



oCPS Fall School, Eindhoven, 2019



Cooperative driving application





Cooperative Adaptive Cruise Control (CACC)

Movie CACC vs ACC

TU/e

Introduction: We live in a hyperconnected world









Connected Vehicles

Smart Grid

Industrial internet



- Use of advanced communication technologies lead to novel applications and better control performance
- However, if communication networks are overloaded, these benefits are lost
 - ► long delays, many packet losses, etc.

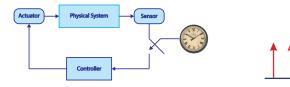


Key question: When to exchange data between sensors, controllers and/or actuators s.t.

- 1. desirable stability and performance properties are guaranteed
- 2. limitations of implementation resources such as bandwidth, power usage, etc. of U/e communication networks are incorporated

Introduction

Control over packet-based communication networks

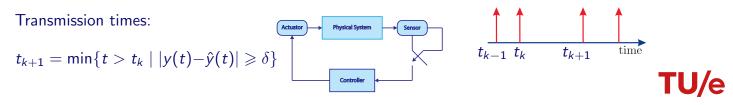


- Time-triggered control: Control tasks executed periodically & triggered by time: $t_{k+1} = t_k + h$
- Time-triggered paradigm "speaks because it has to say something," not because "it has something to say": Inefficient usage of resources

''Wise men speak because they have something to say, fools because they have to say something'' – Plato \sim 370 BC



• Event-triggered control: Only speak when there is something to say: feedback in resource usage



Introduction

Movie inverted pendulum [time-triggered vs event-triggered communication]

 \bullet Same performance as 1 [kHz] time-triggered controller at \sim 1% of transmissions

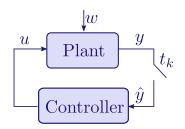


- Event-triggered control (ETC): Requirements
- Modelling systems with packet-based communication: hybrid systems
- Event generators:
 - ► Basic schemes (< 2012)
 - Challenges
 - ► Advanced schemes (≥ 2012)
 - ► Focus: Periodic event-triggered control
- Cooperative driving
- Conclusions

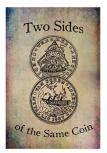
Event-triggered control

Requirements

$$t_{k+1} = \min\{t > t_k \mid |y(t) - \hat{y}(t)| \ge \delta\}$$



• Quality of Control: Stability, Performance, Robustness

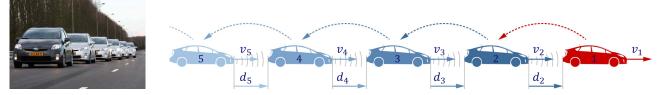


- <u>Resource Utilisation</u>:
 - \blacktriangleright Warning: Zenoness: An infinite number of transmissions in finite time
 - ▶ Strong non-Zenoness: There is T > 0 s.t. transmission intervals $t_{k+1} t_k \ge T$ for all $k \in \mathbb{N}$
 - ► Reduced communication w.r.t. time-triggered control
 - Consistency!

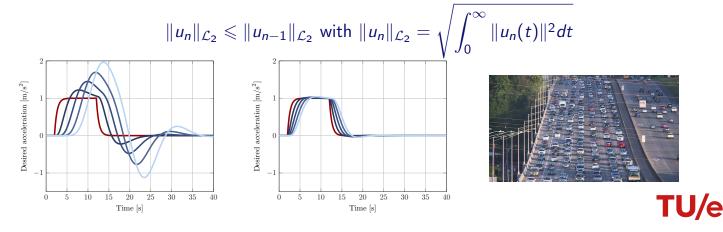


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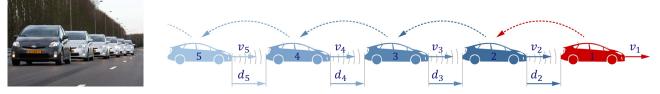
Requirements for CACC



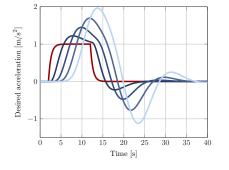
- Vehicle following: Maintain a small 'headway-time' (stability property)
- String stability: disturbance attenuation along the vehicle string (\mathcal{L}_2 -gain $\leqslant 1$)

