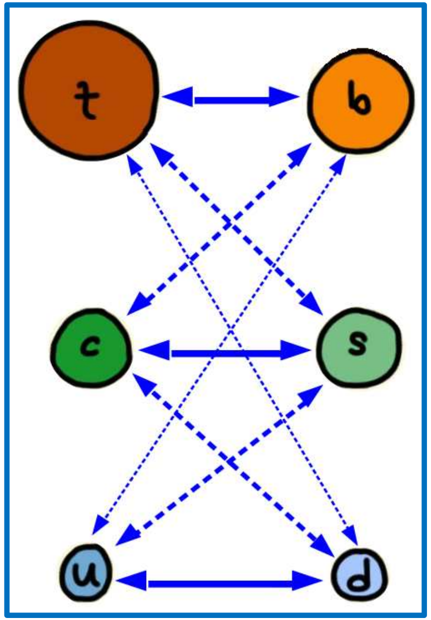
LECTURE 4: RARE DECAYS

Learning goals

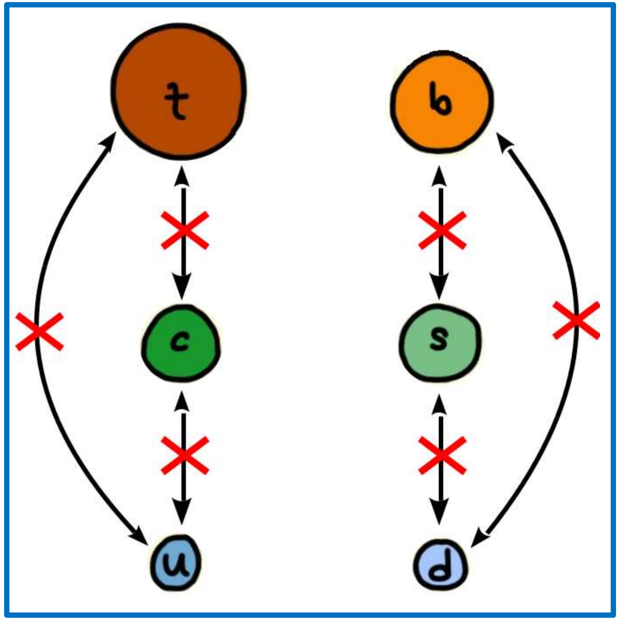
- what are rare decays?
- sketch of theory of rare decays
- some ‘recent’ highlights in rare B decays
    - Bs->mumu
    - Bd->K*gamma
    - lepton-flavour violation tests

Standard Model: “No FCNC at tree level”

CKM: Flavour changing *charged* currents



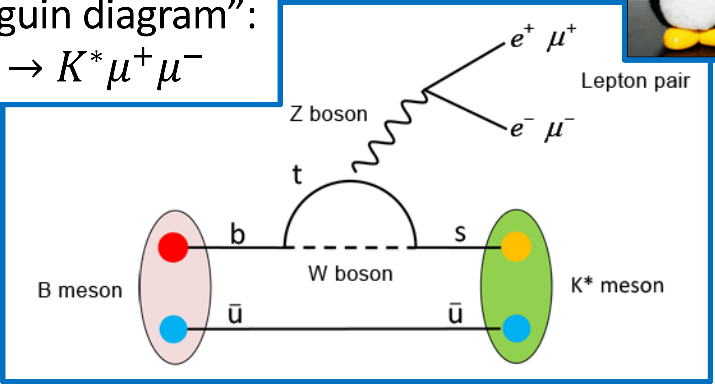
No Flavour changing *neutral* currents



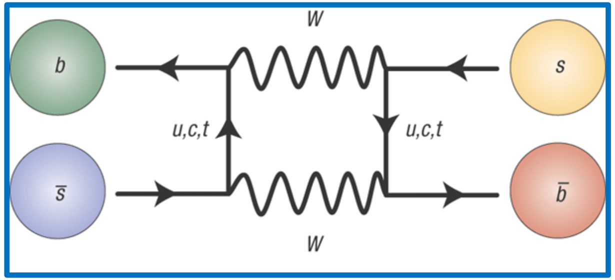
FCNC at loop level

- neutral currents are possible at higher order

Decay via
"Penguin diagram":
$B \rightarrow K^{*}\mu^{+}\mu^{-}$



Flavour Oscillation
via "Box diagram":
$\overline{B_s} \rightarrow B_s$



- we call them 'rare'
  - higher order
  - often 'GIM suppressed' (cancellation due to unitarity)

Examples of rare decays

- very incomplete table

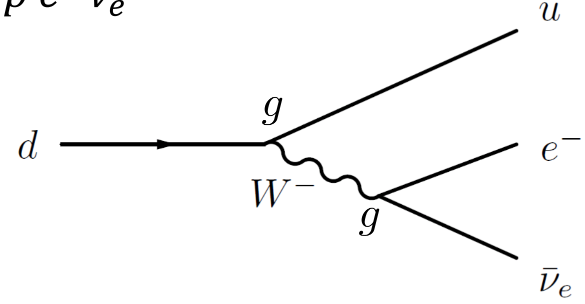
transition	example of decays
$b \to s\gamma$	$B^0 \to K^{*0}\gamma$
$b \to s\ell^+\ell^-$	$B_s \to \mu^+\mu^-$, $B^0 \to K^{*0}\mu^+\mu^-$, $B^+ \to K^+\mu^+\mu^-$
$b \to sq\bar{q}$	$B_d \to K\pi$, $B_s \to \phi\pi$
$b \to d\ell^+\ell^-$	$B^0 \to \rho^0\mu^+\mu^-$, $B_d \to \mu^+\mu^-$
$s \to d\gamma$	$K_L \to \gamma\gamma$
$s \to d\ell^+\ell^-$	$K_L \to \mu^+\mu^-$, $K_L \to \pi^0 e^+ e^-$, $K^+ \to \pi^0\mu^+\mu^-$
$s \to d\nu\bar{\nu}$	$K_L \to \pi^0\nu\bar{\nu}$, $K^+ \to \pi^+\nu\bar{\nu}$

- branching fractions typically smaller than $\sim 10^{-5}$, some much much smaller

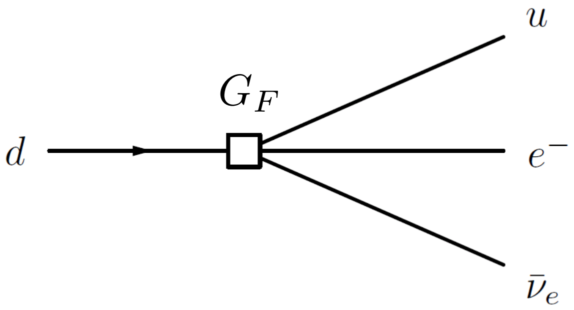
Effective couplings

- Beta decay: "charged current":

$n \to p\, e^- \bar{\nu}_e$



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$



$$\mathcal{M} = \left[\frac{gV_{ud}}{\sqrt{2}}\overline{u}_L\gamma^\mu d_L\right] \frac{g_{\mu\nu} - p_\mu p_\nu/M_W^2}{p^2 - M_W^2} \left[\frac{g}{\sqrt{2}}\overline{e}_L\gamma^\nu \nu_L\right]$$

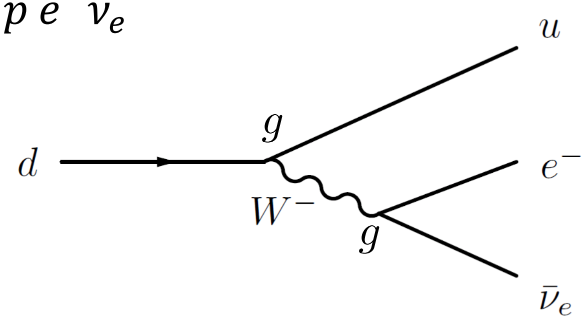
$$\mathcal{M} = \frac{V_{ud}G_F}{\sqrt{2}} \left[\overline{u}_L\gamma^\mu d_L\right] \left[\overline{e}_L\gamma_\mu \nu_L\right]$$

Effective theory: exploit *separation of scales*

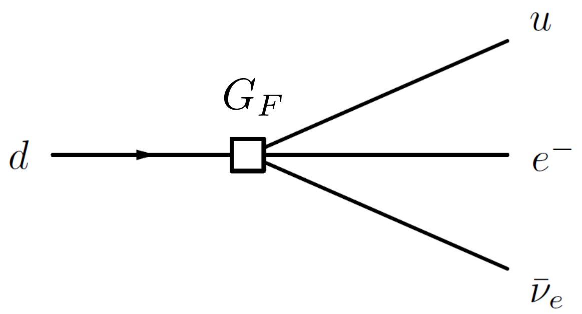
Effective couplings

- Beta decay: "charged current":

$n \rightarrow p\, e^{-} \overline{\nu}_e$

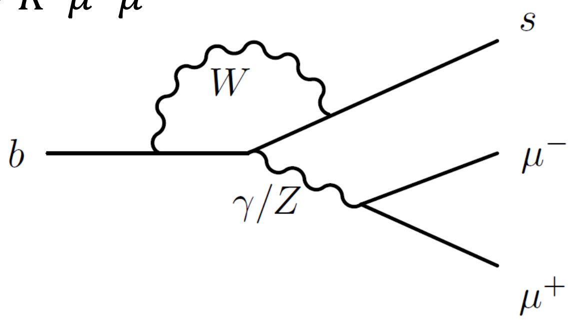


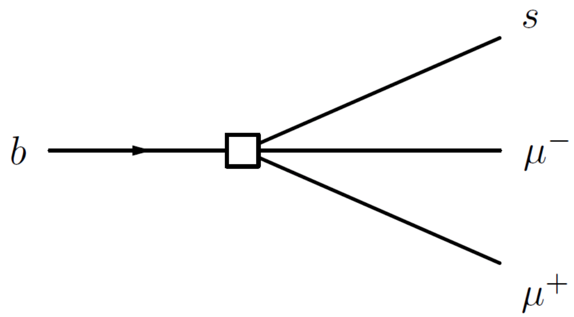
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$



