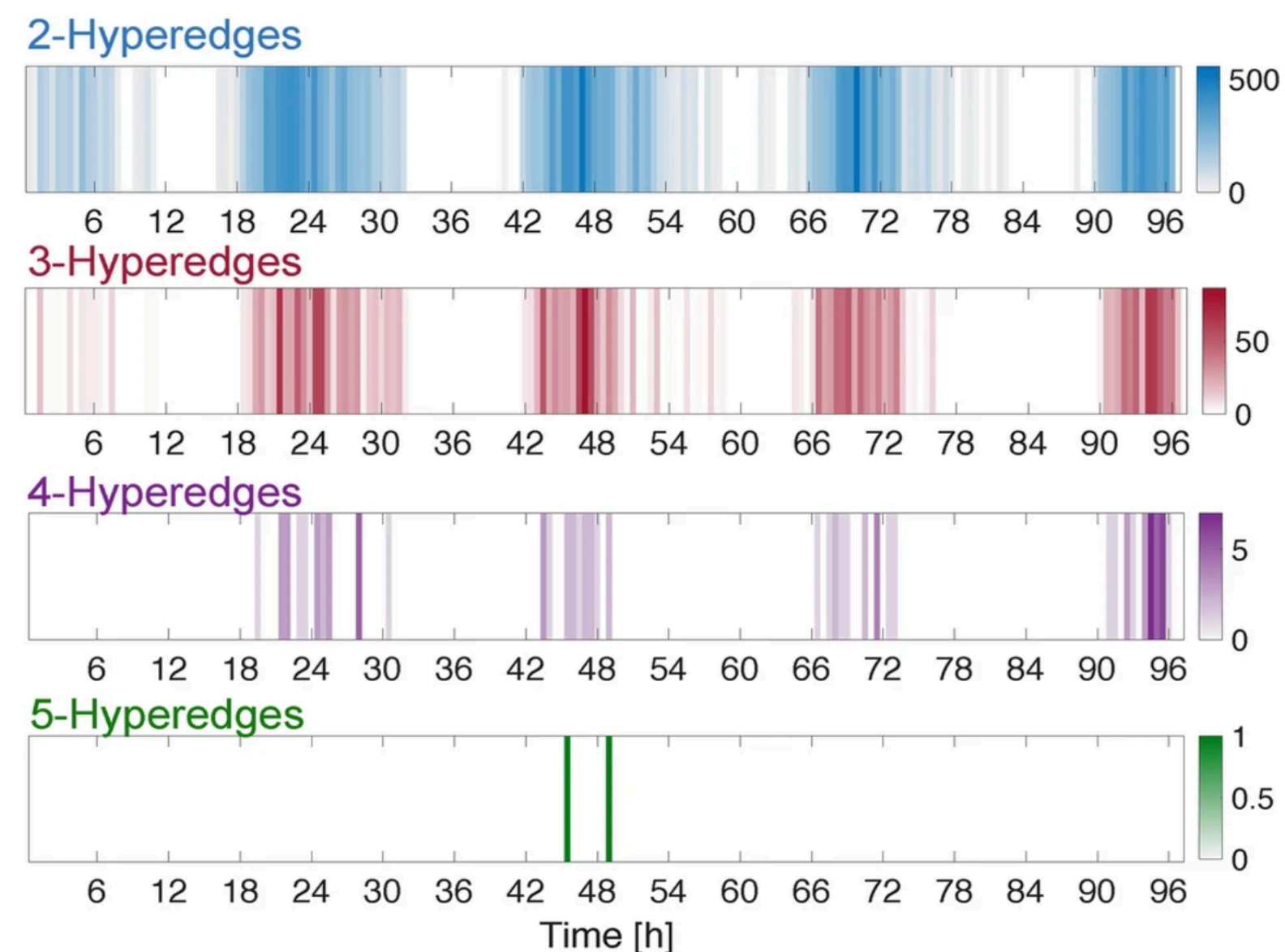
- Characterization of properties of (higher-order) time-aggregated topology (**ignores time!!**)
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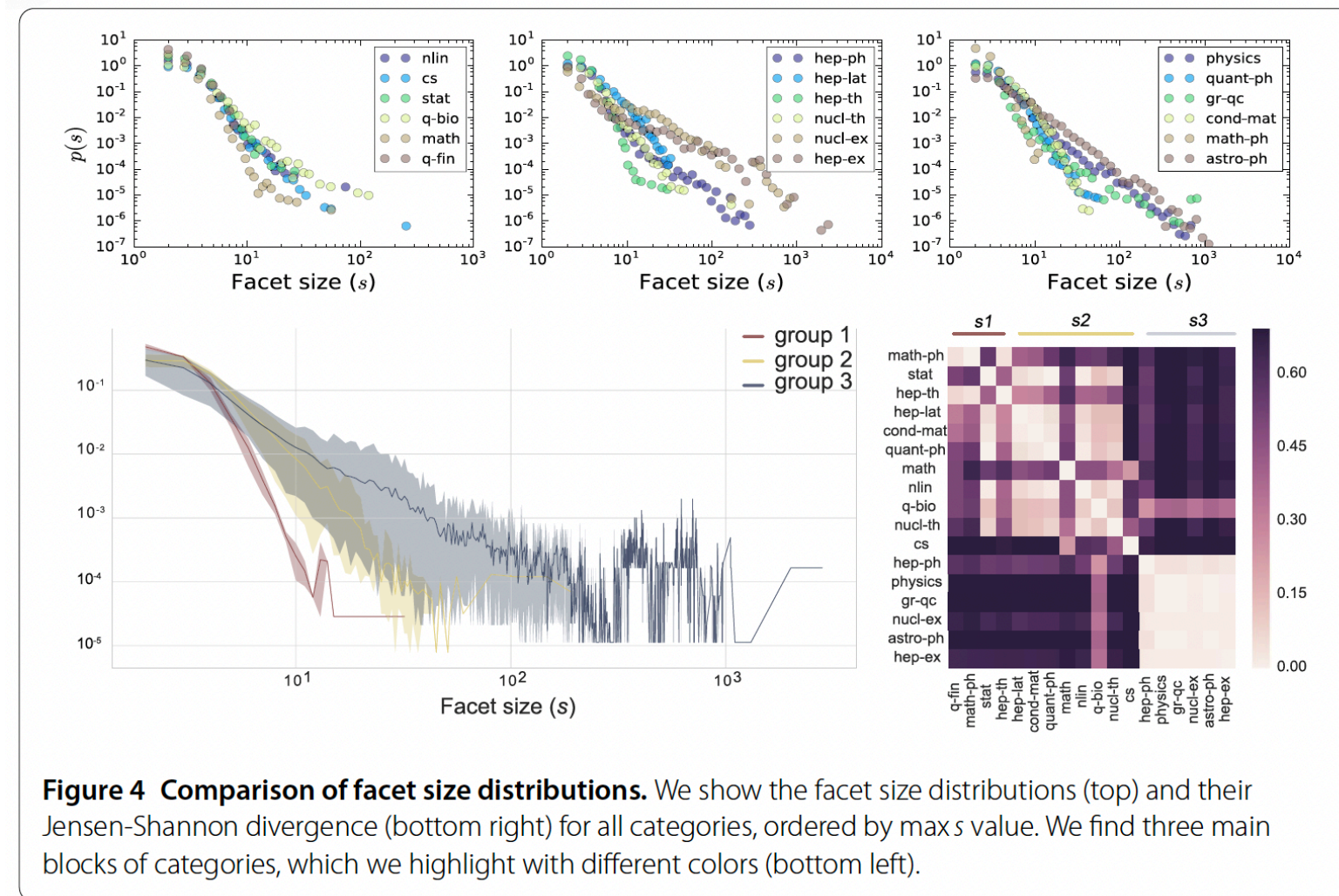


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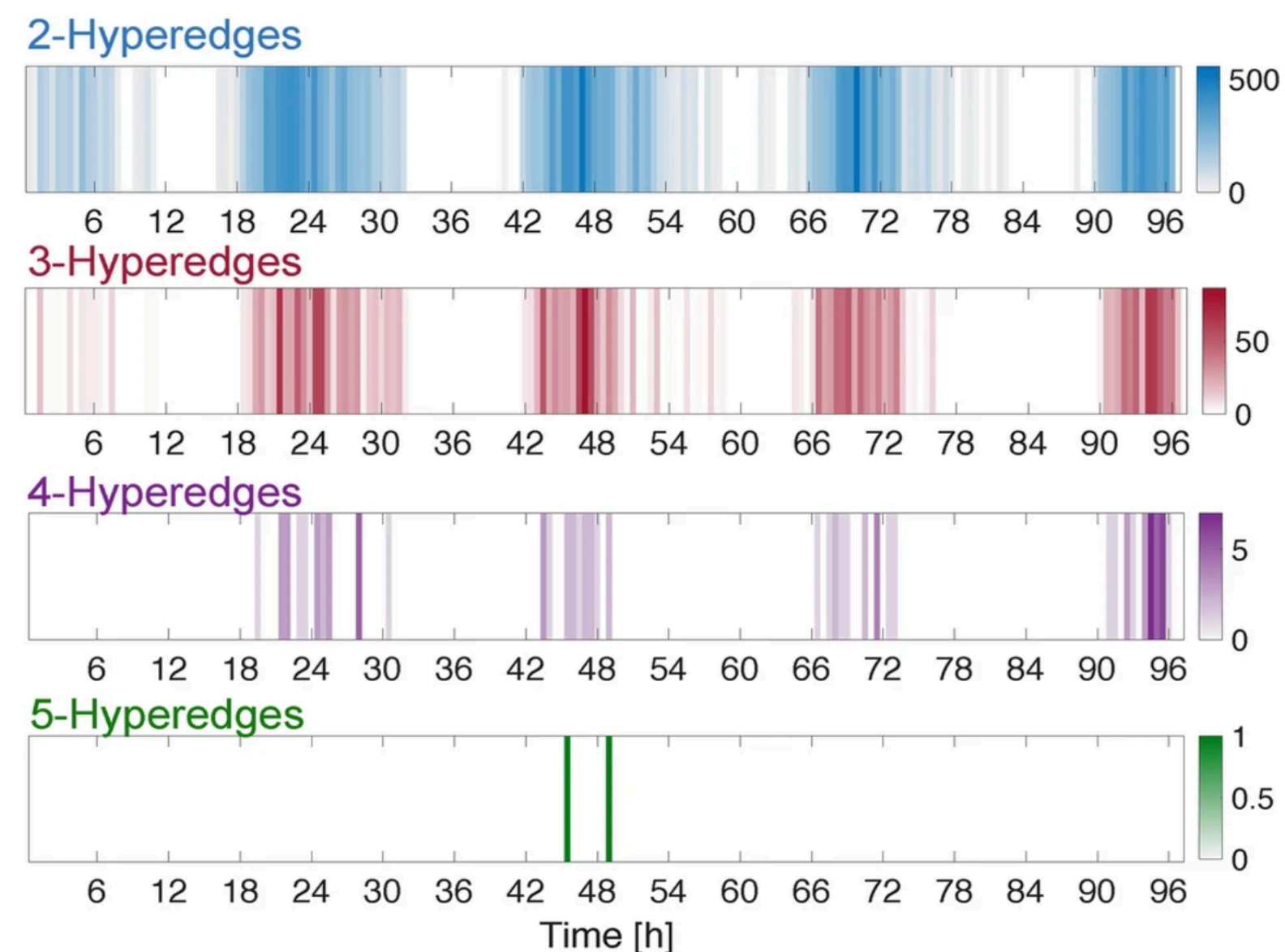
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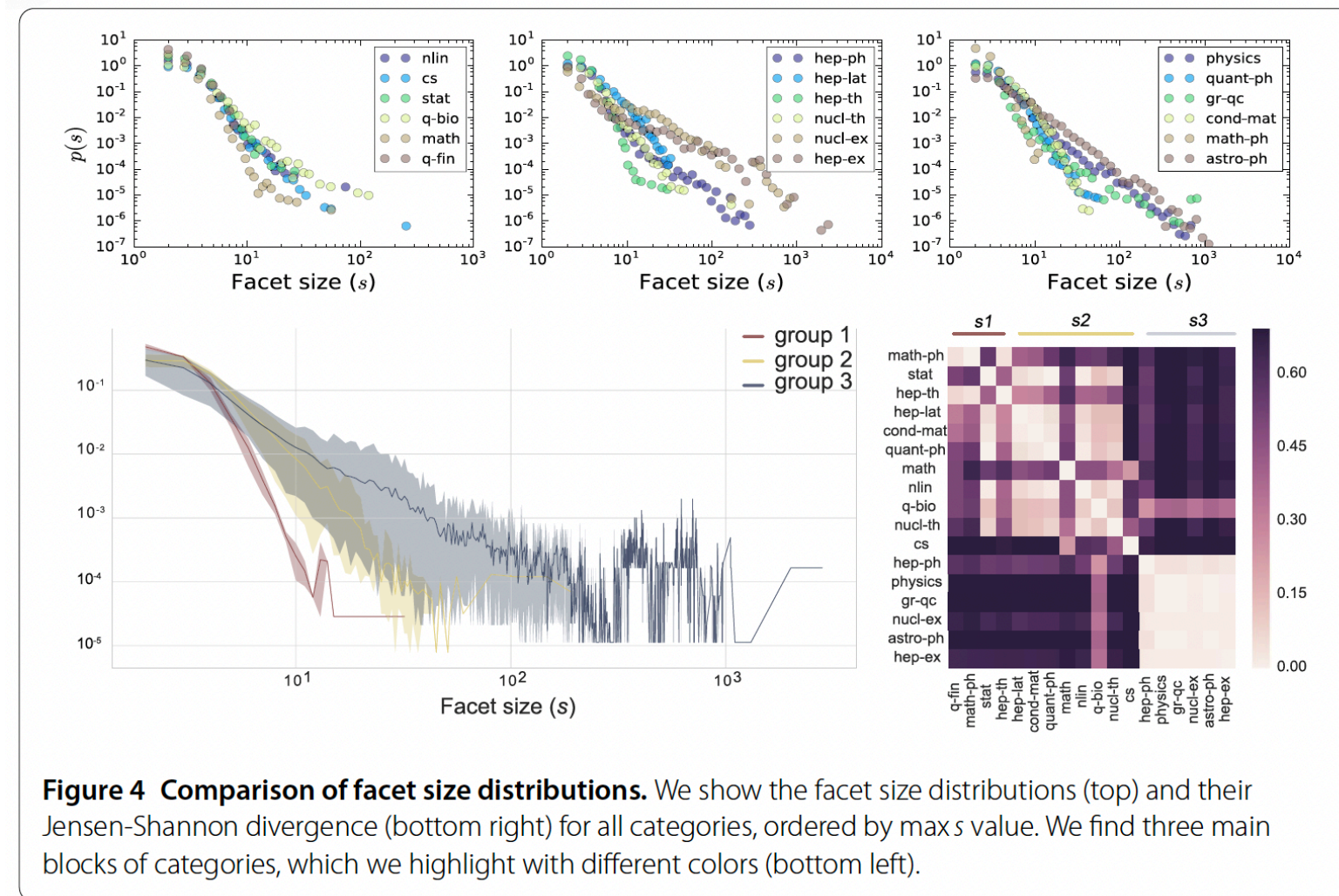


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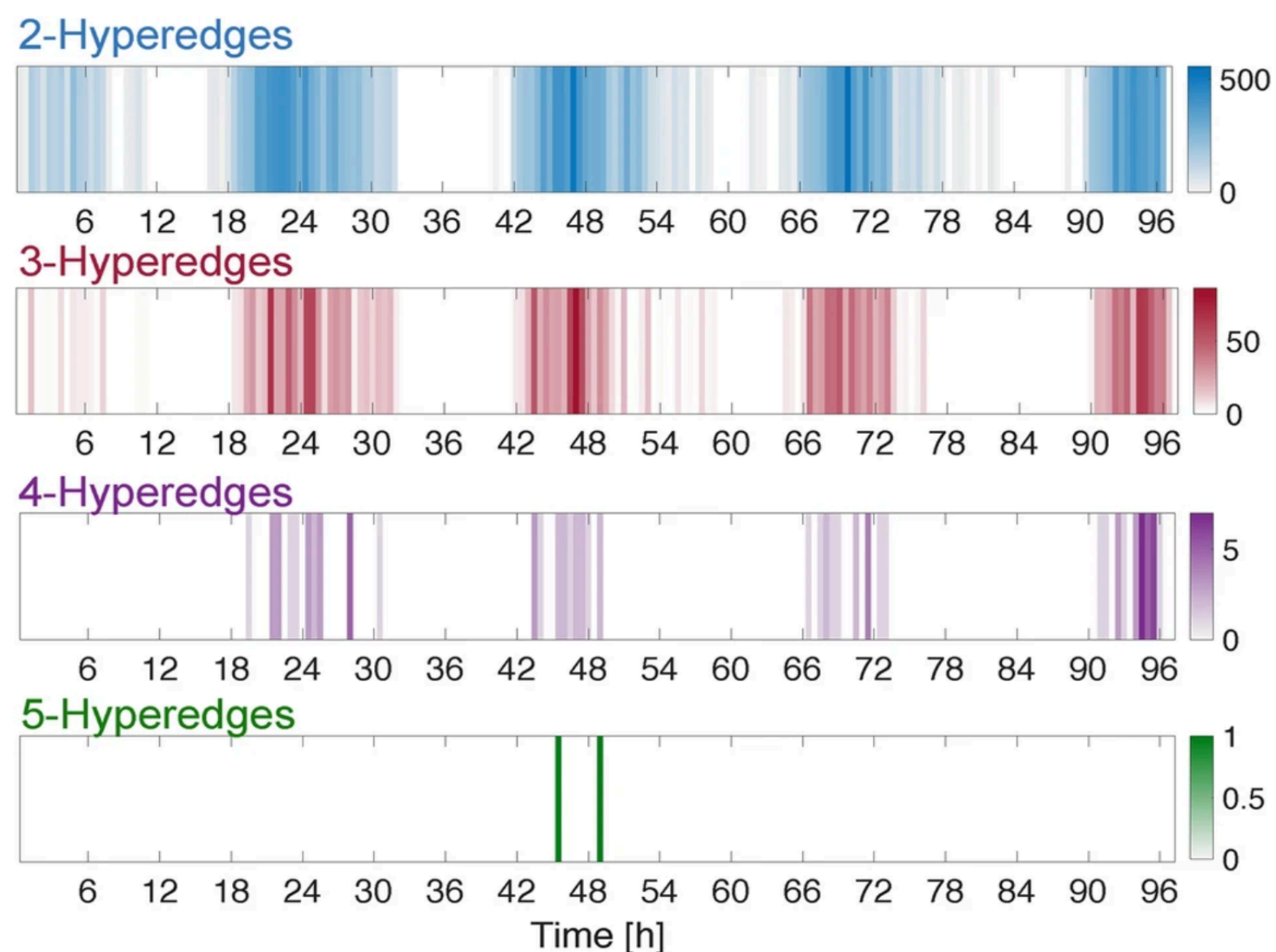
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**Here, we propose methods to characterize simultaneously the temporal and topological relations of events of arbitrary orders.**

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**Do events close in time  
occur close in topology too?**

# Characterization of temporal and topological properties of events

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- Relation of topological and temporal distance of events with different orders

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# Characterization of temporal and topological properties of events

- Relation of topological and temporal distance of events with different orders
- Analysis of the topological overlap of events with different orders
- Temporal correlation of events with different orders overlapping in topology



# Which higher-order evolving networks?

Network	$ \mathcal{N} $	$ \mathcal{L} $	$ \mathcal{E} $	$T$	$dt$	<i>contact type</i>
Primary School (PS)	242	12704	106877	3099	20 s	<i>physical</i>
High School 2013 (HS2013)	327	7818	172031	7371	20 s	<i>physical</i>
Hypertext 2009 (HT2009)	113	2434	19037	7227	20 s	<i>physical</i>
Infectious (Infectious)	410	3350	14275	1422	20 s	<i>physical</i>
Workplace 2015 (WP2)	217	4909	73820	20947	20 s	<i>physical</i>
SFHH Conference (SFHH)	403	10541	54306	3800	20 s	<i>physical</i>
Hospital (Hospital)	75	1825	27835	16027	20 s	<i>physical</i>
High School 2012 (HS2012)	180	2645	42105	14115	20 s	<i>physical</i>
High energy physics, lattice (hep-lat)	10598	11588	18267	10809	1 d	<i>collaboration</i>
Nuclear physics, theory (nucl-th)	25246	27094	39511	10620	1 d	<i>collaboration</i>
Quantitative biology (q-bio)	45645	22978	25973	10704	1 d	<i>collaboration</i>
Quantitative finance (q-fin)	7509	6192	7577	9027	1 d	<i>collaboration</i>
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**SocioPatterns**



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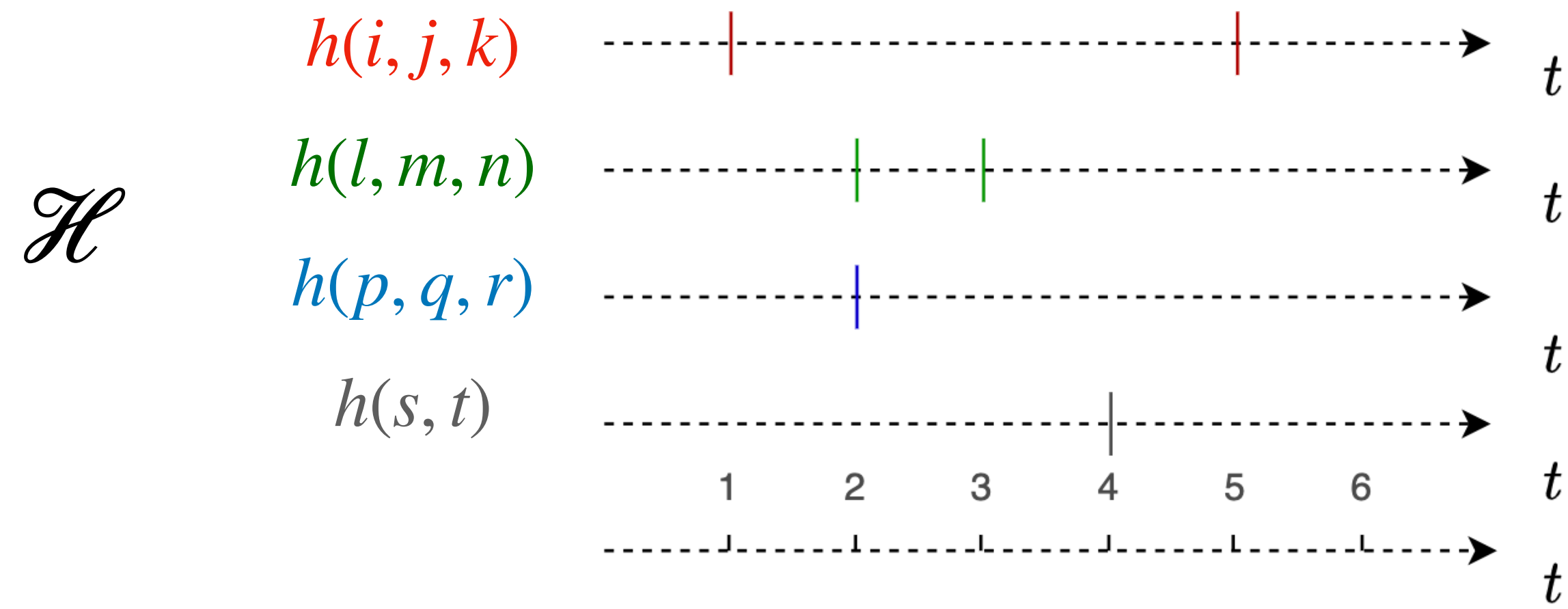
**SocioPatterns**

arXiv

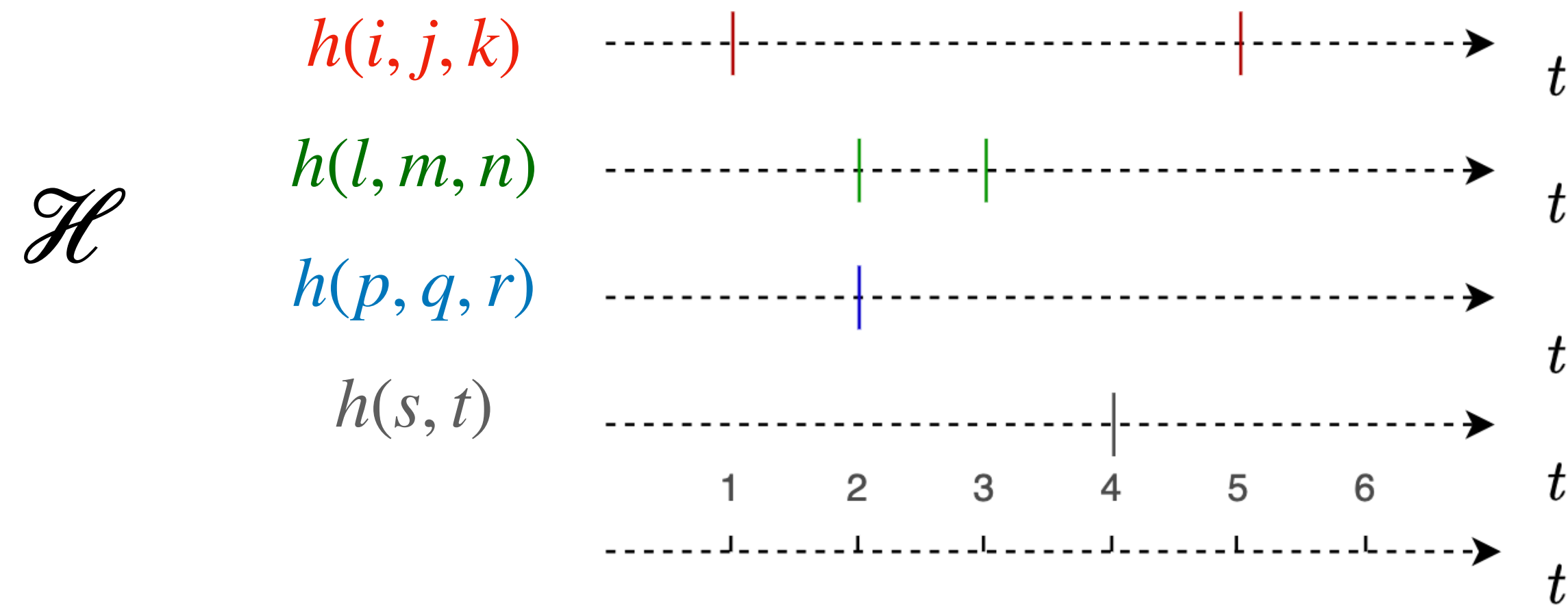


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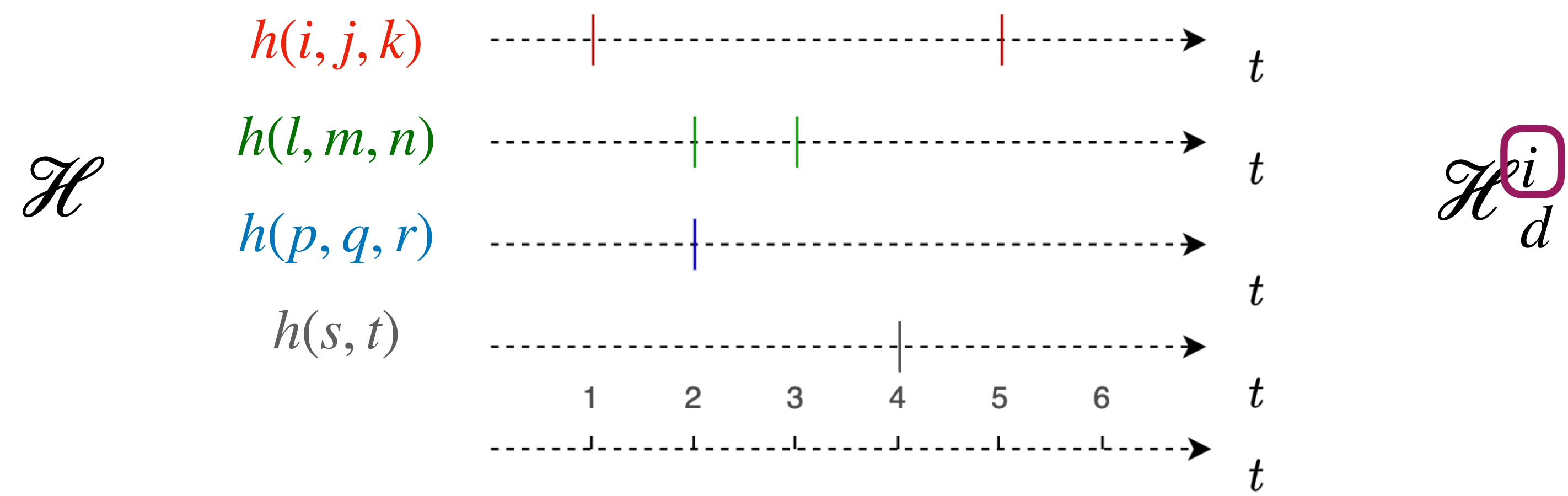


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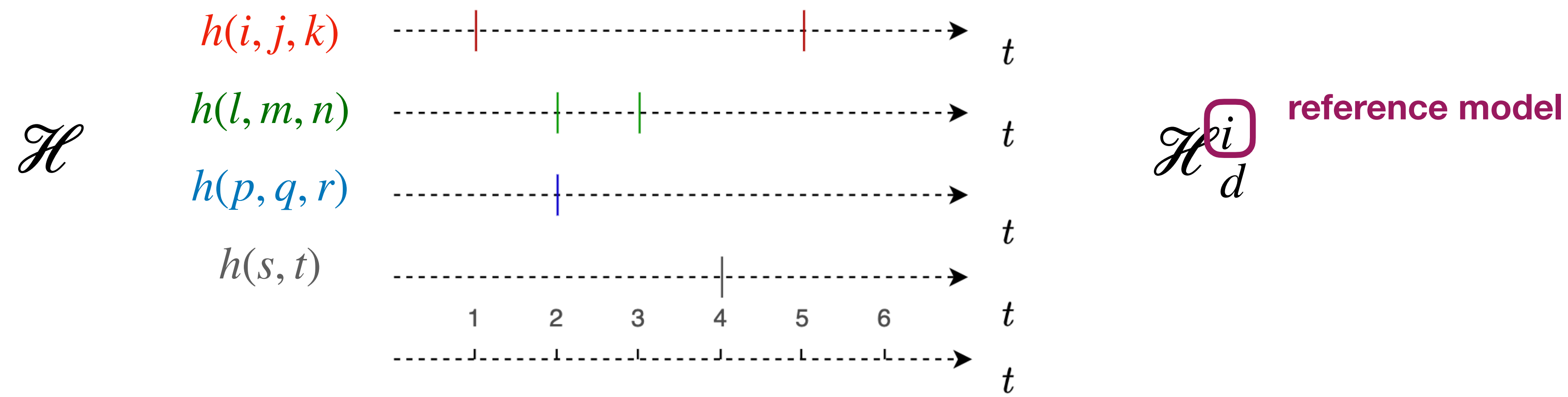


$\mathcal{H}_d^i$

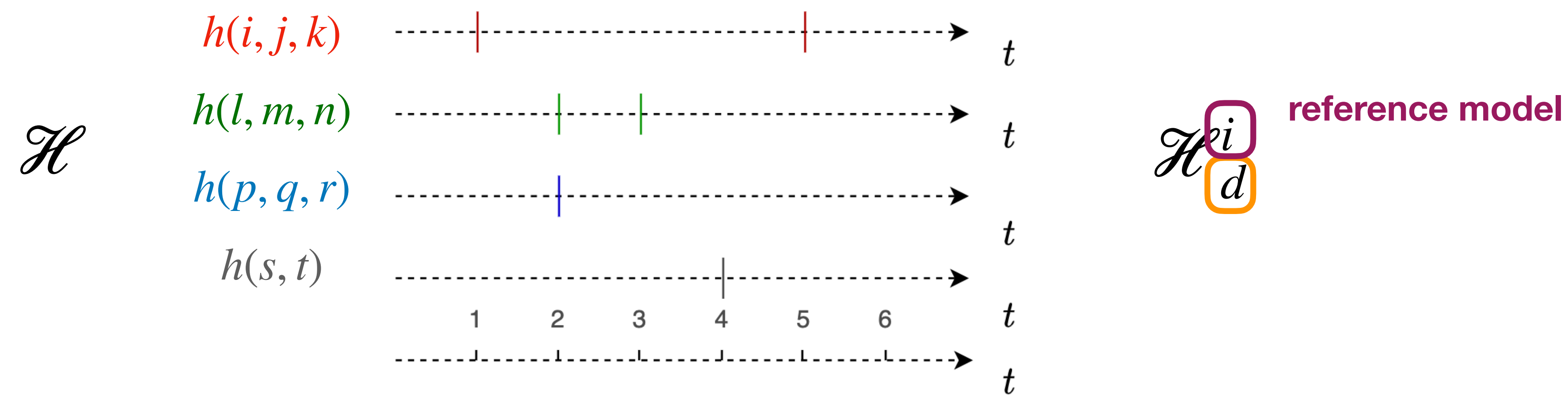
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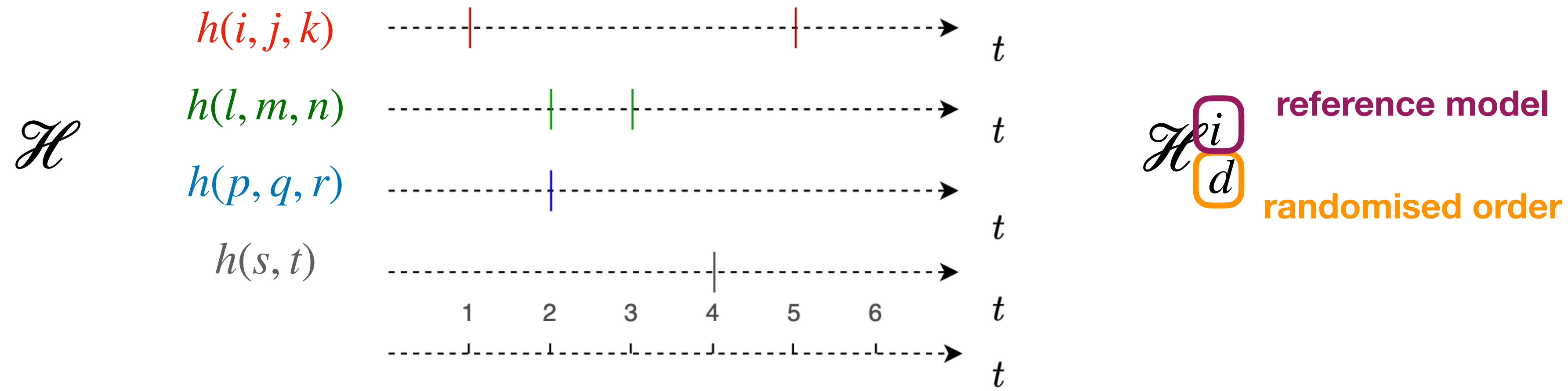
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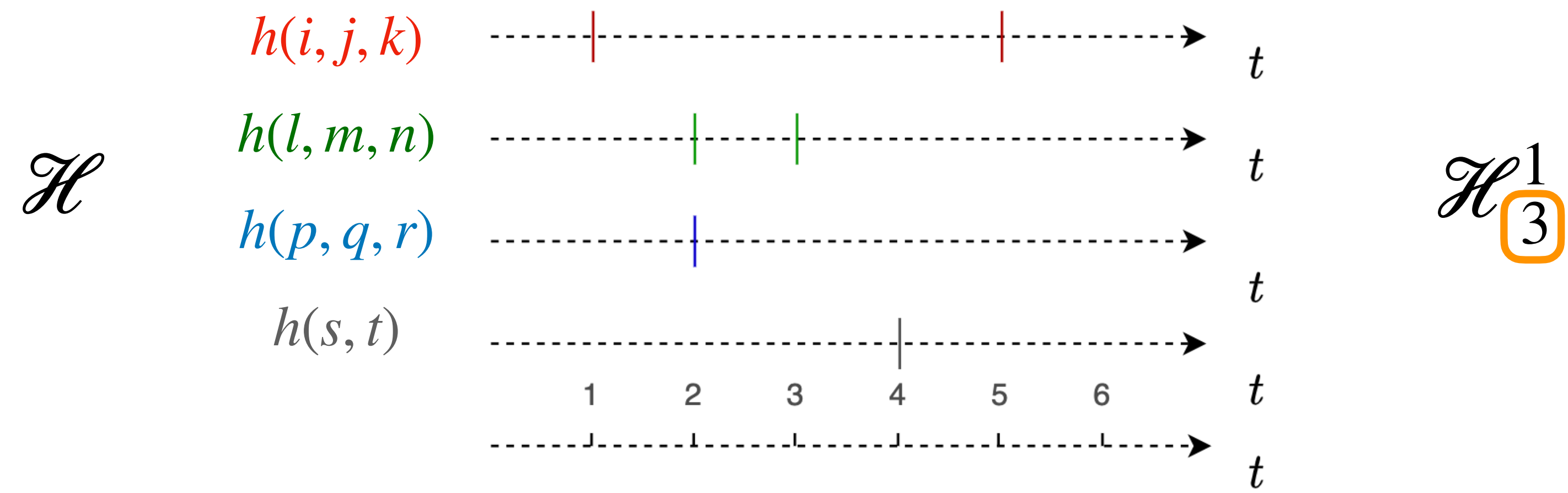


