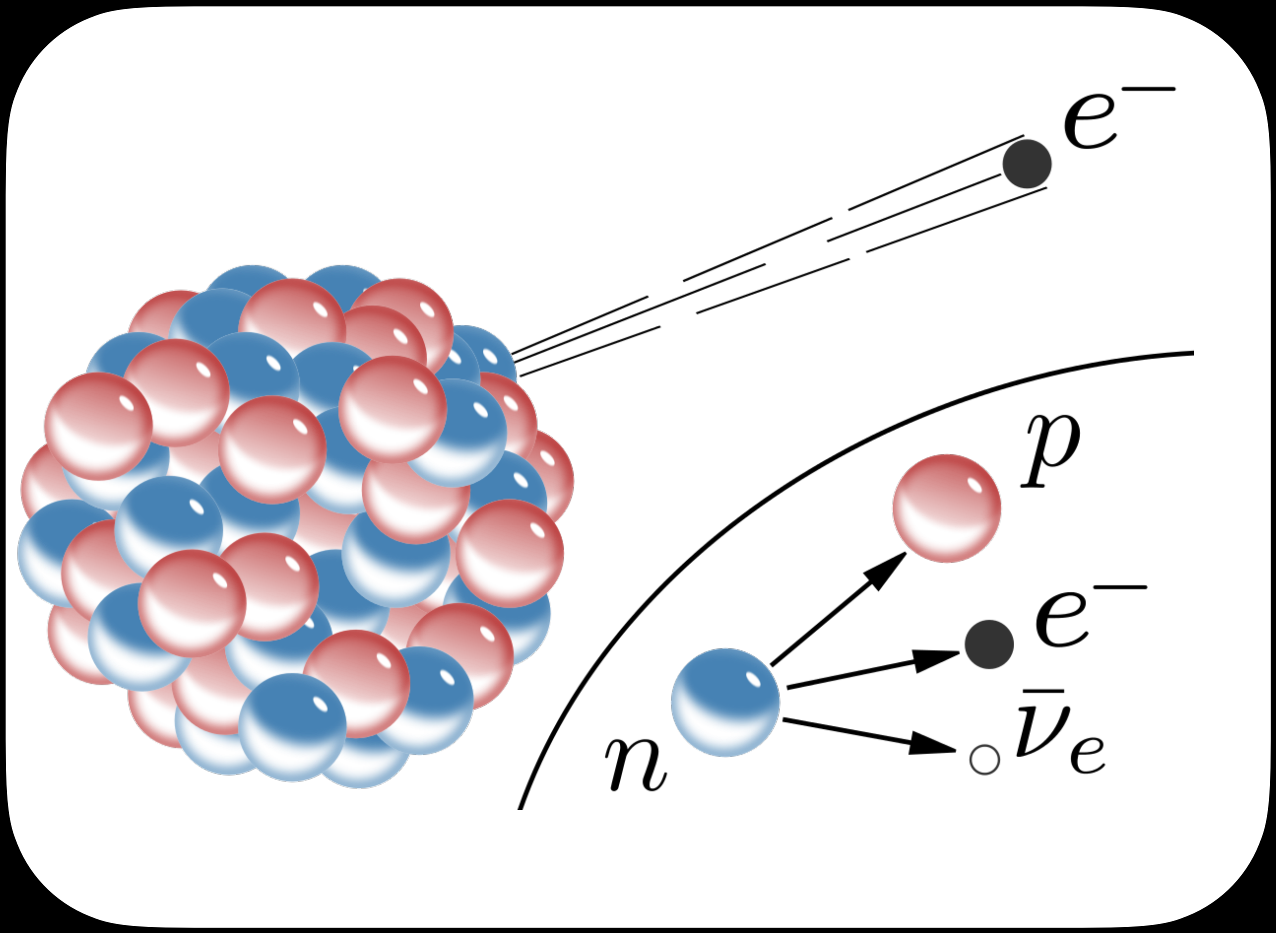
Weak interactions



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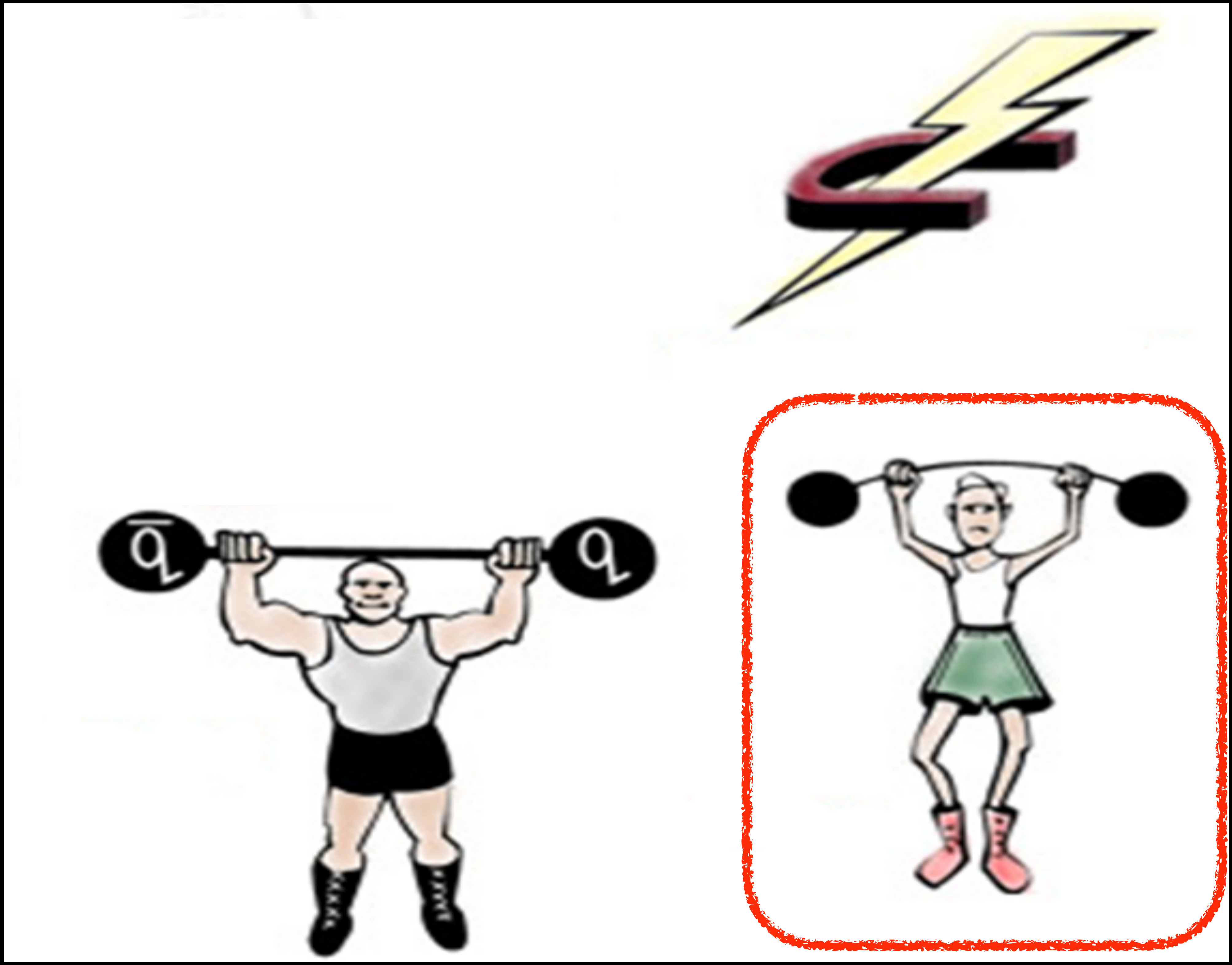
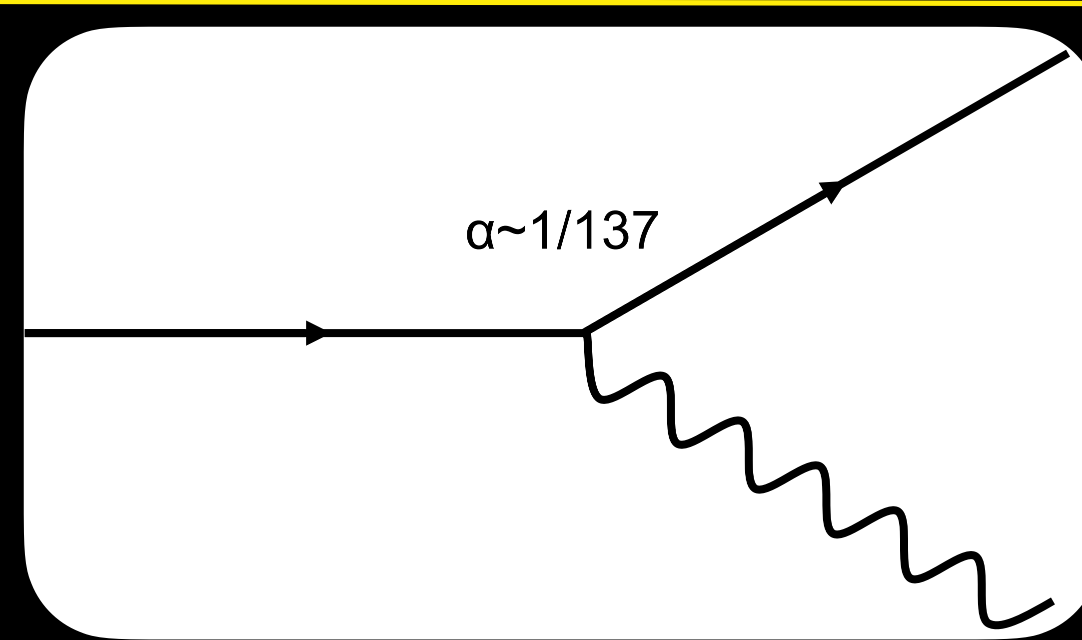
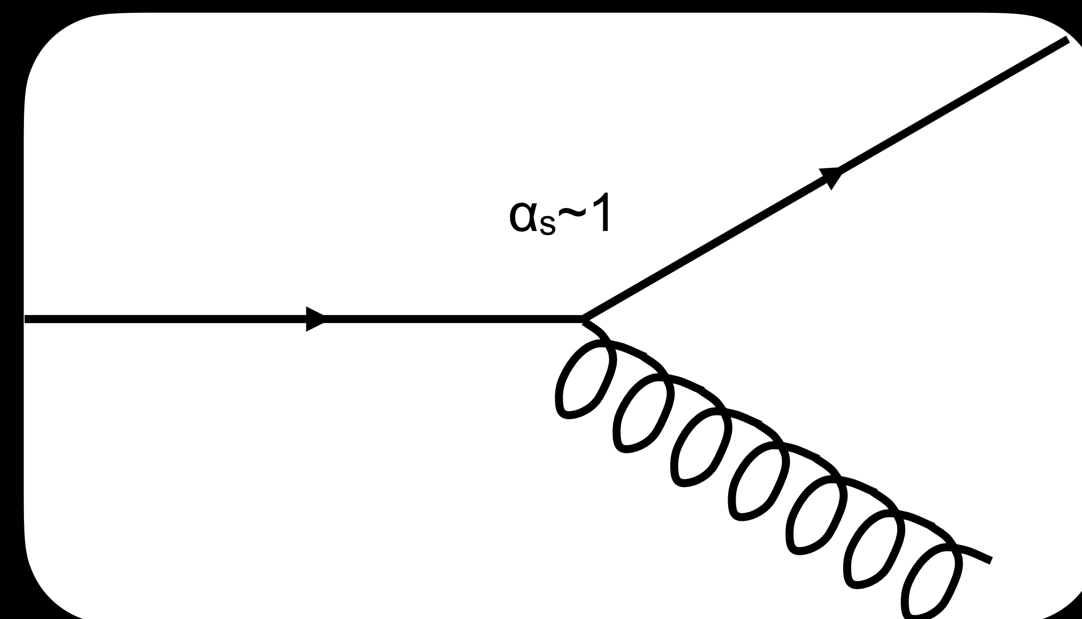
Forces and mediators



Interactions of the Standard Model

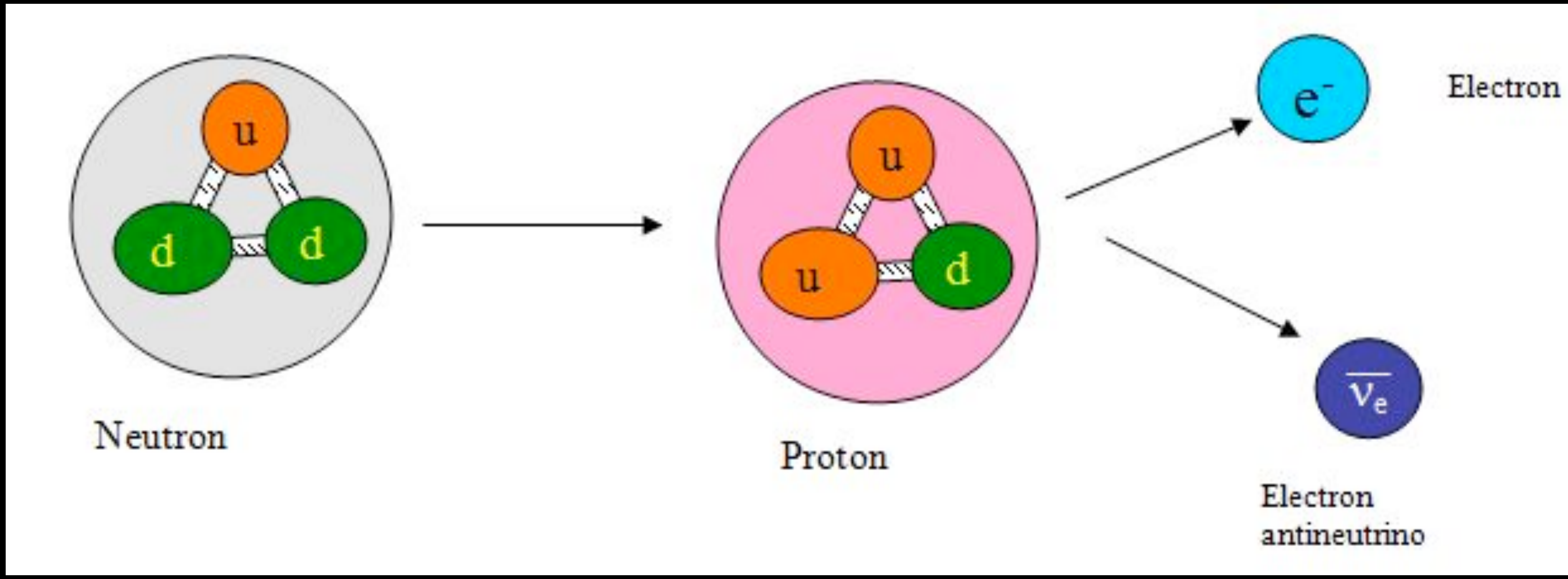
- Electromagnetic interactions
  - Mediated by the massless photon
    - Photon couples to electric charge
  - Act on charged leptons, quarks and antiquarks
  - Do not change flavour
- Strong interactions
  - Mediated by the 8 massless gluons
    - Gluons couple to colour charge
  - Act on quarks, antiquarks and gluons
  - Do not change flavour
- Weak interactions
  - Mediated by the massive W and Z bosons
    - Mediators couple to all particles
  - Change flavour

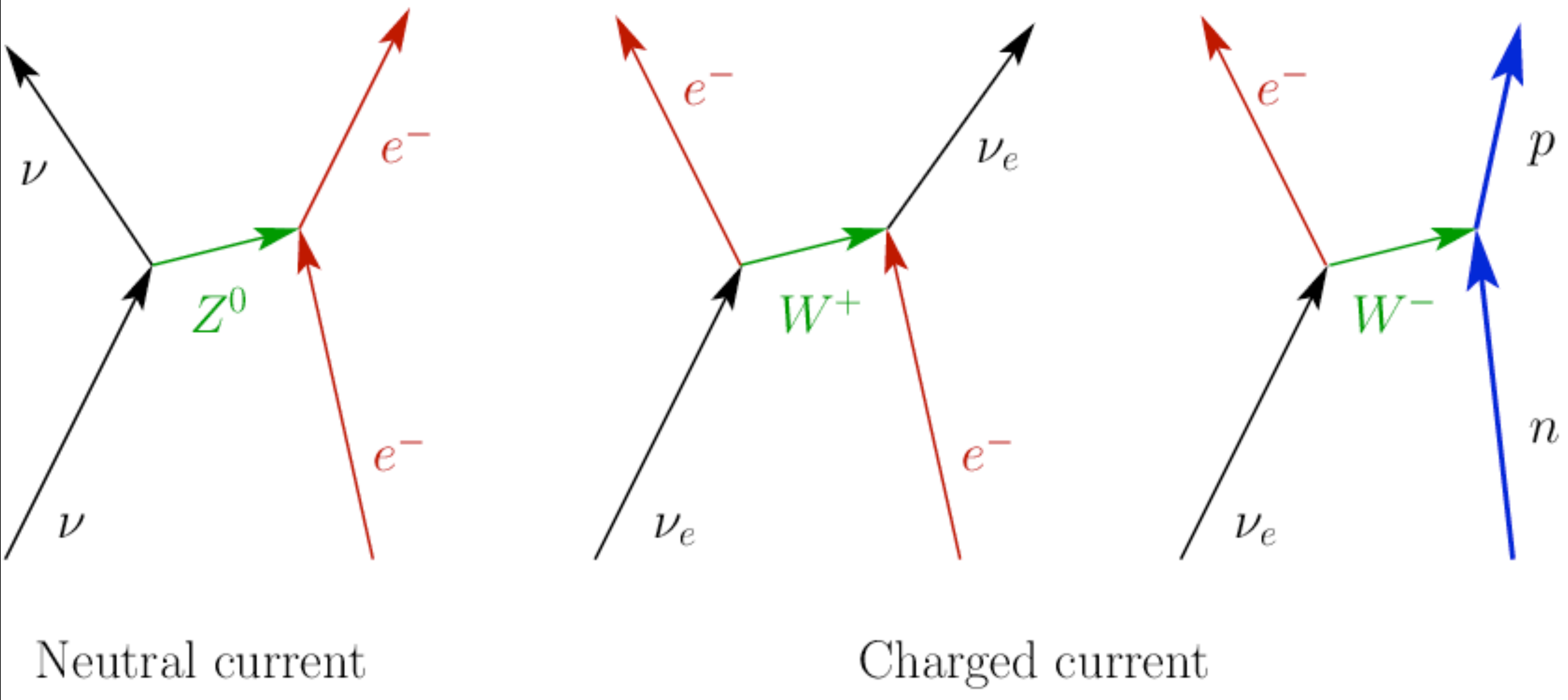




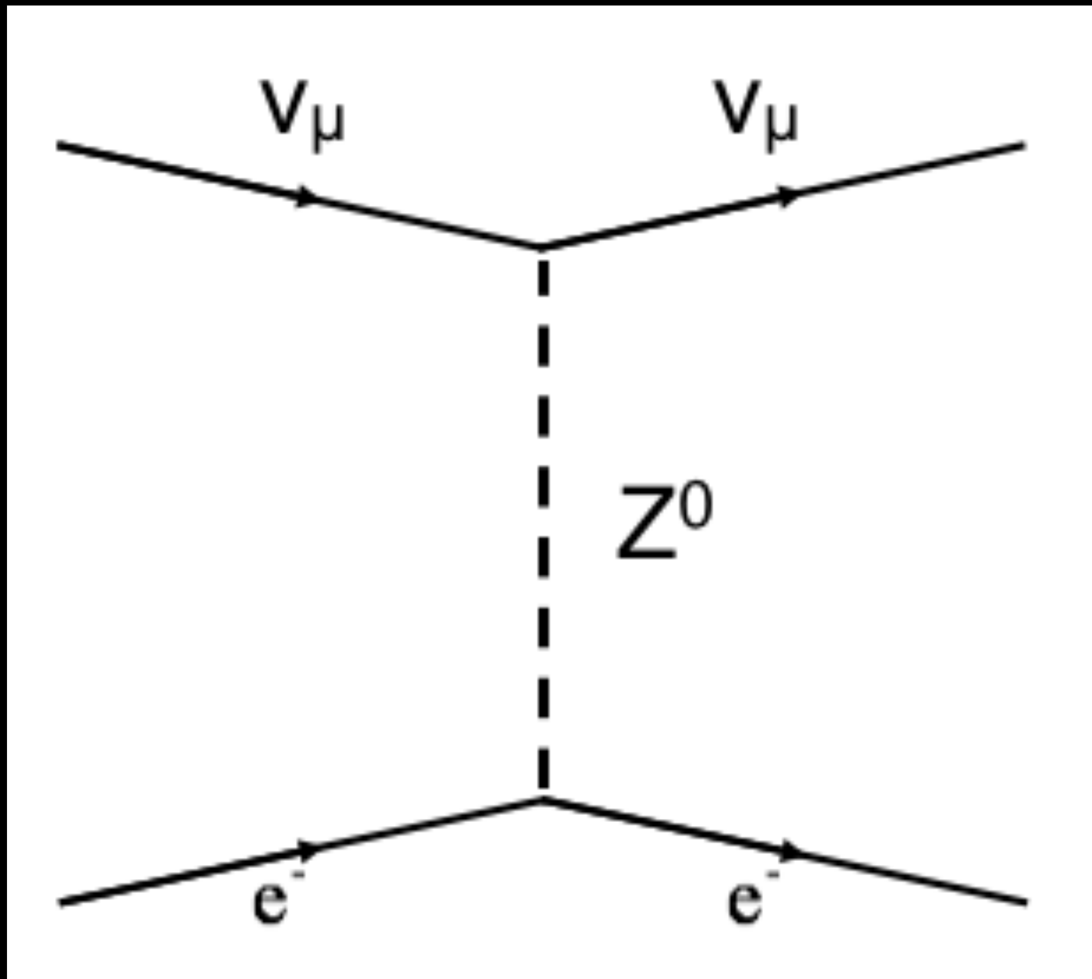
Very different interactions!!!

