Temporal Network
Prediction and Interpretation

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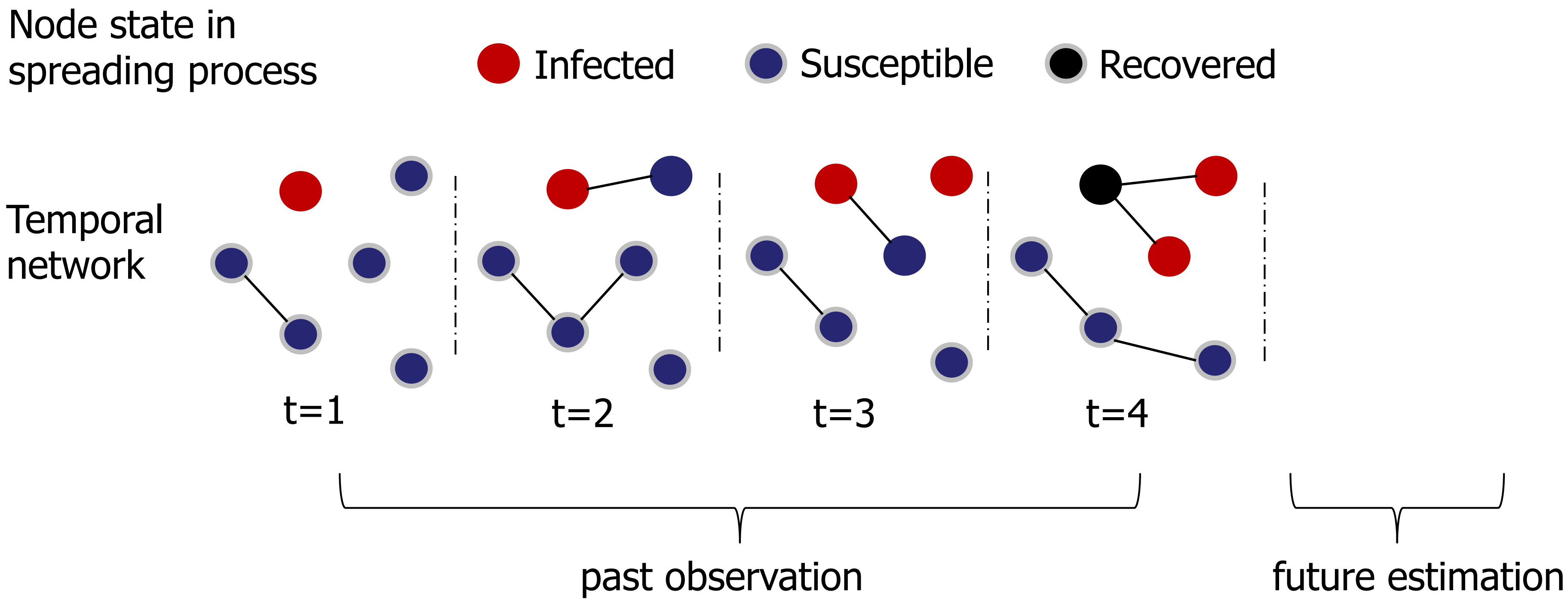
Epidemic Spreading on Temporal Networks



Temporal network: the topology of a network changes over time.

Examples: two individuals are connected when they meet face-to-face (call or collaborate).

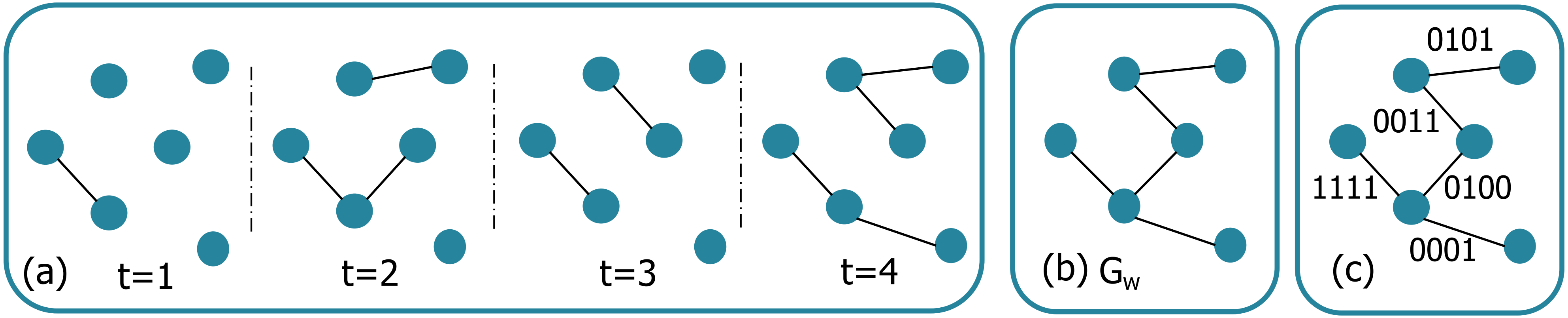
Ambition: Realistic Temporal Network Model
Mechanisms that form temporal networks

Temporal Network Prediction and Interpretation
Intrinsic properties/mechanisms that enable the prediction

Interpretable Learning Algorithms

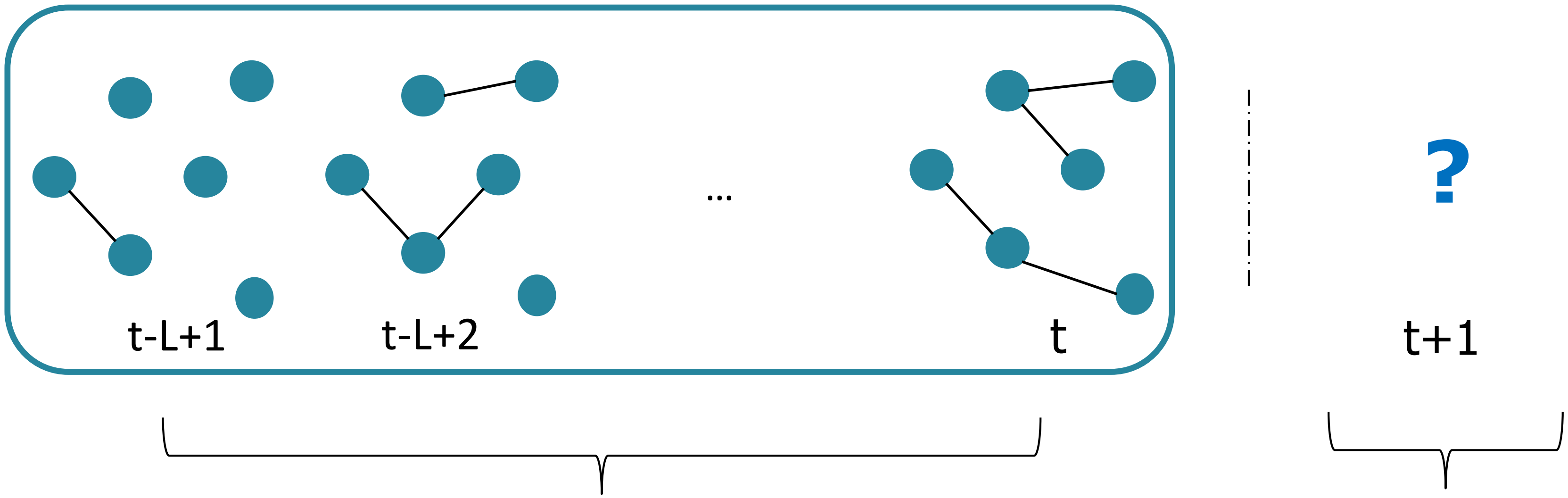
Network based Models

Temporal Network Representation



Representation: aggregated network G_w, in which each link i is specified by a time series $\{x_i(1), x_i(2),..., x_i(T)\}$ that encodes the existence of interaction/contact at each time step.

Temporal Network predication Problem



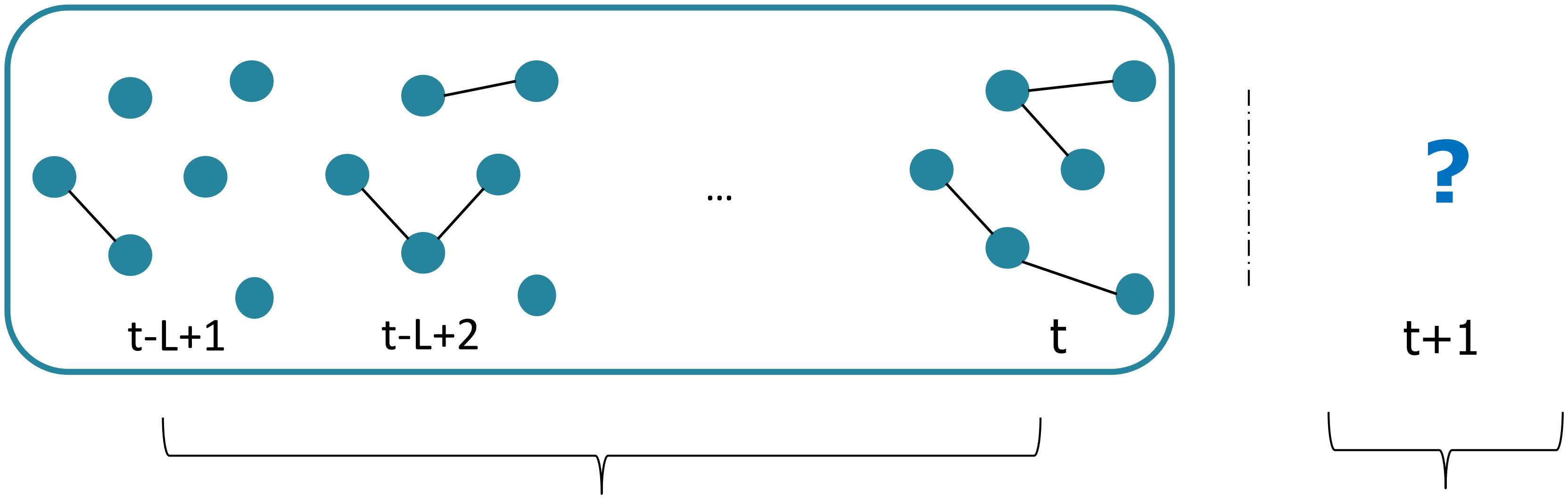
past observation/training set

Future prediction/test set

G_w over [1,T] is known.

