A Differential Fault Attack against Deterministic FALCON Signatures

Sven Bauer and Fabrizio De Santis

Overview

- FALCON [PFH+22] is a post-quantum signature scheme based on the GPV framework [GPV07], a hash-then-sign construction.
- In standard FALCON, hashing the message is randomized. (We will see in a moment why).
- There are use-cases for a deterministic variant. Such a variant has been specified in [LPa17].
- Our attack is based on injecting faults in the trapdoor sampler. This produces different signatures for the same message hash.
- Such signatures lead to relatively short lattice vectors.
- Then, lattice reduction is used to find the private key.

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The GPV signature framework

Defined by Gentry, Peikert and Vaikuntanathan in [GPV07].

Setup

Lattice $\mathcal{L}_q^{\perp}(A) = \mathcal{L}(B) \subset \mathbb{Z}^n$, B with 'short' rows. A will be the public key, B will be the private key. Hash function H maps messages to points in $\mathbb{Z}_q^n$.

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Signing a message M

1. Find $c \in \mathbb{Z}_q^n$ s.t. $cA^T = H(M)$ (linear algebra).

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2. Find random $v \in \mathcal{L}(B) = \mathcal{L}_q^{\perp}(A)$ s.t. v is `close' to $c$ (`pre-image sampling').

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3. Output $s := c - v$. Note that s is `short' and that $sA^T = cA^T - vA^T = H(M) - 0 = H(M)$.

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3. Output $s := c - v$. Note that s is \`short' and that $sA^T = cA^T - vA^T = H(M) - 0 = H(M)$.

Verifying a signature s for M

Check that

1. s is \`short' and
2. $sA^T = H(M)$.

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Check that

1. s is `short' and
2. $sA^T = H(M)$.

Security

Finding v close to c is hard without a short basis like B.

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Sampling twice is problematic

Signing a message M

1. Find $c \in \mathbb{Z}_q^n$ s.t. $cA^T = H(M)$ (linear algebra).
2. Find random $v \in \mathcal{L}(B) = \mathcal{L}_q^{\perp}(A)$ s.t. v is `close' to $c$ (`pre-image sampling').
3. Output $s := c - v$. Note that s is `short' and that $sA^T = cA^T - vA^T = H(M) - 0 = H(M)$.

Suppose we sign the same c twice and sample v, v' in step 2 with $v \neq v'$. Then:

- We obtain two different signatures s, s'.
- An attacker can calculate $s - s'$. Because both s and s' are both short, so is $s - s'$.
- Also, $s - s' \in \mathcal{L}(B)$, because

$$s - s' = (c - v) - (c - v') = v' - v,$$

So an attacker can easily obtain a (relatively) short lattice vector. Repeating this leaks more and more information about the secret basis B. This has already been noted in the original paper about the GPV framework [GPV07].

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Mitigating the risk of repeated sampling and the idea of our attack

Standard Falcon

Randomize the message hash, i.e. $cA^T = H(r, M)$ with some random r [PFH+22].

Deterministic Falcon

It is deterministic! The sampler returns the same v when the same message is signed again [LPa17].

Idea of the attack against deterministic Falcon

- Sign the same message repeatedly.
- Inject faults in the sampler.
- This gives the attacker different (valid) signatures.
- Differences between pairs of such signatures yield (relatively) short lattice vectors.
- Reduce these to find a short basis.

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NTRU lattices [HPS98, PFH+22]

Setup

q prime, $n = 2^k$ and $\phi(X) = X^n + 1$.

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NTRU lattices [HPS98, PFH$^+$22]

Setup

q prime, $n = 2^k$ and $\phi(X) = X^n + 1$.

NTRU private key

$f, g, F, G \in \mathbb{Z}[X]/(\phi)$ with 'small' coefficients such that

$$fG - gF \equiv q \bmod \phi$$

and such that $f^{-1} \mod \phi, q$ exists.