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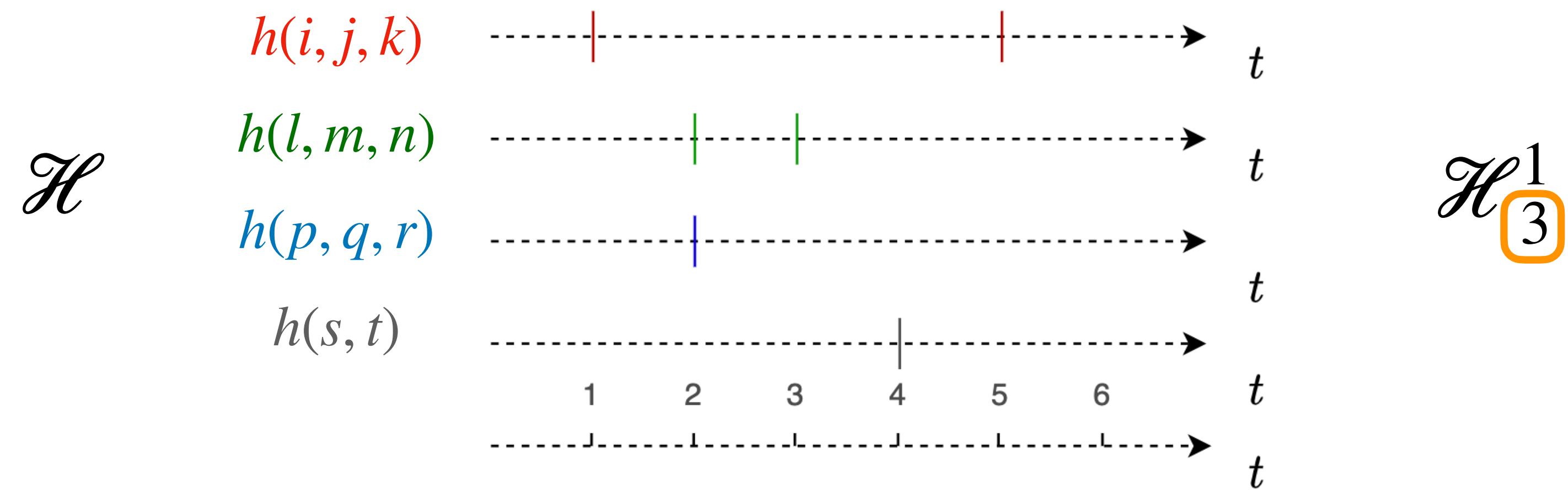
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Hyperlink	$\mathcal{H}$	$\mathcal{H}_3^1$
$h(i, j, k)$	1	2
$h(l, m, n)$	2	5
$h(p, q, r)$	2	3
$h(l, m, n)$	3	1
$h(s, t)$	4	4
$h(i, j, k)$	5	2

reshuffle  
timestamps of  
order 3 events

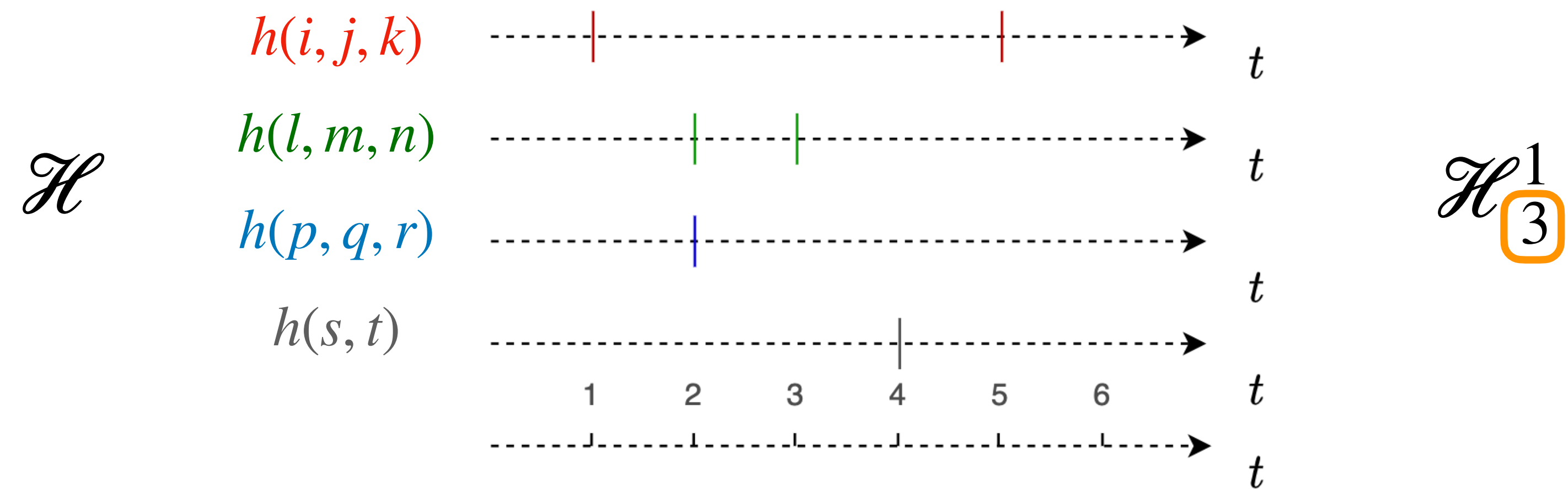
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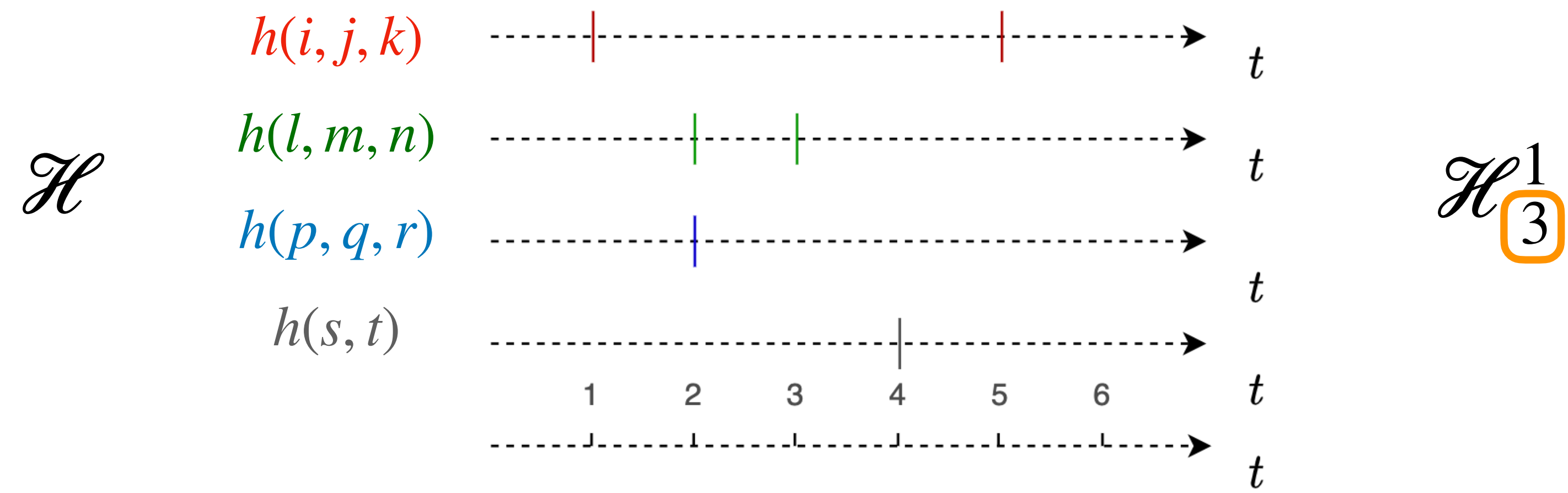


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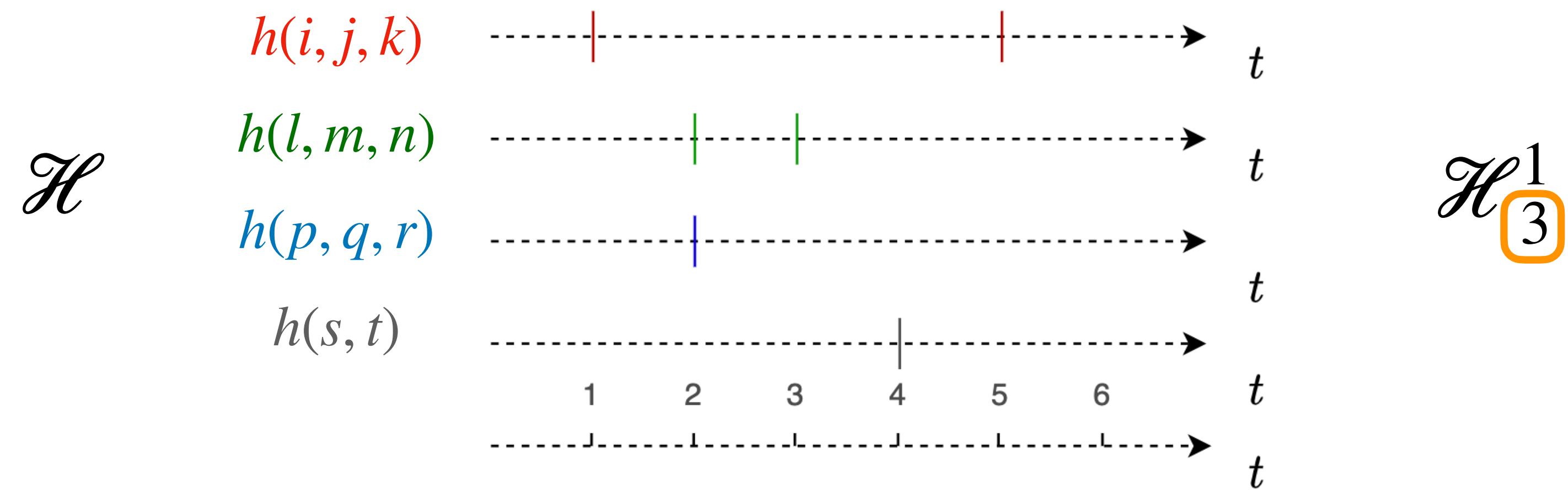
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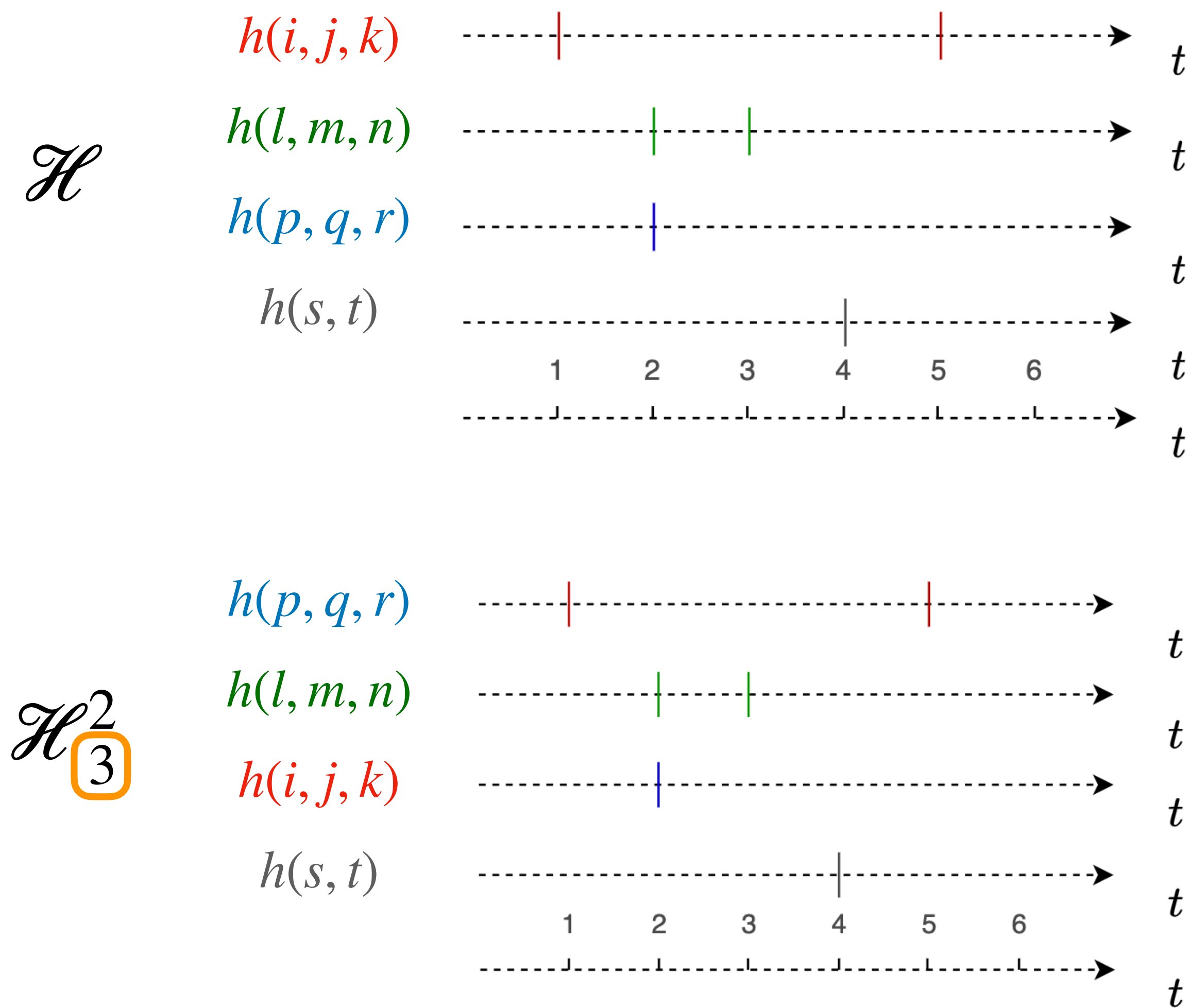
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$\mathcal{H}_3^1$

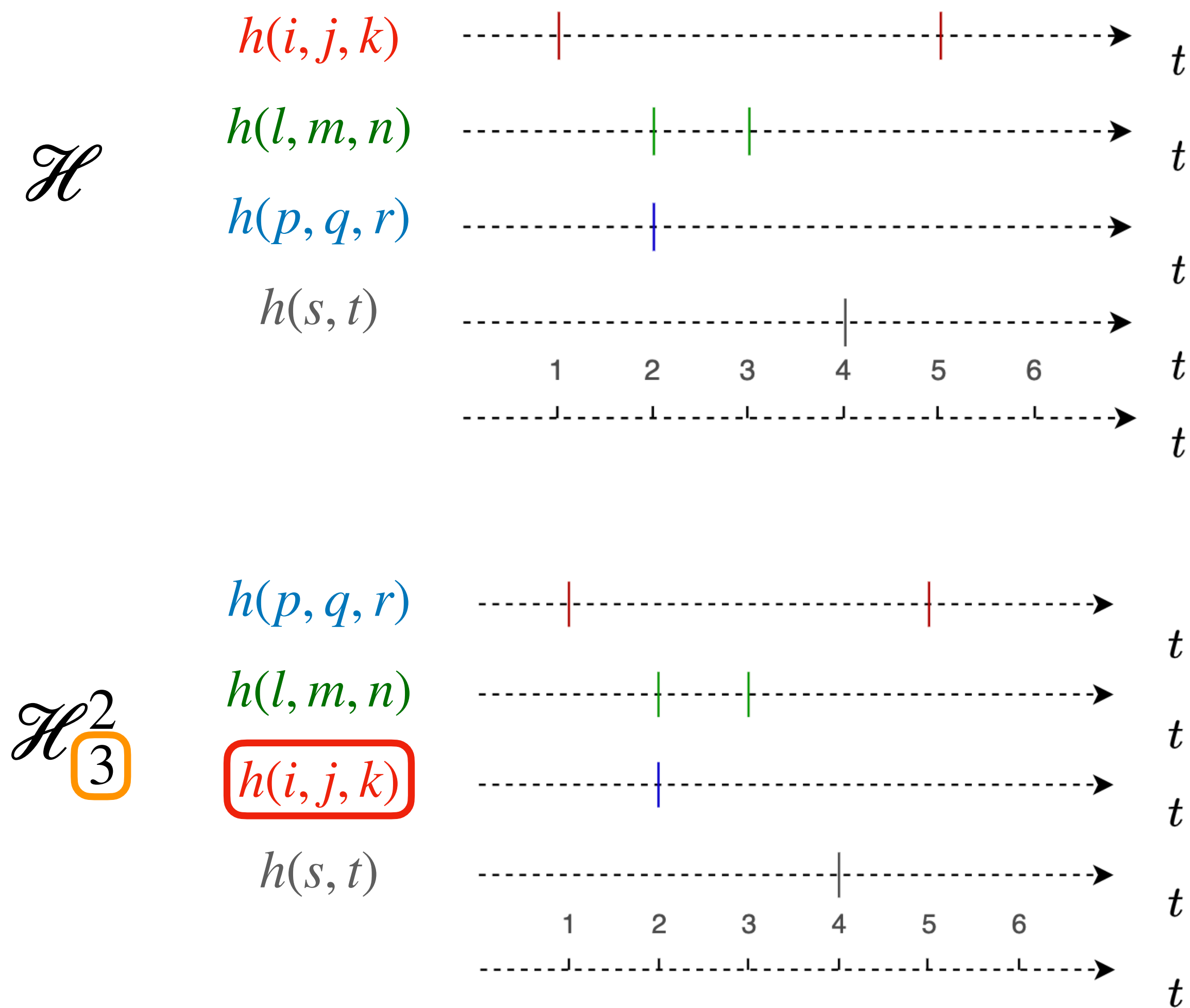
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swap time series of order 3 hyperlinks



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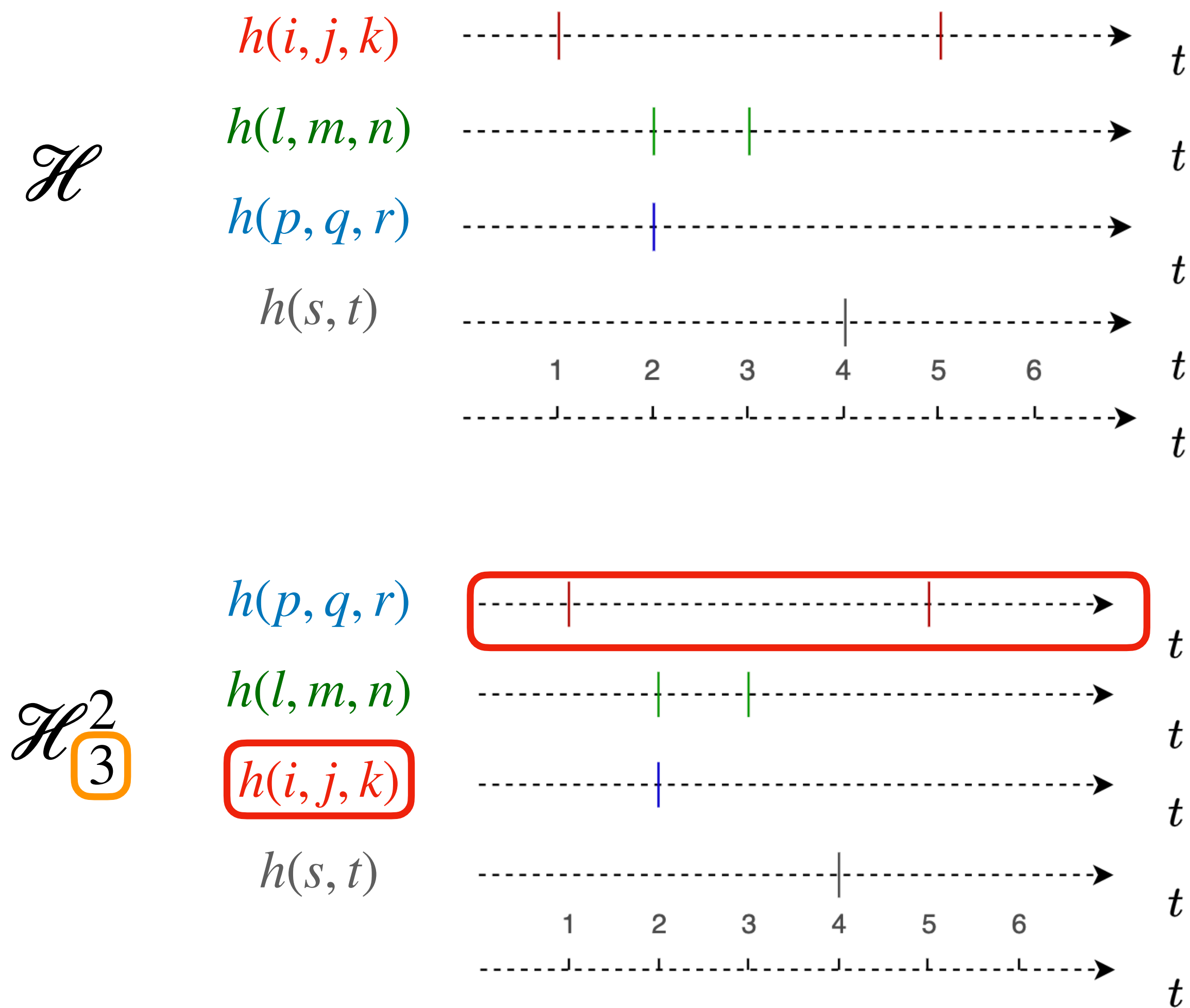
$\mathcal{H}_3^1$

Hyperlink	$\mathcal{H}$	$\mathcal{H}_3^1$
$h(i, j, k)$	1	2
$h(l, m, n)$	2	5
$h(p, q, r)$	2	3
$h(l, m, n)$	3	1
$h(s, t)$	4	4
$h(i, j, k)$	5	2

reshuffle  
timestamps of  
order 3 events

swap time series of order 3 hyperlinks

# Randomised reference models



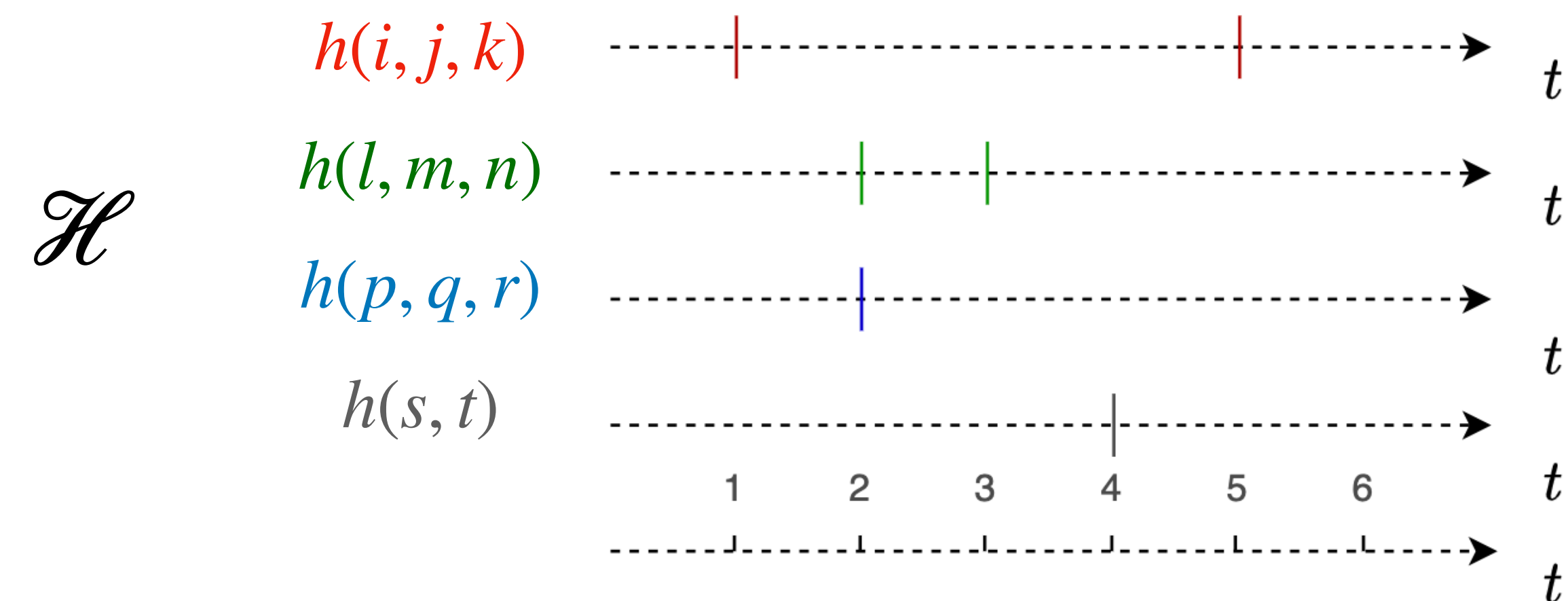
$\mathcal{H}_3^1$

Hyperlink	$\mathcal{H}$	$\mathcal{H}_3^1$
$h(i, j, k)$	1	2
$h(l, m, n)$	2	5
$h(p, q, r)$	2	3
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$h(s, t)$	4	4
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reshuffle  
timestamps of  
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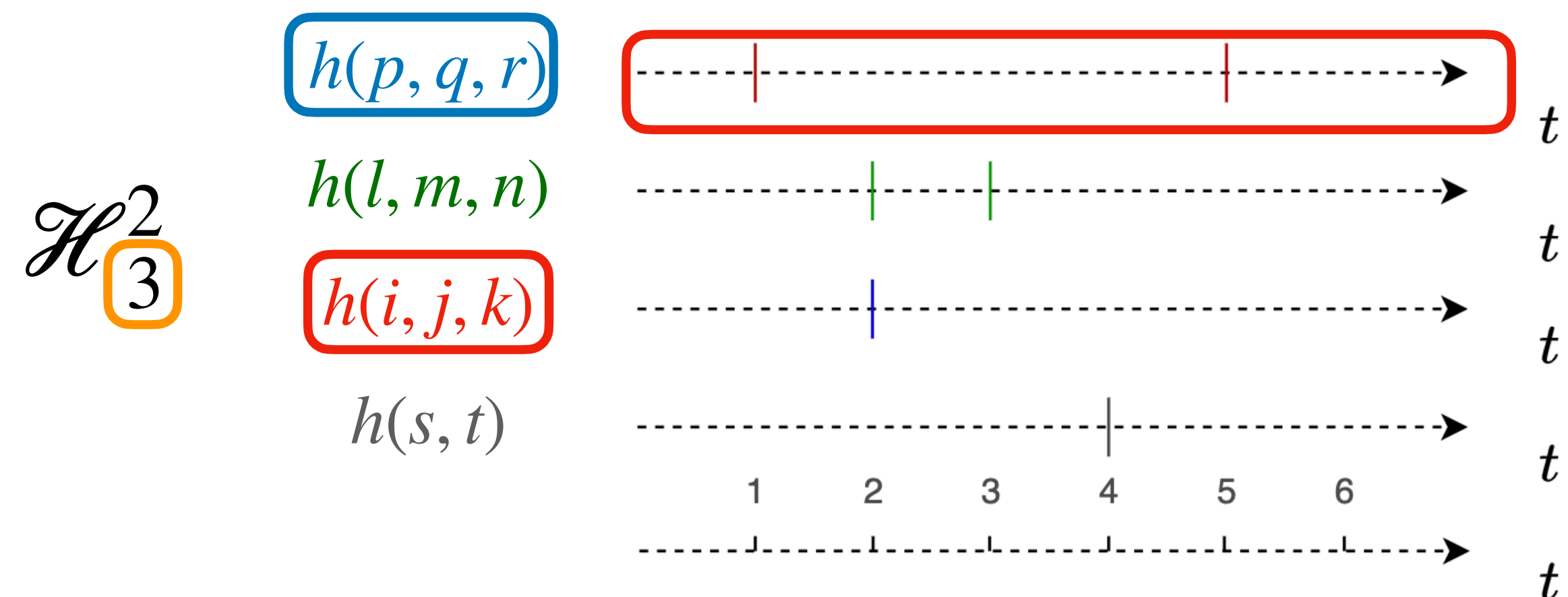
# Randomised reference models



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Hyperlink	$\mathcal{H}$	$\mathcal{H}_3^1$
$h(i, j, k)$	1	2
$h(l, m, n)$	2	5
$h(p, q, r)$	2	3
$h(l, m, n)$	3	1
$h(s, t)$	4	4
$h(i, j, k)$	5	2

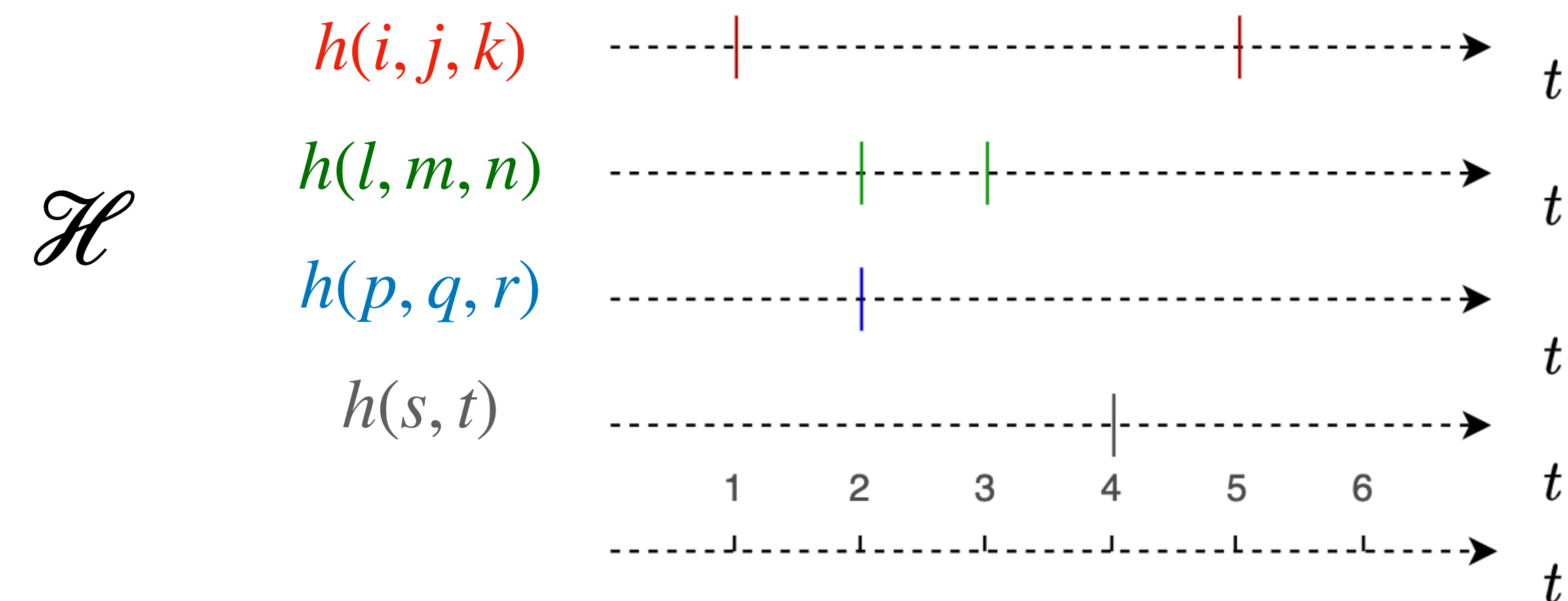
reshuffle  
timestamps of  
order 3 events



swap time series of order 3 hyperlinks



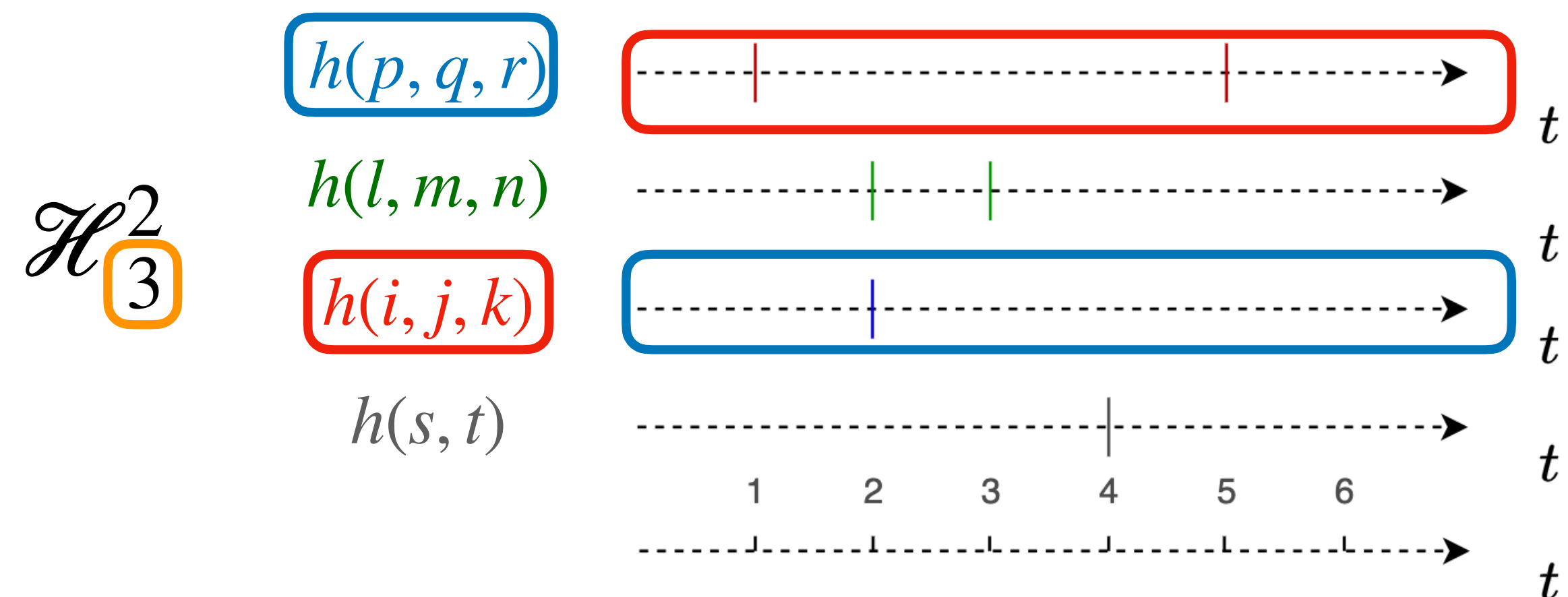
# Randomised reference models



$\mathcal{H}_3^1$

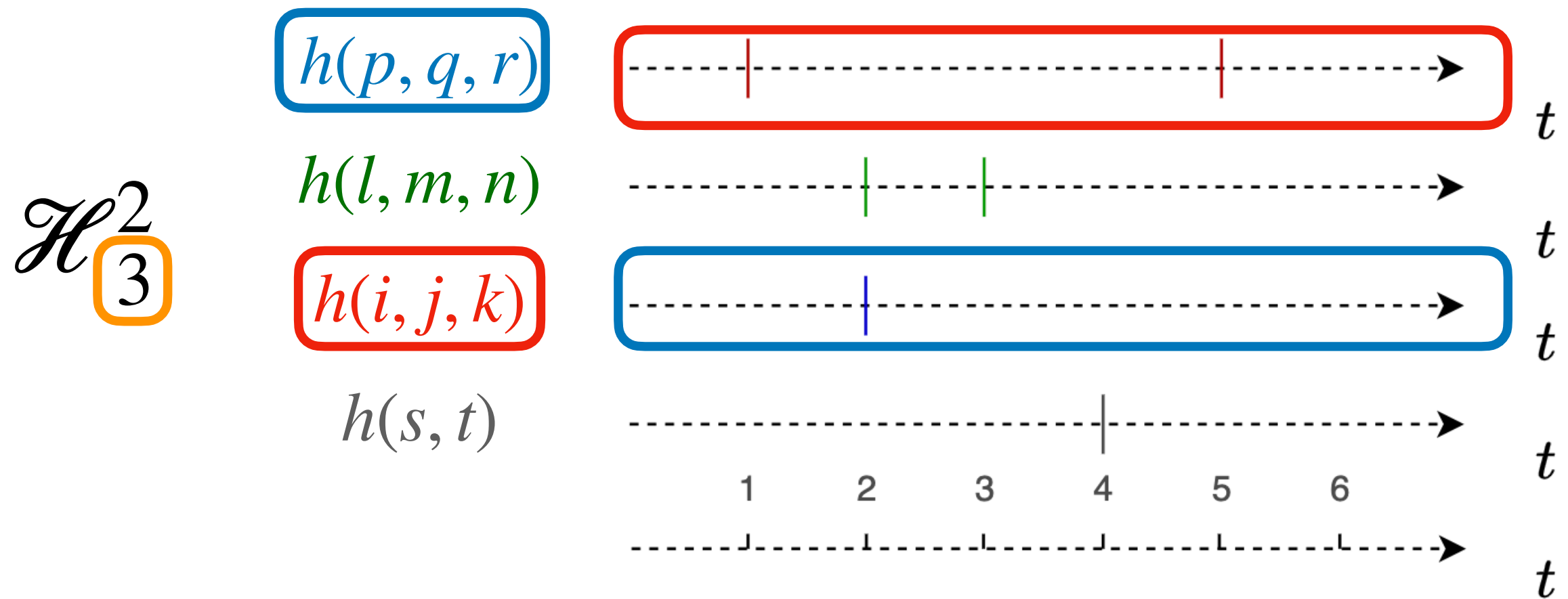
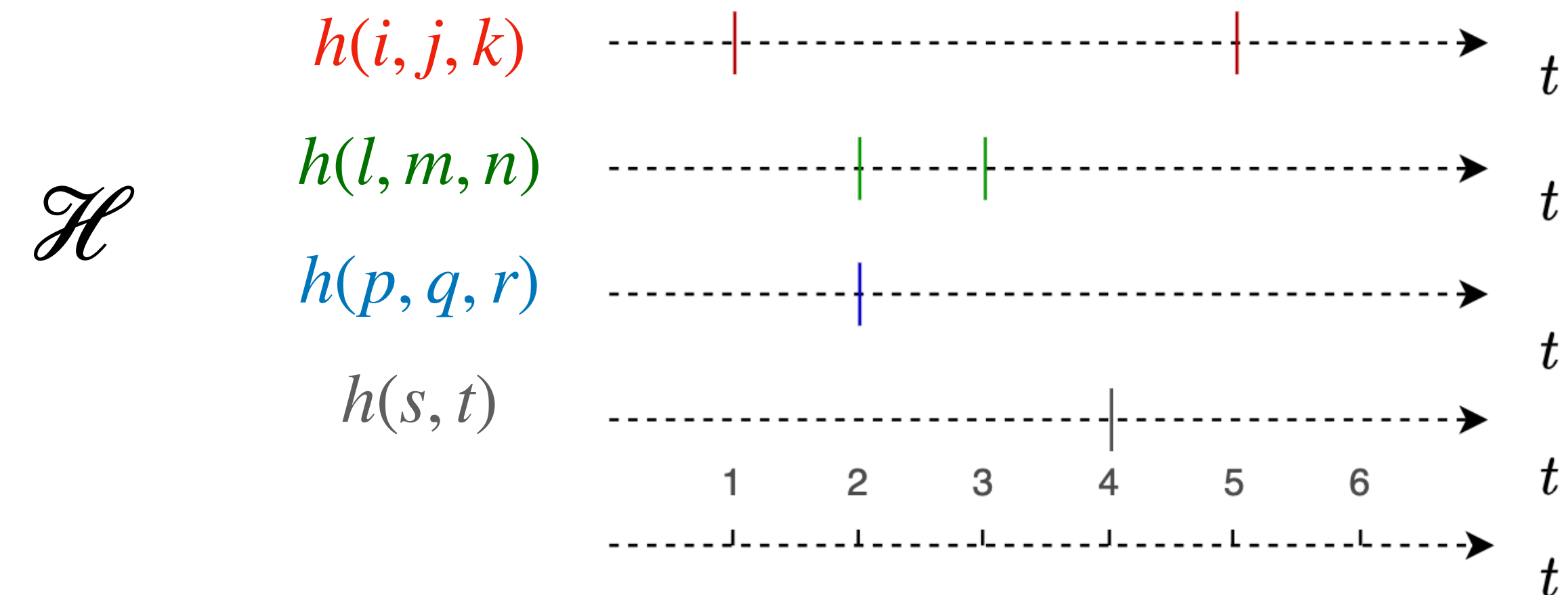
Hyperlink	$\mathcal{H}$	$\mathcal{H}_3^1$
$h(i, j, k)$	1	2
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reshuffle  
timestamps of  
order 3 events



swap time series of order 3 hyperlinks

# Randomised reference models



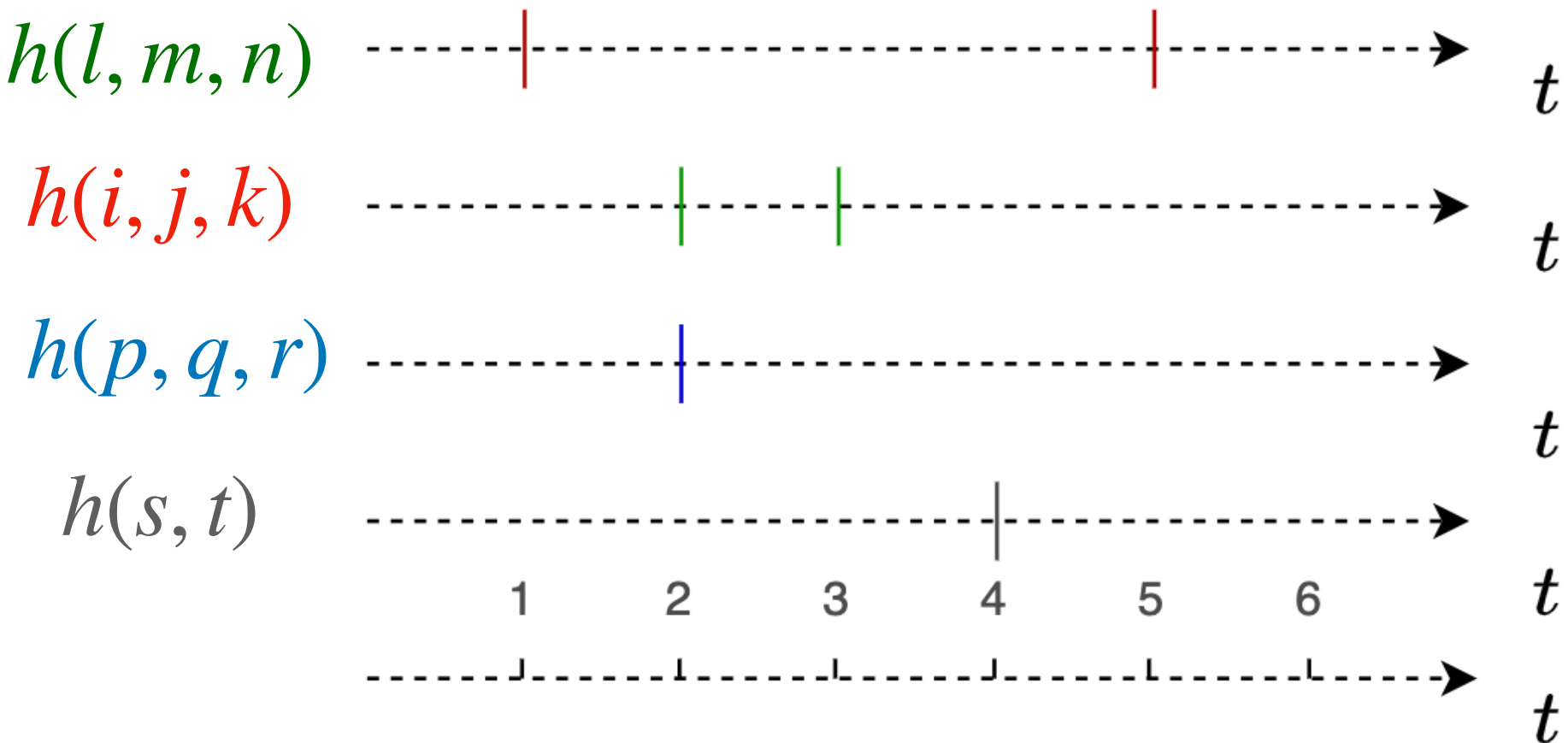
swap time series of order 3 hyperlinks

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Hyperlink	$\mathcal{H}$	$\mathcal{H}_3^1$
$h(i, j, k)$	1	2
$h(l, m, n)$	2	5
$h(p, q, r)$	2	3
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$h(s, t)$	4	4
$h(i, j, k)$	5	2