L. Zou, X. -X. Zhan, J. Sun, A. Hanjalic and H. Wang, "Temporal Network Prediction and Interpretation," in IEEE Transactions on Network Science and Engineering, vol. 9, no. 3, pp. 1215-1224, 2022

Lasso Regression



past observation/training set

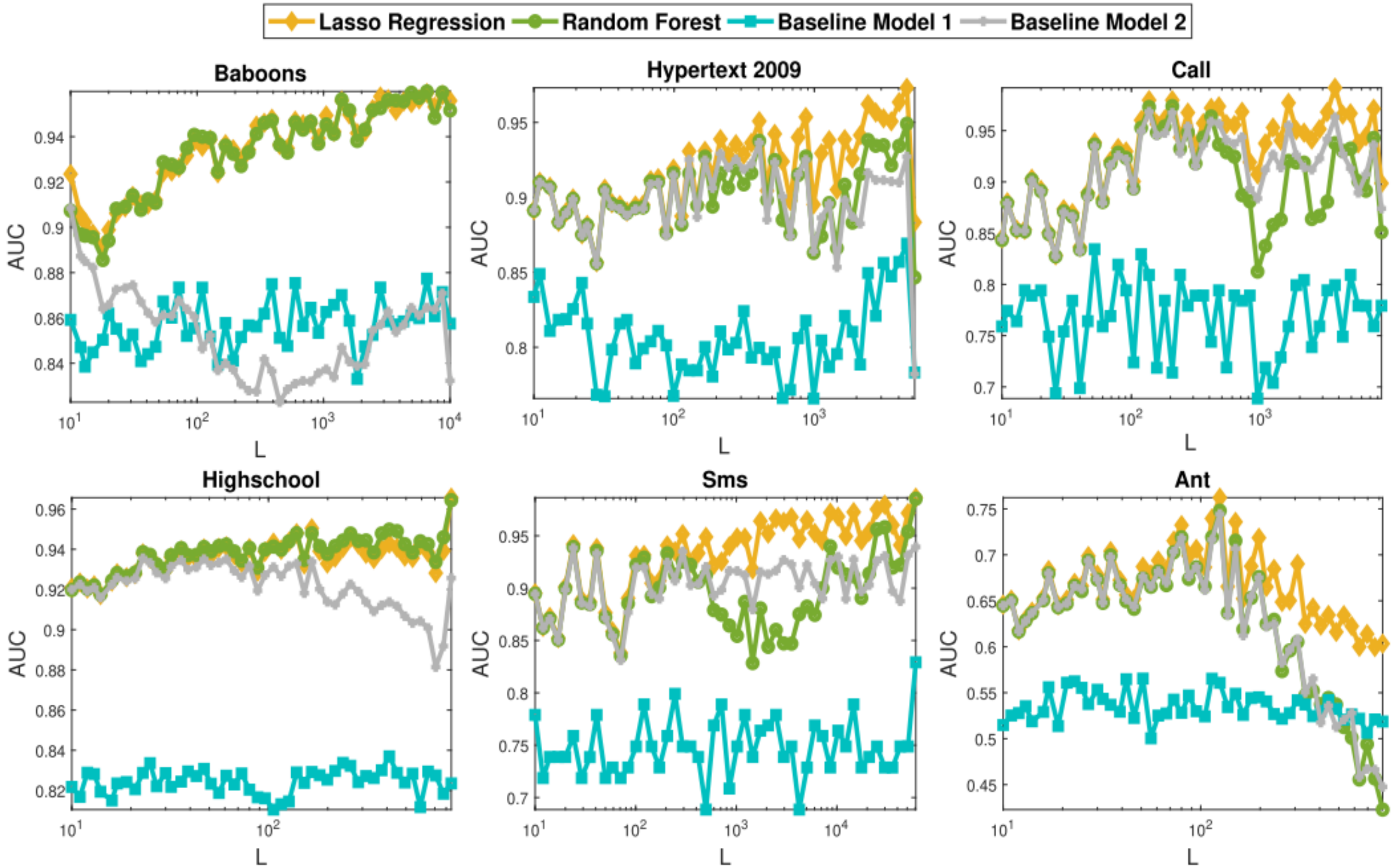
Future prediction/test set

$$x_i(t+1) = \sum_{j=1}^{M} x_j(t)\,\beta_{ji} + c_i$$

Objective:

$$\min_{\{\beta_{1i},\beta_{2i},\dots,\beta_{Mi},c_i\}} \left\{ \sum_{t=p}^{p+L-1} \left(x_i(t+1) - \sum_{j=1}^{M} x_j(t)\,\beta_{ji} - c_i \right)^2 + \alpha \sum_{j=1}^{M} \left|\beta_{ji}\right| \right\}$$

Prediction Quality



Baseline Model 1: $x_i(t+1) = x_i(t)$

Baseline Model 2: $x_i(t+1) = x_i(t)\beta_{ii} + c_i$

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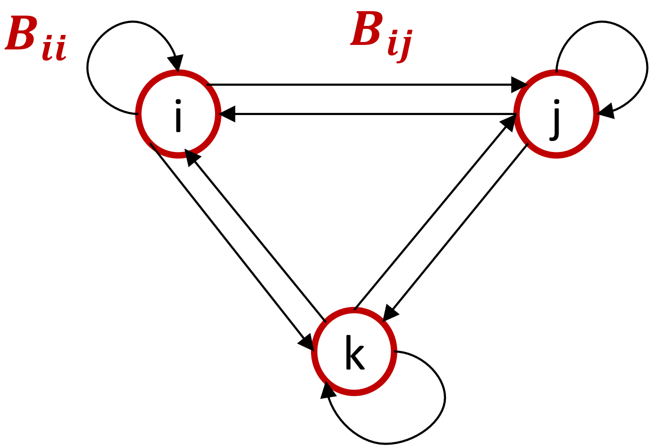
Prediction Backbone Network

M nodes → M links in the aggregated network

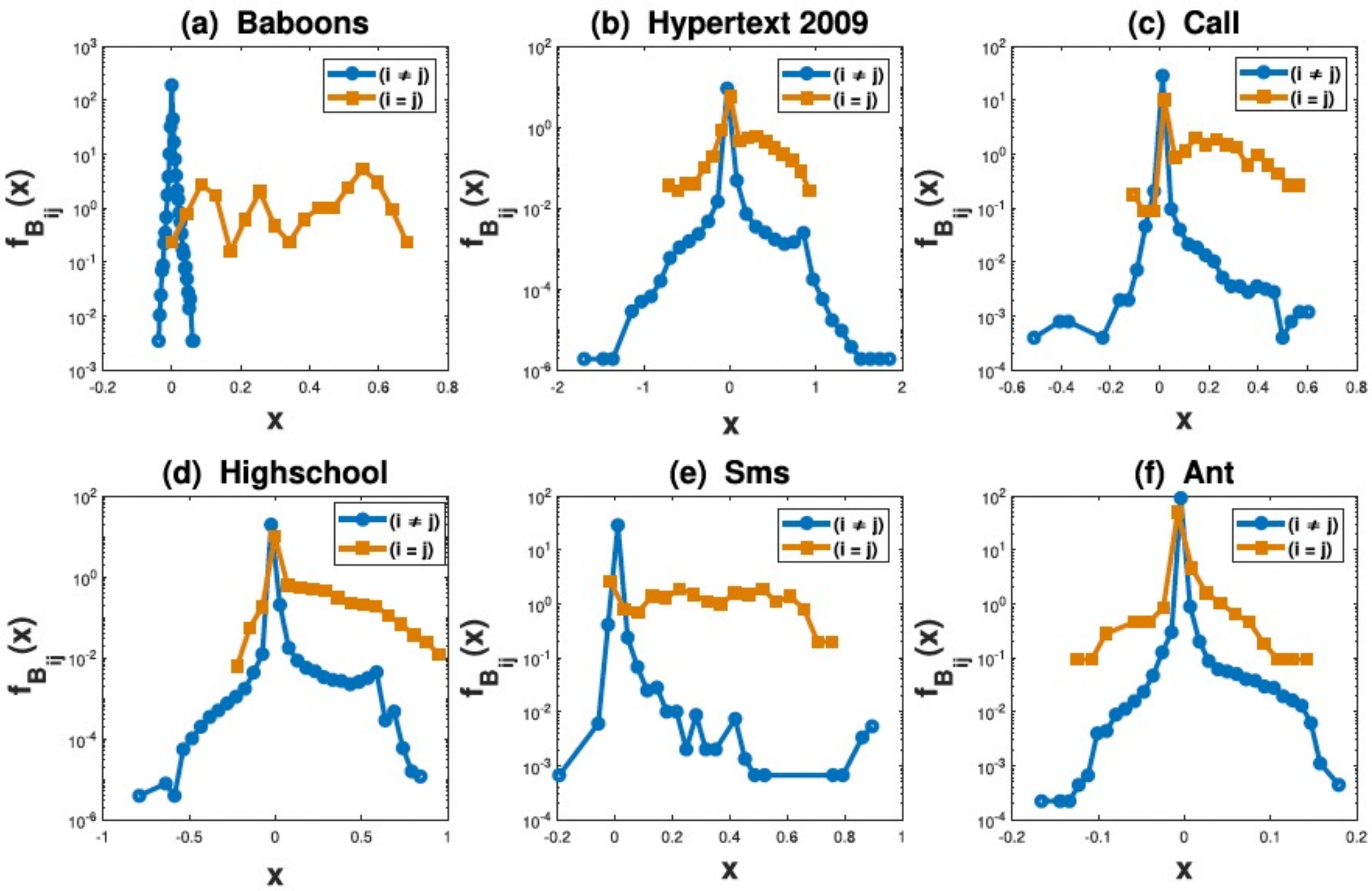
Fully connected, directed, with self loops and weight $\boldsymbol{B_{ij}}$=**E[**$\boldsymbol{\beta_{ij}}$**]** average over training sets



