Online Appendix for "Buying First or Selling First in Housing Markets"

Appendix C: Data and Calibration

C.1 Data description

We use two data sets. The first (EJER) is an ownership register which contains the owners (private individuals and legal entities) of properties in Denmark as of the end of a given calendar year. The data set contains unique identifiers for owners (which, unfortunately, cannot be matched with other data-sets beyond EJER for different years). It also contains unique identifiers for each individual property. The second data set (EJSA) contains a record of all property sales in a given calendar year. The majority of transactions include information on the sale price, sale (agreement), and takeover (closing) dates. Furthermore, they contain the property identifiers used in the EJER data-set, which allows for linking of the two data-sets. The first data set is available from 1986 (recording ownership in 1985) until 2010 (recording ownership at the end of 2009), while the second is available from 1992 to 2010. Therefore, we effectively use data from 1991 (for ownership as of January 1, 1992) to 2009 (for ownership as of January 1, 2010).

We focus on the Copenhagen urban area (Hovedstadsområdet). We take the definition of the Copenhagen urban area as containing the following municipalities (by number): 101, 147, 151, 153, 157, 159, 161, 163, 165, 167, 173, 175, 183, 185, 187, 253, 269.[23]

We restrict attention to private owners and also to the primary owner of a property in a given year (whenever a property has more than one owners). Furthermore, we examine transactions where the new owner is a private individual and which have a non-missing agreement date. We drop

[23]Due to a reform in 2007, which merged some municipalities and created a new one, we omit municipality 190 for consistency.

properties that are recorded to transact more than once in a given year. We also remove property-year observations for which no owner is recorded. This leaves us with a total of 3,312,520 property-year observations. These comprise 199,812 unique properties and 345,943 unique individual owners over our sample period.

To identify an individual owner as a buyer-and-seller we rely on the information from the ownership register across consecutive years. First of all, we use the information on ownership over consecutive years to determine the counterparties for each recorded transaction in our sample. We then identify an individual owner as a buyer-and-seller if he is recorded to buy a new property and sell an old property within the same year or over two consecutive years. An old property is defined as a property which an individual is registered as owning over at least 2 consecutive years.[24] Also, we do not count individuals that are recorded as holding two properties for two or more consecutive years, which we treat as purchases for investment purposes.

We conduct this for individuals that are recorded as owning at most 2 properties at the end of any calendar year in our sample. This comprises the large majority of individual owners in our sample. In particular, in a given year in our sample from 1991-2009 there are on average only around 0.4% of individual owners who own more than two properties in the Copenhagen. Therefore, the majority of individuals hold at most 1 or 2 properties over that period. In particular, on average, around 1.6% of individual owners hold two properties at the end of a calendar year in our sample. Interestingly, around 5% of the recorded owners of two properties at the end of a calendar year are also identified as a buyer-and-seller according to our identification procedure described above with that number going up to almost 14% at the peak of the housing boom in 2006.

For each individual owner that has been identified as buyer-and-seller, we compute the time period (in days) between the agreement data for sale of the old property and the agreement date

[24]We make this restriction in order not to misclassify as a buyer-and-seller an individual who acquires a house, for example as a bequest (which is not recorded as a transaction), which he ends up selling quickly and then buys a new house with the proceeds from the sale. Adding back those agents has a very small effect on the pattern we uncover.

for the purchase of the new property. Similarly, we compute the time period (in days) between the closing date that of the buyer-and-seller's old property by the new owner and the closing date for his new property. We then denote a buyer-and-seller for which the time period between agreement dates is negative (sale date is before purchase date) as "selling first" and a buyer-and-seller for which the time period is positive (sale date is after purchase date) as "buying first". We also do the same classification but based on closing dates rather than agreement dates. Given the way we identify a buyer-and-seller, we have a consistent count for the number of owners who "buy first" vs. "sell first" in a given year for the years 1993 to 2008.

In principle, and as Figures 1 and 2 show, working with either of the two identifications produces similar results. This is not surprising given that the time difference between the agreement dates and closing dates are highly correlated with a correlation coefficient of 0.9313.

C.2 Calibration of γ and g

To calibrate γ and g, we need to remedy the lack of information on owner-renter transitions and on transitions within and across housing markets by housing tenure status in Denmark. We therefore supplement the available mobility information from Denmark with information from the USA in the following way. We first take the one-year mobility rates of owners and renters within and across counties from the 2012 American Community Survey (ACS). We then assume that the ratio of the mobility rates of renters relative to owners within a US county and within a Danish municipality are the same. We assume the same for the renter-owner mobility ratio across US counties and Danish municipalities. These assumptions are reasonable since US counties and Danish municipalities tend to be of similar size and roughly correspond to local housing markets. Also, there is no *a priori* reason to think that Danish owners are more or less mobile within (or across) housing markets relative to renters compares to US owners and renters.

Table C.1: Internal migration by housing tenure for Denmark (a) and the USA (b).

(a)

	Within municipality	Outside municipaity	Total
Owners	4.1*	2.6*	6.7*
Renters	21.4*	10.2*	31.6*
All	9.9	5.1	

(b)

	Within county	Outside county	Total
Owners	3.8	2.7	6.5
Renters	19.7	10.7	30.4
All	9.2	6.1	

Notes. Data source: Statistics Denmark, 2012 ACS and own calculations. An asterisk denotes that the mobility rate is imputed. See text for details.

We then combine these assumptions with the home ownership rate in Denmark and the total mobility of individuals within and across municipalities in 2012 to impute the mobility of owners and renters within and across municipalities. Put differently, we solve for the mobility rate of owners m_{own}^{within} implicitly given in the equation

$$m_{own}^{within} own + \rho^{within,US} m_{own}^{within} (1 - own) = m_{tot}^{within}, \tag{C.1}$$

where own is the home-ownership rate, $\rho^{within,US}$ is the ratio of the mobility of renters to owners in the USA, and m_{tot}^{within} is the total mobility rate within municipalities, and similarly for m_{own}^{across}. Table C.1 shows the resulting mobility rates and compares them against the same mobility rates in the USA. As the table shows, the overall mobility rates in the US and Denmark are quite similar, which given our assumption implies that the imputed mobility rates of owners and renters are also similar.

Second, to get owner-renter transitions, we use the 2003 American Housing Survey (AHS) to compute the fraction of movers that owned a housing unit 2 years before and are renters as of the 2003 survey.[25] We use the 2003 AHS to minimize the effects of the 2002-2010 housing boom-bust cycle. We find that approximately 0.248 of moving owners switch to renters. We assume that this

[25] We do not count movers who lived in an owned unit but were not the owners themselves. This omits some groups of moves that would artificially inflate the number of owner-renter transitions, for example, young people that move out of their parents' house and establish a new household as renters.

is the fraction of owner-renter switches in Denmark as well.

Finally, to get owner-owner transitions within municipalities we reduced the owner mobility rate of 4.1 by the owner-renter transition rate, which is approximately $0.248 \times 4.1 = 1$ percentage point. Similarly, we inflate the owner transition rate across municipalities of 2.6 percentage points with that number. Therefore, we arrive at a 1 year owner-owner transition rate within a housing market of 3.1% and a 1 year transition out of owning housing or across housing markets of 3.6%. We use these transition rates to calibrate γ and g, respectively. Therefore, we set $g = 0.0371$ and $\gamma = 0.0322$.

C.3 Additional results

Figure 1a plots the distribution of the time difference between "sell" and "buy" agreement dates over the whole period 1993-2008. As Figure 2a shows, the share of moving owners that buy first has fluctuated substantially over this period, so it is very likely that the distribution of the difference in sell and buy dates is not stable over time but varies as moving owners switch from selling first to buying first and vice versa. To illustrate this, in Figure C.1 we plot kernel densities for the distribution of the time between "sell" and "buy" transactions in two periods: first, between 2004-2006 when the share of buy first owners was the highest in the sample and, second, between 2007-2008 when the share of buy first owners fell substantially. As the Figure shows, the distribution is shifted to the left during 2007-2008 relative to 2004-2006, which is a direct implication of more homeowners buying first in the first period relative to the second period.

Additionally, in Table 1 we used proxies for average time-to-buy and average time-to-sell based on the average time between the sell (buy) and buy (sell) transactions for owners that sell first (buy first) and who complete their second transaction in a given quarter. We can examine whether the time-to-sell proxy constructed this way comoves with seller time-on-market for the period 2004-

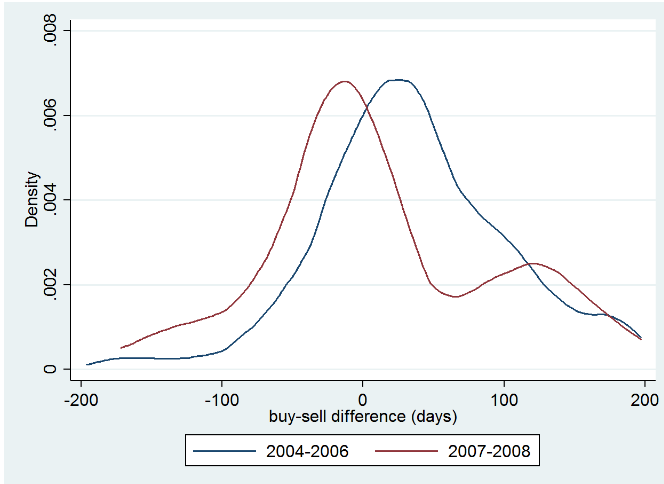

Figure C.1: Distribution of the time difference between "sell" and "buy" agreement dates for 2004-2006 (blue) and 2007-2008 (red). Own calculations based on registry data from Statistics Denmark. The distributions are truncated at ±200 days.

2008. Figure C.2 plots the two series for the period 2004-2008. The two series are nearly identical in the first half of this period. Subsequently, the time-to-sell proxy is consistently above seller time-on-market, though the two series continue to fluctuate closely. One explanation for this difference is that our measure of seller time-on-market does not account for property withdrawals and relistings, which are known to be particularly prevalent during a housing downturn. In that case, the time-to-sell proxy would be a more accurate measure of the true underlying seller time-on-market.

Finally, Figure C.3 plots the dynamics of market tightness θ for a numerical example in which *all* mismatched owners switch from buying first to selling first, with γ, g, and the matching function from Table 2.

Appendix D: Omitted Proofs

Proof of Proposition 4

Below, we use the notation Σ_{ij} to denote the surplus from trade between agents of type i and type j. Also, for brevity, we use the notation $\underline{\theta}$ for $\lim_{g,\gamma\to 0,\frac{g}{\gamma}=\kappa}\underline{\theta}$ and $\bar{\theta}$ for $\lim_{g,\gamma\to 0,\frac{g}{\gamma}=\kappa}\bar{\theta}$.

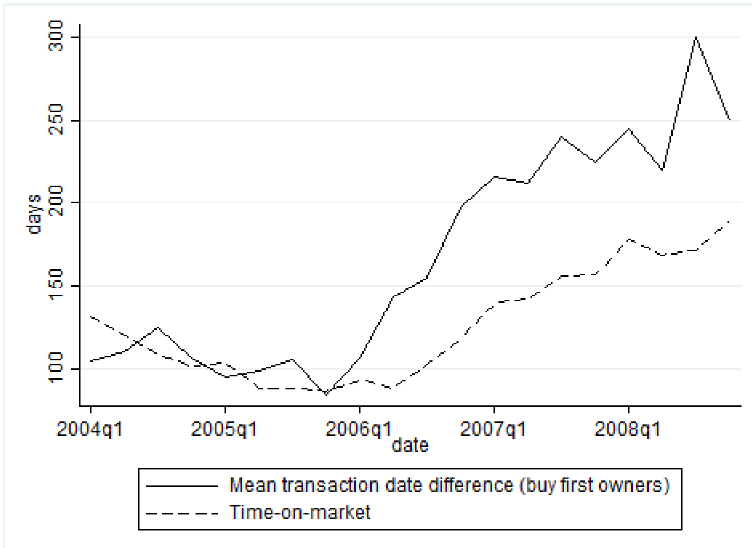

Figure C.2: Comparison of seller time-on-market (dashed line) against time-to-sell proxy (solid line) based on the mean transaction data difference of buy-first owners. Copenhagen, Q1:2004-Q4:2008. Seller time-on-market for Copenhagen is from the Danish Mortgage Banks' Federation (available at http://statistik.realkreditforeningen.dk/BMSDefault.aspx). The time-to-sell proxy is based on own calculations based on registry data from Statistics Denmark. Specifically it is given by the average time between buy and sell transactions for buy-first owners who complete the second transaction in the quarter.

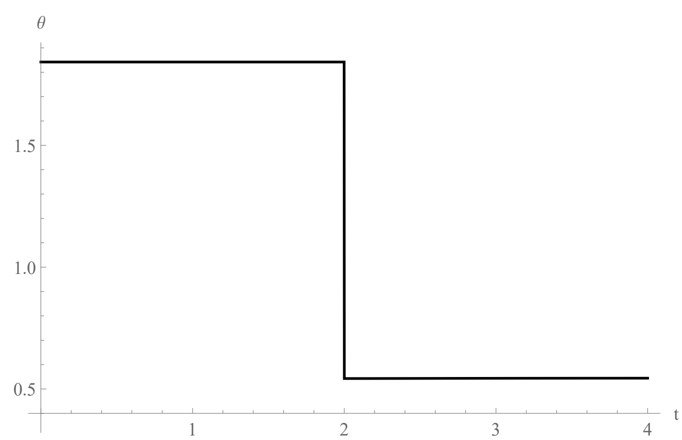

Figure C.3: Dynamics of market tightness θ.

“Sell first” equilibrium existence.

We first show that a “Sell first” equilibrium exists with $\theta = \underline{\theta} < 1$. We proceed in two steps. First, we show that no mismatched owner has an incentive to deviate and buy first when $\theta = \underline{\theta} < 1$. This is verified under the conjecture that $\Sigma_{ij} \geq 0$ for all buyer-seller pairs except for Σ_{S1B1}. Second, we verify the conjecture on the different surpluses.