- Rare B decay: "Flavour changing neutral current":

$B^0 \rightarrow K^{*} \mu^{+} \mu^{-}$

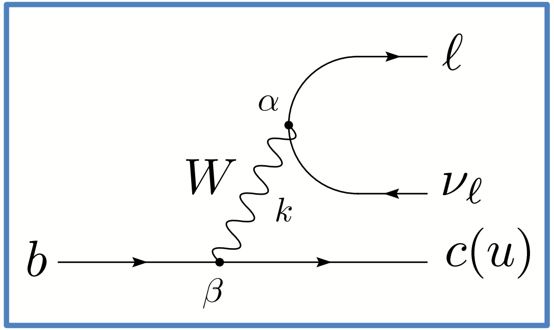




Dealing with bound states

- consider “$B \rightarrow D\, l\, \nu$”

quark level process



$$\mathcal{A}(i \to f) \;=\; \langle f|\,\mathcal{H}\,|i\rangle$$

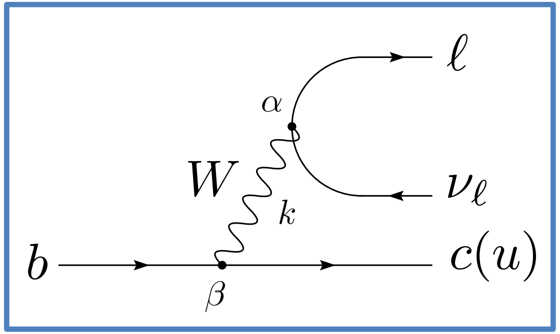
$$\Gamma(i \to f) \;=\; \int \;|\mathcal{A}(i \to f)|^2 \;\; \mathrm{d(phase\ space)}$$

$$\mathcal{A}(b \to c\ell\bar{\nu}) \;=\; \frac{G_F}{\sqrt{2}} V_{cb}\, [\bar{\ell}\gamma^\mu(1-\gamma^5)\nu]\, [\bar{c}\gamma_\mu(1-\gamma^5)b]$$

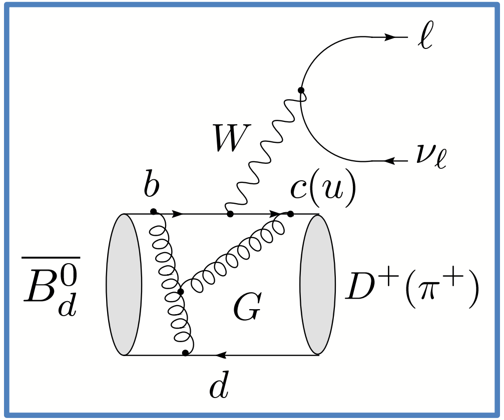
Dealing with bound states

- consider "$B \to D\, l\, \nu$"

quark level process



hadron level process



$$\mathcal{A}(b \to c\ell\bar{\nu}) \;=\; \frac{G_F}{\sqrt{2}} V_{cb} \left[\bar{\ell}\gamma^{\mu}(1-\gamma^5)\nu\right] \left[\bar{c}\gamma_{\mu}(1-\gamma^5)b\right]$$

$$\mathcal{A}(B^0 \to D^+\ell\bar{\nu}) \;=\; \left\langle D^+\ell\bar{\nu}\right| \mathcal{H} \left|B^0\right\rangle \;=\; ?$$

Dealing with bound states

- sketch of solution (no formal theory!)

$$\mathcal{A}(B^0 \to D^+ \ell \bar{\nu}) \;=\; \frac{G_F}{\sqrt{2}} V_{cb} \times \langle D^+ \ell \bar{\nu} | \, [\bar{\ell} \gamma^\mu (1-\gamma^5) \nu] \, [\bar{c} \gamma_\mu (1-\gamma^5) b] \, | B^0 \rangle$$

coefficient:
- can be computed in SM

local operator:
- computation involves QCD

General solution: *operator product expansion*

- approximate H with effective Hamiltonian that integrates out 'all heavy stuff', **not just the W, but also the top**, etc

$$\mathcal{A}(i \to f) = \langle f | \mathcal{H} | i \rangle \to \langle f | \mathcal{H}_{\rm eff} | i \rangle$$

scale μ: usually taken to be the meson mass

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_i V_i^{\rm CKM} \, C_i(\mu) \, O_i(\mu)$$

"short distance" Wilson coefficient:
- stuff having to do with scales > μ

local operator:
- stuff having to do with scales < μ

General solution: *operator product expansion*

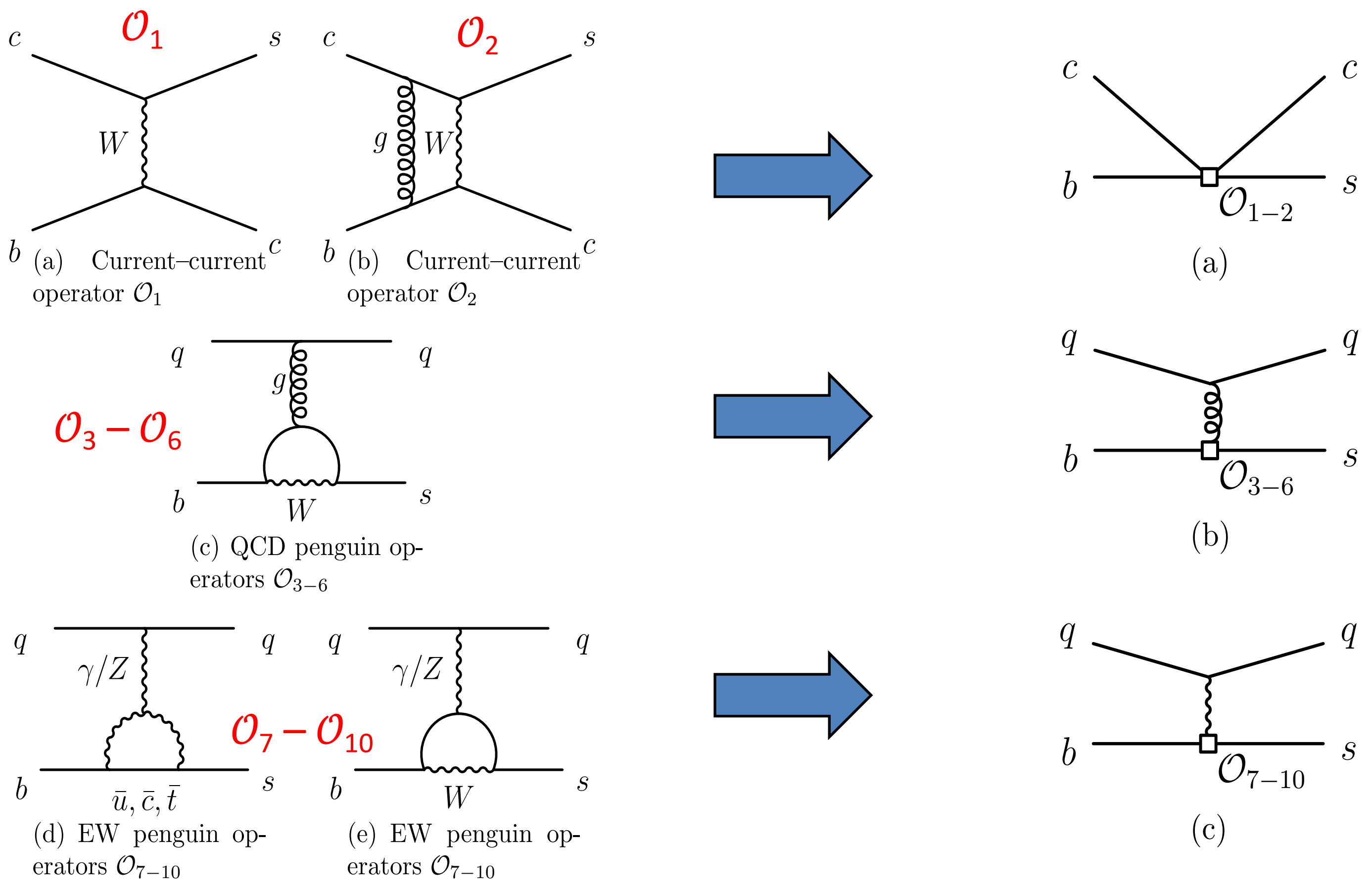
$$\mathcal{A}(i \to f) \;=\; \frac{G_F}{\sqrt{2}} \sum_i \underbrace{V_i^{\rm CKM}\; C_i(\mu)}\;\; \underbrace{\langle f|\, O_i(\mu)\, |i\rangle}$$

“short distance” Wilson coefficient:
- perturbative: SM computation ‘easy’
- sensitive to New Physics

“long distance” matrix element
- non-perturbative: difficult
- not sensitive to New Physics

