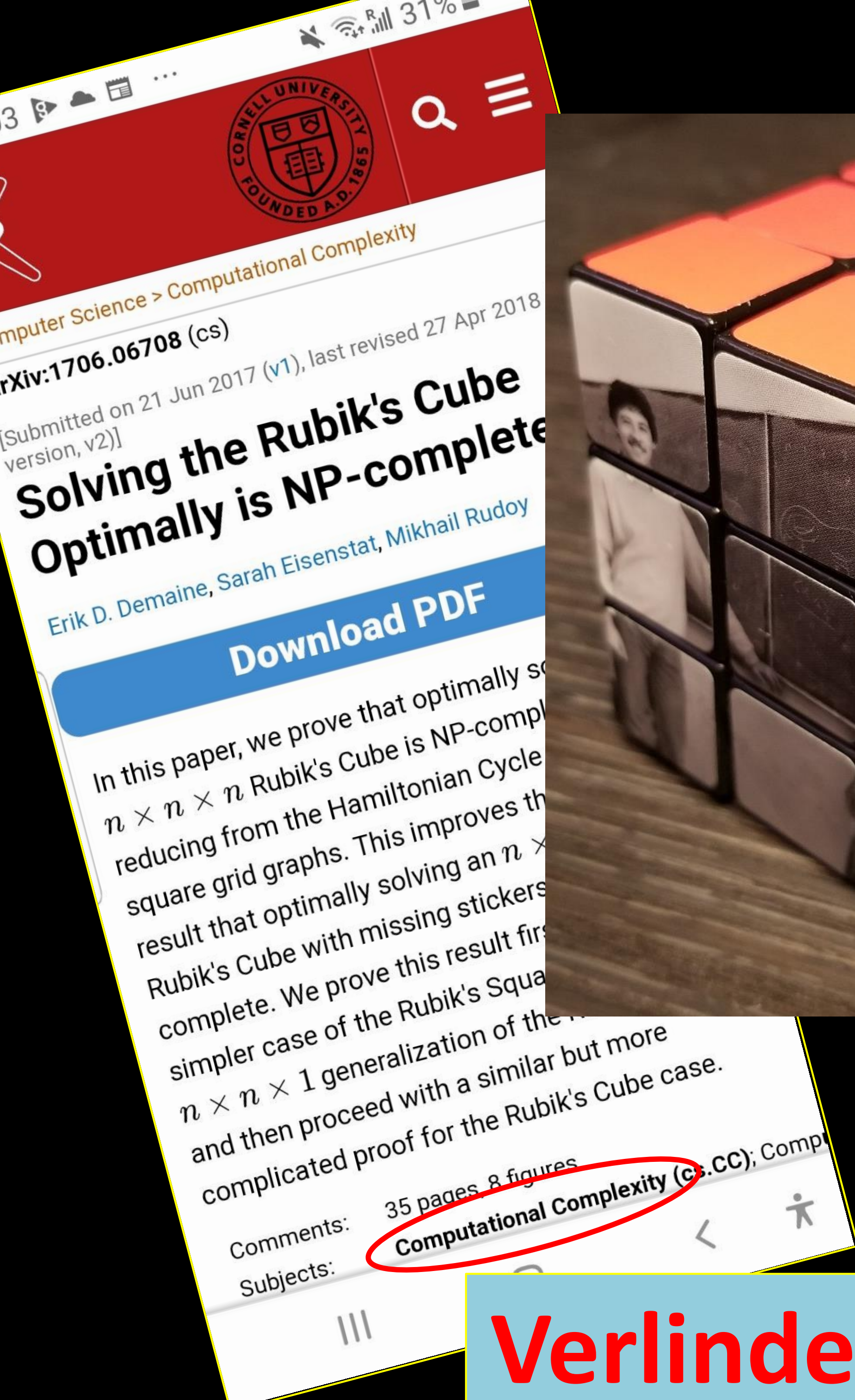
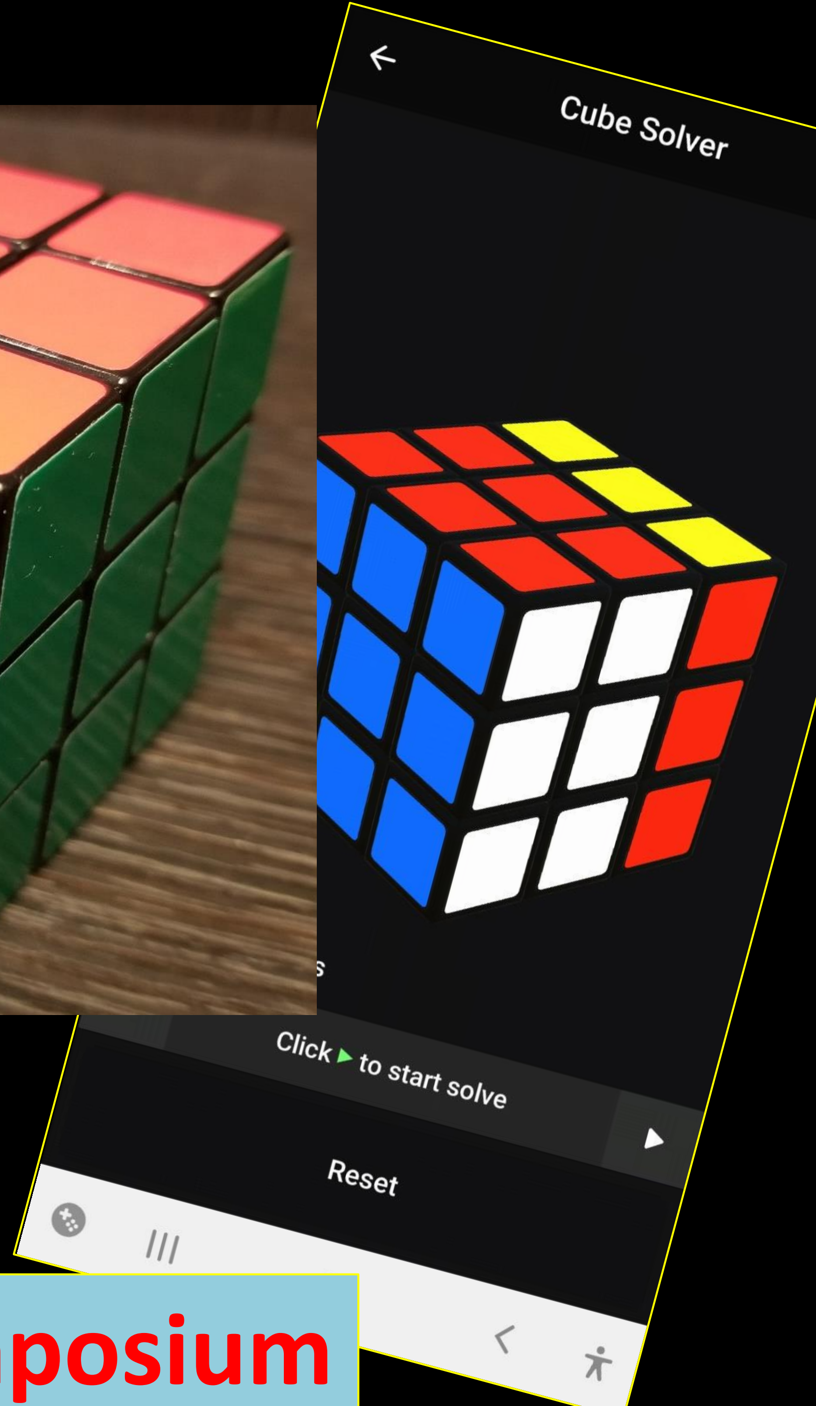