Step 1. In the limit economy with small flows of a “Sell first” equilibrium candidate, the faction of buyers who are forced renters is given by

$$\lim_{g \to 0, \gamma \to 0, \frac{g}{\gamma} = \kappa} \frac{B_0}{B} = \lim_{g \to 0, \gamma \to 0, \frac{g}{\gamma} = \kappa} \frac{q\left(\underline{\theta}\right) - \mu\left(\underline{\theta}\right)}{g + q\left(\underline{\theta}\right)} = 1 - \underline{\theta},$$

where $\underline{\theta} = \frac{1}{1+\kappa}$. Similarly,

$$\lim_{g \to 0, \gamma \to 0, \frac{g}{\gamma} = \kappa} \frac{A}{S} = \underline{\theta}.$$

Thus,

$$\begin{aligned} rV^{B0} &= u_0 - R + \frac{1}{2} q\left(\underline{\theta}\right)\left(\frac{A}{S}\Sigma_{AB0} + \frac{S_1}{S}\Sigma_{S1B0}\right) \\ &= u_0 - R + \frac{1}{2} q\left(\underline{\theta}\right)\left(\underline{\theta}\Sigma_{AB0} + \left(1 - \underline{\theta}\right)\Sigma_{S1B0}\right), \end{aligned}$$

and similarly,

$$rV^{Bn} = u_n - R + \frac{1}{2} q\left(\underline{\theta}\right)\left(\underline{\theta}\Sigma_{ABn} + \left(1 - \underline{\theta}\right)\Sigma_{S1Bn}\right),$$

so

$$V^{Bn} - V^{B0} = \frac{u_n - u_0}{r + \frac{1}{2} q\left(\underline{\theta}\right)}. \tag{D.1}$$

Also,

$$
\begin{aligned}
rV^{A} &= R+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(\frac{B_n}{B}\left(V-V^{Bn}-V^{A}\right)+\frac{B_0}{B}\left(V-V^{B0}-V^{A}\right)\right) \\
&= R+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta}\left(V-V^{Bn}-V^{A}\right)+\left(1-\underline{\theta}\right)\left(V-V^{B0}-V^{A}\right)\right),
\end{aligned}
$$

or

$$
V^{A}=\frac{R}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V-V^{B0}-\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\right).
$$

Analogous to (D.1)

$$
V^{S2}-V^{A}=\frac{u_2+\frac{1}{2}\mu\left(\underline{\theta}\right)V}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}.
$$

This in turn implies that

$$
V-V^{S2}=\frac{rV-u_2}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}-V^{A}=\frac{u-u_2}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}-V^{A}.
$$

Turning to the value functions of mismatched owners, a mismatched seller has a value function given by

$$
rV^{S1}=u-\chi+\frac{1}{2}\mu\left(\underline{\theta}\right)\left(V-V^{S1}-\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}\right),
$$

which can be re-written as

$$
V^{S1}=\frac{u-\chi}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}+\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}V-\frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\underline{\theta}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)}.
$$

For the value function of a deviating mismatched buyer, assuming that trade takes place when he meets a real-estate firm but not when he meets a mismatched seller, writes

$$rV^{B1} = u - \chi + \frac{1}{2} q\left(\underline{\theta}\right) \underline{\theta} \Sigma_{AB1}.$$

Or

$$\left(r + \frac{1}{2}\mu\left(\underline{\theta}\right)\right) V^{B1} = u - \chi + \frac{1}{2}\mu\left(\underline{\theta}\right)\left(V^{S2} - V^{A}\right).$$

Consider the difference between the utilities from buying first compared to selling first. In the limit we consider, we have that

$$\left(r + \frac{1}{2}\mu\left(\underline{\theta}\right)\right)\left(V^{B1} - V^{S1}\right) = \frac{1}{2}\mu\left(\underline{\theta}\right)\left(V^{S2} - V^{A} - V + \underline{\theta}\frac{u_n - u_0}{\rho + \frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Substituting for $V^{S2} - V^{A} - V$, we get that

$$V^{B1} - V^{S1} = \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\left(\frac{u_2 - u}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} + \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Note that at $\underline{\theta} = 1$ (i.e. for $\kappa = 0$),

$$\frac{u_2 - u}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} + \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q\left(\underline{\theta}\right)} = 0,$$

given Assumption B1. As $\underline{\theta}$ moves away from 1 toward 0 (κ increases), we have that $\frac{u_2 - u}{r+\frac{1}{2}\mu(\underline{\theta})} + \underline{\theta}\frac{u_n - u_0}{r+\frac{1}{2}q(\underline{\theta})}$ decreases, so $V^{B1} < V^{S1}$ for $\underline{\theta} < 1$. Therefore, it is not optimal for a mismatched owner to deviate and buy first in an equilibrium in which mismatched owners sell first and $\theta < 1$.

Step 2. We verify that our conjectures for the surpluses are correct. It is clear given our assumptions that $\Sigma_{S2B1} = V - V^{B1} > 0$ and $\Sigma_{S1B0} = V - V^{S1} > 0$. Also, $\Sigma_{ABn} \geq 0$. Next, we show that $\Sigma_{S1Bn} > 0$. In the limit we consider,

$$
\begin{aligned}
\Sigma_{S1Bn} &= V - V^{Bn} + V^{B0} - V^{S1} = V - V^{S1} - \frac{u_n - u_0}{r + \frac{1}{2} q\left(\underline{\theta}\right)} \\
&= \frac{\chi}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} + \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta} - 1\right) - r}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} \frac{u_n - u_0}{r + \frac{1}{2} q\left(\underline{\theta}\right)} \\
&= \frac{r\left(\chi + u_0 - u_n\right) + \frac{1}{2} q\left(\underline{\theta}\right)\chi + \frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta} - 1\right)\left(u_n - u_0\right)}{\left(r + \frac{1}{2} q\left(\underline{\theta}\right)\right)\left(r + \frac{1}{2}\mu\left(\underline{\theta}\right)\right)}.
\end{aligned}
$$

Therefore, at $\underline{\theta} = 1$, $\Sigma_{S1Bn} > 1$ if

$$
r\left(\chi + u_0 - u_n\right) + \frac{1}{2}\mu_0 \chi > 0.
$$

Note that given Assumption B1, this is equivalent to

$$
r\left(u_2 - \left(u - \chi\right)\right) + \frac{1}{2}\mu_0 \chi > 0,
$$

which holds by Assumption B2. Therefore, by continuity of the value functions with respect to θ, it follows that there is a $\kappa_1 > 0$, such that for $\kappa < \kappa_1$, $\Sigma_{S1Bn} > 0$. Next, we show that $\Sigma_{ABn} > 0$. To show, this suppose, toward a contradiction, that $\Sigma_{ABn} < 0$. Then

$$
rV^{Bn} + rV^{A} \leq u_n + \frac{1}{2} q\left(\underline{\theta}\right)\Sigma_{ABn} + \frac{1}{2}\mu\left(\underline{\theta}\right)\Sigma_{ABn},
$$

where the inequality comes from $\Sigma_{ABn} < 0 < \Sigma_{S1Bn}$ and $\Sigma_{ABn} < \Sigma_{AB0}$, since $V^{Bn} > V^{B0}$. Therefore,

$$
\Sigma_{ABn} \geq \frac{rV - u_n}{r + \frac{1}{2} q\left(\underline{\theta}\right) + \frac{1}{2}\mu\left(\underline{\theta}\right)} > 0,
$$

so we arrive at a contradiction. $\Sigma_{ABn} > 0$ also implies that $\Sigma_{AB0} > 0$, since $V^{Bn} > V^{B0}$. Next notice that

$$\Sigma_{S2Bn} = \Sigma_{ABn} + V - V^{S2} + V^{A} = \Sigma_{ABn} + \frac{rV - u_2}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} > 0.$$

Again, this also implies that $\Sigma_{S2B0} > 0$. Next, we show that $\Sigma_{AB1} > 0$. In the limit we consider,

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2} - V^{B1} - V^{A} = V^{S2} - V^{A} - \frac{u-\chi}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V^{S2} - V^{A}\right) \\
&= \frac{r\left(V^{S2} - V^{A}\right) - (u-\chi)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} = \frac{\frac{r}{r+\frac{1}{2}\mu(\underline{\theta})}u_2 + \frac{\frac{1}{2}\mu(\underline{\theta})}{r+\frac{1}{2}\mu(\underline{\theta})}u - (u-\chi)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} \\
&= \frac{r\left(u_2 - (u-\chi)\right) + \frac{1}{2}\mu\left(\underline{\theta}\right)\chi}{\left(r + \frac{1}{2}\mu\left(\underline{\theta}\right)\right)^2}.
\end{aligned}$$

At $\underline{\theta} = 1$, $\Sigma_{AB1} > 0$ if $r\left(u_2 - (u-\chi)\right) + \frac{1}{2}\mu_0\chi > 0$, which is our parametric Assumption B2. Therefore, by continuity of the value functions with respect to $\underline{\theta}$, it follows that there is a $\kappa_2 > 0$, such that for $\kappa < \kappa_2$, $\Sigma_{AB1} > 0$. Finally, in the limit we consider

$$\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - V^{A} \\
&= \Sigma_{AB1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}.
\end{aligned}$$

At $\underline{\theta} = 1$,

$$\begin{aligned}
\frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu_0} &= \frac{\frac{r}{r+\frac{1}{2}\mu_0}\left(u_0 - R\right) + \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}u - \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}rV^{A} - (u-\chi) + R}{r + \frac{1}{2}\mu_0} \\
&= \frac{ru_0 + \frac{1}{2}\mu_0 u - \frac{1}{2}\mu_0\left(rV^{A} - R\right) - \left(r + \frac{1}{2}\mu_0\right)(u-\chi)}{\left(r + \frac{1}{2}\mu_0\right)^2}.
\end{aligned}$$

Substituting for Σ_{AB1}, we get

$$\Sigma_{S1B1} = \frac{r\left(u_0 + u_2 - 2\left(u - \chi\right)\right) + \mu\chi - \frac{1}{2}\mu_0\left(rV^A - R\right)}{\left(r + \frac{1}{2}\mu_0\right)^2}.$$

Therefore, a sufficient condition for $\Sigma_{S1B1} < 0$ at $\underline{\theta} = 1$ is

$$r\left(u_0 + u_2 - 2\left(u - \chi\right)\right) + \mu_0\chi \leq 0,$$

or

$$r\left(u_2 - u_0\right) \geq 2\left[r\left(u_2 - \left(u - \chi\right)\right) + \frac{1}{2}\mu_0\chi\right],$$

which is our parametric Assumption B3. Again by continuity of the value functions with respect to $\underline{\theta}$, we have that there is a $\kappa_3 > 0$, s.t. for $\kappa < \kappa_3$, $\Sigma_{S1B1} < 0$. Taking $\underline{\kappa} = \min\{\kappa_1, \kappa_2, \kappa_3\}$, we have that for $\kappa < \underline{\kappa}$, there is a "Sell first" equilibrium with a market tightness given by $\underline{\theta} = \frac{1}{1+\kappa}$.

"Buy first" equilibrium existence.

We follow the same two steps to show the existence of a "Buy first" equilibrium with $\theta = \bar{\theta} > 1$. Again, we make the same conjectures on the different surpluses as in the case of the "Sell first" equilibrium. In the limit economy, the fraction of buyers who are new entrants is

$$\lim_{g\to 0, \gamma\to 0, \frac{g}{\gamma}=\kappa} \frac{B_n}{B} = \frac{1}{\bar{\theta}},$$

where $\bar{\theta} = 1 + \kappa$. Also,

$$\lim_{g\to 0, \gamma\to 0, \frac{g}{\gamma}=\kappa} \frac{A}{S} = \frac{1}{\bar{\theta}}$$

as well. Therefore, similarly to the value functions in the "Sell first" equilibrium, we have that

$$\left(r + \frac{1}{2}\mu\left(\bar{\theta}\right)\right)V^A = R + \frac{1}{2}\mu\left(\bar{\theta}\right)\left(\frac{1}{\bar{\theta}}\left(V - V^{Bn}\right) + \frac{\bar{\theta} - 1}{\bar{\theta}}\left(V^{S2} - V^{B1}\right)\right),$$

and

$$\left(r+\frac{1}{2}\mu\left(\bar{\theta}\right)\right)V^{S2}=u_2+\left(r+\frac{1}{2}\mu\left(\bar{\theta}\right)\right)V^{A}+\frac{1}{2}\mu\left(\bar{\theta}\right)V.$$

Therefore, as in the "Sell first" equilibrium,

$$V-V^{S2}=\frac{rV-u_2}{r+\frac{1}{2}\mu\left(\bar{\theta}\right)}-V^{A}.$$

Also, as in the "Sell first" equilibrium,

$$V^{Bn}-V^{B0}=\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)}.$$

Turning to the value functions of a mismatched buyer, we have that

$$rV^{B1}=u-\chi+\frac{1}{2}q\left(\bar{\theta}\right)\left(\frac{1}{\bar{\bar{\theta}}}\left(V^{S2}-V^{B1}-V^{A}\right)+(1-\frac{1}{\bar{\bar{\theta}}})\left(V-V^{B1}\right)\right),$$

For the value function of a deviating agent who chooses to sell first, we have that

$$rV^{S1}=u-\chi+\frac{1}{2}\mu\left(\bar{\theta}\right)\left(\frac{1}{\bar{\bar{\theta}}}\Sigma_{S1Bn}\right),$$

since $\Sigma_{S1B1}<0$. Then,

$$\left(r+\frac{1}{2}q\left(\bar{\theta}\right)\right)V^{S1}=u-\chi+\frac{1}{2}q\left(\bar{\theta}\right)V+\frac{1}{2}q\left(\bar{\theta}\right)\frac{u_0-u_n}{r+\frac{1}{2}q\left(\bar{\theta}\right)}.$$

Therefore, the difference between $V^{B1}-V^{S1}$ satisfies

$$\left(r+\frac{1}{2}q\left(\bar{\theta}\right)\right)\left(V^{B1}-V^{S1}\right)=\frac{1}{2}q\left(\bar{\theta}\right)\left(\frac{1}{\bar{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\bar{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)}\right).$$

At $\bar{\theta}=1$, we have that

$$\frac{1}{\bar{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\bar{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)}=0,$$

by Assumption B1. As $\bar{\theta}$ increases, we have that $\frac{1}{\bar{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu(\bar{\theta})}+\frac{u_n-u_0}{r+\frac{1}{2}q(\bar{\theta})}$ increases, so $V^{B1}>V^{S1}$ for $\bar{\theta}>1$. Therefore, it is not optimal for a mismatched owner to deviate and sell first in an equilibrium in which mismatched owners buy first and $\theta>1$.

