

Lecture 8:

Heavy Ion Physics

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Nikhef



Scientific question to be answered:







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Fundamental questions in physics



- What is dark matter?
- What is the nature of dark energy?
- How did the Universe begin and evolve?
- Can we incorporate quantum effects in a general gravitational theory?
- What are the neutrino masses and what is their role in the evolution of the universe?
- How do Cosmic Accelerators work and what are they accelerating?
- Are protons unstable?
- What are the new states of matter at exceedingly high density and temperature?
- Are there additional space-time dimensions?
- How were the elements from iron to uranium made?
- Is a new theory of matter and light needed at the highest energies?



Eleven Science Questions for the New Century

IONAL RESEARCH COUNCI



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There is evidence that during its earliest moments the universe underwent a tremendous burst of expansion, known as inflation, so that the largest objects in the universe had their origins in subatomic quantum fuzz. The underlying physical cause of this inflation is a mystery. In addition, the universe evolved passing through the EW and the strong phase transition, through a state of extreme conditions which are too of a complete mystery.

The theory of how protons and neutrons form the atomic nuclei of the chemical elements is well developed. At higher densities, neutrons and protons may dissolve into an undifferentiated soup of quarks and gluons, which can be probed in heavy-ion accelerators. Densities beyond nuclear densities occur and can be probed in neutron stars, and still higher densities and temperatures existed in the early universe.



Fermi's notes: 2010 physics discussed in ~1950





The birth of QCD

VOLUME 30, NUMBER 26 PHYSICAL REVIEW LETTERS

25 JUNE 1973

Ultraviolet Behavior of Non-Abelian Gauge Theories*

David J. Gross† and Frank Wilczek Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540 (Received 27 April 1973)

It is shown that a wide class of non-Abelian gauge theories have, up to calculable logarithmic corrections, free-field-theory asymptotic behavior. It is suggested that Bjorken scaling may be obtained from strong-interaction dynamics based on non-Abelian gauge symmetry.

Non-Abelian gauge theories have received much attention recently as a means of constructing unified and renormalizable theories of the weak and electromagnetic interactions.¹ In this note we report on an investigation of the ultraviolet (UV) asymptotic behavior of such theories. We have found that they possess the remarkable feature, perhaps unique among renormalizable theories, of asymptotically approaching free-field theory. Such asymptotically free theories will exhibit, for matrix elements of currents between on-mass-shell states, Bjorken scaling. We therefore suggest that one should look to a non-Abelian gauge theory of the strong interactions to provide the explanation for Bjorken scaling, which has so far eluded field-theoretic understanding.

The UV behavior of renormalizable field theories can be discussed using the renormalization-group equations.^{2.3} which for a theory involving one field (say $g\phi^{*}$) are

 $[m\partial/\partial m + \beta(g)\partial/\partial g - n\gamma(g)]\Gamma_{m\gamma}(n)(g; P_1, \dots, P_n) = 0.$

(1)

The Nobel Prize in Physics 2004 David J. Gross, H. David Politzer, Frank Wilczek

The Nobel Prize in Physics 2004



David J. Gross



Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.

Politzer

VOLUME 30, NUMBER 26

PHYSICAL REVIEW LETTERS

25 JUNE 1973

¹⁶Y. Nambu and G. Jona-Lasino, Phys. Rev. <u>122</u>, 345 (1961); S. Coleman and E. Weinberg, Phys. Rev. D <u>7</u>, 1888 (1973).

¹⁵K. Symanzik (to be published) has recently suggested that one consider a $\lambda φ^{\delta}$ theory with a negative λ to achieve UV stability at $\lambda = 0$. However, one can show, using the renormalization-group equations, that in such theory the ground-state energy is unbounded from below (8, Coleman, private communication). ¹⁰W. A. Bardeon, H. Fritzsch, and M. Gell-Mann, CERN Report No. CERN-TH-1538, 1972 (to be published).
¹¹H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28,

¹⁴⁹⁴ (1972); S. Weinberg, Phys. Rev. D 5, 1962 (1972). ¹⁸For a review of this program, see S. L. Adler, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

Reliable Perturbative Results for Strong Interactions?*

H. David Politzer Jefferson Physical Laboratories, Harvard University, Cambridge, Massachusetts 02138 (Received 3 May 1973)

An explicit calculation shows perturbation theory to be arbitrarily good for the deep Euclidean Green's functions of any Yang-Mills theory and of many Yang-Mills theories with fermions. Under the hypothesis that spontaneous symmetry breakdown is of dynamical origin, these symmetric Green's functions are the asymptotic forms of the physically significant spontaneously broken solution, whose coupling could be strong.