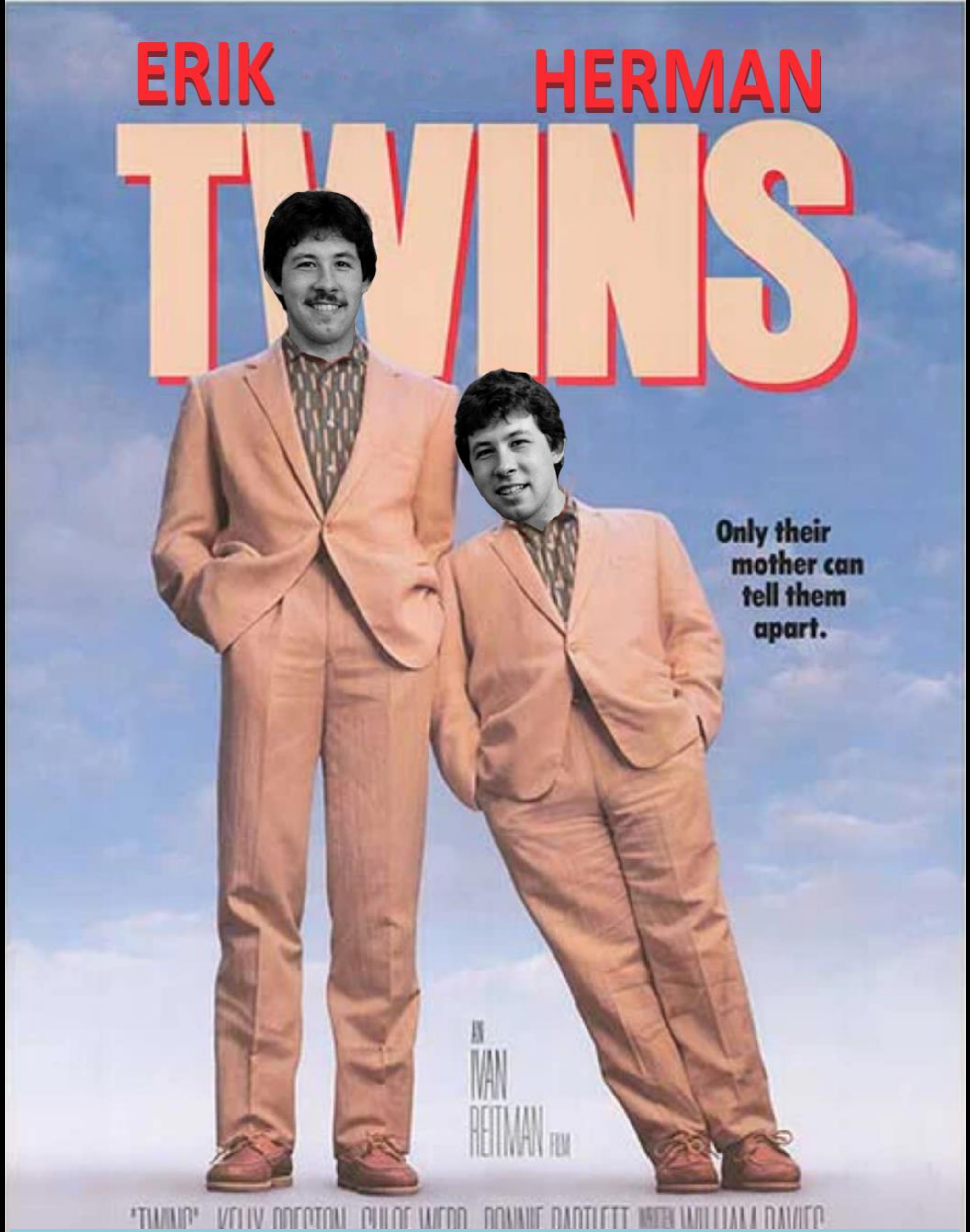