Finally, we verify that our conjectures for the surpluses are correct. As in the "Sell first" case, $\Sigma_{S1B0}>0$ and $\Sigma_{S2B1}>0$. Also, as in the "Sell first" case, in the limit we consider,

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2}-V^{B1}-V^{A}=V^{S2}-V^{A}-V+V \\
&- \frac{u-\chi}{r+\frac{1}{2}q\left(\bar{\theta}\right)}-\frac{\frac{1}{2}q\left(\bar{\theta}\right)}{r+\frac{1}{2}q\left(\bar{\theta}\right)}\left[\frac{1}{\bar{\theta}}\left(V^{S2}-V^{A}-V\right)+V\right] \\
&= \frac{\left(r+\frac{1}{2}q\left(\bar{\theta}\right)\frac{\bar{\theta}-1}{\bar{\theta}}\right)}{r+\frac{1}{2}q\left(\bar{\theta}\right)}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\bar{\theta}\right)}+\frac{\chi}{r+\frac{1}{2}q\left(\bar{\theta}\right)} \\
&= \frac{r\left(u_2-\left(u-\chi\right)\right)+\frac{1}{2}\mu\left(\bar{\theta}\right)\chi+\frac{1}{2}q\left(\bar{\theta}\right)\frac{\bar{\theta}-1}{\bar{\theta}}\left(u_2-u\right)}{\left(r+\frac{1}{2}\mu\left(\bar{\theta}\right)\right)\left(r+\frac{1}{2}q\left(\bar{\theta}\right)\right)}.
\end{aligned}$$

Note that at $\bar{\theta}=1$, Σ_{AB1} in the "Buy first" case is the same as the "Sell first" case. Therefore, there is a $\kappa_4>0$, such that for $\kappa<\kappa_4$ and $\bar{\theta}=1+\kappa$, $\Sigma_{AB1}>0$. Similarly,

$$\begin{aligned}
\Sigma_{S1Bn} &= V-V^{Bn}+V^{B0}-V^{S1}=V-V^{S1}-\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)} \\
&= \frac{\chi}{r+\frac{1}{2}q\left(\bar{\theta}\right)}-\frac{r}{r+\frac{1}{2}q\left(\bar{\theta}\right)}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\bar{\theta}\right)} \\
&= \frac{r\left(\chi+u_0-u_n\right)+\frac{1}{2}q\left(\bar{\theta}\right)\chi}{\left(r+\frac{1}{2}q\left(\bar{\theta}\right)\right)^2},
\end{aligned}$$

which at $\bar{\theta}=1$ is again the same as for the "Sell first" case. Therefore, there is a $\kappa_5>0$, such that for $\kappa < \kappa_5$, $\Sigma_{S1Bn} > 0$. A similar argument to the one for the "Sell first" case also confirms that $\Sigma_{ABn} > 0$, $\Sigma_{AB0} > 0$, $\Sigma_{S2Bn} > 0$ and $\Sigma_{S2B0} > 0$. Finally,

$$\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - (u-\chi) + R + \frac{1}{2}q\left(\bar{\theta}\right)\left(\bar{\theta}-1\right)\left(V^{S2} - V^{B1} - V^{A}\right)}{r + \frac{1}{2}q\left(\bar{\theta}\right)} - V^{A} \\
&= \left(1 + \frac{\frac{1}{2}q\left(\bar{\theta}\right)\left(\bar{\theta}-1\right)}{r + \frac{1}{2}q\left(\bar{\theta}\right)}\right)\Sigma_{AB1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}q\left(\bar{\theta}\right)}.
\end{aligned}$$

At $\bar{\theta} = 1$, showing that $\Sigma_{S1B1} < 0$ in the "Buy first" case therefore follows the "Sell first" case, so that $\Sigma_{S1B1} < 0$ for $\kappa < \kappa_6$, for some $\kappa_6 > 0$. Taking $\bar{\kappa} = \min\{\kappa_4, \kappa_5, \kappa_6\}$, we have that for $\kappa < \bar{\kappa}$, there is a "Buy first" equilibrium with a market tightness given by $\bar{\theta} = 1 + \kappa$. Finally, taking $\kappa^* = \min\{\bar{\kappa}, \underline{\kappa}\}$, we arrive at the desired result. □

Appendix E: Additional Extensions

E.1 Tighter bounds on θ_b and θ_s

We want to derive a set of conditions on the stocks such that if the conditions are satisfied initially, they will be satisfied at all later points in time along the buy-first trajectory. We will then do the same for the sell first trajectory.

Our ultimate goal is to derive a lower bound $\theta_b^{low} > \theta_b^{\min}$. First, we can redefine the lower bound on $B_n(t)$ as

$$B_n^{low} = \frac{g}{g + q\left(\theta_b^{low}\right)} \tag{E.1}$$

To derive a lower bound O_b^{low}, we focus on the the inflow into $O(t)$ from the pool of new entrants only. Define O_b^{low} by the equation

$$B_n^{low} q\left(\theta^{ub}\right) = O_b^{low}\left(\gamma + g\right). \tag{E.2}$$

If $B_{n0}(0) \geq B_n^{low}$ and $O(0) \geq O_b^{low}$, then $O(t) > O_b^{low}$, for all t.

Define $\theta_b^{low} = 1 + \frac{\gamma}{g} O_b^{low} > 1$. From (B.17) it follows that along any buy first trajectory, $\theta(t) > \theta^{low}$ within a finite amount of time. Substituted into (E.2) this gives

$$1 + \frac{q\left(\theta^{ub}\right)}{g + q\left(\theta_b^{low}\right)} \frac{\gamma}{\gamma + g} = \theta_b^{low}. \tag{E.3}$$

This is one equation in one unknown θ_b^{low}. At $\theta_b^{low} = 1$, the left-hand side is strictly greater than the right-hand side. At $\theta_b^{low} = \theta^{ub}$, the opposite is true.[26] Hence $\theta_b^{low} \in (1, \theta^{ub})$. Substituting θ_b^{low} into (E.2) gives O_b^{low}. Note also that for any $\theta < \theta_b^{low}$, the left-hand side of (E.3) is less than the right-hand side.

In Proposition 3 we showed that if $\theta \geq \tilde{\theta}$ along a buy first trajectory, then the buy first trajectory constitutes a dynamic equilibrium. The next lemma follows:

Lemma E.1. *Consider a switch to a buy first trajectory at $t = t'$. Suppose $B_{n0}(t') \geq B_n^{low}$ and that $O(t') \geq O^{low}$. Then the following is true:*

1. *If $\theta_b(t') \geq \theta_b^{low} \geq \tilde{\theta}$, the buy first trajectory constitutes a dynamic equilibrium from t' onward.*

2. *If $\tilde{\theta} \leq \theta_b(t') \leq \theta_b^{low}$, the buy first trajectory constitutes a dynamic equilibrium from t' onward.*

Proof. The first part follows directly from the discussion above. We therefore turn to Item 2. Define $\hat{\theta}^{low} = \theta(t')$, and redefine θ_b^{low} to $\hat{\theta}^{low}$, and redefine O^{low} to $\hat{O}^{low} < O^{low}$ as the lower bound on $O(t)$ when $\theta \geq \hat{\theta}^{low}$. From (E.3) and its properties it follows that the inflow to $O(t)$ is greater than

[26] Since $1 + \frac{q(\theta^{ub})}{g+q(\theta_b^{low})} \frac{\gamma}{\gamma+g} < 1 + \gamma/(g+\gamma) < 1 + \gamma/g = \theta^{ub}$.

$\gamma\hat{\theta}^{low}$, so that $O(t)$ will not decrease below $\hat{O}^{low}$. Hence $\theta(t)$ cannot fall below $\theta(t')$, and the result follows. □

As should be clear from the derivation, θ_b^{low} is a lower bound, and certainly not a greatest lower bound. To get a simpler expression, suppose $q(\theta_b^{low})$ is large relative to g. Then (E.3) simplifies to

$$\mu\left(\theta_b^{low}\right) - q\left(\theta_b^{low}\right) = \frac{\gamma}{g+\gamma} q\left(\theta^{ub}\right) \tag{E.4}$$

where $\theta^{ub} = 1 + \gamma/g$. For example, with $\gamma/g = 1$ and $q(\theta) = \theta^{-1/2}$, we get that $\theta_b^{low} = 1.42$.

We now derive tighter bounds along the sell first trajectory. Hence our goal is to derive an upper bound $\theta_s^{high} < \theta_s^{\max}$ along the sell first trajectory. We use exactly the same approach as in the buy first case. First we define a lower bound on $A_s(t)$ as a function of θ_s^{high}.

$$A_s^{low} = \frac{g}{g + \mu\left(\theta_s^{high}\right)}. \tag{E.5}$$

We proceed to derive a lower bound on $O(t)$, denoted O_s^{low}, given by

$$A_s^{low}\mu\left(\theta^{lb}\right) = O_s^{low}\left(\gamma + g\right). \tag{E.6}$$

If $O(0) > O_s^{low}$, $O(t) > O_s^{low}$, for all t. It follows that $O_s^{low}\gamma/g \leq S_1(t)/A_s(t)$, provided that the inequality holds at $t = 0$. Hence an upper bound on $\theta(t)$ is $\frac{g}{g+\gamma O_s^{low}}$, or substituting for O_s^{low} from (E.6),

$$1 + \frac{\mu\left(\theta^{lb}\right)}{g + \mu\left(\theta_s^{high}\right)}\frac{\gamma}{\gamma + g} = \frac{1}{\theta_s^{\max}}. \tag{E.7}$$

At $\theta_s^{\max} = 1$, the left-hand side is strictly larger than the right-hand side. At $\theta_s^{\max} = \theta^{lb}$, the opposite is true.[27] Furthermore, for any $\theta > \theta_s^{high}$, the left-hand side is greater than the right-hand

[27] At this point, the right-hand side reads $1 + \gamma/g$, which is strictly greater than the left-hand side.

side.

Lemma E.2. *Consider a switch to a sell first trajectory at $t = t'$. Suppose $A_s(t') \geq A_s^{low}$ and that $O(t') \geq O_s^{low}$. Then the following is true:*

1. *If $\theta_s(t') \leq \theta_s^{high} \leq \tilde{\theta}$, the sell first trajectory constitutes a dynamic equilibrium from t' onward.*

2. *If $\tilde{\theta} \geq \theta_s(t') \geq \theta_s^{high}$, the sell first trajectory constitutes a dynamic equilibrium from t' onward.*

Proof. The proof is analogous to the proof of Lemma E.1. □

Also in this case we can get a simpler expression by assuming that $\mu(\theta_s^{high})$ is large relative to g. Then (E.7) simplifies to

$$q\left(\theta_s^{high}\right) - \mu\left(\theta_s^{high}\right) = \frac{\gamma}{g+\gamma}\mu\left(\theta^{lb}\right), \tag{E.8}$$

where $\theta^{lb} = \frac{g}{g+\gamma}$. Continuing the example, with $\gamma/g = 1$ and $q(\theta) = \theta^{-1/2}$, we get that $\theta^{lb} = 1/2$ and $\theta_s^{high} = 1/1.42 = 0.70$.

Finally, if the matching function is symmetric, it follows that $\theta_s^{high} = 1/\theta_b^{low}$. To see this, recall that with a symmetric matching function, it follows that $\mu(\theta) = q(1/\theta)$. Inserting θ_b^{low} into the left-hand side of (E.7) reads (using that $\theta^{lb} = (\theta^{ub})^{-1}$)

$$1 + \frac{\mu\left(\theta^{lb}\right)}{g + \mu\left(\left(\theta^{ub}\right)^{-1}\right)}\frac{\gamma}{\gamma+g} = 1 + \frac{q\left(\theta^{ub}\right)}{g + q\left(\theta_b^{low}\right)}\frac{\gamma}{\gamma+g} = \theta_b^{low}, \tag{E.9}$$

from (E.3). Hence $1/\theta_b^{low}$ satifies (E.7).

E.2 Prices determined by Nash bargaining – additional discussion

In this section we provide an informal discussion of the characterization of the "Buy first" and "Sell first" steady state equilibria when prices are determined by Nash bargaining and of the underlying economic forces.

Consider a "Buy first" steady state equilibrium candidate with a market tightness of $\theta = \bar{\theta} > 1$. In that candidate equilibrium, the sellers with positive measure are the double owners and real-estate firms, while the buyers with positive measure are the mismatched owners and new entrants. In the small flows economy from Section 4.1.2, the outflow rate of mismatched owners is equal to the outflow rate of new entrants, so $B_1/B_n = \gamma/g = \kappa$. Hence, the shares of new entrants and mismatched buyers in the pool of buyers are $1/\bar{\theta}$ and $1 - 1/\bar{\theta}$, respectively. Furthermore, in the limit, as there is no death, the shares of real-estate firms and double owners in the pool of sellers are also $1/\bar{\theta}$ and $1 - 1/\bar{\theta}$, respectively.

Given these shares and since buyers and sellers split the match surplus evenly, the value function of a mismatch buyer is (given $\rho \to r$ in the limit)

$$rV^{B1} = u - \chi + \frac{1}{2}q(\bar{\theta})\left[\frac{1}{\bar{\theta}}\Sigma_{AB1} + (1 - \frac{1}{\bar{\theta}})\Sigma_{S2B1}\right],$$

where $\Sigma_{AB1} = V^{S2} - V^{B1} - V^{A}$ is the match surplus when a mismatched buyer meets a real-estate firm, and $\Sigma_{S2B1} = V - V^{B1}$ is the match surplus when a mismatched buyer meets a double owner.