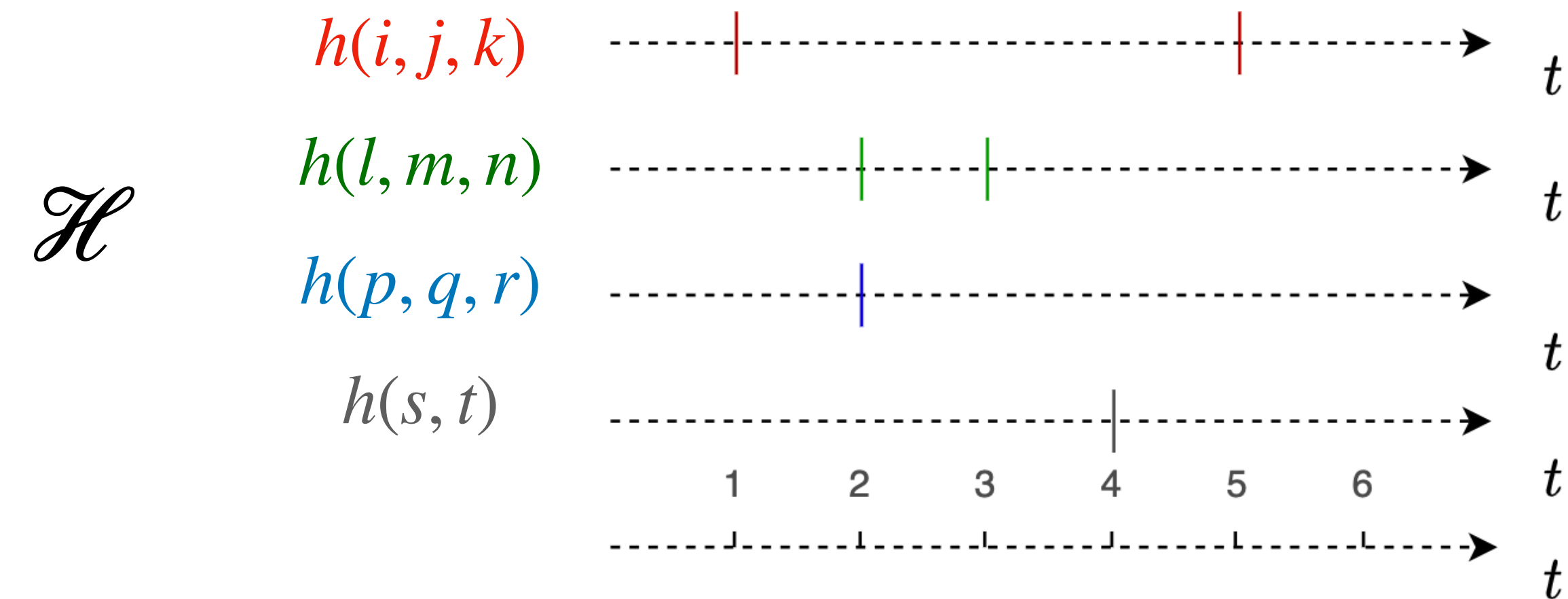
reshuffle  
timestamps of  
order 3 events

$\mathcal{H}_3^3$



swap time series of order 3 hyperlinks  
with same number of activations

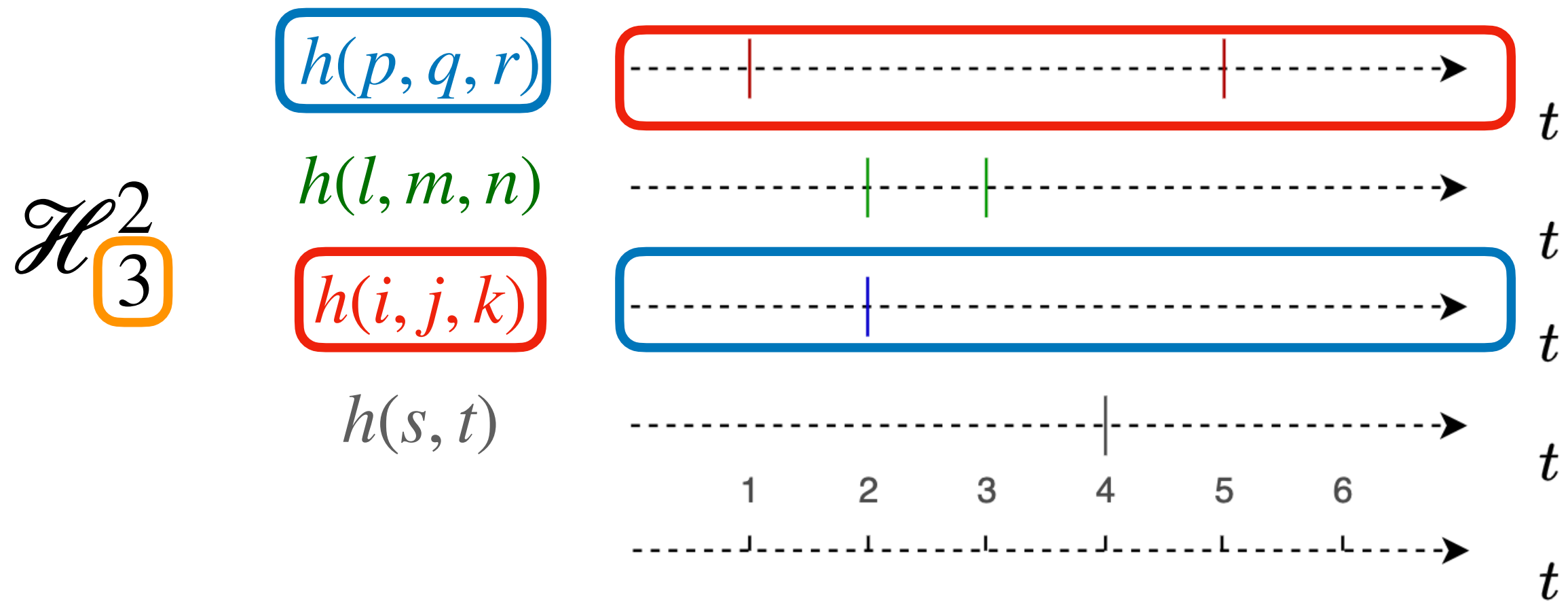
# Randomised reference models



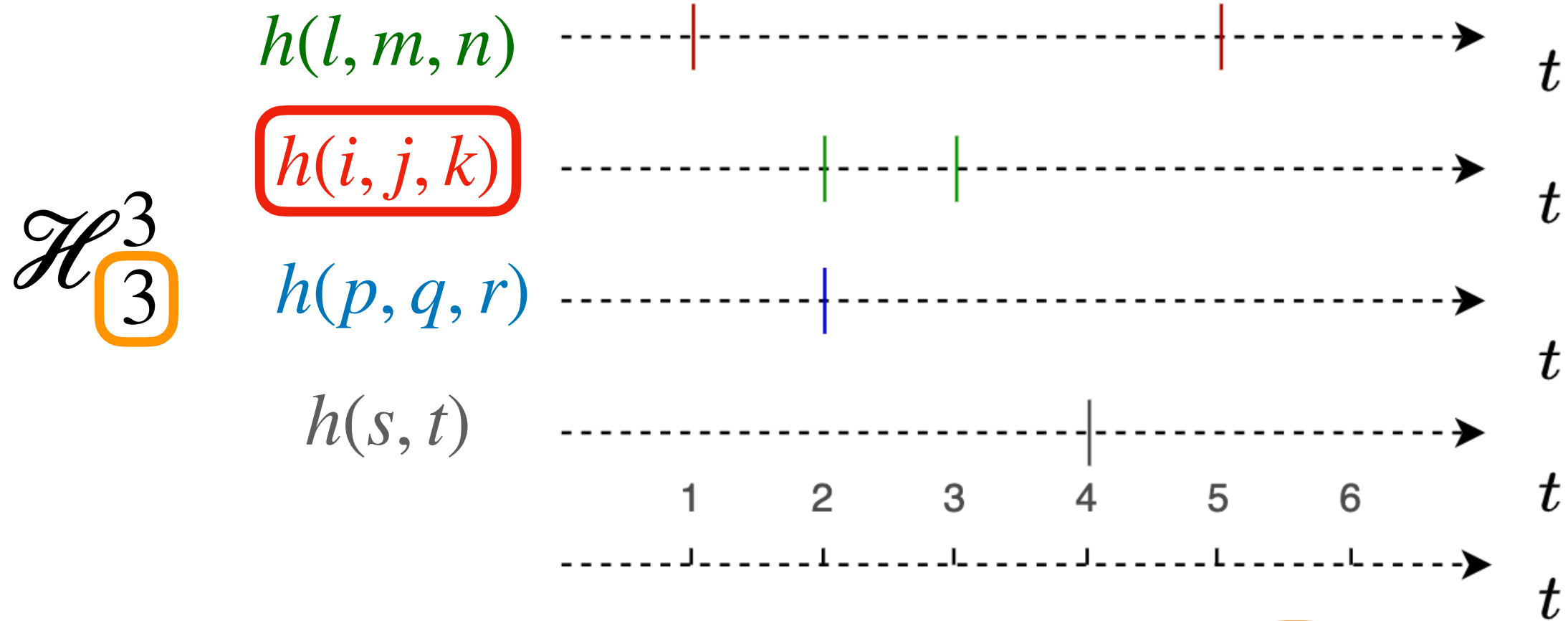
$\mathcal{H}_3^1$

Hyperlink	$\mathcal{H}$	$\mathcal{H}_3^1$
$h(i, j, k)$	1	2
$h(l, m, n)$	2	5
$h(p, q, r)$	2	3
$h(l, m, n)$	3	1
$h(s, t)$	4	4
$h(i, j, k)$	5	2

reshuffle  
timestamps of  
order 3 events



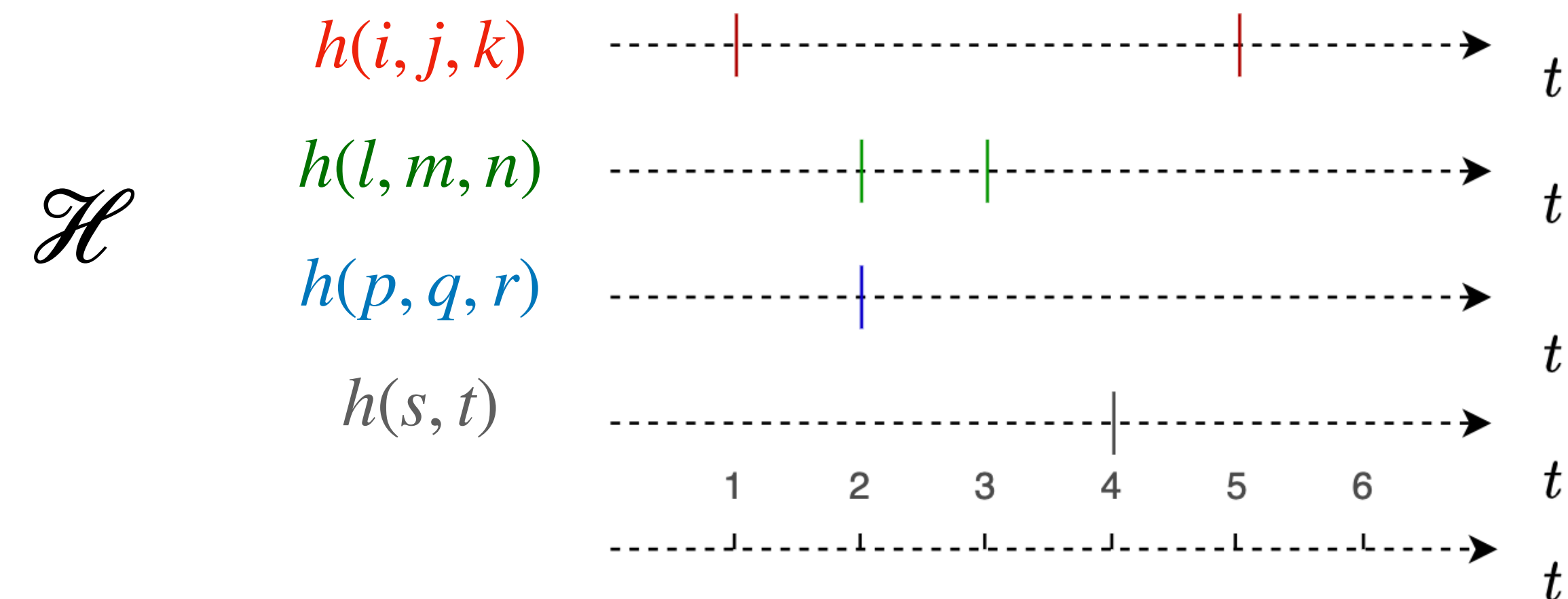
swap time series of order 3 hyperlinks



swap time series of order 3 hyperlinks  
with same number of activations



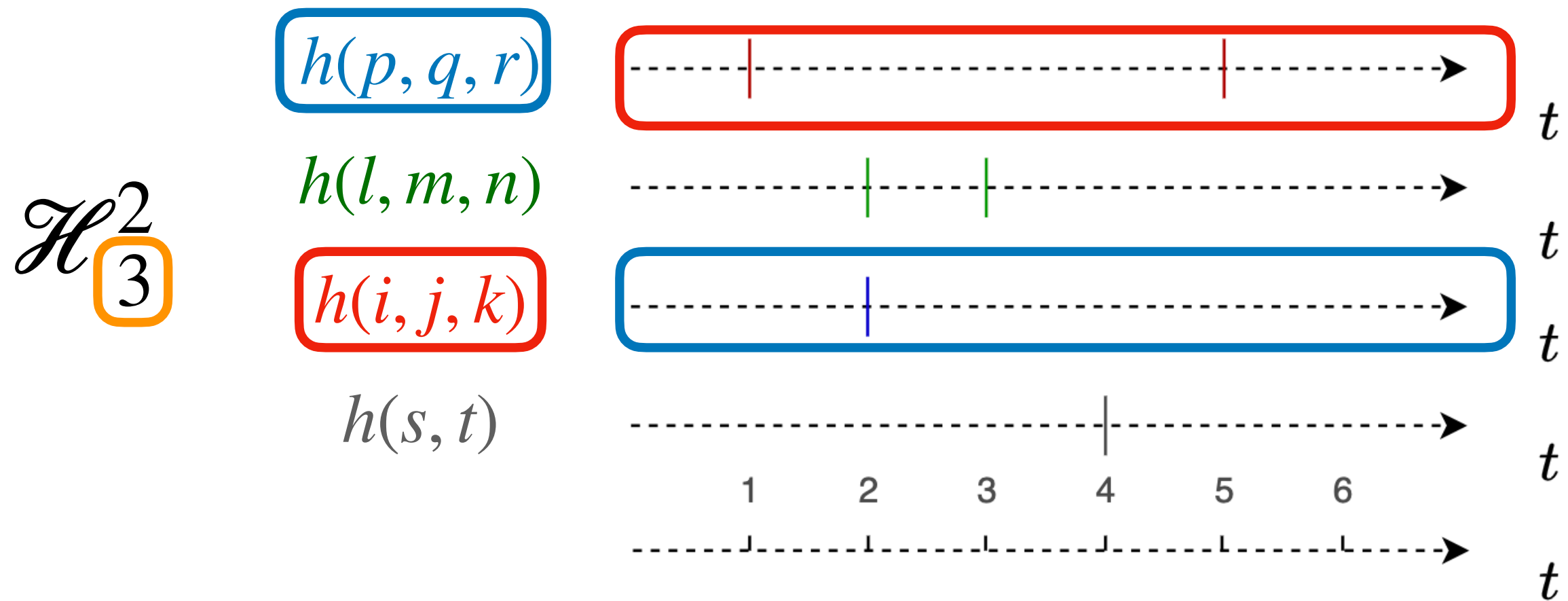
# Randomised reference models



$\mathcal{H}_3^1$

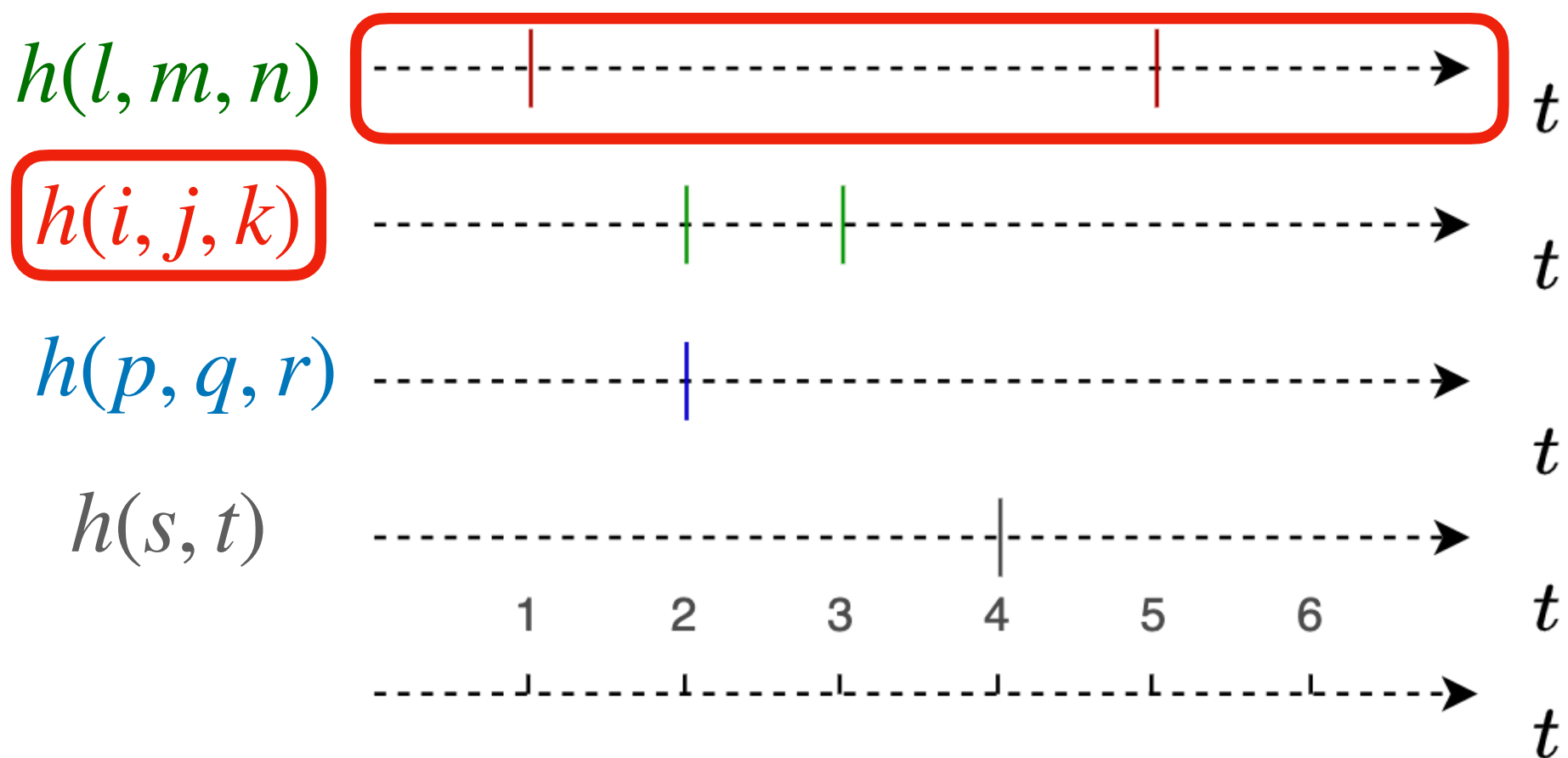
Hyperlink	$\mathcal{H}$	$\mathcal{H}_3^1$
$h(i, j, k)$	1	2
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reshuffle  
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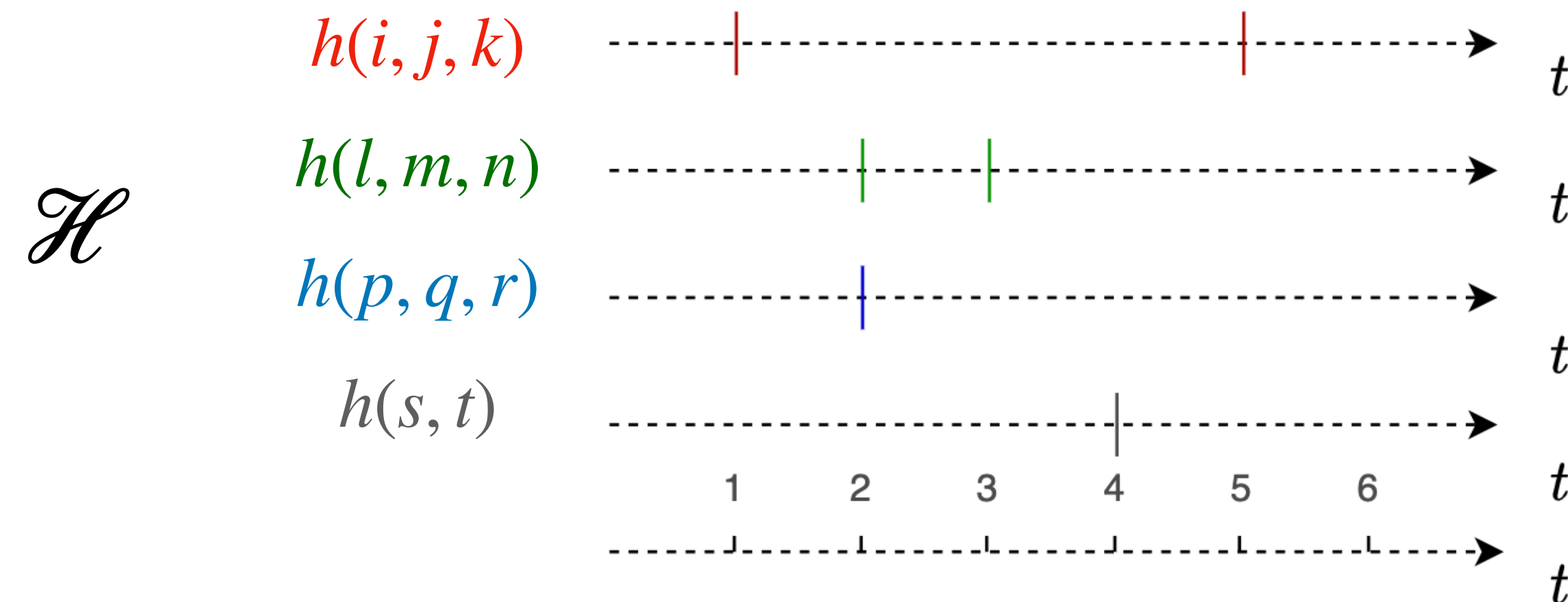
swap time series of order 3 hyperlinks

$\mathcal{H}_3^3$



swap time series of order 3 hyperlinks  
with same number of activations

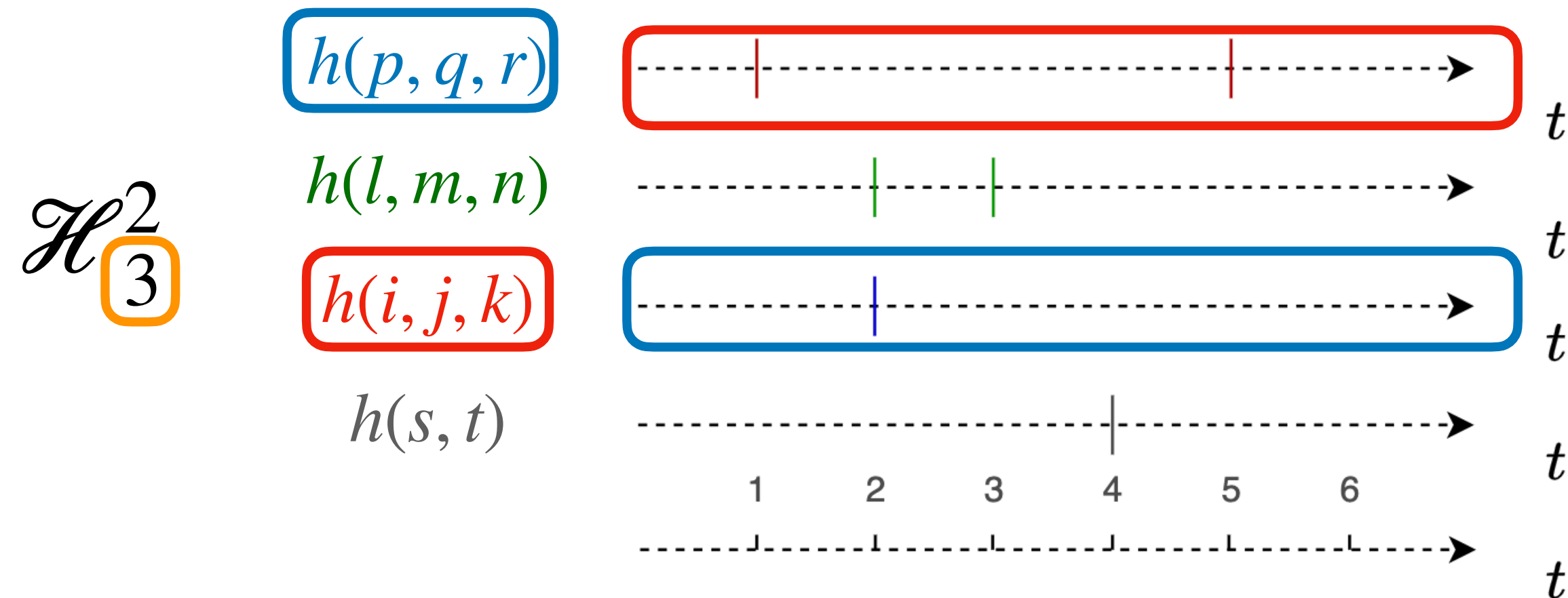
# Randomised reference models



$\mathcal{H}_3^1$

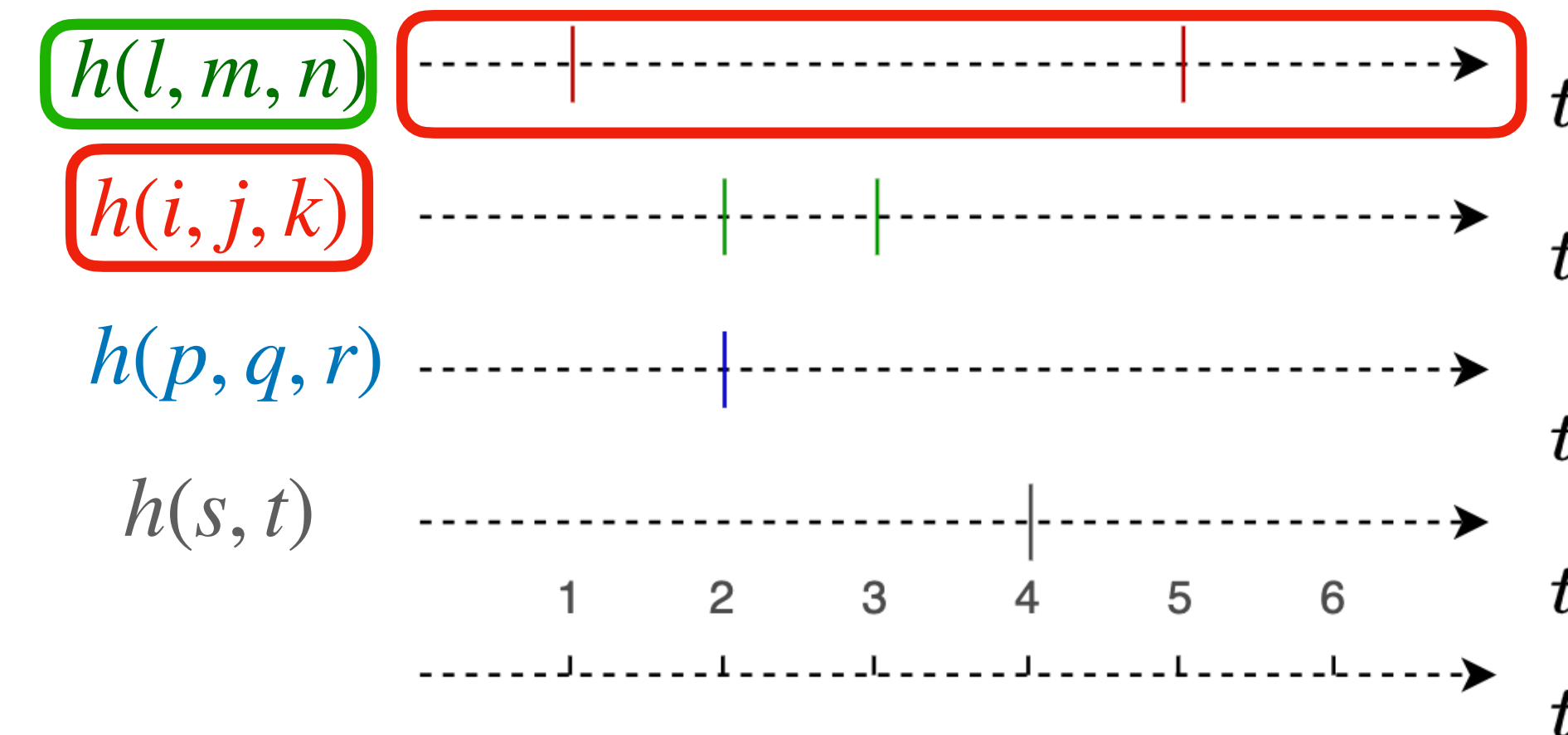
Hyperlink	$\mathcal{H}$	$\mathcal{H}_3^1$
$h(i, j, k)$	1	2
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$h(s, t)$	4	4
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reshuffle  
timestamps of  
order 3 events



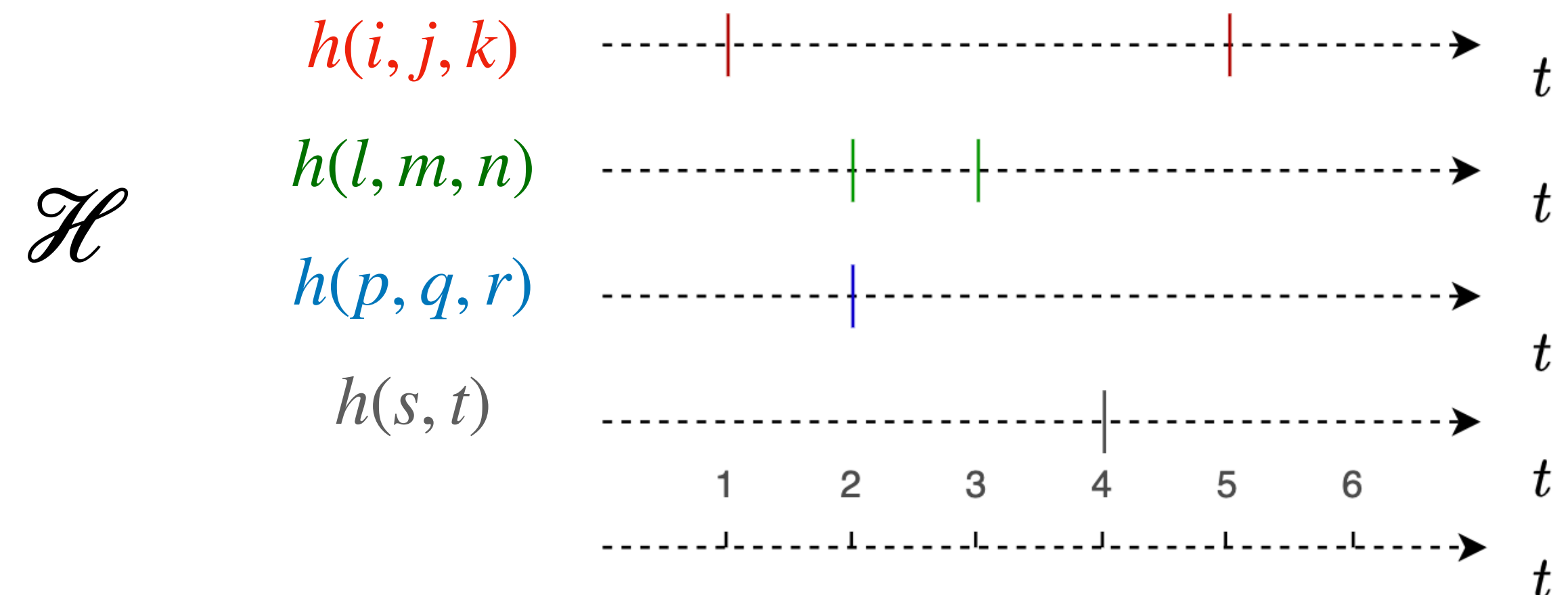
swap time series of order 3 hyperlinks

$\mathcal{H}_3^3$



swap time series of order 3 hyperlinks  
with same number of activations

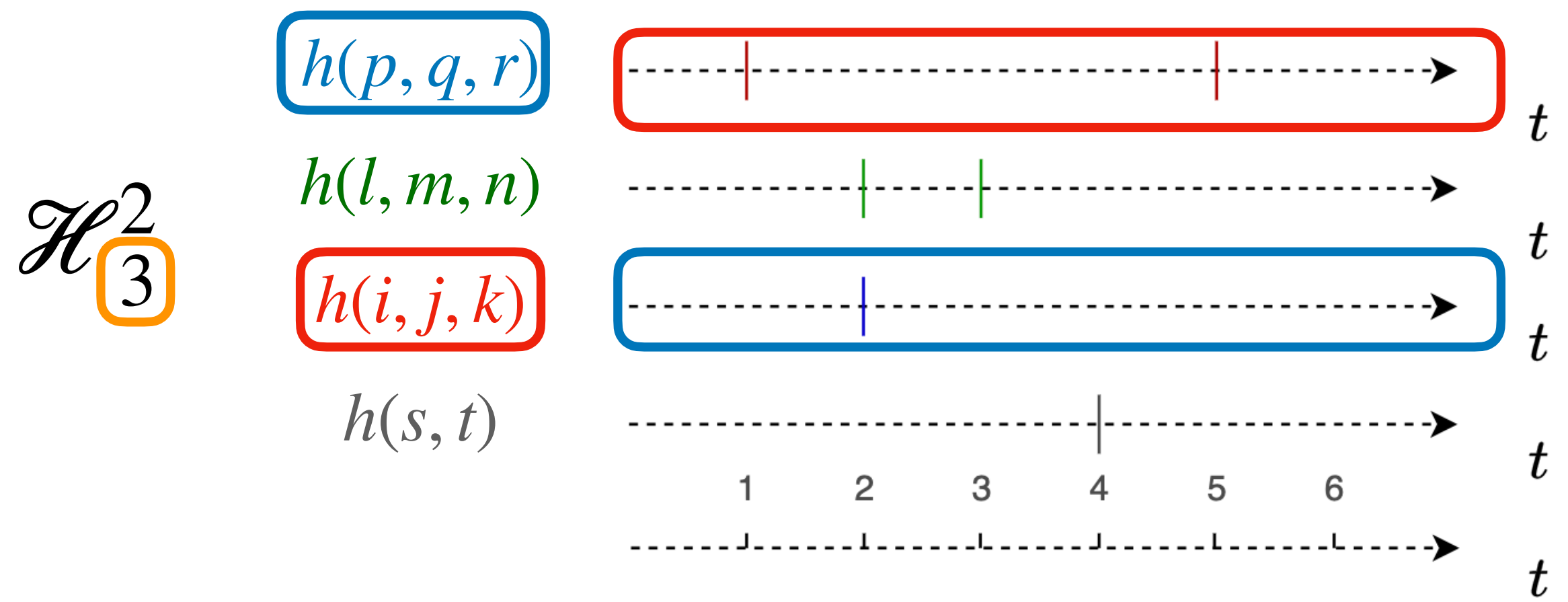
# Randomised reference models



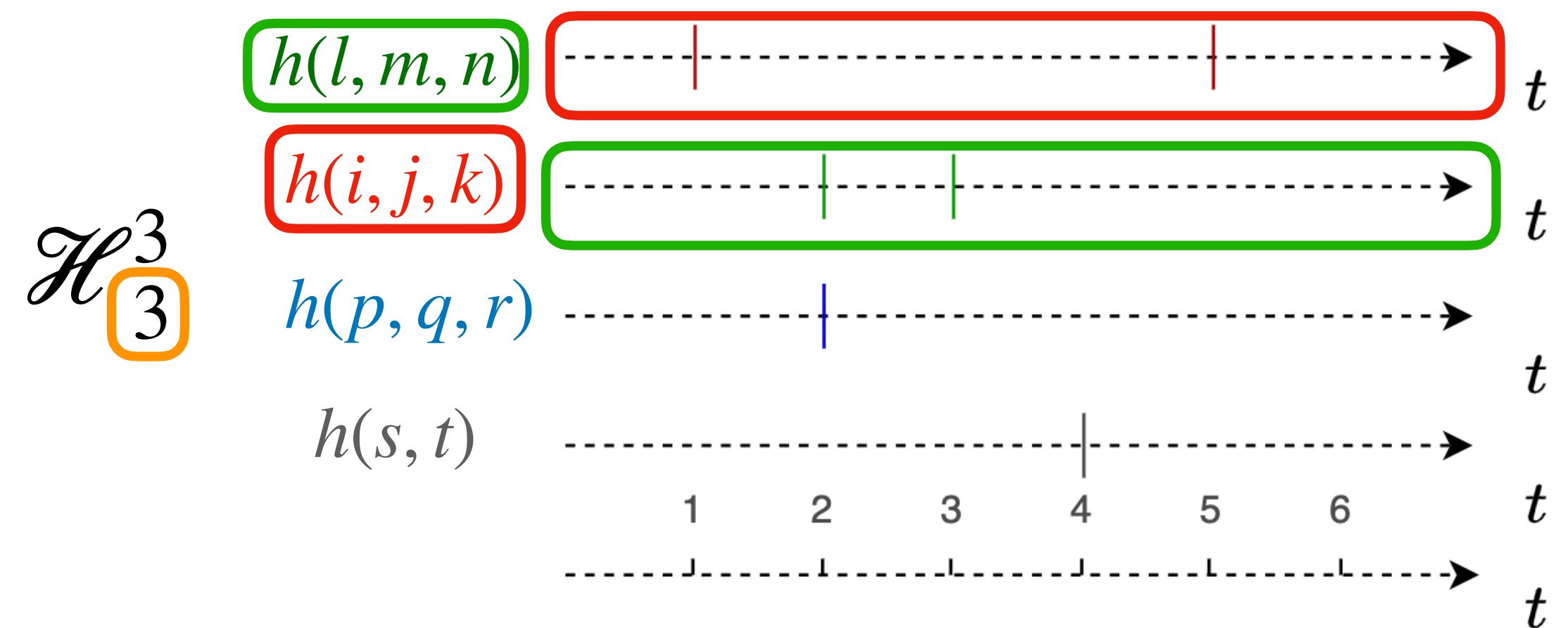
$\mathcal{H}_3^1$

Hyperlink	$\mathcal{H}$	$\mathcal{H}_3^1$
$h(i, j, k)$	1	2
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$h(i, j, k)$	5	2

reshuffle  
timestamps of  
order 3 events



swap time series of order 3 hyperlinks



swap time series of order 3 hyperlinks  
with same number of activations



# Topological and temporal distance of events

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$$\eta( ( h(u_1, \dots, u_d), t_1 ) , ( h(v_1, \dots, v_{d'}), t_2 ) ) =$$