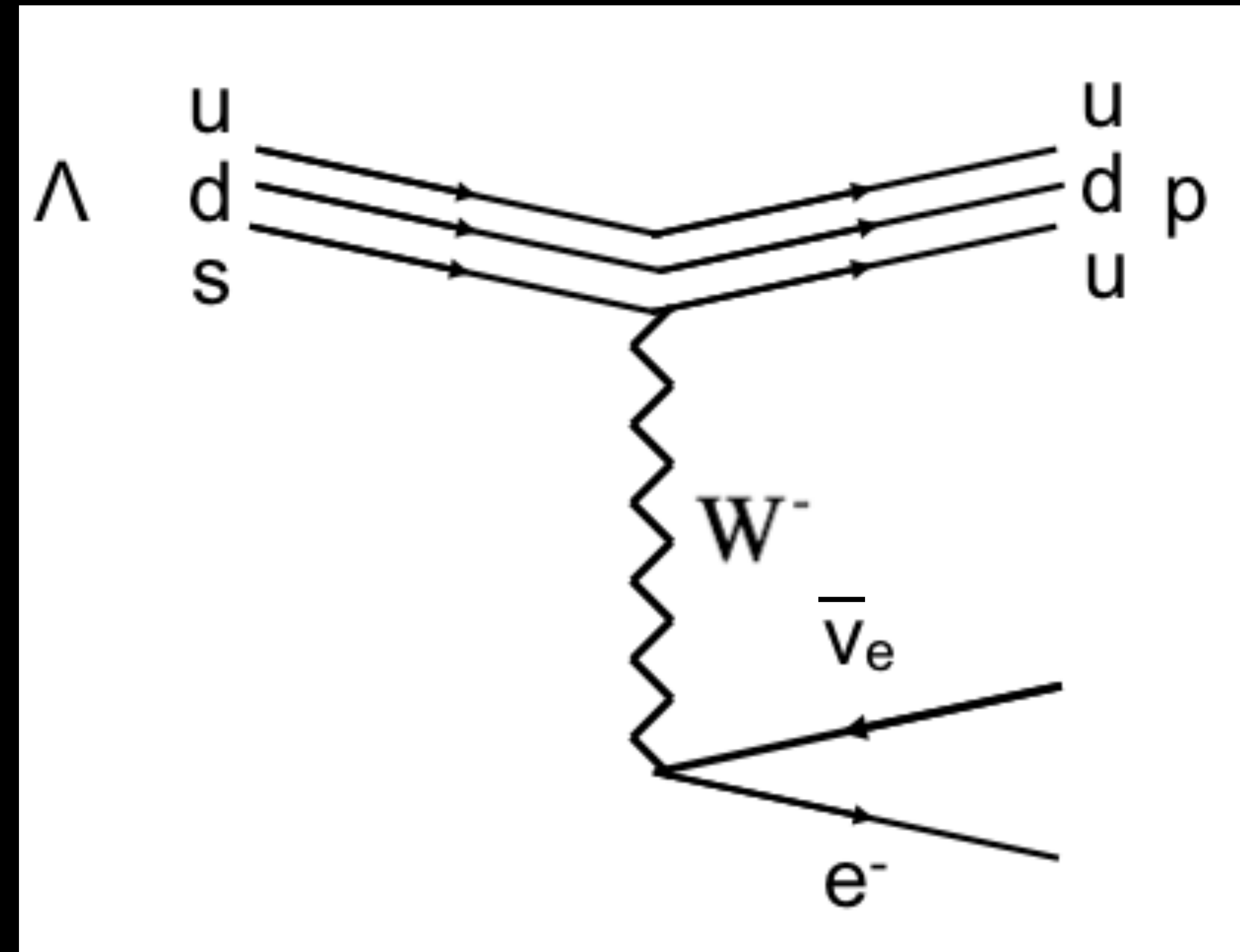
Weak interactions





Categories of weak interactions

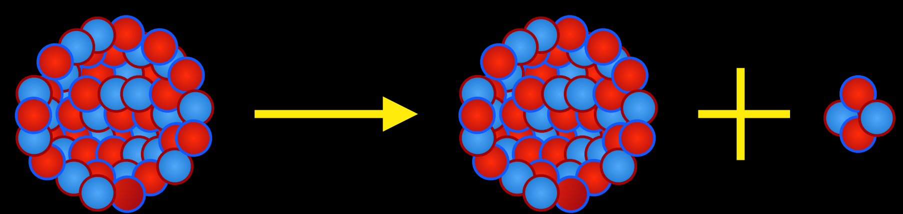


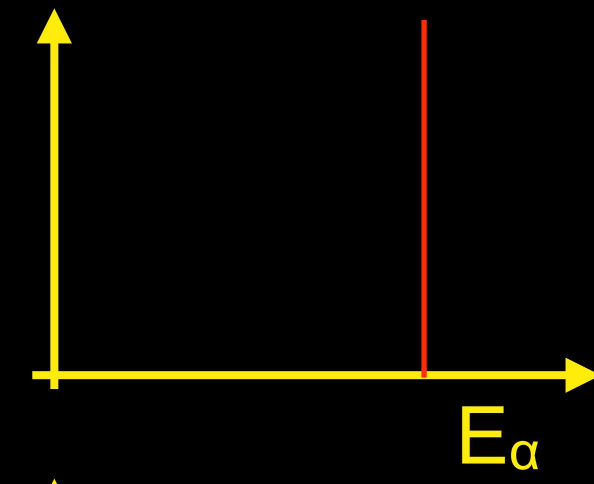


- Mediated by the W (charged) and Z (neutral) bosons which are massive
- Leptonic processes (involve only leptons), semi-leptonic (leptons and hadrons), hadronic (only hadrons)
- Can change the flavour of quarks
  - Long lived decaying particles ~10^{-10}sec

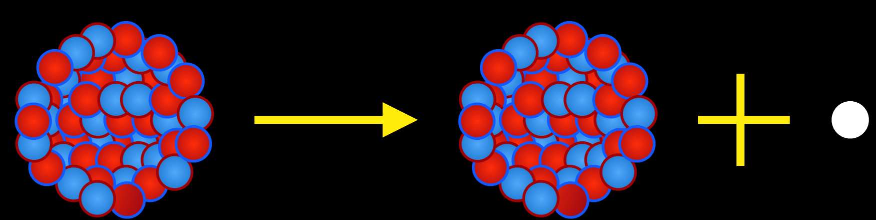
Birth of weak interactions

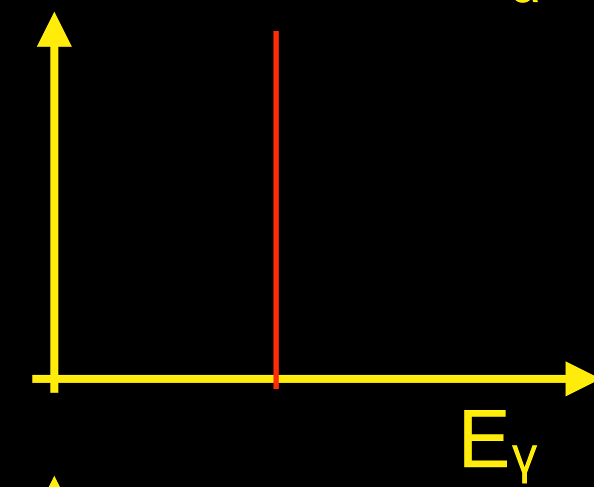
- Three types of nuclear radiation
  - α-decay
    - Nucleus emits a He nucleus
    - The α-particle is emitted with the same energy



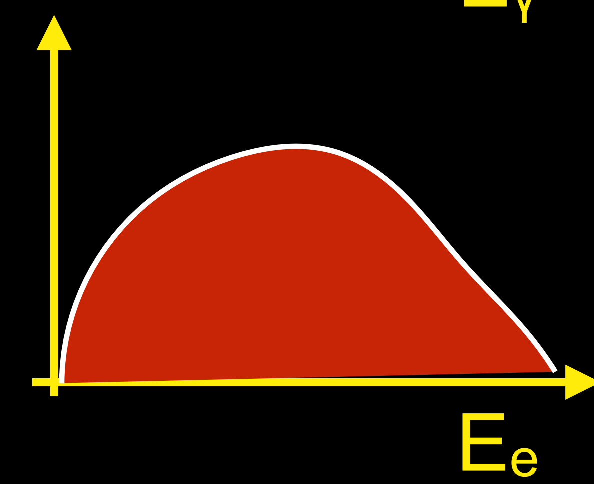


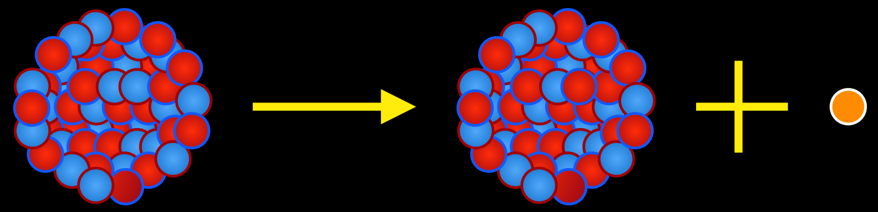
  - γ-decay
    - Nucleus emits a photon
    - The γ-particle has always the same energy





  - β-decay
    - Neutron turns into a proton and emits an electron
    - The electron is emitted with a range of energies





β-decay

- The story of the weak interactions begins already back in 1896
  - By that time Becquerel had discovered radioactivity
  - It soon became clear that non-stable nuclei that decay can emit three types of radiation: α, β and γ rays
- Measurements over the years of β-decays proved that the β-particles were electrons emitted without a discrete but with a continuous energy spectrum

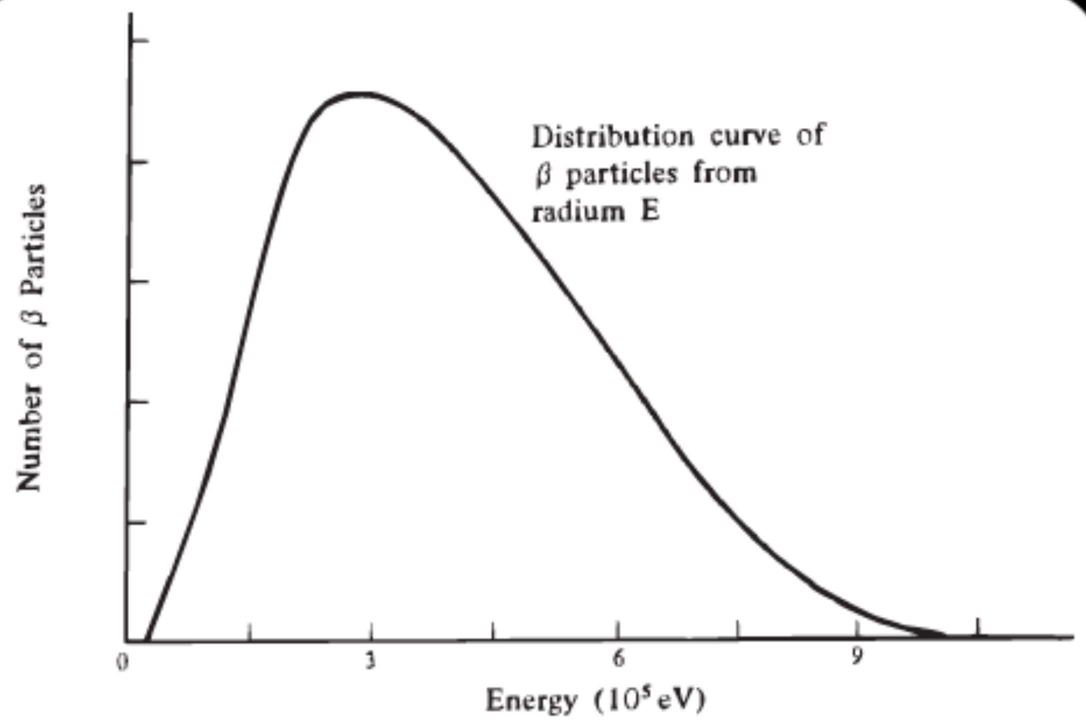
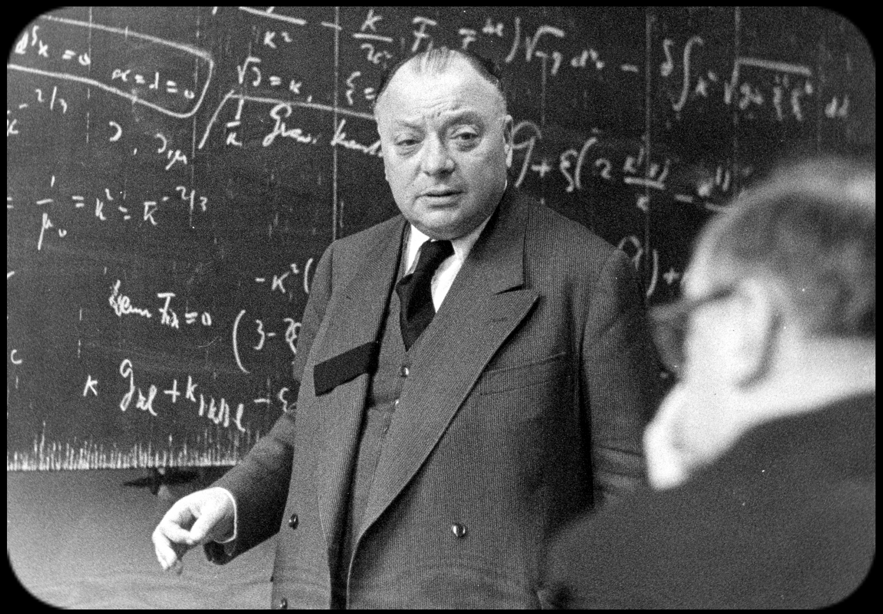
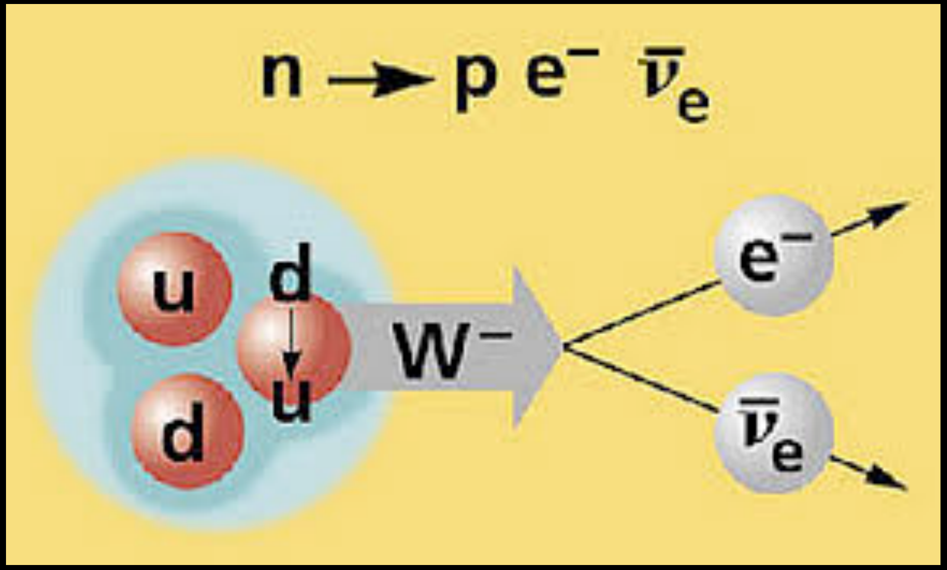


Figure 11.1: Example of a beta spectrum. [This figure is taken from one of the classic papers: C.D. Ellis and W.A. Wooster, *Proc. R. Soc.* (London) **A117**, 109 (1927).] Present experimental techniques yield more accurate energy spectra, but all essential aspects are already contained in the curve reprinted here.

β-decay (cont.)



- The existence of quantised energy levels for the nuclei imposed a question related to the continuum energy spectrum of electrons in β-decays
- The puzzle was solved by Pauli around 1930 who theorised the existence of a light, uncharged, penetrating particles, the (anti)neutrino that was emitted together with the e^- in β-decay
- The neutrino (ν) had to be neutral and weakly interacting
  - it hadn't been detected !
  - Neutrinos were first detected in 1956
- Soon later, Fermi made detailed calculations on the mechanism based on which a neutron is transformed into a proton with the parallel emission of an electron and its antineutrino
  - The electron and the antineutrino share the decay energy and some times the former gets a small part and some other times a larger part of it
    - The continuous energy spectrum is qualitatively explained



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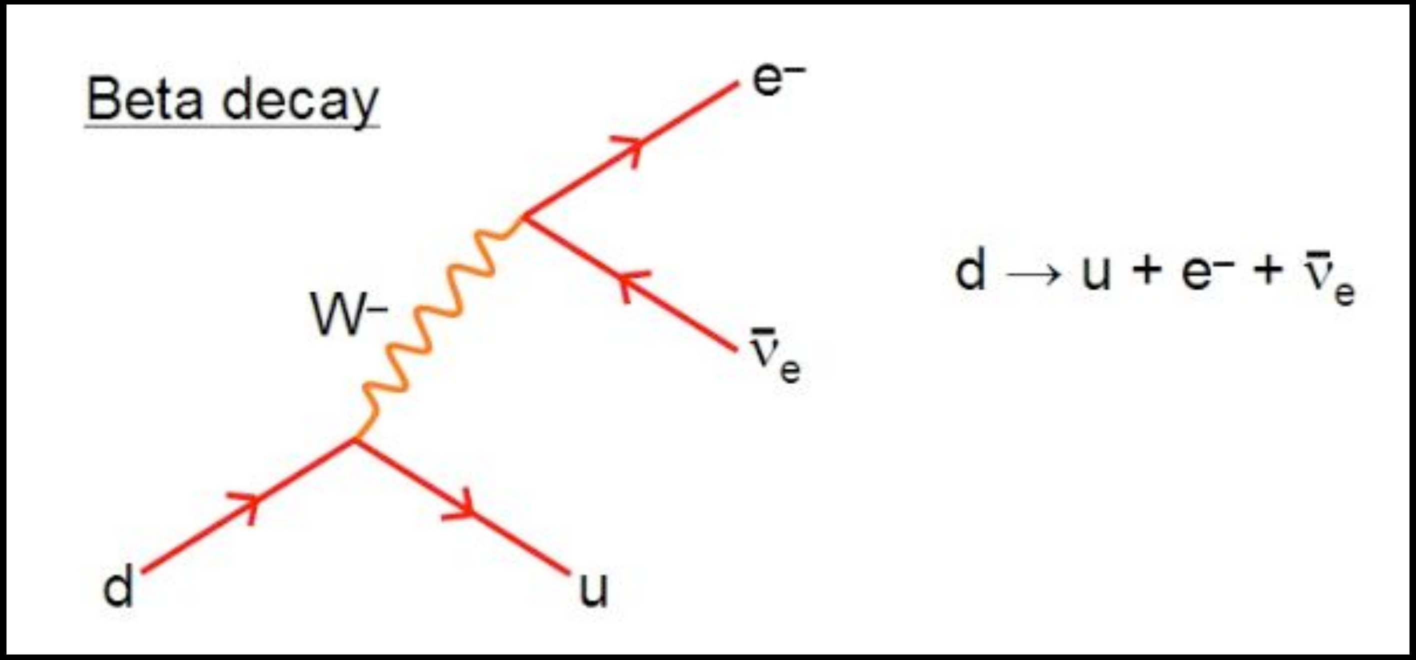
β-decay (cont.)