Consider a mismatched owners who deviates (permanently) and sells first.[28] Since a meeting between a mismatched buyer and a mismatched seller is assumed to lead to negative surplus, the value function of a deviator is simply

$$rV^{S1} = u - \chi + \frac{1}{2}\mu\left(\bar{\theta}\right)\frac{1}{\bar{\theta}}\Sigma_{S1Bn},$$

and so, the difference between the value of buying first and selling first, $D(\bar{\theta}) = V^{B1} - V^{S1}$, can be

[28]Studying permanent deviations is without loss of generality, since a temporary deviation can dominate a permanent deviation if and only if no deviation dominates the permanent deviation.

written as

$$D(\bar{\theta}) = \frac{\frac{1}{2}q\left(\bar{\theta}\right)}{r + \frac{1}{2}q\left(\bar{\theta}\right)}\left(\frac{u_n - u_0}{r + \frac{1}{2}q\left(\bar{\theta}\right)} - \frac{1}{\bar{\theta}}\frac{u - u_2}{r + \frac{1}{2}\mu\left(\bar{\theta}\right)}\right). \tag{E.10}$$

Given our assumptions on utility flows, $D\left(\bar{\theta} = 1\right) = 0$ for $\kappa = 0$. An increase in $\bar{\theta}$ (equivalently, an increase in κ) leads to an increase in $D\left(\bar{\theta}\right)$, since the expression in parenthesis increases. This increase comes from two effects. First, $\mu\left(\bar{\theta}\right)$ increases and $q\left(\bar{\theta}\right)$ decreases, so the second term in the parenthesis decreases (given that $u_2 < u - \chi < u$) and the first term increases (since then $u_n > u_0$). This effect is tightly linked to the queue-length effect from Section 4. Specifically, as before, an increase in $\bar{\theta}$ increases the value of buying first given a lower expected time-on-market for double owners, while it decreases the value of selling first, given a higher expected time-on-market for forced renters.

Second, the fraction of new entrants and real-estate firms, $1/\bar{\theta}$, decreases. Therefore, buyers are more likely to meet double owners and sellers are more likely to meet mismatched buyers. However, the trading surplus for a buyer is higher when matched with a double owner compared to a match with a real-estate firm. Similarly, the trading surplus is lower for a seller when matched with a mismatched buyer compared to a new entrant. This compositional effect on both sides of the market strengthens the incentives to buy first.

Finally, when trading between a mismatched buyer and seller is not profitable for $\bar{\theta}$ close to 1, the discounting effect arising from higher prices is dominated by both the queue-length and compositional effects. Thus, $D\left(\bar{\theta}\right)$ in (E.10) unambiguously increases in $\bar{\theta}$.

Consider a “Sell first” equilibrium candidate with a market tightness of $\theta = \underline{\theta} < 1$. In that candidate equilibrium, the sellers with positive measure are the mismatched owners and real-estate firms, while the buyers are the forced renters and new entrants. In the limit economy, the shares of forced renters and new entrants in the pool of buyers are $\underline{\theta}$ and $1 - \underline{\theta}$, respectively. These are also the respective shares of real-estate firms and mismatched owners.

In this equilibrium candidate, the gain from deviating to (permanently) buying first for a mismatched owner is $D(\underline{\theta}) = V^{B1} - V^{S1}$, which is given by

$$D\left(\underline{\theta}\right) = \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\left(\underline{\theta}\frac{u_n - u_0}{r+\frac{1}{2}q\left(\underline{\theta}\right)} - \frac{u - u_2}{r+\frac{1}{2}\mu\left(\underline{\theta}\right)}\right). \tag{E.11}$$

Given our assumptions on the utility flows, $D(\underline{\theta} = 1) = 0$ for $\kappa = 0$. Decreasing $\underline{\theta}$ (increasing κ) decreases D, and hence, makes it more attractive to sell first.

E.3 House Price Expectations

So far, we assumed that mismatched owners do not expect house prices to change. In this section we examine the implications of expected changes in prices for the behavior of mismatched owners. To focus on the effect of expected capital gains or losses rather than the discounting effect explained above, we study the benchmark case in which $R = \rho p$. To simplify the exposition, we also assume that $u_0 = u_2 = c$.

Consider a simple, exogenous process for the price p. With rate λ, the house price p changes to a permanent new level p_N.[29] We compare the utility from buying first relative to selling first for a mismatched owner before the price change. If the price change occurs between the two transactions, the mismatched owner will make a capital gain of $p_N - p$ if he buys first and a capital loss of the same amount if he sells first. If the shock happens before the first or after the second transaction, it will not influence the decision to buy first or sell first.

The price risk associated with the transaction sequence decision creates asymmetry in the payoff from buying first or selling first. Specifically, at $\theta = 1$, the difference between the two value functions

[29] Since we assume that $p = \frac{R}{\rho}$, one can think of a permanent change in the equilibrium rental rate to R_N, which leads to a house price change to $p_N = \frac{R_N}{\rho}$. Also, for this exercise, we implicitly assume that $\gamma \to 0$, so that V is independent of p.

$D(\theta) = V^{B1} - V^{S1}$ is

$$D(1) = \frac{\mu(1)}{(\rho + q(1) + \lambda)(\rho + \mu(1) + \lambda)} 2\lambda (p_N - p). \tag{E.12}$$

An expected price decrease leads to a higher value of V^{S1} relative to V^{B1}, even if matching rates for a buyer and a seller are the same. Consequently, $V^{S1} > V^{B1}$ even for some values of $\theta > 1$. If the expected price decrease (increase) is sufficiently large, so that $D(\bar{\theta}) < 0$ ($D(\underline{\theta}) > 0$), then selling (buying) first will dominate buying (selling) first for any value of θ that is consistent with equilibrium.

Proposition E.1. *Consider the economy with $u_0 = u_2$, $R = \rho p$, and an exogenous and permanent house price change to p_N at rate λ. Then for every $\lambda > 0$ and every steady state market tightness, a mismatched owner prefers to "sell first" for sufficiently low values of p_N. Analogously, a mismatched owner prefers to "buy first" for sufficiently high values of p^N.*

Proof. Consider the difference between the two value functions, $D(\theta) = V^{B1} - V^{S1}$ assuming that the mismatched owner transacts in both cases, and denote the value of a mismatched owner after the price change by $\bar{V}_N$:

$$\begin{aligned} D(\theta) = &\frac{\mu(\theta)\left(1 - \frac{1}{\theta}\right)\left(u - \chi - c + \lambda\left(\bar{V}_N - v^{B0}\right)\right)}{(\rho + q(\theta) + \lambda)(\rho + \mu(\theta) + \lambda)} \\ &+ \frac{\frac{\lambda\mu(\theta)\left(1-\frac{1}{\theta}\right)q(\theta)}{(r+\mu(\theta))(r+q(\theta))}[\rho V - c] + \mu(\theta)\left(1 + \frac{1}{\theta}\right)\lambda(p_N - p)}{(\rho + q(\theta) + \lambda)(\rho + \mu(\theta) + \lambda)}. \end{aligned} \tag{E.13}$$

The proof of Proposition 1 implies that $\bar{\theta}$ is the highest possible steady state market tightness, so consider the case of $1 < \theta \leq \bar{\theta}$. In this case, $\bar{V}_N = V_N^{B1}$, where V_N^{B1} denotes the value of buying first after the price change, this difference simplifies further to

$$D(\theta) = \frac{\mu(\theta)\left[\left(1 - \frac{1}{\theta}\right)\left(1 + \frac{\lambda}{\rho + q(\theta)}\right)(u - \chi - c) + \left(1 + \frac{1}{\theta}\right)\lambda(p_N - p)\right]}{(\rho + q(\theta) + \lambda)(\rho + \mu(\theta) + \lambda)}. \tag{E.14}$$

Suppose that $p_N < p$ and define θ_{B1}^{PR} as the solution to

$$\frac{\theta_{B1}^{PR}-1}{\theta_{B1}^{PR}+1}\left(1+\frac{\lambda}{\rho+q\left(\theta_{B1}^{PR}\right)}\right)=\frac{\lambda\left(p-p_N\right)}{\left(u-\chi-c\right)}. \tag{E.15}$$

Therefore, θ_{B1}^{PR} is the value of θ that leaves a mismatched owner indifferent between buying first and selling first he anticipates a price change of $p_N - p$ and a market tightness of $\theta > 1$ after the price change. Note that θ_{B1}^{PR} is increasing in $p - p_N$ if $\theta_{B1}^{PR} \geq 1$. Therefore, a sufficient condition for mismatched owners to prefer to sell first, given $1 < \theta \leq \bar{\theta}$, is that $\theta_{B1}^{PR} > \bar{\theta}$.

Similarly, the proof of Proposition 1 implies that $\underline{\theta}$ is the lowest possible steady state market tightness, so consider the case of $\underline{\theta} \leq \theta < 1$. In this case, $\bar{V}_N = V_N^{S1}$, where V_N^{S1} denotes the value of selling first after the price change. In that case the difference in value functions can be written as

$$D\left(\theta\right)=\frac{\mu\left(\theta\right)\left[\left(1-\frac{1}{\theta}\right)\left(1+\frac{\lambda}{\rho+\mu(\theta)}\right)\left(u-\chi-c\right)+\left(1+\frac{1}{\theta}\right)\lambda\left(p_N-p\right)\right]}{\left(\rho+q\left(\theta\right)+\lambda\right)\left(\rho+\mu\left(\theta\right)+\lambda\right)}. \tag{E.16}$$

Suppose that $p_N > p$ and define θ_{S1}^{PR} as the solution to

$$\frac{\theta_{S1}^{PR}-1}{\theta_{S1}^{PR}+1}\left(1+\frac{\lambda}{\rho+\mu\left(\theta_{S1}^{PR}\right)}\right)=\frac{\lambda\left(p-p_N\right)}{\left(u-\chi-c\right)}. \tag{E.17}$$

Similarly, to the case of θ_{B1}^{PR}, θ_{S1}^{PR} is increasing in $p - p_N$ if $\theta_{S1}^{PR} \leq 1$. Then, a sufficient condition for mismatched owner to prefer to buy first, given $\underline{\theta} \leq \theta < 1$ is that $\theta_{S1}^{PR} < \underline{\theta}$. □

In the next section we show that such house price expectations can exert a destabilizing force on the housing market when prices move with market tightness, and study dynamic equilibria that feature switches in the transaction sequence decision.

E.4 Equilibrium switches

Consider the limit economy introduced in Section 4.1.2, where $g \to 0$ and $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$, $\bar{\theta} = 1 + \kappa$, and $\underline{\theta} = \frac{1}{1+\kappa} = \frac{1}{\bar{\theta}}$. Suppose that the economy starts in a "Buy first" equilibrium. In that case

$$\theta = \bar{\theta} = \frac{\bar{B}}{\bar{S}} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n}, \tag{E.18}$$

where $\bar{B}$ and $\bar{S}$ denote the stocks of buyers and sellers in the "Buy first" equilibrium. Suppose that the whole stock of mismatched owners, B_1, decide to sell first rather than buy first, and so, moves to the seller side of the market. In that case, the new market tightness becomes

$$\theta' = \frac{B'}{S'} = \frac{B_n}{B_n + B_1} = \underline{\theta},$$

where B' and S' denote the stocks of buyers and sellers immediately after the switch. Hence, the tightness jumps directly to its new steady state value with no dynamic adjustment in θ.

We can use this property of the limit economy to construct (approximate) dynamic equilibria, in which prices and rents move with tightness and in tandem according to $R = \rho p$. Suppose that $X(t) \in \{0, 1\}$ follows a two-state Markov chain. $X(t)$ starts in $X(t) = 0$ and with Poisson rate λ transitions permanently to $X(t) = 1$. The realization of $X(t)$ plays the role of a sunspot variable. The price in state $X(t) = 1$ is given by a smooth and increasing function $p_1 = f(\theta_1)$. The price in state 0 is implicitly given by a smooth function $p_0 = f(\theta_0, \lambda(p_1 - p_0))$, increasing in both arguments, and with $f(\theta, 0) \equiv f(\theta)$. As in Section 5.1, we take these relationships as exogenous and reduced-form to illustrate the equilibrium consequences of the interaction of housing prices and market liquidity conditions with the transaction decisions of mismatched owners.

We consider a regime-switching equilibrium in which the economy starts out in a "Buy first" regime ($X(t) = 0$), in which 1) mismatched owners prefer to buy first and the market tightness is $\theta_0 = \bar{\theta}$, and 2) agents expect that with rate λ, the economy permanently switches to a "Sell first" regime with market tightness $\theta_1 = \underline{\theta}$. In that second regime, 1) mismatched owners strictly prefer to sell first, and 2) agents expect that the economy will remain in the "Sell first" regime forever. As $\lambda \to 0$, the payoffs from buying first and selling first converge to the payoffs without regime switching. Hence, in the limit, buying first in state 0 is an equilibrium strategy if $\bar{\theta} > \tilde{\theta}$, while selling first is an equilibrium strategy in state 1 if $\underline{\theta} < \tilde{\theta}$, where $\tilde{\theta}$ is defined by Proposition 1. The following proposition therefore shows that self-fulfilling fluctuations in prices and tightness can exist if $\underline{\theta} < \tilde{\theta} < \bar{\theta}$ and agents don't expect them to happen too often.