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$$\eta( (h(u_1, \dots, u_d), t_1), (h(v_1, \dots, v_{d'}), t_2) ) =$$
$$= \begin{cases} \min_{u \in \{u_1, \dots, u_d\}, v \in \{v_1, \dots, v_{d'}\}} (\delta(u, v) + 1) & h(u_1, \dots, u_d) \neq h(v_1, \dots, v_{d'}) \\ 0 & h(u_1, \dots, u_d) = h(v_1, \dots, v_{d'}) \end{cases}$$

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- **Temporal** distance of two events:

$$\mathcal{T}( (h(u_1, \dots, u_d), t_1), (h(v_1, \dots, v_{d'}), t_2) ) = |t_1 - t_2|$$

# Topological and temporal distance of events

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$$\eta( (h(u_1, \dots, u_d), t_1), (h(v_1, \dots, v_{d'}), t_2) ) =$$

$$= \begin{cases} \min_{u \in \{u_1, \dots, u_d\}, v \in \{v_1, \dots, v_{d'}\}} (\delta(u, v) + 1) & h(u_1, \dots, u_d) \neq h(v_1, \dots, v_{d'}) \\ 0 & h(u_1, \dots, u_d) = h(v_1, \dots, v_{d'}) \end{cases}$$

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$$\eta( (h(u_1, \dots, u_d), t_1), (h(v_1, \dots, v_{d'}), t_2) ) =$$

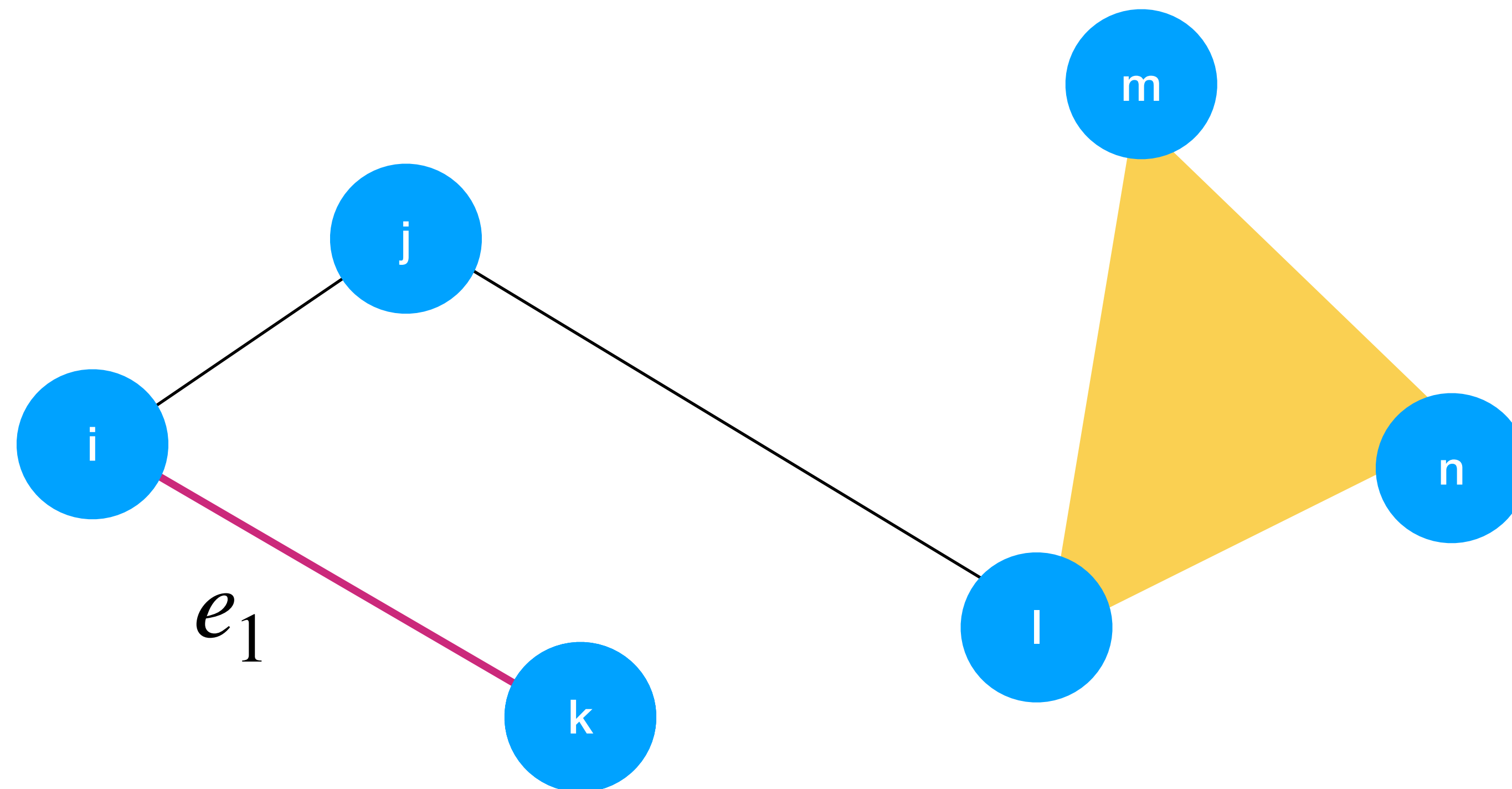
$$= \begin{cases} \min_{u \in \{u_1, \dots, u_d\}, v \in \{v_1, \dots, v_{d'}\}} (\delta(u, v) + 1) & h(u_1, \dots, u_d) \neq h(v_1, \dots, v_{d'}) \\ 0 & h(u_1, \dots, u_d) = h(v_1, \dots, v_{d'}) \end{cases}$$

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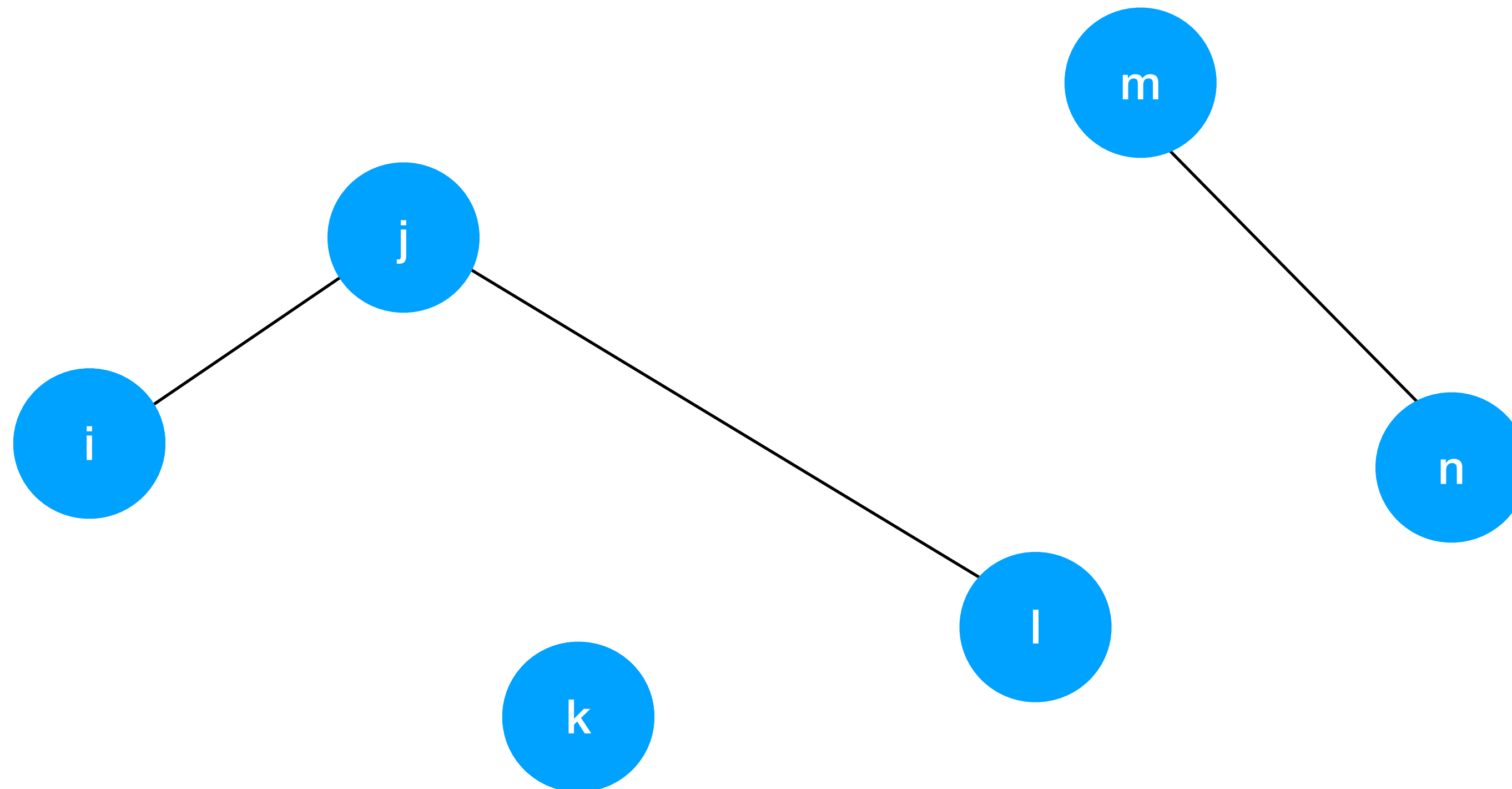
# Example

h 18:00



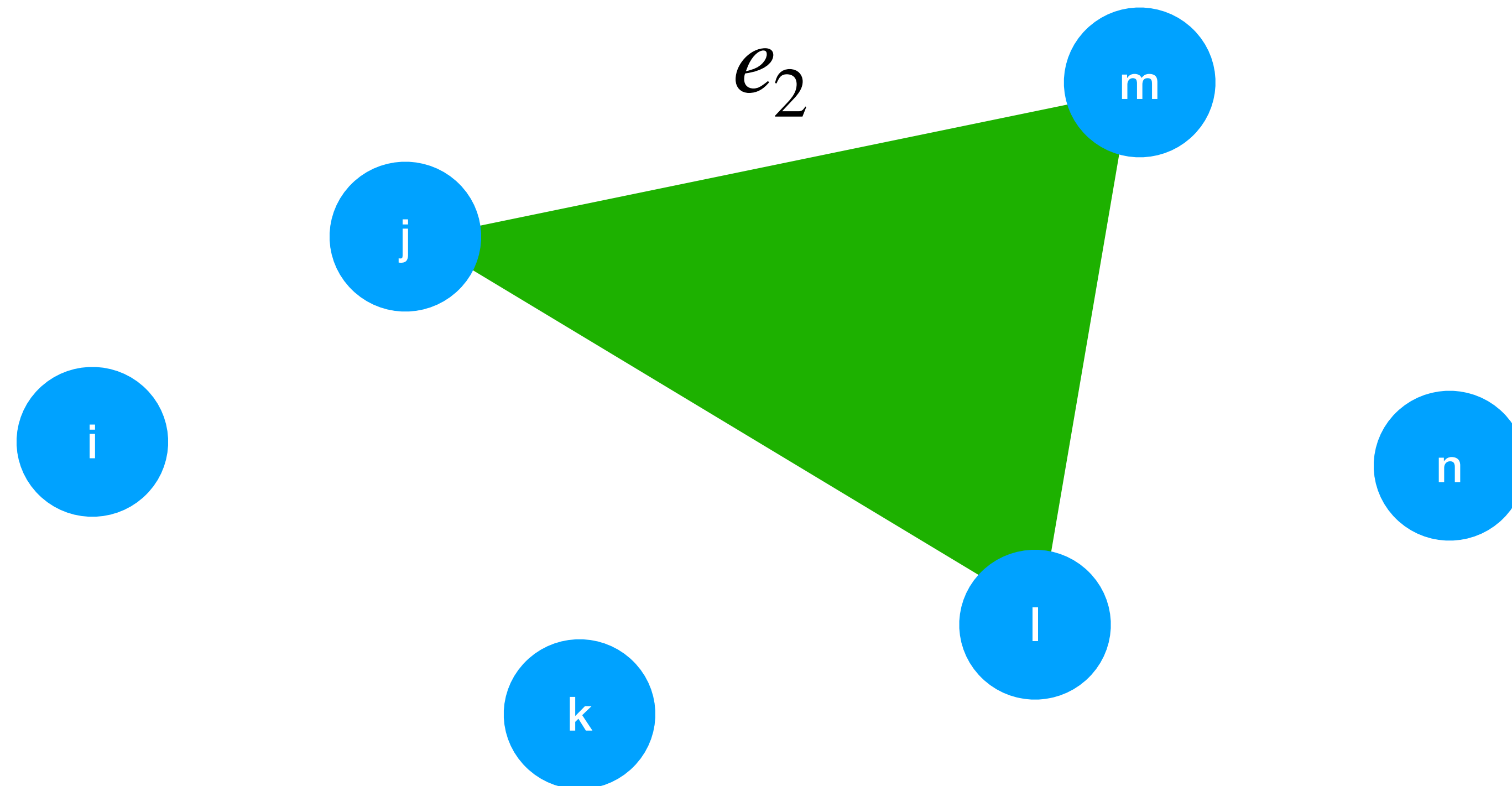
# Example

**h 18:30**



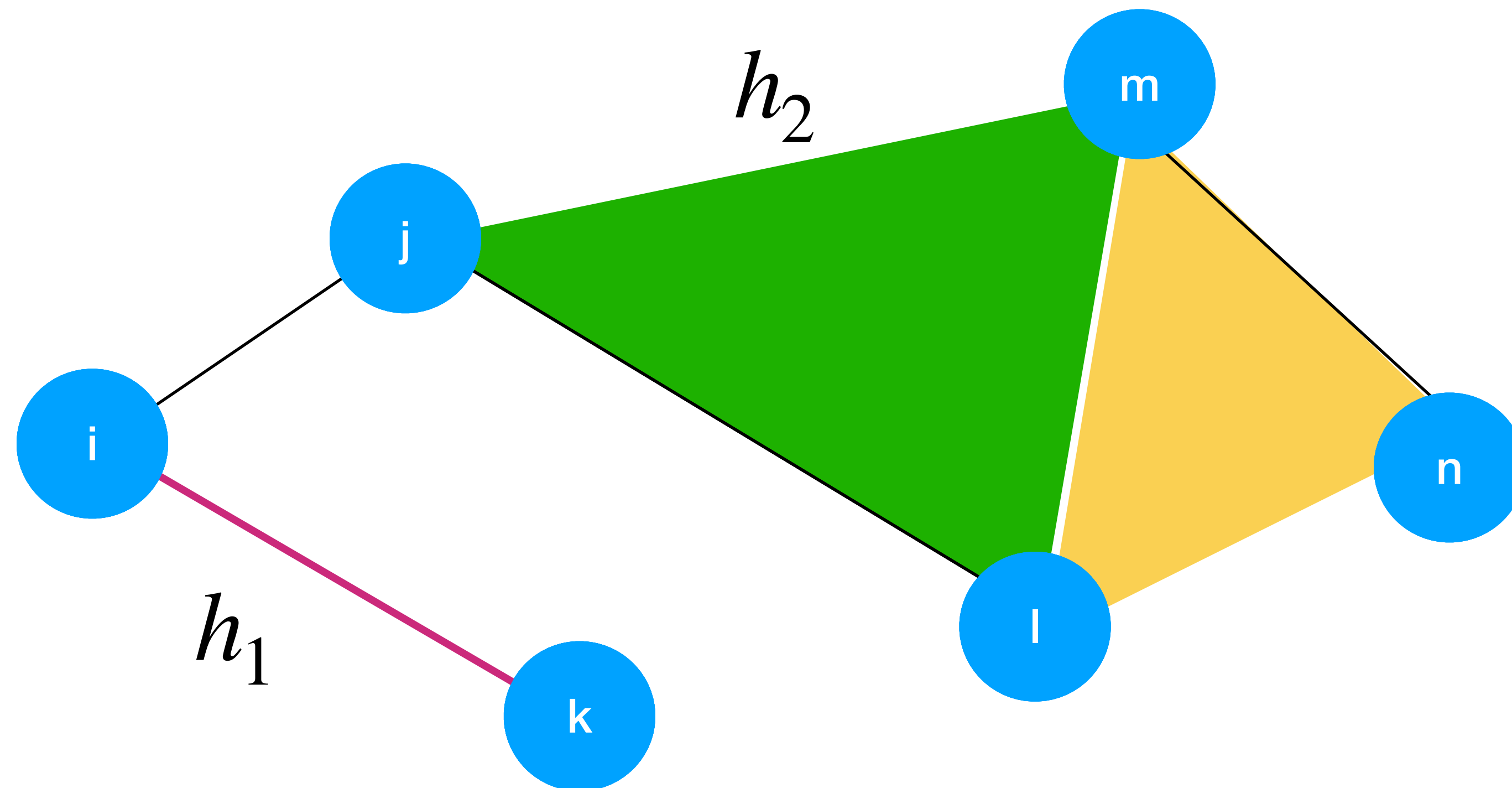
# Example

h 19:00

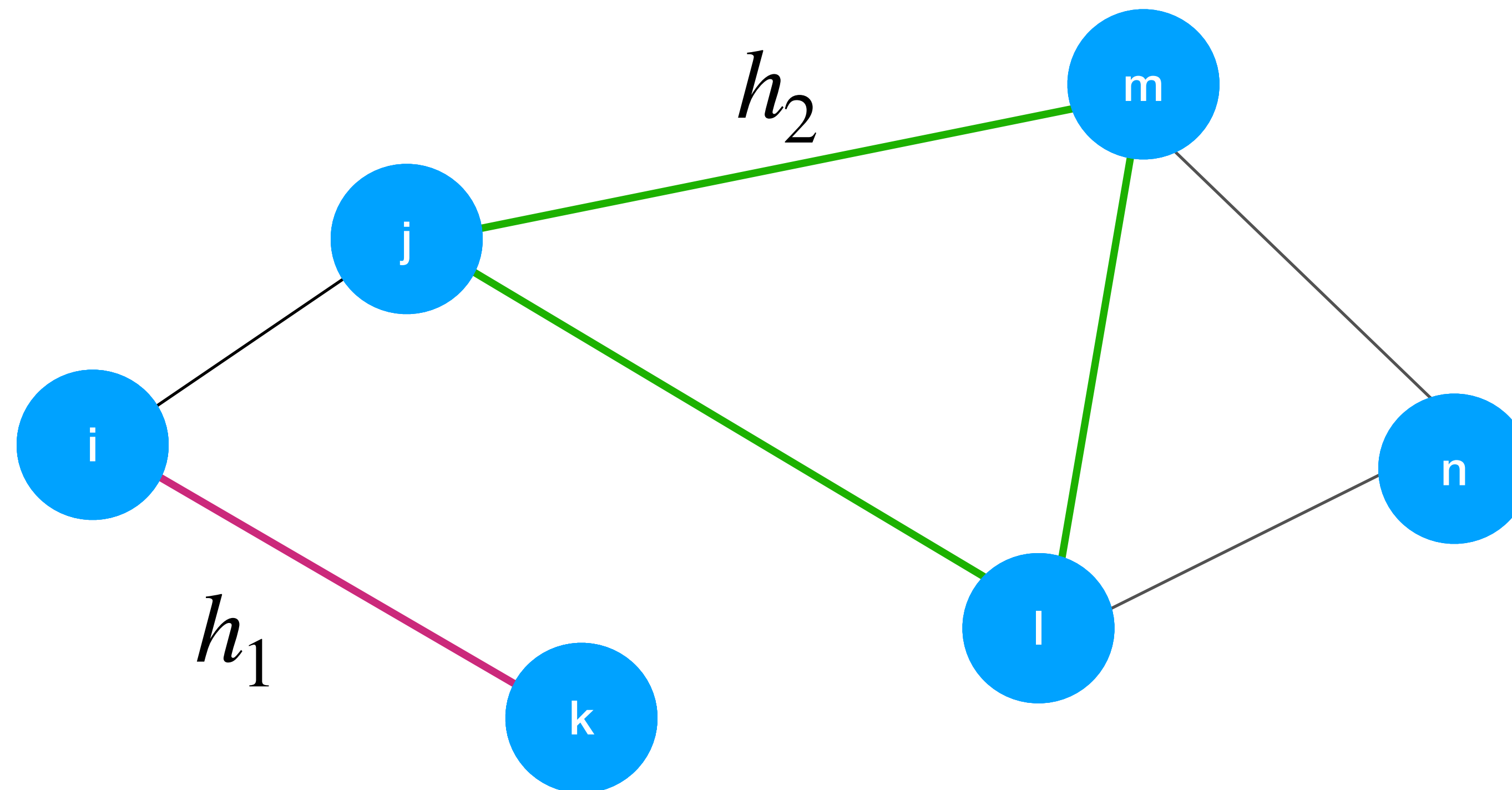




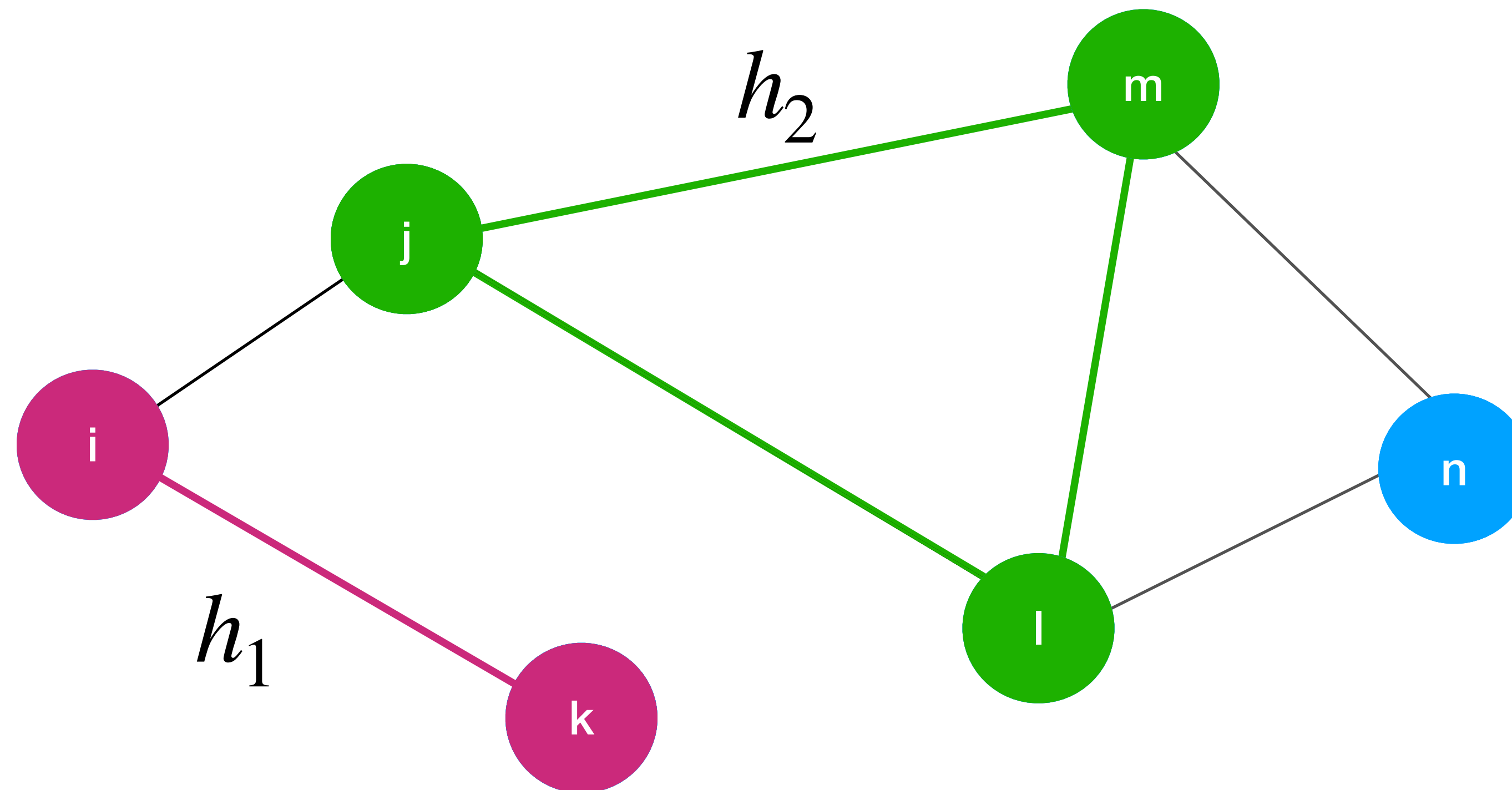
# Example



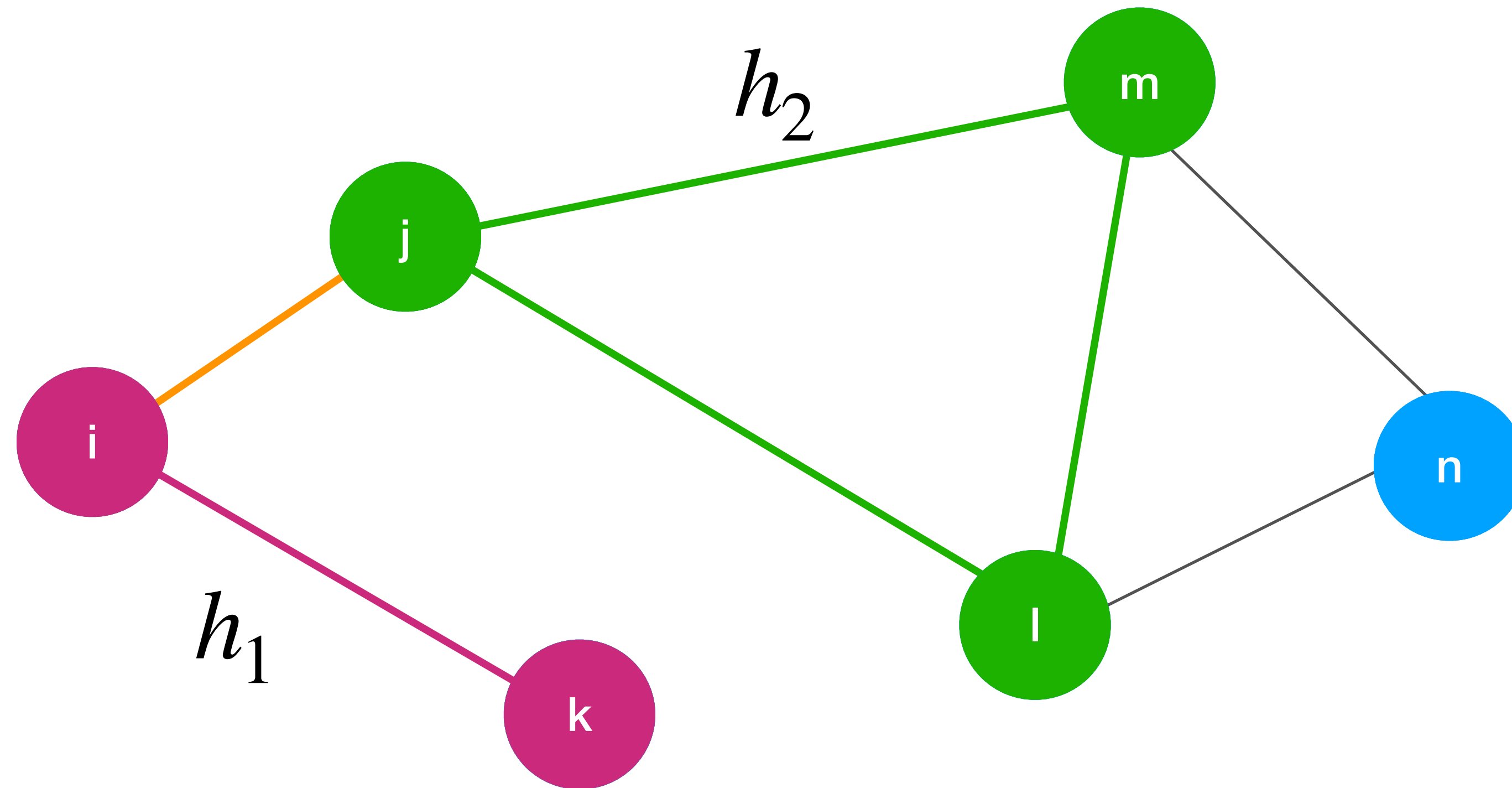
# Example



# Example



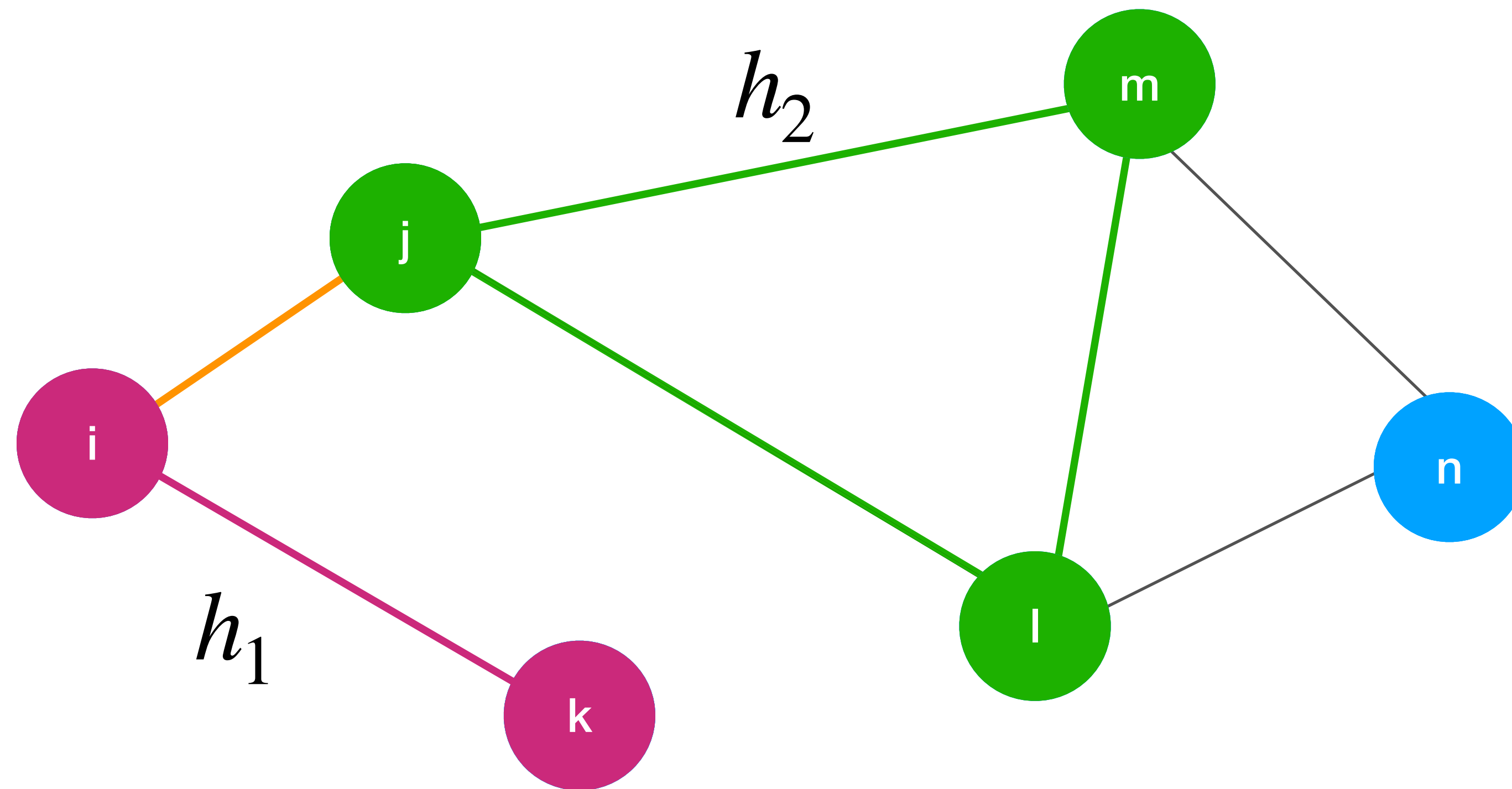
# Example



**Topological distance:**  $\eta(e_1, e_2) = 2$



# Example



**Topological distance:**  $\eta(e_1, e_2) = 2$

**Temporal distance:**  $\mathcal{T}(e_1, e_2) = 1h$

# Relation of topological and temporal distance of events with different orders

# Relation of topological and temporal distance of events with different orders

## average topological distance

Average topological distance

$$\mu_d(\Delta t) = E[\eta(e, e') \mid \quad ]$$

# Relation of topological and temporal distance of events with different orders

average topological distance of events with different orders

Average topological distance

Events with order d  
vs  
events with different orders

$$\mu_d(\Delta t) = E[\eta(e, e')] \mid$$

$$e \in \mathcal{E}_d, e' \in \mathcal{E} \setminus \mathcal{E}_d]$$



# Relation of topological and temporal distance of events with different orders

average topological distance of events with different orders  
and temporal distance  $< \Delta t$

Average topological distance

Temporal distance

Events with order d  
vs  
events with different orders

$$\mu_d(\Delta t) = E[\eta(e, e') \mid \mathcal{T}(e, e') < \Delta t, e \in \mathcal{E}_d, e' \in \mathcal{E} \setminus \mathcal{E}_d]$$

# Relation of topological and temporal distance of events with different orders

Normalised average topological distance of events with different orders  
and temporal distance  $< \Delta t$