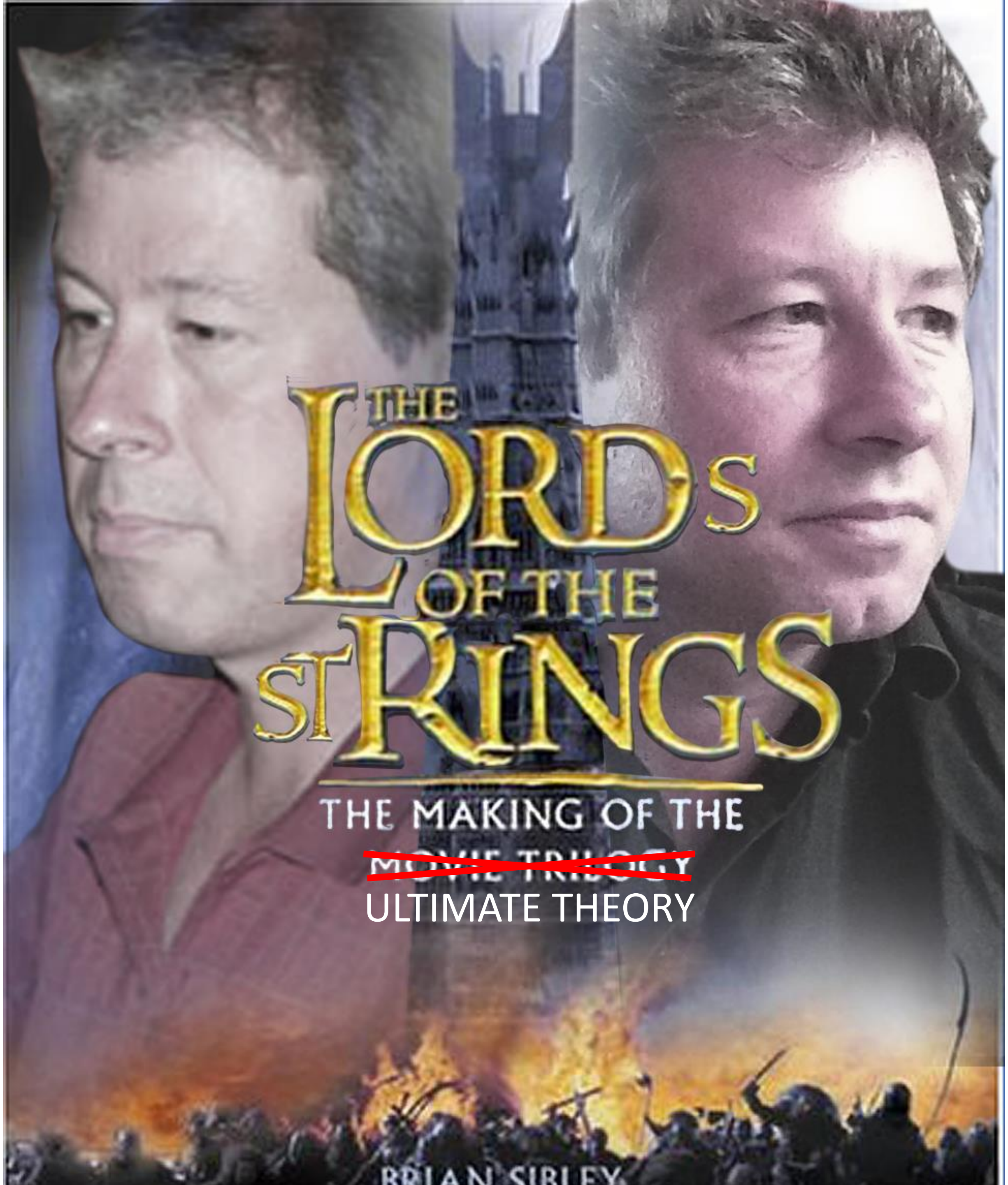


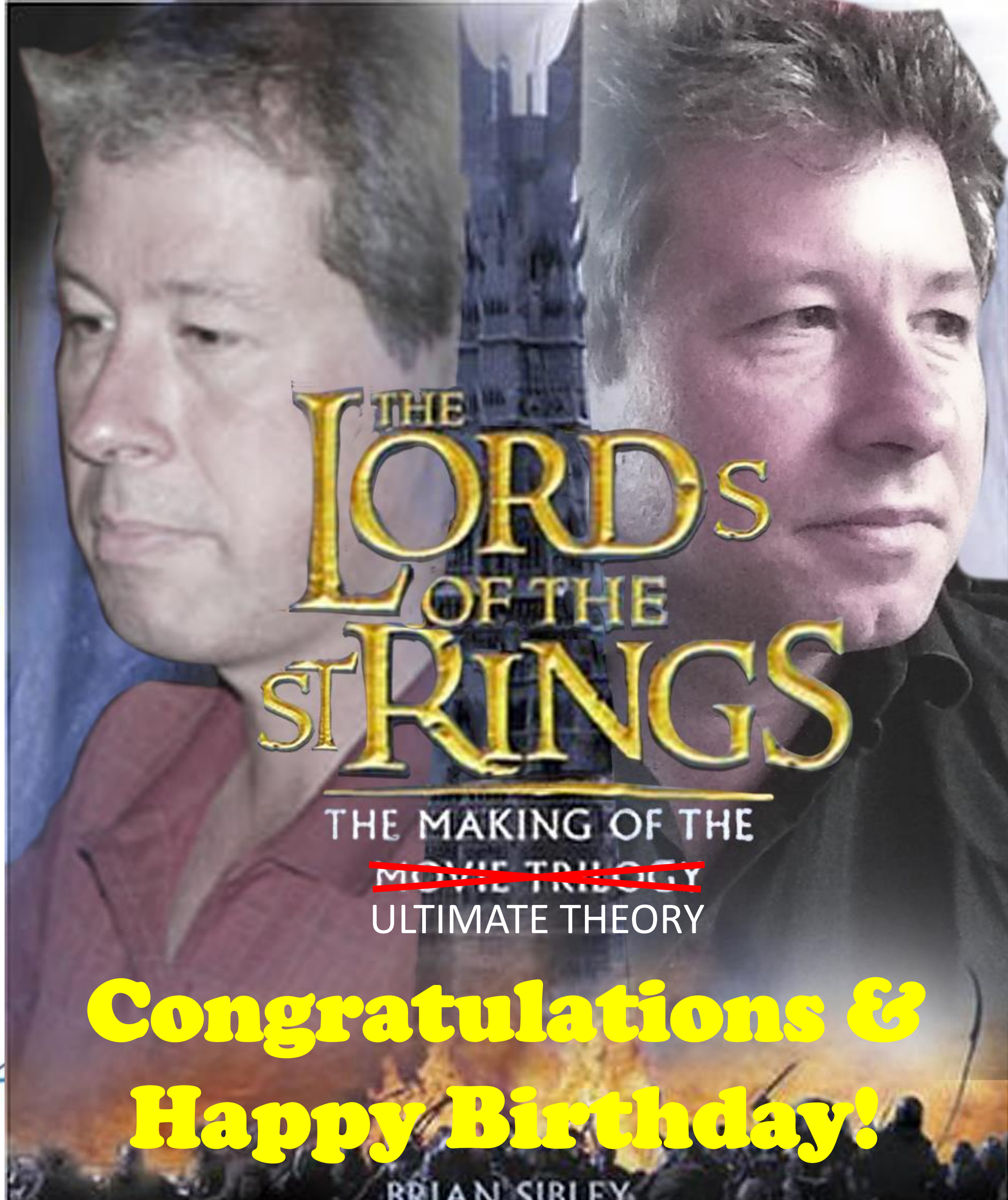
Verlinde^2 Symposium



Verlinde^2 Symposium


FROM THE ACADEMY AWARD® WINNING SCREENWRITER OF
AMSTERDAM ~~L.A.~~ CONFIDENTIAL
AND THE PRODUCERS OF
THE THEORY OF EVERYTHING
HERMAN
ERIK
LEGEND
THE NOTORIOUS TRUE STORY
OF THE ~~KRAY~~ TWINS
VERLINDE


THE LORDs OF THE stRINGS
THE MAKING OF THE
ULTIMATE THEORY
BRIAN SIBLEY


THE LORDs OF THE stRINGS
THE MAKING OF THE
ULTIMATE THEORY
Congratulations &
Happy Birthday!

Complexity = Anything

Alexandre Belin, RCM, Shan-Ming Ruan, Gabor Sarosi & Antony Speranza

[arXiv:2111.02429; arXiv:2208.0xxxx]