- We now understand β-decay in terms of the weak force which is mediated by a massive (80 GeV) W-boson
- Here the weak interaction vertex changes a d into a u quark and then "pair-produces" an e^- and a $\overline{\nu}_e$ In fact, the transition that takes place involves the constituents of the neutron and the proton, the quarks
  - A d-quark (from the neutron) is transformed into a u-quark and together with the remaining d and u quarks of the initial neutron form a proton (uud)
    - with the parallel emission of an electron needed to conserve the electric charge in the entire process
    - And an electron antineutrino needed to conserve the electron lepton number

$$n \to p e^- \overline{\nu}_e \quad \longrightarrow \quad d \to u e^- \overline{\nu}_e$$



Kurie plot

$$\left(\frac{dw_{\beta\alpha}}{p_e^2\, dp_e}\right)^{1/2} = \text{const.}\left(\overline{|\langle pe^-\bar{\nu}|H_w|n\rangle|^2}\right)^{1/2}(E_{\max}-E_e). \qquad (11.6)$$

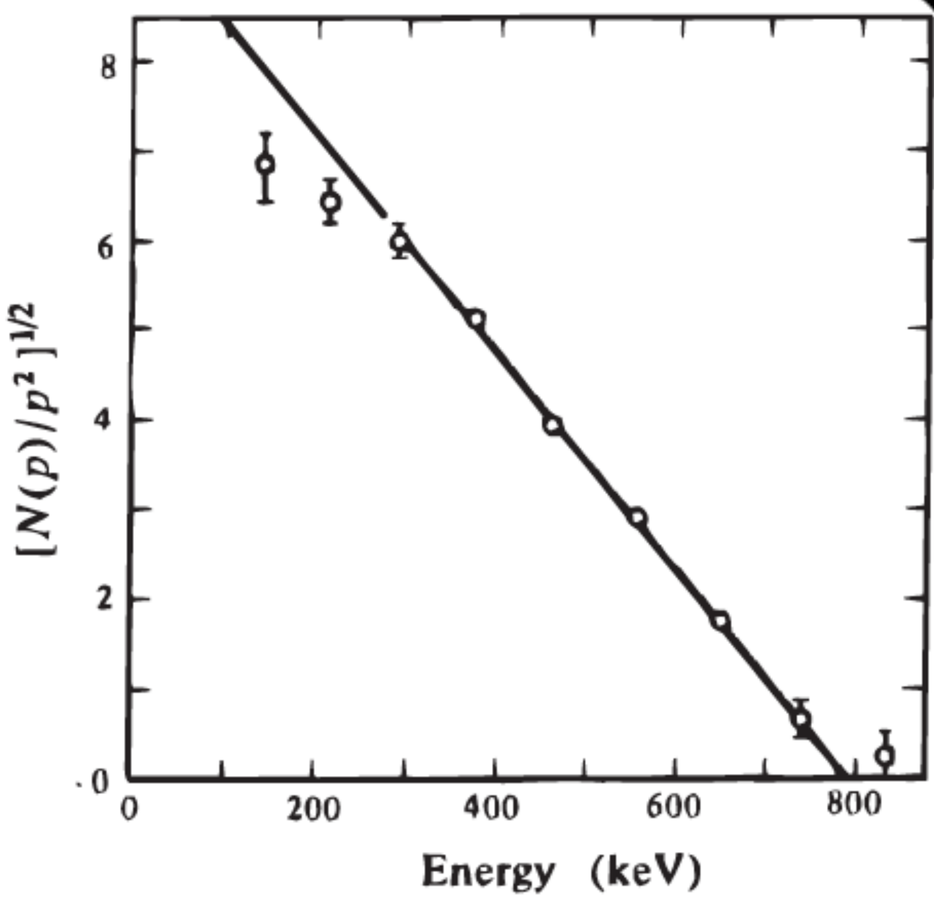
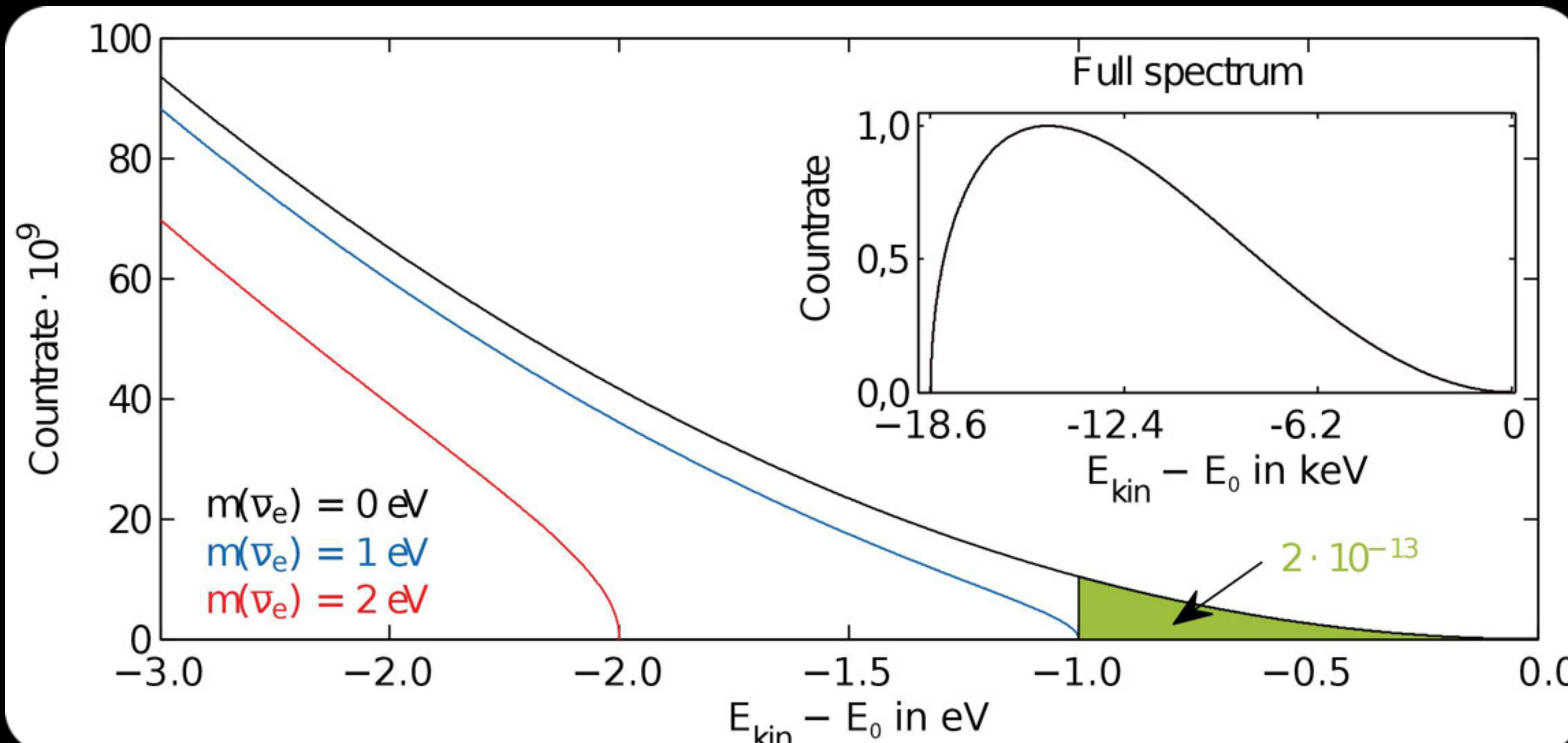


Figure 11.2: Kurie plot for the neutron decay. [From J. M. Robson, *Phys. Rev.* **83**, 349 (1951).] Here $N(p)$ corresponds to dw/dp_e in Eq. (11.6).

- ✓ The square root of the number of beta particles whose momenta (or energy) lie within a certain narrow range vs the energy of the electron
- ✓ The x-axis intercept of a Kurie plot corresponds to the maximum energy given to the electron/positron

Kurie plot (cont.)

- ✓ With a Kurie plot one can find the limit on the effective mass of a neutrino
- ✓ A mass larger than zero will modify the shape of the energy spectrum of the emitted β-electron near the high-energy endpoint



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Particle decays

- Consider a number of independent particles, each having probability λ to decay
- The number of decayed particle within dt is given by

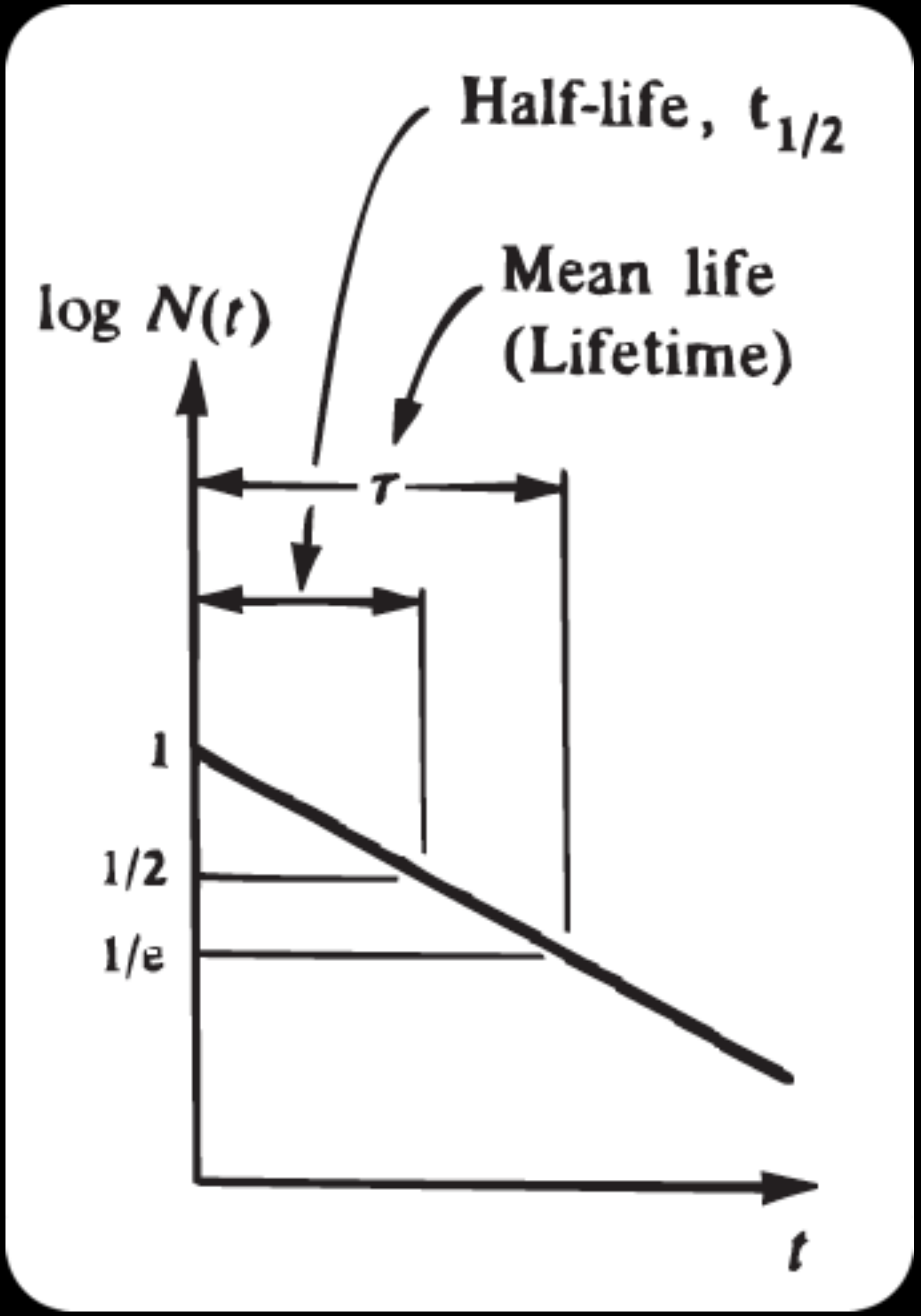
$$dN=-\lambda N(t)dt$$

$$\int_{N_0}^{N(t)} NdN = \int_0^t -\lambda dt$$

$$N(t)=N_0 e^{-\lambda t}$$

- Decay half life ($t_{1/2}$): the time it takes for half of the sample of particles to decay
- Mean lifetime (τ): The average time a particle exists before decaying

$$\tau=\frac{1}{\lambda}=\frac{t_{1/2}}{ln2}$$



β-decay lifetime

- Information about the magnitude of the matrix element can be obtained from the lifetimes of β-emitters
- The matrix element is given by $|\overline{M_{if}}|^2 = \frac{2\pi^3}{ft_{1/2}} \frac{\hbar^7}{m_e^5 c^4}$
- The value of $ft_{1/2}$ is called **comparative half-time**
- The comparative half-time spans a wide range of values in nature from 10^3 up to 10^{23} sec
- This variation in time is not due to the weak interaction itself but rather to the nuclear wave functions that are a key component in the matrix element calculation

Table 11.1: Comparative Half-Lives of a Few Beta Decays.

Decay	Spin-parity Sequence	$t_{1/2}$	E_{max} (MeV)	$ft_{1/2}$ (sec)
$n \to p$	$\frac{1}{2}^+ \to \frac{1}{2}^+$	10.2 min	0.782	1054
$^6He \to {}^6Li$	$0^+ \to 1^+$	0.807 sec	3.50	800
$^{14}O \to {}^{14}N$	$0^+ \to 0^+$	70.6 sec	1.752	3039

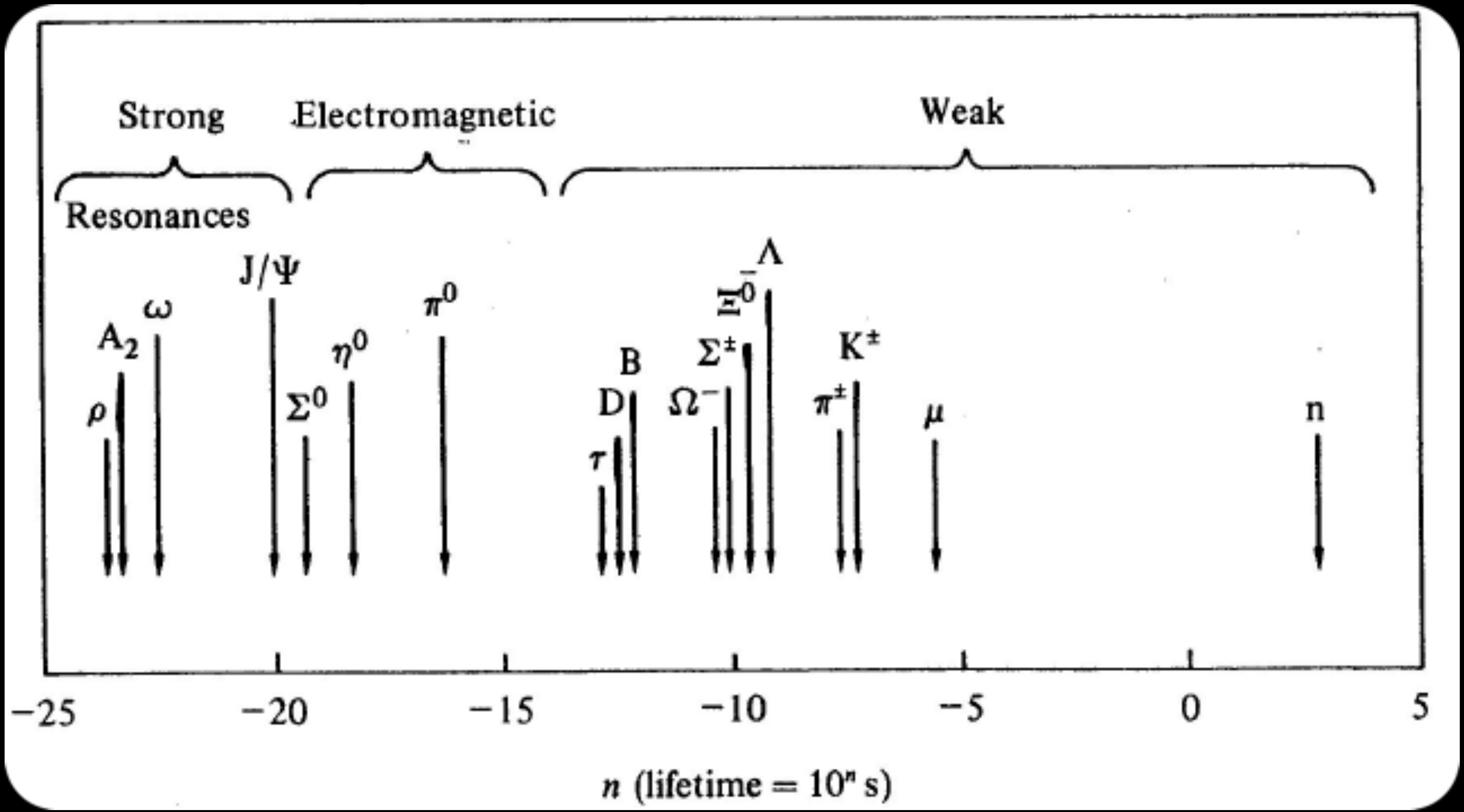
$$|\overline{M_{if}}|^2_{n \to pe^- \bar{\nu}_e} \approx 2 \cdot 10^{-4} MeV fm^3$$

Matrix element: proportional to volume

$$H_w \approx 10^{-4} MeV$$

Weak decays

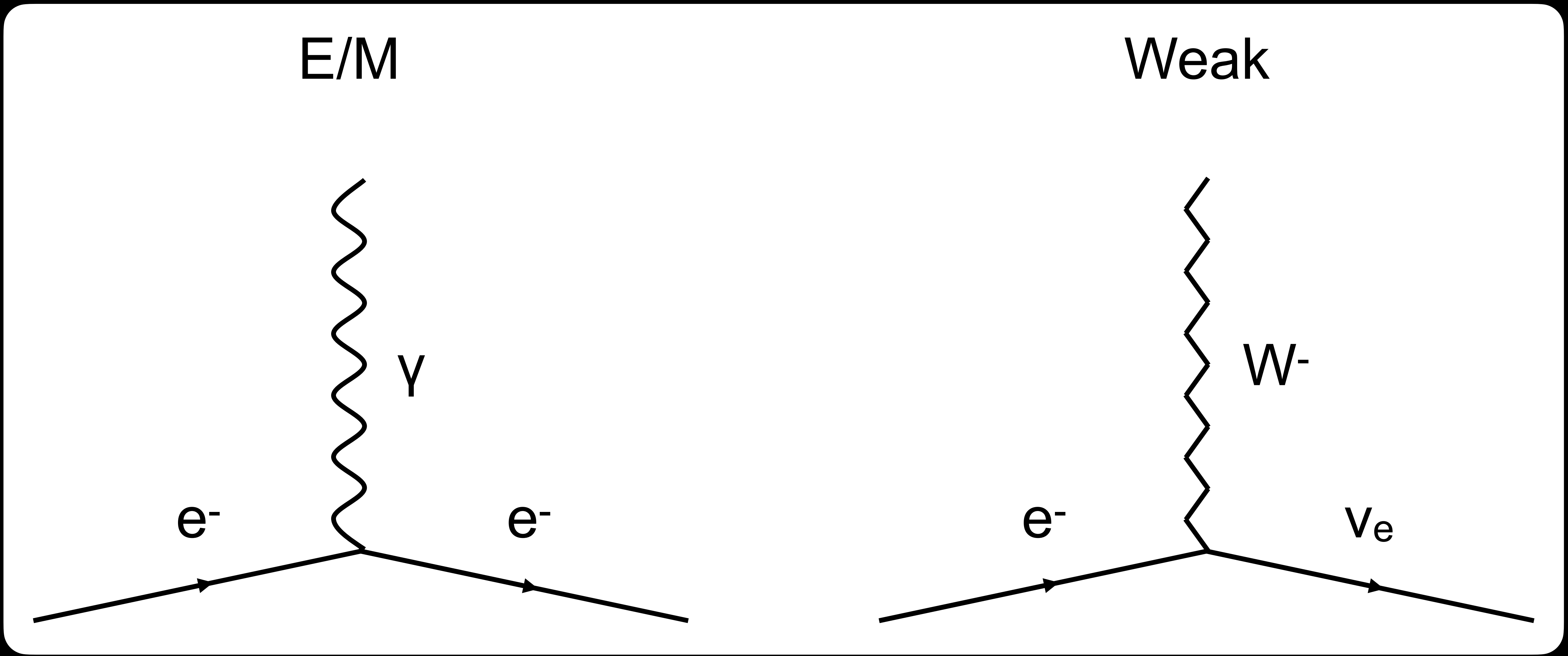
- Because the weak interaction changes flavour it is responsible for the majority of particle decays



- Because the weak interaction is a weak force particle lifetimes are relatively long

Parenthesis: Propagator for weak decays (Feynman rules)

- The main difference between E/M and weak interaction in terms of Feynman diagrams and rules is the form of the propagator

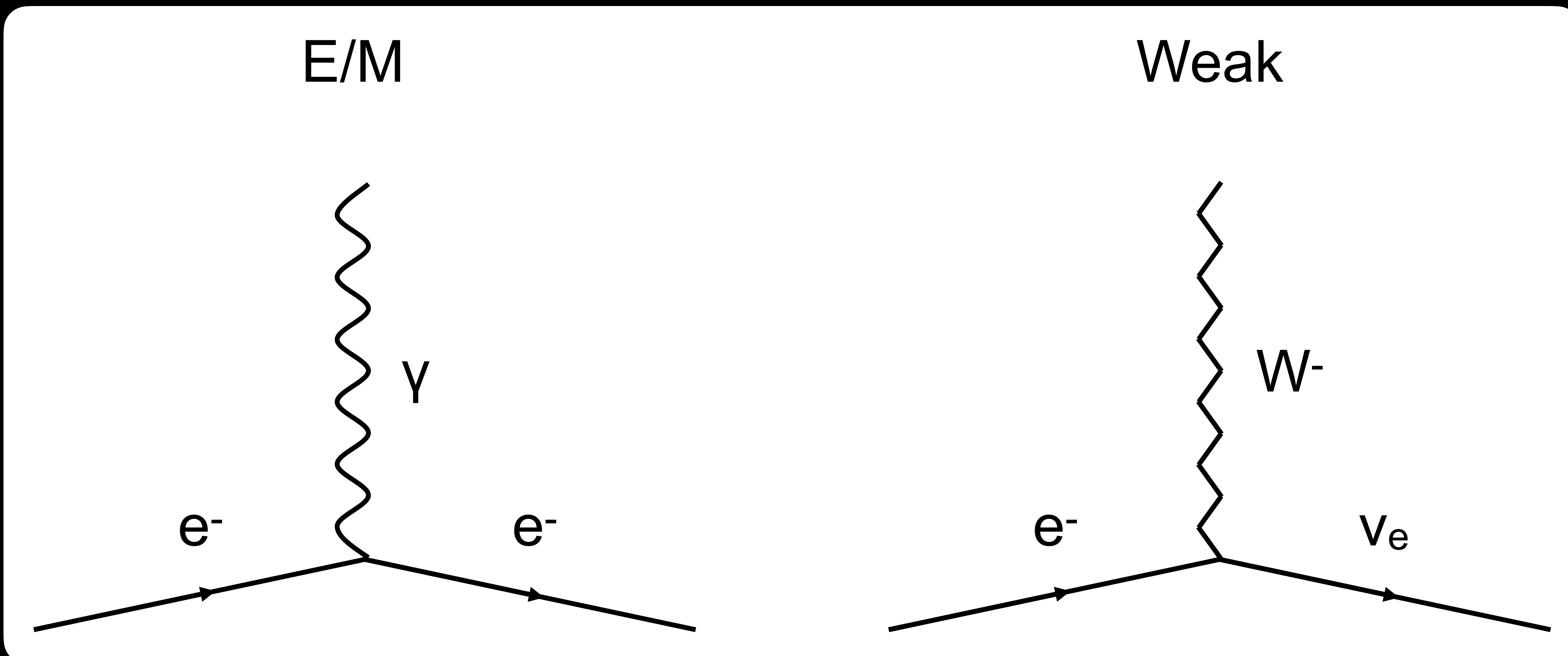


$$-\frac{ig_{\mu\nu}}{q^2}$$

$$-\frac{i\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{M^2}\right)}{q^2-M^2}$$