1973: QCD



It looks like QED, no?

 $\mathcal{J} = \frac{1}{4g^2} G_{\mu\nu} G_{\mu\nu} + \frac{5}{7} \overline{g}_i (18^{\mu} D_{\mu} + m_i) q_i$ where Guy = du A, -d, A, + for A, A, and Du= que + it An That's it!



It looks like QED, no?

 $f = \frac{1}{4g^2} G_{\mu\nu} G_{\mu\nu} + \frac{5}{7} \overline{g}_i (i \delta^{\mu} D_{\mu} + m_i) g_i$ where $G_{\mu\nu}^{\alpha} \equiv \partial_{\mu} H_{\nu}^{\alpha} - \partial_{\mu} H_{\mu}^{\alpha} + i \int_{\partial \alpha} H_{\mu}^{\beta} H_{\mu}^{\alpha}$ and Du= du + it An That's it! mm Well...it's not!!!



8

8.

8.

QCD distinct features





- The running coupling constant suggests the possibility of creating a new state of QCD matter where quarks and gluons are "free" The strong phase transition
- This transition can happen at sufficiently large:
 - ★ temperature T
 - \star energy density ε
- Which T and ϵ ?
 - ★ for massless quarks and gluons the only physical scale in QCD is the confinement scale ~1 fm
 - T ~ 200 MeV
 - \bigcirc $\epsilon \sim 1 \text{ GeV/fm}^3$
- At these scales the strong coupling constant becomes large
 - Perturbation theory can not be applied and analytical calculations are notoriously difficult to be made
 - Confinement is still poorly understood from first principles!
- Better understanding of this non-perturbative domain comes from lattice QCD calculations



Lattice QCD calculations







Lattice QCD: beyond a critical temperature there is a rapid rise in the number of degrees of freedom ➡ phase transition to a deconfined state of quarks and gluons!!!

Quark-Gluon Plasma (QGP)



Going back in time: few µsec after the Big-Bang

Major Events **Time Since** Since Big Bang **Big Bang** Humans present observe stars, the cosmos. galaxies Era of and clusters Galaxies (made of atoms and plasma) 1 billion **First galaxies** atoms and years form. plasma Era of (stars Atoms begin Atoms form: to form) photons fly free 500,000 and become years plasma of microwave background. hydrogen and Era of helium nuclei 0 Nuclei Fusion ceases: plus electrons o normal matter is G 3 minutes 75% hydrogen, protons, neutrons, 0 25% helium, by Era of O electrons, neutrinos Nucleosynthesis mass. (antimatter rare) Matter annihilates 0.001 seconds elementary particles antimatter. Particle Era (antimatter Electromagnetic and weak common) 10⁻¹⁰ seconds forces become distinct. elementary **Electroweak Era** Strong force becomes particles 10⁻³⁸ seconds distinct, perhaps causing inflation of elementary GUT Era universe. particles 10⁻⁴³ seconds **Planck Era** ???? electron neutron antielectrons 420 quarks neutrino antineutron proton Copyright @ Addison Wesley.



Going back in time: few µsec after the Big-Bang

Major Events **Time Since** Since Big Bang **Big Bang** Humans present observe stars, the cosmos. galaxies Era of and clusters Galaxies (made of atoms and plasma) 1 billion **First galaxies** atoms and years form. plasma Era of (stars Atoms begin Atoms form: to form) photons fly free 500,000 and become years plasma of microwave background. hydrogen and Era of helium nuclei 0 Nuclei Fusion ceases: plus electrons 0 normal matter is 0 3 minutes 75% hydrogen, protons, neutrons, 0 25% helium, by Era of G electrons, neutrinos Nucleosynthesis mass. (antimatter rare) a Matter annihilates 0.001 seconds elementary particles antimatter. Particle Era (antimatter Electromagnetic and weak common) 10⁻¹⁰ seconds forces become distinct. elementary **Electroweak Era** Strong force becomes particles 10⁻³⁸ seconds distinct, perhaps causing inflation of elementary GUT Era universe. particles 10⁻⁴³ seconds **Planck Era** ???? electron neutron antielectrons 420 quarks neutrino antineutron proton Copyright @ Addison Wesley.











