

### Model-driven Performance Engineering of FMS

About saving time ...

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### Just a few questions ...

- What is the productivity of this machine?
- Optimal production schedule?
- Productivity bottlenecks?
- Optimal controllers?
- Design alternatives?

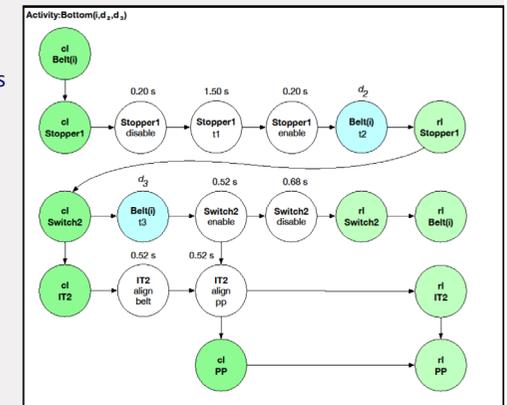
How would **you** approach this?



### Activity Modeling

### Activity – Deterministic, Atomic Functionality

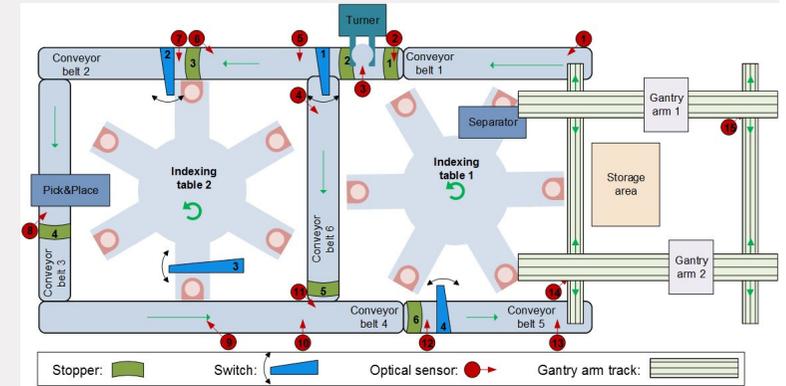
- Actions, precedences
- (Worst-case) action execution times
- Resource claims, releases
- Consistent



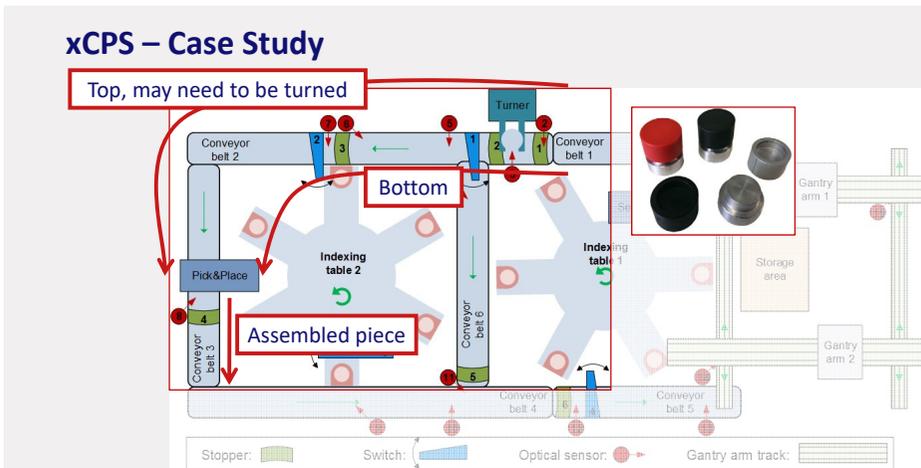
# xCPS – eXplore Cyber-Physical Systems



# xCPS – Case Study

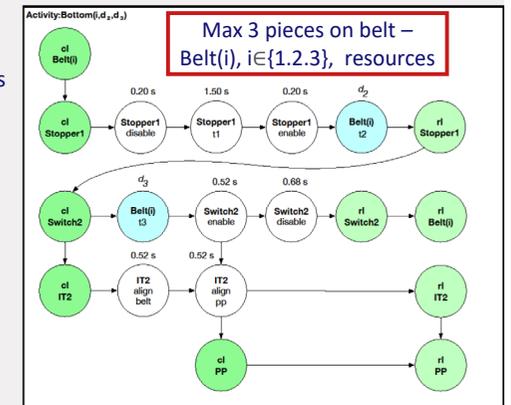
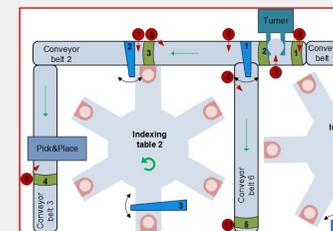


# xCPS – Case Study

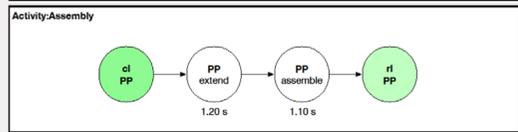
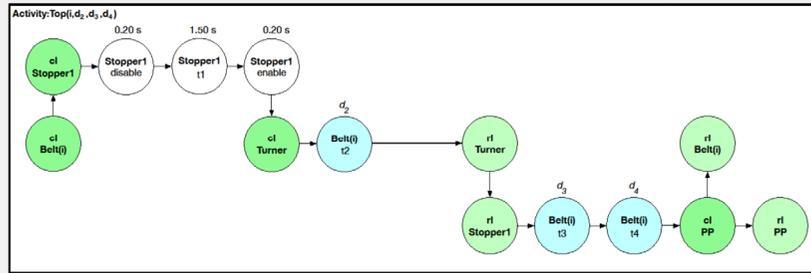