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NTRU public key

$h = gf^{-1} \bmod \phi, q$, coefficients of h are typically 'large'.

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NTRU – from polynomials to integer lattices

Identify polynomials $a(X) = \sum_{i=0}^{n-1} a_i X^i \in \mathbb{Z}[X]/(\phi)$ with their coefficient vector $a = (a_0, a_1, \ldots, a_{n-1}) \in \mathbb{Z}^n$.

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Polynomial multiplication $a(X)b(X)$ is linear, so we can identify b with a matrix $B \in \mathbb{Z}^{n \times n}$:

$$B = \begin{pmatrix} b_0 & b_1 & \ldots & b_{n-1} \\ -b_{n-1} & b_0 & \ldots & b_{n-2} \\ -b_{n-2} & -b_{n-1} & \ldots & b_{n-3} \\ \vdots & \vdots & \ddots & \vdots \\ -b_1 & -b_2 & \ldots & b_0 \end{pmatrix}$$

(Homework: Check that $a \cdot B \leftrightarrow a(X)b(X)$.)

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(Homework: Check that $a \cdot B \leftrightarrow a(X)b(X)$.)

With this identification, the NTRU-lattice is:

$$\mathcal{L}\begin{pmatrix} g & -f \\ G & -F \end{pmatrix} = \mathcal{L}_q^{\perp}\begin{pmatrix} 1 & h^T \end{pmatrix}$$

(Homework: Check this equality.)

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Signing with deterministic FALCON (simplified)

Require: A message M, a secret key $\mathtt{sk}$ consisting of $B = \begin{pmatrix} g & -f \\ G & -F \end{pmatrix}$ and some pre-computed data structure T.
Ensure: A valid $\textsc{Falcon}$ signature s of M.

1: **procedure** $\textsc{Falcon-Sign}(\mathrm{M}, \mathtt{sk})$
2: $c \leftarrow \mathtt{HashToPoint}(\mathtt{salt}, M)$
3: $(t_0, t_1) \leftarrow (c, 0)\mathbf{B}^{-1}$
4: **do**
5: $(z_0, z_1) \leftarrow \mathtt{ffSampling}_n((t_0, t_1), T)$ ▷ Note: the sampling is deterministic!
6: $(s_1, s_2) \leftarrow ((t_0, t_1) - (z_0, z_1))\mathbf{B}$
7: **while** $\|(s_1, s_2)\|^2 > \lfloor \beta^2 \rfloor$
8: **return** (s_1, s_2)

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1: **procedure** FALCON-SIGN(M,sk)
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7: **while** $\|(s_1, s_2)\|^2 > \lfloor \beta^2 \rfloor$
8: **return** (s_1, s_2)

inject fault in this subroutine

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Signing with deterministic Falcon (simplified)

Require: A message M, a secret key sk consisting of $B = \begin{pmatrix} g & -f \\ G & -F \end{pmatrix}$ and some pre-computed data structure T.
Ensure: A valid Falcon signature s of M.

procedure Falcon-Sign(M,sk)
  $c \leftarrow \texttt{HashToPoint}(\texttt{salt}, M)$
  $(t_0, t_1) \leftarrow (c, 0)\mathbf{B}^{-1}$
  do
    $(z_0, z_1) \leftarrow \boxed{\texttt{ffSampling}_n((t_0, t_1), T)}$ ▷ Note: the sampling is deterministic!
    $(s_1, s_2) \leftarrow ((t_0, t_1) - (z_0, z_1))\mathbf{B}$
  while $\|(s_1, s_2)\|^2 > \lfloor \beta^2 \rfloor$
  return (s_1, s_2)

inject fault in this subroutine

Inject fault such that only z_0 is affected! (Details in the paper)