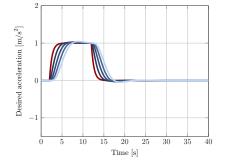


Requirements for CACC



- Vehicle following: Maintain a small 'headway-time' (stability property)
- String stability: disturbance attenuation along the vehicle string (\mathcal{L}_2 -gain $\leqslant 1$)
- Reduced communication to avoid overload (strong non-Zenoness)
- Partial information case: Local triggers not having access to full state of platoon









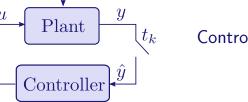
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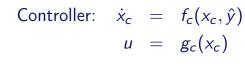
Modelling systems with packet-based communication

|w|

Hybrid systems

Plant: $\dot{x}_p = f_p(x_p, u, w)$ $y = g_p(x_p)$





- $\hat{y}(t_k) = y(t_k)$ at transmission times $t_k, \ k \in \mathbb{N}$
- Predictor for \hat{y} between transmissions: $\frac{d}{dt}\hat{y} = f_y(\hat{y}, u)$ (= 0 when ZOH)
- Sampling-induced error $e = \hat{y} y$ u Plant yController \hat{y} \hat{y} \hat

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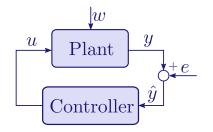
Modelling systems with packet-based communication

Hybrid systems

Plant: $\dot{x}_p = f_p(x_p, u, w)$ $y = g_p(x_p)$



- $\hat{y}(t_k) = y(t_k)$ at transmission times t_k , $k \in \mathbb{N}$
- Predictor for \hat{y} between transmissions: $\frac{\mathrm{d}}{\mathrm{d}t}\hat{y} = f_y(\hat{y}, u)$
- Sampling-induced error $e = \hat{y} y$



Physical part with $x = (x_p, x_c)$: $\dot{x} = f(x, e, w) = \begin{bmatrix} f_p(x_p, g_c(x_c), w) \\ f_c(x_c, g_p(x_p) + e) \end{bmatrix}$

Modelling systems with packet-based communication

Hybrid systems

Plant: $\dot{x}_p = f_p(x_p, u, w)$ $y = g_p(x_p)$ uPlant y t_k Controller: $\dot{x}_c = f_c(x_c, \hat{y})$ $u = g_c(x_c)$

- $\hat{y}(t_k) = y(t_k)$ at transmission times t_k , $k \in \mathbb{N}$
- Predictor for \hat{y} between transmissions: $\frac{d}{dt}\hat{y} = f_y(\hat{y}, u)$ (= 0 when ZOH)
- Sampling-induced error $e = \hat{y} y$

Cyber part: (ZOH)

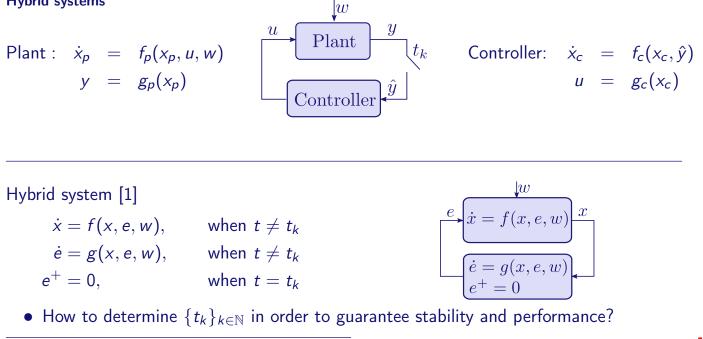
$$\dot{e} = g(x, e, w) := -\frac{\partial g_p}{\partial x_p} f_p(x_p, g_c(x_c), w), \qquad \text{when } t \neq t_k$$
$$e^+ = 0, \qquad \text{when } t = t_k$$

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Modelling systems with packet-based communication





[1] Goebel, Sanfelice, Teel, Hybrid Dynamical Systems, Princeton, 2012.