# Activity – Deterministic, Atomic Functionality

- Actions, precedences
- (Worst-case) action execution times
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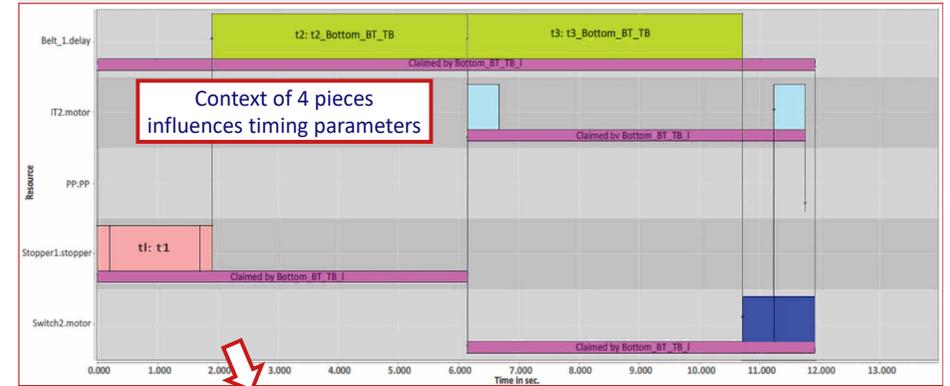


### Activity – Deterministic, Atomic Functionality



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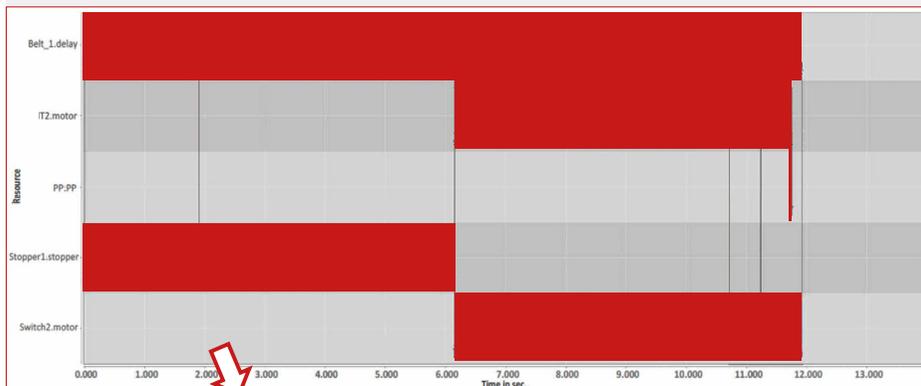
### Action Execution – Bottom(1,4.24,4.57)



10

Looks a bit like a tetris block

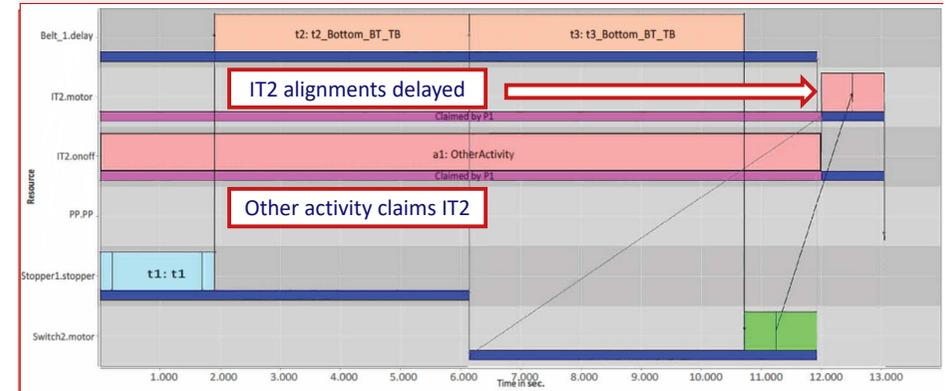
### Action Execution – Bottom(1,4.24,4.57)



11

Looks a bit like a tetris block

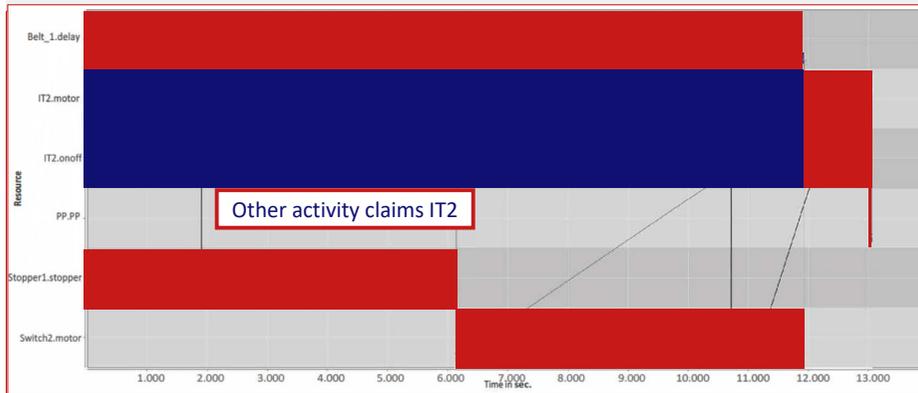
### Action Execution – Bottom(1,4.24,4.57)