$\boldsymbol{B_{ij}}$: influence of link i on link j in determining link j's activity.

$$x_i(t+1) = \sum_{i=1}^{M} x_j(t)\,\beta_{ji} + c_i$$

Prediction Backbone Network

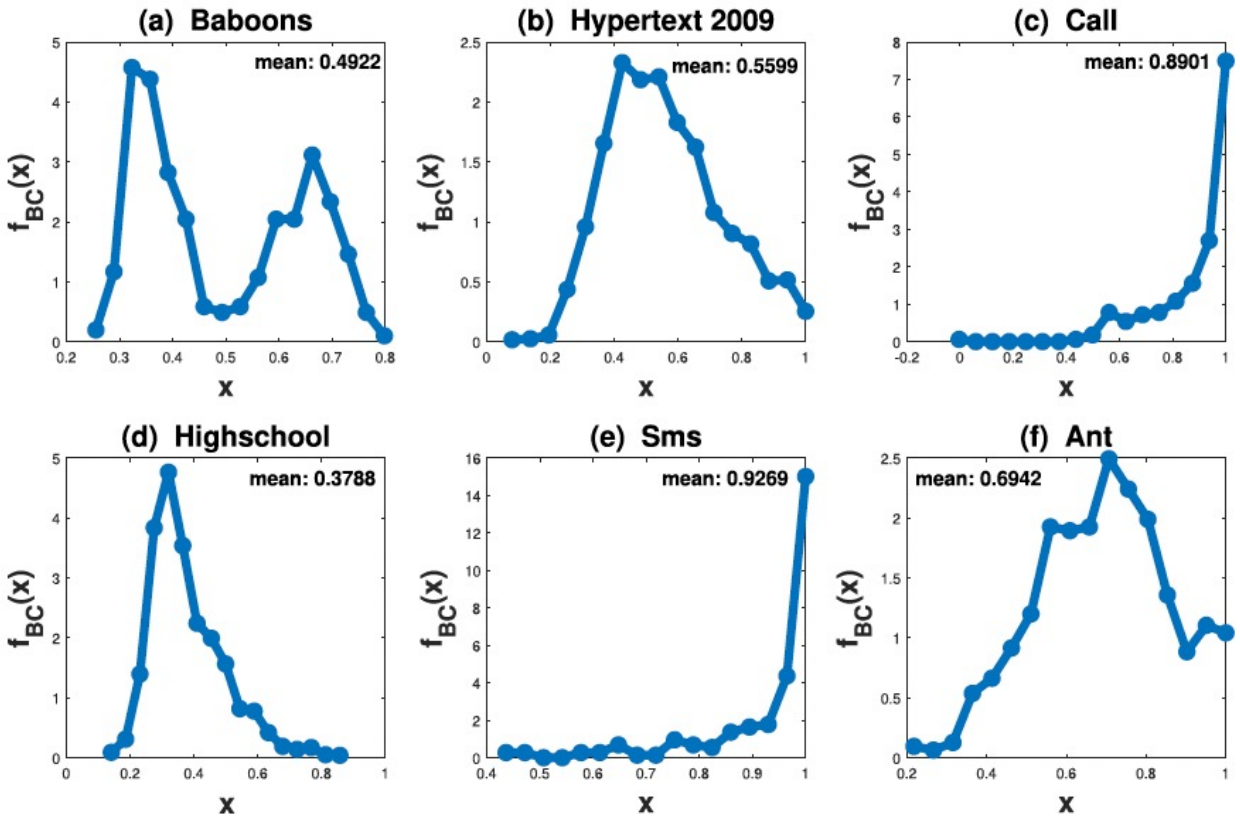


Probability density function of B_{ij}

A link's activity is influenced more by the activity of its own than activities of other links.

Backbone in Relation to Time Series

Pearson correlation $R_{x_i x_j}(t-1,t)$ between $\{x_i(t)\}_{t=1,2,\ldots,T-1}$ and $\{x_j(t)\}_{t=2,3,\ldots,T}$



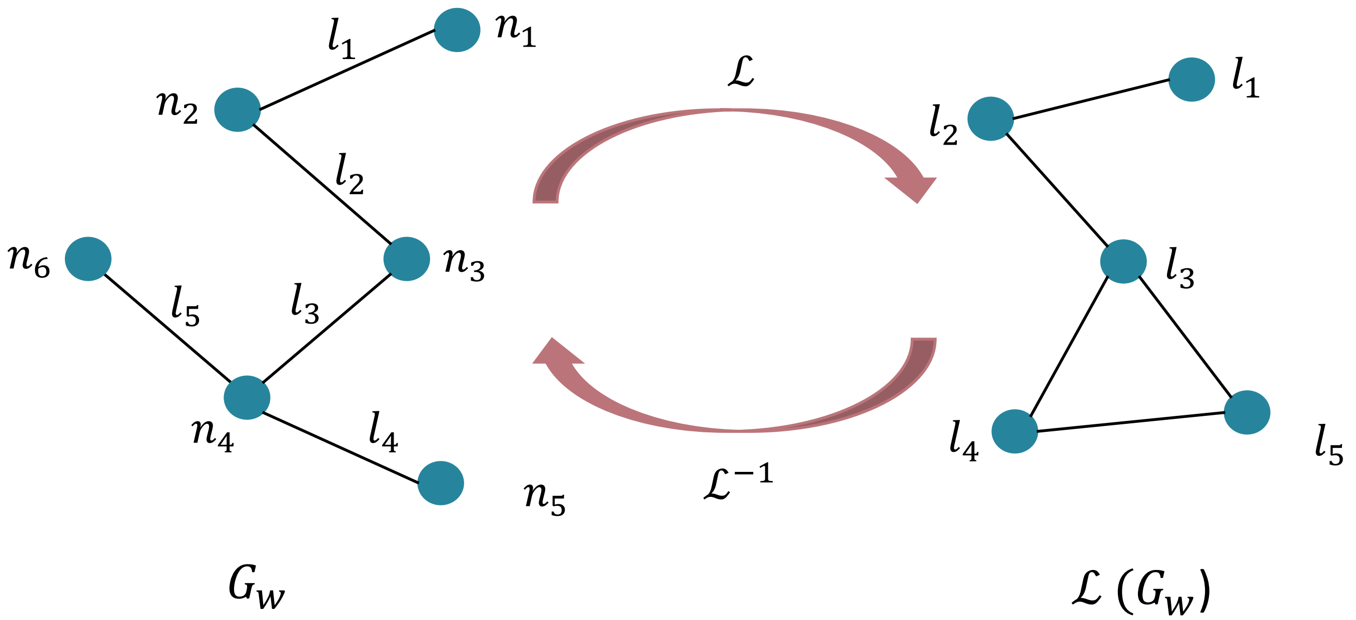
Correlation BC between B_{ij} and $R_{x_i x_j}(t-1,t)$