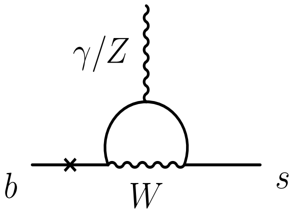
- Wilson coefficients and matrix elements depend on scale ‘mu’
  - computations need to ‘match’, such that mu-dependence cancels
- matrix elements are hard to compute but effective approximations available: “heavy quark effective theory”, “lattice calculations”, etc

Rare B-decays and effective couplings: $b \to sq\bar{q}$



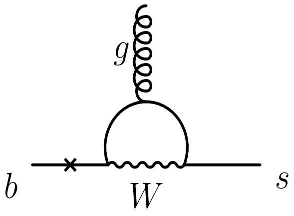
Rare B-decays and effective couplings: $b \to s l^+ l^-$

$\mathcal{O}_{7\gamma}$

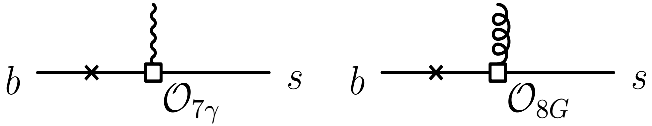


(f) Electromagnetic penguin operator $\mathcal{O}_{7\gamma}$

$\mathcal{O}_{8G}$

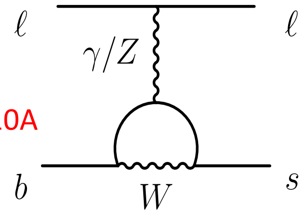


(g) Chromomagnetic penguin operator $\mathcal{O}_{8G}$

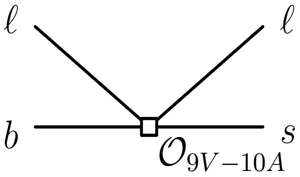


(d) (e)

$\mathcal{O}_{9V} - \mathcal{O}_{10A}$



(h) Semi-leptonic operators $\mathcal{O}_{9V-10A}$



(f)

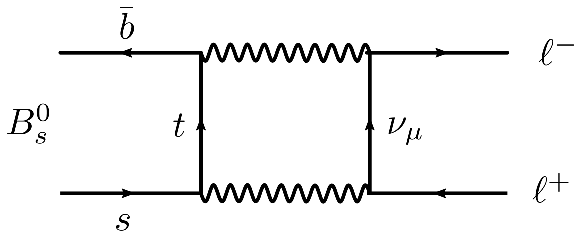
Effects of 'new physics'

$$\mathcal{A}(i \to f) = \frac{G_F}{\sqrt{2}} \sum_i V_i^{\mathrm{CKM}} \, C_i(\mu) \, \langle f | \, O_i(\mu) \, | i \rangle$$

- new 'heavy' particles only affect scales > mu
  - → change Wilson coefficients

- new physics may also lead to local operators that are absent in SM
  - e.g. with scalar bosons or right-handed currents
  - lead to different `kinematics' of final state particles

Fully leptonic

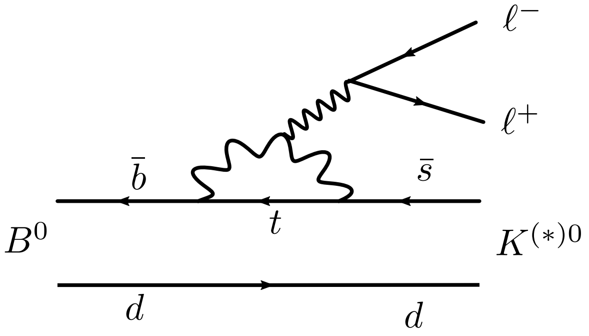


Very rare! $\mathcal{B} \lesssim 10^{-9}$

- Theoretically clean
- Mostly clean to reconstruct

Sensitive mainly to $\mathcal{C}_{10}^{(\prime)}$**.**

Semi-leptonic

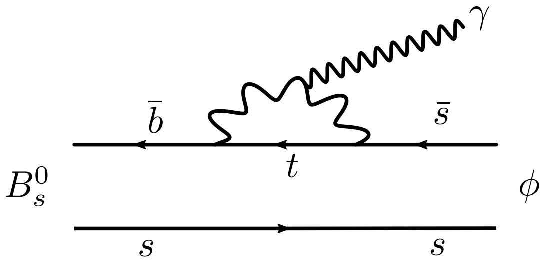


Quite rare, $\mathcal{B} \sim 10^{-6}$

- Hadronic pollution.
- Mostly clean to reconstruct.
- Electron reconstruction very challenging.

Sensitive to $\mathcal{C}_7^{(\prime)}$**,** $\mathcal{C}_9^{(\prime)}$ **and** $\mathcal{C}_{10}^{(\prime)}$ depending on $q^2 \equiv m^2_{\ell^+\ell^-}$ region.

Radiative



Fairly rare, $\mathcal{B} \sim 10^{-5}$

- Similar to semi-leptonic.
- Experimental resolution not great.

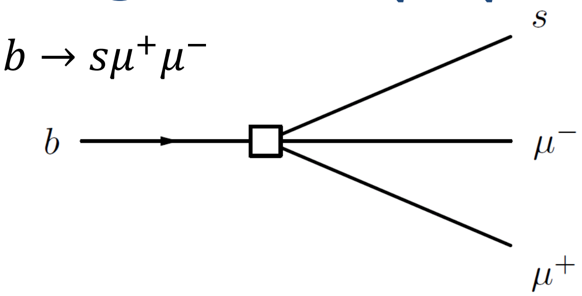
Sensitive to $\mathcal{C}_7^{(\prime)}$**.**

(from Riley Henderson at FPCP'23)

Rare B-decays and effective couplings: $b \to s\mu^+\mu^-$

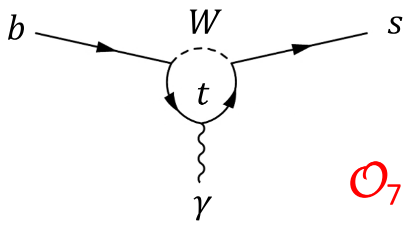
- Effective 4-fermion coupling:

$$\mathcal{H}_{eff} = -\frac{4\,G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} \mathcal{C}_i \mathcal{O}_i$$

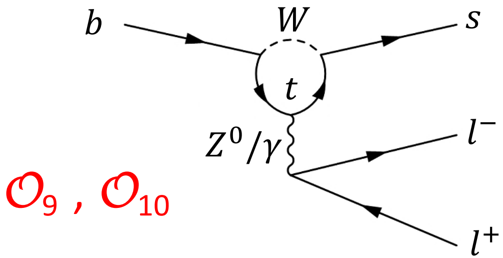


- Standard Model diagrams:

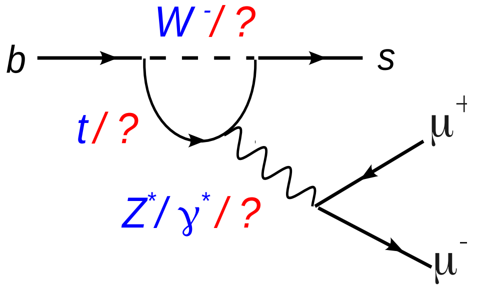
Photon penguin:



Vector, Axial vector:



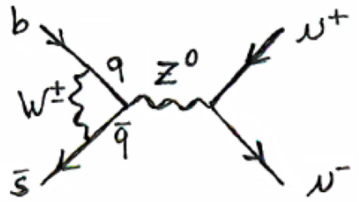
- Beyond Standard Model:



- Experimental test: Compare calculable C_i coefficients to experimental data
  - Sensitivity for NP in Wilson coefficients C_7, C_9, C_{10}

$B_{s,d} \rightarrow \mu^+\mu^-$

- very rare decay in SM: FCNC, helicity suppressed
- precise SM calculation (update!)



- $B(B_s^0 \rightarrow \mu^+\mu^-) = (3.66 \pm 0.14) \times 10^{-9}$
- $B(B^0 \rightarrow \mu^+\mu^-) = (1.03 \pm 0.05) \times 10^{-10}$

PRL 112 (2014) 101801
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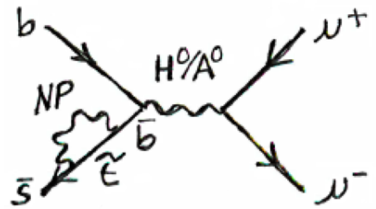
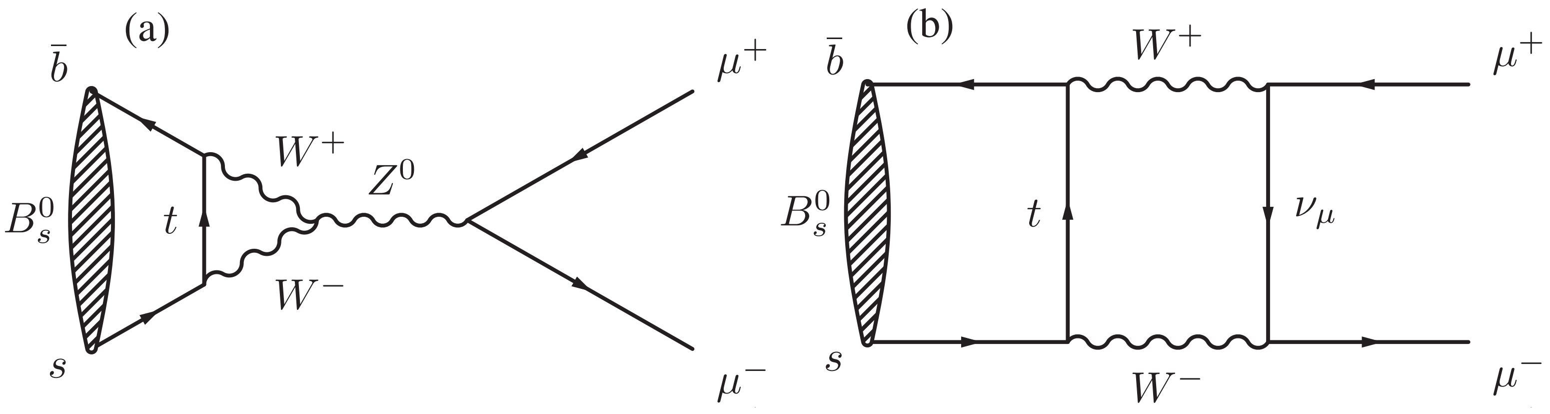