12

Looks a bit like a tetris block

## Action Execution – Bottom(1,4.24,4.57)

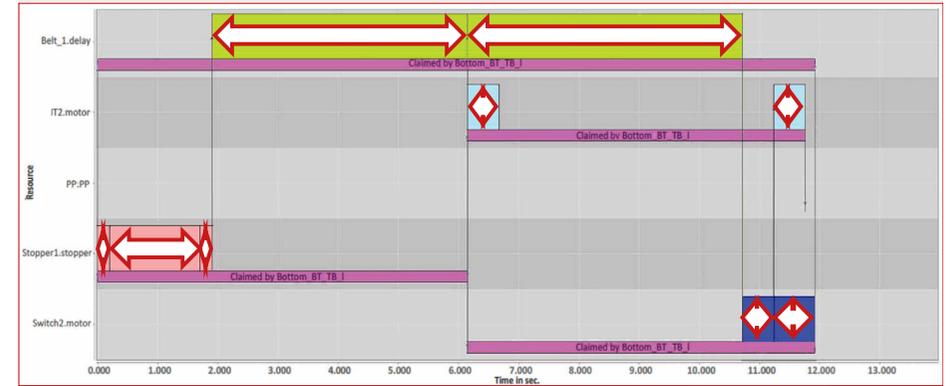


13

Looks a bit like a tetris block

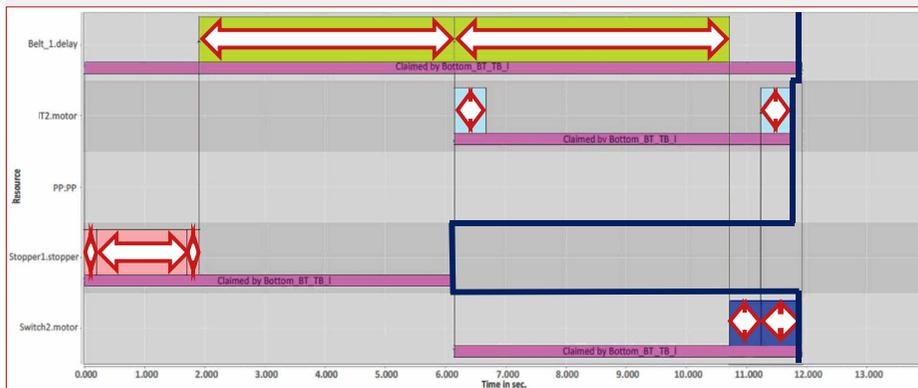
But flexible !

## The Constant – Action Execution Times



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## The Variable – Resource Availability Times



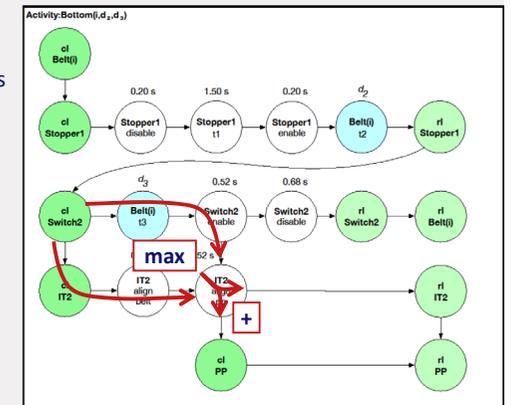
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## Activity – Computing Resource Release Times

- Actions, precedences
- (Worst-case) action execution times
- Resource claims, releases
- Consistent

### (max,+) algebra

Semiring  $(\mathbb{R} \cup \{-\infty\}, \oplus, \otimes)$  with  
 $x \oplus y = \max(x, y)$   
 $x \otimes y = x + y$



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**(max,+) tetris**  
courtesy Marc Geilen

- a
- b
- dependencies
- rotate
- clear

**(max,+) Matrix Characterization of Activity Timing**

Bottom(1,d<sub>2</sub>,d<sub>3</sub>)