B_{ij} and $R_{x_i x_j}(t-1,t)$ are postively correlated at each j

Self influence is larger in networks with stronger activity autocorrelation

Backbone in Relation to Aggregated Network

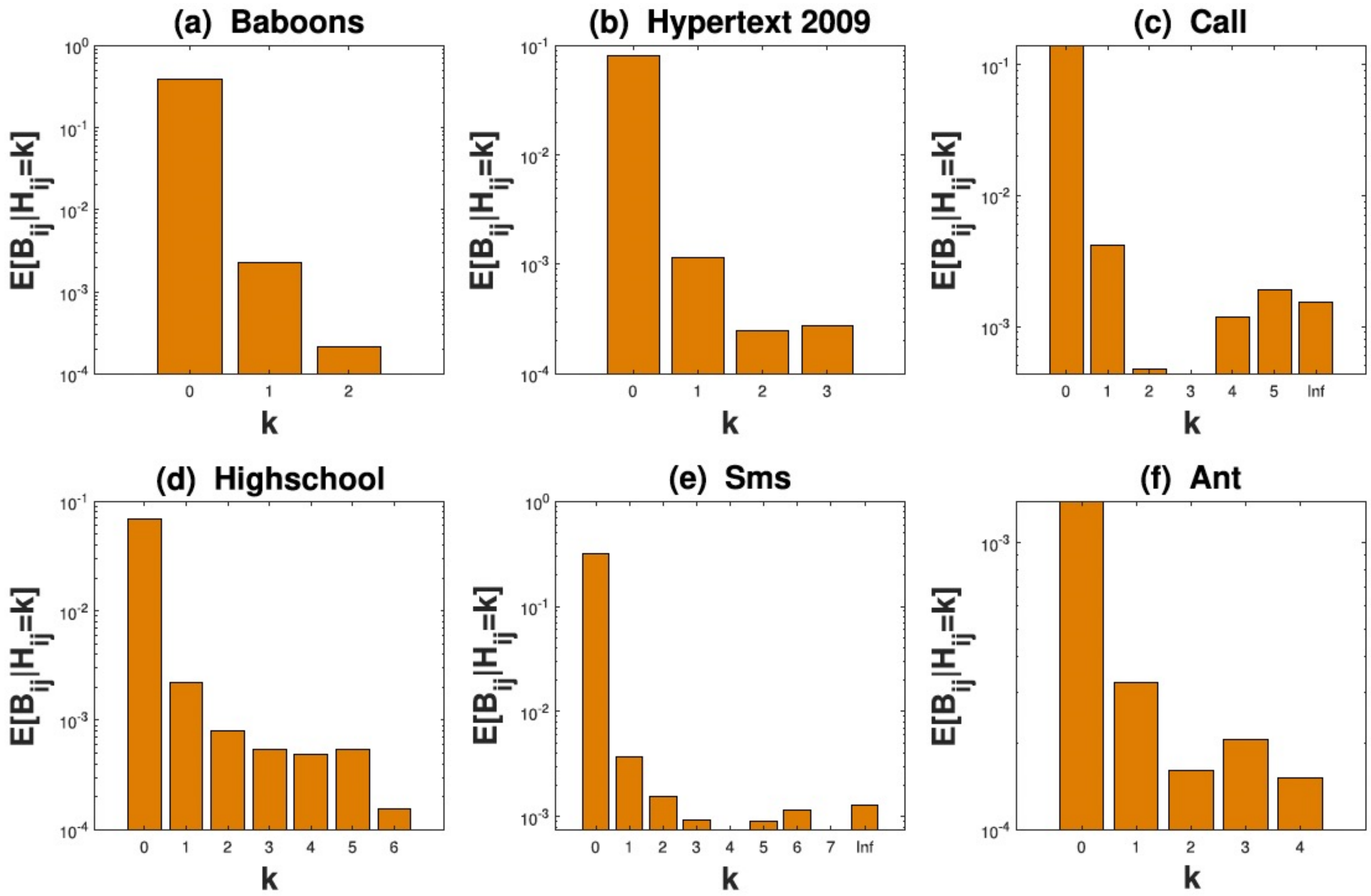
Links that are close in the aggregated network tend to have a high influence on each other?



Distance of two links in the aggregated network G_w is the hopcount between their corresponding nodes in the line graph $\mathcal{L}\,(G_w)$ of G_w.

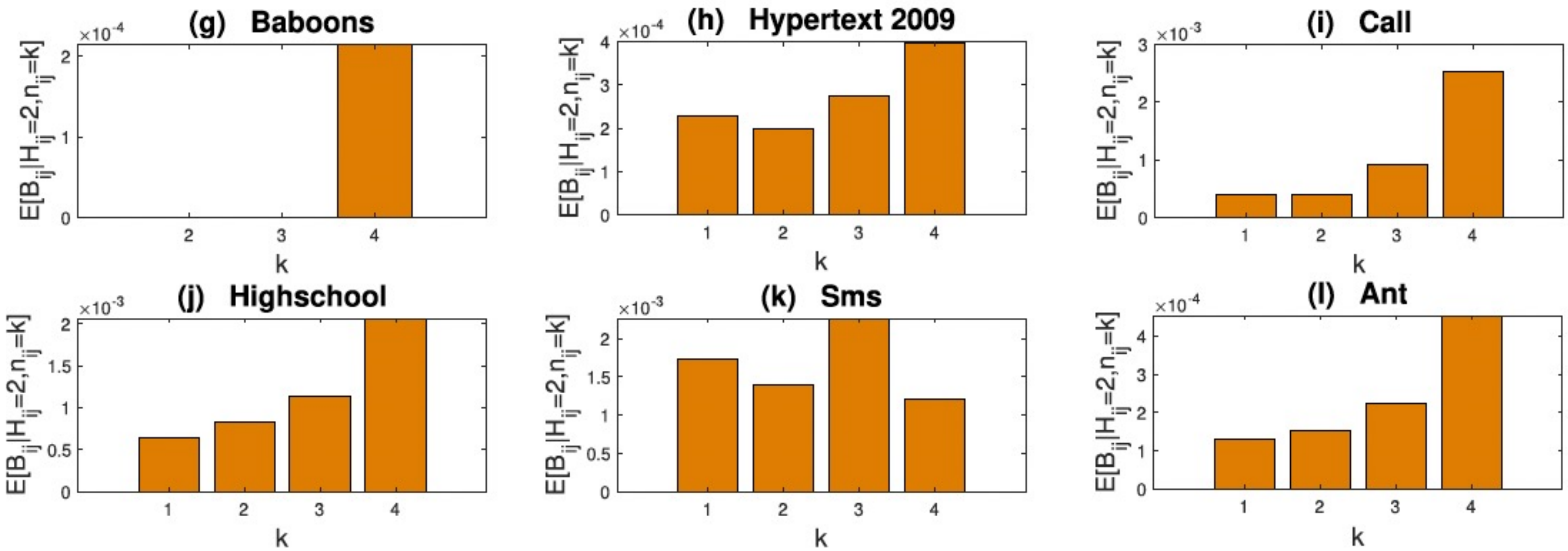
It is also the minimal hopcount between two end nodes from the two links respectively in G_w plus one

Backbone in Relation to Aggregated Network



Links that are close in the aggregated network tend to have a high influence on each other

Backbone in Relation to Aggregated Network



Links (nodes in line graph) that have more shortest paths, tend to have a higher influence on each other

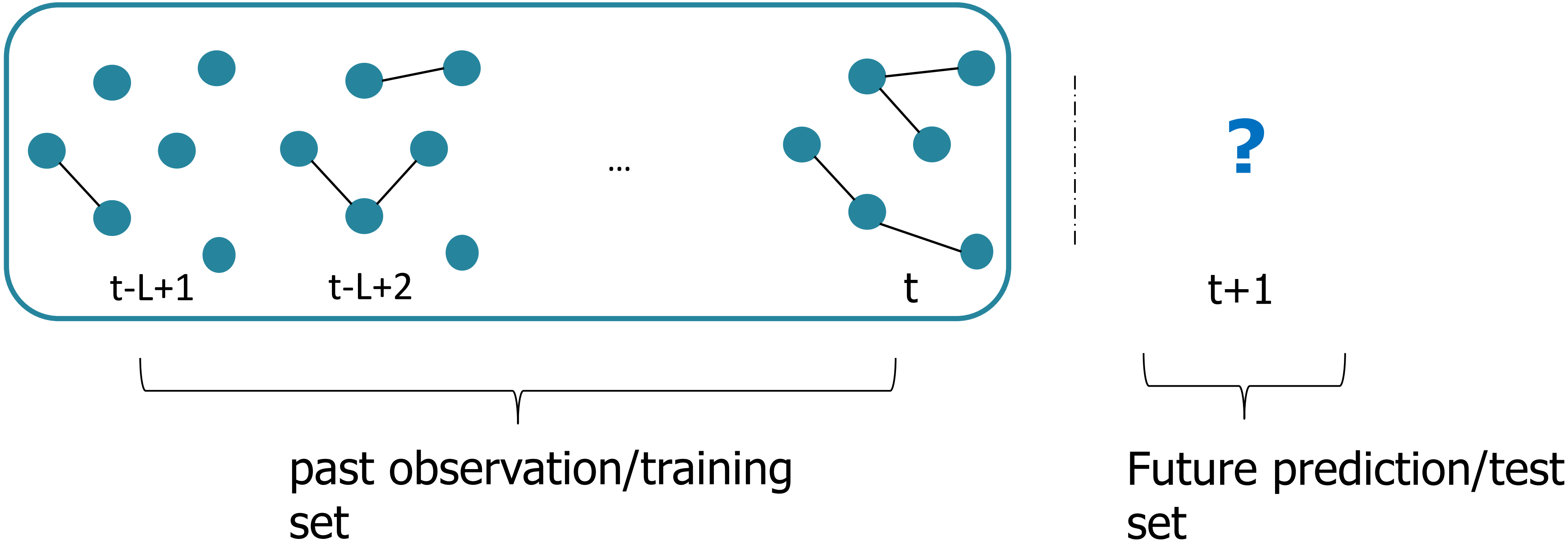
Conclusion

A link's next step activity is mainly influenced by the current activity of the link itself and of other links that are close to the link and/or correlated with the link in activity time series.