COMPLEXITY=ANYTHING
&
EVERYTHING EVERYWHERE ALL AT ONCE
天馬行空
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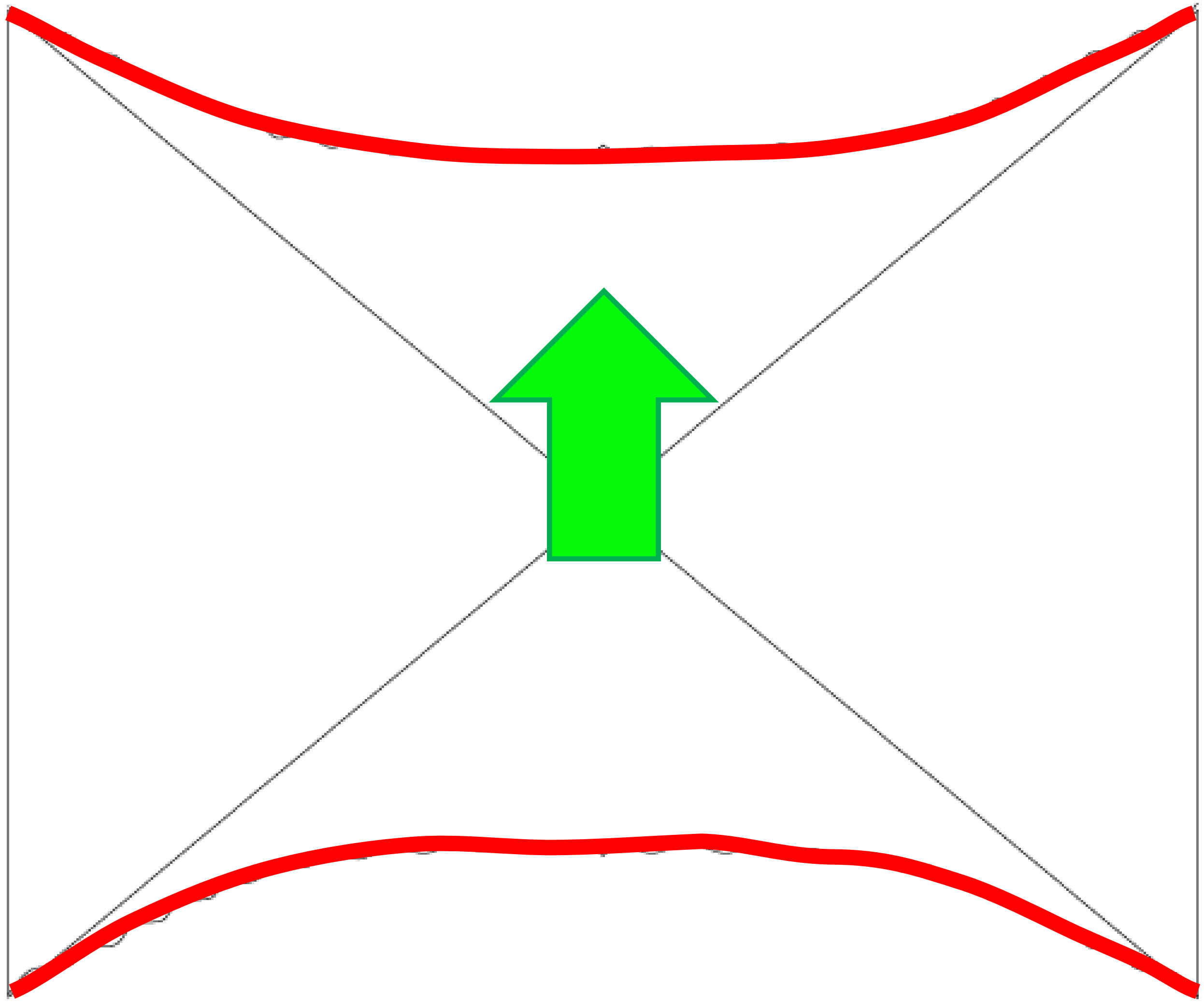
<u>Susskind</u>: Entanglement ^Entropy is not enough!

- "to understand the rich geometric structures that exist behind the horizon and which are predicted by general relativity."



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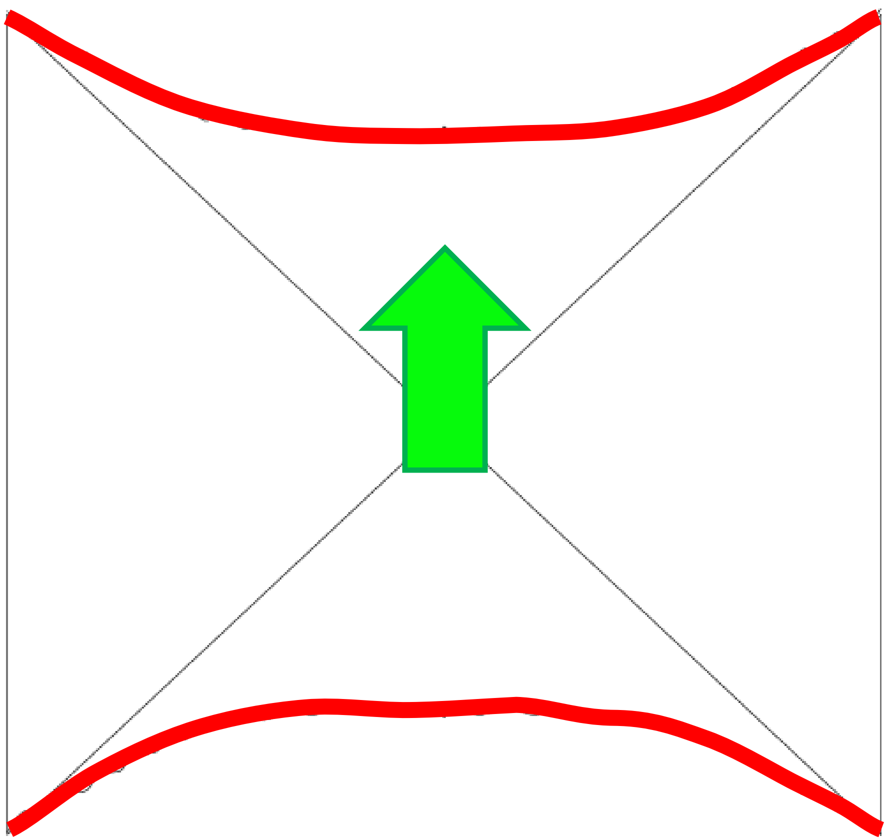