From the Big-Bang to the Little-Bangs...





- ★ existed few µs after the Big-Bang (the universe crossed this phase after expanding and cooling down): Studying the strong phase transition → study primordial matter
- QCD: Phase transition beyond a critical temperature (~170 MeV) and energy density (~0.5 GeV/fm³) → accessible in the laboratory → heavy-ion collisions

Can we constrain the equation of state and the transport properties of QGP?

 $T_{(QGP-transition)} 10^5 x T_{(Sun's core)}$



What are the properties of the QGP? EoS, transport properties (density, viscosity...)



Need to study as many observables as possible as a function of centrality







- Heavy ions are not point-like objects
- Collisions can create systems with different properties depending on whether they are head-on (i.e. large overlap region) or if the nuclei graze each other (i.e. small overlap region)
- Centrality defined geometrically by the impact parameter b
 - ★ Distance between the centers of the two nuclei
 - **Perpendicular** to the beam axis
- Centrality related to the fraction of the geometrical cross-section that overlaps





















ALICE Collaboration, Phys. Rev. Lett. **105**, 252301 (2010)



~1600 particles in the central region (not the whole phase space) in central Pb-Pb collisions!!!



ALICE Collaboration, Phys. Lett. B754 (2016) 373





ALICE Collaboration, Phys. Lett. B754 (2016) 373





- Number of participants (N_{part}): nucleons undergoing at least one collision
 - ★ Scale with volume ~2A
- Number of binary collisions (N_{coll}): inelastic collisions between a nucleon of one nucleus and at least one nucleon of the other nucleus
 - **★** Scale with $AxA^{1/3}=A^{4/3}$
- Number of spectators (N_{spec}): nucleons that do not lie in the overlap region and thus fly away without interacting









- Experimentally neither the impact parameter nor the N_{part}/N_{spec} can be measured
- Have to rely on experimental measurements:
 - Multiplicity (central or/and forward regions)
 - Large (small) for central (peripheral) collisions
 - Zero degree calorimeters (energy deposited by spectator nucleons)
 - EzDC small (large) for central (peripheral) collisions
- Expressed as the percentage of the total nuclear interaction cross section
 - e.g. 5% most central Pb-Pb (or Au-Au) collisions are the 5% with the highest multiplicity







Heavy-ion physics: SPS (CERN) (~1990)



Fixed target experiments (event display courtesy of NA49)





Heavy-ion physics: RHIC (BNL) (~2000)



Collider experiments (event displays courtesy of PHENIX and STAR)







Heavy-ion physics: LHC (CERN) (2010)









Experimental setup





Two main ways to probe the QGP properties:





Studying the properties of the QGP

- Two main ways to probe the QGP properties:
 - Through bulk observables
 - The vast majority of particles are produced with $p_T < 2 \text{ GeV/}c$





- Two main ways to probe the QGP properties:
 - Through bulk observables
 - The vast majority of particles are produced with $p_T < 2 \text{ GeV/}c$
 - ★ Hard (rare) probes with $p_T > 6$ GeV/c

 - Solution
 - Heavy flavour (e.g. charmmesons)







M. Roirdan and W. Zajc, Scientific American 34A May (2006)

EVIDENCE FOR A DENSE LIQUID

Two phenomena in particular point to the quark-gluon medium being a dense liquid state of matter: jet quenching and elliptic flow. Jet quenching implies the quarks and gluons are closely packed, and elliptic flow would not occur if the medium were a gas.