Learning algorithms capture **network properties and correlation in activity time series,** which are usually used by network-based prediction methods.

L. Zou, X. -X. Zhan, J. Sun, A. Hanjalic and H. Wang, "Temporal Network Prediction and Interpretation," in IEEE Transactions on Network Science and Engineering, vol. 9, no. 3, pp. 1215-1224, 2022

Network based Temporal Network predication

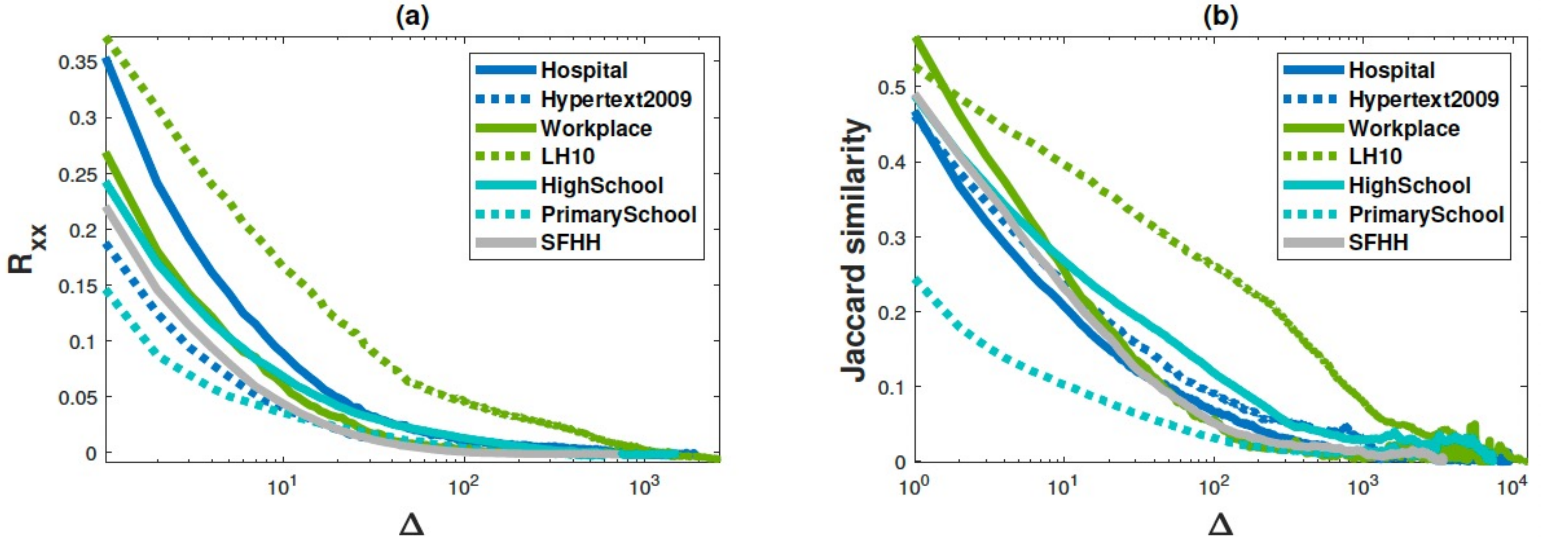


Learning models : $x_i(t+1) = f_i\{x_1(t), x_2(t), \ldots, x_M(t)\}$

Network based models: $x_i(t+1) = \boldsymbol{f}\{G_t, G_{t-1}, \ldots, G_{t-L+1}\}$

L. Zou, A. Ceria and H. Wang, "Short- and long-term temporal network prediction based on network memory", Applied Network Science 8, 76, 2023.

Memory in Temporal Networks



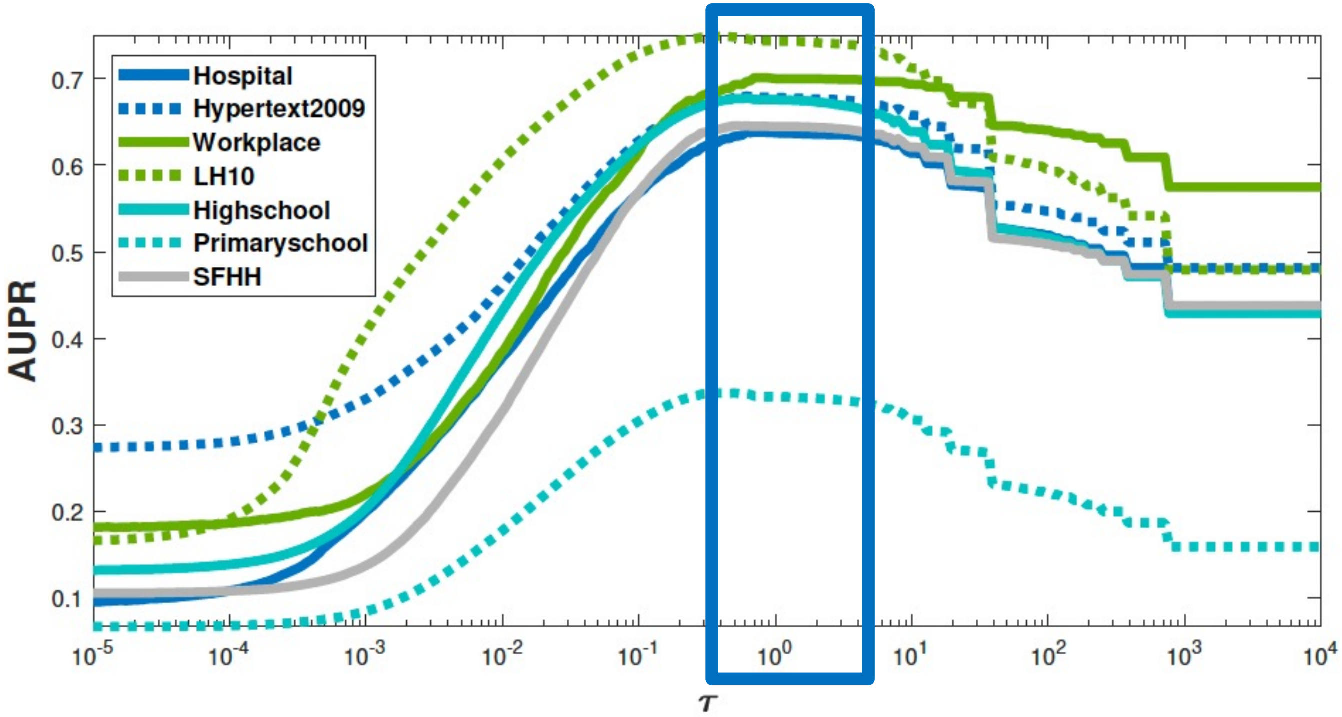
Pearson correlation $R_{x_i x_i}(\Delta)$ between $\{x_i(t)\}_{t=1,2,\ldots,T-\Delta}$ and $\{x_i(t)\}_{t=1+\Delta,2+\Delta,\ldots,T}$

Jaccard similarity $(G_t, G_{t+\Delta}) = \frac{|E_t \cap E_{t+\Delta}|}{|E_t \cup E_{t+\Delta}|}$

Both decay with time lag Δ -> time-decaying memory

Self-driven (SD) model

SD tendency: $w_i(t+1) = \sum_{k=t-L+1}^{k=t} e^{-\tau(t-k)}\, x_i(k)$



static $\tau \in [0.5, 5]$
~ optimal for all networks

static τ or dynamic τ for samples of prediction step t+1

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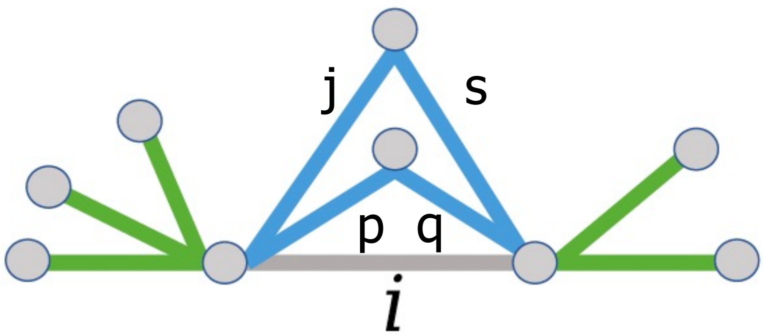
Self and Cross-driven (SCD) model