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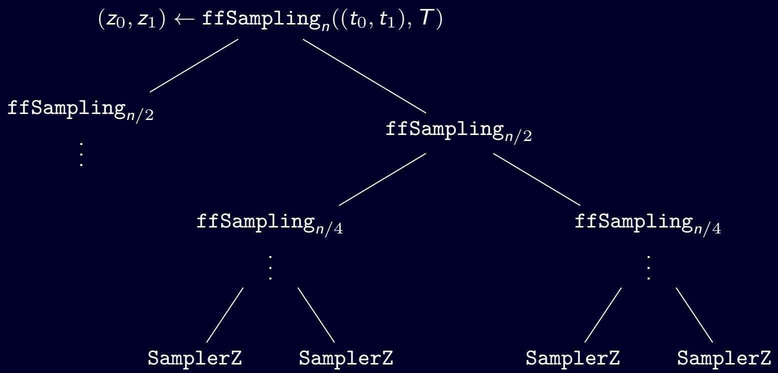
Sampling in FALCON is defined via recursion (simplified)



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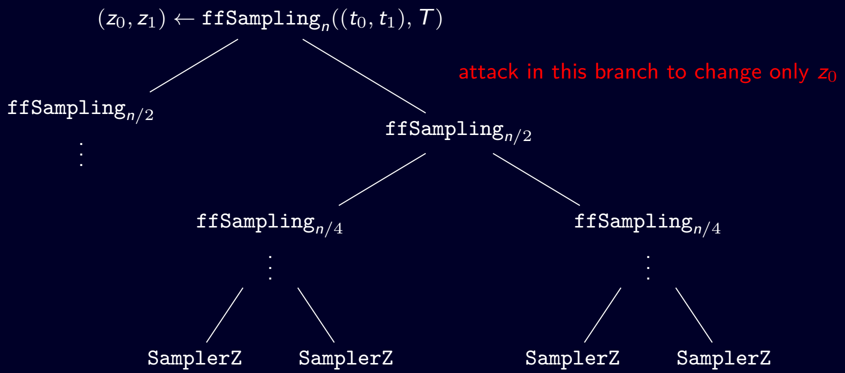

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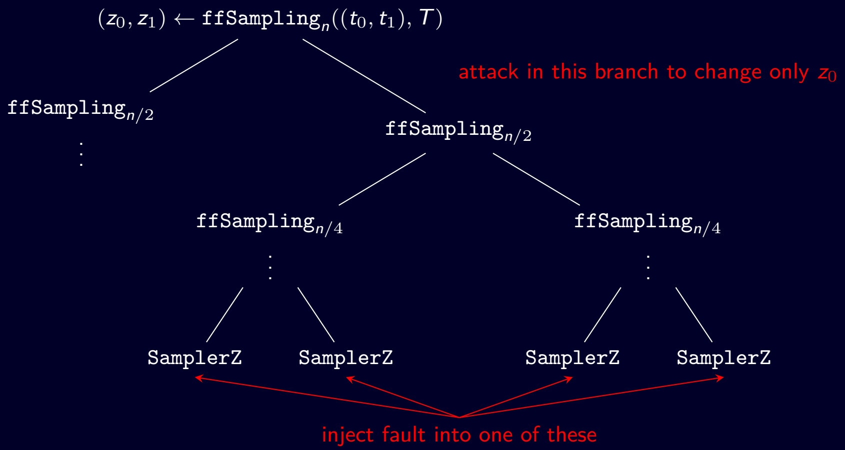

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Attack step one: Fault injection

Recall from previous slide that for a signature (s_1, s_2):

$$(s_1, s_2) = ((t_0, t_1) - (z_0, z_1))\begin{pmatrix} g & -f \\ G & -F \end{pmatrix}$$

Hence

$$s_2 = (z_0 - t_0)f + (z_1 - t_1)F$$

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Re-sign with a fault in z_0 to obtain (s_1', s_2'). Then

$$s_2 - s_2' = (z_0 - z_0')f$$

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Re-sign with a fault in z_0 to obtain (s_1', s_2'). Then

$$s_2 - s_2' = (z_0 - z_0')f$$

Doing this repeatedly, the attacker obtains a set:

$$\Delta = \{\delta_1 f, \delta_2 f, \ldots, \delta_m f\}$$

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Attack step two: Lattice reduction