Parenthesis: Propagator for weak decays (Feynman rules)

- The propagator ends up in the matrix element from where it appears in the cross-section
- Usually processes take place for $q^2 \ll M^2$ so that the propagator turns



$$-\frac{ig_{\mu\nu}}{q^2} \qquad \frac{ig_{\mu\nu}}{M^2} \longleftarrow -\frac{i\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{M^2}\right)}{q^2-M^2}$$

Muon decay: the weak coupling constant

- The total transition rate for the muon decay is related to the E/M coupling constant can be determined by studying processes that involve charged particles that interact via the exchange of virtual photons or electrons
  - Rutherford scattering
  - Mott scattering
  - …
- For the weak coupling constant one of the best ways to constrain its value is via the measurement of the total lifetime of μ
  - Muon decays involve no hadronic states ⇒ calculations not affected by complications introduced by hadrons
  - Possibility to perform a very accurate measurement

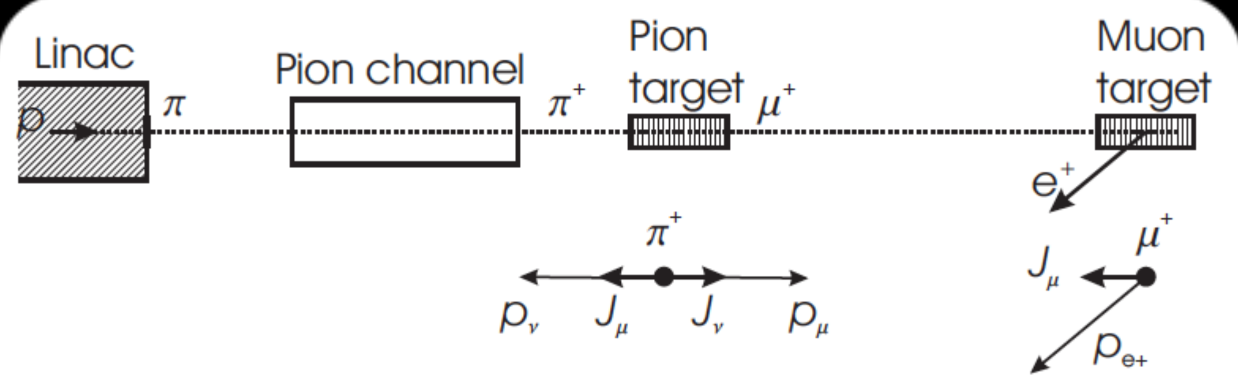
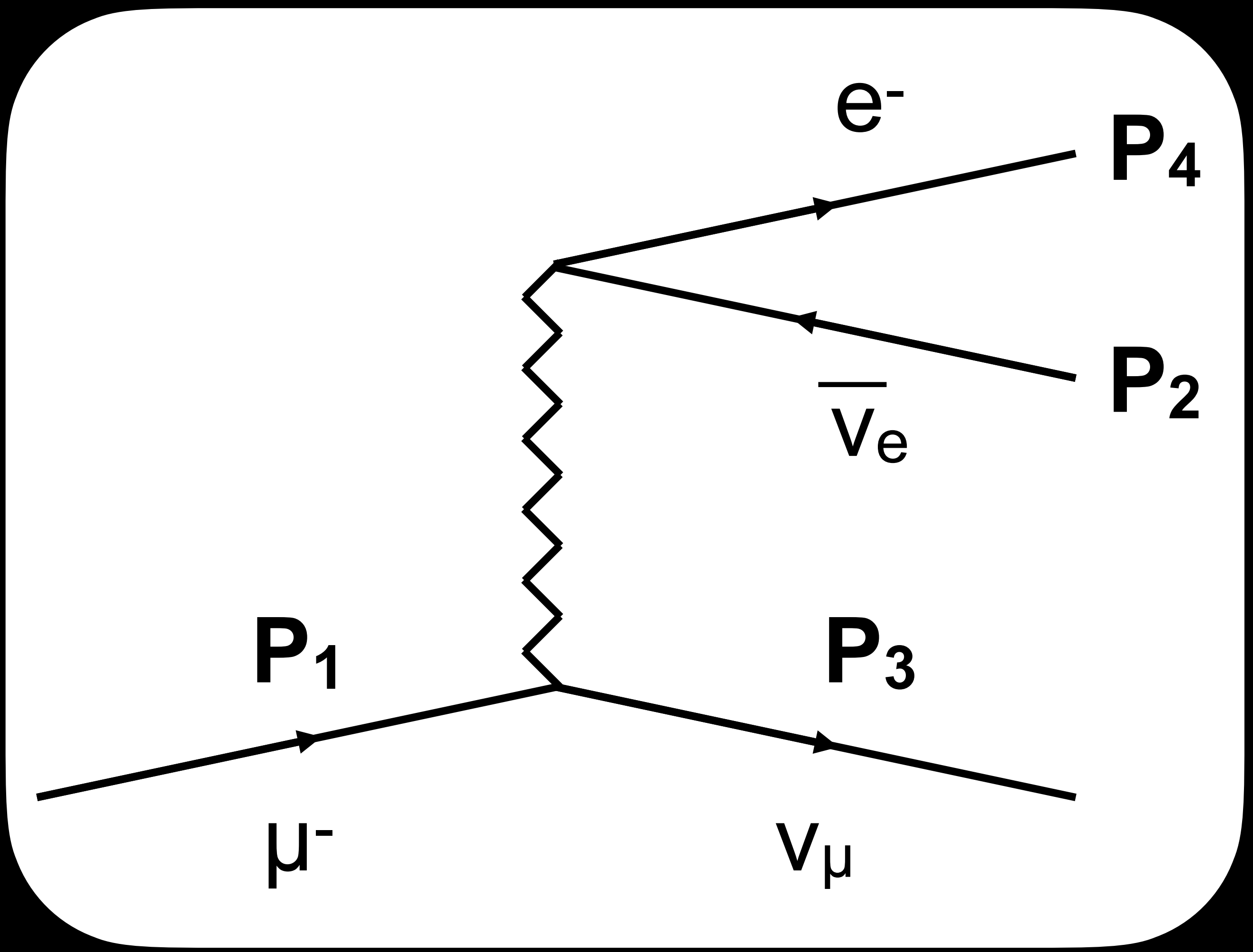


Figure 11.7: A positive pion is selected in the pion channel and comes to rest in the pion target. The pion decay results in a fully polarized muon. The muon escapes from the pion target and comes to rest in the muon target. Its spin points in the direction from which it came. The decay electron is then observed.

Muon decay



Matrix element

$$M_{if}=\frac{g_W^2}{8M_W^2}\left[\bar{u}_3\gamma^\mu(1-\gamma^5)u_1\right]\left[\bar{u}_4\gamma_\mu(1-\gamma^5)v_2\right]$$

Decay rate

$$d\Gamma=\left(\frac{g_W}{4\pi M_W}\right)^4 m_\mu\left(\frac{m_\mu}{2}-\frac{2}{3}P_4\right)d^3P_4$$

$$\frac{d\Gamma}{dE}=\left(\frac{g_W}{M_W}\right)^4\frac{m_\mu^2E^2}{2(4\pi)^3}\left(1-\frac{4E}{3m_\mu}\right)$$

$$\Gamma=\left(\frac{m_\mu g_W}{M_W}\right)^4\frac{m_\mu}{12(8\pi)^3}$$

Lifetime

$$\tau=\frac{1}{\Gamma}=\left(\frac{M_W}{m_\mu g_W}\right)^4\frac{12(8\pi)^3}{m_\mu}$$

Both g_W and M_W appear as their ratio and not independently!

Muon decay (cont.)

- Introducing the Fermi coupling constant $G_F=\frac{\sqrt{2}}{8}\left(\frac{g_W}{M_W}\right)^2$

$$\tau=\frac{1}{\Gamma}=\left(\frac{M_W}{m_\mu g_W}\right)^4\frac{12(8\pi)^3}{m_\mu} \longrightarrow \tau=\frac{192\pi^3}{G_F^2 m_\mu^5}$$

- Plugging in the value of the lifetime for the muon we get

$$g_W\approx 0.653$$

$$\alpha_W=\frac{g_W^2}{4\pi}\approx\frac{1}{29.5} \longrightarrow \alpha_W>\alpha\approx\frac{1}{137}$$

- The weak interactions are weak not because α_W is small but mainly because the mediators are so massive!

- If we work at low energies, then the denominator of the propagator becomes

$$-\frac{i\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{M^2}\right)}{q^2-M^2} \longrightarrow \frac{ig_{\mu\nu}}{M^2}$$

  - This makes the cross-section of the weak process so small!
  - For processes where $q^2 > M_W^2$ the weak interactions become stronger than the E/M ones!

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Weak processes

- ✓ A classification of the weak processes can be made by separating the weak current into a leptonic and a hadronic part
- ✓ Leptonic processes, characterised by a Hamiltonian expressed by a weak current

hadronic

$$J_{weak} = J^{l}_{weak} + J^{h}_{weak}$$

leptonic

$$H_{weak} = \frac{G_F}{\sqrt{2}\, c^2} \int d^3x \left[c^2 \rho_{weak} \rho^{\dagger}_{weak} - J_{weak} J^{\dagger}_{weak} \right]$$

$$J^{\mu}_{weak} = (c\rho_{weak}, J_{weak})$$

Probability density and current for the weak interactions

Weak leptonic processes

- Characterised by a weak leptonic current
  - They involve only leptons (and anti-leptons) in both the initial and final states

$e^- + \nu_e \rightarrow e^- + \nu_e$ $\mu^- + \nu_e \rightarrow \mu^- + \nu_e$ $\tau^- + \nu_e \rightarrow \tau^- + \nu_e$

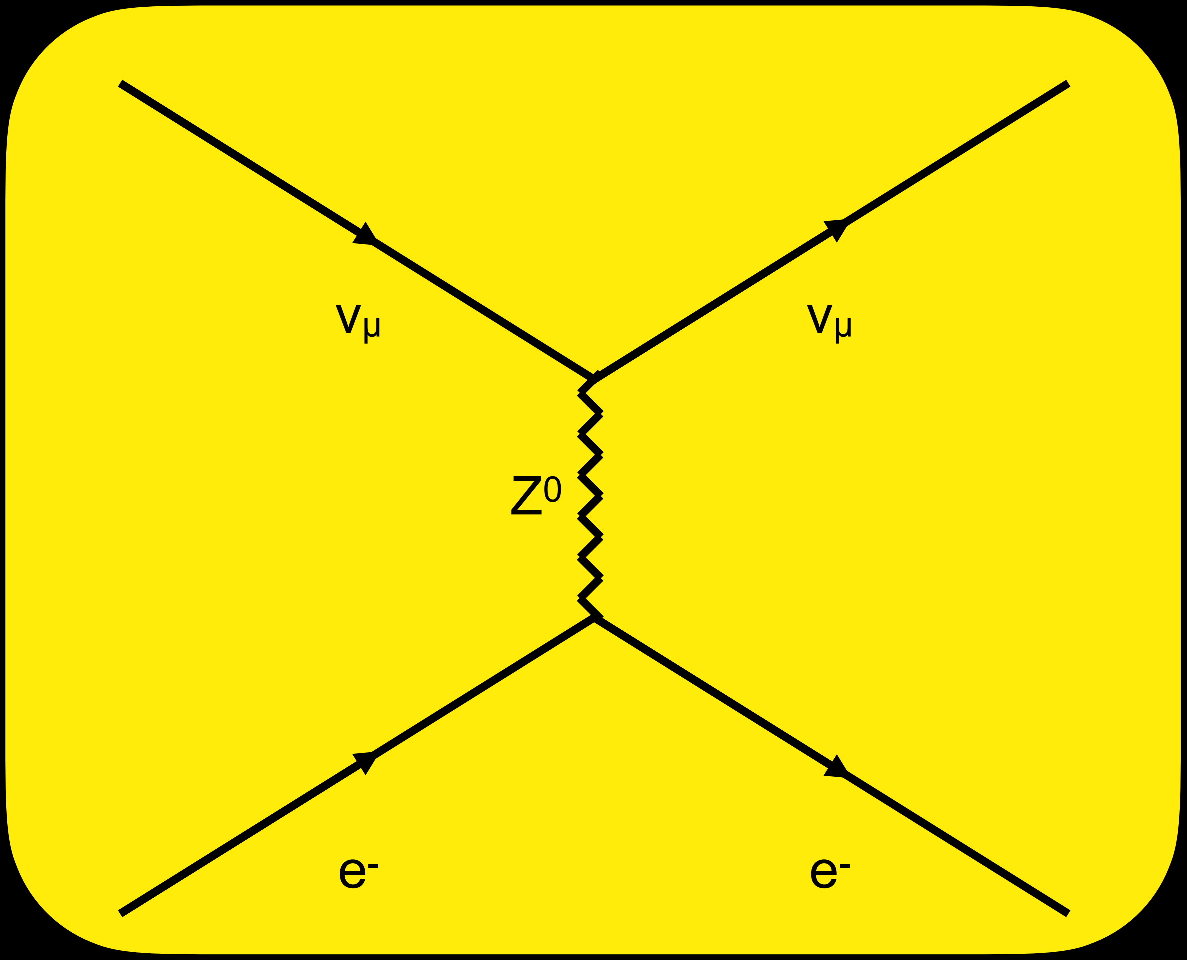
$e^- + \nu_\mu \rightarrow e^- + \nu_\mu$ $\mu^- + \nu_\mu \rightarrow \mu^- + \nu_\mu$ $\tau^- + \nu_\mu \rightarrow \tau^- + \nu_\mu$

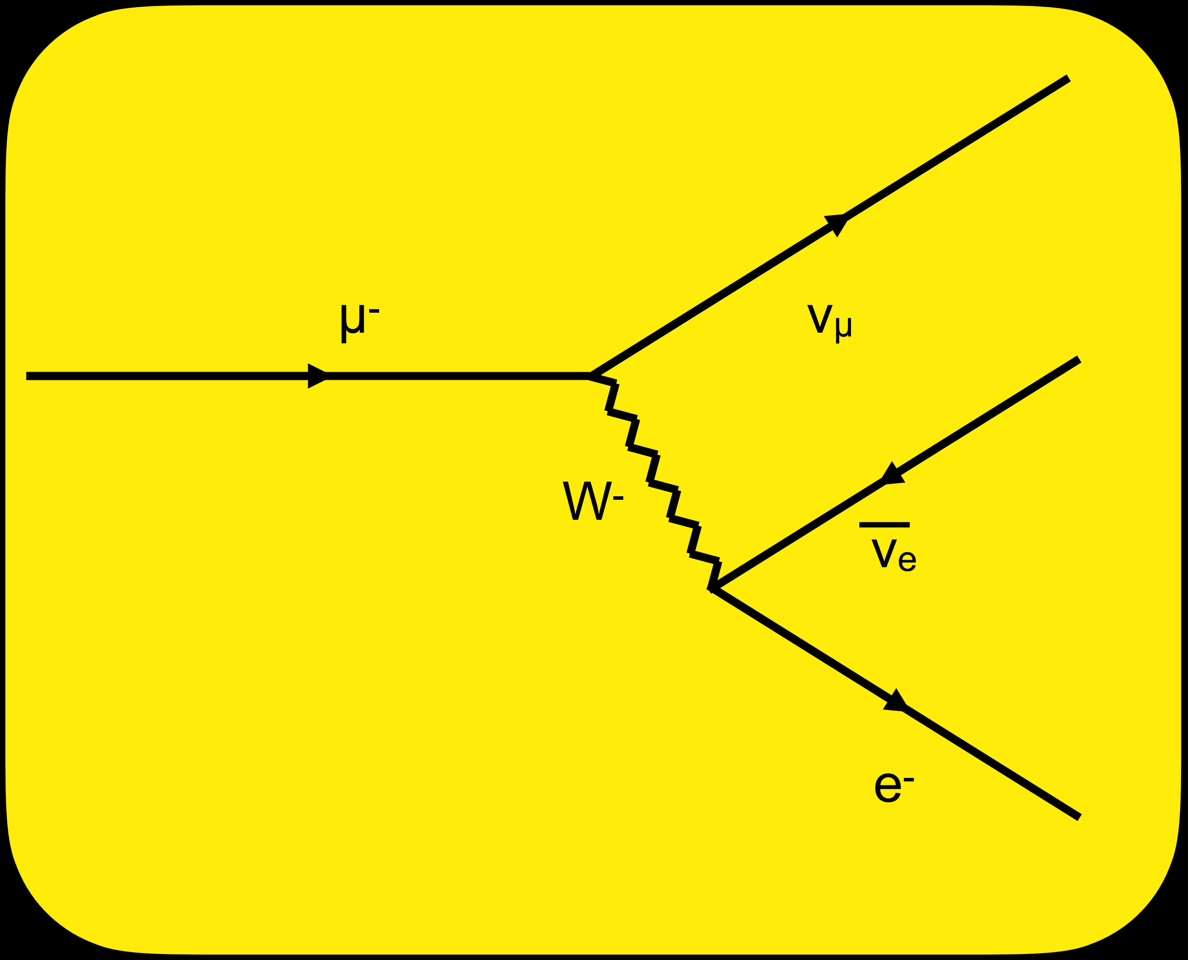
$e^- + \nu_\tau \rightarrow e^- + \nu_\tau$ $\mu^- + \nu_\tau \rightarrow \mu^- + \nu_\tau$ $\tau^- + \nu_\tau \rightarrow \tau^- + \nu_\tau$

$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$

$\tau^- \rightarrow \nu_\tau + e^- + \bar{\nu}_e$

$\tau^- \rightarrow \nu_\tau + \mu^- + \bar{\nu}_\mu$





Weak semi-leptonic processes

- Characterised by both a weak leptonic and a weak hadronic current
  - They involve both leptons (and anti-leptons) and hadrons in the initial and final states