- considered very sensitive to new physics (SUSY etc)

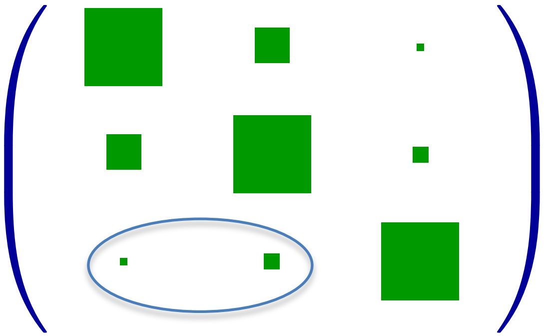
- “clean/easy” experimental signature: just count!

Q: why is the Bd decay more rare than the Bs decay?



$\mathbf{B_s}$: $V_{ts}\, V_{tb}^{*}$

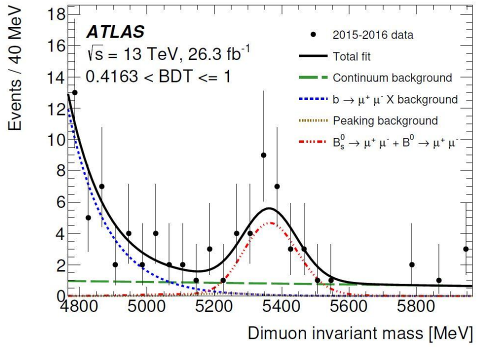
$\mathbf{B_d}$: $V_{td}\, V_{tb}^{*}$



$B_{s,d} \to \mu^+\mu^-$

- observed signals at the LHC:

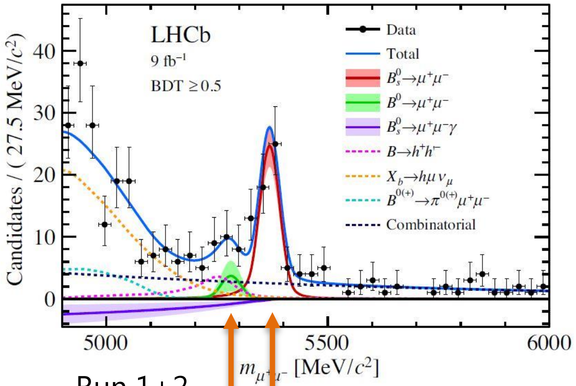
JHEP04(2019)098



2015+16 data

PRL128(2022)041801

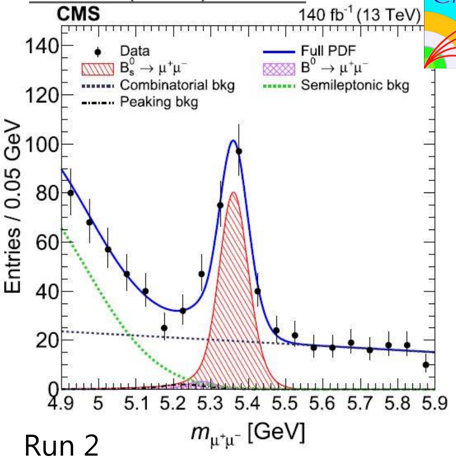
PRD105(2022)012010



Run 1+2

$B_d \to \mu^+\mu^-$ | $B_s \to \mu^+\mu^-$

PLB842(2023)137955

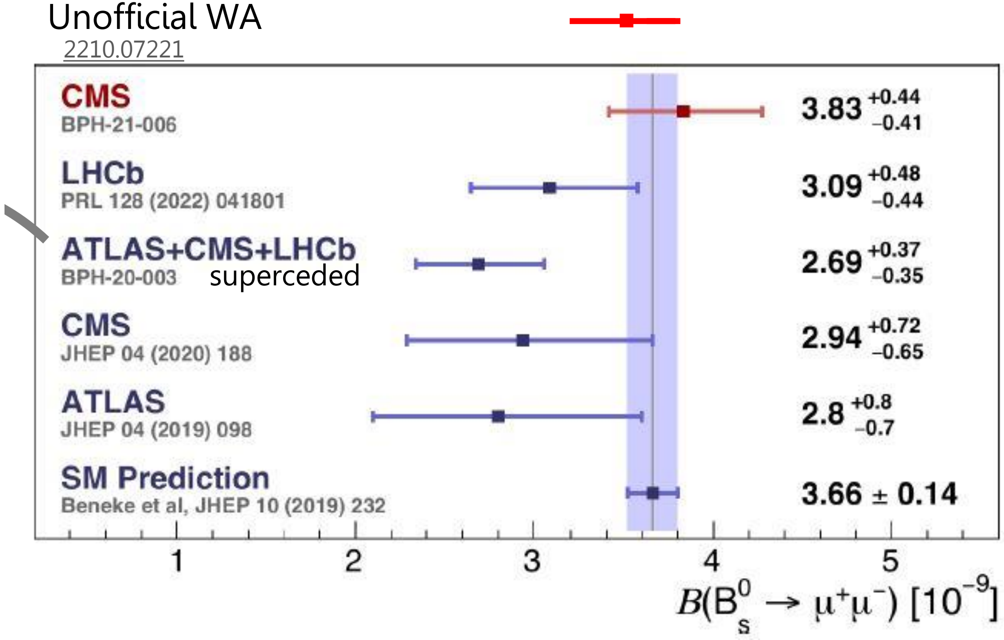


Run 2

- note the importance of good mass resolution!

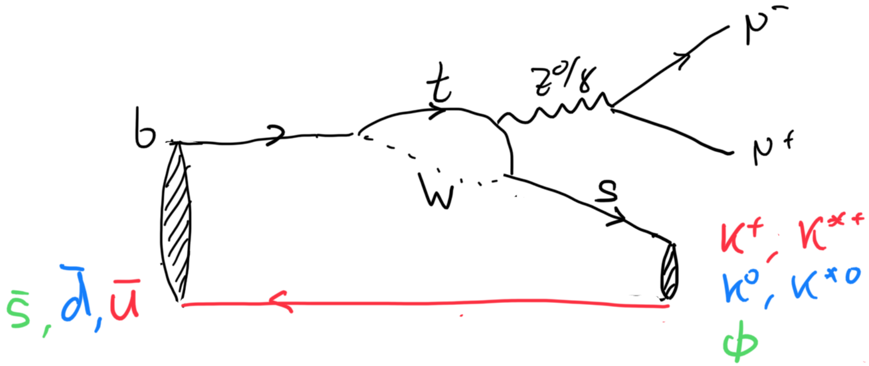
$B_{s,d} \to \mu^+\mu^-$

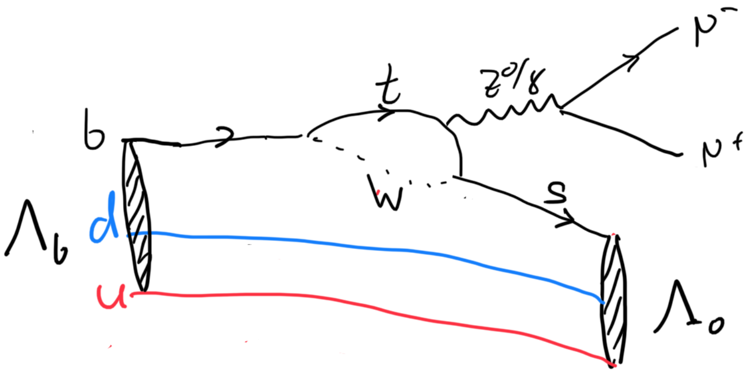
Unofficial WA
2210.07221



- good agreement between experiment and theory
- non-SM contributions not much more than 15%
  - → strong contraints on BSM physics
- no clear evidence yet for Bd->μμ

$B_d \rightarrow K^{*0}\mu^+\mu^-$ (and alike)