Compute the SD tendency for each link:

$$w_i(t+1) = \sum_{k=t-L+1}^{k=t} e^{-\tau(t-k)}\, x_i(k)$$



Network	β_1^*	β_2^*	β_3^*	cc
Hospital	0.31	0.07	0.00	0.37
Hypertext2009	0.32	-0.02	0.00	0.32
Workplace	0.32	0.00	0.00	0.28
LH10	0.32	0.21	0.00	0.41
HighSchool	0.33	0.04	0.00	0.38
PrimarySchool	0.24	0.48	-0.02	0.54
SFHH	0.32	0.03	0.01	0.21

$$h_i(t+1) = \beta_0^* + \boldsymbol{\beta_1^* w_i(t+1)} + \boldsymbol{\beta_2^* u_i(t+1)} + \boldsymbol{\beta_3^* g_i(t+1)}$$

$$u_i(t+1) = \frac{1}{2}\left(\sqrt{w_j(t+1)w_s(t+1)} + \sqrt{w_p(t+1)w_q(t+1)}\right)$$

averge geometric mean

$g_i(t+1)$ average SD tendency

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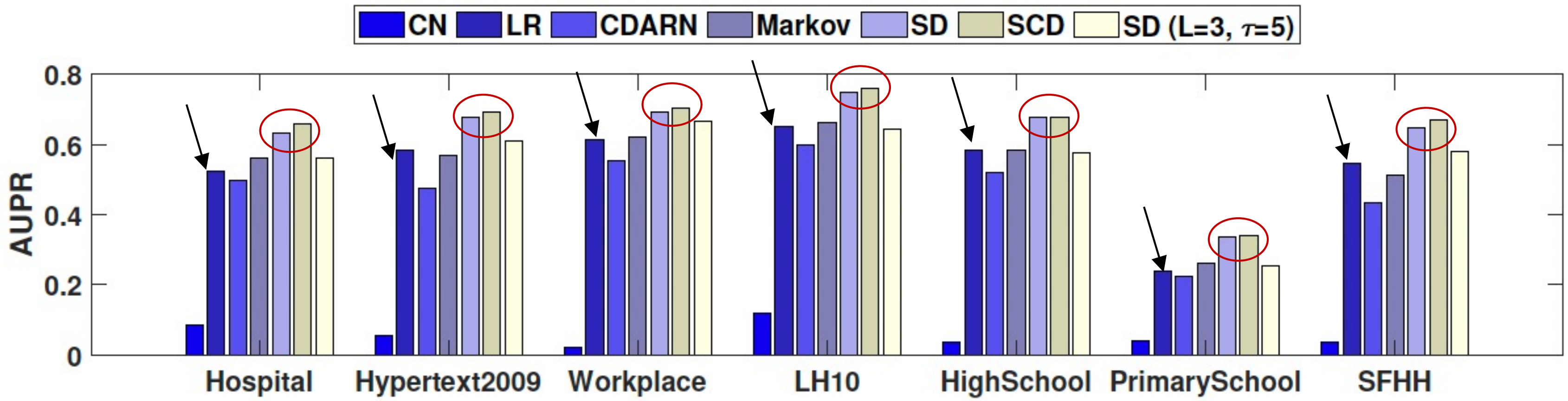
Model Comparison



CN: sum of common neighbors over previous L snapshots
LR: Lasso Regression
CDARN: Correlated Discrete Auto-Regression Network
SD, SCD: τ = 0.5
L = T/2 for all models besides SD (L=3, τ = 5)

Williams, O. E., Mazzarisi, P., Lillo, F., Latora, V.,
Phys. Rev. E. 105, 034301 (2022).

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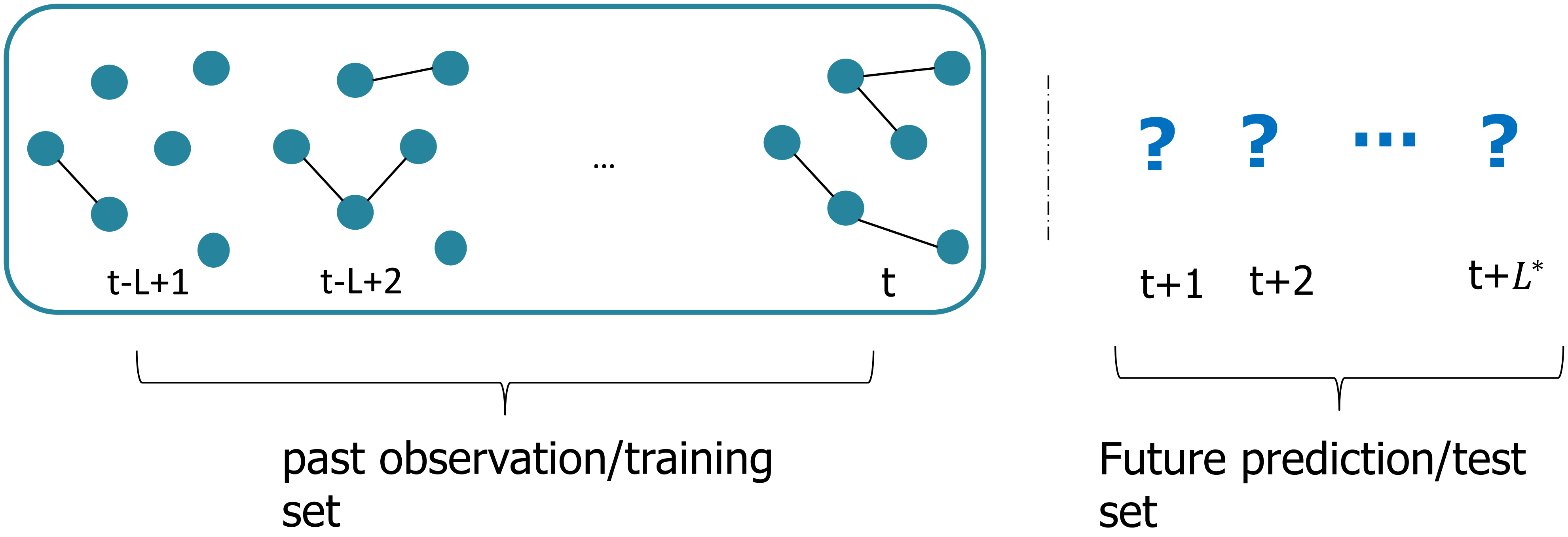
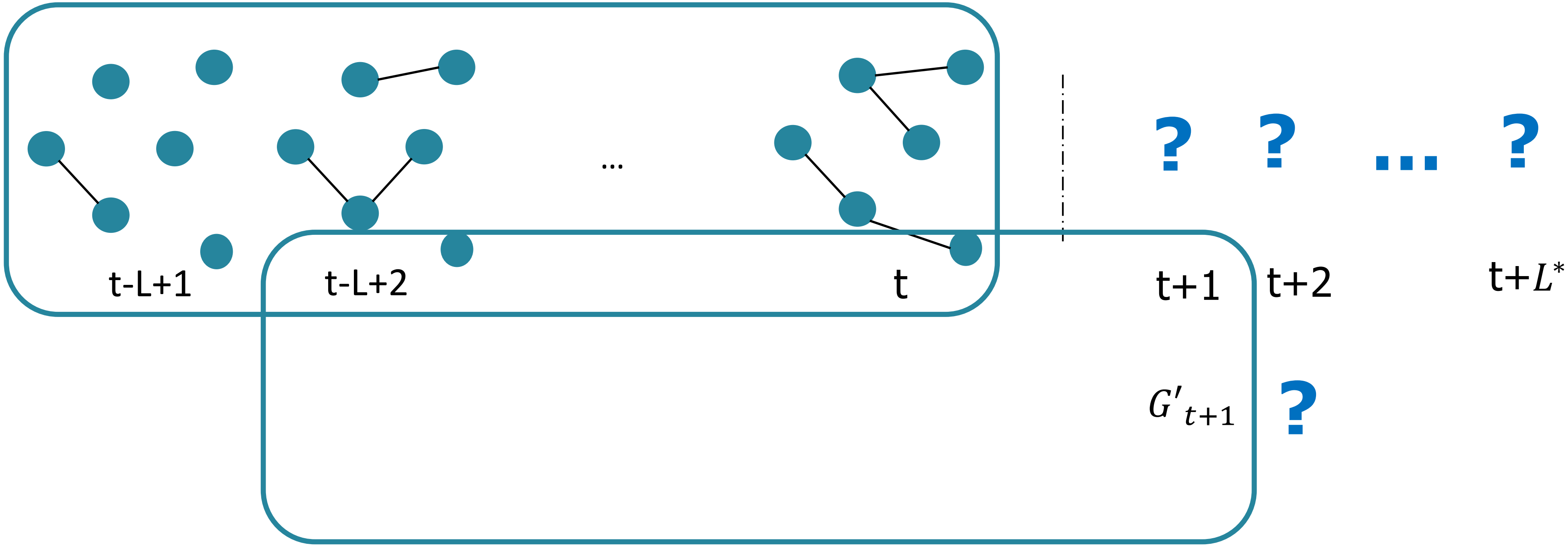
Long-term Temporal Network Predication



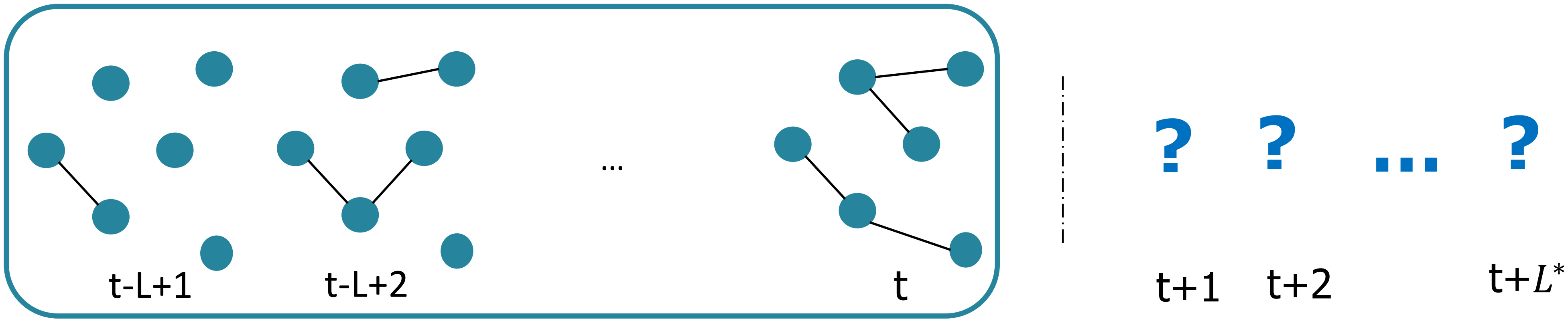
G_w and number of contacts m(t+1) at each time is known.