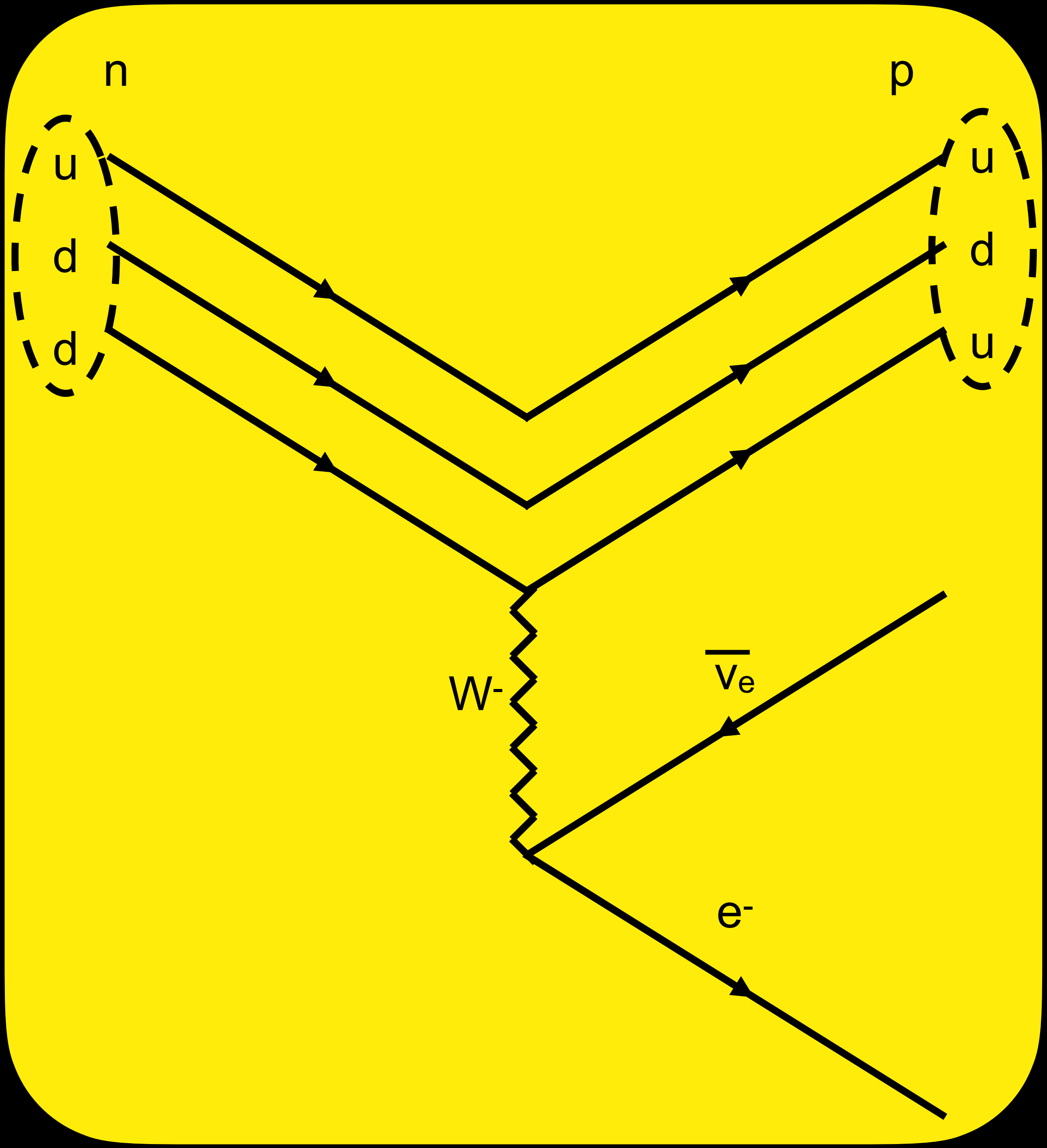


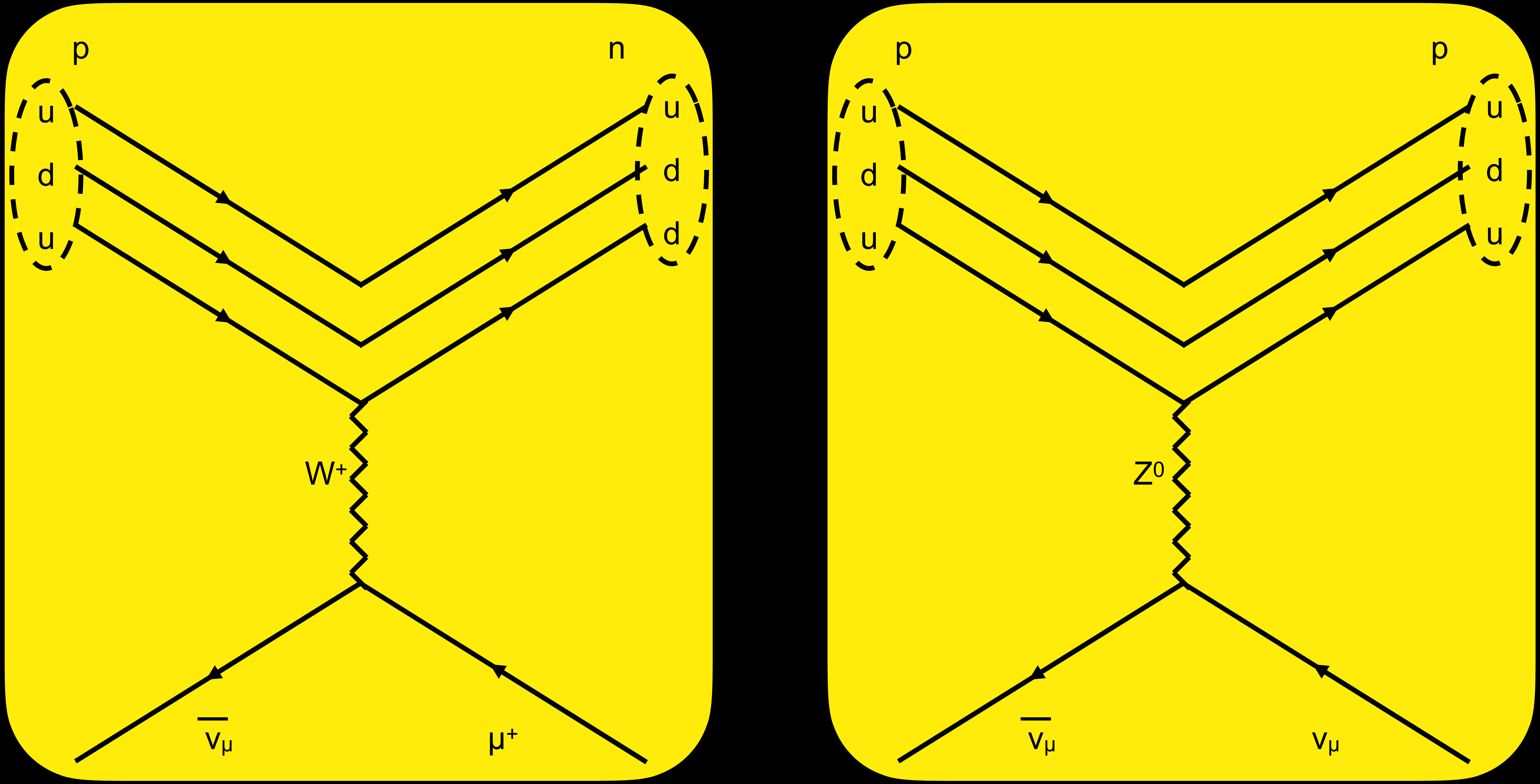
Table 11.2: Decay Properties of Three Semileptonic Decays.

Decay	Spin-parity Sequence	$t_{1/2}{}^{\dagger}$ (sec)	$E_{\max}$ (MeV)	$ft_{1/2}$ (sec)
$\pi^{\pm} \to \pi^0 e\nu$	$0^- \to 0^-$	1.76	4.1	3.1×10^3
$n^0 \to pe\bar{\nu}$	$\frac{1}{2}^+ \to \frac{1}{2}^+$	612	0.78	1.1×10^3
$\Sigma^- \to \Lambda^0 e^- \bar{\nu}$	$\frac{1}{2}^+ \to \frac{1}{2}^+$	1.8×10^{-6}	81	6×10^3

†Partial half-life.

Weak semi-leptonic processes (cont.)

- Characterised by both a weak leptonic and a weak hadronic current
  - They involve both leptons (and anti-leptons) and hadrons in the initial and final states



Weak semi-leptonic processes (cont.)

- Characterised by both a weak leptonic and a weak hadronic current
    - They involve both leptons (and anti-leptons) and hadrons in the initial and final states
- Weak semi-leptonic processes include also processes that change strangeness

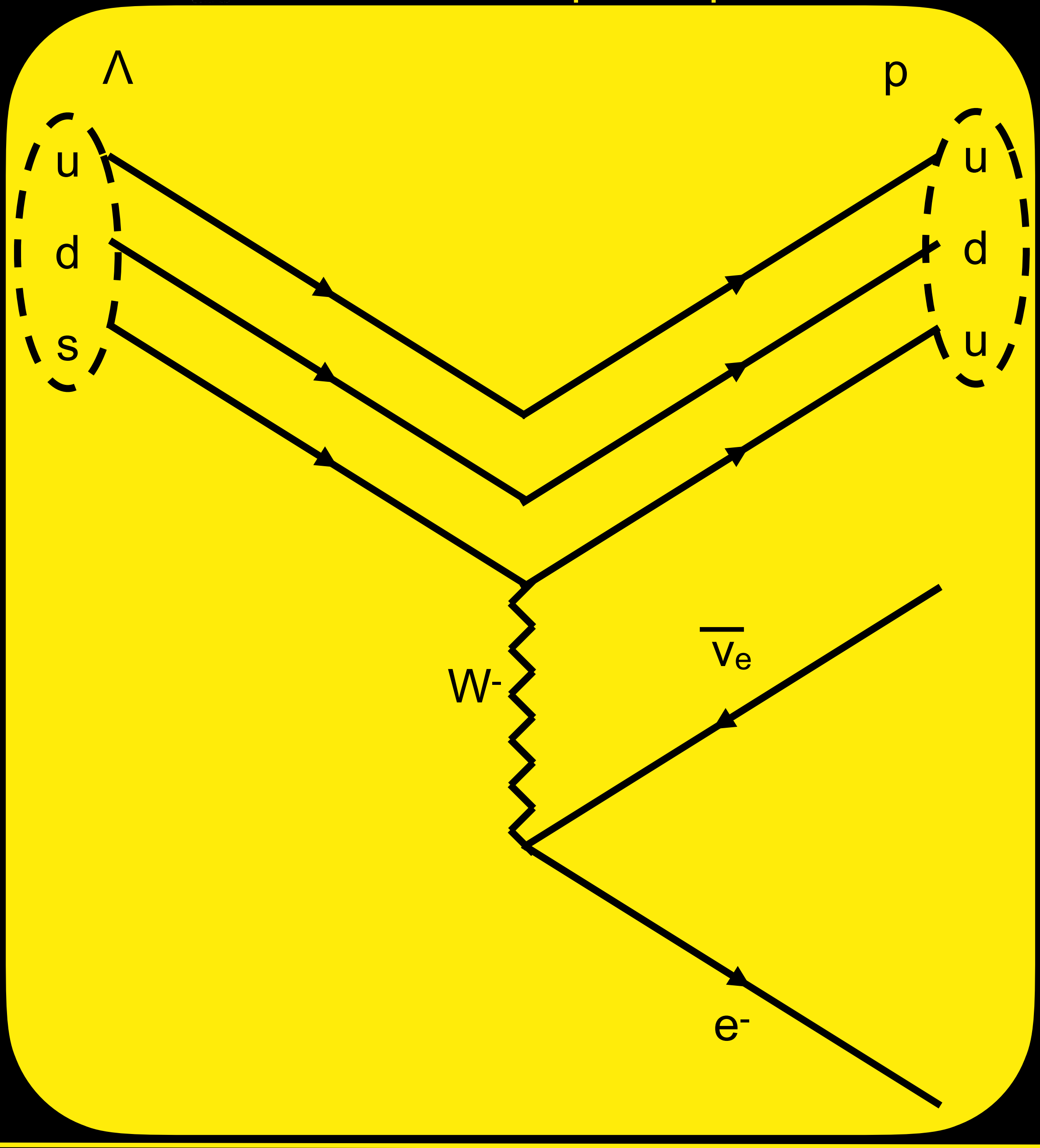


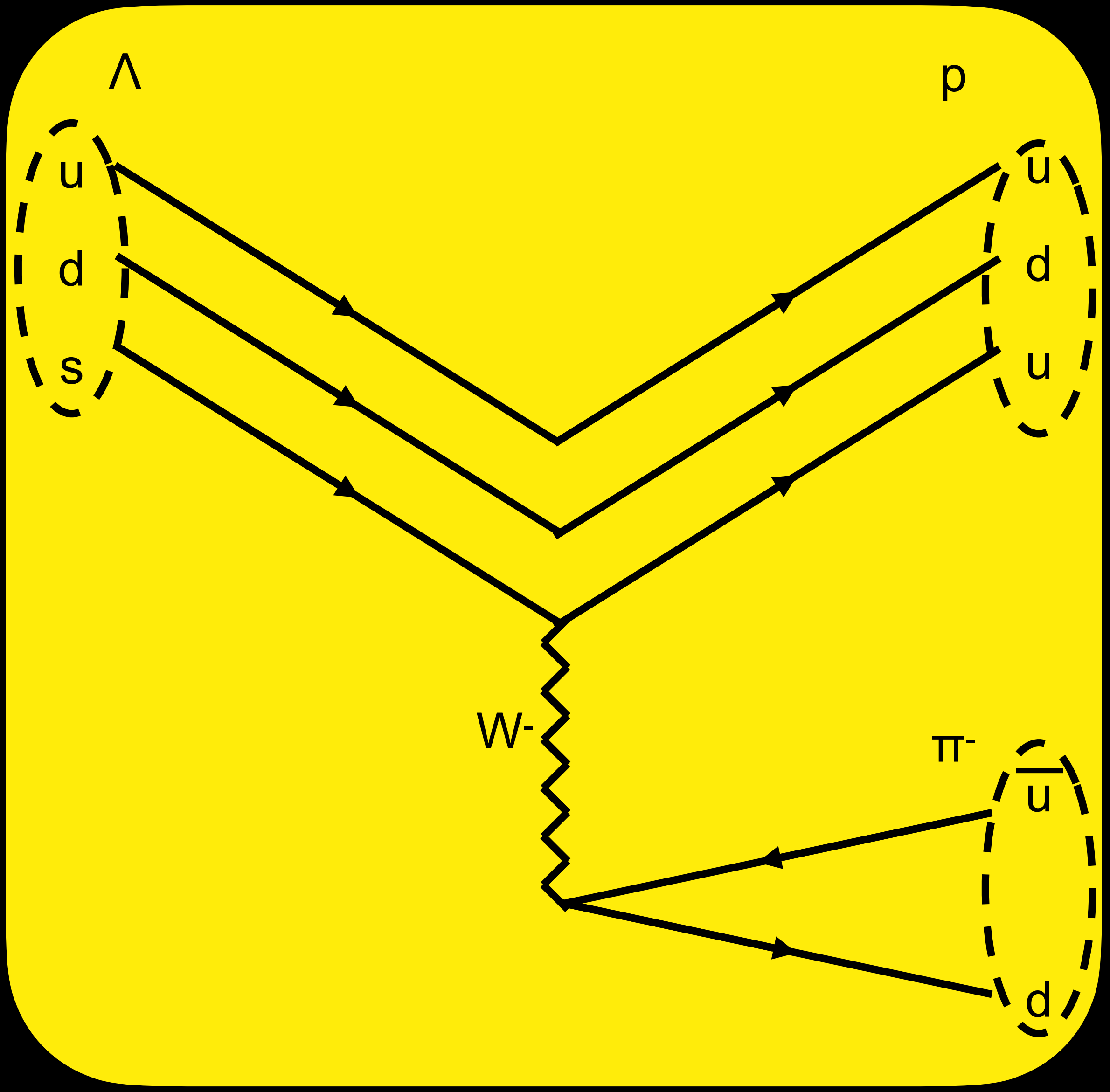
Table 11.3: STRANGENESS-CHANGING SEMILEPTONIC DECAYS.

Decay	Spin-parity Sequence (of hadron)	$t_{1/2}{}^{\dagger}$ (sec)	$E_{max}(e)$ (MeV)	$ft_{1/2}$ (sec)
$K^+ \to \pi^0 e^+ \nu_e$	$0^- \to 0^-$	1.8×10^{-7}	358	1×10^6
$\Lambda^0 \to p e^- \bar{\nu}_e$	$\frac{1}{2}^+ \to \frac{1}{2}^+$	2.2×10^{-7}	177	2×10^4
$\Sigma^- \to n e^- \bar{\nu}_e$	$\frac{1}{2}^+ \to \frac{1}{2}^+$	1.0×10^{-7}	257	1×10^5

†Partial half-life.

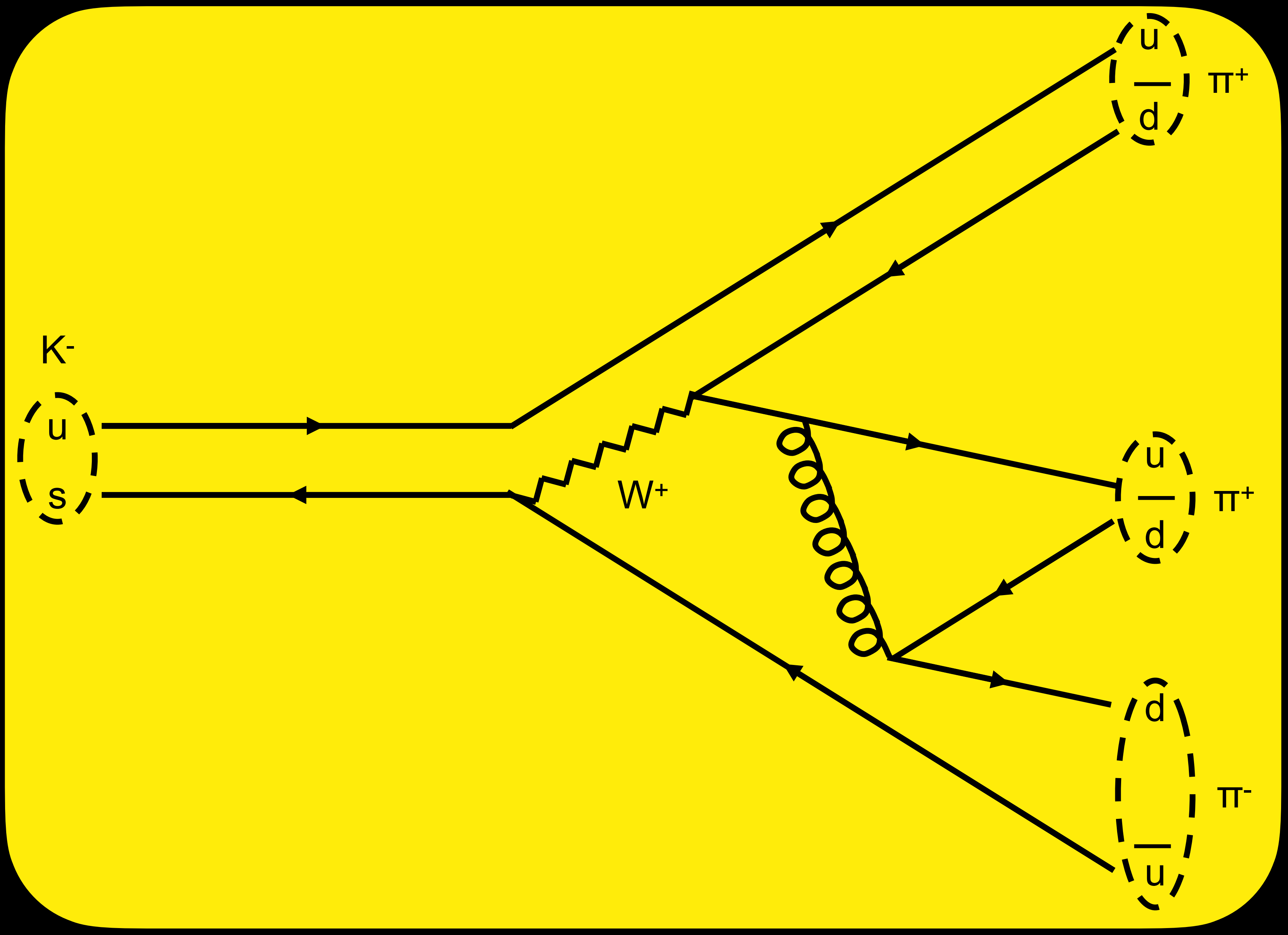
Weak hadronic processes

- Characterised by a weak hadronic current
  - They involve only hadrons in the initial and final states
  - They also involve processes that change strangeness



Weak hadronic processes (cont.)

- Characterised by a weak hadronic current
  - They involve only hadrons in the initial and final states
  - They also involve processes that change strangeness



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CKM matrix

- In case of leptons the coupling to W± takes place strictly within a given generation

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

$$e^- \longrightarrow W^- + \nu_e$$
$$\mu^- \longrightarrow W^- + \nu_\mu$$
$$\tau^- \longrightarrow W^- + \nu_\tau$$

- Quarks are also introduced in three generations
  - the top row contains all quarks with Q=2e/3
  - the bottom row has all quarks with Q=-e/3

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

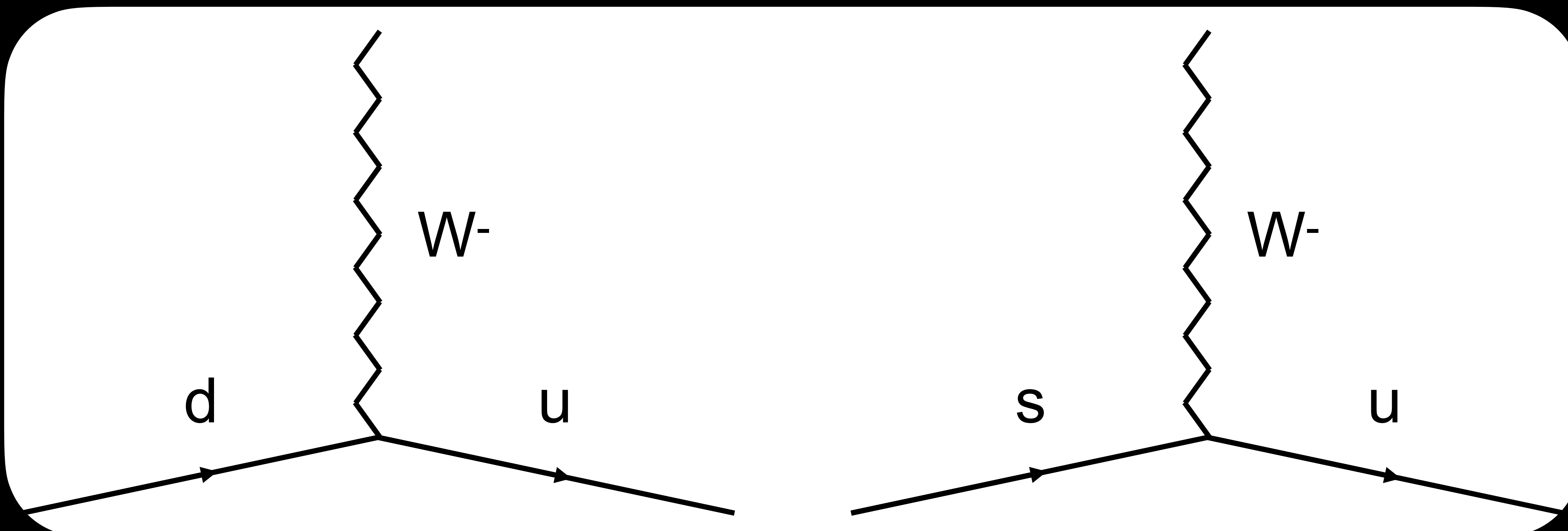
- The coupling of quarks to W± takes place between generations

$$d \longrightarrow W^- + u$$
$$s \longrightarrow W^- + u$$
$$b \longrightarrow W^- + u$$

- The transition was found to be less probable (weaker) the biggest the mass difference between the initial and the final quark was

CKM matrix (cont.)