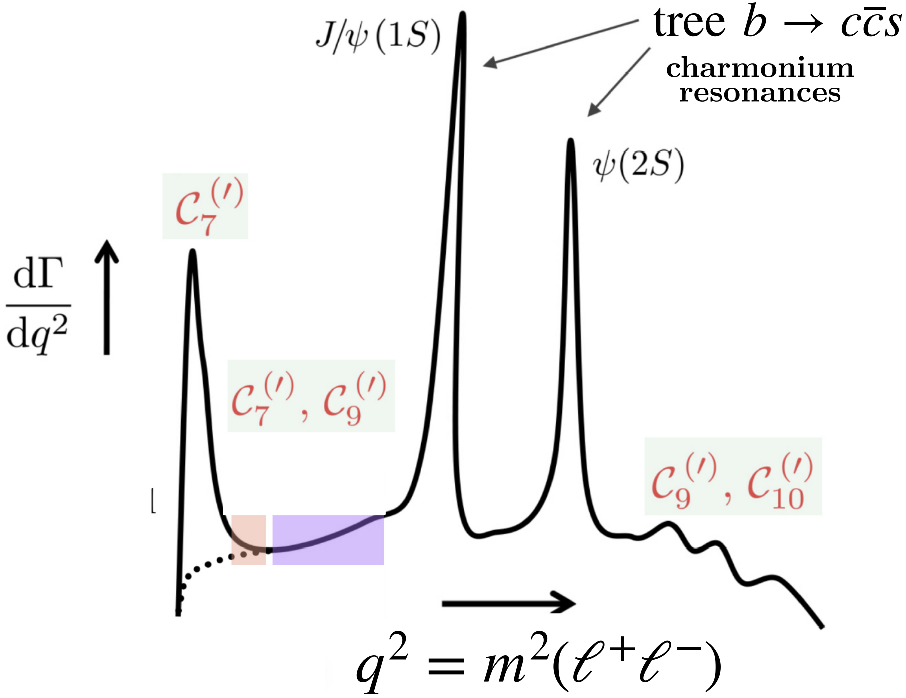


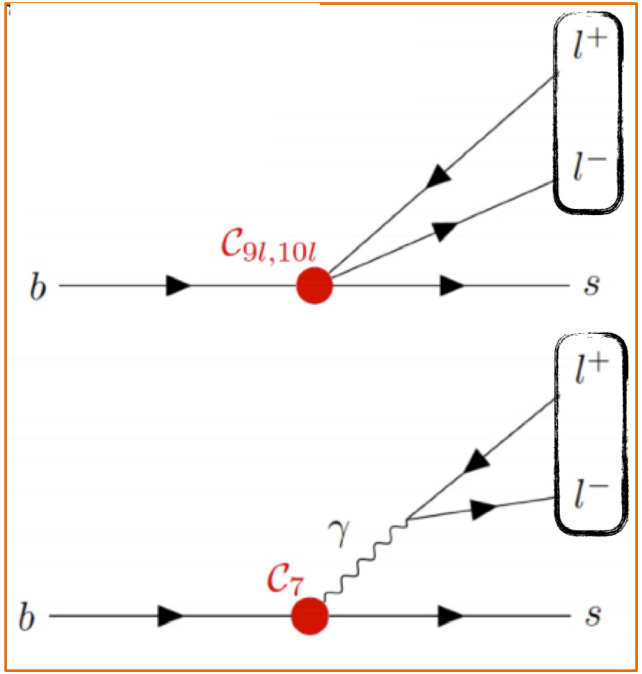


- invariant mass-squared of μμ pair is called “q-squared”:

$$q^2 = \left|p^\mu(\ell^+) + p^\mu(\ell^-)\right|^2$$

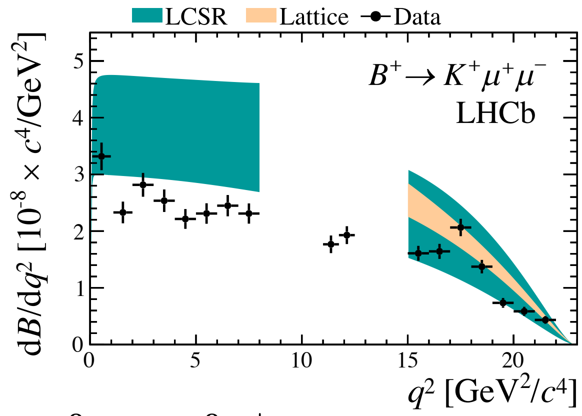
Contribution of operators depends on q^2



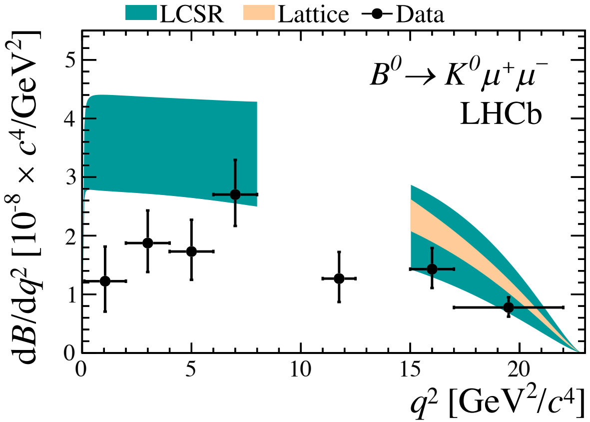


Branching fractions of Rare Decays: $b \to s\,\mu^+\mu^-$

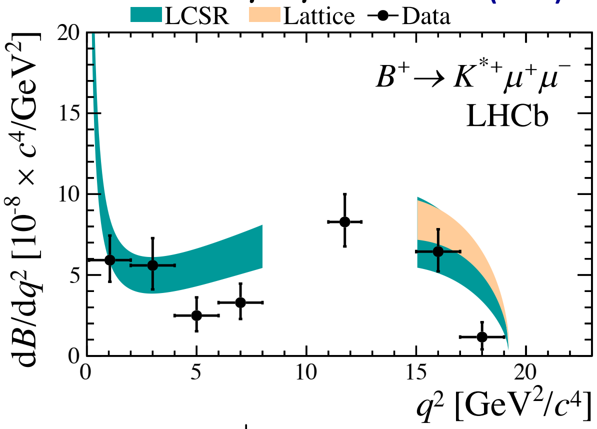
$B^+ \to K^+\mu^+\mu^-$ **JHEP 06 (2014) 133**



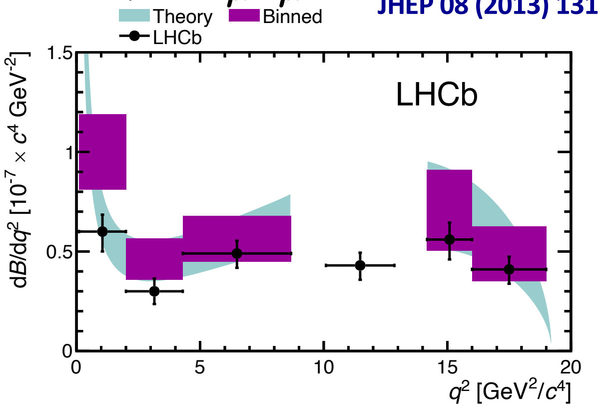
$B^0 \to K^0\mu^+\mu^-$ **JHEP 06 (2014) 133**



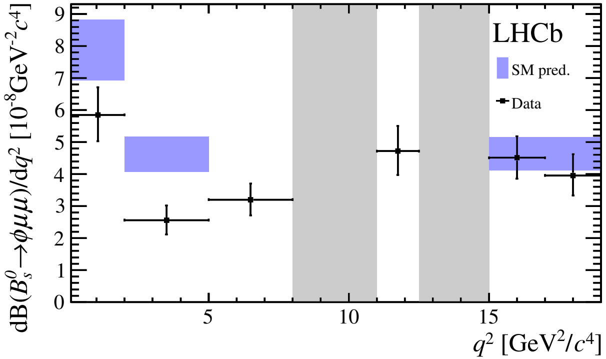
$B^+ \to K^{*+}\mu^+\mu^-$ **JHEP 06 (2014) 133**



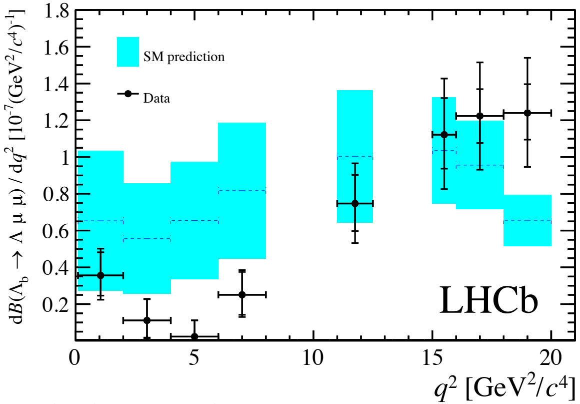
$B^0 \to K^{*0}\mu^+\mu^-$ **JHEP 08 (2013) 131**



$B_s \to \phi\mu^+\mu^-$ **JHEP 09 (2015) 179**



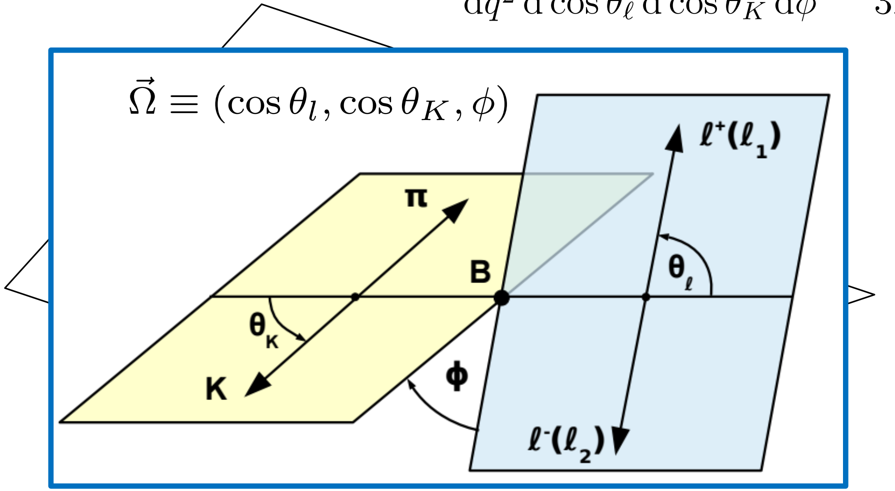
$\Lambda_b \to \Lambda\mu^+\mu^-$ **JHEP 06 (2015) 115**



- Branching fractions related to $b \to s\,\mu^+\mu^-$ transition *consistently lower* than predicted.