- recall S_{EE} only probes the eigenvalues of the density matrix

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\begin{aligned}
S_{EE} &= -Tr\left[\rho_A \log \rho_A\right] \\
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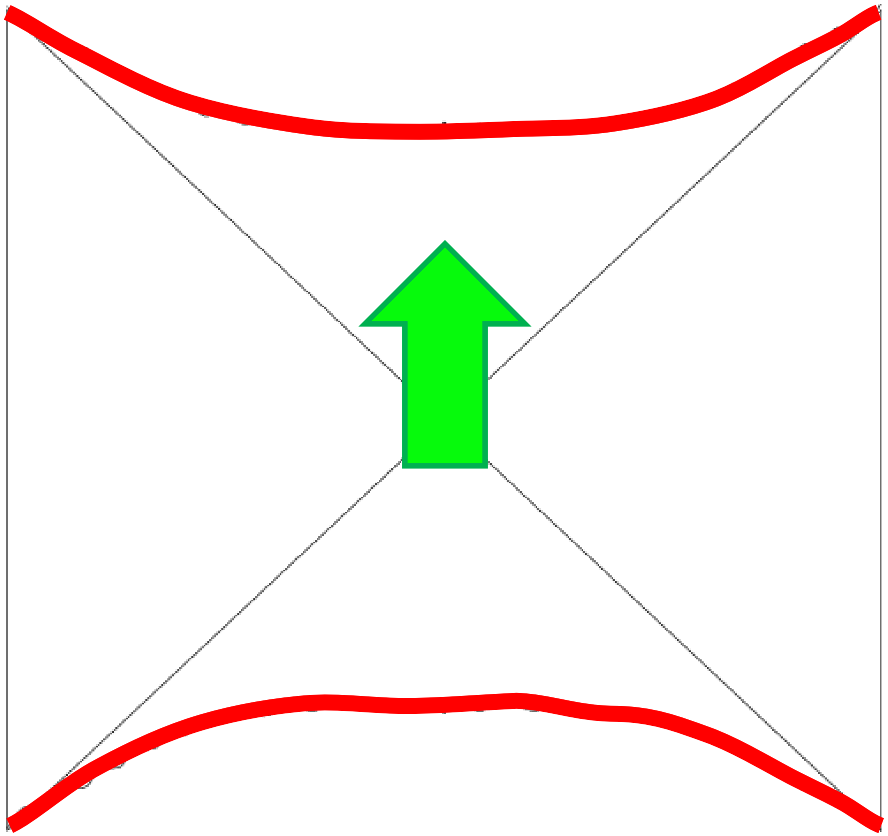

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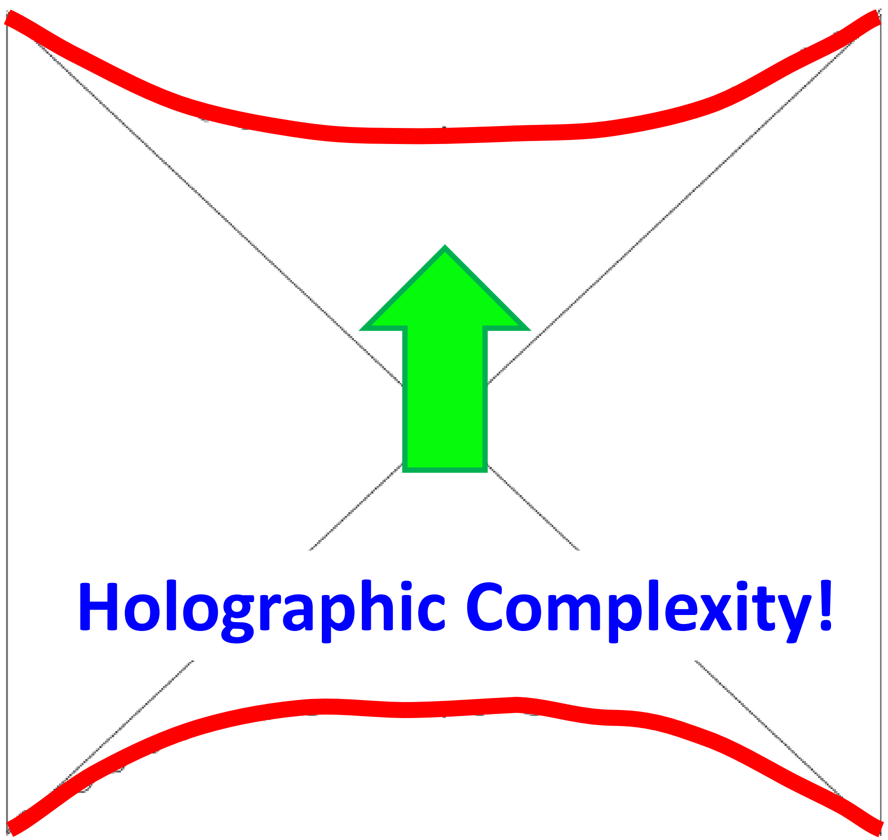