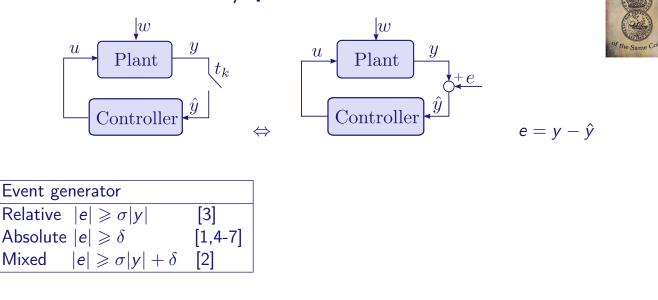
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ETC for disturbance / partial information case



[1] Heemels, Sandee, van den Bosch, Analysis of event-driven controllers for linear systems, IJC 2008

[2] Donkers, Heemels, Output-Based Event-Triggered Control with Guaranteed L_{∞} -gain ..., TAC 2012

3] Tabuada, Event-triggered real-time scheduling of stabilizing control tasks, TAC 2007

[4] Yook, Tilbury, Soparkar, Trading computation for bandwidth: Reducing communication in distributed control ..., TCST 2002

[5] Miskowicz, Send-on-delta concept: An event-based data-reporting strategy, Sensors 2006
[6] Kofman, Braslavsky, Level crossing sampling in feedback stabilization under data-rate constraints, CDC 2006

[0] Kotman, Braslavsky, Level crossing sampling in feedback stabilization under data-rate constraint
[7] Lunze and Lehmann, A state-feedback approach to event-based control, Automatica 2010

ETC for full (state) information case (no disturbances)

• No disturbance w = 0 and full information y = x

Event generator	non-Zenoness	
Relative $ e \ge \sigma x $	global	
Absolute $ e \ge \delta$	semi-global	
Mixed $ e \ge \sigma x + \delta$	global	

- Strong non-Zeno: there is T > 0 s.t. transmissions intervals $t_{k+1} t_k \ge T$, $k \in \mathbb{N}$
- Global: lower bound on transmission intervals holds for all initial states
- Semi-global: lower bound on transmission interval depends on size of initial state (and goes to zero for large states)

Two Sides

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ETC for full (state) information case (no disturbances)

• No disturbance w = 0 and full information y = x

Event generator	non-Zenoness	control properties	$u \rightarrow Plant y$
Relative $ e \ge \sigma x $	global	asymptotic stability	
Absolute $ e \ge \delta$	semi-global	practical stability	$\bigcirc Controller \stackrel{\hat{y}}{\checkmark}$
Mixed $ e \ge \sigma x + \delta$	global	practical stability	

W

- Asymptotic stability $x(t) \rightarrow 0$ when $t \rightarrow \infty$
- Practical stability $x(t) \rightarrow \mathcal{B}(0, \varepsilon)$ when $t \rightarrow \infty$ with $\varepsilon > 0$
- Analysis can be found in [1,2,3]
- Relative triggering looks promising, but fragile...

[1] Borgers, H., Event-Separation Properties of Event-Triggered Control Systems, TAC 2014

[2] Donkers, H., Output-Based Event-Triggered Control with Guaranteed L_{∞} -gain ..., TAC 2012 [3] Tabuada, Event-triggered real-time scheduling of stabilizing control tasks, TAC 2007

ETC for disturbance / partial information case

Event generator			
Relative $ e \ge \sigma y $			
Absolute $ e \ge \delta$			
$Mixed e \geqslant \sigma y + \delta$			

ETC for disturbance / partial information case

Event generator	non-Zenoness
Relative $ e \ge \sigma y $	Х
Absolute $ e \ge \delta$	semi-global
Mixed $ e \ge \sigma y + \delta$	semi-global

• Semi-global: lower bound on transmission interval depends on size of initial state (and goes to zero for large states)

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ETC for disturbance / partial information case

Event generator	non-Zenoness	control properties	
Relative $ e \ge \sigma y $	Х	Х	$ \qquad \longrightarrow \qquad \text{Plant} \qquad \int g \\ \downarrow + c \\ \downarrow +$
Absolute $ e \ge \delta$	semi-global	practical stability	
Mixed $ e \ge \sigma y + \delta$	semi-global	practical stability	\Box Controller \checkmark

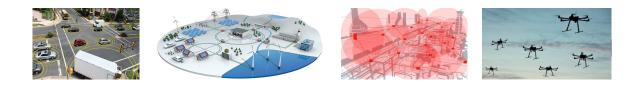
- Analysis can be found in [1,2]
- Use of absolute thresholds is partial solution
 - Semi-global strong non-Zenoness
 - ▶ No asymptotic stability $(x(t) \not\rightarrow 0)$ and no finite \mathcal{L}_2 -gain

$$||x||_{\mathcal{L}_{2}} \leq \beta(|x(0)|) + \gamma ||w||_{\mathcal{L}_{2}} \text{ with } ||x||_{\mathcal{L}_{2}} = \sqrt{\int_{0}^{\infty} ||x(t)||^{2} dt}$$