Proposition E.2. *Consider the limit economy with $g \to 0$, $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$, and the sunspot process described above. Suppose further that $R = \rho p$ and that $\underline{\theta} < \tilde{\theta} < \bar{\theta}$. Then there is a $\bar{\lambda}$, such that for $\lambda < \bar{\lambda}$, there exists a regime-switching equilibrium characterized by two regimes $x \in \{0, 1\}$. In the first regime, $\theta_0 = \bar{\theta}$ and mismatched owners buy first. In the second regime, tightness is $\theta_1 = \underline{\theta}$, mismatched owners sell first, and $p_1 < p_0$. The economy starts in regime 0 and transitions to regime 1 with rate λ.*

Proof. Consider the first regime in which tightness $\theta_0 = \bar{\theta}$. The value function of a mismatched buyer (who transacts) in the first regime is given by

$$V_0^{B1} = \frac{u - \chi}{\rho + q\left(\bar{\theta}\right) + \lambda} + \frac{q\left(\bar{\theta}\right)}{\rho + q\left(\bar{\theta}\right) + \lambda}\left(V_0^{S2} - p_0\right) + \frac{\lambda}{\rho + q\left(\bar{\theta}\right) + \lambda} V^{S1},$$

where

$$V_0^{S2} = v^{S2}\left(\bar{\theta}\right) + \frac{\lambda}{\rho + \mu\left(\bar{\theta}\right) + \lambda}\left(v^{S2}\left(\underline{\theta}\right) - v^{S2}\left(\bar{\theta}\right) + p_1 - p_0\right) + p_0,$$

with

$$v^{S2}(\theta) = \frac{c}{\rho + \mu(\theta)} + \frac{\mu(\theta)}{\rho + \mu(\theta)} V,$$

where V^{S1} is given in (6) with (7) substituted in, which arises since in the second regime a mismatched owner sells first. For the value of selling first we have

$$V_0^{S1}(\bar{\theta}) = \frac{u - \chi}{\rho + \mu(\bar{\theta}) + \lambda} + \frac{\mu(\bar{\theta})}{\rho + \mu(\bar{\theta}) + \lambda}\left(V_0^{B0} + p_0\right) + \frac{\lambda}{\rho + \mu(\bar{\theta}) + \lambda} V^{S1},$$

where

$$V_0^{B0} = v^{B0}(\bar{\theta}) + \frac{\lambda}{\rho + q(\bar{\theta}) + \lambda}\left(v^{B0}(\underline{\theta}) - v^{B0}(\bar{\theta}) + p_0 - p_1\right) - p_0,$$

with

$$v^{B0}(\theta) = \frac{c}{\rho + q(\theta)} + \frac{q(\theta)}{\rho + q(\theta)} V.$$

Consider the difference $D_0(\bar{\theta}) = V_0^{B1}(\bar{\theta}) - V_0^{S1}(\bar{\theta})$, and note that

$$\lim_{\lambda \to 0} D_0(\bar{\theta}) = \frac{\mu(\theta) - q(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))}(u - \chi - c) > 0.$$

Since $V_0^{B1}(\bar{\theta})$ and $V_0^{S1}(\bar{\theta})$ are continuous in λ, it follows that $D_0(\bar{\theta})$ is continuous in λ, as well, so that $D_0(\bar{\theta}) > 0$ will also be the case for λ sufficiently close to 0. Therefore, there exists a $\bar{\lambda}$ such that for $\lambda < \bar{\lambda}$, $V_0^{B1}(\bar{\theta}) > V_0^{S1}(\bar{\theta})$ and mismatched owners prefer to buy first. Also, by Lemma 2, $\bar{\theta}$ is consistent with the behavior of mismatched owners and given by $\bar{\theta} = (B_n + B_1)/B_n$.

Upon $X(t) = 1$, the whole stock of mismatched owners, B_1, sells first and, so, moves to the seller side of the market. In that case, the new market tightness becomes $B_n/(B_n + B_1) = 1/\bar{\theta} = 1/(1 + \kappa) = \underline{\theta}$. Since Lemma 2 shows that $\underline{\theta}$ obtains in steady state when all mismatched owners sell first, tightness jumps directly to its value θ_1 without any dynamic adjustment in θ. In that regime agents' payoffs are as in Section 3.2, and therefore, by Lemma 1, mismatched owners prefer to sell first.

Finally, since $\bar{\theta} > \underline{\theta}$, it follows that $p_0 > p_1$. To see this, suppose $p_0 \leq p_1$. Then $p_0 = f(\bar{\theta}, \lambda(p_1 - p_0)) \geq f(\bar{\theta})$. But then $p_0 \geq f(\bar{\theta}) > f(\underline{\theta}) = p_1$, which is a contradiction. □

As a result, there exist dynamic equilibria in which prices and tightness move together. The expectation that prices will fall, induces mismatched owners to sell first, which leads to a fall in market tightness and thus prices. The reason that λ cannot be too high is that if agents expect the change in regimes to occur sufficiently soon, then from Proposition E.1, it can be optimal for mismatched owners to sell first in the first regime despite the high market tightness, speculating on regimes changing in between their two transactions. This, however, is inconsistent with equilibrium. Therefore, a regime-switching equilibrium exists only for (sufficiently) low values of λ.

Upon the switch, average seller time-on-market for sellers, $\frac{1}{\mu(\theta)}$, increases. Second, consider the ratio of the stock of sellers before and after the switch. That ratio is exactly $\underline{\theta}$, which is less than 1. Therefore, there is an increase in the for-sale stock, since some of the previous buyers become sellers. Finally, transaction volume may also fall depending on the shape of the matching function. Specifically, consider a Cobb-Douglas matching function, $m(B,S) = \mu_0 B^\alpha S^{1-\alpha}$, for $0 < \alpha < 1$. The ratio of transaction volumes before and after the switch is

$$\frac{\mu(\bar{\theta})}{q(\underline{\theta})} = \frac{\mu_0 \bar{\theta}^\alpha}{\mu_0 \underline{\theta}^{\alpha-1}} = (1+\kappa)^{2\alpha-1}.$$

Hence, transaction volume falls after the switch if $\alpha > \frac{1}{2}$ and increases if $\alpha < \frac{1}{2}$. The reason is that for $\alpha > \frac{1}{2}$ buyers are more important than sellers in generating transactions. When mismatched owners switch from buying first to selling first, this leads to a reduction in the number of buyers and an increase in the number of sellers, and hence, to a fall in the transaction rate. As discussed in Section 4.3, Genesove and Han (2012) estimate a value of $\alpha = 0.84$. At that value, transaction volume would drop after the switch.

Although transaction volume falls immediately after the switch, it fully recovers over time. To see this, consider the ratio of transaction volumes in the buy first and sell first steady state equilibria in the limit economy. Denoting the total mass of buyers and sellers in the buy first and sell first steady state equilibria by $\bar{B}$ and $\underline{S}$, respectively, we can write that ratio as

$$\frac{q\left(\bar{\theta}\right)\bar{B}}{\mu\left(\underline{\theta}\right)\underline{S}} = \frac{q\left(\bar{\theta}\right)\left(g+\gamma\bar{O}\right)/\left(g+q\left(\bar{\theta}\right)\right)}{\mu\left(\underline{\theta}\right)\left(g+\gamma\underline{O}\right)/\left(g+\mu\left(\underline{\theta}\right)\right)}$$
$$= \frac{q\left(\bar{\theta}\right)}{\left(g+q\left(\bar{\theta}\right)\right)}\frac{\left(g+\mu\left(\underline{\theta}\right)\right)}{\mu\left(\underline{\theta}\right)}\frac{\left(1+\kappa\bar{O}\right)}{\left(1+\kappa\underline{O}\right)},$$

where $\bar{O}$ and $\underline{O}$ denote the stock of matched owners in the buy first and sell first steady state equilbiria, respectively. Next, note that

$$\lim_{\gamma,g\to 0,\gamma/g=\kappa}\frac{q\left(\bar{\theta}\right)\bar{B}}{\mu\left(\underline{\theta}\right)\underline{S}} = \lim_{\gamma,g\to 0,\gamma/g=\kappa}\frac{q\left(\bar{\theta}\right)}{\left(g+q\left(\bar{\theta}\right)\right)}\frac{\left(g+\mu\left(\underline{\theta}\right)\right)}{\mu\left(\underline{\theta}\right)}\frac{\left(1+\kappa\bar{O}\right)}{\left(1+\kappa\underline{O}\right)} = 1.$$

Therefore, in an economy with small flows, transaction volumes in the two steady state equilibria are (approximately) the same. Consequently, even if transaction volume falls upon a switch in mismatched owners' behavior, it eventually recovers (almost) fully. This property of the small flows economy is consistent with the transitional dynamics in our numerical example in Section 4.3.

E.5 A model with competitive search

In competitive search equilibrium, sellers post prices, and buyers direct their search towards the sellers they find most attractive, taking into account that better terms of trade mean a longer expected waiting time before trade occurs. The market splits up in submarkets, and the different agents choose which submarket to enter. As shown in Garibaldi et al. (2016), the most patient buyers (who are most willing to trade off a short waiting time for a low price) will search for the most impatient sellers (who are most willing to trade off a low price for a short waiting time). Analogously, the least patient buyers search for the most patient sellers.

We first define a competitive search equilibrium for our economy. Let $(\mathcal{P}, \Theta)$ denote the active market segments in the economy, i.e. segments that attract a positive measure of buyers and sellers. The following equations describe the steady state value functions of agents. For new entrants we have:

$$\rho V^{Bn} = u_n - R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q\left(\theta\right)\left(-p + V - V^{Bn}\right)\right\}. \tag{E.19}$$

Similarly, for a real estate firm, we have

$$\rho V^{A} = R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ \mu\left(\theta\right)\left(p - V^{A}\right)\right\}. \tag{E.20}$$

For mismatched owners that buy first, we have

$$\rho V^{B1} = u - \chi + \max\left\{0, \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q\left(\theta\right)\left(-p + V^{S2} - V^{B1}\right)\right\}\right\}, \tag{E.21}$$

where the value function takes into account the possibility that a mismatched buyer may be better off not searching. Similarly, if the mismatched owner sells first, we have

$$\rho V^{S1} = u - \chi + \max\left\{0, \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ \mu\left(\theta\right)\left(p + V^{B0} - V^{S1}\right)\right\}\right\}. \tag{E.22}$$

A double owner solves

$$\rho V^{S2} = u_2 + R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ \mu\left(\theta\right)\left(p + V - V^{S2}\right)\right\}, \tag{E.23}$$

while a forced renter solves

$$\rho V^{B0} = u_0 - R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q(\theta)\left(-p + V - V^{B0}\right)\right\}. \tag{E.24}$$

Finally, for a matched owner we have

$$\rho V = u + \gamma \left(\max\left\{V^{B1}, V^{S1}\right\} - V\right). \tag{E.25}$$

Next, we describe the steady state stock-flow conditions. Let

$$\left(p^{Bn}, \theta^{Bn}\right) \in \left(\mathcal{P}^{Bn}, \Theta^{Bn}\right) \equiv \arg\max_{(p,\theta)} \left\{ q(\theta)\left(-p + V - V^{Bn}\right)\right\} \subset (\mathcal{P}, \Theta) \tag{E.26}$$

denote a market segment that maximizes the value of searching for a new entrant. We define $\left(p^j, \theta^j\right)$ and $\left(\mathcal{P}^j, \Theta^j\right)$ analogously for an agent type $j \in \{A, B1, S1, B0, S2\}$. For agents $j \in \{B1, S1\}$, we adopt the convention that $\Theta^j = \emptyset$ if they choose not to search.

We have the following stock-flow conditions

$$g = \left(\sum_{\theta\in\Theta} x^{Bn}(\theta)\, q(\theta) + g\right) B_n, \tag{E.27}$$

$$\sum_{\theta\in\Theta} x^{S1}(\theta)\, \mu(\theta)\, S_1 = \left(\sum_{\theta\in\Theta} x^{B0}(\theta)\, q(\theta) + g\right) B_0, \tag{E.28}$$

$$\gamma x_b O = \left(\sum_{\theta\in\Theta} x^{B1}(\theta)\, q(\theta) + g\right) B_1, \tag{E.29}$$

$$\gamma x_s O = \left(\sum_{\theta\in\Theta} x^{S1}(\theta)\, \mu(\theta) + g\right) S_1, \tag{E.30}$$

$$\sum_{\theta\in\Theta} x^{B1}(\theta)\, q(\theta)\, B_1 = \left(\sum_{\theta\in\Theta} x^{S2}(\theta)\, \mu(\theta) + g\right) S_2, \tag{E.31}$$

$$g\left(O+B_1+S_1+2S_2\right)=\sum_{\theta\in\Theta}x^A\left(\theta\right)\mu\left(\theta\right)A, \tag{E.32}$$

$$x_b+x_s=1, \tag{E.33}$$

with

$$\sum_{\theta\in\Theta}x^j\left(\theta\right)=1\;\forall j\in\left\{Bn,A,B0,S2\right\}, \tag{E.34}$$

where $x^j\left(\theta\right)=0$ if $\theta\notin\Theta^j$ and, if a mismatched buyer/seller chooses to search,

$$\sum_{\theta\in\Theta}x^j\left(\theta\right)=1 \text{ for } j\in\left\{B1,S1\right\}, \tag{E.35}$$

with $x^j\left(\theta\right)=0$ if $\theta\notin\Theta^j$. In the above expressions $\mathbf{x}^j\left(\theta\right)\geq 0$ is the vector of mixing probabilities over segments in Θ for an agent $j\in\{Bn,A,B1,S1,B0,S2\}$. Market tightnesses in each segment are given by

$$\theta=\frac{x^{Bn}\left(\theta\right)B_n+x^{B1}\left(\theta\right)B_1+x^{B0}\left(\theta\right)B_0}{x^A\left(\theta\right)A+x^{S2}\left(\theta\right)S_2+x^{S1}\left(\theta\right)S_1}, \tag{E.36}$$

where $x^j\left(\theta\right)=0$ if $\theta\notin\Theta^j$.

Finally, we have the population constancy and housing ownership conditions

$$B_n+B_0+B_1+S_1+S_2+O=1, \tag{E.37}$$

and

$$O+B_1+S_1+A+2S_2=1. \tag{E.38}$$

Following Moen (1997), we additionally require that the active market segments $(\mathcal{P},\Theta)$ are such that the equilibrium allocation is a "no-surplus allocation". Formally, we make the following requirement.

No-surplus allocation Let $\mathcal{B} \subset \{Bn, B1, B0\}$ and $\mathcal{S} \subset \{A, S1, S2\}$ denote the sets of *active* buyers and sellers in a steady state equilibrium, that is agents that have a strictly positive measure in steady state. Given the set of active segments $(\mathcal{P}, \Theta)$ and agents' steady state value functions $\left\{V^{Bn}, V^{B1}, V^{B0}, V^{A}, V^{S1}, V^{S2}\right\}$, there exists no pair $(p, \theta) \notin (\mathcal{P}, \Theta)$, such that $V^{i}(p, \theta) > V^{i}$, for some $i \in \{Bn, B1, B0\}$, and $V^{j}(p, \theta) \geq V^{j}$ for some $j \in \mathcal{S}$, or $V^{i}(p, \theta) > V^{i}$, for some $i \in \{A, S1, S2\}$, and $V^{j}(p, \theta) \geq V^{j}$ for some $j \in \mathcal{B}$, where $V^{i}(p, \theta)$ denotes the steady state value function of an agent that trades in segment (p, θ), for $i \in \{Bn, B1, B0, A, S1, S2\}$.