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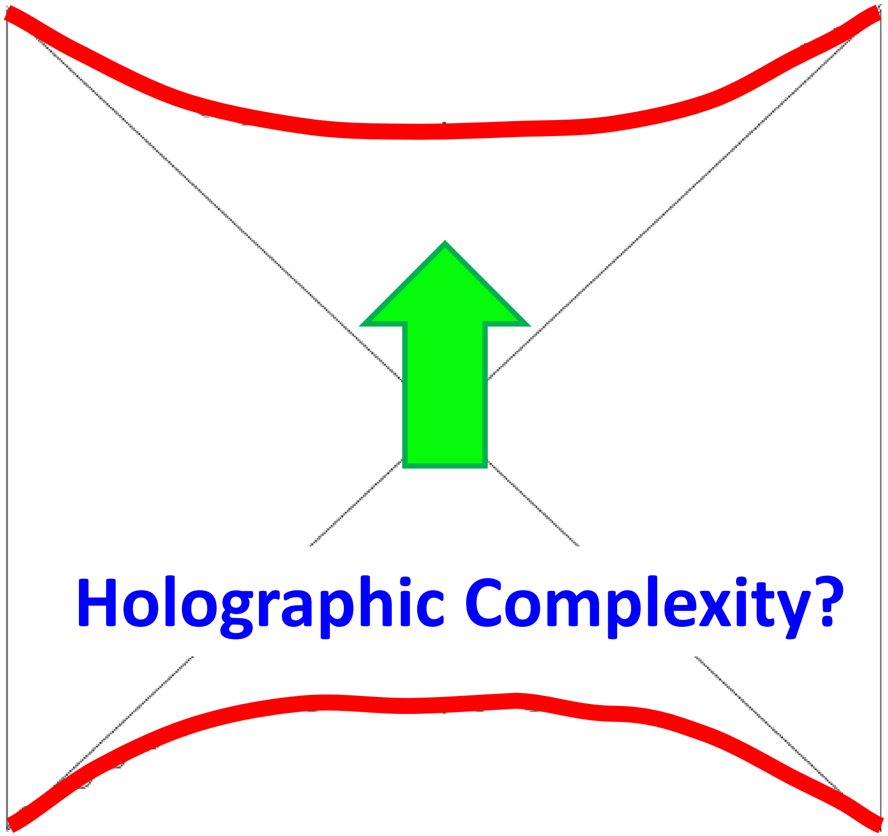

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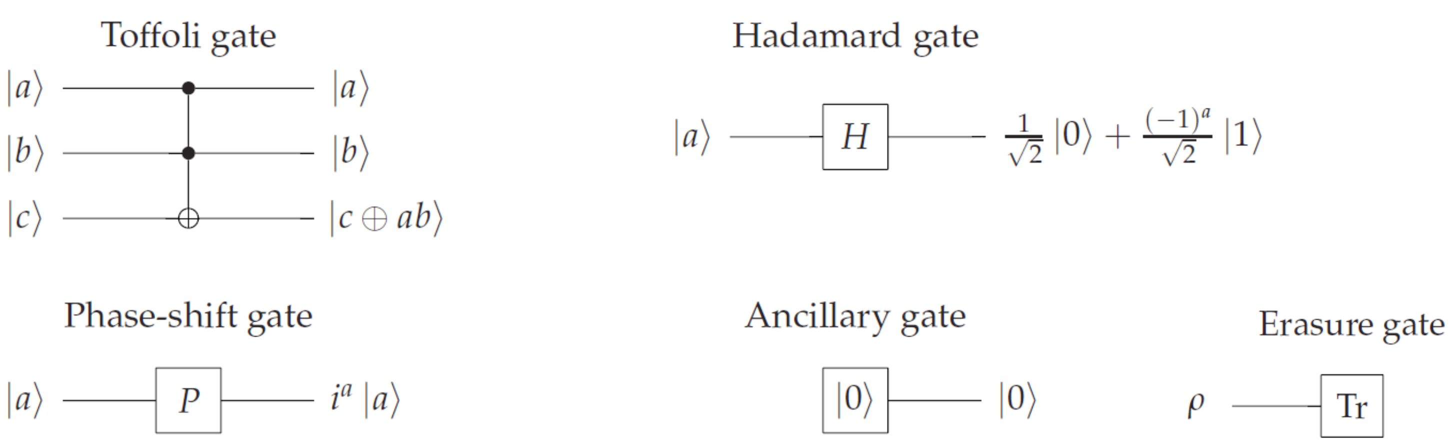

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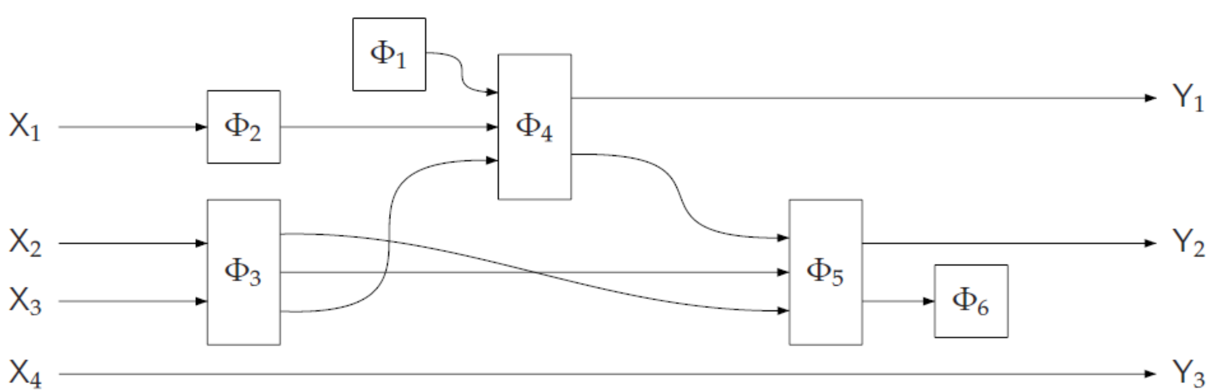