[1] Borgers, H., Event-Separation Properties of Event-Triggered Control Systems, TAC 2014 [2] Donkers, H., Output-Based Event-Triggered Control with Guaranteed L_{∞} -gain ..., TAC 2012

Challenge for networked systems (with disturbances)





Challenge: What about global strong non-Zeno and GAS $(x(t) \rightarrow 0)$ / finite \mathcal{L}_2 -gains ?

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Absolute triggering with time-varying threshold [0-2]

 $t_{k+1} = \min\{t \ge t_k \mid |y(t) - \hat{y}(t)| \ge \delta(t_k)\}$

Enforcing minimal inter-event/waiting time [3,6]

 $t_{k+1} = \min\{t \ge t_k + T \mid |y(t) - \hat{y}(t)| \ge \sigma |y(t)|\}$

• Periodic Event-Triggered Control (PETC) [2-5]: Events only at $kh, k \in \mathbb{N}$

 $t_{k+1} = \min\{t > t_k \mid |y(t) - \hat{y}(t)| \ge \sigma |y(t)| \land t = kh, \ k \in \mathbb{N}\}$

ightarrow Combining the best of two worlds !?

[0] Seyboth, Dimarogonas, Johansson, Control of Multi-Agent Systems via Event-based Communication, IFAC WC 2011

[1] Mazo Jr., Cao, Asynchronous decentralized event-triggered control, Automatica 2013

Postoyan, Tabuada, Nešić, Anta, Framework for the Event-Triggered Stabilization of Nonlinear Systems, TAC15 Heemels, Sandee, van den Bosch, Analysis of event-driven controllers for linear systems, IJC 2008 Heemels, Donkers, Teel, Periodic Event-Triggered Control for Linear Systems, TAC 2013 Henningsson, Johannesson, Cervin, Sporadic event-based control of first-order linear stochastic ..., Automatica 2008 Tallapragada, Chopra, Event-triggered decentralized dynamic output .. LTI systems, NECSYS 2012

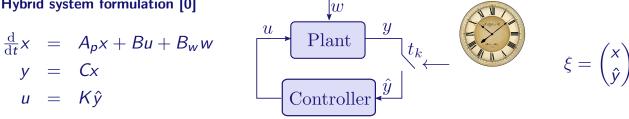
- [4]
- [5] [6]



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PETC for linear systems

Hybrid system formulation [0]



 $t_{k+1} = \min\{t > t_k \mid |y(t) - \hat{y}(t)| > \sigma|y(t)| \land t = kh, \ k \in \mathbb{N}\}$

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \xi \\ \tau \end{bmatrix} = \begin{bmatrix} A\xi + Bw \\ 1 \end{bmatrix}, \tau \in [0, h] \qquad \qquad \phi(\xi) = \begin{cases} J_1 \xi & \text{when } \xi^\top Q\xi > 0 \\ J_2 \xi & \text{when } \xi^\top Q\xi \leqslant 0 \end{cases}$$
$$\begin{bmatrix} \xi^+ \\ \tau^+ \end{bmatrix} = \begin{bmatrix} \phi(\xi) \\ 0 \end{bmatrix}, \quad \tau = h \qquad \qquad \text{with } A := \begin{bmatrix} A_p & BK \\ 0 & 0 \end{bmatrix}, \quad B := \begin{bmatrix} B_w \\ 0 \end{bmatrix}, \quad J_1 := \begin{bmatrix} I & 0 \\ C & 0 \end{bmatrix}, \quad J_2 := \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

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[0] Goebel, Sanfelice, Teel, Hybrid dynamical systems: Modeling, Stability and Robustness, 2012

[1] H., Dullerud, Teel, \mathcal{L}_2 -gain Analysis for a Class of Hybrid Systems with Applications to ... Event-triggered Control.., TAC 2016 [2] H., Donkers, Teel, Periodic Event-Triggered Control for Linear Systems, TAC 2013

Periodic event-triggered control (PETC)