The attacker now has a set $\Delta = \{\delta_1 f, \delta_2 f, \ldots, \delta_m f\}$.
Consider the lattice $\Lambda(\Delta)$ generated by Δ, and note

$$\Lambda(\Delta) \subset \Lambda(f)$$

Now:

- If Δ is large enough, then perhaps $f \in \Lambda(\Delta)$.
- If the rank of $\Lambda(\Delta)$ is small enough, the attacker may find f with lattice reduction.
- The other private key components can be calculated from f and the public key.

Making the lattice reduction feasible

The size of the attacker's lattice

- If the attacker's lattice $\Lambda(\Delta)$ is large, there is a better chance it contains the secret key component f.
- If it is smaller, lattice reduction is easier.
- Injecting a fault in one of the final five PRNG calls apparently works.

Two types of faults

Note: A fault can either affect the output of only one call to the PRNG or its internal state and hence also all future calls to the PRNG.

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Properties of the attack

Liberal fault model

Any fault that changes something in the PRNG works (instruction skip, data fault).

Combination with exhaustive search

Recall that the attacker works in $\Lambda(\Delta) \subset \Lambda(f)$.

So lattice reduction with Δ may not yield f but cf, with some polynomial c.

If $c(X) = X^k$, then cf is just a rotation of f. In this case, the attacker obtains an equivalent private key.

If c is sparse, the attacker can try to find c (and hence f) by exhaustive search.

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Code example on a Cortex-M4

Code from [LPa17], file `rng.c`, function `falcon_inner_prng_refill()`.

C code:

```
uint64_t cc;
uint32_t state[16];
...
state[14] ^= (uint32_t)cc;
```

Assembly code:

```
ldr     r2, [sp, #32]
ldr     r3, [sp, #136]
...
eors    r3, r2
```

Randomly changing r2 before eors is a suitable fault for the attack.
(Tested with gdb on a Cortex-M4 target. Successful key recovery with 100 faulty signatures. More details in the paper.)

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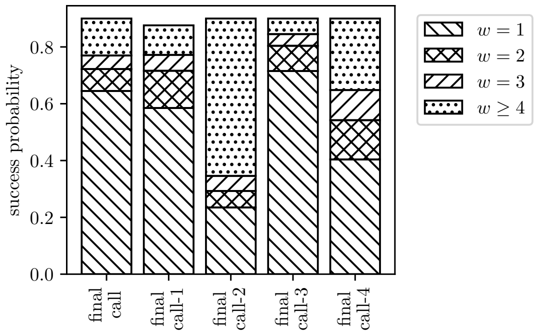
Simulation results

Attack with 50 faults against FALCON-512 simulated on a PC, the faults affected the PRNG seed, i.e. had a persistent effect. w is the number of non-zero coefficients of c, where cf is the polynomial recovered by lattice reduction.



More pictures in the paper.

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Countermeasures

- Note: Faulty signatures are valid. So, verification does not work as a countermeasure.
- Re-calculation of signatures or at least ffSampling is expensive and works only if the attacker cannot inject the same fault twice.
- Calculate a checksum over the PRNG output, then re-run the PRNG, re-calculate the checksum and compare.

Summary and outlook

Summary

- We have seen a fault attack against deterministic FALCON signature generation.
- The attack works under a very liberal fault model (any fault in the PRNG used for sampling will do).
- It has a high success rate.
- The attack can be combined with an exhaustive search step to reduce the number of faults required or increase the success rate.

Future work

- Transfer results to other signatures based on GPV framework (Mitaka, ModFalcon)
- Investigate suppression of entropy in standard FALCON to make the attack applicable in the non-deterministic setting.

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