	Stopper1	Switch2	IT2	PP	Turner	Belt(1)	Belt(2)	Belt(3)
Stopper1	1.9 + d <sub>2</sub>	-∞	-∞	-∞	-∞	1.9 + d <sub>2</sub>	-∞	-∞
Switch2	3.1 + d <sub>2</sub> + d <sub>3</sub>	d <sub>3</sub> + 1.2	-∞	-∞	-∞	3.1 + d <sub>2</sub> + d <sub>3</sub>	-∞	-∞
IT2	2.94 + d <sub>2</sub> + d <sub>3</sub>	d <sub>3</sub> + 1.04	1.04	0	-∞	2.94 + d <sub>2</sub> + d <sub>3</sub>	-∞	-∞
PP	2.94 + d <sub>2</sub> + d <sub>3</sub>	d <sub>3</sub> + 1.04	1.04	0	-∞	2.94 + d <sub>2</sub> + d <sub>3</sub>	-∞	-∞
Turner	-∞	-∞	-∞	-∞	0	-∞	-∞	-∞
Belt(1)	3.1 + d <sub>2</sub> + d <sub>3</sub>	d <sub>3</sub> + 1.2	-∞	-∞	-∞	3.1 + d <sub>2</sub> + d <sub>3</sub>	-∞	-∞
Belt(2)	-∞	-∞	-∞	∞	-∞	-∞	0	-∞
Belt(3)	-∞	-∞	-∞	∞	-∞	-∞	-∞	0

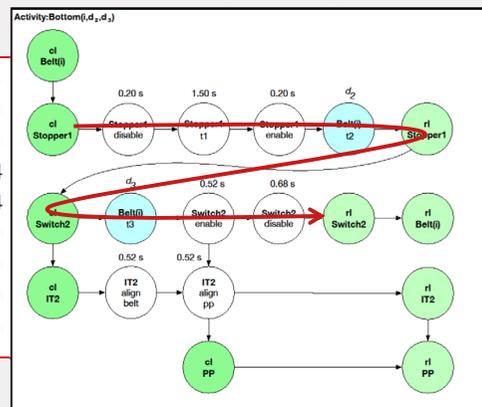
... till release

**(max,+) Matrix Characterization of Activity Timing**

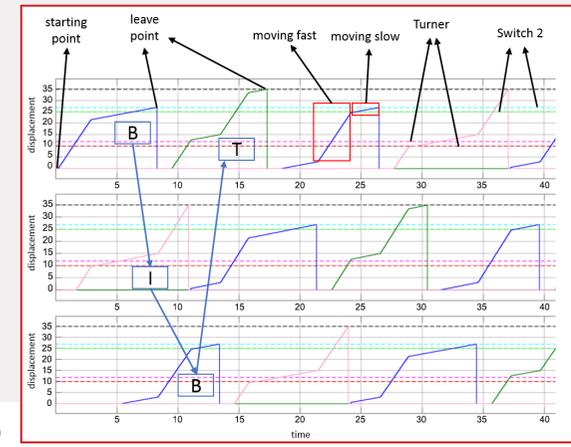
From availability ...

	Stopper1	Switch2	IT2
Stopper1	1.9 + d <sub>2</sub>	-∞	-∞
Switch2	3.1 + d <sub>2</sub> + d <sub>3</sub>	d <sub>3</sub> + 1.2	-∞
IT2	2.94 + d <sub>2</sub> + d <sub>3</sub>	d <sub>3</sub> + 1.04	1.04
PP	2.94 + d <sub>2</sub> + d <sub>3</sub>	d <sub>3</sub> + 1.04	1.04
Turner	-∞	-∞	-∞
Belt(1)	no dependency	+ 1.2	-∞
Belt(2)	-∞	-∞	-∞
Belt(3)	-∞	-∞	-∞

... till release



**Action Execution Times – Simulations, Measurements, ...**



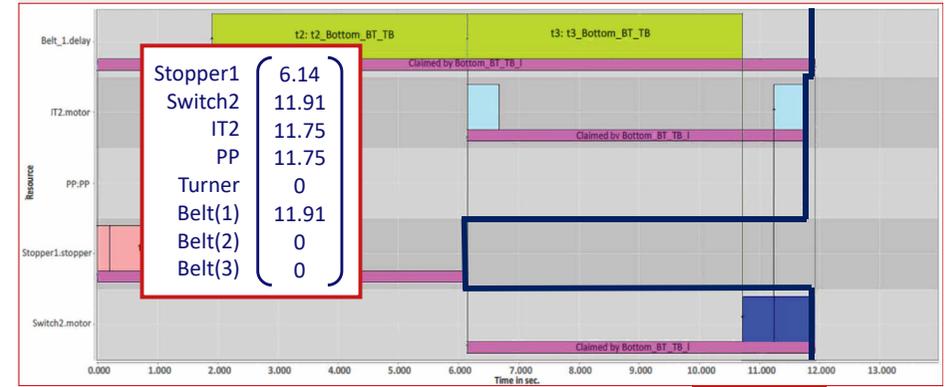
## Timing of Activity Execution – (max,+) Matrix Multiplication

Bottom(1,4.24,4.57)

$$\begin{array}{l}
 \text{Stopper1} \\
 \text{Switch2} \\
 \text{IT2} \\
 \text{PP} \\
 \text{Turner} \\
 \text{Belt(1)} \\
 \text{Belt(2)} \\
 \text{Belt(3)}
 \end{array}
 \begin{pmatrix}
 6.14 \\
 11.91 \\
 11.75 \\
 11.75 \\
 0 \\
 11.91 \\
 0 \\
 0
 \end{pmatrix}
 =
 \begin{pmatrix}
 6.14 & -\infty & -\infty & \dots \\
 11.91 & 5.77 & \dots & \\
 11.75 & \dots & & \\
 \dots & & & 
 \end{pmatrix}
 \begin{pmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{pmatrix}
 \begin{array}{l}
 \text{Stopper1} \\
 \text{Switch2} \\
 \text{IT2} \\
 \text{PP} \\
 \text{Turner} \\
 \text{Belt(1)} \\
 \text{Belt(2)} \\
 \text{Belt(3)}
 \end{array}$$