Discretization

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \xi \\ \tau \end{bmatrix} = \begin{bmatrix} A\xi + Bw \\ 1 \end{bmatrix}, \tau \in [0, h] \qquad \qquad \phi(\xi) = \begin{cases} J_1 \xi & \text{when } \xi^\top Q\xi > 0 \\ J_2 \xi & \text{when } \xi^\top Q\xi \leqslant 0 \end{cases}$$
$$\begin{bmatrix} \xi^+ \\ \tau^+ \end{bmatrix} = \begin{bmatrix} \phi(\xi) \\ 0 \end{bmatrix}, \qquad \tau = h$$

Discretization at sampling times kh, $k \in \mathbb{N}$, with $\xi_k = \xi(kh^+)$, when (w = 0) leads to discrete-time piecewise linear system:

$$\xi_{k+1} = \phi(e^{Ah}\xi_k) = \begin{cases} J_1 e^{Ah}\xi_k, & \text{when } \xi_k^\top e^{A^\top h} Q e^{Ah}\xi_k > 0, \\ J_2 e^{Ah}\xi_k, & \text{when } \xi_k^\top e^{A^\top h} Q e^{Ah}\xi_k \leqslant 0 \end{cases}$$

- LMI-based analysis using piecewise quadratic Lyapunov functions [1,2]
- Also \mathcal{L}_2 -gain analysis based on discrete-time PWL systems [3] (lifting)

^[1] Heemels, Donkers, Teel, Periodic Event-Triggered Control for Linear Systems, TAC 2013

^[2] Heemels, Donkers, Model-based Periodic Event-Triggered Control for Linear Systems, Automatica 2013

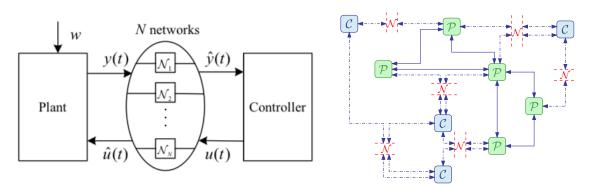
^[3] Heemels, Dullerud, Teel, \mathcal{L}_2 -gain Analysis for a Class of Hybrid Systems with Applications to ... Event-triggered Control, TAC 2016



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PETC: Interconnected nonlinear systems

- Asynchronous distributed PETC for nonlinear systems [1]
 - Network \mathcal{N}_i has sampling times s_j^i with $s_{j+1}^i s_j^i \leqslant T_i$
 - $t_{k+1}^{i} = \min\{t > t_{k}^{i} \mid ||y_{i} \hat{y}_{i}|| > \sigma_{i}||y_{i}|| \land t = s_{i}^{i}, j \in \mathbb{N}\}$



[1] Wang, R. Postoyan, D. Nesic, H., Periodic event-triggered control for nonlinear networked control systems, TAC 2020

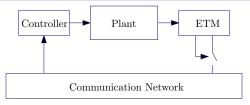


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Is ETC better than TTC?

Consistency: [1] Better tradeoff between

- Quality of Control using some performance measure
- Cost of implementation (e.g. average transmission rate)



- LQG setting: Linear continuous-time plant $\dot{x} = Ax + Bu + w$
- Performance measure: $V_{\pi} := \limsup_{T \to \infty} \frac{1}{T} \mathbb{E} [\int_0^T x(t)^{\intercal} Q x(t) + u(t)^{\intercal} R u(t) dt]$
- PETC controller/scheduler policy π:

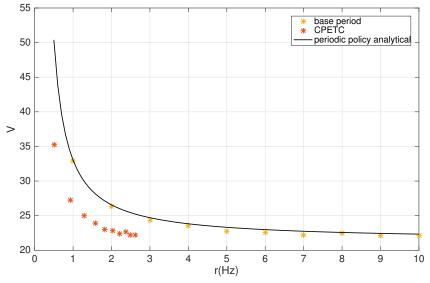
checking at $s_k = kh$ to transmit ($\sigma_k = 1$) or not ($\sigma_k = 0$)

• Average transmission rate: $r_{\pi} := \frac{1}{h} \times \limsup_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} \mathbb{E}[\sigma_k]$

 ^[1] Asadi Khashooei, Antunes, Heemels, A Consistent Threshold-based Policy for Event-triggered Control, IEEE Control Systems Letters 2018
[2] Antunes, Asadi Khashooei, Consistent dynamic event-triggered policies for linear quadratic control, CONES, 2017
[3] Antunes, Heemels, Rollout event-triggered control: Beyond periodic performance, TAC 14
[4] Astrom & Bernhardsson (IFAC WC 1999, CDC 2002)