Angular distributions

- in >2-body decays, also ”angular distributions” sensitive to NP

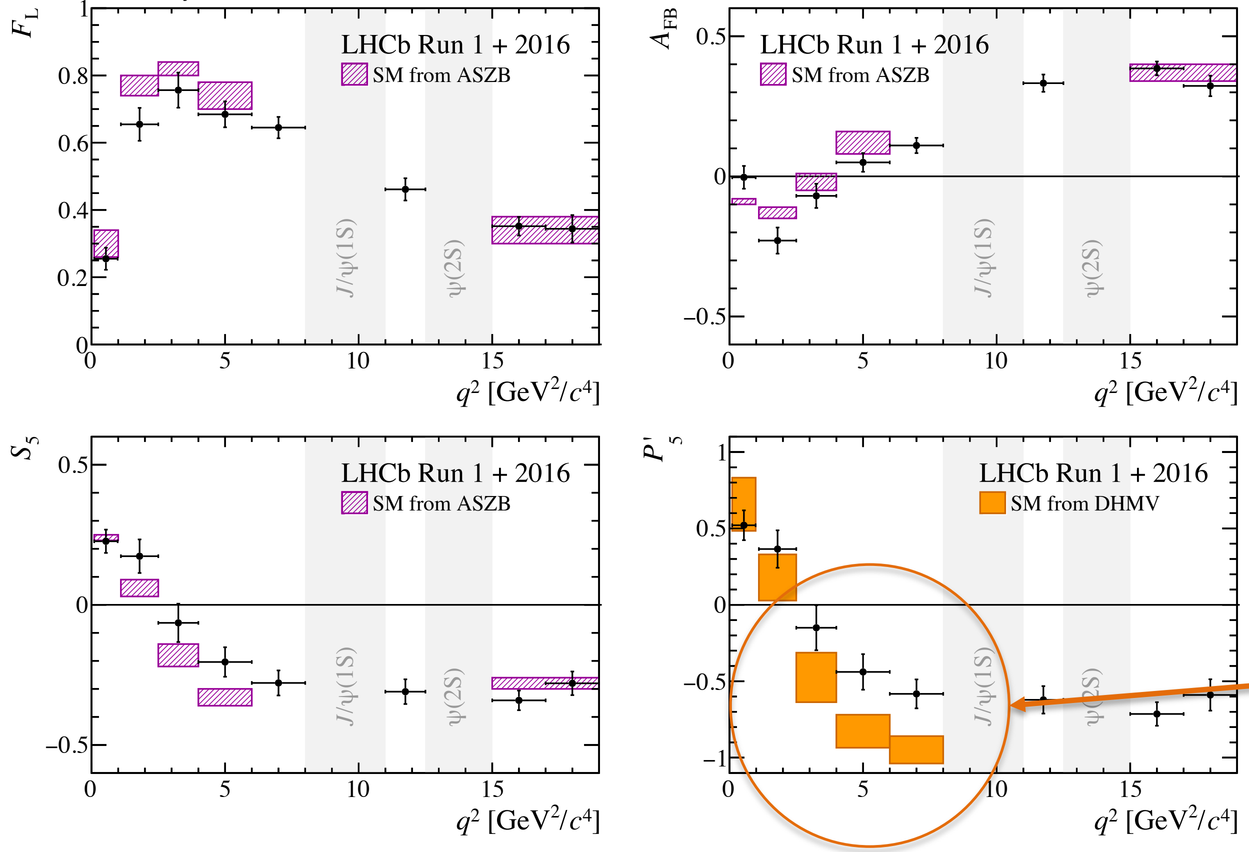


$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi}\Big[I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K + I_2^s \sin^2\theta_K \cos 2\theta_\ell + I_2^c \cos^2\theta_K \cos 2\theta_\ell + I_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + I_5 \sin 2\theta_K \sin\theta_\ell \cos\phi + I_6 \sin^2\theta_K \cos\theta_\ell + I_7 \sin 2\theta_K \sin\theta_\ell \sin\phi + I_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + I_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi \Big] ,$$

- experimental challenge: backgrounds and **angular efficiency**
- theoretical challenge: choose **observables with small hadronic uncertainties**

Example: angular distributions in $B^0 \rightarrow K^{*0}\mu\mu$

LHCb, arXiv:2003.04831

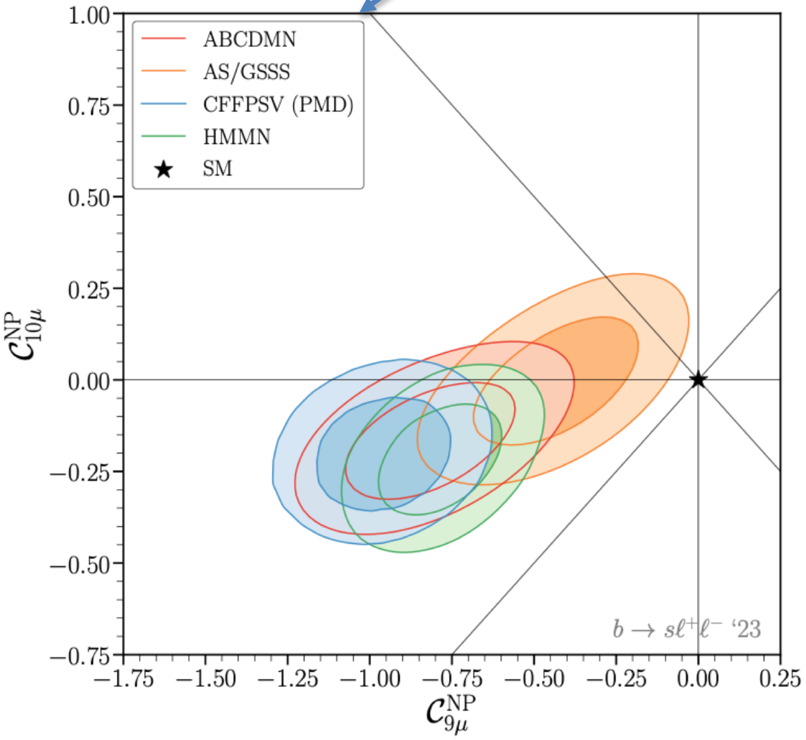


not great agreement with SM prediction

Global Fit of $b \to s\,\mu^+\mu^-$

different theoretical collaborations

- global fits: perform fits so all b->sll data, allowing for NP contributions to Wilson coefficients
- the fit seems to indicate new contributions to ‘C9’
- the ‘pull’ of the SM is about 4 sigma



Lepton universality

- SM: all leptons have ‘universal couplings’
- well tested with $W^{\pm} \to l^{\pm}\nu$ and $Z^{0} \to l^{+}l^{-}$ (e.g. at LEP and SLC)

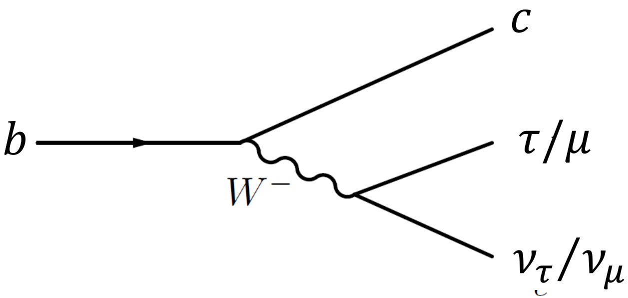
for example, branching fractions of Z to leptons from PDG:

Γ_1	e^+e^-	[1]	$(3.3632 \pm 0.0042)\%$
Γ_2	$\mu^+\mu^-$	[1]	$(3.3662 \pm 0.0066)\%$
Γ_3	$\tau^+\tau^-$	[1]	$(3.3696 \pm 0.0083)\%$

- meson decays provide additional tests, e.g. sensitivity to new forces between quarks and leptons (“lepto-quarks”)