- In 1963 Nicola Cabibbo tried to address this by postulating that
  - the vertex $d \rightarrow W^- + u$ carries a factor of $\cos\theta_c$
  - the vertex $s \rightarrow W^- + u$ carries a factor of $\sin\theta_c$

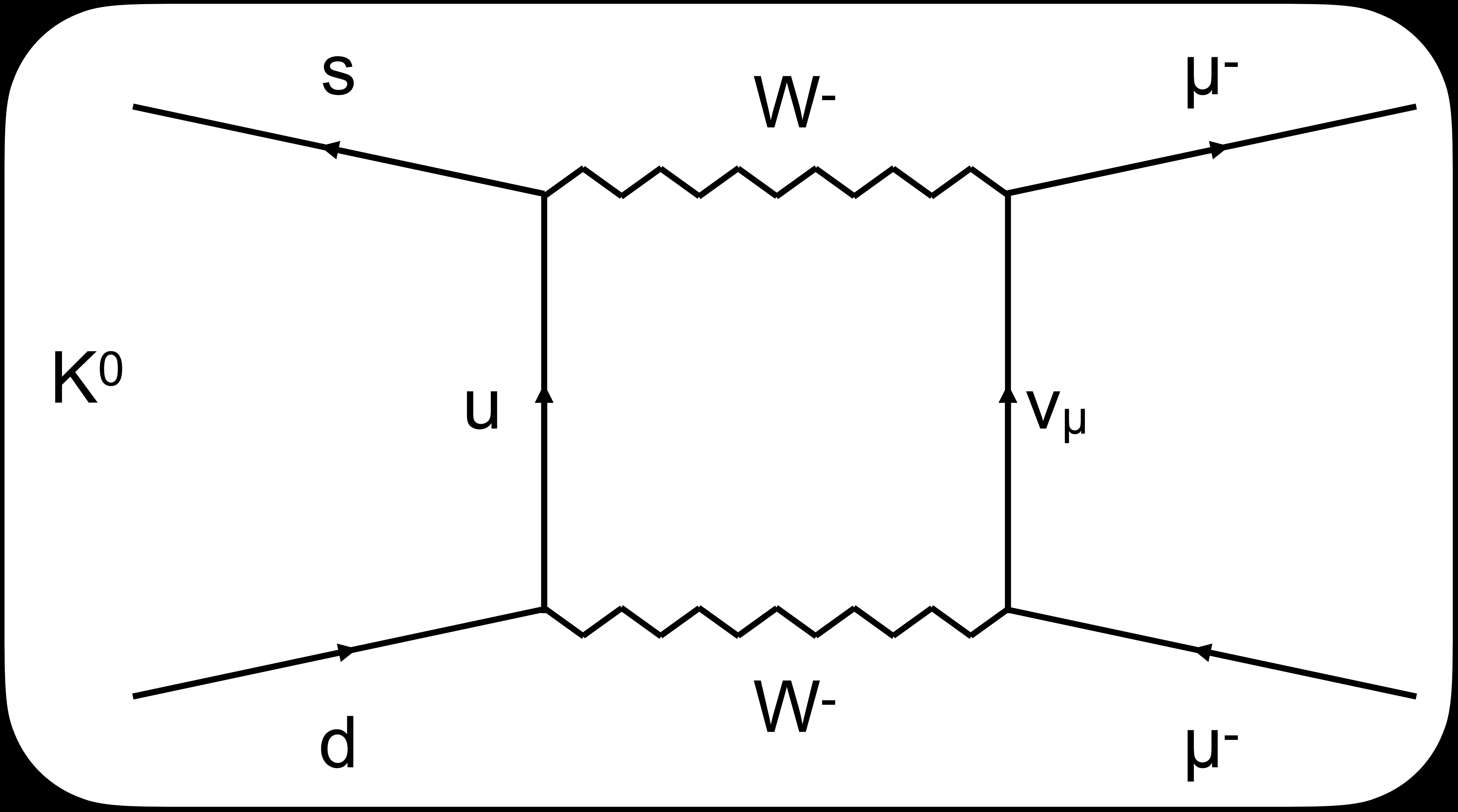


$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\cos\theta_c \qquad \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\sin\theta_c$$

- Since the s → u transition was weaker than the d → u one the value of θ_c must be small
  - Experimental result: $\theta_c \approx 13.15^o$

CKM matrix (cont.)

- The "theory" worked well except for the case of the decay of the K0 into a pair of muons

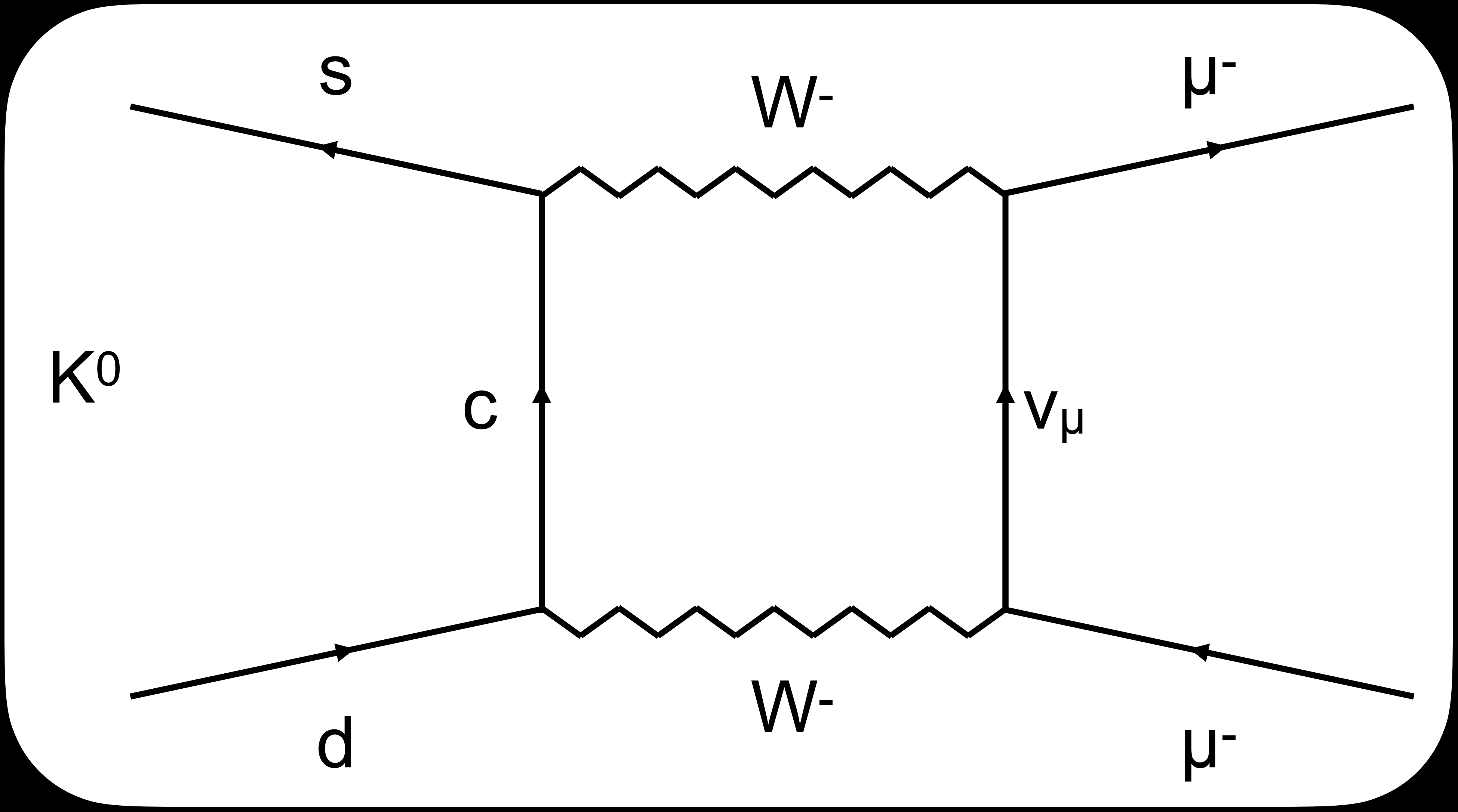


- The amplitude should be proportional to $\sin\theta_c\cos\theta_c$, but it was calculated to be greater than the experimental limit

- A solution was proposed by Glashow, Iliopoulos and Maiani with their GIM mechanism

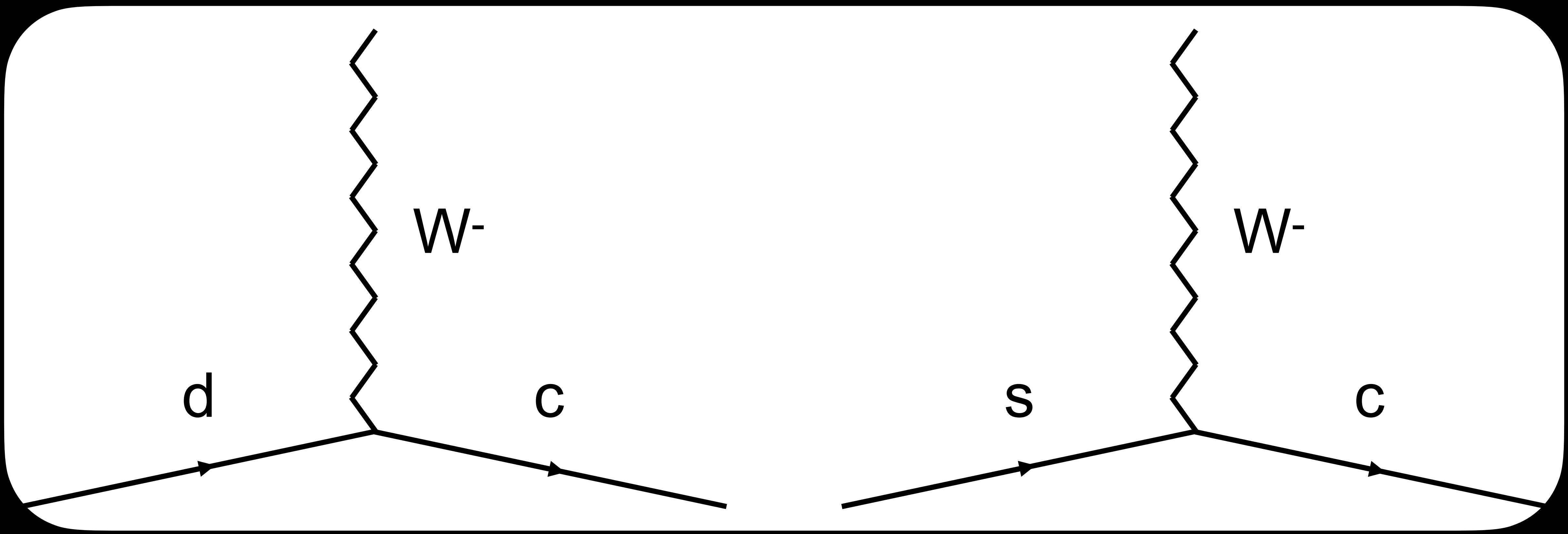
The GIM mechanism

- ✓ They introduced a 4th quark (charm) whose coupling to d and s is different



The GIM mechanism (cont.)

- ✓ The coupling of c to d comes with ($-\sin\theta_c$) and with s with $\cos\theta_c$



$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)(-\sin\theta_c)$$

$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\cos\theta_c$$

The GIM mechanism (cont.)

- The Cabibbo-GIM scheme dictates that instead of the physical quarks d and s, the weak interactions take place between the “rotated” quantum states d’ and s’

$$\left\{ \begin{array}{l} d' = d\cos\theta_c + s\sin\theta_c \\ s' = -d\sin\theta_c + s\cos\theta_c \end{array} \right\} \Rightarrow \begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- The $W^{\pm}$ do not couple to the physical states but to the Cabibbo rotated quantum states

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$$

  - In the same way they couple to the lepton generations

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

- This was generalised via the Cabibbo-Kobayashi-Maskawa matrix into three quark generations

CKM matrix: weak decays of quarks

- All charged weak decays can be explained assuming that all transitions that change row are allowed
  - A down quark can change into an up with the emission of a W^-
- Introducing the prime quark representations, implies that transitions are contained within the same column

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \begin{pmatrix} t \\ b' \end{pmatrix}$$

  - The new, prime quarks are linear combinations of the physics quark states

Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

V_{ij}: specifies the coupling between quark i and j

The Standard Model does not give any insight into the CKM matrix and its elements!!!

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(1/2) Nobel Prize of 2008

The Nobel Prize in Physics 2008
Yoichiro Nambu, Makoto Kobayashi, Toshihide Maskawa

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The Nobel Prize in Physics 2008

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Toshihide Maskawa
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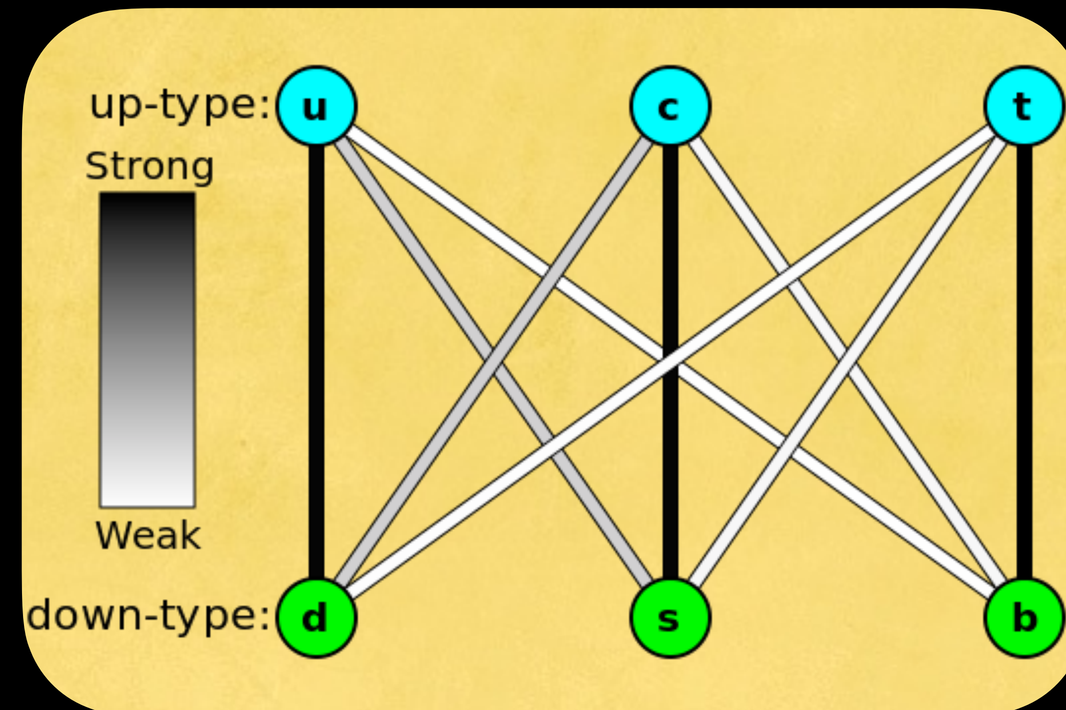
The Nobel Prize in Physics 2008 was divided, one half awarded to Yoichiro Nambu *"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"*, the other half jointly to Makoto Kobayashi and Toshihide Maskawa *"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"*.

Nicola Cabibbo (1935-2010)

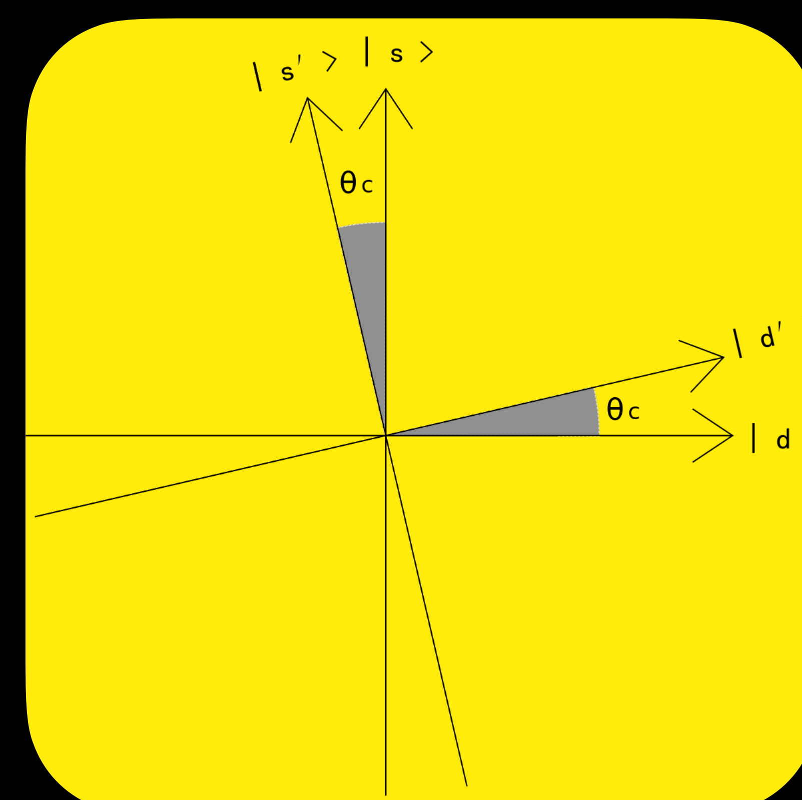
CKM matrix: weak decays of quarks (cont.)

- Weak decays between quarks have a different effective coupling constant depending on which species are involved $V_{ij}G_F$
  - ($u \Longleftrightarrow d$) $V_{ud} \sim 0.97$
  - ($u \Longleftrightarrow s$) $V_{us} \sim 0.22$



$$V_{\rm CKM} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

- The CKM matrix produces a rotation from the quark mass eigenstates to the ones of the weak interaction
  - The matrix should be unitary e.g. $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=1$
  - Any deviation from unity could reveal the existence of new physics



CKM matrix: weak decays of quarks (cont.)

- The CKM matrix can be parametrised by three mixing angles θ_{ij} and by the CP violating KM phase δ
- The δ-phase is responsible for all the CP-violating phenomena in flavour changing processes of the Standard Model

$$V_{CKM}=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad s_{ij}=\sin(\theta_{ij}) \quad c_{ij}=\cos(\theta_{ij})$$

- Experimentally it is known that $s_{13} \ll s_{23} \ll s_{12} \ll 1$
- Using the Wolfenstein parametrisation, we get a more suitable representation of the basic parameters of the Standard Model

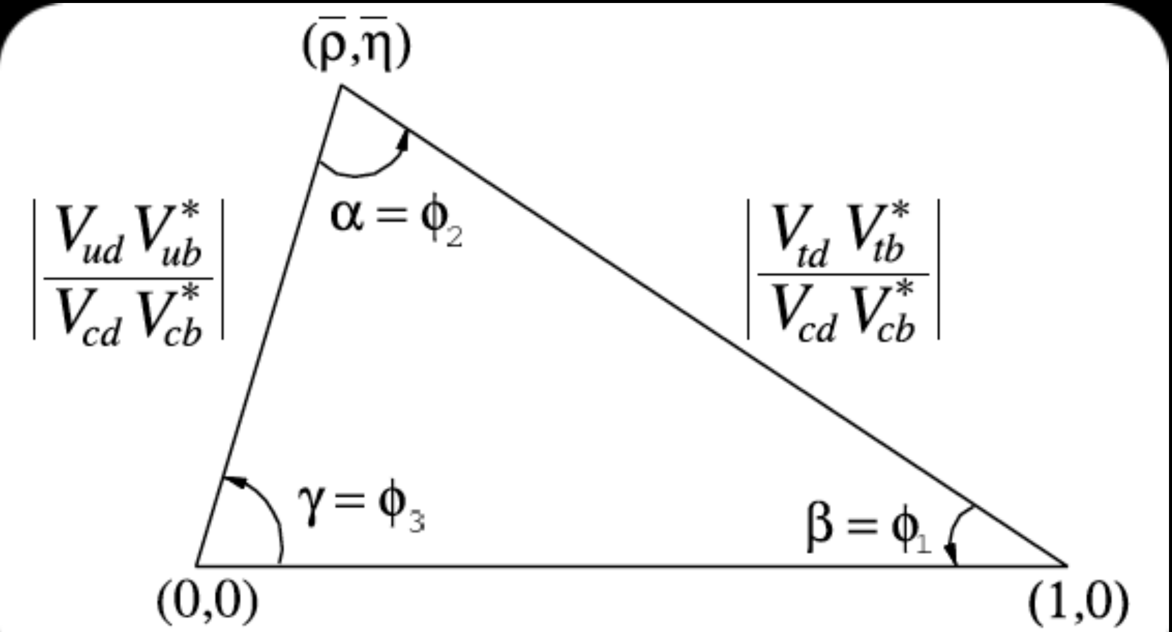


Figure 11.1: Sketch of the unitarity triangle.

Constrains on the Standard Model parameters

- $|V_{ud}|$: The most precise measurements come from nuclear β-decays and from neutron lifetime measurements
  - Very small theoretical uncertainties related to nuclear Coulomb distortions and radiative corrections
- $|V_{us}|$: Constrained by the K^0_L decay $K^0_L \rightarrow \pi^+ + e^- + \overline{\nu}_e$
  - Small uncertainties stemming from different experimental measurements that are combined to give the final value
- $|V_{cd}|$: Mainly determined from semi-leptonic charm decays $D^0 \rightarrow K^+ + e^- + \overline{\nu}_e$
  - Uncertainties stemming from theoretical (QCD) calculations $D^0 \rightarrow \pi^+ + e^- + \overline{\nu}_e$
- $|V_{cs}|$: Mainly comes from measurements of W^+ decays to a c and s-bar quark or from semi-leptonic and leptonic D and D_s mesons
- $|V_{cb}|$: Can be determined from exclusive and inclusive semi-leptonic decays of B-mesons to charm
- $|V_{ub}|$: Mainly determined from semi-leptonic decays of B-mesons to particles containing u quarks with the parallel emission of a lepton and a (anti)neutrino

Constrains on the Standard Model parameters (cont.)