Informally, the no-surplus allocation condition requires that in equilibrium there are no agents that would be strictly better off from deviating and opening a new market segment that would be at least as attractive for some active agents (buyers or sellers) compared to their equilibrium values.

We can now define a symmetric steady state competitive search equilibrium of this economy as follows

Definition E.1. A symmetric steady state competitive search equilibrium of this economy consists of a set of active market segments $(\mathcal{P}, \Theta)$, steady state value functions V^{Bn}, V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^{A}, fractions of mismatched owners that choose to buy first and sell first, x_b, and x_s, aggregate stock variables, B_n, B_0, B_1, S_1, S_2, O, and A, distributions of agent types over active market segments $\left\{\mathbf{x}^{j}\right\}_{j \in \{Bn, A, B1, S1, B0, S2\}}$, and set of active buyers and sellers, $\mathcal{B}$ and $\mathcal{S}$, such that

1. The value functions satisfy equations (E.19) - (E.25) and the mixing distributions $\left\{\mathbf{x}^{j}\right\}_{j}$ are consistent with the agents' optimization problems.

2. Mismatched owners choose to buy first or sell first, to maximize $\bar{V} = \max\left\{V^{B1}, V^{S1}\right\}$ and

the fractions x_b, and x_s reflect that choice, i.e.

$$x_b = \int_i I\{x_i = b\}\, di,$$

where $i \in [0,1]$ indexes the i-th mismatched owner, and similarly for x_s;

3. The aggregate stock variables B_n, B_0, B_1, S_1, S_2, O, and A, solve (E.27)-(E.32) and (E.37)-(E.38) given Θ, $\{\mathbf{x}^j\}_j$ and mismatched owners' optimal decisions, reflected in x_b and x_s.

4. Every $\theta \in \Theta$ satisfies equation (E.36) given B_n, B_0, B_1, S_1, S_2, O, A, and $\{\mathbf{x}^j\}_j$;

5. The set of active buyers and sellers, $\mathcal{B}$ and $\mathcal{S}$, is consistent with mismatched owners' optimal decisions;

6. $(\mathcal{P}, \Theta)$ and agents' steady state value functions satisfy the no-surplus allocation condition.

Next, we characterize competitive search equilibria when the cost of being mismatched, χ, is low, and so is the flow utility of being a double owner, u_2.[30] Also, as in Section 5.2 we assume that new entrants enjoy a strictly higher flow utility than forced renters: $u_n > u_0$.

In the "Buy first" equilibrium, the buyers are mismatched owners and new entrants, while the sellers are real estate firms and double owners. Figure E.1a shows the market constellations in this equilibrium, where blue indicates sellers and red buyers. The lightly-shaded rectangles and dashed lines indicate a deviating agent. The most patient buyer is the mismatched owner, while the most impatient sellers are the double owners. Hence, these agents always transact. The least patient buyers are the new entrants, while the most patient sellers are the real estate firms. Hence, submarkets for real estate firms and new entrants will always exist. In addition, a market for new entrants and double owners will also open.

[30] We have no reason to believe that the condition on χ is necessary to obtain multiple equilibria. However, without it the model becomes less tractable, as it is not clear from the outset what market constellations will then be realized.

Note that the match surplus Σ_{B1S2} between a mismatched buyer and a double owner is given by

$$\Sigma_{B1S2} = V + V^{S2} - V^{B1} - V^{S2} = V - V^{B1} > 0,$$

so that there is trade in this market. Now, consider a mismatched owner that deviates and sells first. For a small χ, this seller will be more patient than both the real estate firms and the double owners, and will, therefore, transact with the most impatient buyers among the non-deviating buyers, namely, the new entrants. He will then become a new buyer type – a forced renter – that is even more impatient than the new entrant because $u_n > u_0$. A forced renter will , therefore, transact with the most patient sellers, which are the real estate firms. Given that a forced renter is more impatient than a new entrant, real estate firms are willing to open a new submarket for the deviating agent. In particular, the value from being a forced renter, V^{B0}, maximizes his gain from search given the value of the real estate firm.

The match surplus between the deviating mismatched owner and the new entrant, Σ_{BnS1}, can be written as

$$\Sigma_{BnS1} = V + V^{B0} - V^{Bn} - V^{S1}.$$

Note that also $\lim_{\chi \to 0} V^{S1} = \lim_{\chi \to 0} V$. Moreover, for $u_0 < u_n$, V^{B0} is strictly lower than V^{Bn}, also in the limit as $\chi \to 0$. As a result, the match surplus Σ_{BnS1} is negative for small values of χ, so that the mismatched owner cannot gain by deviating and the "Buy first" equilibrium exists.

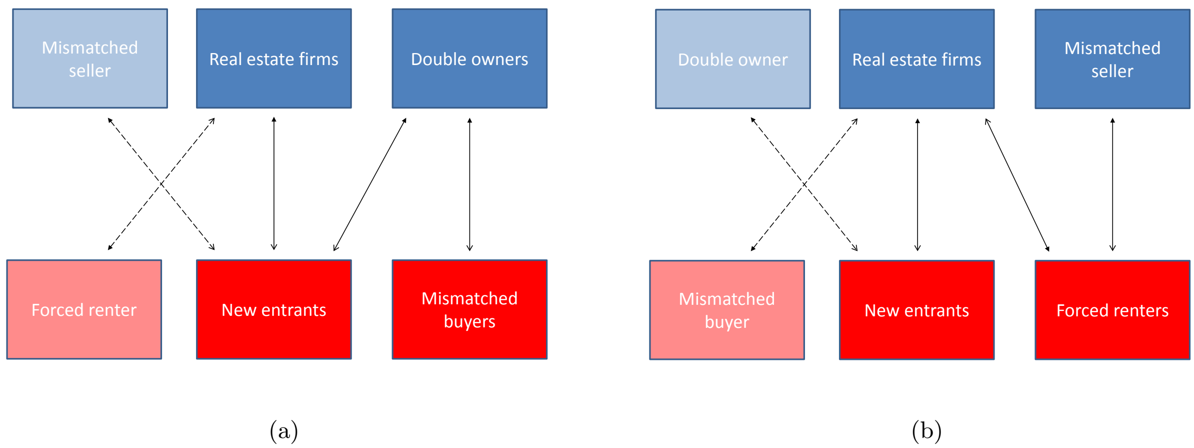

Figure E.1: Equilibrium market segments (solid colors) and deviators (weaker colors) for "Buy first" (a) and "Sell first" (b) competitive search equilibria.

In the "Sell first" equilibrium, the sellers are mismatched owners and real estate firms, while the buyers are new entrants and forced renters. The most patient buyers are the new entrants, and the most patient sellers are the mismatched owners. The active submarkets will be between new entrants and real estate firms, mismatched buyers and forced renters, and forced renters and real estate firms. The markets (together with a deviating agent) are illustrated in Figure E.1b.

The asset values in the market segment for real estate firms and new entrants are as in the "Buy first" equilibrium, so this market is active. For a real estate firm, the surplus of a transaction with a (more impatient) forced renter is even larger than with a new entrant, so that there are benefits to trading in that market as well. Hence, a forced renter obtains the same value V^{B0} as the (deviating) forced renter in the "Buy first" equilibrium. The match surplus between a forced renter and a mismatched seller, Σ_{B0S1}, is given by

$$\Sigma_{B0S1} = V + V^{B0} - V^{S1} - V^{B0} = V - V^{S1} > 0.$$

Now, consider a mismatched agent that deviates and buys first. This buyer will be more patient than both new entrants and forced renters, and will thus transact with the real estate firm – the most impatient seller. He will then become a double owner, thus, becoming more impatient than the real estate firm, and will therefore transact with the new entrants. A new submarket will open up, where the asset value V^{S2} is the same as for a double owner in the "Buy first" equilibrium. However, for the same reasons as in the "Buy first" equilibrium, the match surplus between the mismatched buyer and the real estate firm, Σ_{B1A}, is negative for low values of u_2. It follows that the deviation is unprofitable. We conclude that the model still exhibits multiple equilibria, as stated in the following proposition.

Proposition E.3. *Consider the economy with competitive search, and suppose that χ and u_2 are small and that $u_0 < u_n$. Then the economy exhibits multiple equilibria. In one equilibrium, all mismatched owners buy first. In another equilibrium, all mismatched owners sell first.*

Proof. Consider first the "Buy first" equilibrium as described above. In this "Buy first" equilibrium there are three active market segments characterized by prices $p_1^{B1} > p_2^{B1} > p_3^{B1}$ and market tightnesses $\theta_1^{B1} < \theta_2^{B1} < \theta_3^{B1}$. New entrants trade with real estate agents in market 1 and with double owners in market 2, while the latter also trade with mismatched buyers in market 3. Let x^{Bn} denote the probability with which a new entrant visits segment $\left(p_1^{B1}, \theta_1^{B1}\right)$, and x^{S2} the probability with which a double owner visits segment $\left(p_2^{B1}, \theta_2^{B1}\right)$. The stock-flow conditions for this equilibrium are

$$B_n = \frac{g}{x^{Bn} q\left(\theta_1^{B1}\right) + \left(1 - x^{Bn}\right) q\left(\theta_2^{B1}\right) + g}, \tag{E.39}$$

$$A = \frac{g}{\mu\left(\theta_1^{B1}\right) + g}, \tag{E.40}$$

$$B_1 = \frac{\gamma O}{q\left(\theta_3^{B1}\right) + g}, \tag{E.41}$$

$$S_2 = \frac{q\left(\theta_3^{B1}\right) B_1}{x^{S2}\mu\left(\theta_2^{B1}\right) + \left(1 - x^{S2}\right)\mu\left(\theta_3^{B1}\right) + g}, \tag{E.42}$$

$$B_n + B_1 + S_2 + O = 1, \tag{E.43}$$

and

$$B_n = A + S_2. \tag{E.44}$$

The market tightnesses in each active segment satisfy

$$\theta_1^{B1} = \frac{x^{Bn} B_n}{A}, \tag{E.45}$$

$$\theta_2^{B1} = \frac{\left(1 - x^{Bn}\right) B_n}{x^{S2} S_2}, \tag{E.46}$$

and

$$\theta_3^{B1} = \frac{B_1}{\left(1 - x^{S2}\right) S_2}. \tag{E.47}$$

Observe that (E.39), (E.40), (E.44), and (E.45) imply that $x^{Bn} < 1$, as otherwise, (E.39), (E.40) and (E.45) give

$$\theta_1^{B1} = \frac{B_n}{A} = \frac{\mu\left(\theta_1^{B1}\right) + g}{q\left(\theta_1^{B1}\right) + g}, \tag{E.48}$$

which has a unique solution at $\theta_1^{B1} = 1$. However, this is inconsistent with (E.44).

Let Σ_{ij}, for $i \in \{B_n, B_0, B_1\}$ and $j \in \{A, S_1, S_2\}$ denote the match surplus from trading between a buyer i and seller j. The no-surplus allocation condition determines the equilibrium prices in each segment as a function of the steady state values of agents. Define

$$\begin{aligned}\bar{V}^{Bn} &= q\left(\theta_1^{B1}\right)\left(-p_1^{B1} + V - V^{Bn}\right) \\ &= q\left(\theta_2^{B1}\right)\left(-p_2^{B1} + V - V^{Bn}\right),\end{aligned}$$

$$\bar{V}^{A} = \mu\left(\theta_1^{B1}\right)\left(p_1^{B1} - V^{A}\right),$$

$$\bar{V}^{B1} = q\left(\theta_3^{B1}\right)\left(-p_3^{B1} + V^{S2} - V^{B1}\right),$$

and

$$\begin{aligned}\bar{V}^{S2} &= \mu\left(\theta_2^{B1}\right)\left(p_2^{B1} + V - V^{S2}\right)\\ &= \mu\left(\theta_3^{B1}\right)\left(p_3^{B1} + V - V^{S2}\right),\end{aligned}$$

as the maximized value of searching for each trader. The no-surplus allocation condition implies that

$$\left(p_1^{B1}, \theta_1^{B1}\right) = \arg\max_{p,\theta} \mu\left(\theta\right)\left(p - V^{A}\right),$$

$$\text{s.t. } q\left(\theta\right)\left(-p + V - V^{Bn}\right) \geq \bar{V}^{Bn}.$$

Denote the elasticity of the matching function with respect to buyers by α (which may depend on θ). Solving for p_1^{B1} and θ_1^{B1} gives the well-known Hosios rule (Hosios (1990)),

$$p_1^{B1} - V^{A} = \left(1 - \alpha\right)\Sigma_{BnA},$$

or equivalently,

$$p_1^{B1} = \left(1 - \alpha\right)\left(V - V^{Bn}\right) + \alpha V^{A}.$$

Therefore,

$$\bar{V}^{Bn} = \alpha q\left(\theta_1^{B1}\right)\Sigma_{BnA} = \alpha q\left(\theta_2^{B1}\right)\Sigma_{BnS2},$$

or

$$q\left(\theta_1^{B1}\right)\Sigma_{BnA} = q\left(\theta_2^{B1}\right)\Sigma_{BnS2}. \tag{E.49}$$

We have similar surplus sharing rules between the other trading pairs, which determine p_2^{B1} and p_3^{B1}. There is one more indifference condition for a double owner that relates θ_2^{B1} and θ_3^{B1}. Specifically,

$$\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} = \mu\left(\theta_3^{B1}\right)\Sigma_{B1S2}. \tag{E.50}$$

These surplus sharing rules imply that the value functions of active agents satisfy the equations

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_2^{B1}\right)\Sigma_{BnS2}, \tag{E.51}$$

$$\rho V^{A} = R + (1-\alpha)\mu\left(\theta_1^{B1}\right)\Sigma_{BnA}, \tag{E.52}$$

$$\rho V^{B1} = u - \chi + \alpha q\left(\theta_3^{B1}\right)\Sigma_{B1S2}, \tag{E.53}$$

$$\rho V^{S2} = u_2 + R + (1-\alpha)\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2}, \tag{E.54}$$

and

$$\rho V = u + \gamma\left(V^{B1} - V\right). \tag{E.55}$$

Finally, use V^{Bn} and V^{S2} from (E.51) and (E.54) to solve for

$$\Sigma_{BnS2} = \frac{2\rho V - u_n - u_2}{\rho + \alpha q\left(\theta_2^{B1}\right) + (1-\alpha)\mu\left(\theta_2^{B1}\right)}. \tag{E.56}$$

Similarly, using V^{Bn} and V^{A} from (E.51) and (E.20), combined with indifference condition (E.49), to solve for

$$\Sigma_{BnA} = V - V^{Bn} - V^{A} = \frac{\rho V - u_n}{\rho + \alpha q\left(\theta_1^{B1}\right) + (1-\alpha)\mu\left(\theta_1^{B1}\right)}. \tag{E.57}$$

Solving for V^{B1} from equation (E.53), we get

$$V^{B1} = \frac{u - \chi}{\rho + \alpha q\left(\theta_3^{B1}\right)} + \frac{\alpha q\left(\theta_3^{B1}\right)}{\rho + \alpha q\left(\theta_3^{B1}\right)} V,$$

so

$$\Sigma_{B1S2} = V - V^{B1} = \frac{\rho V - (u - \chi)}{\rho + \alpha q\left(\theta_3^{B1}\right)}. \tag{E.58}$$

Therefore, equations (E.39)-(E.47), combined with the two indifference conditions (E.49) and (E.50), and the value function equations (E.51)-(E.55) with surpluses (E.56)-(E.58) jointly determine the equilibrium stocks of agents, market tightnesses, mixing probabilities x^{Bn} and x^{S2}, and active agent value functions in a "Buy first" equilibrium.