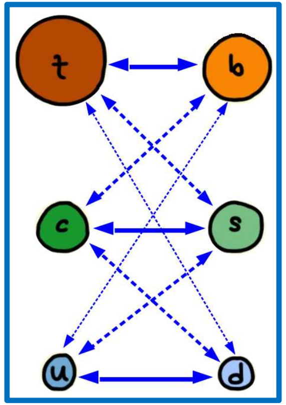
B-decays and lepton universality

- $b \to c\, l\, \nu$ charged current: "Allowed" → large decay rates

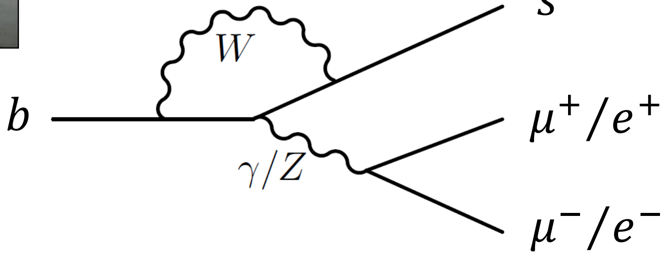


$$R_D = \frac{B \to D\tau\nu}{B \to D\mu\nu}$$

$$R_{D^*} = \frac{B \to D^*\tau\nu}{B \to D^*\mu\nu}$$

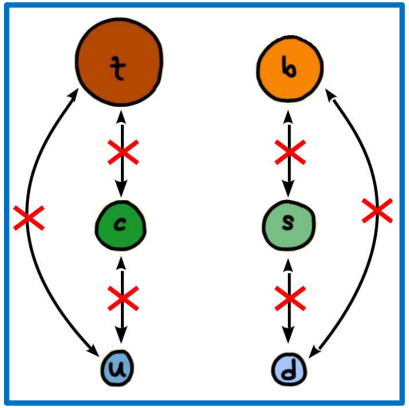


- $b \to s l^+ l^-$ neutral current: "Forbidden" → rare decays



$$R_K = \frac{B^+ \to K^+\mu^+\mu^-}{B^+ \to K^+e^+e^-}$$

$$R_{K^*} = \frac{B^0 \to K^{*0}\mu^+\mu^-}{B^0 \to K^{*0}e^+e^-}$$



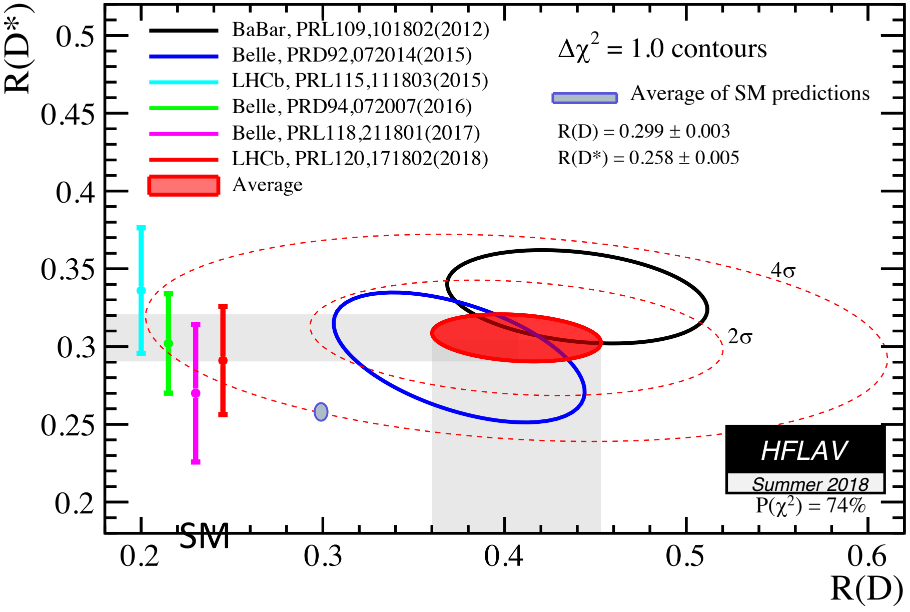
R_D and R_{D^*}

- $b \to c\, l\, \nu$ allowed
  charged current (tree level)

$$R(D^{(*)}) = \frac{BR(B \to D^{(*)}\tau\nu)}{BR(B \to D^{(*)}\mu\nu)}$$

➔Involves leptons of 2nd and 3rd generation

~ 4σ deviation



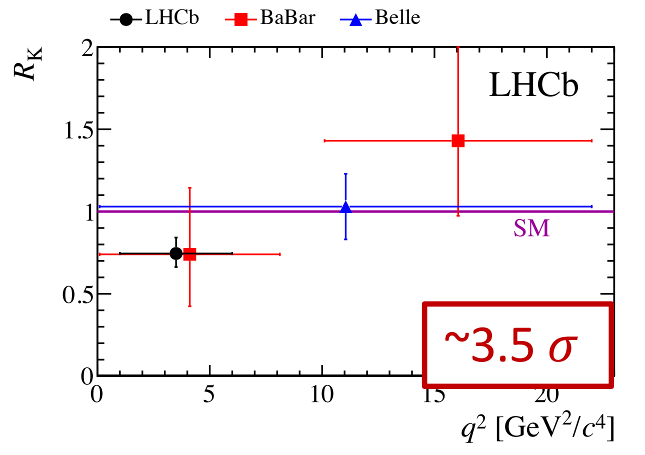
R_K and R_{K^*}

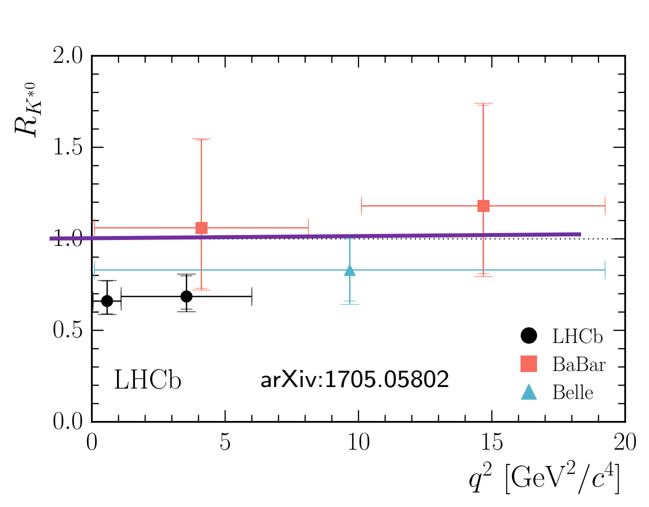
- $b \rightarrow s\, l^+ l^-$ suppressed neutral current

$$R(K) = \frac{BR(B^+ \rightarrow K^+ \mu^+ \mu^-)}{BR(B^+ \rightarrow K^+ e^+ e^-)}$$

$$R(K^*) = \frac{BR(B^0 \rightarrow K^* \mu^+ \mu^-)}{BR(B^0 \rightarrow K^* e^+ e^-)}$$

OLD





- caused some excitement in past, because LHCb seemingly found deviations from 1

- latest LHCb results are perfectly compatible with expectations

➔ involves leptons of 1st and 2nd generation

R_K and R_{K^*}

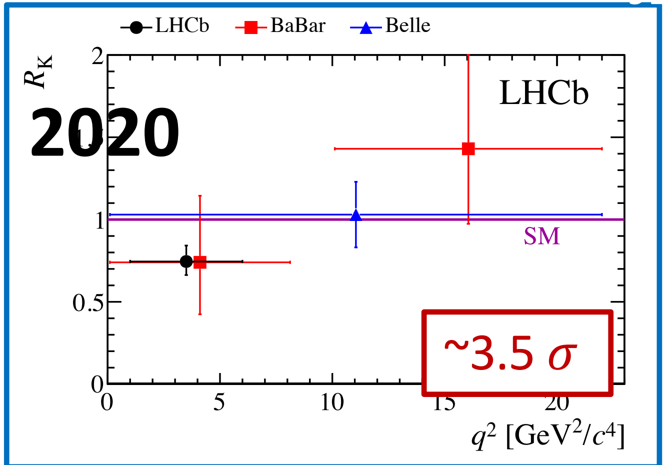
- $b \to s\, l^+ l^-$ suppressed neutral current

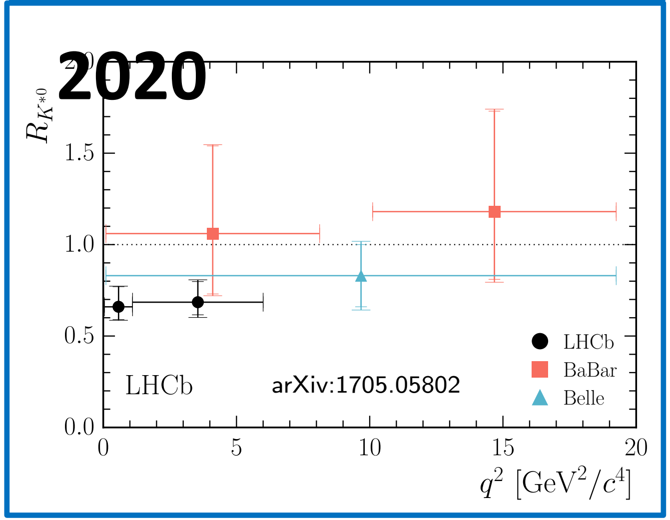
$$R(K) = \frac{BR(B^+ \to K^+ \mu^+ \mu^-)}{BR(B^+ \to K^+ e^+ e^-)}$$

$$R(K^*) = \frac{BR(B^0 \to K^* \mu^+ \mu^-)}{BR(B^0 \to K^* e^+ e^-)}$$

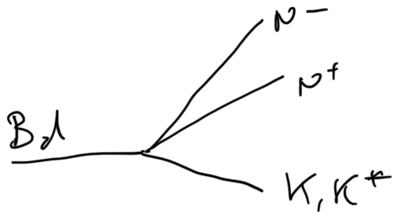
➔Involves leptons of 1st and 2nd generation

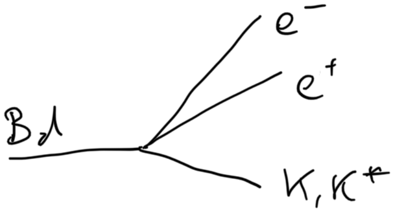
- situation until 2022: >3 sigma deviation from expectation