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## Timing of Activity Execution – Bottom(1,4.24,4.57)



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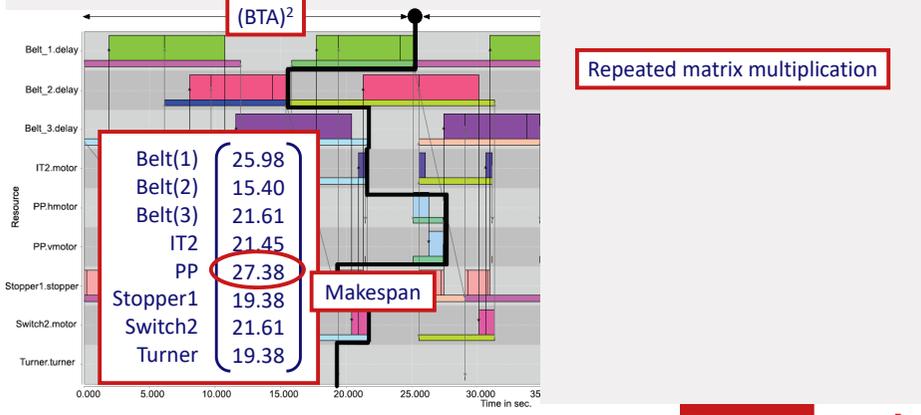
Worst-case timing analysis of distributed systems:  
(max,+) algebra

Timing Analysis

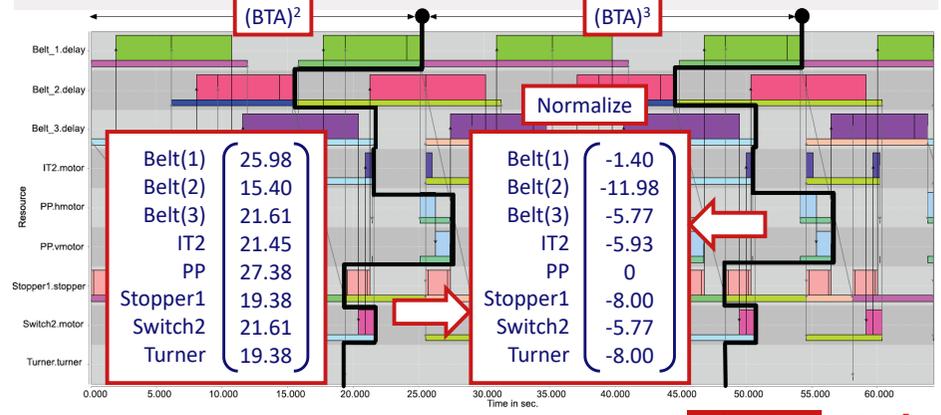
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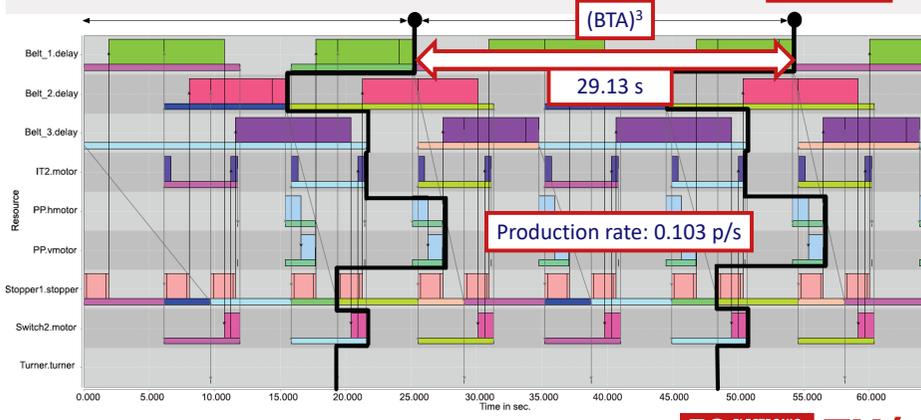
### Timing of Activity Sequences – BTABTABTA...



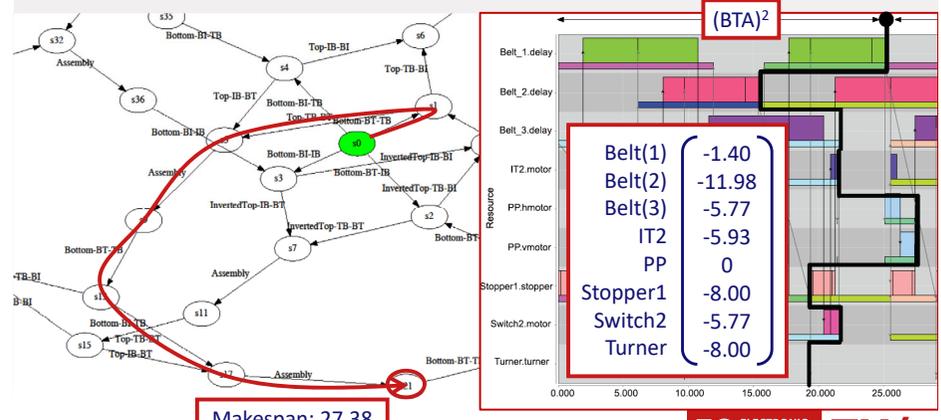
### Timing of Activity Sequences – BTABTABTA...