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

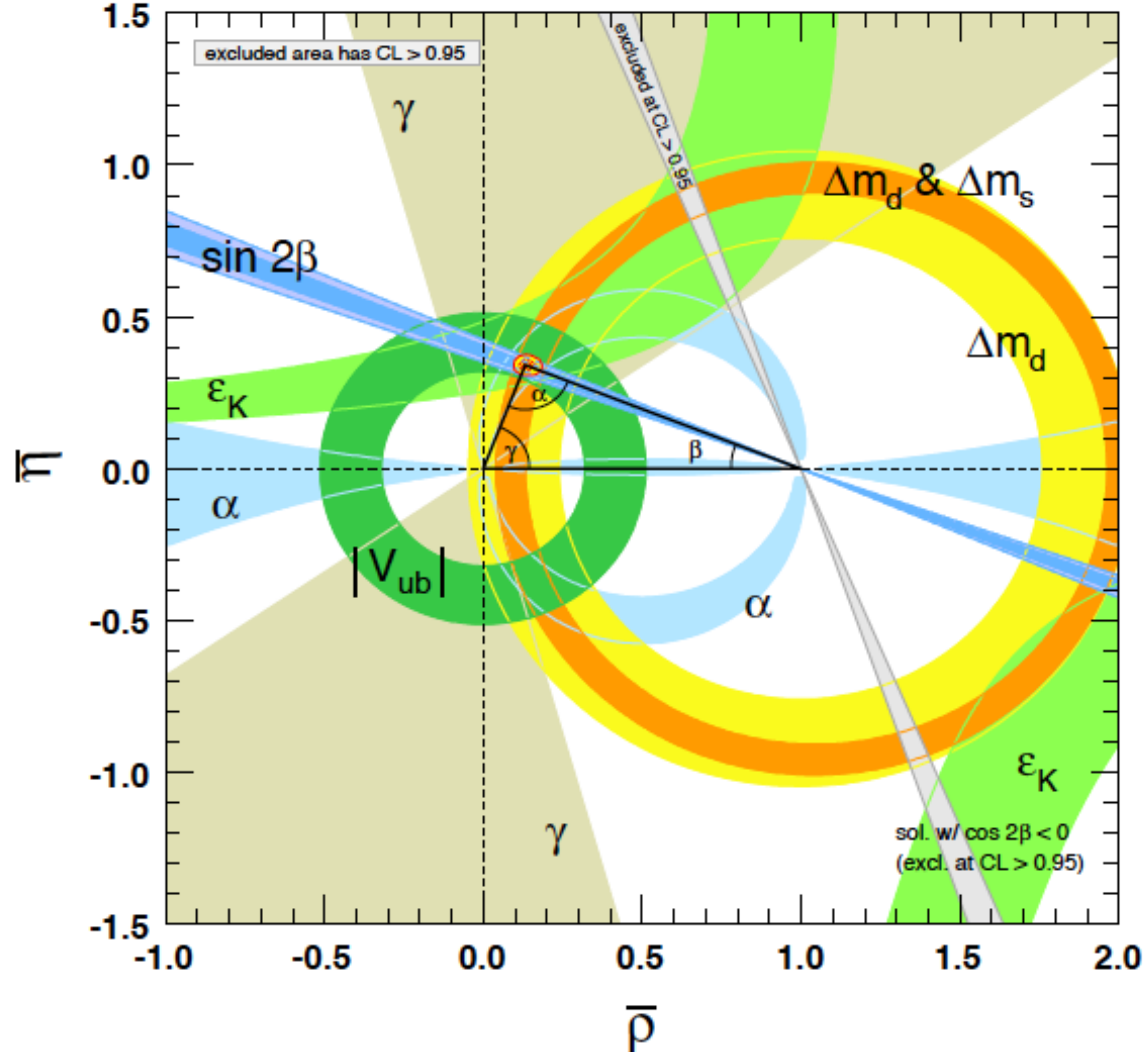


Figure 11.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL. Color version at end of book.

Lepton universality of the weak coupling constant

- Is the weak coupling, let's say to W-bosons, the same for all fermions that couple to these bosons?
  - The answer is yes for leptons and no for quarks

$$\mu^- \rightarrow \nu_\mu + e^- + \overline{\nu}_e \qquad \tau^- \rightarrow \nu_\tau + e^- + \overline{\nu}_e$$

- The universality of the coupling constant between leptons can be studied via the decays of μ and the τ

- The transition rate for the muon decay is given by $\Gamma(\mu \rightarrow e\nu_e\overline{\nu}_\mu) = \frac{G_F^2 m_\mu^5 c^4}{192\pi^3\hbar^7}$

- From the formula above, replacing muon ($m_\mu \sim 105.658$ MeV) with the tau mass ($m_\mu \sim 1777$ MeV) and considering that the branching ratio of the tau decay is ~17.8% we get:

$$\left(\frac{g_\tau}{g_\mu}\right)^4 = B(\tau \rightarrow e\nu_e\overline{\nu}_e)\left(\frac{m_\mu}{m_\tau}\right)^5\left(\frac{\tau_\mu}{\tau_\tau}\right) \qquad \frac{g_\tau}{g_\mu} = 0.999 \pm 0.003$$

Lepton universality of the weak coupling constant (cont.)

- From the relative branching ratios for the tau and muon decays one can also test the universality between muons and electrons

$$\frac{g_\mu}{g_e} = 1.001 \pm 0.004$$

- which is in agreement with measurements based on the decay of pions

$$\pi \to e\nu \qquad \pi \to \mu\nu$$

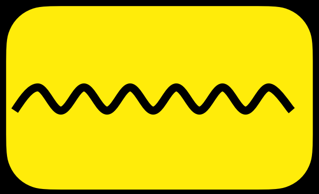
$$\frac{g_\mu}{g_e} = 1.001 \pm 0.002$$

- This universality holds for weak decays that take place via the exchange/decay of a Z-boson

$$Z^0 \to e^+e^- : \mu^+\mu^- : \tau^+\tau^- = 1 : 1.000 \pm 0.004 : 0.999 \pm 0.005$$

Symmetries and gauge tranformations (Weak)

Conserved quantities	Gauge transformation	Symmetry group	Field
Charge: ψ	$\Psi \rightarrow \Psi' = e^{ia}\Psi$	U(1) (QED)	A_μ (1 photon)



Symmetries and gauge tranformations (Weak)

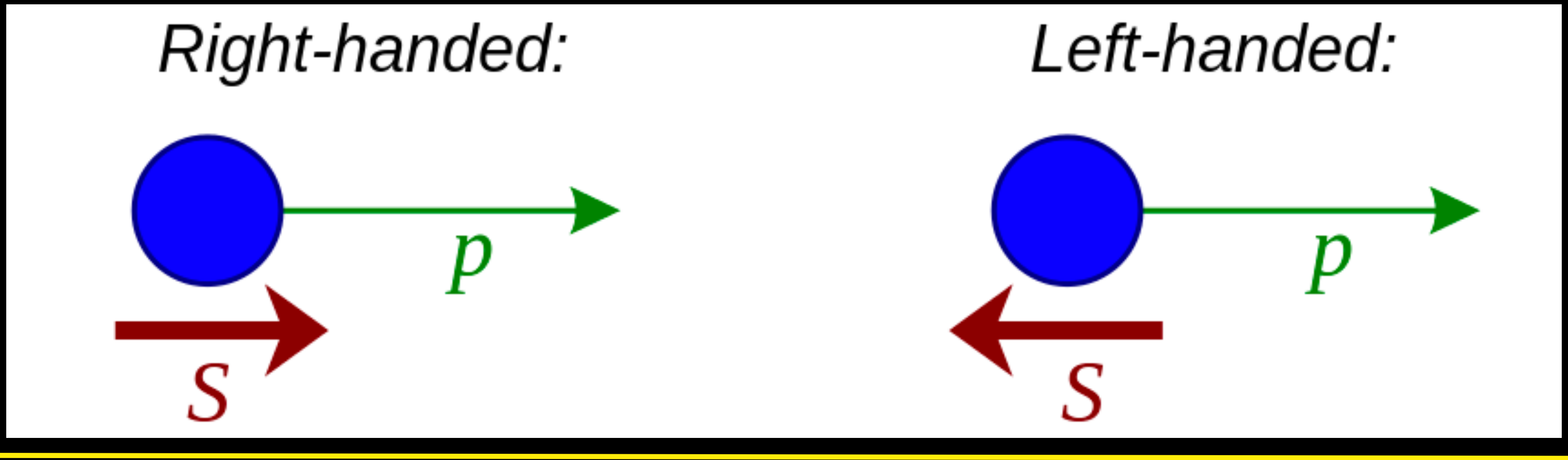
Conserved quantities	Gauge transformation	Symmetry group	Field
Charge: ψ	$\Psi \to \Psi' = e^{i\alpha}\Psi$	U(1) (QED)	A_μ (1 photon)
Weak isospin $\psi = \begin{pmatrix} \nu \\ e \end{pmatrix}$ $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$	$\Psi \to \Psi' = e^{\frac{i\vec{\tau}\vec{\alpha}}{2}}\Psi$	SU(2) (weak)	A_μ (3 bosons, $W^\pm$, Z^0)

Parity operator

- Parity (P):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$

  - Converts right-handed systems to left-handed ones
  - Vectors change sign but axial vectors remain unchanged
- Helicity: In particle physics and for massless particles, helicity is the projection of the angular momentum onto the direction of momentum.
  - The angular momentum vector J is the sum of the orbital momentum vector L and the intrinsic angular momentum or spin S
  - The helicity of a particle is
    - right-handed if the direction of its spin is the same as the direction of its motion
    - left-handed if the directions of spin and motion are opposite.
    - Mathematically, helicity is the sign of the projection of the spin vector onto the momentum vector: left is negative, right is positive.



Parity violation in weak interactions

- In the early 1950s physicists were faced with a problem known as the "τ−θ" puzzle.
- Based on one set of criteria, that of mass and lifetime, two elementary particles (the τ and the θ) appeared to be the same, whereas on another set of criteria, that of spin and intrinsic parity, they appeared to be different.

$$\theta^+ \longrightarrow \pi^+ + \pi^0 \qquad \tau^+ \longrightarrow \pi^+ + \pi^+ + \pi^0$$

- The intrinsic parity of pions is P = -1 and parity is a multiplicative quantum number
  - The parity of θ was +1 while the one of τ was -1 ⇒ they must be different particles
- Detailed measurements about their masses and lifetimes though indicated that there were no significant differences between the two particles ⇒ they seem to be the same particles
  - How can one particle have two different parity values at the same time?

Parity violation in weak interactions (cont.)

Symmetries are part of the building blocks of particle physics. However their validity rests on **experimental verification**!!!

- T.D. Lee and C.N. Yang (1956) realised that the problem would be solved, and that the two particles would be different decay modes of the same particle, if parity were not conserved in the decay of the particles, a weak interaction.

- They examined the evidence for parity conservation and found, to their surprise, that although there was strong evidence that parity
  - was conserved in the strong (nuclear) and electromagnetic interactions,
  - there was no supporting evidence that it was conserved in the weak interaction.
    - It had never been tested.

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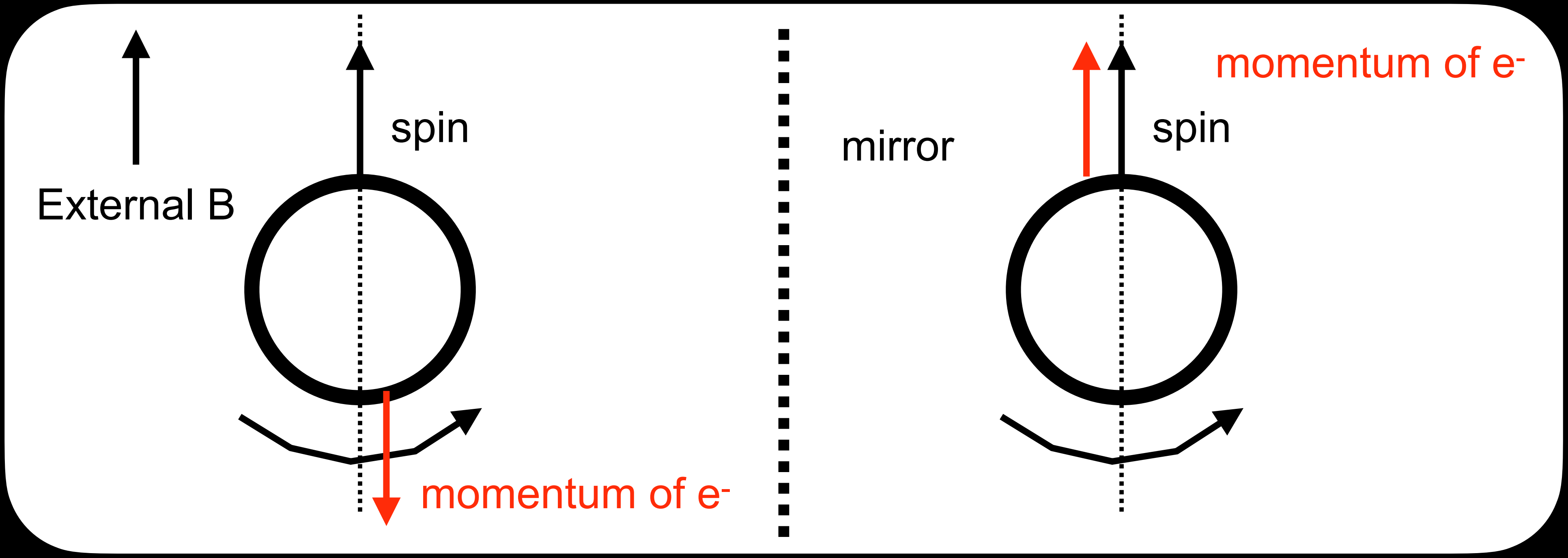
Parity violation in weak interactions (cont.)

- The proposed way to test it is summarised as follows
  - Consider a collection of radioactive nuclei, all of whose spins point in the same direction.
  - Suppose also that the electron given off in the radioactive decay of the nucleus is always emitted in a direction opposite to the spin of the nucleus
  - In the mirror the electron is emitted in the same direction as the spin.
    - The mirror image of the decay is different from the real decay.
    - This would violate parity conservation, or mirror symmetry.
    - Parity would be conserved only if, in the decay of a collection of nuclei, equal numbers of electrons were emitted in both directions.

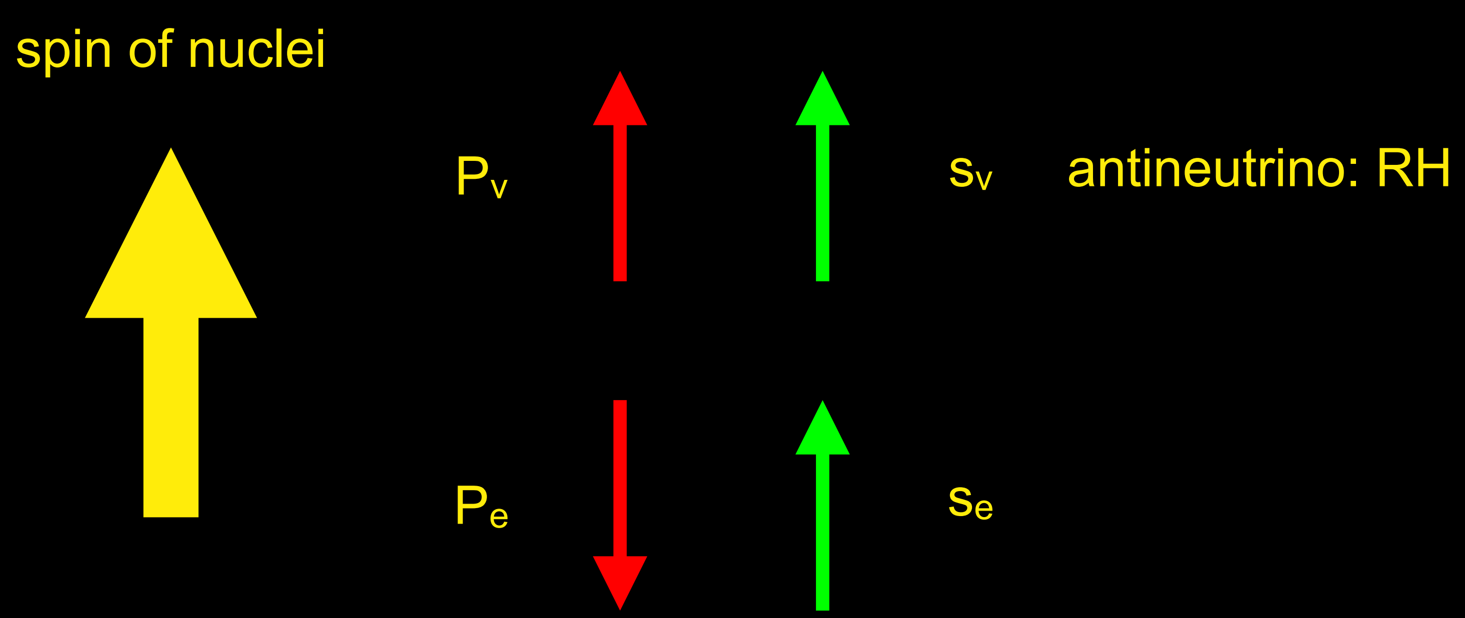


Parity violation: The Wu experiment

- Test parity conservation by observing a dependence of a decay rate (or cross section) on a term that changes sign under the parity operation. If decay rate or cross section changes under parity operation, then the parity is not conserved
- Parity reverses momenta and positions but not angular momenta (or spins). Spin is an axial vector and does not change sign under parity operation.
- Parity (P):
  - Converts right-handed systems to left-handed ones
  - Vectors change sign but axial vectors remain unchanged

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$

$$\vec{x} \to -\vec{x}\,,\ \vec{p} \to -\vec{p}\,,\ \vec{L} = \vec{x} \times \vec{p} \to \vec{L}$$



Parity violation: The Wu experiment

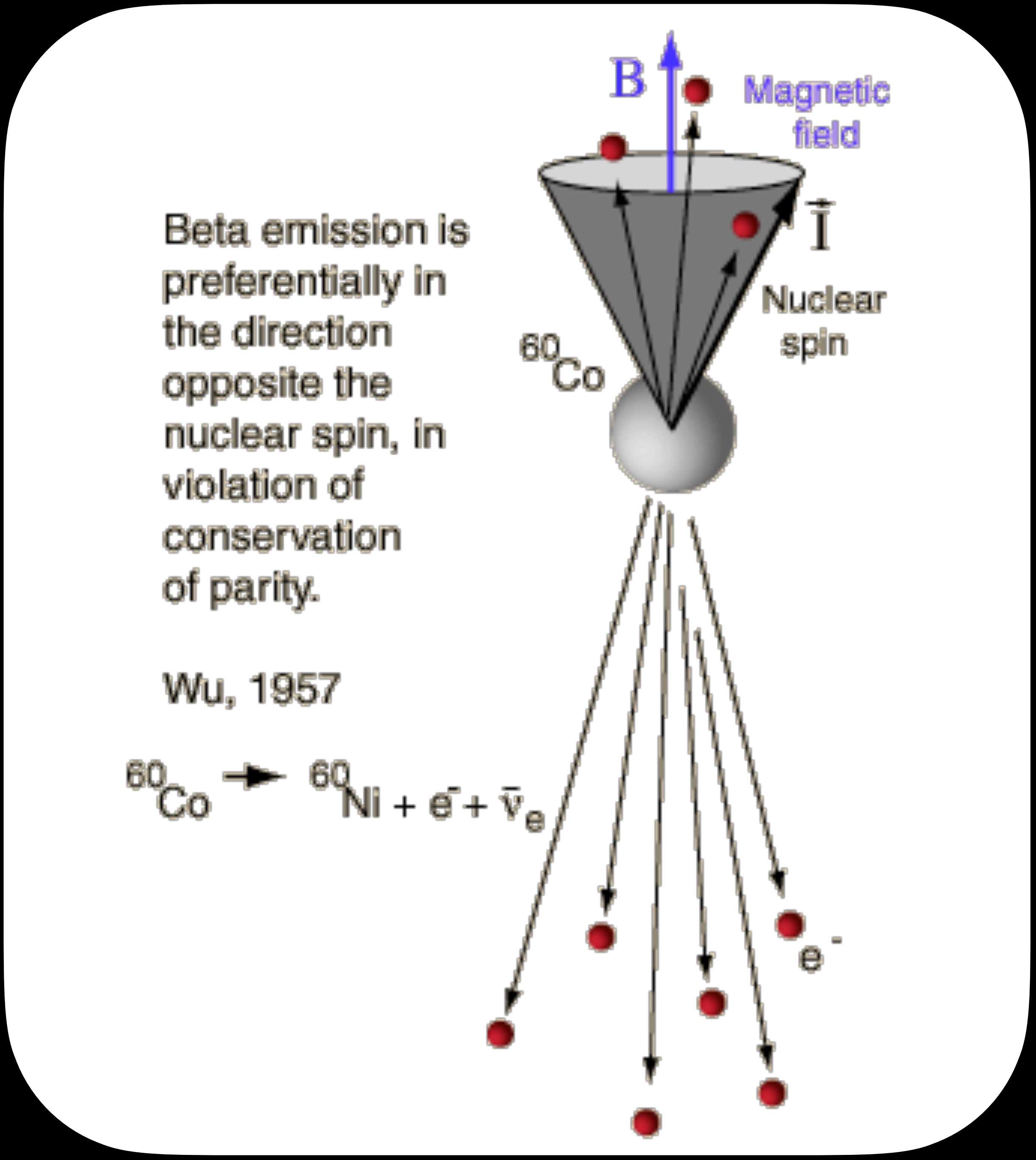
- Decay of cobalt-60 atoms at very small temperature and aligned in a uniform magnetic field.
- ^{60}Co is an unstable isotope that decays (β-decay) to the stable isotope nickel-60 (^{60}Ni).
  - A neutron of the ^{60}Co nucleus decays into a proton by emitting an electron and an electron antineutrino
  - The ^{60}Ni nucleus ends up in an excited state and decays to its ground state by emitting two gamma rays . Hence the overall nuclear equation of the reaction is:

$$^{60}Co \rightarrow {}^{60}Ni + e^{-} + \overline{\nu}_e$$
$$\hookrightarrow 2\gamma$$

- This γ-ray emission is an electromagnetic (EM) process
  - EM was known to respect P-conservation
  - emitted gamma rays acted as a control for the polarization of the emitted electrons via the weak interaction

Parity violation: The Wu experiment (cont.)