Hard probes: Jet quenching





- Hard process scale: $Q >> \Lambda_{QCD}$
- High p_T parton with Q ~ p_T
- These partons are formed early during the evolution of the system
 - \star They fragment and create jets and high transverse momentum hadrons
 - ★ These processes can be calculated in perturbative QCD
- During the propagation through the medium, these objects interact with the medium and lose energy either via collisional or radiative energy loss
- Experimental consequence:
 - **★** Suppression of high transverse momentum particles
 - ★ Attenuation of energy of jets
 - ★ Modification of soft particle production



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CMS Collaboration, EPJC 72 (2012) 1945



Jet quenching at the LHC: RAA

CMS Collaboration, EPJC 72 (2012) 1945





CMS Collaboration, EPJC 72 (2012) 1945



Particles not interacting with the medium (e.g. γ , Z⁰, W) do not show any in medium effects





$$C(\Delta\phi) = \left[1 + \sum_{n=1}^{\infty} 2v_n^2 \cos n\Delta\phi\right]$$





To flow or not to flow?



To flow or not to flow?





To flow or not to flow?













 $\frac{\langle y^2 - x^2}{\langle y^2 + x \rangle}$ ε=

















Collectivity





E=

Collectivity

Courtesy of Mike Lisa



More and faster particles in-plane than outof-plane





Elliptic flow









Elliptic flow

Superposition of independent pp collisions







Evolution as a bulk system







- In non-central collisions the coordinate space configuration is anisotropic (e.g. almond shape)
 - The initial momentum distribution is isotropic (spherically symmetric)
- The interactions among constituents generate a pressure gradient that transforms the initial coordinate space anisotropy into the observed momentum space anisotropy
 - Azimuthal anisotropy quantified by a Fourier expansion
 - 6
 - v_1 : directed flow, v_2 : elliptic flow, v_3 : triangular flow, v_4 : quadrangular flow,...
- Connection to equation of state and to the system's transport properties (e.g. η/S)

$$E\frac{d^{3}N}{d^{3}\overrightarrow{p}} = \frac{1}{2\pi} \frac{d^{2}N}{p_{T}dp_{T}dy} \left(1 + \sum_{n=1}^{\infty} 2v_{n}cos[n(\varphi - \Psi_{RP})]\right)$$
$$v_{n} = \langle cos[n(\varphi - \Psi_{RP})] \rangle$$

S. Voloshin and Y. Zhang, Z. Phys. **C70**, 665 (1996)



The "perfect liquid" at RHIC and LHC







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RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

Monday, April 18, 2005

TAMPA, FL – The four detector groups conducting research at the <u>Relativistic Heavy Ion Collider</u> (RHIC) – a glant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory – say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In <u>peer-reviewed papers</u> summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a liquid. First Indirect Evidence of So-Far Undetected Strange Baryons

Other RHIC News

RHIC Featured in 'How The Universe Works' on the Science Channel

A New Look for RHIC & Sharper View of QCD: Looking Back at the 2014 RHIC-AGS Users' Meeting

RHIC Run 14: A Flawless 'Run of Firsts'



The "perfect liquid" at RHIC and LHC







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RHIC

0.0

 $(T-T_0)/T_0$

 H_2O

QGP

1.0

0.5

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The "perfect liquid" at RHIC and LHC









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Elliptic flow @ the LHC





- Initial geometry not described by the (ideal) almond shape
 - Fluctuations of the initial energy/pressure distributions lead to "irregular" shapes that fluctuate from one event to the other
 - Higher order (odd) harmonics develop, each one having its own symmetry plane



Higher order harmonics more sensitive to the value of η/S

- But initial conditions not known precisely enough (model dependent)
 - The second seco

 $v_n(\eta, p_T) = \langle cos[n(\varphi - \Psi_n)] \rangle$



And there is even more...: higher harmonics!







And there is even more...: higher harmonics!





Due to the low value of η/s , higher harmonics survive at the final state Allow the study of initial conditions of heavy-ion collisions for the first time!

Naghmeh Mohammadi



You knew this was coming...





EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH





CERN-PH-EP-2015-XXX Day Month 2015

Higher harmonic flow coefficients of identified hadrons in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

ALICE Collaboration*

Abstract

The anisotropic flow coefficients, v_2 , v_3 , v_4 and v_5 , of π^+ , K^+ and $p_{+}\overline{p}$ in Pb–Pb collisions at $\sqrt{s_{PN}}$ – 2.76 TeV were measured with the ALICE detector at the Large Hadron Collider (LHC). The results were obtained with the Scalar Product method using a pseudo-rapidity gap of $|A\eta| > 0.0$ between 0 10 the identified hadron under study and the reference particles. Correlations not related to the common symmetry plane, known also as non-flow, were estimated and subtracted from the measurement using 11 the measured correlations in pp collisions. The values of these flow coefficients exhibit a clear mass 12 ordering for pT < 3 GeV/c for all harmonics. For transverse momentum values larger than about 13 3 GeV/c, particles lend to group at an approximate level according to their type, suggesting that ы coalescence might be the relevant particle production mechanism in this region. The experimental 1a data are described fairly well by a model that describes the hydrodynamical expansion of the fireball 16 using a given set of initial conditions and a value of $\eta/S = 0.08$, close to the lower bound, coupled 1. to a hadronic cascade model. Finally, the comparison of the measurements with A Multi Phase Transport model (AMPT) illustrates the significance of this late hadronic rescattering stage to the 19 30 development of the observed mass ordering at low values of p_T and of coalesence as a particle production mechanism for the particle type grouping at intermediate values of pT for all harmonics. 71

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*See Appendix A for the list of collaboration members

	Hi	gher l	armonic flow at the LHC ALICE Collaboration	on
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ALICE physics program extends to smaller systems!!!











ALICE physics program extends to smaller systems!!!











We are leading the field with a number of interesting physics projects that could easily lead to an advanced stage (e.g. publication)

Feel free to pass by my office @ Nikhef (N328) if you are interested!!!



Thank you for your attention!







Backup







- Correlations not connected to the common symmetry plane
 - resonances, jets, femtoscopic correlations,...
- Suppressed utilizing multi-particle techniques, η-gap analysis (e.g. event plane or SP methods with forward detectors), different charge combinations,...

$$C_{n}\{2\} = \langle \langle e^{in(\varphi_{1}-\varphi_{2})} \rangle \rangle = \langle v_{n}^{2} \rangle + \delta_{2}$$

$$C_{n}\{4\} = \langle \langle e^{in(\varphi_{1}+\varphi_{2}-\varphi_{3}-\varphi_{4})} \rangle \rangle - 2 \langle \langle e^{in(\varphi_{1}-\varphi_{2})} \rangle \rangle^{2} =$$

$$= \langle v_{n}^{4} \rangle + 4 \langle v_{n}^{2} \rangle \delta_{2} + 2\delta_{2}^{2} - 2(\langle v_{n}^{2} \rangle + \delta_{2})^{2} + \delta_{4}$$

$$= - \langle v_{n}^{4} \rangle + \delta_{4}$$

$$\delta_2 \sim \frac{l}{M} \Rightarrow v_n \gg \frac{l}{\sqrt{M}} \qquad \qquad \delta_4 \sim \frac{l}{(M-3)(M-2)(M-1)} \sim \frac{l}{M^3} \Rightarrow v_n \gg \frac{l}{\sqrt[4]{M^3}}$$

For a typical Pb-Pb LHC collision at 2.76 TeV, M~400-500 for 30-40% centrality

- \star v_n >> 5% for the 2-particle correlation technique
- \star v_n >> 1% for the 4-particle correlation technique


Cumulants and fluctuations



$$v_{2}\{2\} = \sqrt{\langle v_{2}^{2} \rangle} = \dots = \langle v_{2} \rangle + \frac{1}{2} \frac{\sigma_{v}^{2}}{\langle v_{2} \rangle}$$
$$v_{2}\{4\} = \sqrt[4]{2} \langle v_{2}^{2} \rangle^{2} - \langle v_{2}^{4} \rangle = \dots = \langle v_{2} \rangle - \frac{1}{2} \frac{\sigma_{v}^{2}}{\langle v_{2} \rangle}$$

×_{PP} ,

 x_{RP}

$$v_{2}\left\{6\right\} = \sqrt[6]{\frac{1}{4}\left(\left\langle v_{2}^{6}\right\rangle - 9\left\langle v_{2}^{2}\right\rangle\left\langle v_{2}^{4}\right\rangle + 12\left\langle v_{2}^{2}\right\rangle^{3}\right)} = \dots = \left\langle v_{2}\right\rangle - \frac{1}{2}\frac{\sigma_{v}^{2}}{\left\langle v_{2}\right\rangle}$$