We now prove existence of this equilibrium when χ and u_2 are small, and u_0 is strictly smaller than u_n. Note that $\Sigma^{S2B1} = V - V^{B1} > 0$ for any u_2, but that $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{B1} = \frac{u}{\rho}$, so that $\lim_{\chi\to 0} \Sigma_{B1S2} = 0$. This in turn implies that $\lim_{\chi\to 0} \theta_3^{B1} = \infty$ and $\lim_{\chi\to 0} x^{S2} = 1$. To see this, suppose to the contrary that as $\chi \to 0$, θ_3^{B1} remains bounded and thus x^{S2} is strictly below one. Therefore, $\mu\left(\theta_3^{B1}\right)\Sigma_{B1S2} \to 0$, so indifference condition (E.50) implies that $\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} \to 0$. Given (E.56), this in turn means that $\theta_2^{B1} \to 0$ and thus $x^{Bn} \to 1$. However,

$$\lim_{\theta_2^{B1}\to 0} q\left(\theta_2^{B1}\right)\Sigma_{BnS2} = \frac{2\rho V - u_n - u_2}{\alpha},$$

which is inconsistent with $x^{Bn} \to 1$. To see this, remember from (E.48) that $\theta_1^{B1} \to 1$ as $x^{Bn} \to 1$. Because

$$\lim_{\theta_1^{B1}\to 1} q\left(\theta_1^{B1}\right)\Sigma_{BnA} = \frac{\rho V - u_n}{\frac{\rho}{q(1)} + 1} < \rho V - u_n < \frac{2\rho V - u_n - u_2}{\alpha},$$

in this case new entrants would be strictly better off participating in the second market segment. Thus, we arrive at a contradiction.

As $\theta_3^{B1} \to \infty$, $q\left(\theta_3^{B1}\right) \to 0$, and mismatched owners do not buy to become double owners: $S_2 \to 0$. Without trading partners in market 2, all new entrants visit market 1: $x^{Bn} \to 1$ and thus $\theta_1^{B1} \to \frac{B_n}{A} \to 1$. In this case, V^{Bn} from (E.51) is given by

$$\lim_{\chi \to 0} \rho V^{Bn} = u_n + \frac{\alpha q(1)}{\rho + q(1)}(u - u_n) - R, \tag{E.59}$$

which is strictly between 0 and ρV, as long as R is not too large. Similarly, using that $\mu(1) = q(1)$, V^A is given by

$$\lim_{\chi \to 0} \rho V^A = R + \frac{(1-\alpha)q(1)}{\rho + q(1)}(u - u_n), \tag{E.60}$$

which is also strictly between 0 and ρV if R is not too large. As a result,

$$\lim_{\chi \to 0} \left(V^A + V^{Bn}\right) = \frac{u_n}{\rho} + \frac{q(1)}{\rho + q(1)} \frac{u - u_n}{\rho},$$

which is strictly between 0 and V. By continuity, there exists a $\bar{\chi}_1 > 0$ such that for $\chi \in (0, \bar{\chi}_1)$, it is the case that $\Sigma_{BnA} > 0$, $x^{Bn} \in (0,1)$ and $\theta_1^{B1} \in (0,1)$, but also $\Sigma_{B1S2} > 0$.

With V^{Bn} as defined in (E.59) above, it follows that V^{S2} is uniquely determined as

$$\rho V^{S2} = \max_{p,\theta} \left\{ u_2 + R + \mu(\theta)(V + p - V^{S2}) \right\},$$

subject to $u_n - R + q(\theta)(V - p - V^{Bn}) = \rho V^{Bn}$. Note that V^{S2} goes to negative infinity for any $\chi > 0$ when u_2 does. To see this, suppose to the contrary that V^{S2} remains bounded when u_2 goes to negative infinity. Then $\mu(\theta)$ must go to infinity, and hence θ must go to infinity. But then $q(\theta)$ goes to zero, and for the new entrants to get their outside option, p must go to $-\infty$. In this case V^{S2} still goes to negative infinity, so that we arrive at a contradiction. Consequently, for any $\chi > 0$ there exists a $\bar{u}_2^{B1}$ such that for $u_2 < \bar{u}_2^{B1}$, V^{S2} is sufficiently low such that both

$\Sigma_{B1A} = V^{S2} - V^{A} - V^{B1} < 0$ and $\Sigma_{BnS2} = 2V - V^{Bn} - V^{S2} > \Sigma_{BnA} > 0$. We can then conclude that there is trade in markets 1, 2, and 3, but that real estate agents and mismatched buyers do not open a fourth market.[31]

Furthermore, it follows that for any $u_2 < \bar{u}_2^{B1}$ there exists a $\bar{\chi}_2 > 0$ such that for $\chi \in (0, \bar{\chi}_2)$, it is the case that $\Sigma_{BnS2} > \Sigma_{B1S2}$. Given this ranking and the fact that $\Sigma_{BnA} < \Sigma_{BnS2}$, the ranking of tightnesses across segments then follows from the indifference conditions (E.49) and (E.50). Having established the ranking of tightnesses, the ranking of prices across segments immediately follows from the indifference conditions as well. Specifically, (E.49) implies that

$$q\left(\theta_1^{B1}\right)\left(-p_1^{B1} + V - V^{Bn}\right) = q\left(\theta_2^{B1}\right)\left(-p_2^{B1} + V - V^{Bn}\right),$$

or

$$\frac{q\left(\theta_1^{B1}\right)}{q\left(\theta_2^{B1}\right)} = \frac{-p_2^{B1} + V - V^{Bn}}{-p_1^{B1} + V - V^{Bn}}.$$

$\theta_1^{B1} < \theta_2^{B1}$ and $q(.)$ decreasing imply that $p_1^{B1} > p_2^{B1}$. Similarly, (E.50) implies that $p_2^{B1} > p_3^{B1}$. Finally, note that $\Sigma_{BnS2} > \Sigma_{B1S2}$ implies that $V - V^{Bn} > V^{S2} - V^{B1}$, so a new entrant is more impatient than a mismatched buyer in the sense that the direct utility gain from transacting is higher for a new entrant compared to a mismatched buyer.[32]

Consider now a mismatched owner that deviates and sells first, and upon trade becomes a forced renter. We allow both a mismatched seller and a forced renter to open new market segments with active agents as counterparties. First, observe that $V^{Bn} > V^{B0}$ for any $\chi > 0$, that is, a new entrant is always better off than a forced renter. This ranking comes from the assumption that $u_0 < u_n$ and from a revealed preference argument. Specifically, suppose to the contrary that

[31] For market 2 to be active, it is sufficient for u_2 to be low enough to ensure $\Sigma_{BnS2} > 0$, even if $\Sigma_{BnS2} < \Sigma_{BnA}$. However, $u_2 < \bar{u}_2^{B1}$ ensures the ranking of tightnesses and prices proven next.

[32] This also implies that a new entrant has steeper sloped indifference curves in the $\theta - p$ space, so he is willing to trade-off a higher price for the same decrease in market tightness compared to a mismatched buyer.

$V^{B0} > V^{Bn}$. Suppose also that it is optimal for a forced renter to trade with a real estate firm (the argument for the case where the forced renter trades with a double owner is analogous). The no-surplus allocation condition again implies that the Hosios condition holds, so

$$\rho V^{B0} = u_0 - R + \alpha q\left(\tilde{\theta}\right)\left(V - V^{B0} - V^A\right),$$

where $\tilde{\theta}$ is such that a real estate firm is indifferent between trading in this new segment and trading in the segment with a tightness of θ_1^{B1} and a price of p_1^{B1}. In contrast, we have that

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^A\right).$$

Since $u_0 < u_n$ but $V^{B0} > V^{Bn}$, it follows that $q\left(\tilde{\theta}\right)\left(V - V^{B0} - V^A\right) > q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^A\right)$ and so $\tilde{\theta} < \theta_1^{B1}$. But then a new entrant is better off deviating and trading in the segment with tightness $\tilde{\theta}$, since $q\left(\tilde{\theta}\right)\left(V - V^{Bn} - V^A\right) > q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^A\right)$. Furthermore, given that $V^{B0} > V^{Bn}$, $\Sigma_{BnA} > \Sigma_{B0A}$, so a real estate firm is in fact also strictly better off trading with a new entrant in the segment with tightness $\tilde{\theta}$. However, this is not consistent with $\left(p_1^{B1}, \theta_1^{B1}\right)$ not violating the no-surplus allocation condition. Therefore, in an equilibrium where $\left(p_1^{B1}, \theta_1^{B1}\right)$ are consistent with the no-surplus allocation, we must have $q\left(\tilde{\theta}\right) < q\left(\theta_1^{B1}\right)$. However, this means that $V^{B0} < V^{Bn}$, and we arrive at a contradiction.

We conclude that $V^{B0} > V^{Bn}$ and $\Sigma_{BnA} < \Sigma_{B0A}$, so that a forced renter is the most impatient of the buyers. The forced renter will therefore trade with a real estate agent, the most patient of the sellers. A new submarket opens up, and real estate firms flow into this submarket up to the point where they are indifferent between selling to the deviator and to a new agent. Now suppose the deviating mismatched owner sells to a new entrant. Then the match surplus reads

$$\Sigma_{BnS1} = V - V^{Bn} + V^{B0} - V^{S1} \leq V - V^{Bn} + V^{B0} - \frac{u-\chi}{\rho}.$$

Given that $V^{Bn} - V^{B0}$ is bounded away from zero for any $\chi > 0$, there exists a $\bar{\chi}_3 > 0$ such that for $\chi \in (0, \bar{\chi}_3)$, it is the case that $\Sigma_{BnS1} < 0$. Note, however, that $\Sigma_{BnS1} > \Sigma_{B1S1}$ for $\chi < \bar{\chi}_2$, since, as shown above, in that case $V - V^{Bn} > V^{S2} - V^{B1}$, meaning that a new entrant is more impatient than a mismatched buyer. Therefore, for $\chi < \min\{\bar{\chi}_1, \bar{\chi}_2, \bar{\chi}_3\}$, $\Sigma_{B1S1} < \Sigma_{BnS1} < 0$ and $\Sigma_{B1S2} > 0$. In that case a mismatched owner that deviates and sells first is better off not trading. However, not trading is dominated by buying first since $V^{B1} > \frac{u-\chi}{\rho}$. Therefore, a mismatched owner is never better off deviating from buying first in a “Buy first” equilibrium.