Recursive Long-term Predication



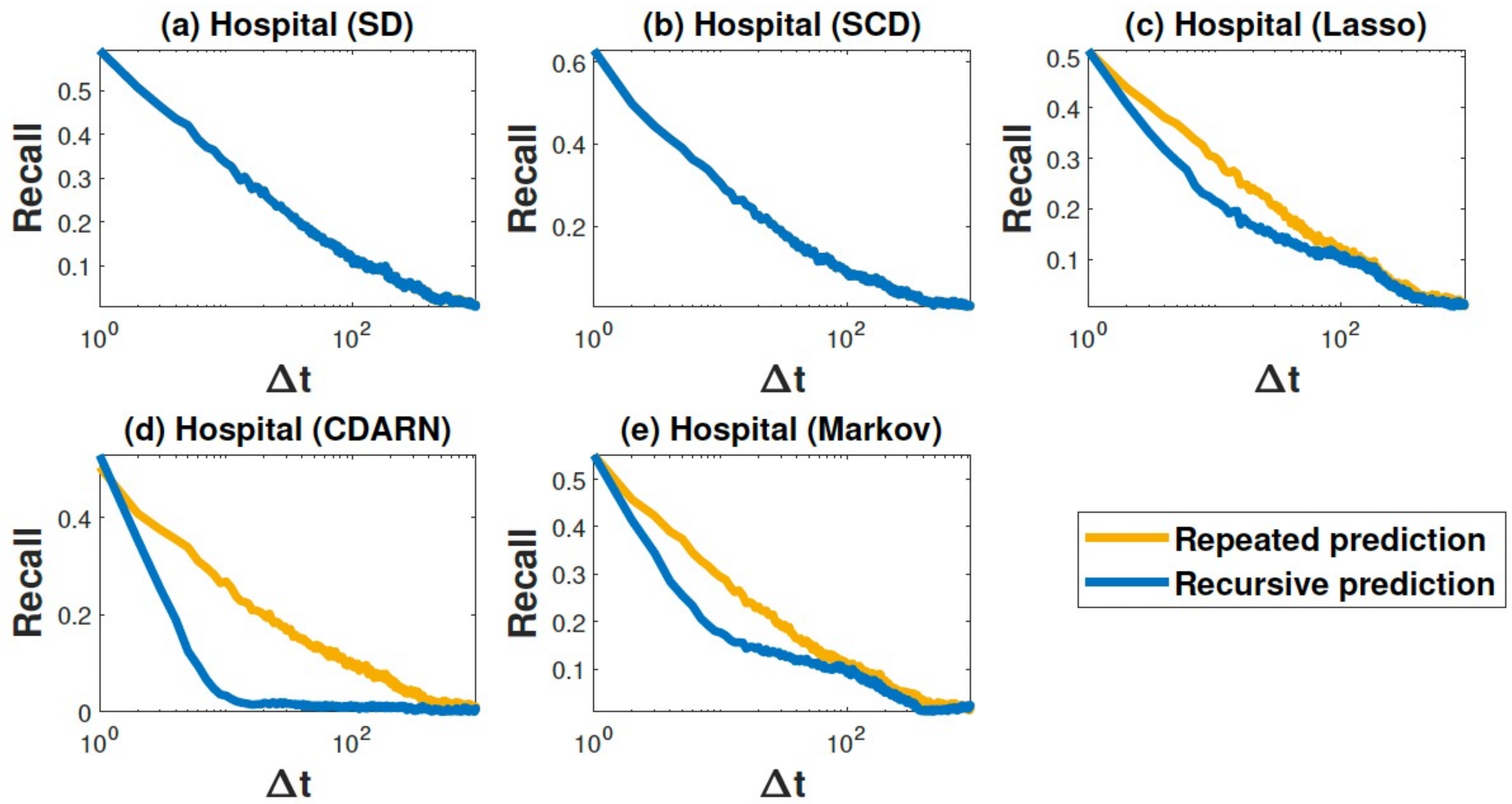
Derive connection tendency $\{w_i(t+1)\}$ using any short-term prediction model
Derive G'_{t+1} based on $\{w_i(t+1)\}$ and m$(t+1)$

Repeated Long-term Predication



Derive connection tendency $\{w_i(t+1)\}$ using any short-term prediction model
Derive G'_{t+1} based on $\{w_i(t+1)\}$ and $m_i(t+1)$
Assume $\{w_i(t+1)\}$ does not change in the prediction period [t+1, t+L^*]