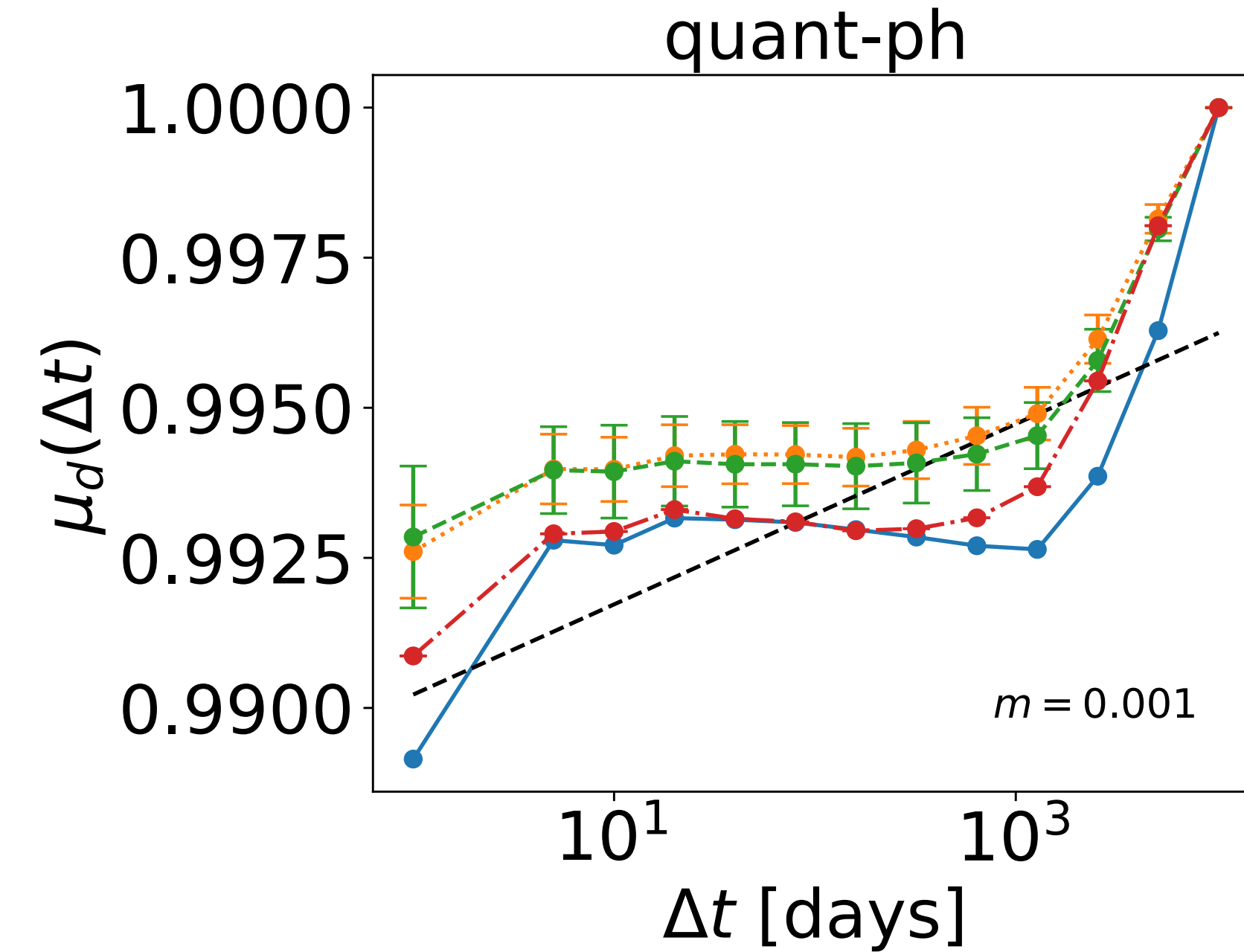
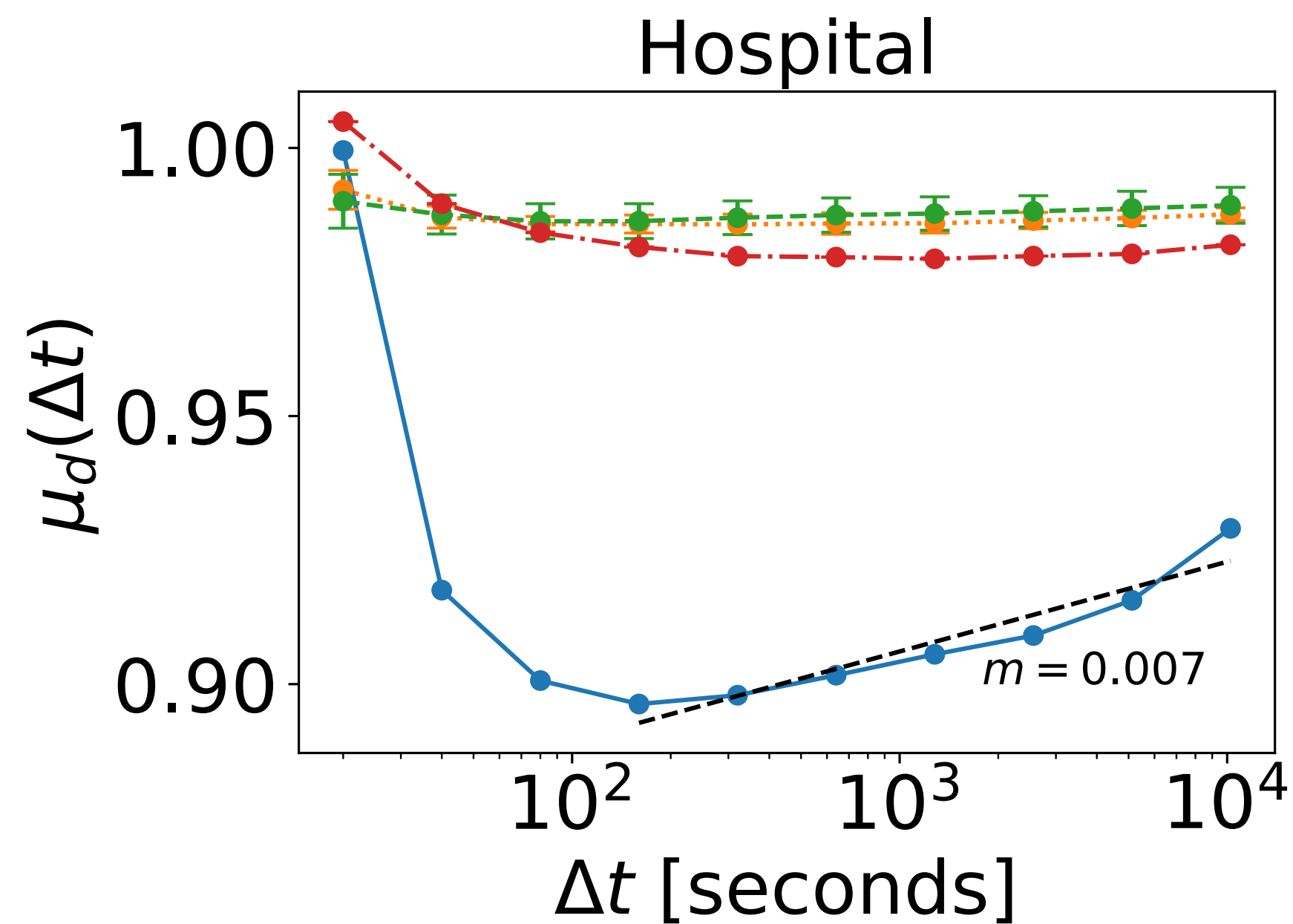
Average topological distance      Temporal distance      Events with order d  
vs  
events with different orders

$$\mu_d(\Delta t) = \frac{E[\eta(e, e') \mid \mathcal{T}(e, e') < \Delta t, e \in \mathcal{E}_d, e' \in \mathcal{E} \setminus \mathcal{E}_d]}{E[\eta(e, e') \mid e \in \mathcal{E}_d, e' \in \mathcal{E} \setminus \mathcal{E}_d]}$$

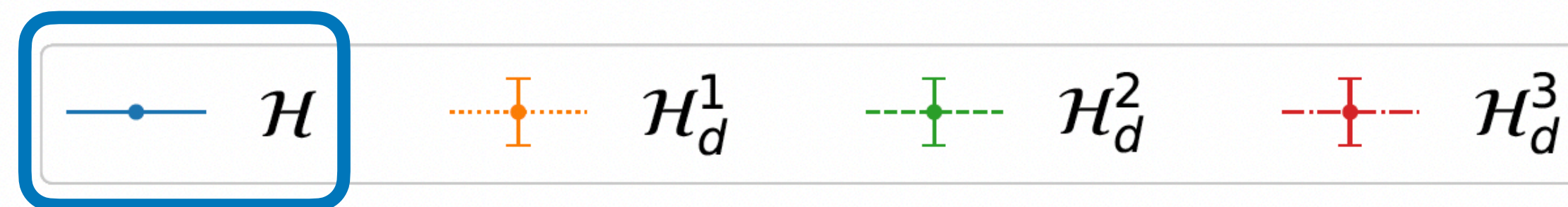
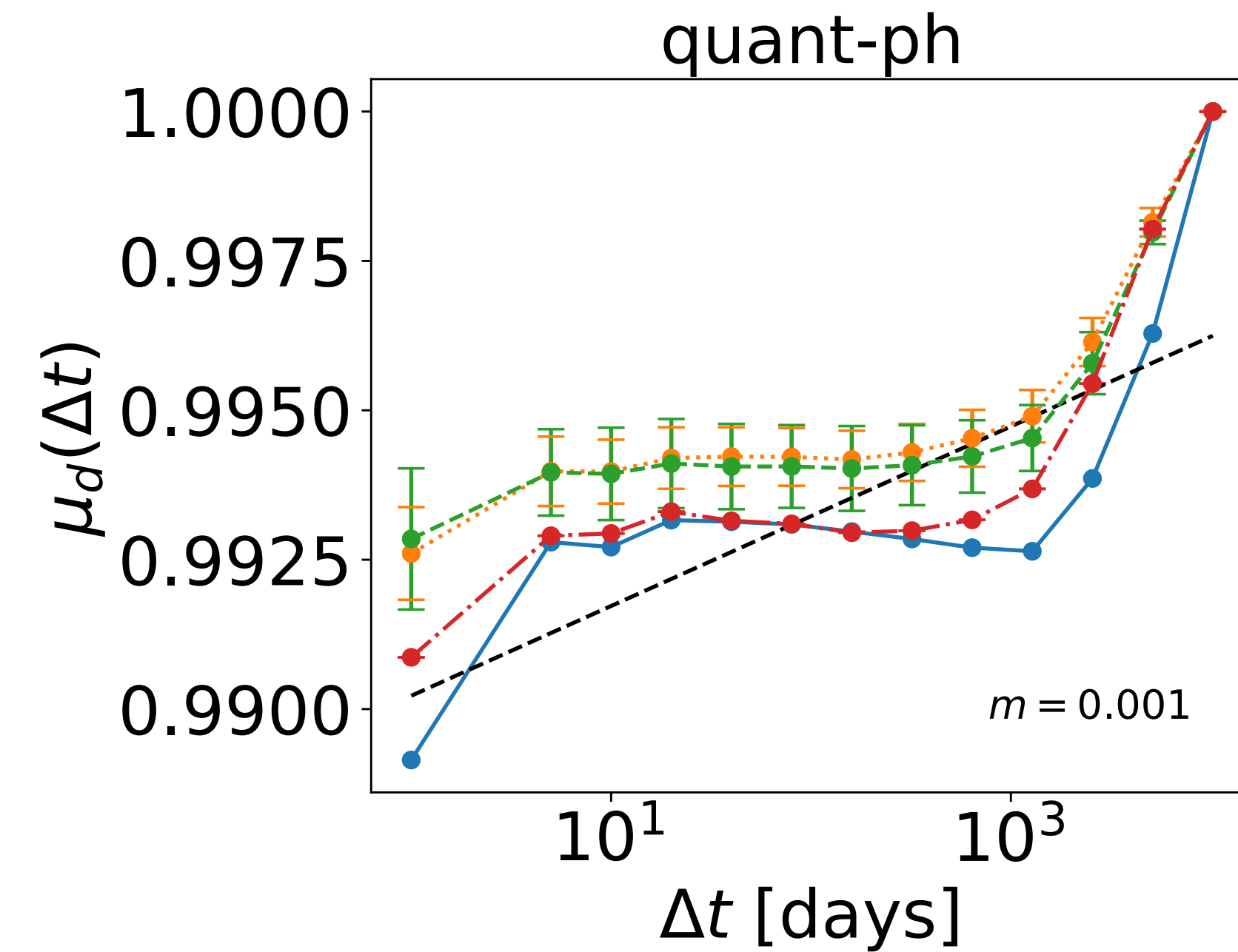
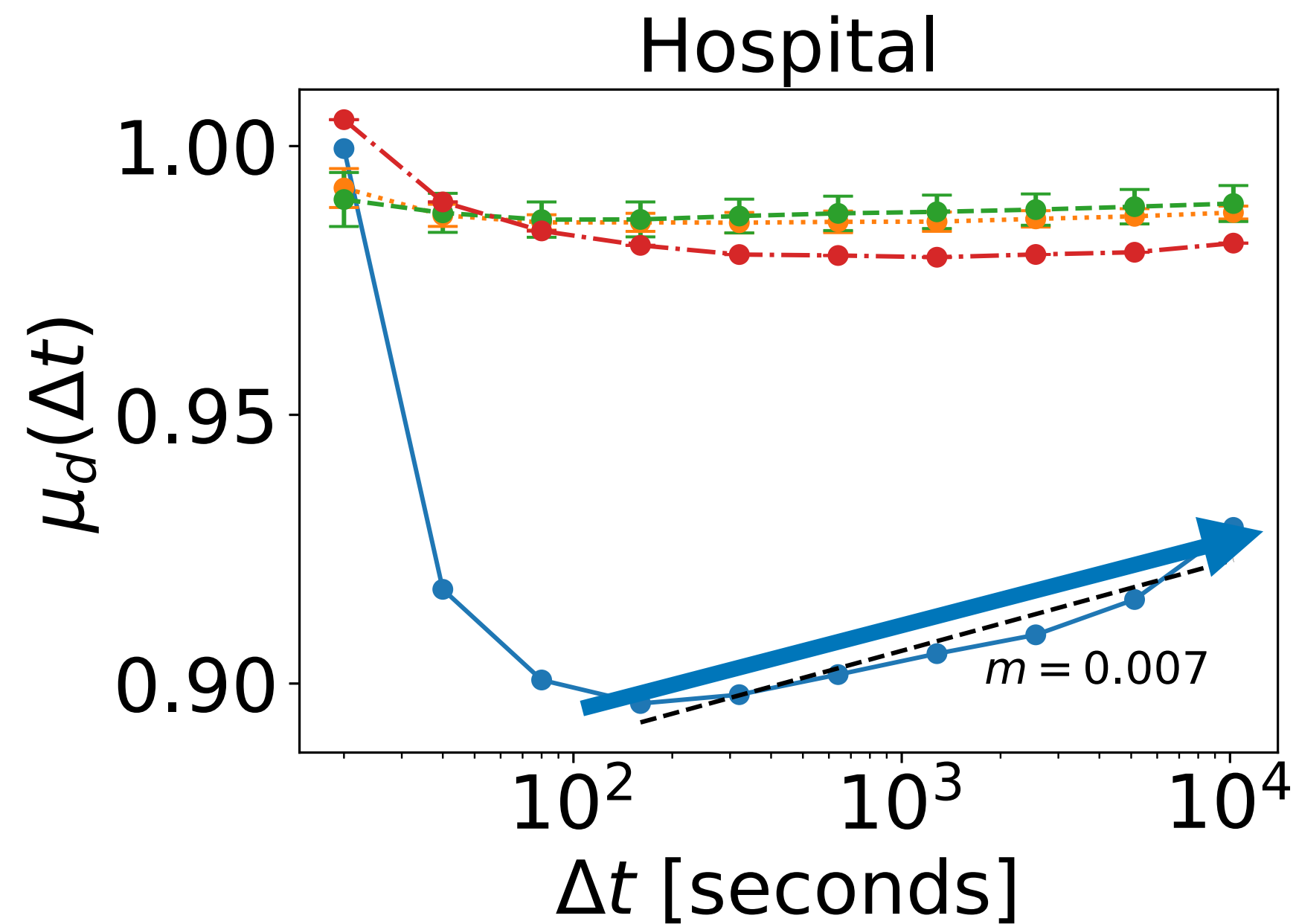
Average topological distance of order d events  
with events of different orders

# Relation of topological and temporal distance of events with different orders ( $d=3$ )

# Relation of topological and temporal distance of events with different orders (d=3)

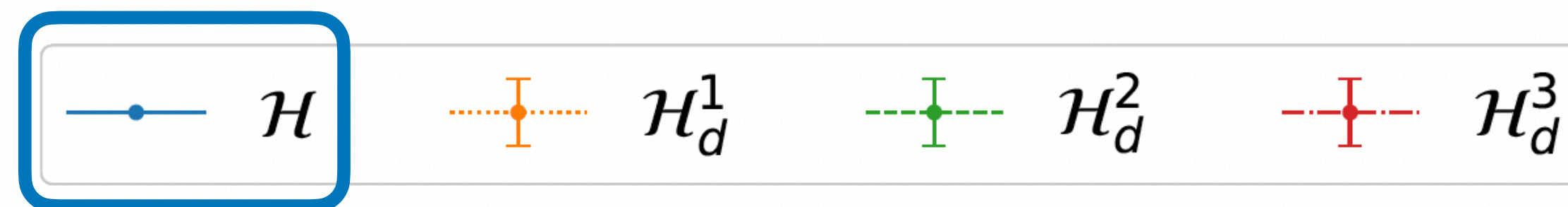
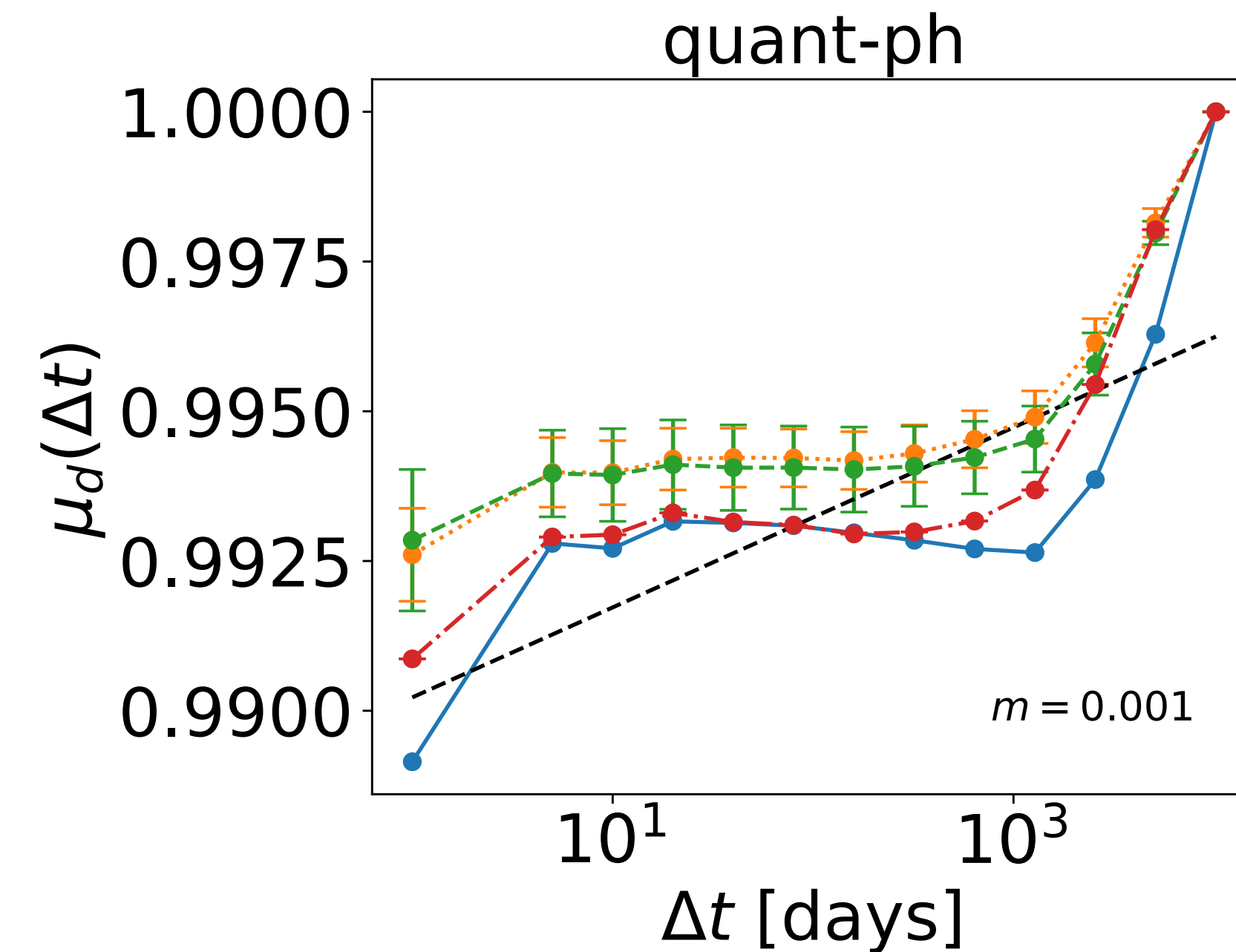
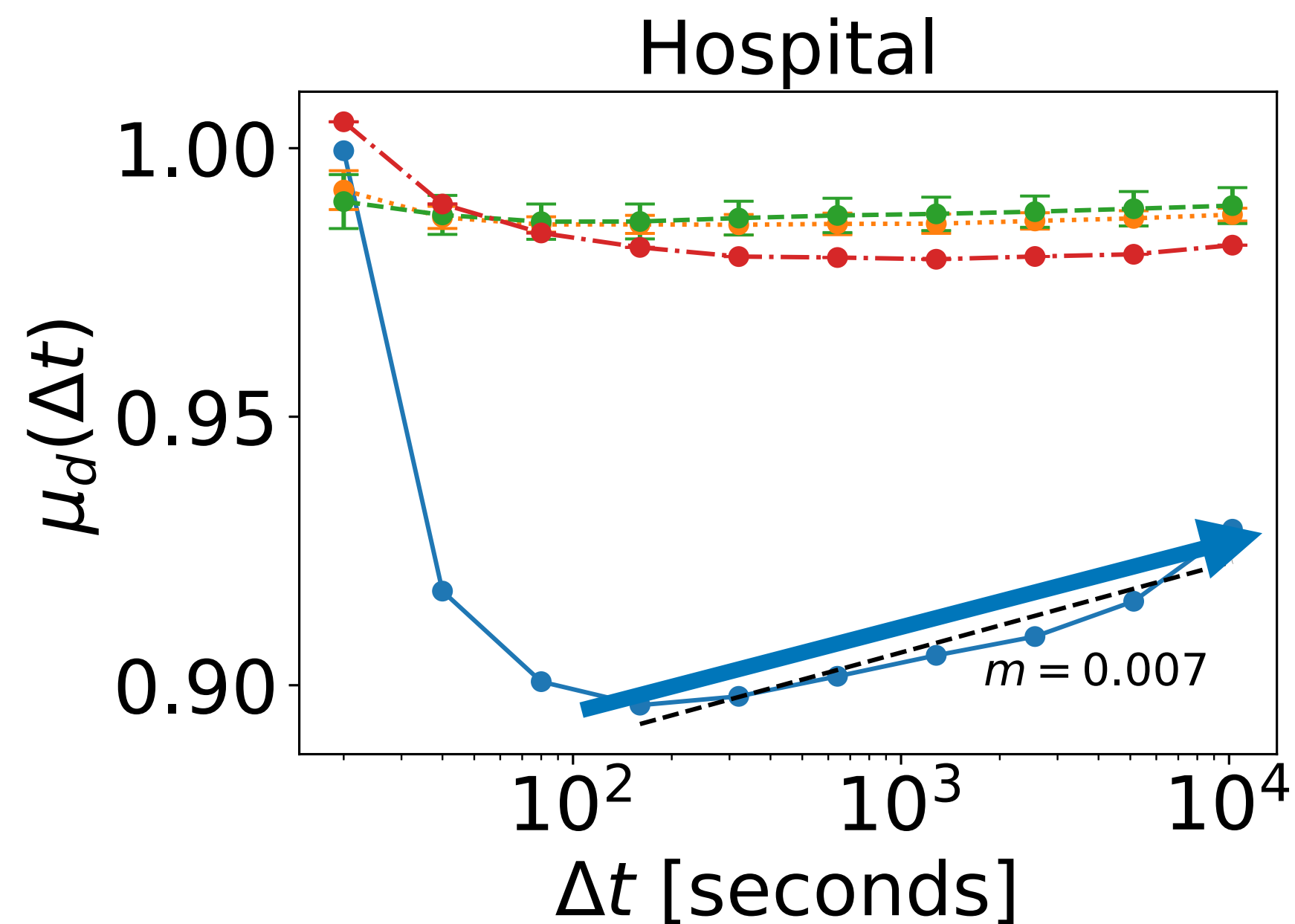


# Relation of topological and temporal distance of events with different orders (d=3)





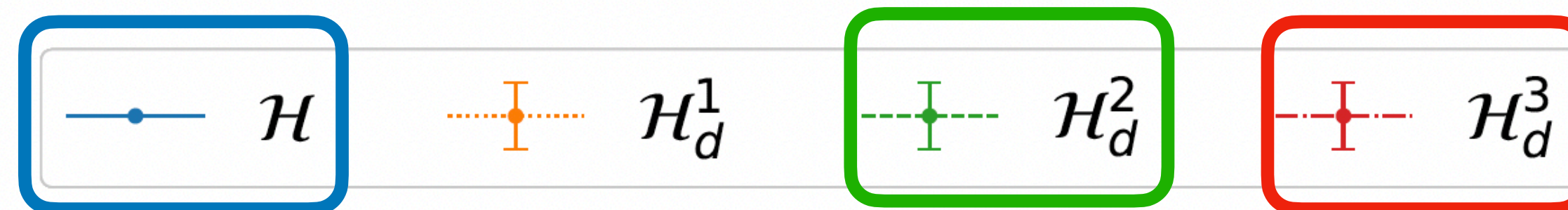
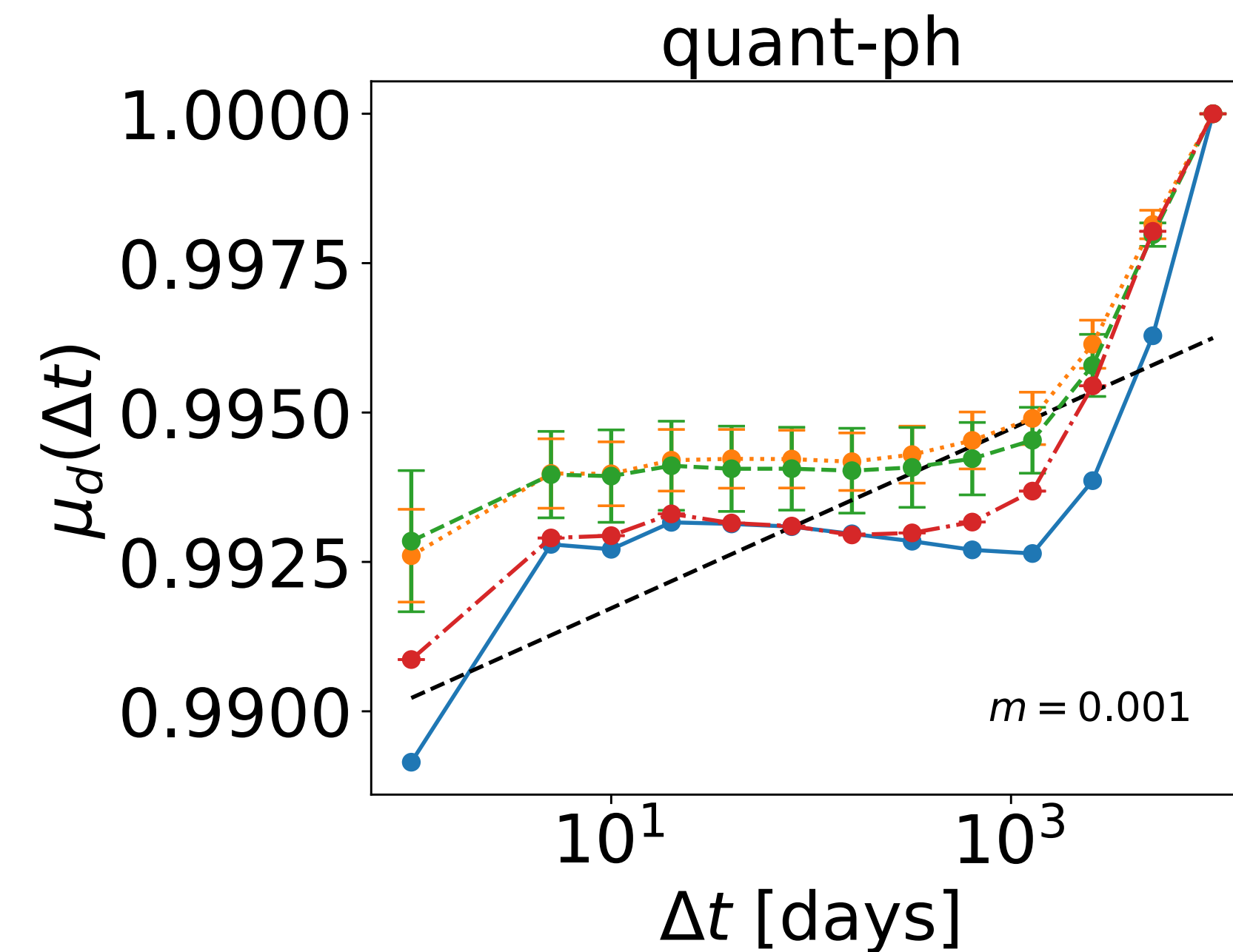
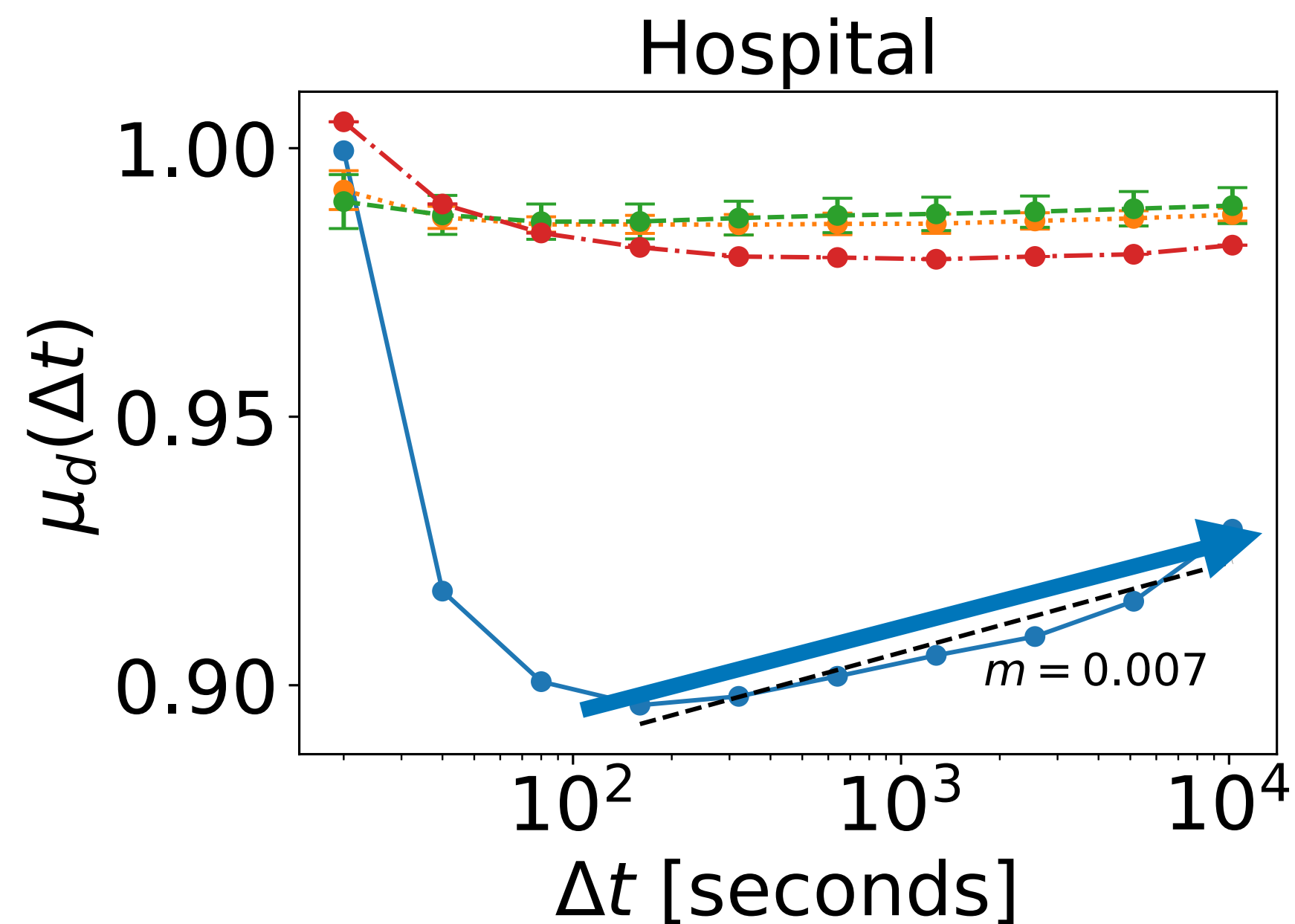
# Relation of topological and temporal distance of events with different orders (d=3)



**In physical contacts, events with different orders close in time are also close in topology!!**



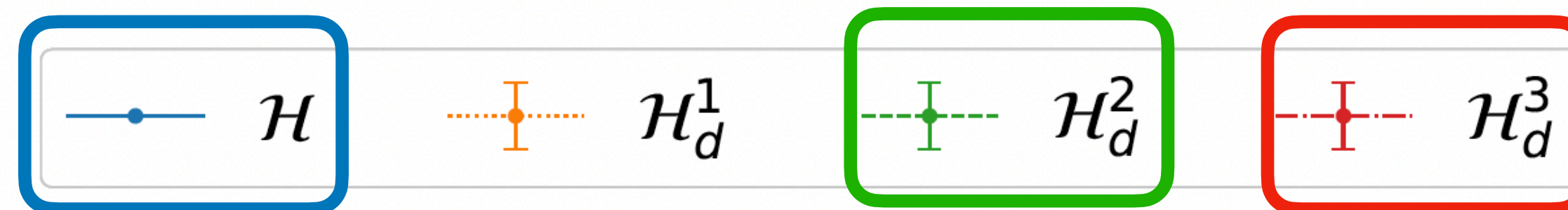
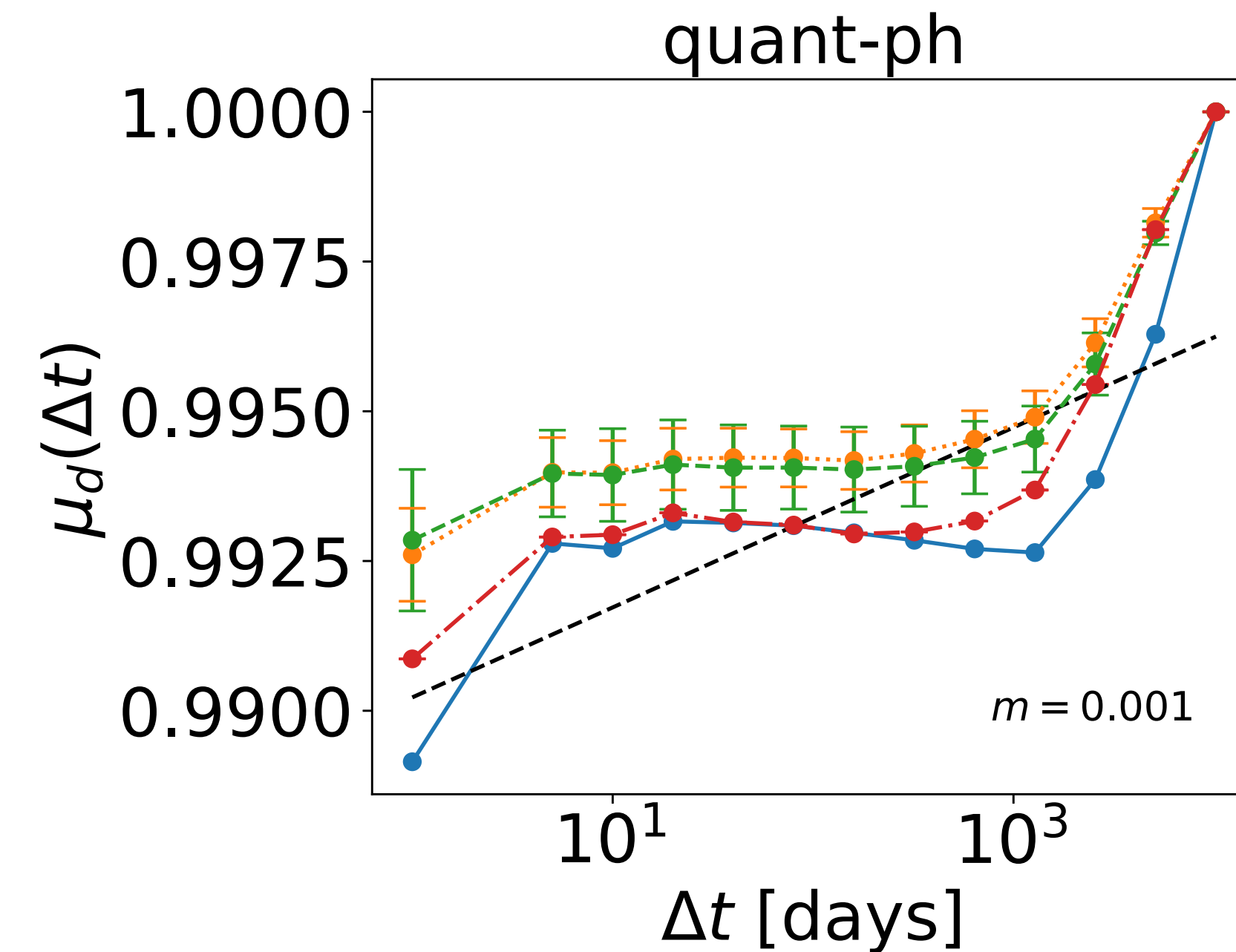
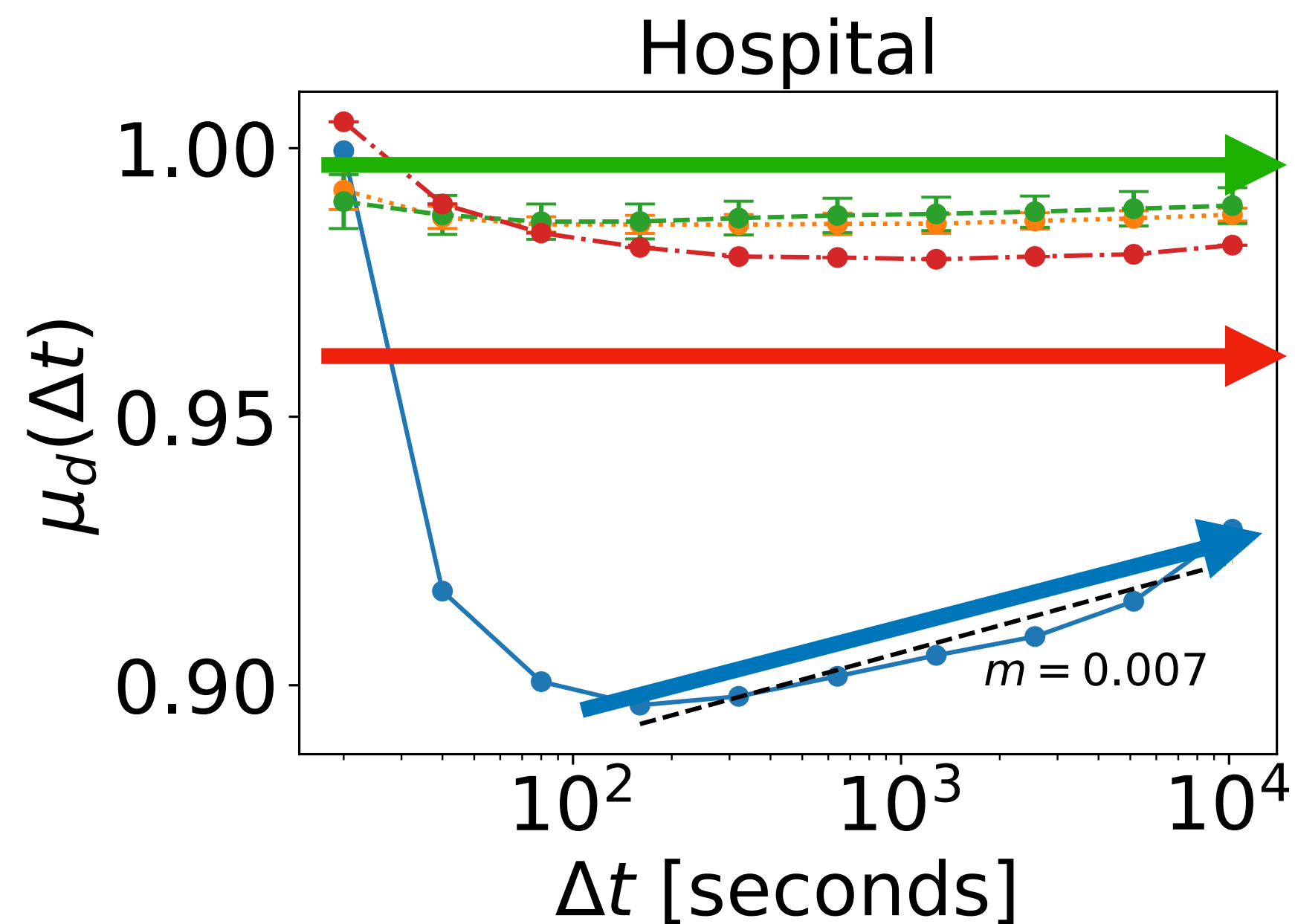
# Relation of topological and temporal distance of events with different orders (d=3)