Constructing a “Sell first” equilibrium follows similar steps. In this equilibrium there are three active market segments characterized by prices $p_1^{S1} < p_2^{S1} < p_3^{S1}$ and market tightnesses $\theta_1^{S1} > \theta_2^{S1} > \theta_3^{S1}$. Real estate agents trade with new entrants in market 1 and with forced renters in market 2, while the latter also trade with mismatched sellers in market 3. Let x^A denote the probability with which a real estate firm visits segment $\left(p_1^{S1}, \theta_1^{S1}\right)$, and x^{B0} the probability with which a forced renter visits segment $\left(p_2^{S1}, \theta_2^{S1}\right)$. The stock-flow conditions in this case become

$$B_n = \frac{g}{q\left(\theta_1^{S1}\right) + g}, \tag{E.61}$$

$$A = \frac{g}{x^A \mu\left(\theta_1^{S1}\right) + \left(1 - x^A\right)\mu\left(\theta_2^{S1}\right) + g}, \tag{E.62}$$

$$S_1 = \frac{\gamma O}{\mu\left(\theta_3^{S1}\right) + g}, \tag{E.63}$$

$$B_0 = \frac{\mu\left(\theta_3^{S1}\right) S_1}{x^{B0} q\left(\theta_2^{S1}\right) + \left(1 - x^{B0}\right) q\left(\theta_3^{S1}\right) + g}, \tag{E.64}$$

$$B_n + B_0 + S_1 + O = 1, \tag{E.65}$$

and

$$B_n + B_0 = A. \tag{E.66}$$

The market tightnesses in each active segment satisfy

$$\theta_1^{S1} = \frac{B_n}{x^A A}, \tag{E.67}$$

$$\theta_2^{S1} = \frac{x^{B0} B_0}{\left(1 - x^A\right) A}. \tag{E.68}$$

and

$$\theta_3^{S1} = \frac{\left(1 - x^{B0}\right) B_0}{S_1}, \tag{E.69}$$

Similarly to before, observe that (E.61), (E.62), (E.66), and (E.69) imply that $x^A < 1$, as otherwise, (E.61), (E.62) and (E.69) give

$$\theta_1^{S1} = \frac{B_n}{A} = \frac{\mu\left(\theta_1^{S1}\right) + g}{q\left(\theta_1^{S1}\right) + g},$$

which has a unique solution at $\theta_1^{S1} = 1$. However, this is inconsistent with (E.66). As before, the no-surplus allocation implies that the match surpluses between trading pairs are split according to the Hosios rule. Consequently, there are two indifference conditions for real estate firms and forced renters given by

$$\mu\left(\theta_1^{S1}\right) \Sigma_{BnA} = \mu\left(\theta_2^{S1}\right) \Sigma_{B0A}, \tag{E.70}$$

and

$$q\left(\theta_2^{S1}\right) \Sigma_{B0A} = q\left(\theta_3^{S1}\right) \Sigma_{B0S1}, \tag{E.71}$$

respectively. In addition, the surplus sharing rules imply that the value functions of active agents satisfy the equations

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_1^{S1}\right)\Sigma_{BnA}, \tag{E.72}$$

$$\rho V^{A} = R + (1-\alpha)\mu\left(\theta_1^{S1}\right)\Sigma_{BnA}, \tag{E.73}$$

$$\rho V^{S1} = u - \chi + (1-\alpha) q\left(\theta_3^{S1}\right)\Sigma_{B0S1}, \tag{E.74}$$

$$\rho V^{B0} = u_0 - R + \alpha\mu\left(\theta_3^{S1}\right)\Sigma_{B0S1}, \tag{E.75}$$

and

$$\rho V = u + \gamma\left(V^{S1} - V\right). \tag{E.76}$$

Finally, the above value functions allow us to solve for the surpluses as follows:

$$\Sigma_{BnA} = V - V^{Bn} - V^{A} = \frac{\rho V - u_n}{\rho + \alpha q\left(\theta_1^{S1}\right) + (1-\alpha)\mu\left(\theta_1^{S1}\right)}. \tag{E.77}$$

$$\Sigma_{B0S1} = V - V^{S1} = \frac{\rho V - (u - \chi)}{\rho + (1-\alpha)\mu\left(\theta_3^{S1}\right)}, \tag{E.78}$$

and

$$\Sigma_{B0A} = \frac{\rho V - u_0}{\rho + \alpha q\left(\theta_2^{S1}\right) + (1-\alpha)\mu\left(\theta_2^{S1}\right)}. \tag{E.79}$$

The stock-flow and market tightness equations (E.61)-(E.69), combined with the two indifference conditions (E.70) and (E.71), and value functions and surpluses (E.72)-(E.79) fully characterize the equilibrium stocks of agents, market tightnesses, mixing probabilities x^A and x^{B0}, and active agent value functions in a "Sell first" equilibrium. We now prove existence of this equilibrium when χ and u_2 are small, and u_0 is strictly smaller than u_n.

Note that $\Sigma_{B0S1} = V - V^{S1} > 0$ for any u_0, but that $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{S1} = \frac{u}{\rho}$, so that $\lim_{\chi\to 0} \Sigma_{B0S1} = 0$. Then, a set of arguments similar to the case of the "Buy first" equilibrium shows that $\lim_{\chi\to 0} \theta_3^{B1} = 0$ and $\lim_{\chi\to 0} x^{B0} = 1$, so that $\lim_{\chi\to 0} \mu\left(\theta_3^{B1}\right) = 0$ and $\lim_{\chi\to 0} B_0 = 0$, implying that $\lim_{\chi\to 0} x^A = 1$ and $\lim_{\chi\to 0} \theta_1^{B1} = 1$. As a result, $\lim_{\chi\to 0} V^{Bn}$ and $\lim_{\chi\to 0} V^A$ are the same as in the "Buy first" equilibrium, and there exists a $\bar{\chi}_4 > 0$ such that for $\chi \in (0, \bar{\chi}_4)$, it is the case that $\Sigma_{BnA} > 0$, $x^A \in (0,1)$ and $\theta_1^{S1} > 1$ but remains bounded, while $\Sigma_{B0S1} > 0$. As a result, markets 1 and 3 are active.

Following the same arguments as in the "Buy first" equilibrium, it is then the case that $V^{Bn} > V^{B0}$, also as $\chi \to 0$, so that $0 < \Sigma_{BnA} < \Sigma_{B0A}$ and market 2 is active. Furthermore, there must exist a $\bar{\chi}_5 > 0$ such that for $\chi \in (0, \bar{\chi}_5)$, it is the case that $\Sigma_{B0A} > \Sigma_{B0S1}$, since $\lim_{\chi\to 0} \Sigma_{B0S1} = 0$. This ranking implies that $-V^A > V^{B0} - V^{S1}$, so that a real estate firm is more impatient than a mismatched seller in the sense that the direct utility gain from transacting is higher for a real estate firm compared to a mismatched seller. The ranking of tightnesses and prices across segments then follows from the indifference conditions (E.70) and (E.71), similar to the case of a "Buy first" equilibrium. The fact that $V^{Bn} - V^{B0}$ is bounded away from zero also implies that there exists a exist a $\bar{\chi}_6 > 0$ such that for $\chi \in (0, \bar{\chi}_6)$, a mismatched owner and a new entrant will not open a fourth market, because $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{S1} = \frac{u}{\rho}$ and thus $\Sigma_{BnS1} = V - V^{S1} + V^{B0} - V^{Bn} < 0$ for a sufficiently small χ.

Now consider a mismatched owner that deviates and buys first. Potential sellers are real estate firms and mismatched homeowners, and upon trade the deviator becomes a double owner, who can open up new market segments with new entrants and forced renters. Note that V^{S2} falls without bounds as u_2 does, because a deviating double owner has to offer new entrants or forced renters their market value, following a similar argument as in the "Buy first" equilibrium. Then there exists a $\bar{u}_2^{S1}$ such that for all $u_2 < \bar{u}_2^{S1}$ it is the case that $\Sigma_{B1A} = V^{S2} - V^A - V^m < 0$, so that a deviating mismatched owner does not buy from a real estate agent. Note, however, that $\Sigma_{B1S1} < \Sigma_{B1A} < 0$ for $\chi < \bar{\chi}_5$ since, as shown above, in that case $-V^A > V^{B0} - V^{S1}$, meaning that a new entrant is more impatient than a mismatched buyer. As a result, a mismatched owner that deviates and buys first is better off not trading. However, not trading is dominated by selling first since $V^{B1} > \frac{u-\chi}{\rho}$. Therefore, a mismatched owner is never better off deviating from selling first in a "Sell first" equilibrium.

Finally, setting $\bar{\chi} = \min\{\bar{\chi}_1, \bar{\chi}_2, \bar{\chi}_3, \bar{\chi}_4, \bar{\chi}_5, \bar{\chi}_6\}$ and $\bar{u}_2 = \min\left\{\bar{u}_2^{B1}, \bar{u}_2^{S1}\right\}$, we arrive at our result. □

E.6 Simultaneous Entry as Buyer and Seller

We assume that a mismatched owner can allocate a fixed amount of time (normalized to 1 unit) to search in the housing market as a buyer or a seller. A mismatched owner that chooses to enter as a buyer or seller only allocates all of his time to one activity. Otherwise, a mismatched owner that enters as both a buyer and a seller can allocate a fraction $\phi \in (0,1)$ of his time to searching as buyer, and searches the remaining $1-\phi$ of his time as seller. For a given market tightness θ, the value function V^{SB} for a mismatched owner that enters as both buyer and seller satisfies the following equation in a steady state equilibrium:

$$\rho V^{SB} = u - \chi + (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{SB}\right\} + \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{SB}\right\}.$$

We then show the following

Proposition E.4. *For $\theta \in \left(0, \tilde{\theta}\right)$, $V^{S1} > V^{SB}$, for any $\phi \in (0,1)$. Also, for $\theta \in \left(\tilde{\theta}, \infty\right)$, $V^{B1} > V^{SB}$, for any $\phi \in (0,1)$.*

Proof. To show the first part, suppose the opposite, so $V^{S1} \leq V^{SB}$. Then

$$\begin{aligned} \mu(\theta) \max\left\{0, p + V^{B0} - V^{S1}\right\} &\leq (1-\phi)\mu(\theta) \max\left\{0, p + V^{B0} - V^{SB}\right\} \\ &+ \phi q(\theta) \max\left\{0, -p + V^{S2} - V^{SB}\right\}. \end{aligned}$$

Under the assumption that $V^{S1} \leq V^{SB}$, and since we know from Lemma 1 that $V^{B1} < V^{S1}$ for $\theta \in \left(0, \tilde{\theta}\right)$, it must then be the case that

$$\begin{aligned} \mu(\theta) \max\left\{0, p + V^{B0} - V^{S1}\right\} &\leq (1-\phi)\mu(\theta) \max\left\{0, p + V^{B0} - V^{S1}\right\} \\ &+ \phi q(\theta) \max\left\{0, -p + V^{S2} - V^{B1}\right\}, \end{aligned}$$

which does not hold because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) > 0$ for $\theta \in \left(0, \tilde{\theta}\right)$ by Assumption A3, and because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) > q(\theta)\left(-p + V^{S2} - V^{B1}\right)$ for $\theta \in \left(0, \tilde{\theta}\right)$ by Lemma 1.

To show the second part, suppose the opposite, so $V^{B1} \leq V^{SB}$. Then

$$\begin{aligned} q(\theta) \max\left\{0, p + V^{S2} - V^{B1}\right\} &\leq (1-\phi)\mu(\theta) \max\left\{0, p + V^{B0} - V^{SB}\right\} \\ &+ \phi q(\theta) \max\left\{0, -p + V^{S2} - V^{SB}\right\}. \end{aligned}$$

Under the assumption that $V^{B1} \leq V^{SB}$, and since we know from Lemma 1 that $V^{S1} < V^{B1}$ for $\theta \in \left(\tilde{\theta}, \infty\right)$, it must then be the case that

$$\begin{aligned} q(\theta) \max\left\{0, p + V^{S2} - V^{B1}\right\} &\leq (1-\phi)\mu(\theta) \max\left\{0, p + V^{B0} - V^{S1}\right\} \\ &+ \phi q(\theta) \max\left\{0, -p + V^{S2} - V^{B1}\right\}, \end{aligned}$$

which does not hold because $q(\theta)\left(-p + V^{S2} - V^{B1}\right) > 0$ for $\theta \in \left(\tilde{\theta}, \infty\right)$ by Assumption A3, and because $\mu(\theta)\left(p+V^{B0}-V^{S1}\right)<q(\theta)\left(-p+V^{S2}-V^{B1}\right)$ for $\theta\in\left(\tilde{\theta},\infty\right)$ by Lemma 1.[33] □

Finally, note that under payoff symmetry (i.e. $\tilde{u}_0=\tilde{u}_2=c$) the possibility to enter as both buyer and seller while allocating each an equal amount of time can result in an equilibrium with a market tightness of $\theta=1$. Specifically, at $\theta=1$, $\mu(\theta)=q(\theta)=\mu(1)$. At these flow rates it can easily be seen that if $\tilde{u}_0=\tilde{u}_2=c$, then $V^{B1}=V^{S1}=V^{SB}$ for any ϕ. Finally, a tightness of $\theta=1$ can result from mismatched owners entering as buyers and sellers simultaneously and allocating each an equal amount of time (so $\phi=0.5$).

This is analogous to the equilibrium described in Proposition 1, with the only difference that now agents follow symmetric strategies compared to asymmetric strategies with one half of mismatched owners buying first and the other half selling first.

E.7 Homeowners compensated for their housing unit upon exit

Suppose that upon exit homeowners receive bids for their housing unit(s) from a set of competitive real estate firms. Therefore, given that the value of a housing unit to a real estate firm is $V^A(\theta)$, homeowners receive $V^A(\theta)$ for each housing unit that they own. Again, we consider a steady state equilibrium with a fixed market tightness θ. We define $\tilde{u}_0(\theta,g)\equiv u_0+\triangle-gV^A(\theta)$ and $\tilde{u}_2(\theta,g)=u_2-\triangle+gV^A(\theta)$. Note that $V^A(\theta)$ is (weakly) increasing in θ, so $\tilde{u}_2$ is increasing in θ and $\tilde{u}_0$ is decreasing in θ;

Given this definition, the difference between the values from buying first and selling first (assuming a mismatched owner transacts in both cases), $D(\theta)\equiv V^{B1}-V^{S1}$, is equal to

[33] Note also that for $\theta=0$ and $\theta\to\infty$, mismatched owners are indifferent between remaining mismatched and any search strategy, because $V^{B1}=V^{S1}=V^{SB}=\frac{u-\chi}{\rho}$, but that such tightnesses cannot occur in steady state by Lemma 2.

$$D(\theta) = \frac{\mu(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))}\left[\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2(\theta, g)) - \tilde{u}_0(\theta, g) + \tilde{u}_2(\theta, g)\right].$$

Let $\tilde{\theta}$ be defined implicitly by

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2\left(\tilde{\theta}, g\right)}{u - \chi - \tilde{u}_0\left(\tilde{\theta}, g\right)},$$

whenever that equation has a solution.[34] Note that in the limit as $g \to 0$, assumption A3 will hold. Therefore, for g sufficiently close to zero, we will have that $u - \chi > \max\{\tilde{u}_0(\theta, g), \tilde{u}_2(\theta, g)\}$, for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, and so a version of Lemma 1 will hold in this case as well. Given this result one can then easily construct multiple steady state equilibria as in Proposition 1.

[34]Note that the above equation for $\tilde{\theta}$, whenever it has a solution, has a unique solution for any $g \geq 0$, since given the properties of $\tilde{u}_0$ and $\tilde{u}_2$, it follows that the right hand side of this expression is (weakly) decreasing in θ. Furthermore, the right hand side is strictly decreasing in g for any $\theta > 0$, so by the implicit function theorem, $\tilde{\theta}$ is decreasing in g.