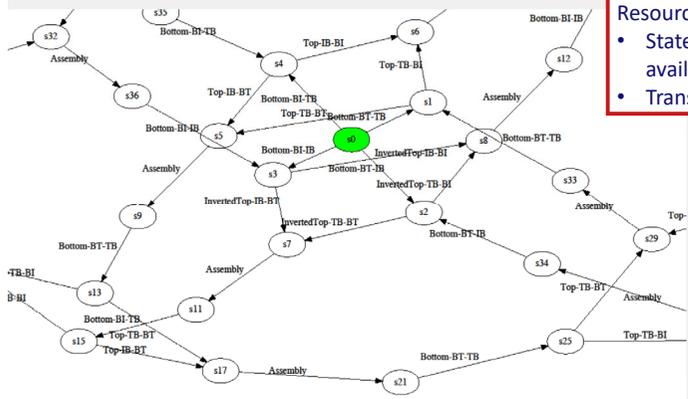
### Timing of Activity Sequences – BTABTABTA...



### Throughput Bounds – Consider All Activity Sequences



## Throughput Lower and Upper Bounds



Resource-availability state space

- States: Normalized resource-availability vectors
- Transitions: Delay

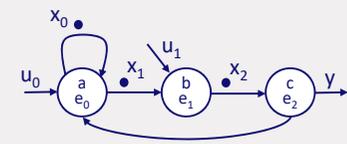
Max/Min Cycle Mean provide bounds (and schedules)

Max: 0.103 p/s  
Min: 0.080 p/s

Analysis time: < 9 ms

Other advanced techniques available

## In General: (max,+) Linear Systems



A dataflow graph

- Actors a,b,c
- Actor execution times  $e_i$
- Channels
- inputs  $u_i$
- output  $y$
- Tokens  $x_i$

Timing semantics: (max,+) linear system

$$x[k+1] = A \otimes x[k] \oplus B \otimes u[k]$$

$$y[k] = C \otimes x[k] \oplus D \otimes u[k]$$

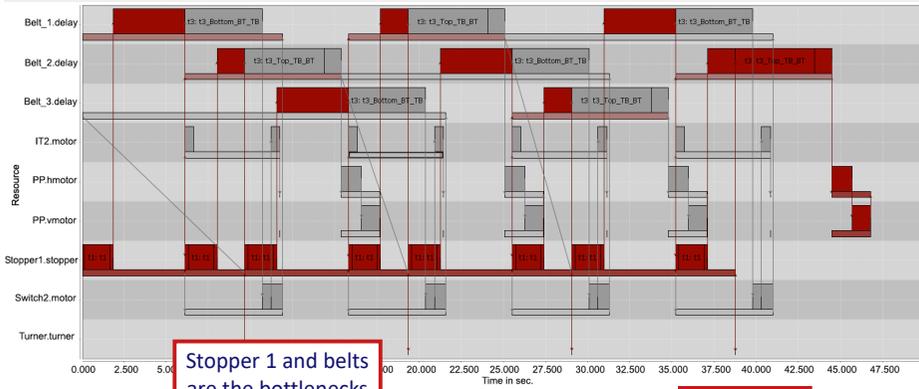
A,B,C,D can be obtained by 'executing the graph once'

Earlier: closed system,  $x[k+1] = Ax[k]$

Worst-case timing of distributed systems:  
(max,+) linear system

Bottleneck Analysis and Redesign

## Critical Path in BTA<sup>ω</sup>



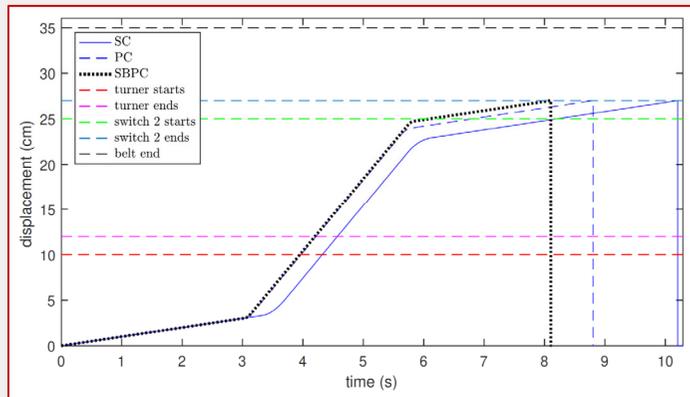
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## Redesigning Critical Resources – Belt Controller

	Sequential Controller	Pipelined Controller	Switched Pipelined Controller
$d_2$	4.55	4.21	4.19
$d_3$	5.62	4.57	3.89

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## Redesigning Critical Resources – Belt Controller