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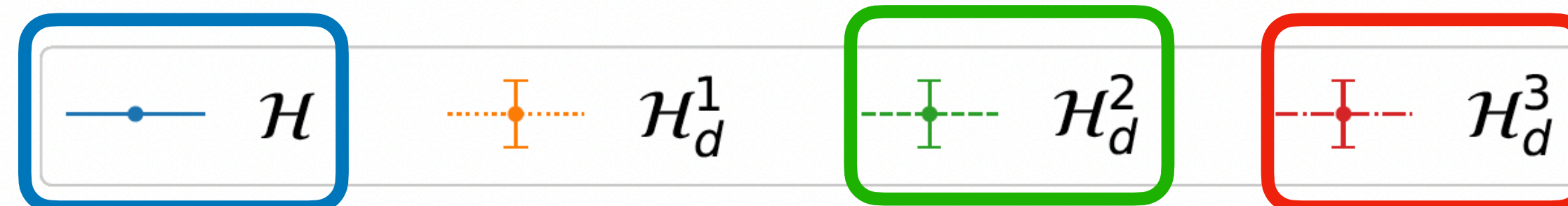
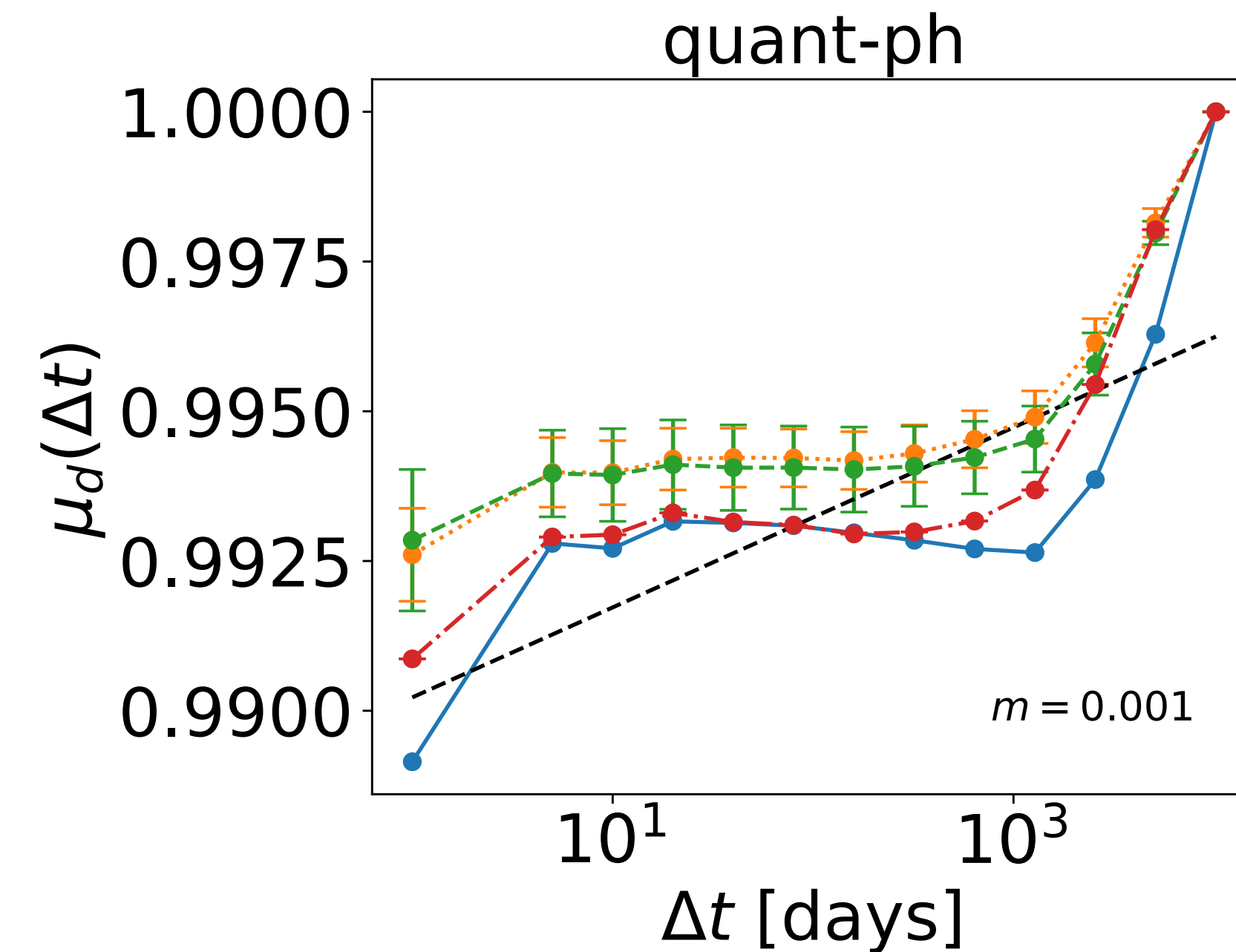
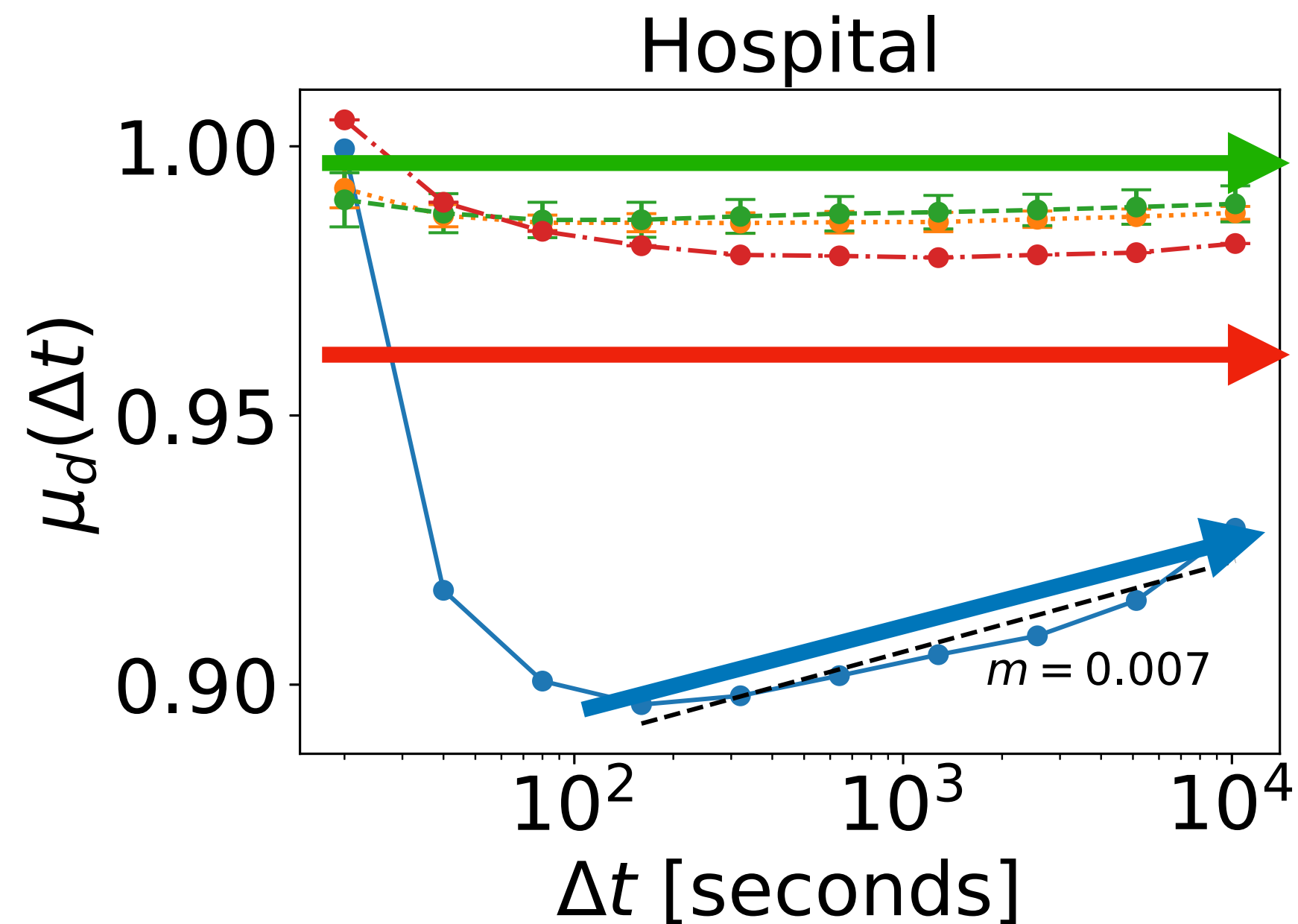


In physical contacts, events with different orders close in time are also close in topology!!





# Relation of topological and temporal distance of events with different orders (d=3)



Temporal properties of time series of activity of single hyperlinks (d=3) **cannot** explain this trend



# Event overlap in topology



# Event overlap in topology

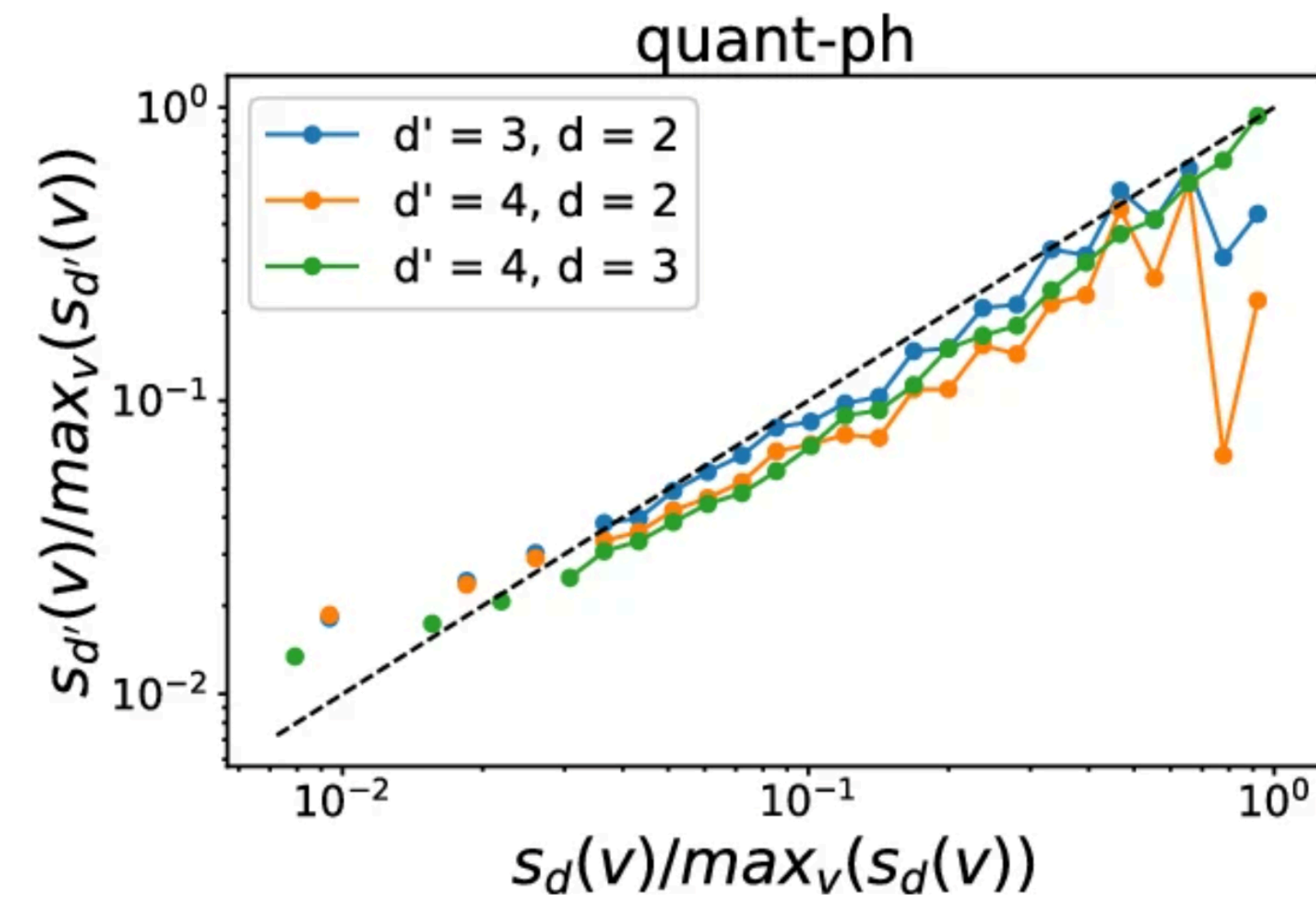
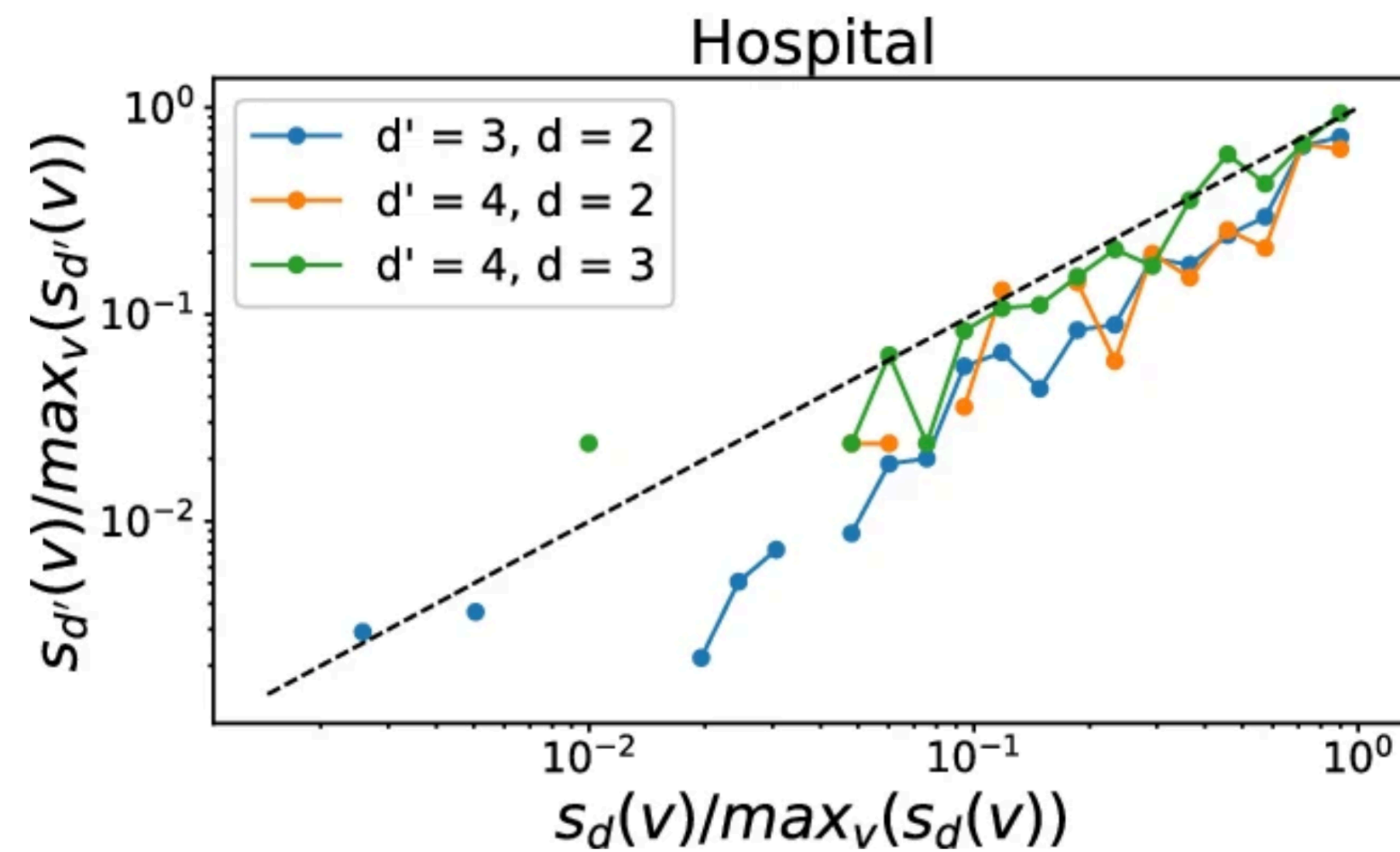
d-strength  $s_d(v)$  of a node  $v$ :

**# events of order  $d$**  which node  $v$  is involved in

# Event overlap in topology

d-strength  $s_d(v)$  of a node  $v$ :

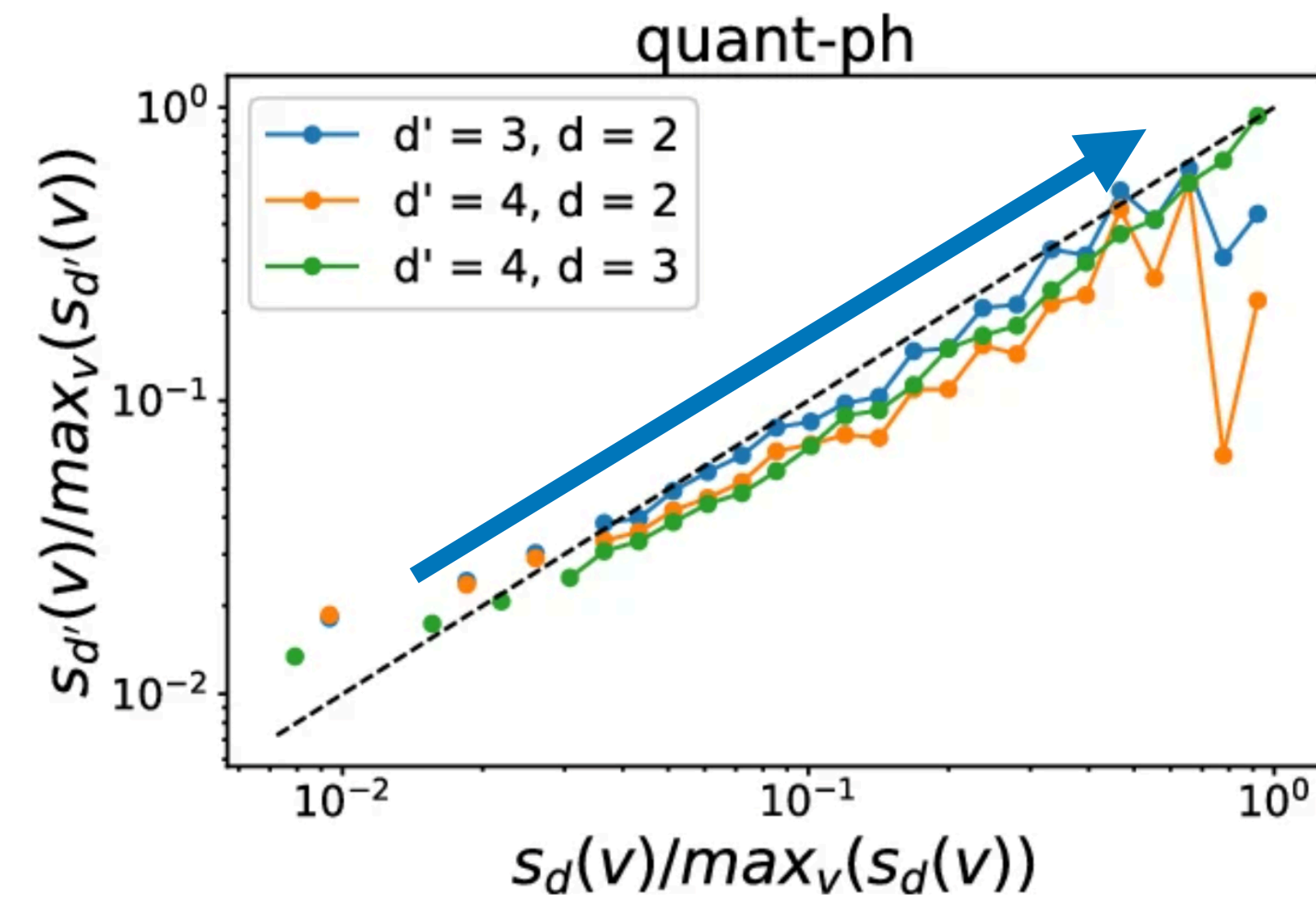
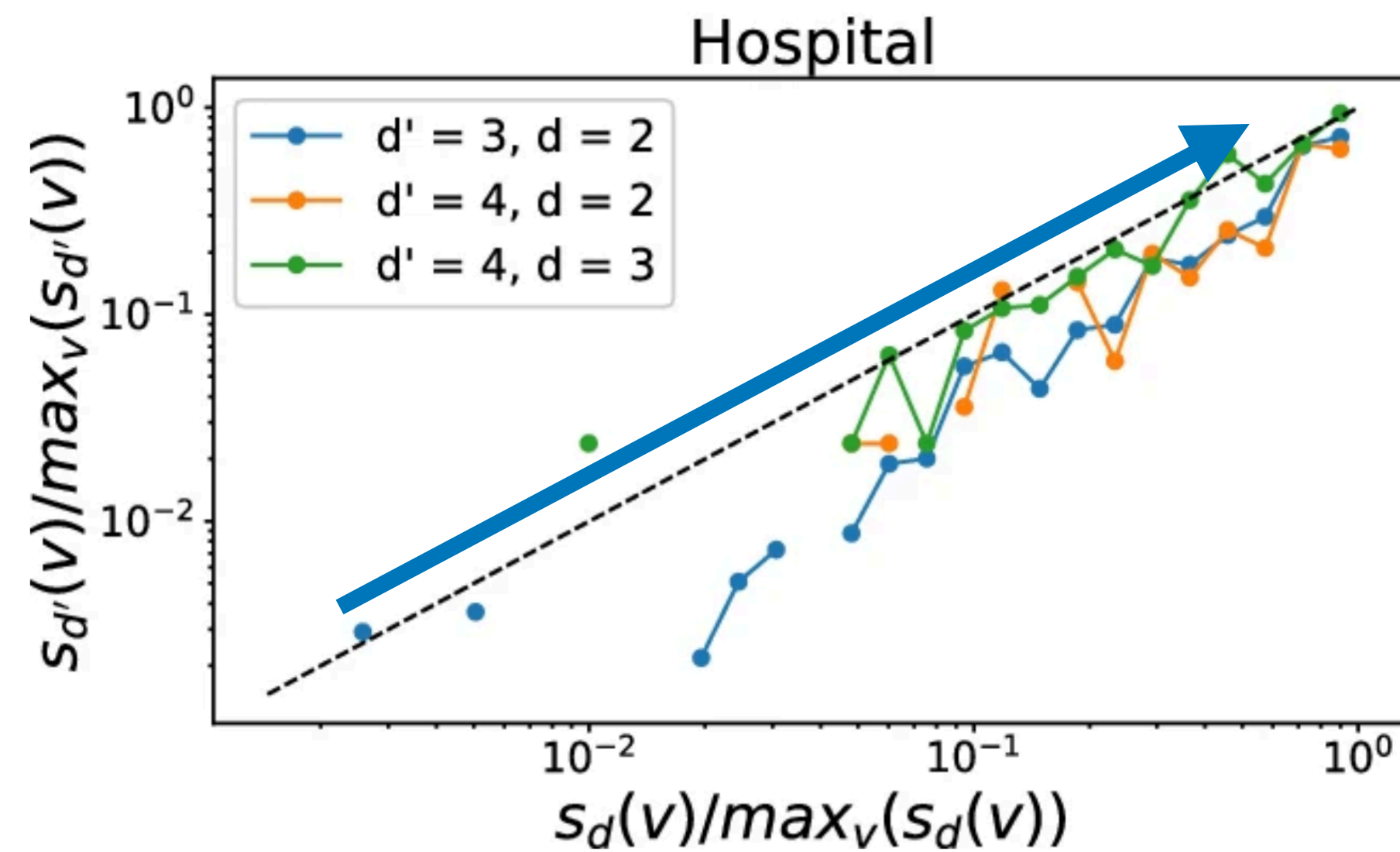
# events of order  $d$  which node  $v$  is involved in



# Event overlap in topology

d-strength  $s_d(v)$  of a node  $v$ :

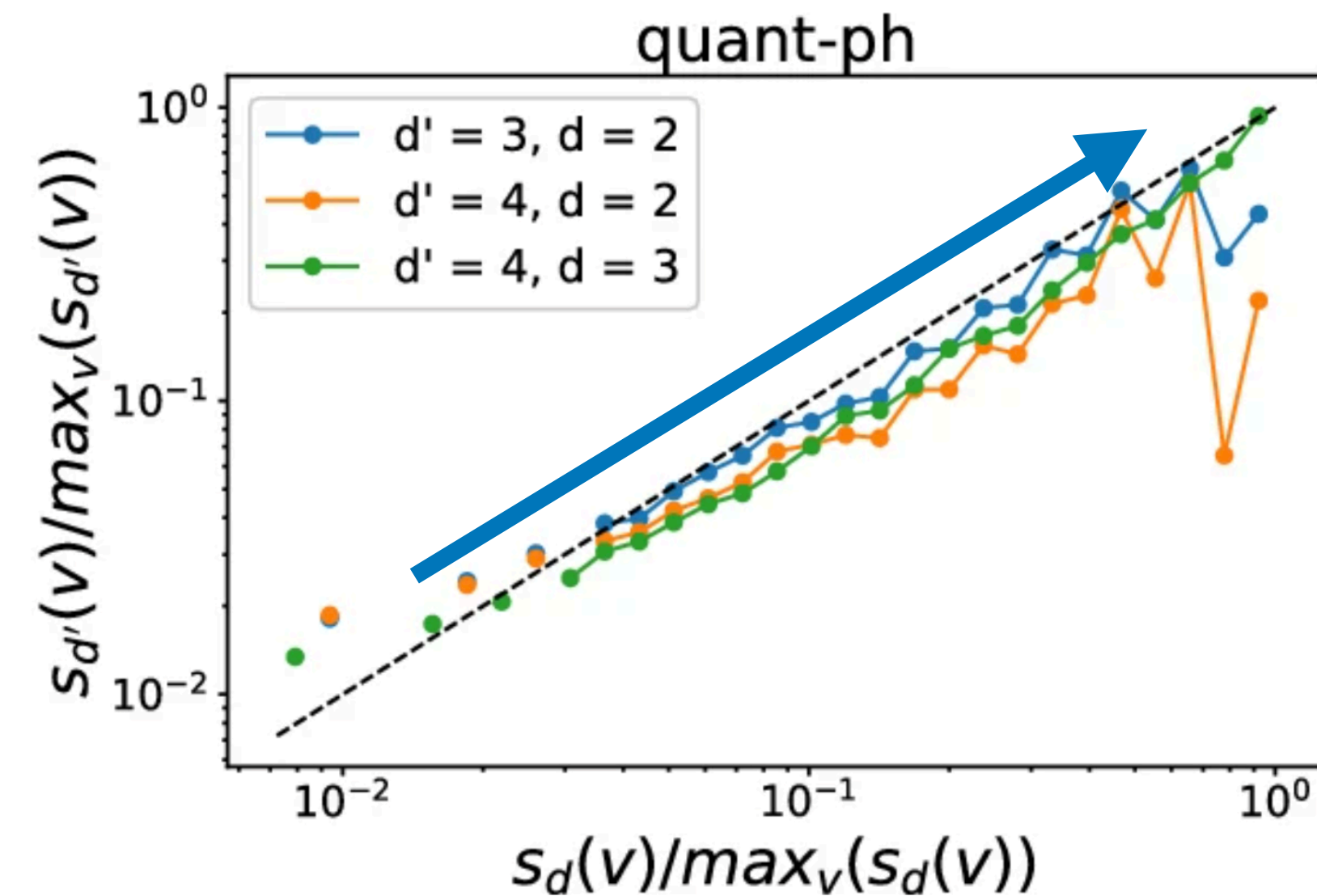
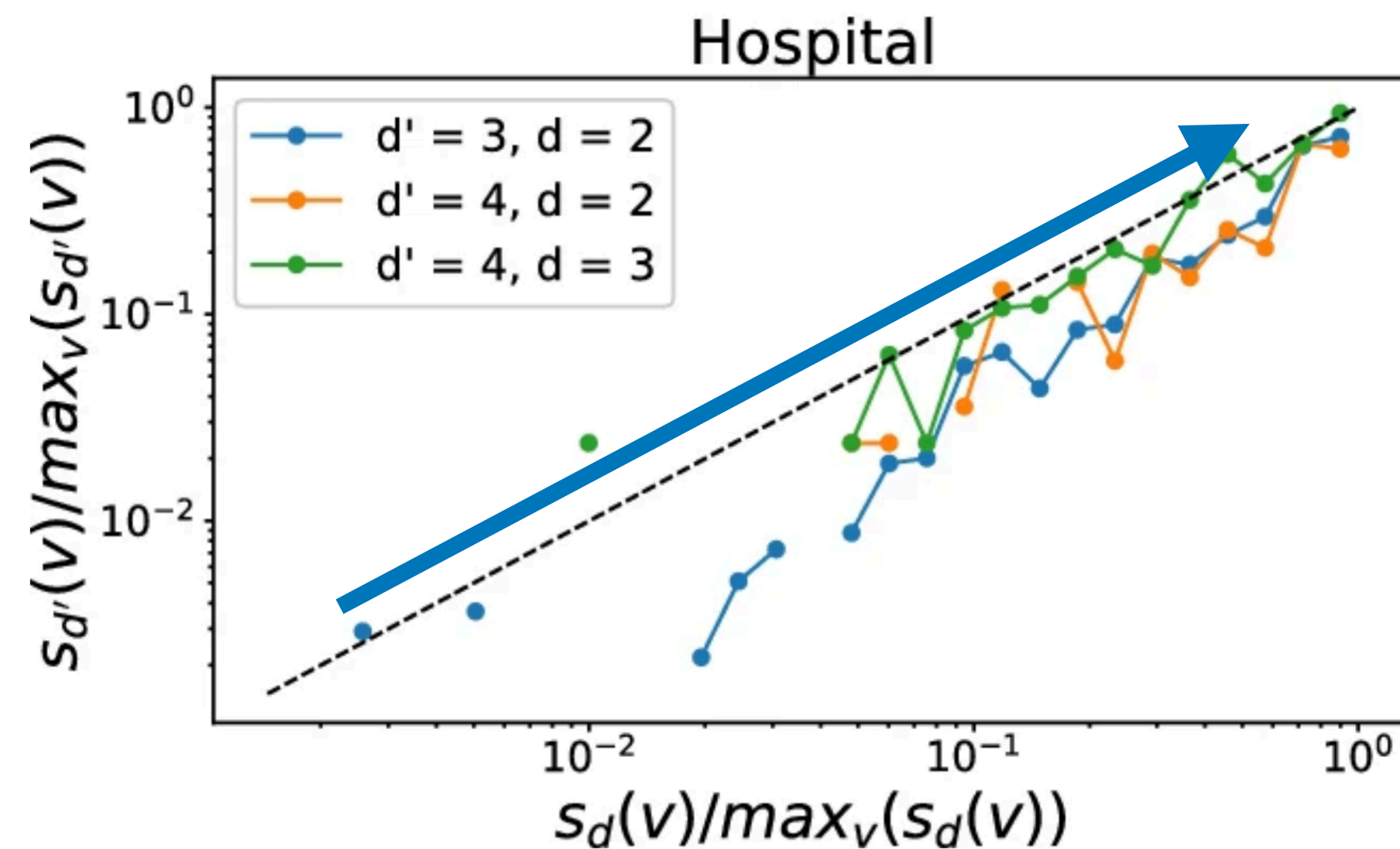
# events of order  $d$  which node  $v$  is involved in



# Event overlap in topology

d-strength  $s_d(v)$  of a node  $v$ :

# events of order  $d$  which node  $v$  is involved in



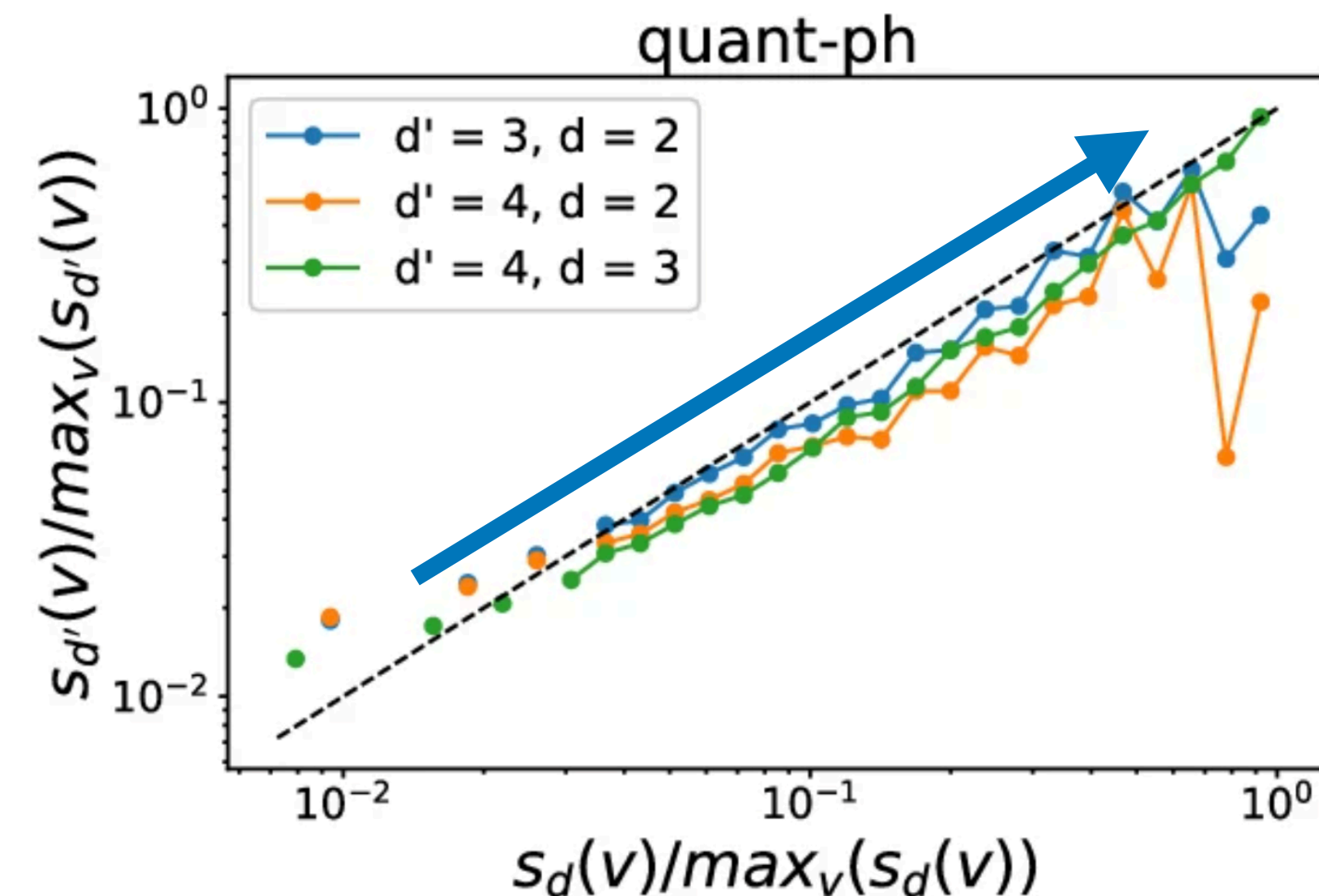
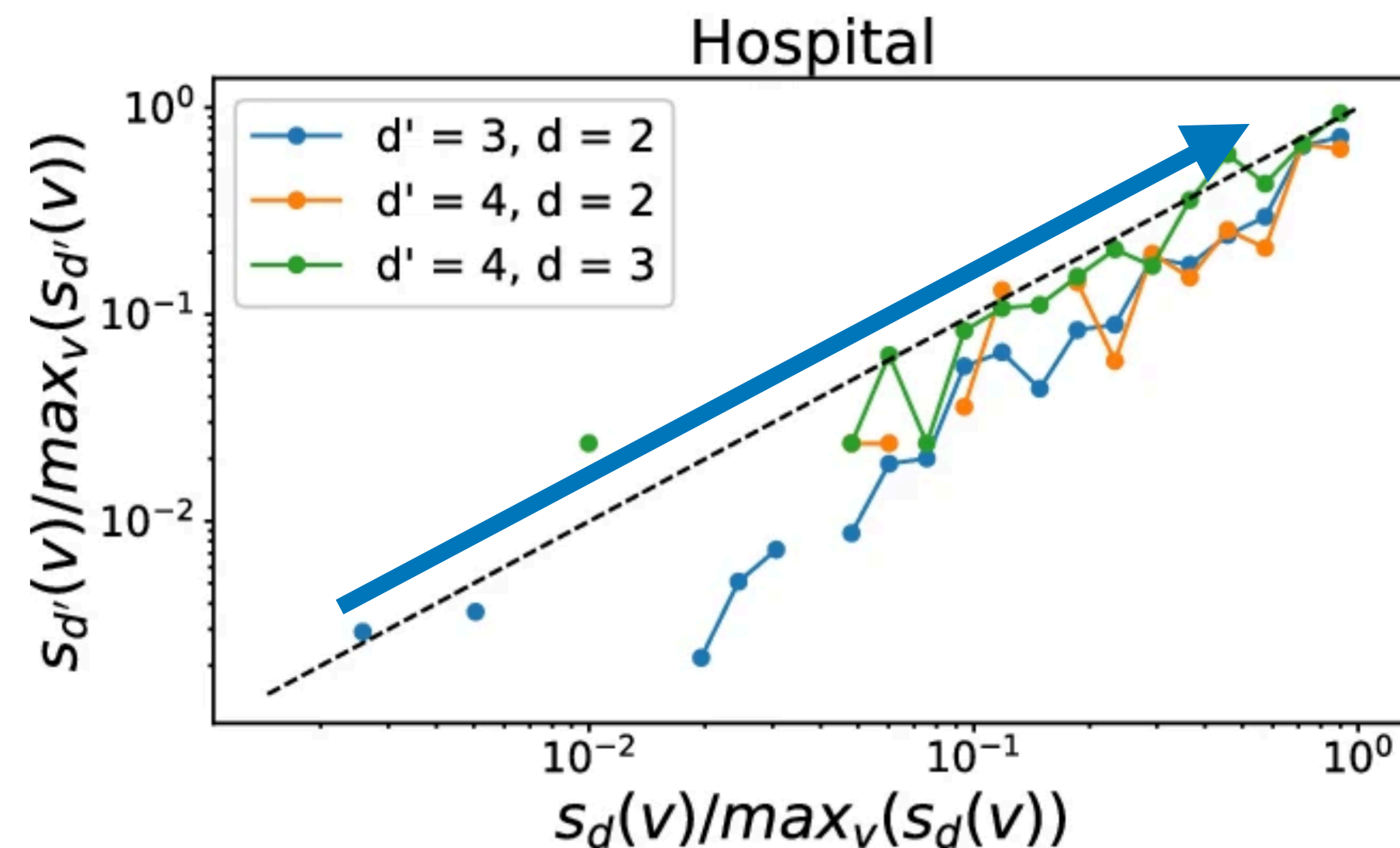
- 1) Events of different order **overlap in component nodes**