- Comparison between the distribution of γ and electron emissions with the nuclear spins in opposite orientations.
  - **if parity is conserved:** electrons always found to be emitted in the same direction and in the same proportion as the gamma rays
  - If there were a bias in the direction of decays, that is, if the distribution of electrons did not follow the distribution of the gamma rays, then P-violation would be established.



Parity violation: The Wu experiment (cont.)

- Parity conservation:
  - electrons would have no preferred direction of decay relative to the nuclear spin
- However, electrons were emitted in a direction preferentially opposite to that of the gamma rays.
  - most of the electrons favoured a specific direction of decay, opposite to that of the nuclear spin

The Nobel Prize in Physics 1957
Chen Ning Yang, Tsung-Dao Lee

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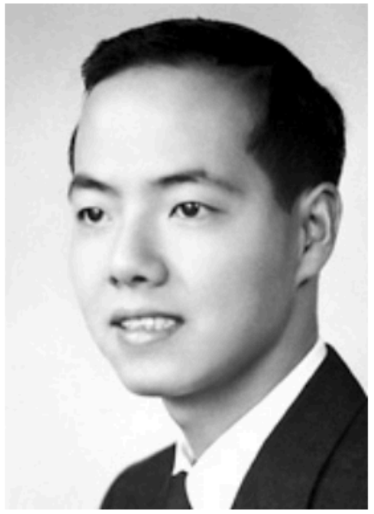
The Nobel Prize in Physics 1957

Chen Ning Yang
Prize share: 1/2



Tsung-Dao (T.D.) Lee
Prize share: 1/2

The Nobel Prize in Physics 1957 was awarded jointly to Chen Ning Yang and Tsung-Dao (T.D.) Lee *"for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles"*

Parity violation: The Wu experiment (cont.)

- ✓ Parity conservation:
  - electrons would have no preferred direction of decay relative to the nuclear spin
- ✓ However, electrons were emitted in a direction preferentially opposite to that of the gamma rays.
  - most of the electrons favoured a specific direction of decay, opposite to that of the nuclear spin

The Nobel Prize in Physics 1957
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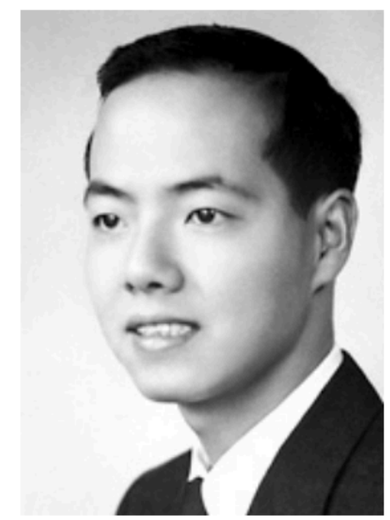
The Nobel Prize in Physics 1957

Chen Ning Yang
Prize share: 1/2



Tsung-Dao (T.D.) Lee
Prize share: 1/2

Chien-Shiung Wu

The Nobel Prize in Physics 1957 was awarded jointly to Chen Ning Yang and Tsung-Dao (T.D.) Lee *"for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles"*

Neutrinos

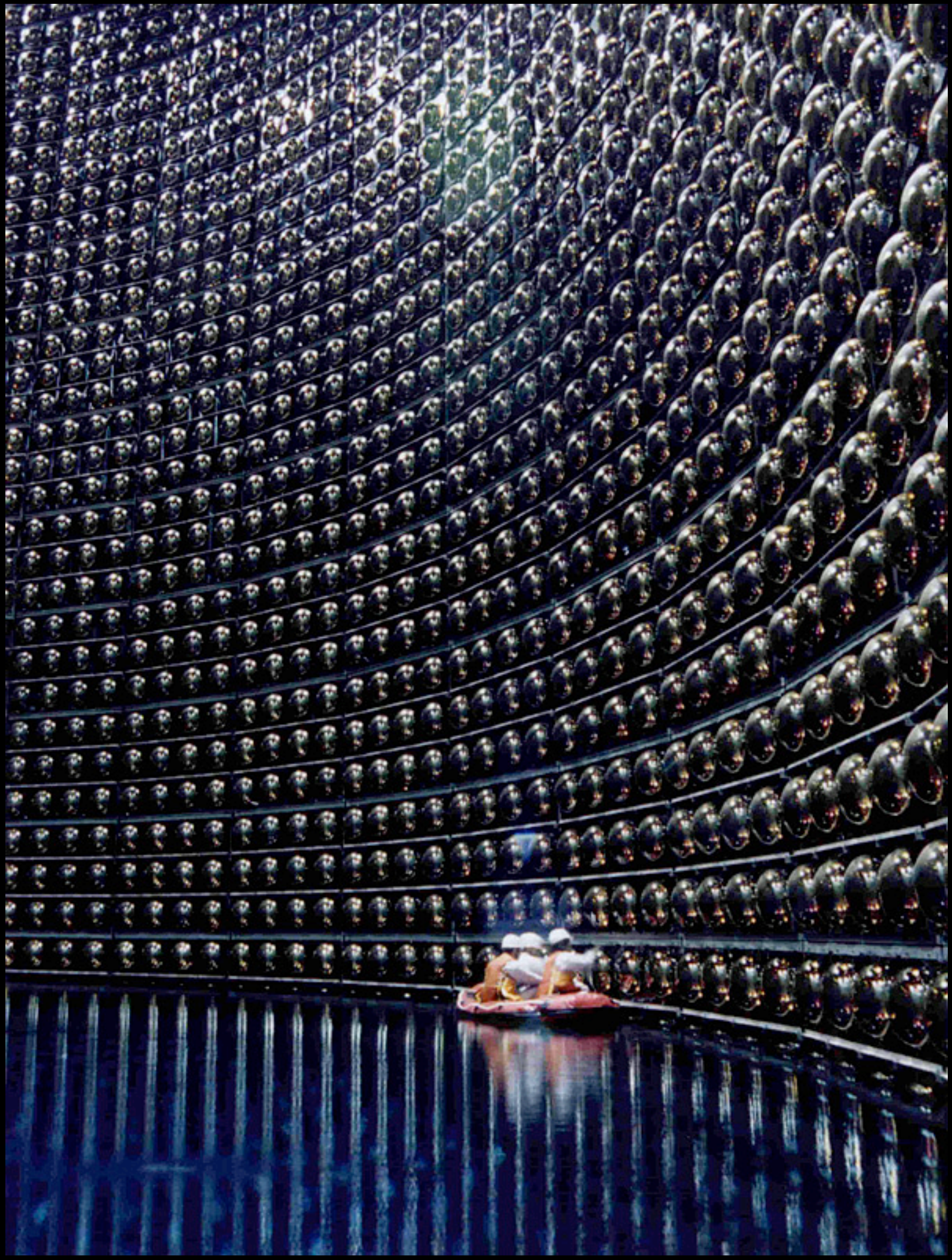
- Neutrinos were considered up until now nearly massless with the latest values for their mass being
  - $\nu_e < 2.2$ eV/c^2
  - $\nu_\mu < 0.19$ MeV/c^2
  - $\nu_\tau < 18.2$ MeV/c^2

Neutrinos: **left-handed** helicity

Antineutrinos: **right-handed** helicity

- Results from two experiments showed something unexpected
  - The IMB-detector
    - built to study the decay of protons did not find any evidence for proton decay
    - detected and identified the flavor of atmospheric neutrinos, neutrinos produced in the upper atmosphere .
    - determined that the ratio of muon neutrinos to electron neutrinos from the upper atmosphere was approximately a factor of 2 too small compared to expectations
  - Homestake-mine Cl detector
    - set up to detect neutrinos from the Sun
    - confirm the mechanisms for production of solar energy
    - found only ≈ 1/ 3 of the electron neutrinos expected from the Sun

Super-Kamioka Neutrino Detector Experiment

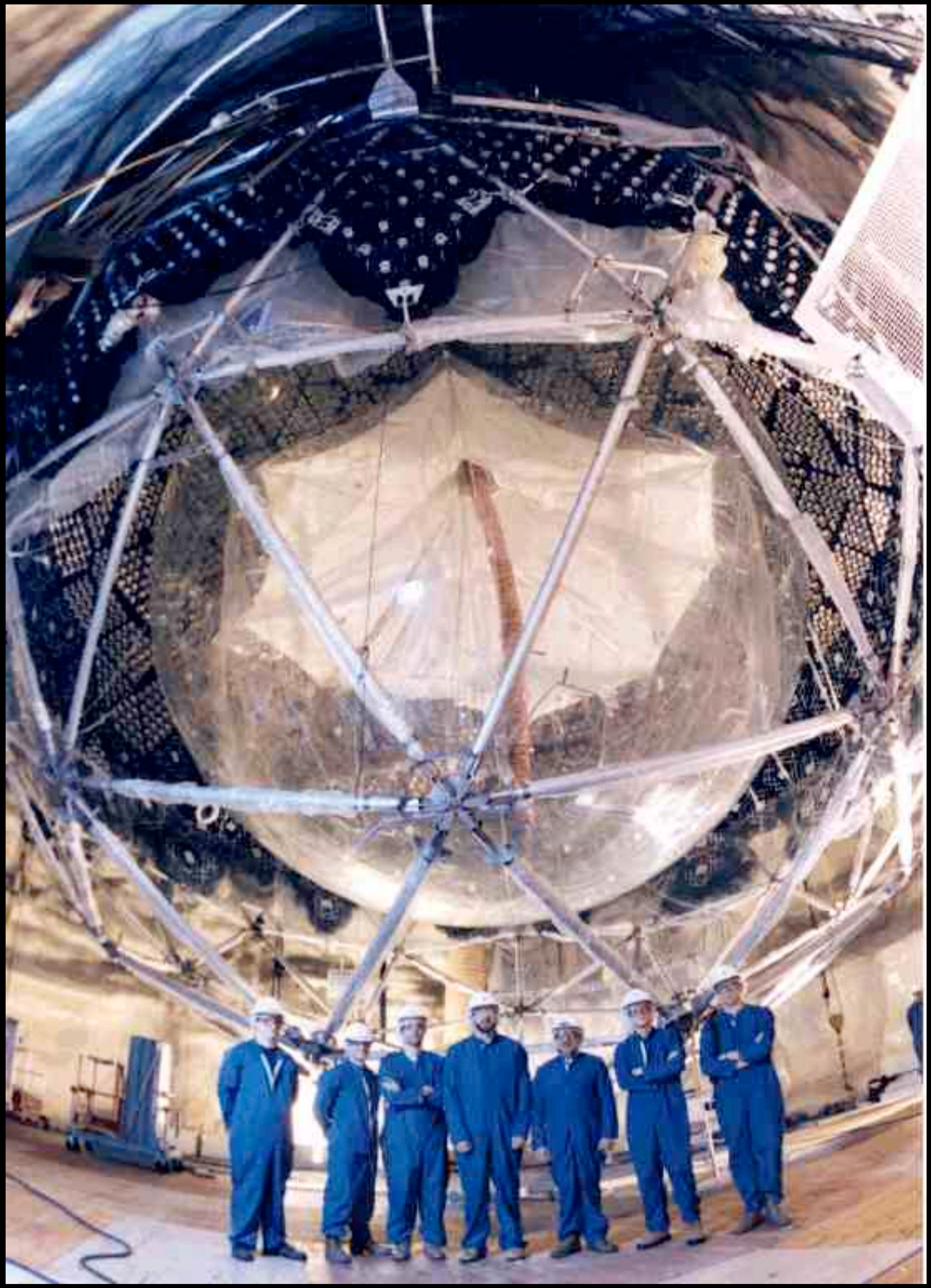


- ✓ Located under the mount Ikeno in Japan
- ✓ Designed to study
  - the decay of protons
  - atmospheric and solar neutrinos
  - Supernovae explosions in the Milky Way

- ✓ Super-K is a huge Cherenkov detector
- ✓ The outer tank is cylindrical, made of stainless-steel tank with 39 m in diameter and 42 m in height.
  - The capacity of the tank exceeds 50 ktons of water.
- ✓ The basic detector module is a PMT (~6000 of them in total)

Discovered atmospheric neutrino oscillations

Sudbury Neutrino Observatory



- ✓ Located ~2Km underground at the Creighton Mine in Sudbury, Ontario, Canada.
- ✓ Designed to study solar neutrinos
  - Rate
  - Flavor

- ✓ Huge tank of “heavy-water” (containing a large amount of deuterium)
- ✓ 1t of heavy-water
- ✓ in a 6m radius vessel
- ✓ The basic detector module is a PMT (~6000 of them in total)

Discovered solar neutrino oscillations

Majorana vs Dirac neutrinos

- Why the neutrino masses are so small compared to those of the other particles.
- Solution: "see-saw" mechanism that involves an extremely heavy neutrino with mass M_{RH} postulated in a grand unified theory.
  - In this theory, the e, μ, τ-neutrinos have masses of the order of $m_\nu \sim m^2_L/M_{RH}$, where m_L is the mass of a lepton or that of the W.
  - The small mass could then be a signal of grand unification.
  - However, in order for this mechanism to exist neutrinos should be identical to their anti-particles.
  - In this case they would be called Majorana particles.
- Fermions that are distinguishable from their anti-particles are called Dirac particles.
  - All charged fermions are Dirac particles, but neutral particles can in principle have either identity
    - The π^0 is indistinguishable from its anti-partner, but the K^0 is distinct from the bar-K^0

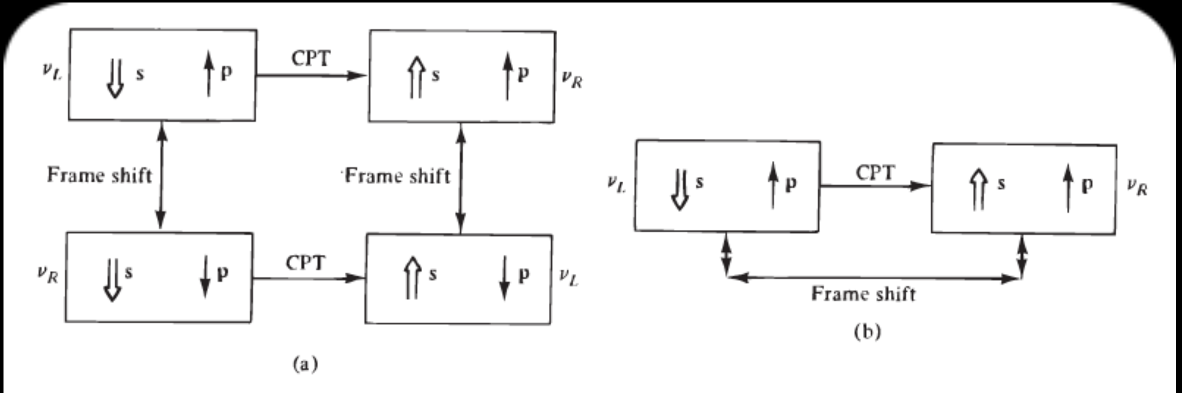


Figure 11.10: (a) Four states of a Dirac neutrino; (b) two states of a Majorana neutrino.

Neutrinoless double-β decay (0νββ)

- Some nuclei do not have enough energy for an ordinary β-decay, but the energy difference of nuclei with Z and Z + 2 protons may be sufficient to allow a decay with the emission of two electrons and two anti-neutrinos:

$$(Z,N) \rightarrow (Z+2,N-2) + 2e^- + 2\overline{\nu}_e$$

- Two neutrons from the original nucleus simultaneously undergo β-decay
  - Slow process but has been observed in nature
- If neutrinos are Majorana then the decay proceeds without the emission of neutrinos

Germanium Detector Array (GERDA)

- Located at the underground Laboratori Nazionali del Gran Sasso (LNGS), Italy
- Designed to study
  - neutrinoless double beta decay (0νββ) in Ge-76

BACKUP

The two (neutral) kaon states

- The Kaon family consists of the charged and neutral particles
  - K^+ consists of a u and an anti-s quarks
  - K^- consists of an anti-u and s quarks
  - K^0 ($\overline{K^0}$) consisting of a d (anti-d) and an anti-s (s) quarks
- The neutral kaons, although they are mainly produced via the strong interaction they decay weakly
- Strangeness and thus hypercharge are the quantum numbers that allow to distinguish between the neutral kaon and its antiparticle
  - both are conserved in the E/M and strong interactions
    - The two neutral kaons are two distinctly different particles based on these two forces
  - violated in the weak sector
    - Transitions between the two states can occur in the weak interactions

$$K^0 \leftrightarrows \pi^+ + \pi^- \leftrightarrows \overline{K}^0$$

The two (neutral) kaon states (cont.)

- Acting with the parity operator on each of these two particles gives (note the intrinsic parity of kaons is -1):

$$\hat{P}|K^0\rangle = -|K^0\rangle \qquad \hat{P}|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$

- The combined action of parity and charge conjugation gives:

$$\hat{C}\hat{P}|K^0\rangle = -|\overline{K}^0\rangle \qquad \hat{C}\hat{P}|\overline{K}^0\rangle = -|K^0\rangle$$

- If the total Hamiltonian conserves CP, then the eigenstates of H can be chosen to be the eigenstates of CP eigenstates
  - These eigenstates can be written as a linear combination the two neutral kaons

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad |K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$

  - These two states, when CP acts on them give

$$\hat{C}\hat{P}|K_1^0\rangle = \frac{1}{\sqrt{2}}(\hat{C}\hat{P}|K^0\rangle - \hat{C}\hat{P}|\overline{K}^0\rangle) = \frac{1}{\sqrt{2}}(-|\overline{K}^0\rangle + |K^0\rangle) = +|K_1^0\rangle$$

$$\hat{C}\hat{P}|K_2^0\rangle = \frac{1}{\sqrt{2}}(\hat{C}\hat{P}|K^0\rangle + \hat{C}\hat{P}|\overline{K}^0\rangle) = \frac{1}{\sqrt{2}}(-|\overline{K}^0\rangle - |K^0\rangle) = -|K_2^0\rangle$$

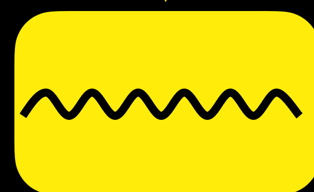
The two (neutral) kaon states (cont.)

- $\overline{K}^0$ is the antiparticle of K^0 (and vice-versa)
  - These two have the same mass and lifetime
- This is not the case for K^0_1 and K^0_2 which are the two weak eigenstates
  - These two particles can have different properties
    - K^0_L (“K-long”, the long-lived neutral kaon, with a mean lifetime of 5.2×10^{-8} s), with the main decay channel being the one into three pions
      - Since the intrinsic parity of pions is -1, the three-pion final state has CP = -1
      - K^0_S is associated with the K^0_2
    - K^0_S (“K-short”, the short-lived neutral kaon, with a mean lifetime of 9.6×10^{-11} s), with the main decay channel being the one into two pions
      - Since the intrinsic parity of pions is -1, the two-pion final state has CP = +1
      - K^0_S is associated with the K^0_1
- It turns out that CP is also not a good symmetry of the Standard Model
  - K-long and K-short are not pure eigenstates of CP…uff too complicated…

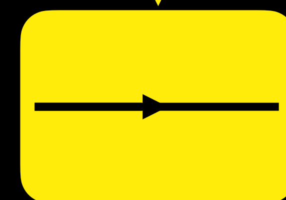
Weak Lagrangian

QED

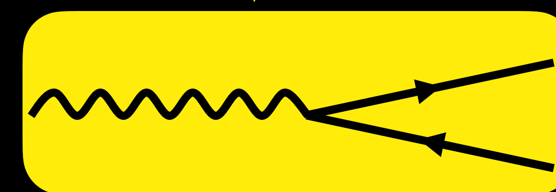
$$\mathscr{L}_{QED} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\,(i\gamma_\mu \partial^\mu - m)\psi - qA_\mu \bar{\psi}\,\gamma^\mu \psi$$



Free photon field A^μ



Fermion ψ with mass m

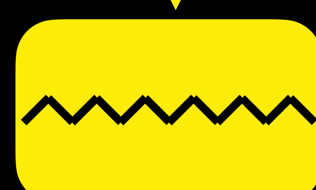


Interaction with coupling α

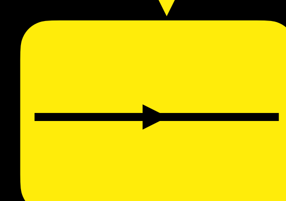
Weak

$$\mathscr{L}_{weak} = -\frac{1}{4} \vec{F}_{\mu\nu}\vec{F}^{\mu\nu} + \bar{\psi}\,(i\gamma_\mu D^\mu - m)\psi$$

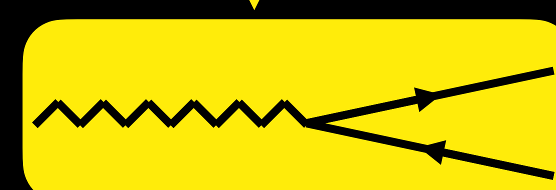
$$= -\frac{1}{4} \vec{F}_{\mu\nu}\vec{F}^{\mu\nu} + I\,\bar{\psi}\,(i\gamma_\mu \partial^\mu - m)\psi - \frac{g}{2}\,\bar{\psi}\,\gamma_\mu \vec{\tau}\,\vec{\alpha}^\mu \psi$$



Three gauge boson fields A^μ ($W^\pm$ Z^0)



Fermion ψ with mass m



Interaction with coupling g

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