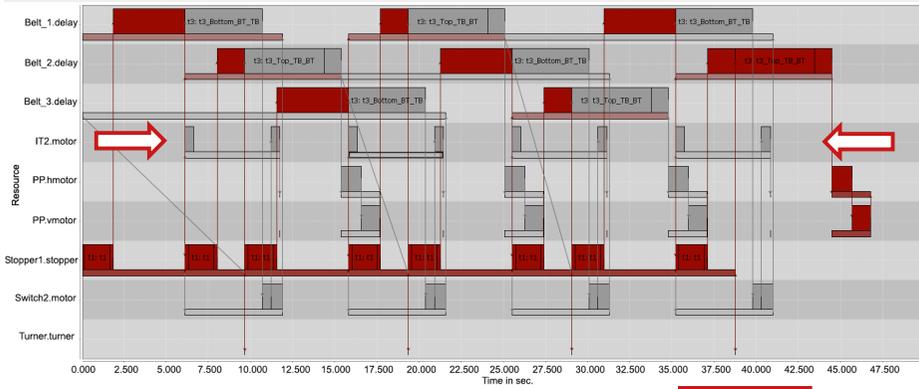
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## Redesigning Critical Resources – Belt Controller

	Sequential Controller	Pipelined Controller	Switched Pipelined Controller
max throughput	0.091 p/s	0.103 p/s	0.111 p/s

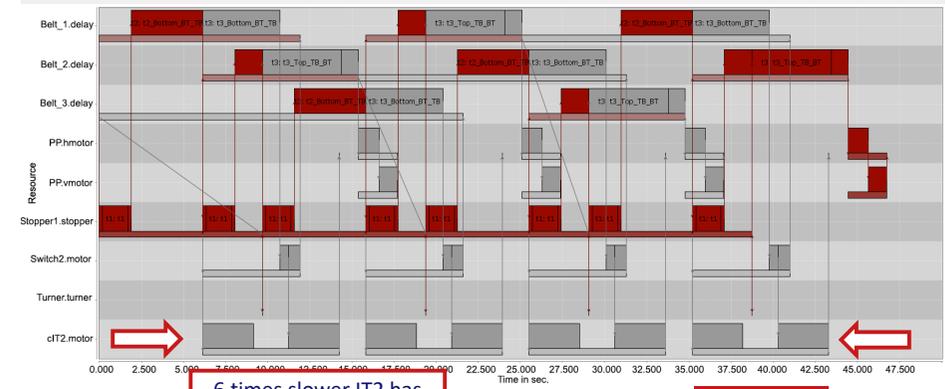
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## Redesigning Non-Critical Resources – Indexing Table



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## Redesigning Non-Critical Resources – Indexing Table



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Activity Models allow  
Fast Design-Space Exploration

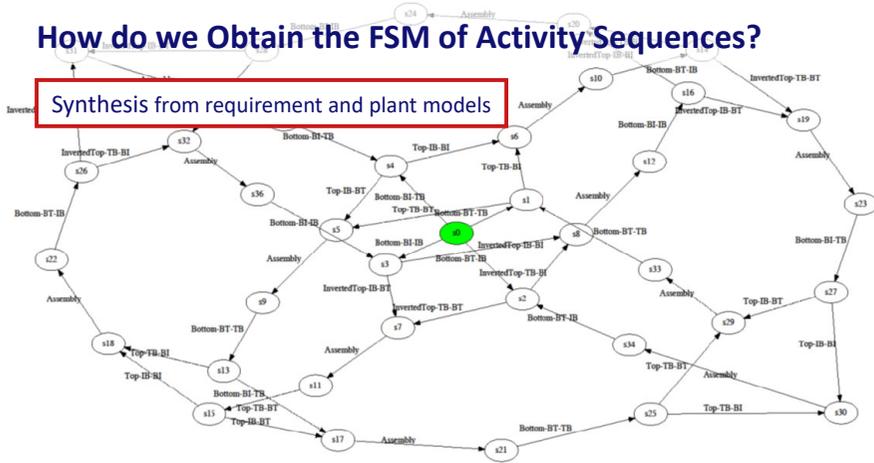
Supervisory Control

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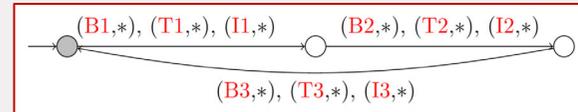
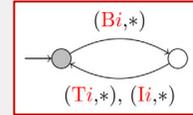
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## How do we Obtain the FSM of Activity Sequences?