# Event overlap in topology

d-strength  $s_d(v)$  of a node  $v$ :

# events of order  $d$  which node  $v$  is involved in



- 1) Events of different order **overlap in component nodes**
- 2) Individuals tend to be **consistently active or inactive** in events **across orders**

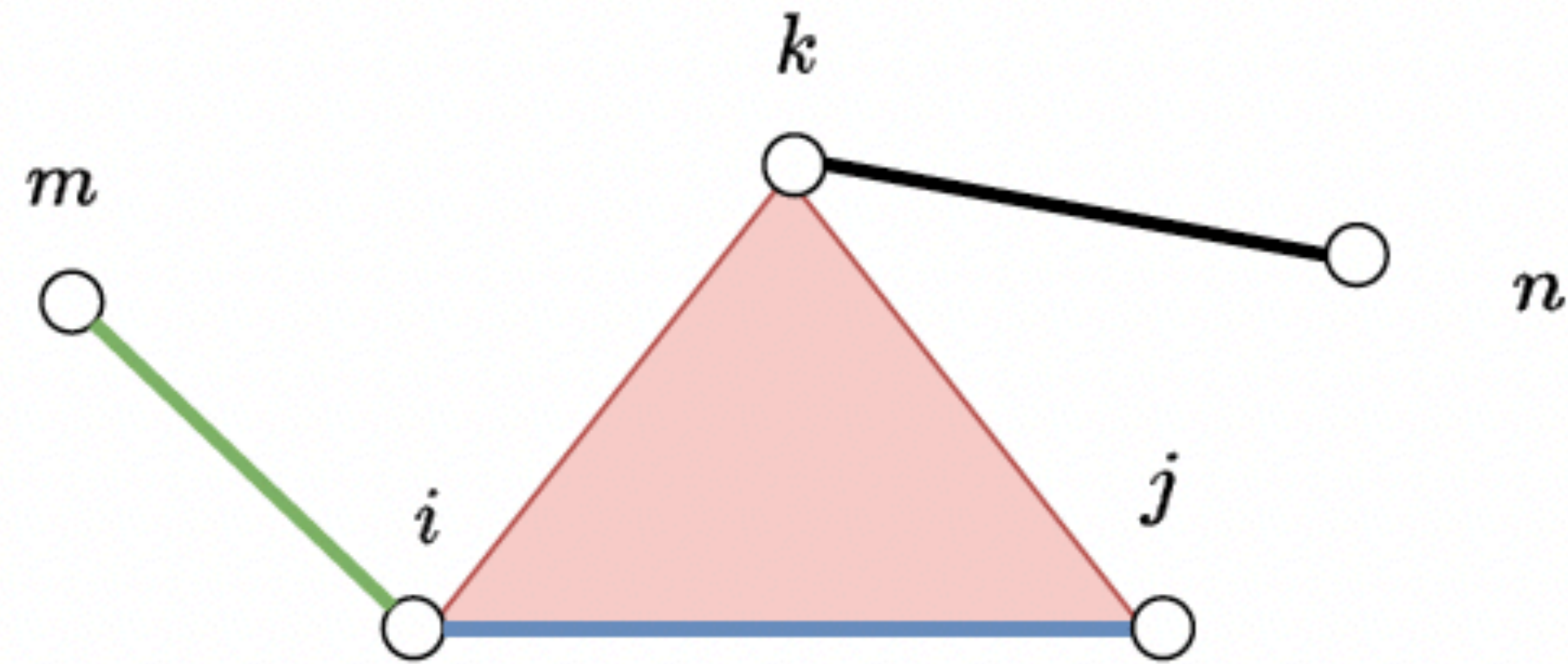


# Temporal correlation of events with different orders overlapping in topology

# Temporal correlation of events with different orders overlapping in topology

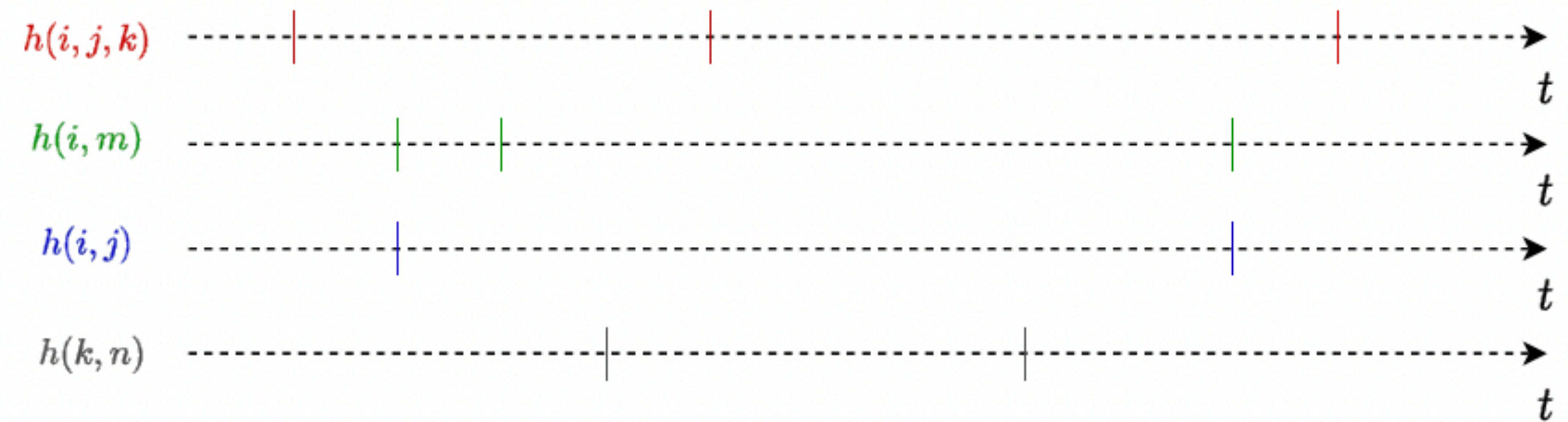
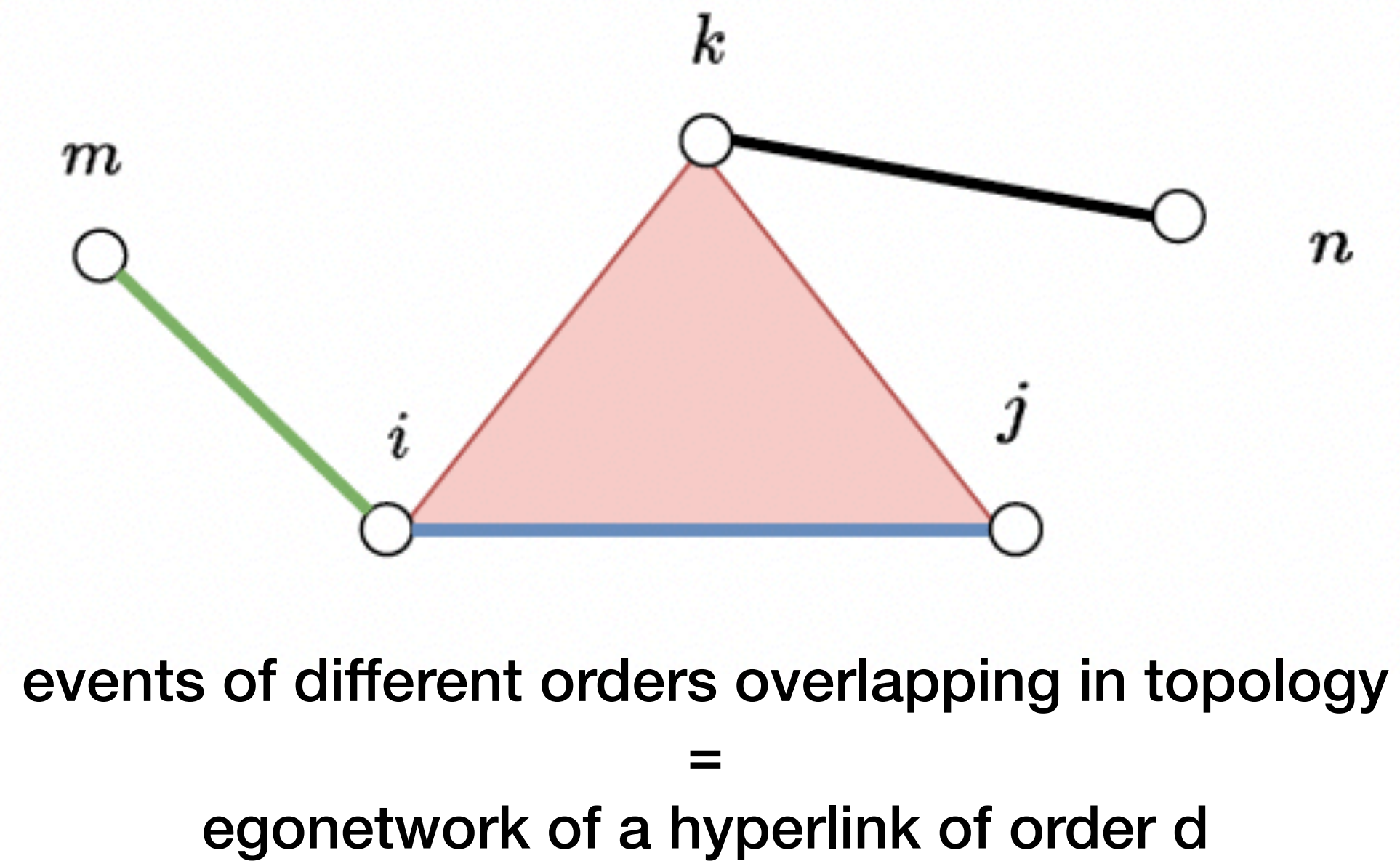
events of different orders overlapping in topology  
=  
egonetwork of a hyperlink of order d

# Temporal correlation of events with different orders overlapping in topology



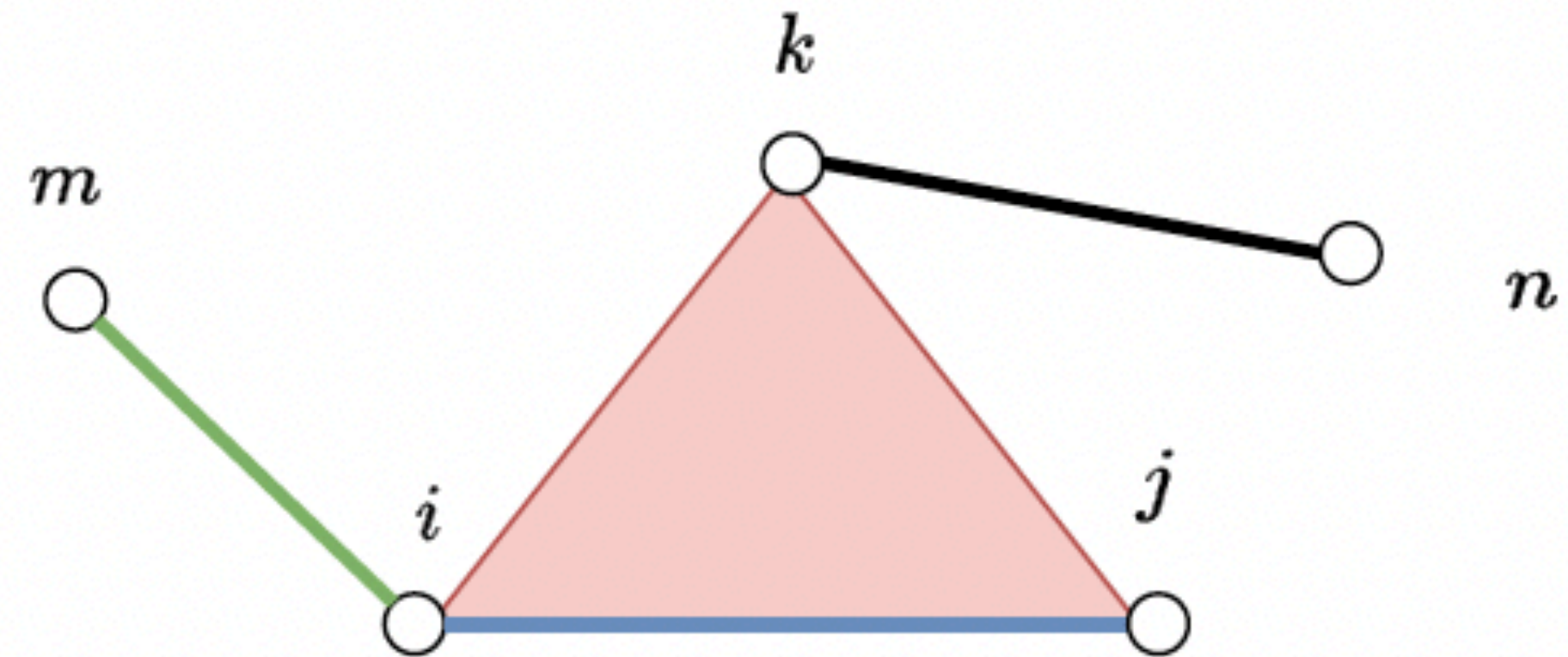
events of different orders overlapping in topology  
=  
egonetwork of a hyperlink of order  $d$

# Temporal correlation of events with different orders overlapping in topology





# Temporal correlation of events with different orders overlapping in topology



events of different orders overlapping in topology

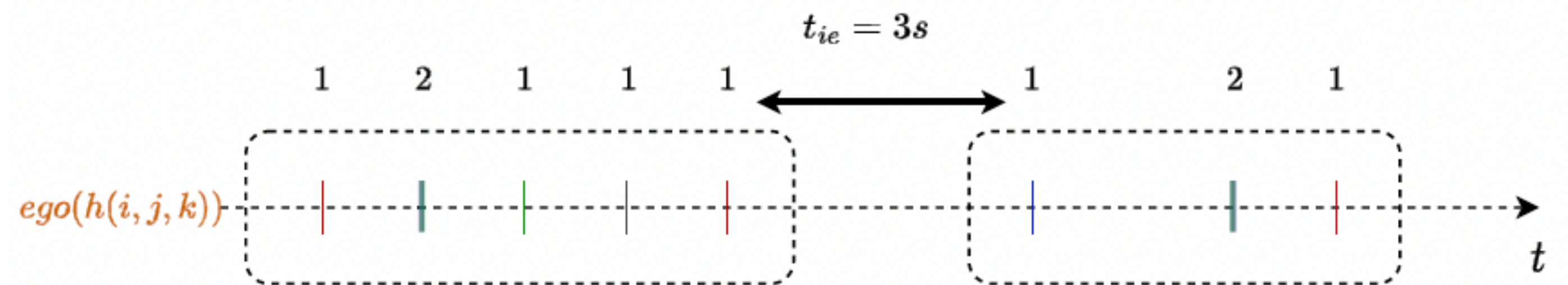
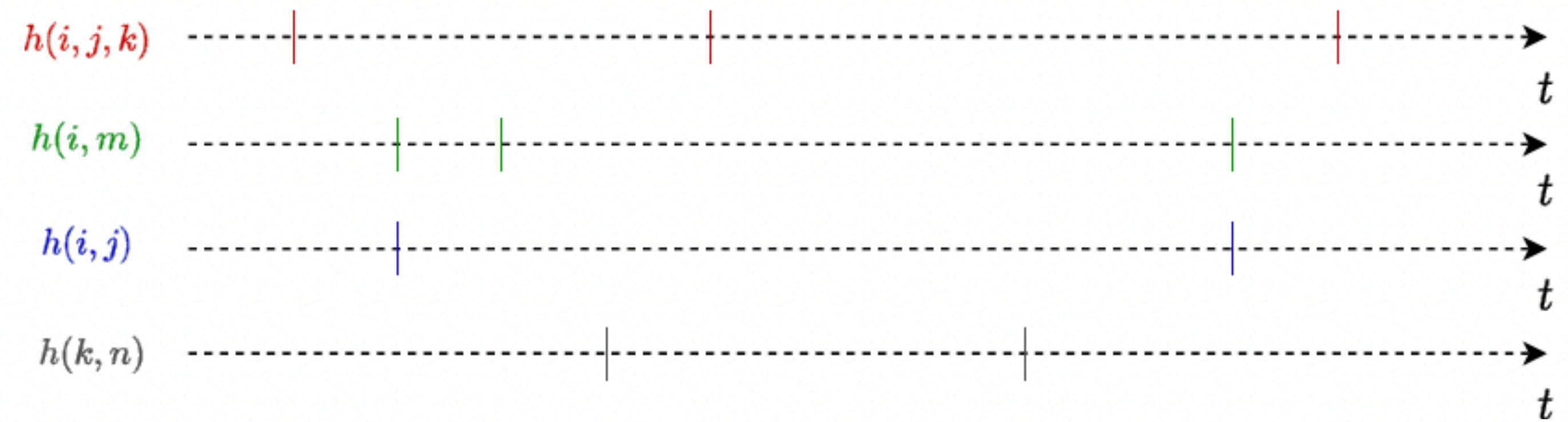
=

egonetwork of a hyperlink of order d

temporal correlation of egonetwork activity

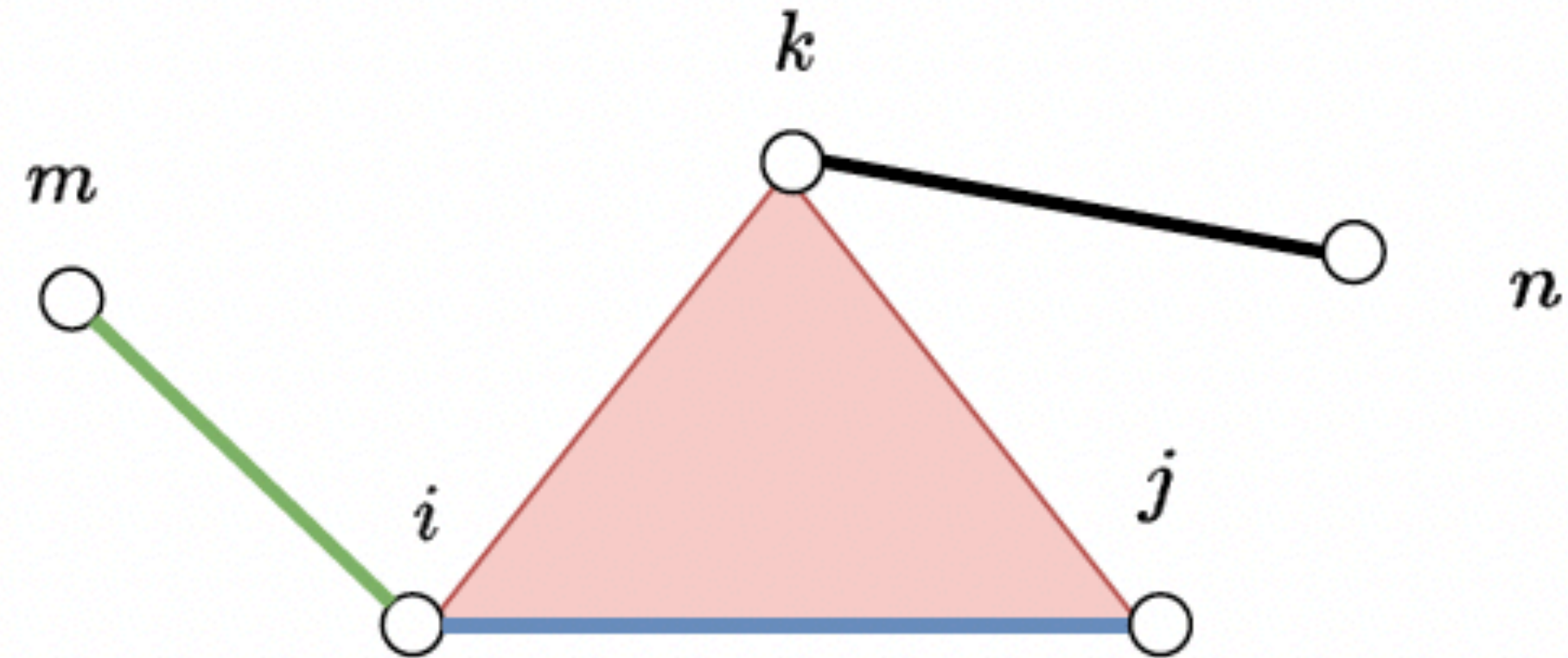
=

train size distribution  
of egonetwork activity





# Temporal correlation of events with different orders overlapping in topology



events of different orders overlapping in topology

=

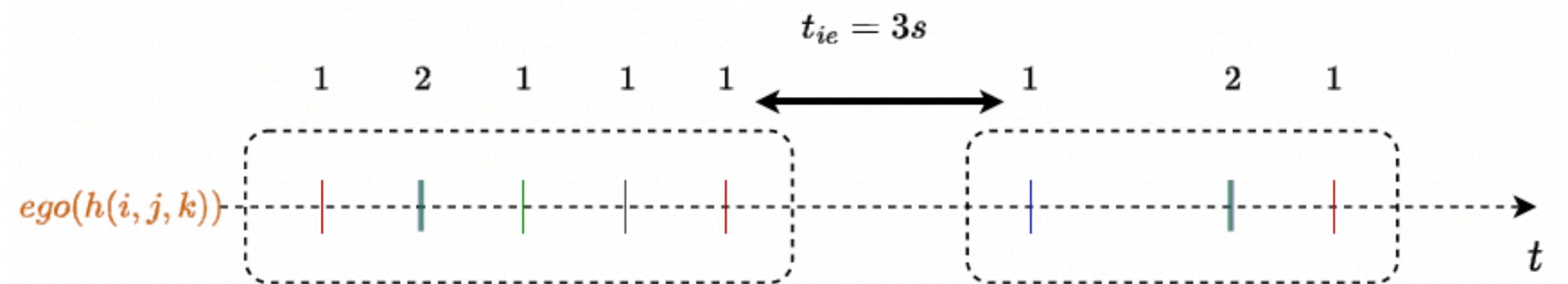
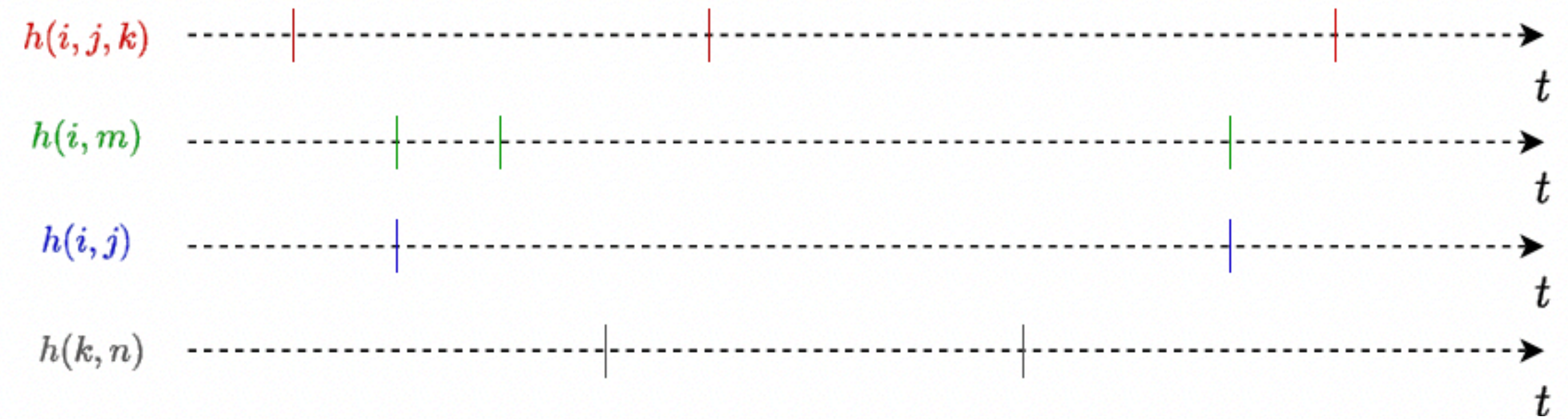
egonetwork of a hyperlink of order d

temporal correlation of egonetwork activity

=

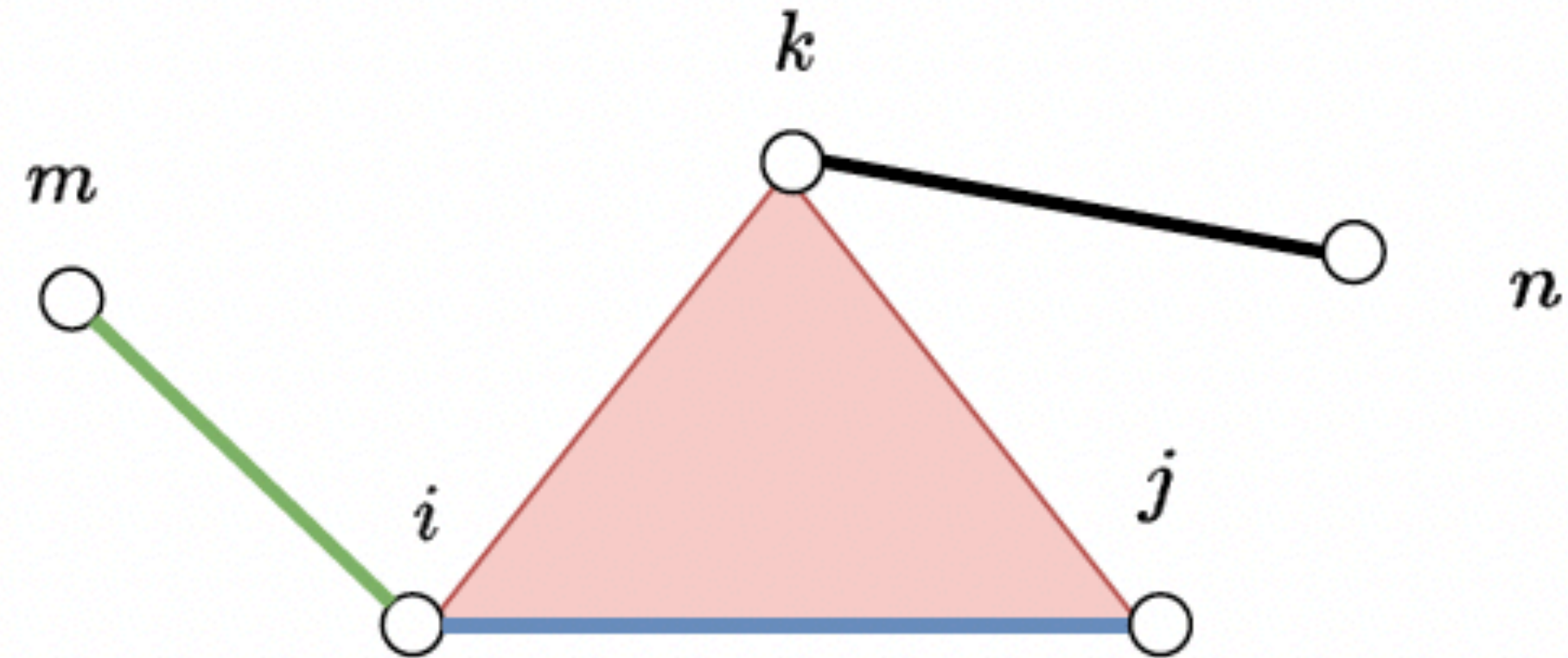
train size distribution  
of egonetwork activity

$$\Delta t = 2s$$





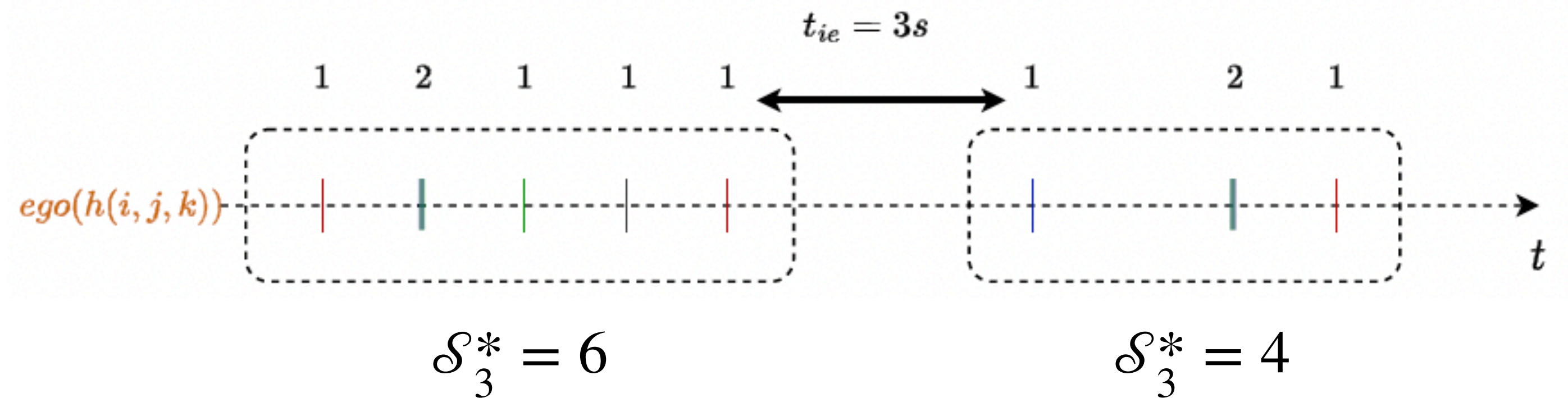
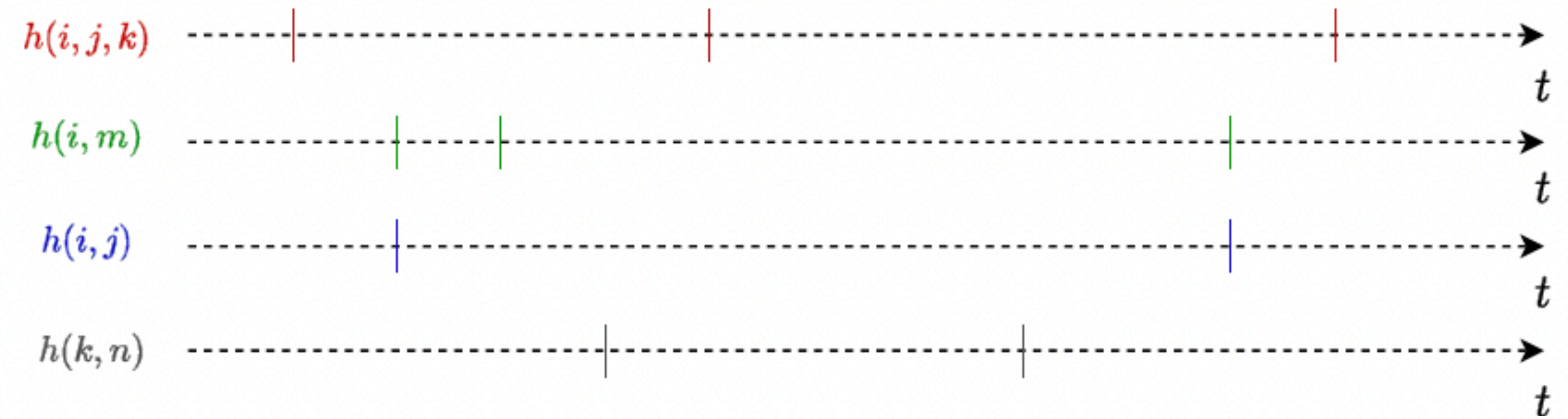
# Temporal correlation of events with different orders overlapping in topology



events of different orders overlapping in topology  
=  
egonetwork of a hyperlink of order d

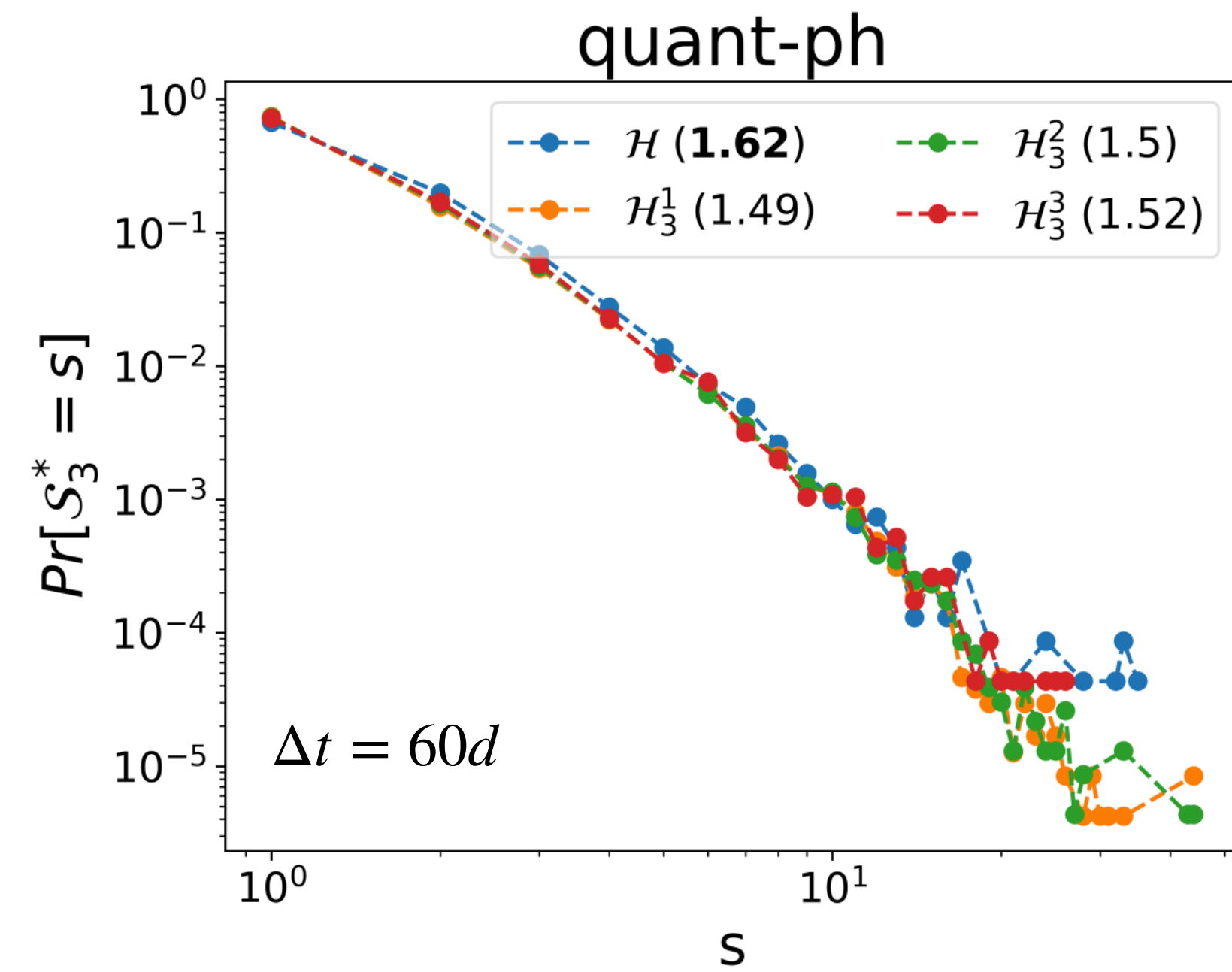
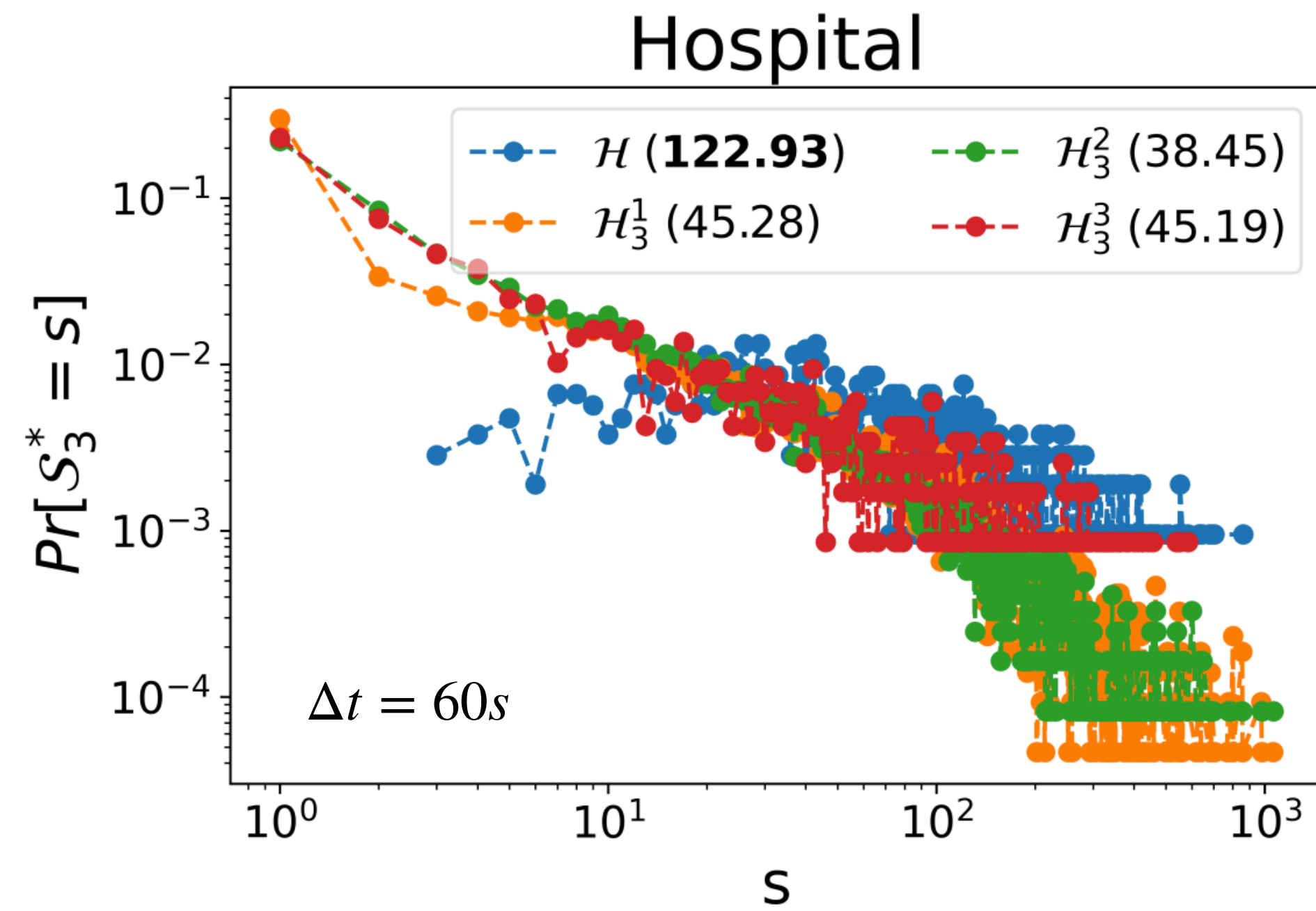
temporal correlation of egonetwork activity  
=  
train size distribution  
of egonetwork activity