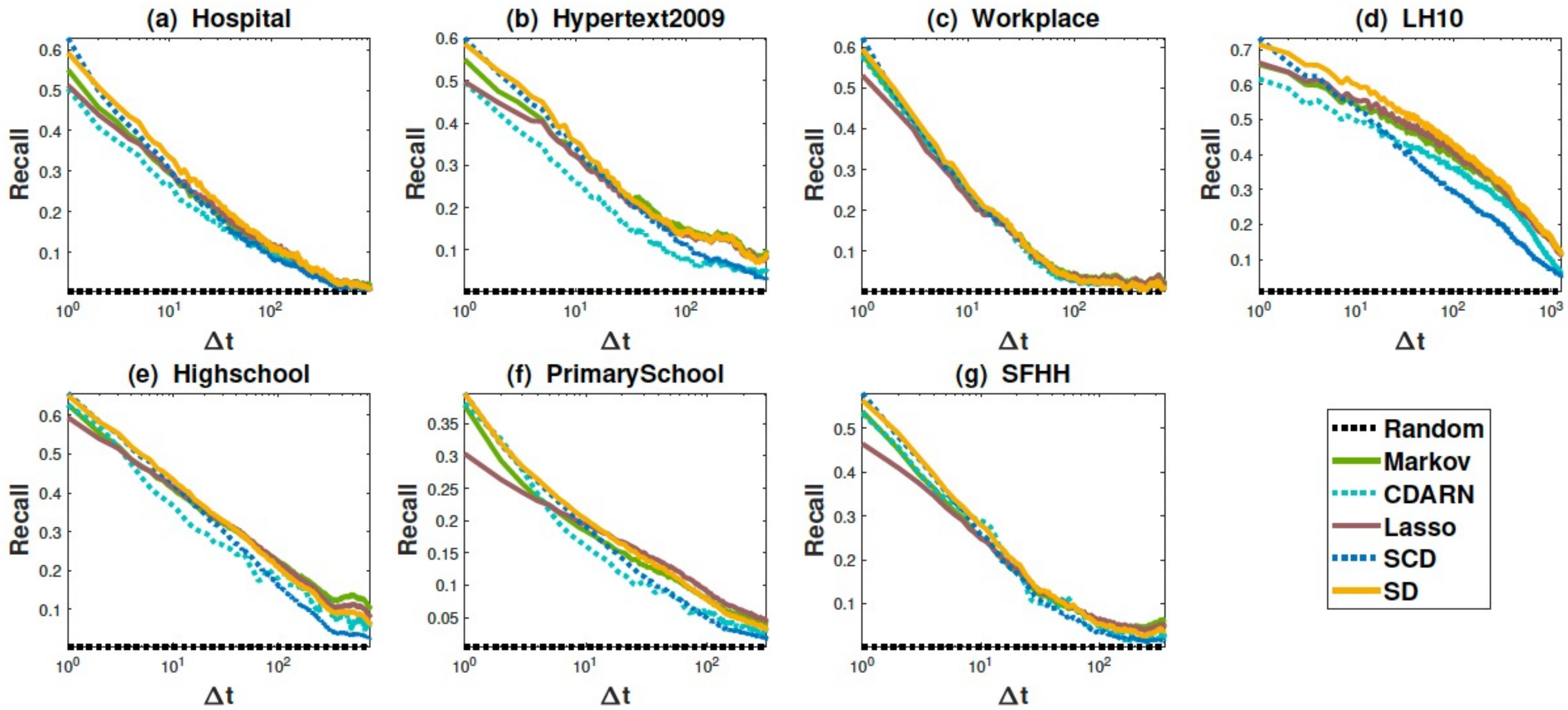
Predication quality per snapshot



Recursive and repeated predication perform similarly for SD, SCD

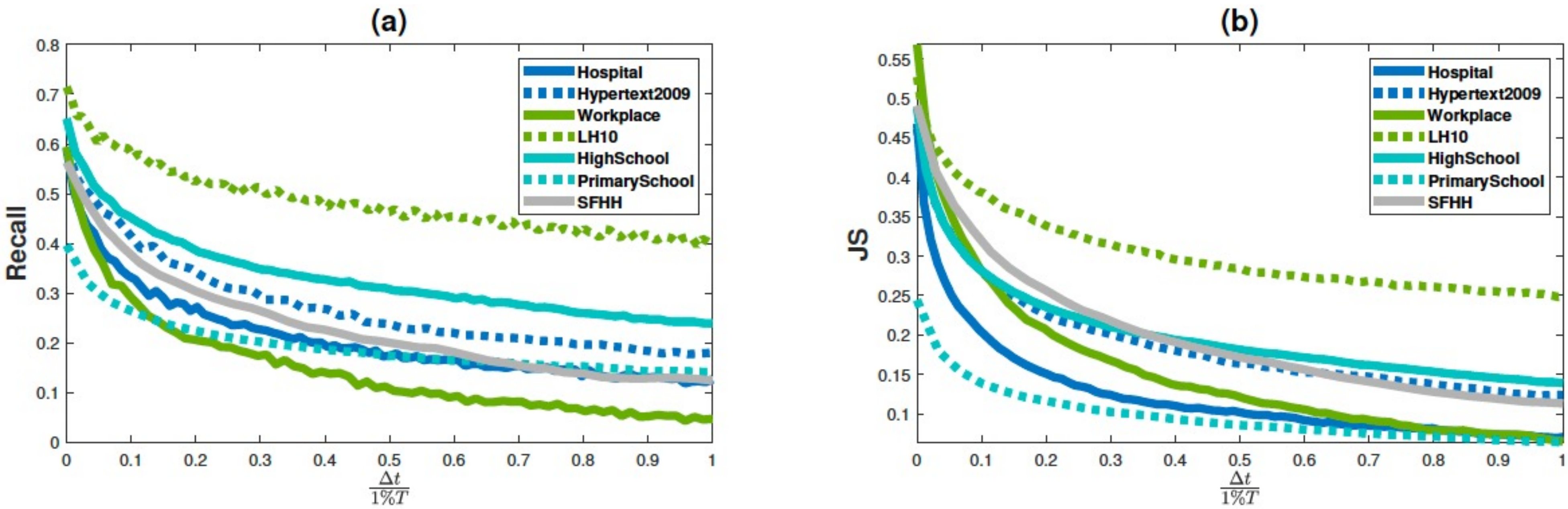
Repeated performs better for other models

Predication quality per snapshot



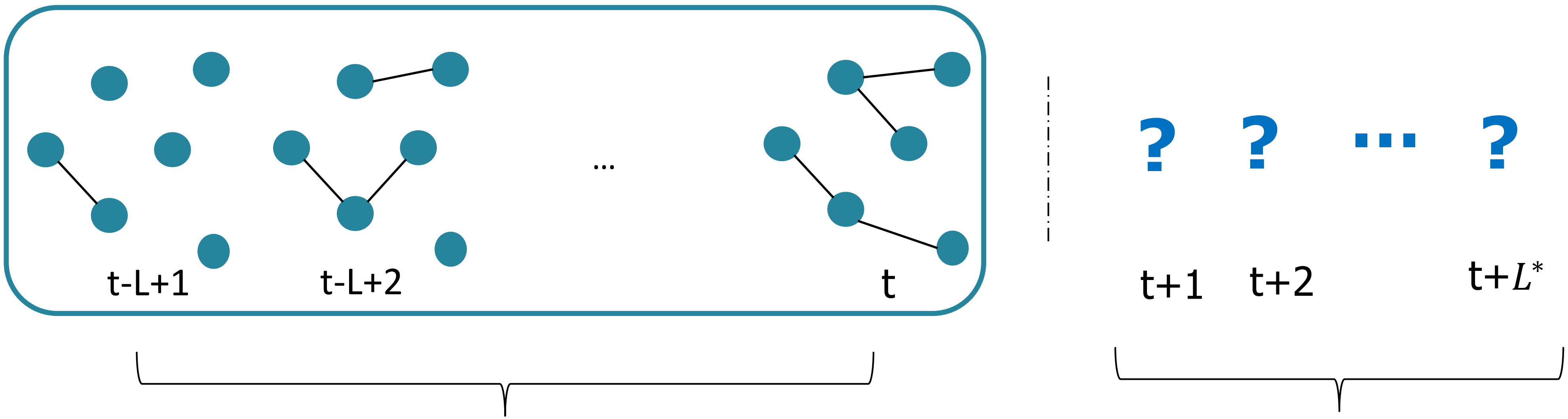
SD performs roughly the best

Predication quality per snapshot



Prediction quality of SD model decays fast in networks with fast decaying memory.

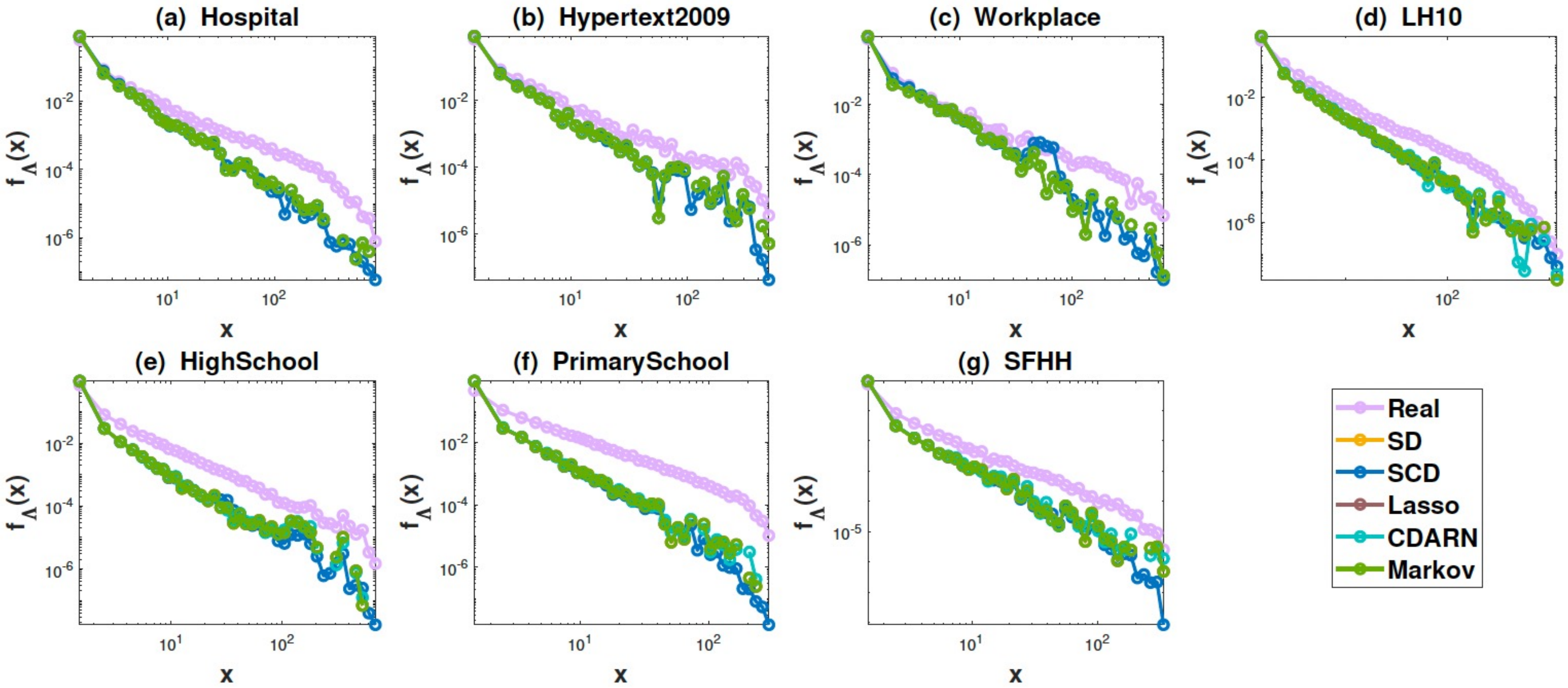
Predication quality in aggregated network



$G'_w(t+1, \text{t+}L^*)$ versus $G_w(t+1, \text{t+}L^*)$

Predication quality in inter-event time Λ

Λ is the time between two consecutive contacts of a link



distribution derived from all links (or class of links with similar total number of contacts)

Heterogenous -> burstness

Conclusion and Discussion

Temporal Network Prediction and Interpretation

Intrinsic properties/mechanisms that facilitate the prediction

Interpretable Learning Algorithms

Network based Models

Susceptible-infected-spreading-based network embedding in static and temporal networks
XX Zhan, Z Li, N Masuda, P Holme, H Wang,
EPJ Data Science 9 (1), 30, 2020.

Conclusion and Discussion

Towards realistic temporal network modeling:

- dynamic processes to model the states of links
- temporal network characterizations to identify key network properties to validate models

A. Ceria, S. Havlin, A. Hanjalic, H. Wang, Topological–temporal properties of evolving networks, *Journal of Complex Networks*, Volume 10, Issue 5, cnac041, (2022)
A. Ceria, H. Wang, Temporal-topological properties of higher-order evolving networks, Scientific Reports 13, 5885 (2023)





Thank you!



kpn

=exact

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