Synthesis from requirement and plant models



## Plant Models



+ some context juggling

## Requirement Models

Assembly do  $c := c - 1$    $(Bi,*)$  when  $c < 1$  do  $c := c + 1$

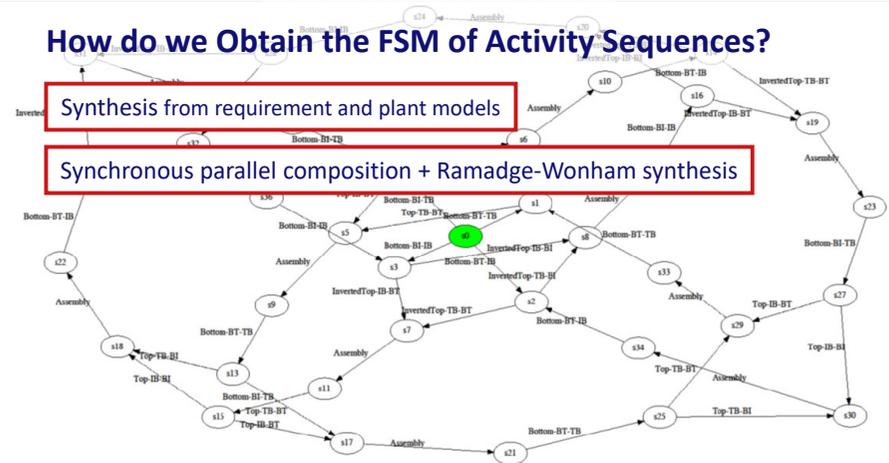
Assembly do  $c := c - 1$    $(Ti,*), (Ii,*)$  when  $c < 3$  do  $c := c + 1$

Assembly  $\Rightarrow$   
 IndexTableBuffer.c > 0 and BeltBuffer.c > 0

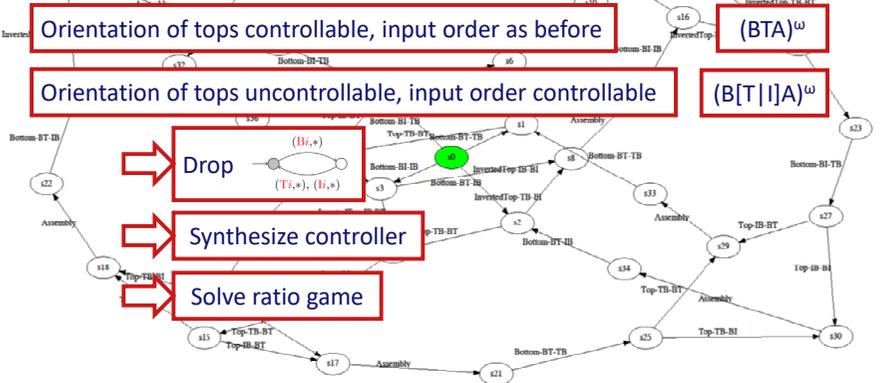
## How do we Obtain the FSM of Activity Sequences?

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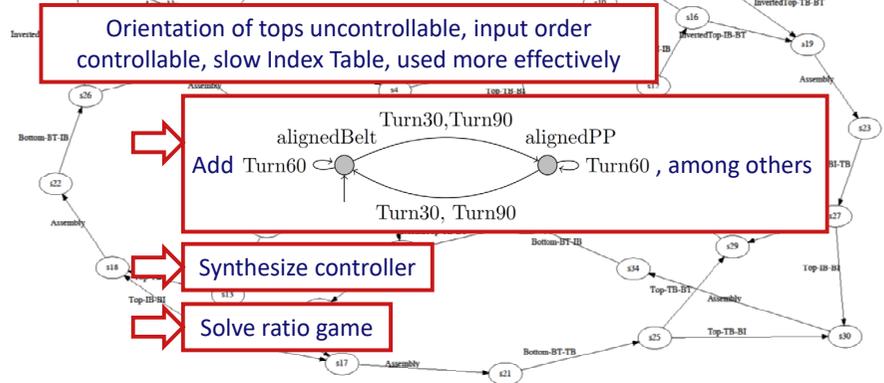
Synchronous parallel composition + Ramadge-Wonham synthesis



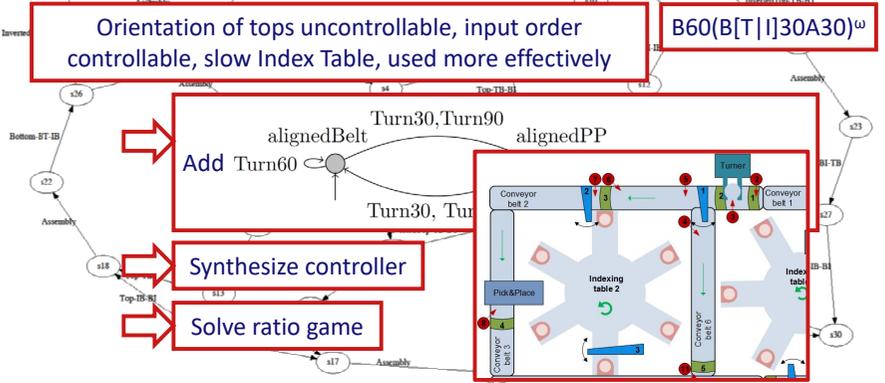
### Optimal Throughput?



### Optimal Throughput?



### Optimal Throughput?



### Supervisory Controller Synthesis from Activity Models

- scales better than Synthesis from Action Models
- allows Performance Optimization

## Conclusions

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## Remember the Take Aways?

Think (max,+)

Activity models

- improve scalability of performance analysis and controller synthesis
- allow integral supervisory controller synthesis and performance optimization

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## Questions ?

- What is the productivity of this machine?
- Optimal production schedule?
- Productivity bottlenecks?
- Optimal controllers?
- Design alternatives?

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