$$\Delta t = 2s$$



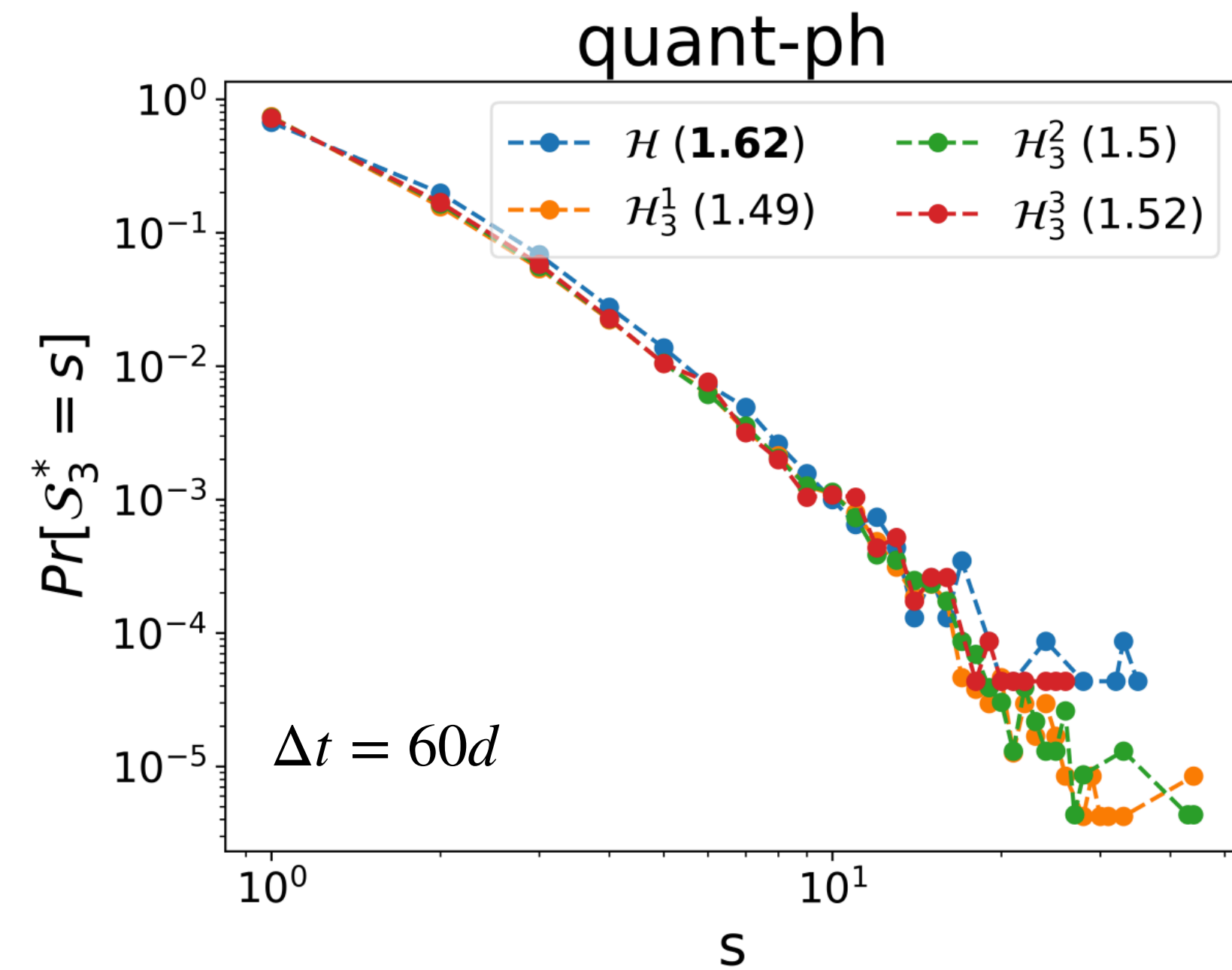
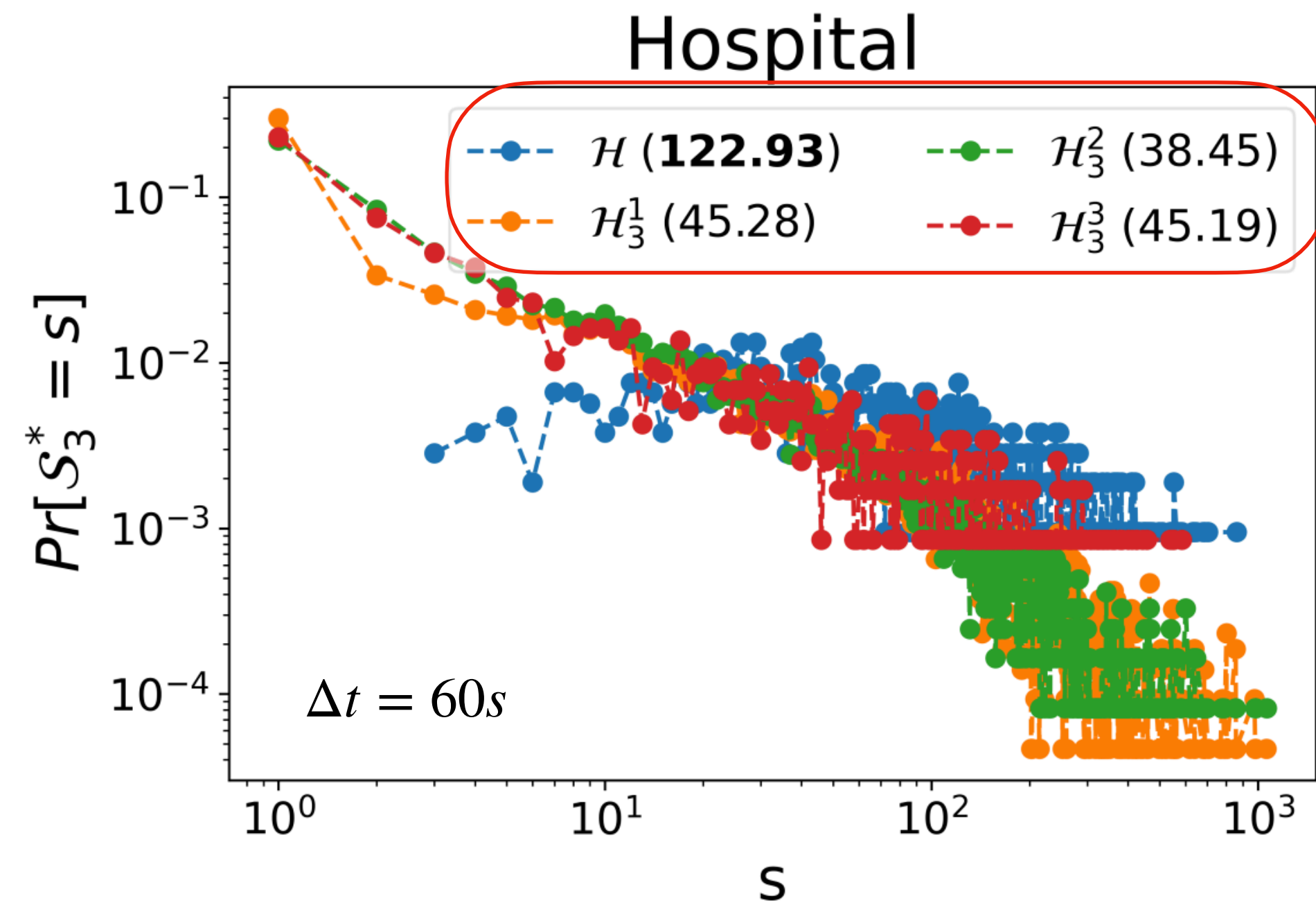
# Temporal correlation of events with different orders overlapping in topology

# Temporal correlation of events with different orders overlapping in topology



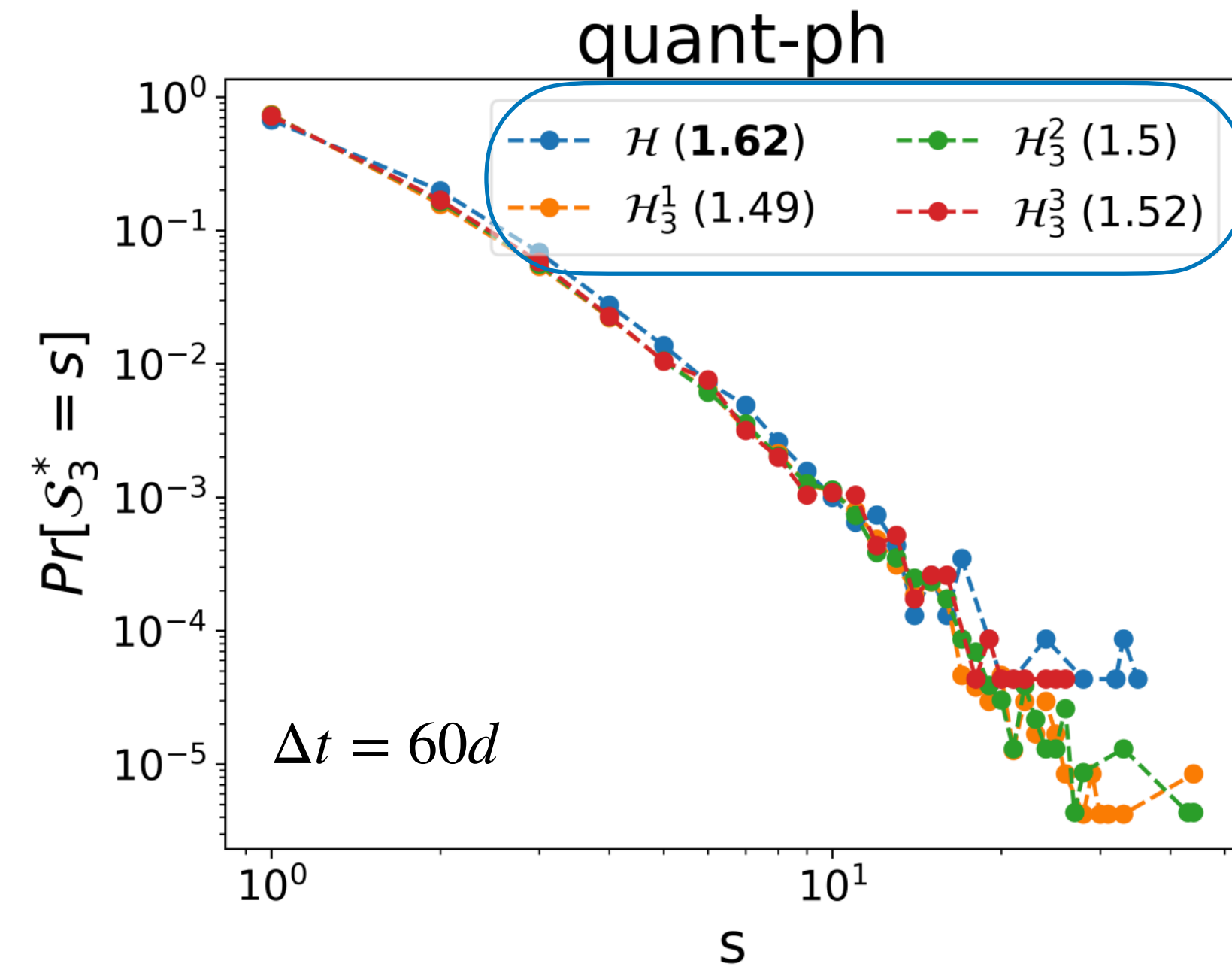
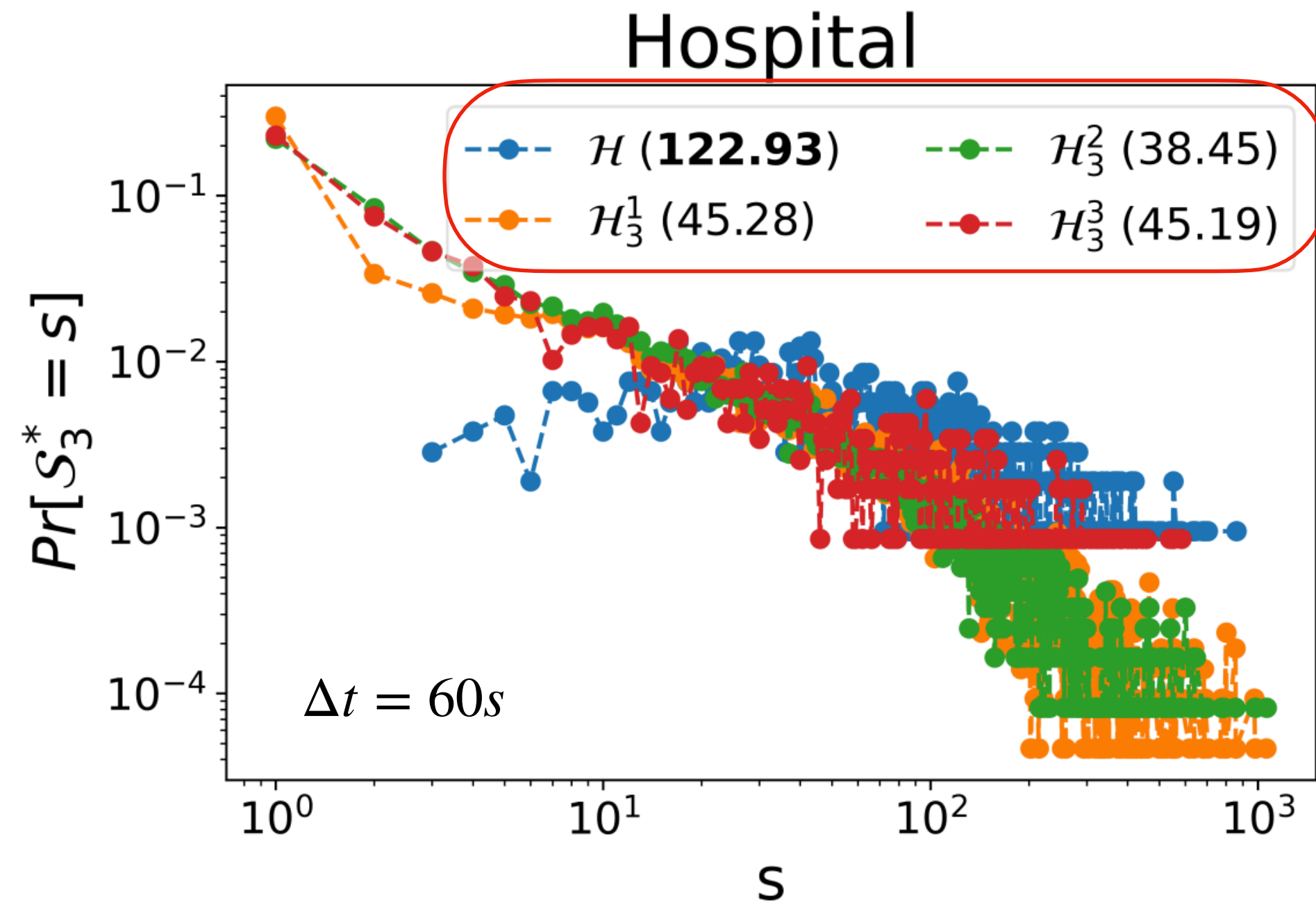


# Temporal correlation of events with different orders overlapping in topology

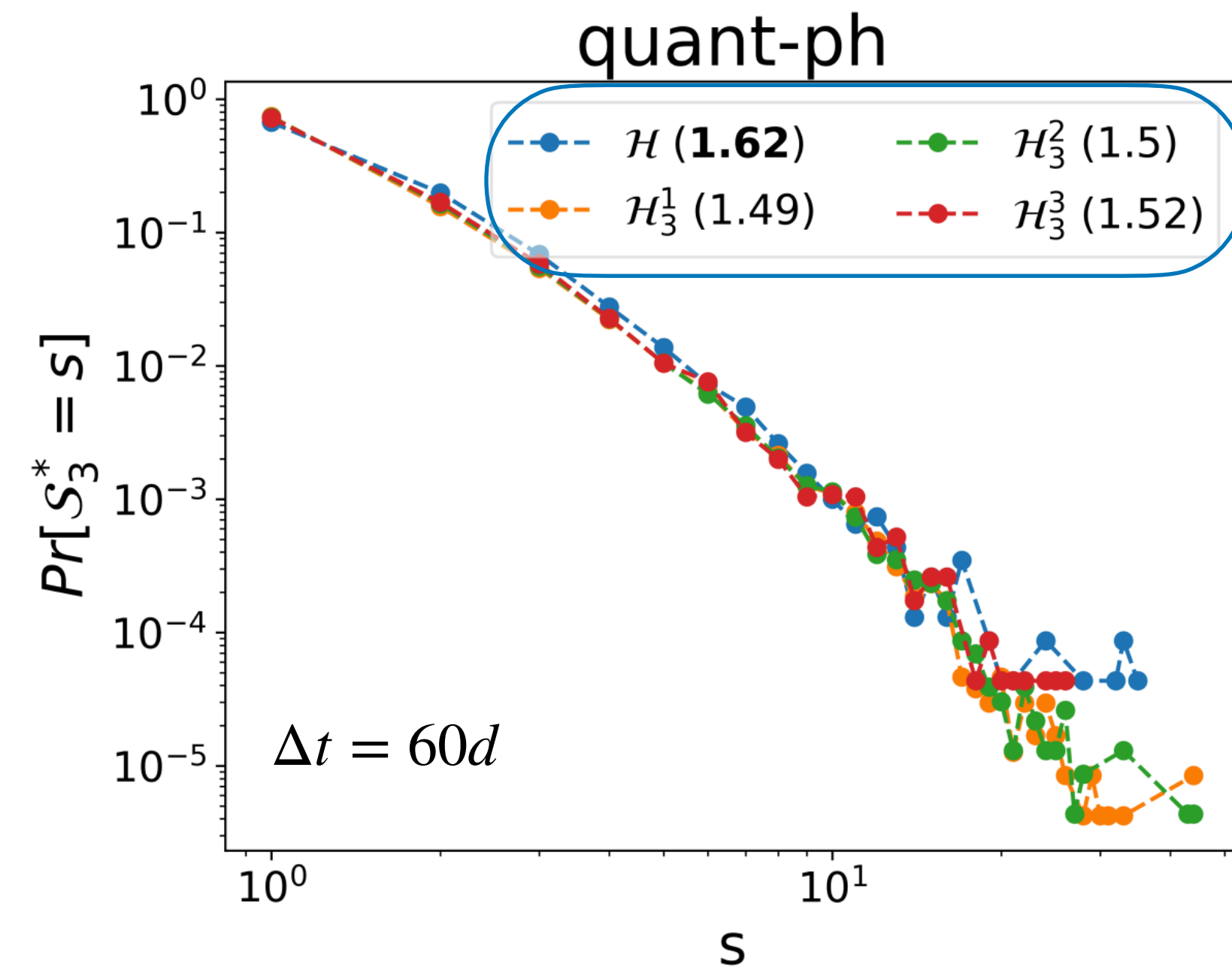
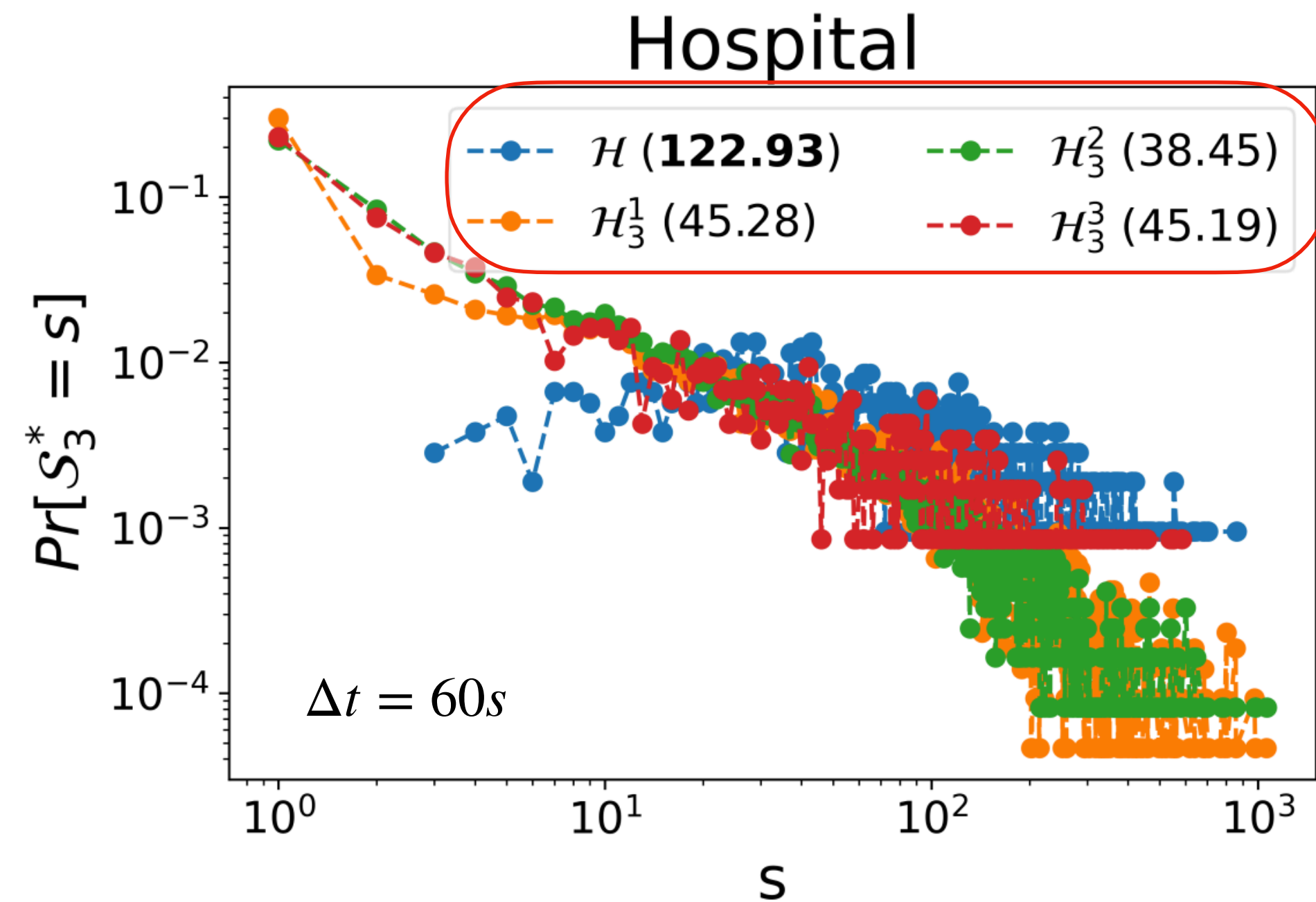




# Temporal correlation of events with different orders overlapping in topology

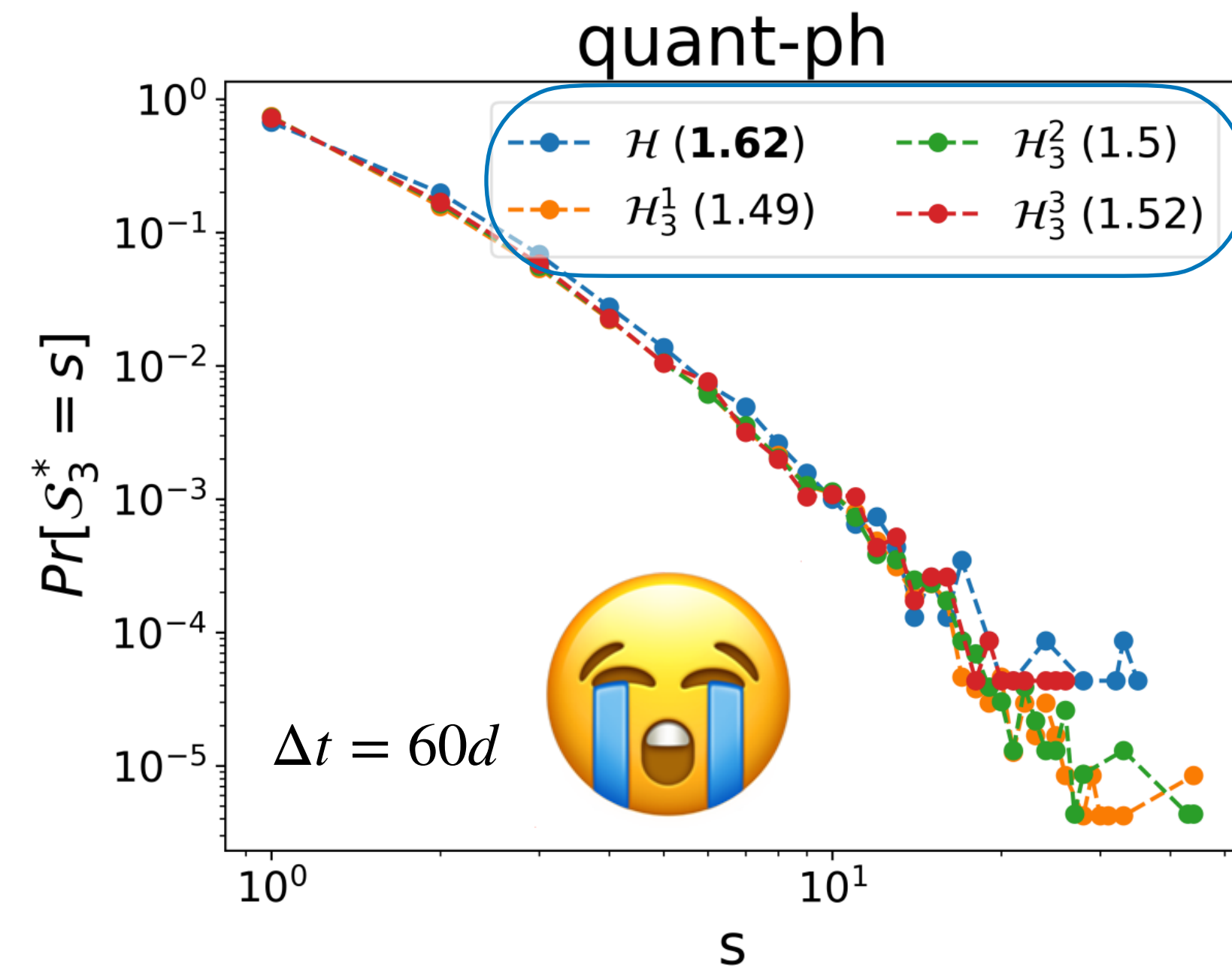
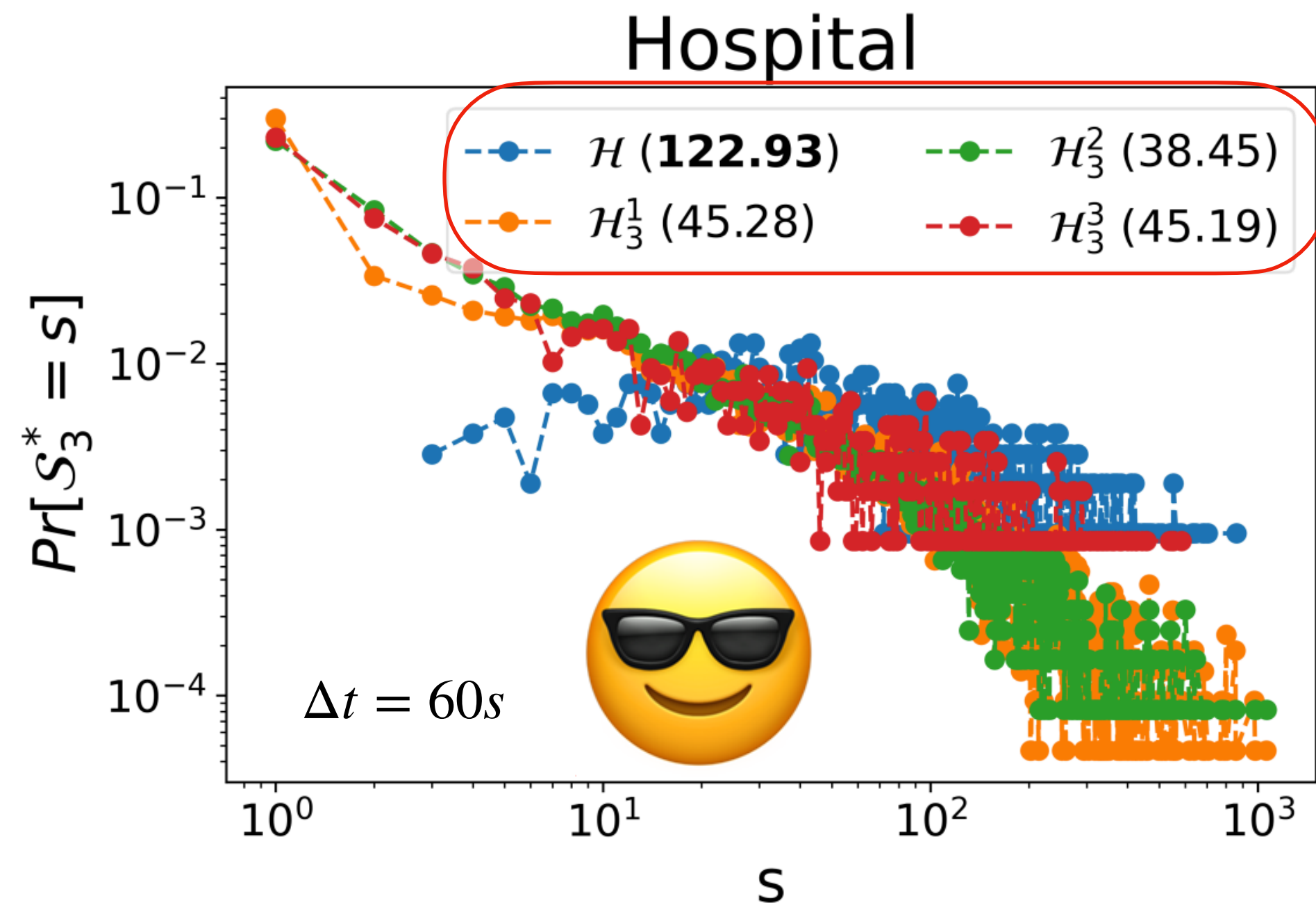


# Temporal correlation of events with different orders overlapping in topology



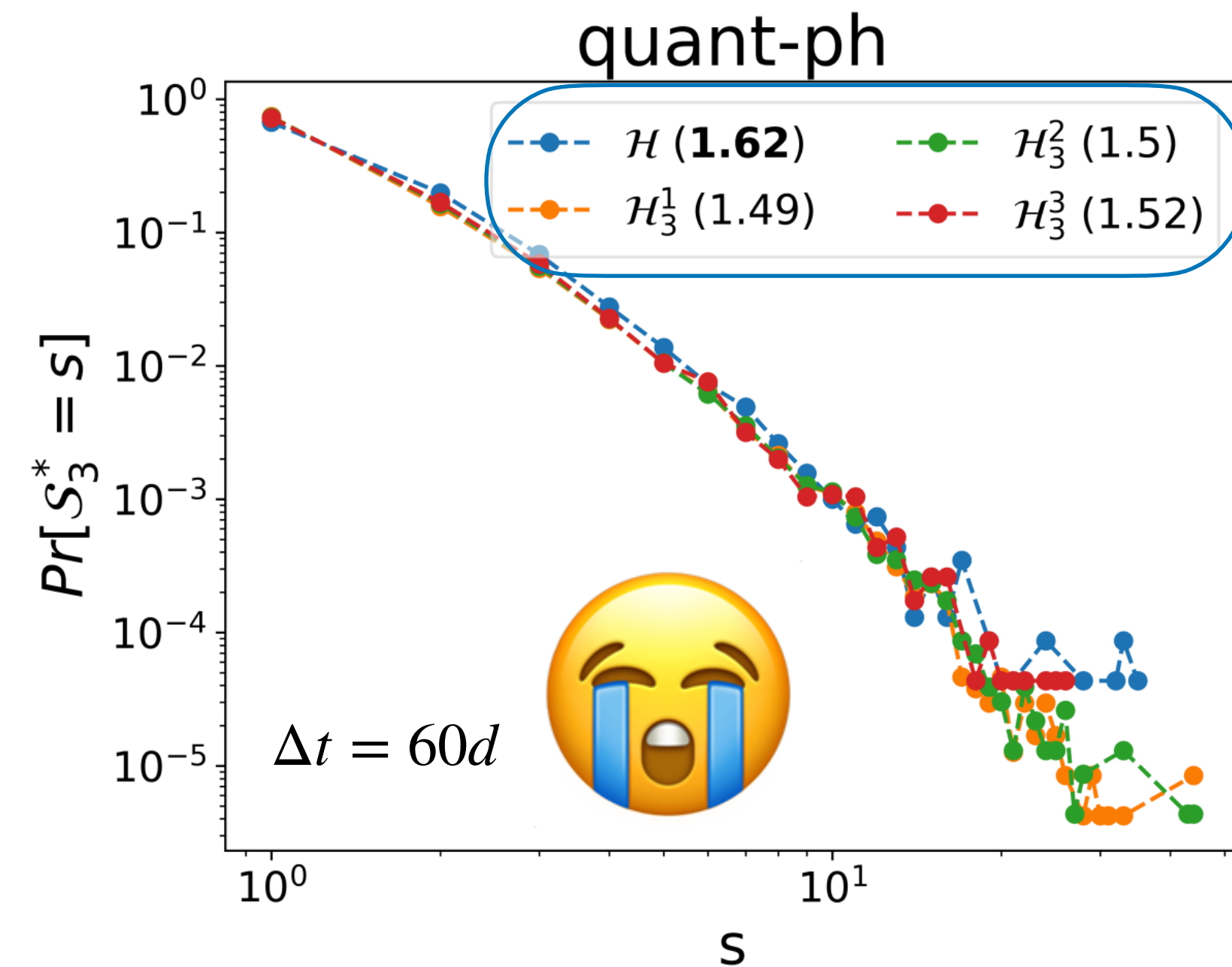
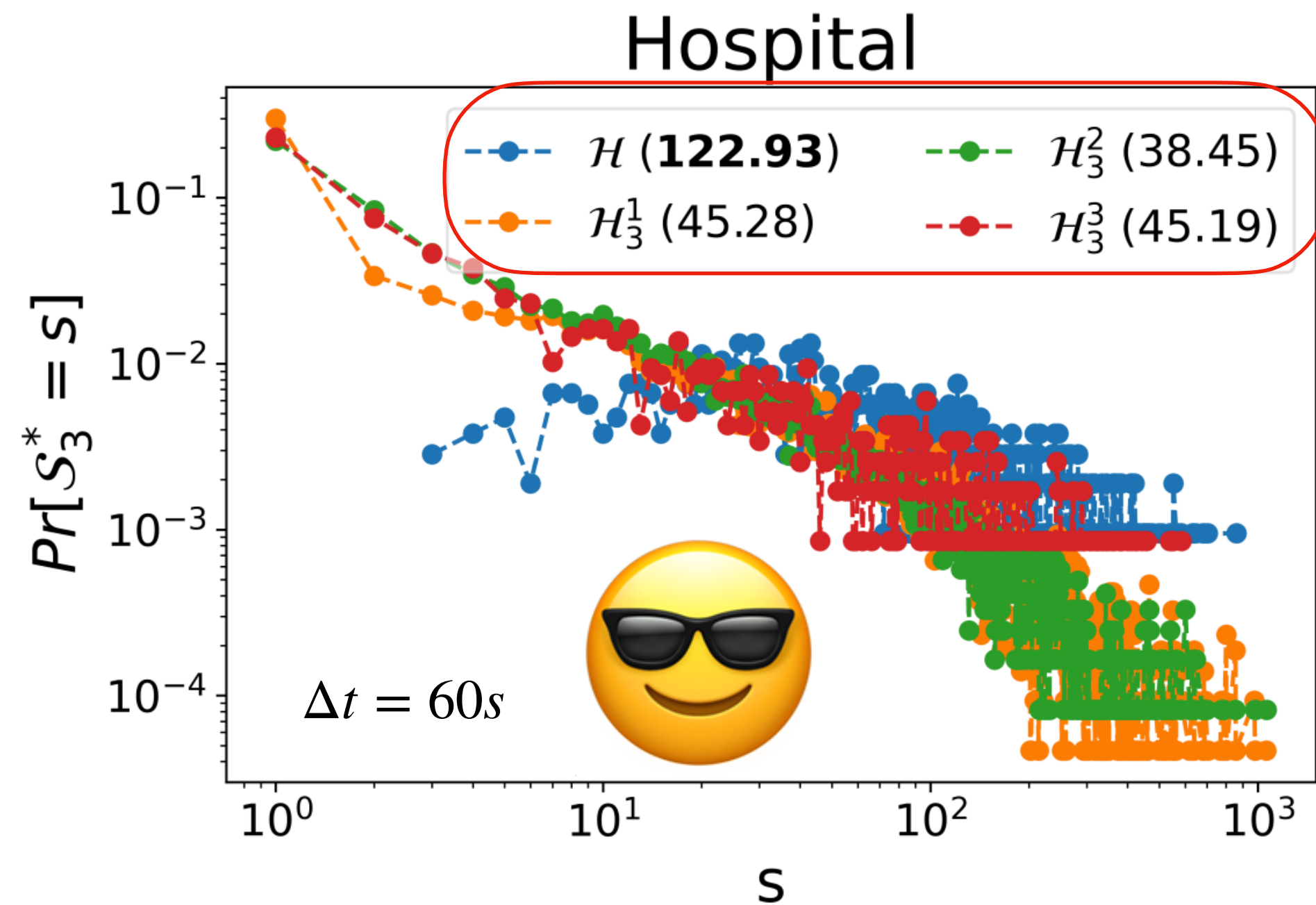
temporal correlation of events overlapping in topology  
 =  
 longer trains on average

# Temporal correlation of events with different orders overlapping in topology



temporal correlation of events overlapping in topology  
=  
longer trains on average

# Temporal correlation of events with different orders overlapping in topology



temporal correlation of events overlapping in topology  
 =  
 longer trains on average



# Take home messages

- In physical contacts, **events** of different orders **close in time** tend to happen also **close in topology**
- In every considered networks, **events of different orders overlap in component nodes**
- In physical contacts, **events** with different orders **overlapping in component nodes are correlated in time**





Dr. Huijuan Wang



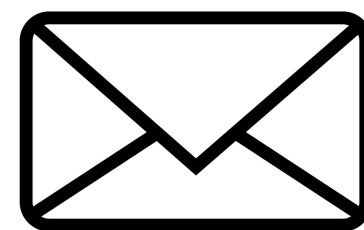
## scientific reports

 Check for updates

OPEN

### Temporal-topological properties of higher-order evolving networks

Alberto Ceria<sup>✉</sup> & Huijuan Wang



**ceriaac@liacs.leidenuniv.nl**



**@cerialbo**

# Dank je wel!