Periodic event-triggered control (PETC)

• PETC outperforms time-triggered control in general LQG setting [1]: Same cost $\lim_{T\to\infty} \frac{1}{T} \mathbb{E}[\int_0^T x(t)^\intercal Qx(t) + u(t)^\intercal Ru(t)dt]$, lower average communication rate:



[1] Asadi Khashooei, Antunes, H., A Consistent Threshold-based Policy for Event-triggered Control, IEEE Control Systems Letters 2018

Consistent Event-triggered Control

Theorem: The following control/transmission policy is consistent:

• Model-based control-input generator (CIG) using the linear predictor

Controller	Plant		ETM	
Communication Network				

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t), \quad t \neq s_k \text{ with } \sigma_k = 1$$

 $\hat{x}(t) = x(t), \quad t = s_k \text{ with } \sigma_k = 1$
 $u(t) = K\hat{x}(t)$

• Scheduling policy (ETM):

 $\sigma_k = \begin{cases} 1, & \text{when } e_k^\mathsf{T} \mathsf{\Gamma} e_k > \delta \\ 0, & \text{otherwise,} \end{cases}$

where $e_k := e(s_k^-) = x(s_k^-) - \hat{x}(s_k^-)$, $s_k = kh, \ k \in \mathbb{N}$

• "I know what you know principle"

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Design for networked systems with disturbances TU/e UNIVERSITY OF

• Periodic Event-Triggered Control (PETC) : Events only at kh, $k \in \mathbb{N}$

$$t_{k+1} = \min\{t > t_k \mid |y(t) - \hat{y}(t)| \ge \sigma |y(t)| \land t = kh, \ k \in \mathbb{N}\}$$

• Enforcing minimal inter-event/waiting time [1,2]

$$t_{k+1} = \min\{t \ge t_k + T \mid |\underbrace{y(t) - \hat{y}(t)}_{e(t)}| \ge \sigma |y(t)|\}$$

 $t_{k+1} = \min\{t \ge t_k + T \mid \eta(t) \le 0\}$ with local dynamic variable η given by

$$\dot{\eta} \sim \begin{cases} \Psi(e, y) \ge 0, & \text{when } t_k \le t \le t_k + T \\ -\beta \eta + \sigma^2 |y|^2 - |e|^2, & \text{when } t_k + T \le t \le t_{k+1} \end{cases}$$

"low-pass filtered version of relative trigger $\sigma^2 |y|^2 - |e|^2 \leq 0$ "

Dolk, Borgers, H., Dynamic event-triggered control with time regularization ..., IEEE Trans. Automatic Control 2017
Dolk, Ploeg, H., Event-triggered Control for String-Stable Vehicle Platooning, IEEE Trans. Intell. Transp. Systems 2017

Cooperative Adaptive Cruise Control (CACC)

Movie event-triggered and time-triggered CACC [simulations]

Dolk, Ploeg, H., Event-triggered Control for String-Stable Vehicle Platooning, IEEE Trans. Intell. Transp. Systems 2017

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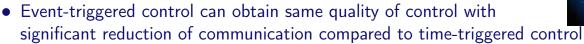
Cooperative Adaptive Cruise Control (CACC)

Movie event-triggered and time-triggered CACC [real-life experiments]

• 75% savings in communication while virtually same performance

Dolk, Ploeg, H., Event-triggered Control for String-Stable Vehicle Platooning, IEEE Trans. Intell. Transp. Systems 2017

Conclusions











- Tip of the iceberg: Expanding field, still very much in motion:
 - ► Different system classes: stochastic systems, constrained systems (resource-aware MPC), discrete-time systems, distributed optimisation, ...
 - Different design approaches: delay-system approach, set-based approaches, optimal control approaches, learning-based approaches, self-triggered control, ...
 - ► Different aspects: quantization, (denial-of-service) attacks, packet losses, delays, etc.
- see www.heemels.tue.nl for preprints on these topics
- Future work: Building complete theory of ETC
 - Formal proofs ETC better than time-triggered (study of transmission intervals!)
 - Co-design of controller and event generators (for nonlinear systems)
 - Unification of the field
 - Multi-agent and distributed systems
 - Applications

Acknowledgements



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