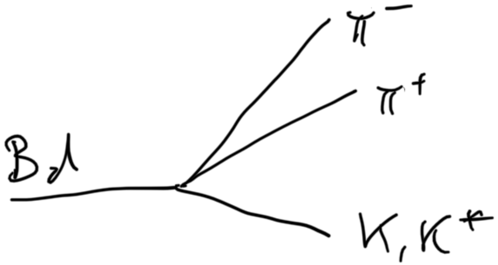
$R =$

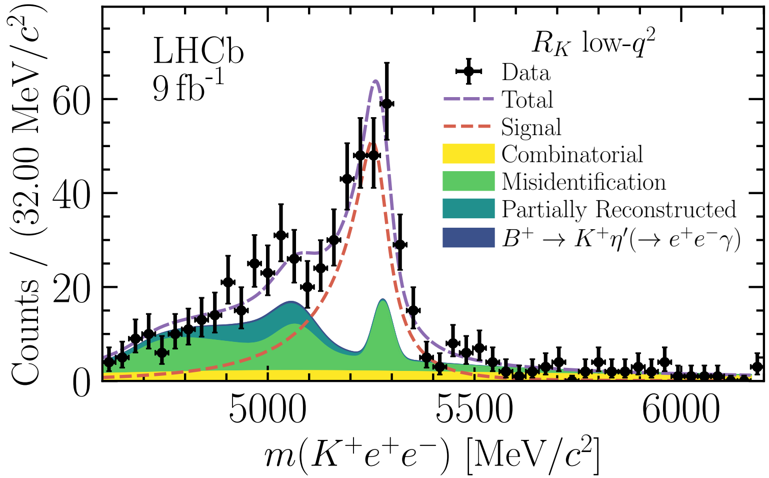




$$= \frac{\left[N^{obs}(\mu\mu) - N^{bkg}(\mu\mu)\right] / \varepsilon_{\mu\mu}}{\left[N^{obs}(ee) - N^{bkg}(ee)\right] / \varepsilon_{ee}}$$

Underestimated background:





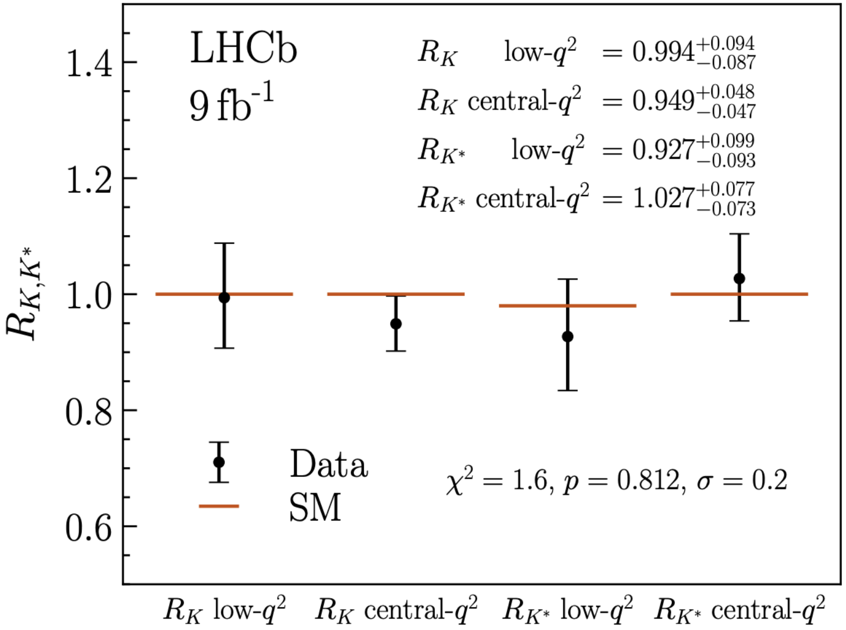
R_K and R_{K^*}

- $b \to s\, l^+ l^-$ suppressed neutral current

$$R(K) = \frac{BR(B^+ \to K^+ \mu^+ \mu^-)}{BR(B^+ \to K^+ e^+ e^-)}$$

$$R(K^*) = \frac{BR(B^0 \to K^* \mu^+ \mu^-)}{BR(B^0 \to K^* e^+ e^-)}$$

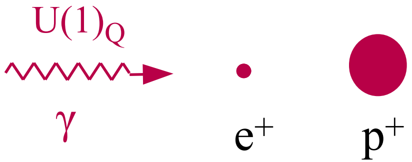
➔Involves leptons of 1st and 2nd generation



- situation today : good agreement with expectation *in this observable*

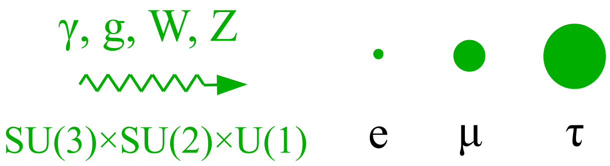
why we should keep testing "Lepton universality"

Suppose we could test matter only with long wave-length photons...



We would conclude that these two particles are "identical copies" but for their mass ...

This is exactly the same (*potentially misleading*) argument we use to infer LFU in the SM...



These three (families) of particles seems to be "identical copies" but for their mass ...

The SM quantum numbers of the three families could be an "accidental" low-energy property: the different families may well have a very different behavior at high energies, as signaled by their different mass

Isidori, 2019

Implications for low-energy flavor physics

If the anomalies are due to NP, we should expect to see several other BSM effects in low-energy observables

E.g.: correlations among down-type FCNCs [*using the results of* U(2)*-based EFT*]:

	μμ (ee)	ττ	νν	τμ	μe
$b \to s$	R_K, R_{K^*} O(20%)	$B \to K^{(*)} \tau\tau$ → 100×SM	$B \to K^{(*)} \nu\nu$ O(1)	$B \to K \tau\mu$ → ~10^{-5}	$B \to K \mu e$???
$b \to d$	$B_d \to \mu\mu$ $B \to \pi \mu\mu$ $B_s \to K^{(*)} \mu\mu$ O(20%) [R_K=R_π]	$B \to \pi \tau\tau$ → 100×SM	$B \to \pi \nu\nu$ O(1)	$B \to \pi \tau\mu$ → ~10^{-7}	$B \to \pi \mu e$???
$s \to d$	*long-distance pollution*	*NA*	$K \to \pi \nu\nu$ O(1)	*NA*	$K \to \mu e$???

Isidori, 2019

S. Fajfer, ICHEP2018

Models at TeV scale explaining both B anomalies

Scalar LQ as pseudo-Nambu-Goldstone boson

Gripaios et al, 1010.3962,
Gripaios et al., 1412.1791,
Marzocca 1803.10972…

Models with scalar LQs

Hiller & Schmaltz, 1408.1627,
Becirevic et al. 1608.08501, SF and Kosnik, 1511.06024, Becirevic et al., 1503.09024,
Dorsner et al, 1706.07779,
Cox et al., 1612.03923,
Crivellin et al.,1703.09226…

W', Z' in warped space

Megias et al.,1707.08014

Vector resonances (from techni-fermions)

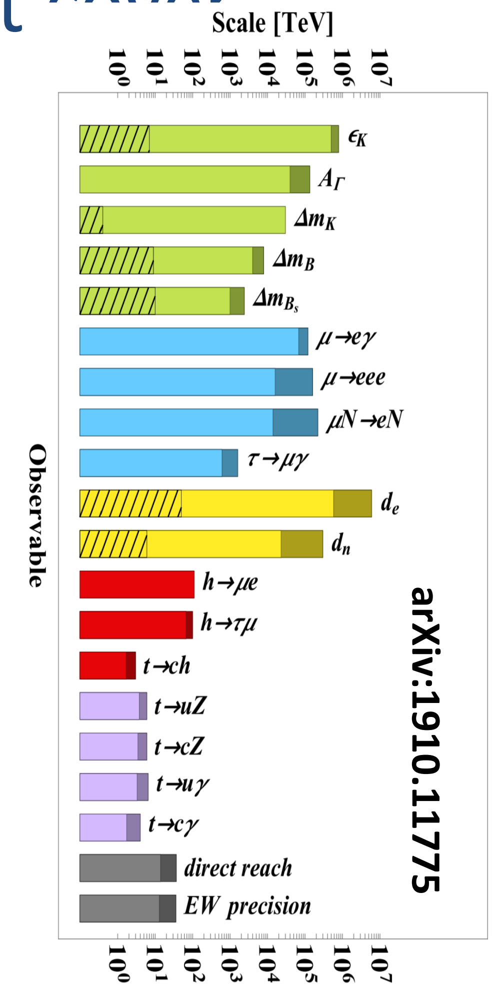
Barbieri et al.,1506.09201, Buttazzo et al. 1604.03940,
Barbieri et al., 1611.04930
Blanke & Crivellin, 1801.07256,…

Gauge bosons

Greljo et al., 1804.04642
Cline, Camalich, 1706.08510
Calibbi et al.,1709.00692
Assad et al., 1708.06350
Di Luzio et al.,1708.08450
Bordone et al.,1712.01368, 1805.09328…

Some of the things that I did not cover

- CP violation and rare decays in the Kaon sector
- lepton-number violation (e.g. $\mu \to e\gamma, \mu \to eee$)
- electric dipole moments
- g-2
- majorana neutrinos
- …



Closing remarks

- low energy measurements can be sensitive to very high mass scales

- several ‘quark flavour physics’ measurements show tension with SM predictions
  - experimental effects?
  - theoretical understanding?
  - new physics?

  → Belle-II/LHC measurements will improve a lot over coming decade

- lot’s of other exciting experiments ongoing: watch tight!