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- does the answer depend on the choices?? **YES!!** go beyond questions like `poly vs exp'

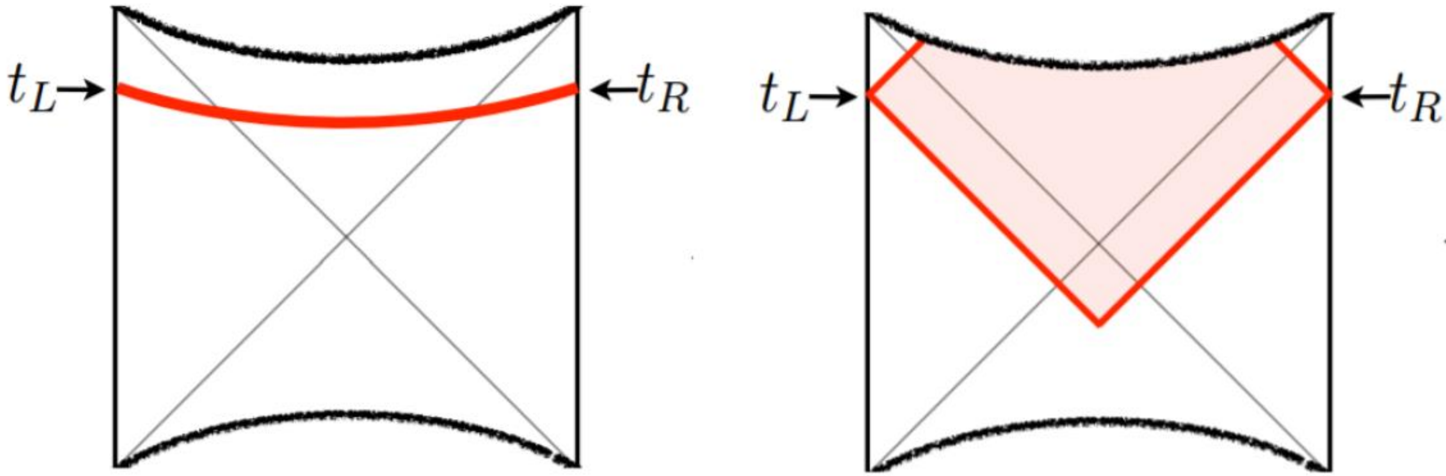
Holographic Complexity: A Tale of Two Dualities

- complexity=volume: evaluate proper volume of extremal codim-one surface connecting Cauchy surfaces in boundary theory (cf holo EE)

(Stanford & Susskind)

Complexity = Volume

Complexity = Action



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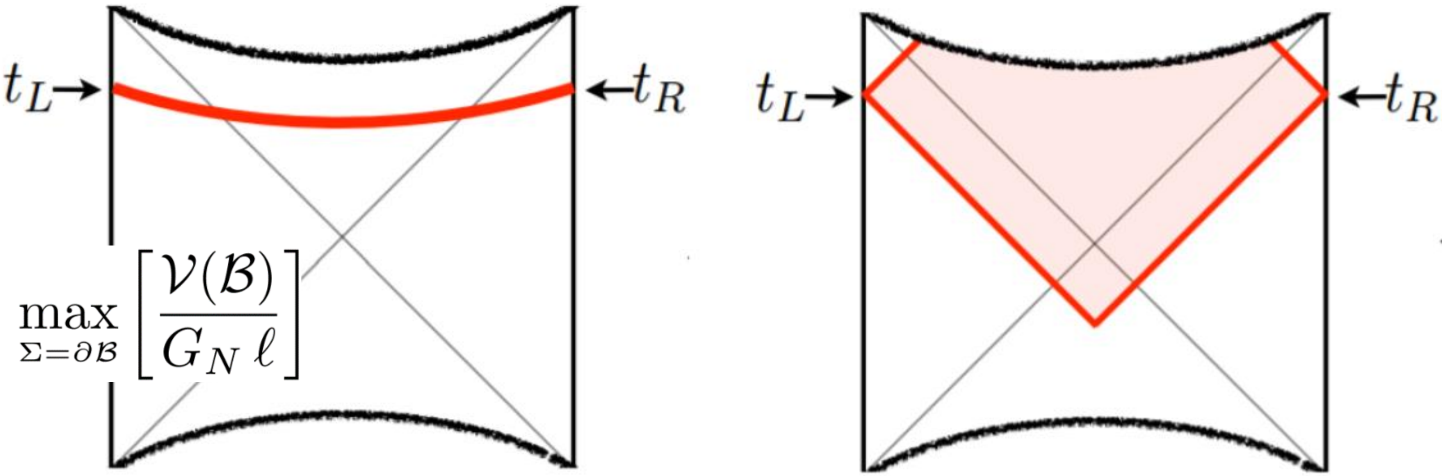

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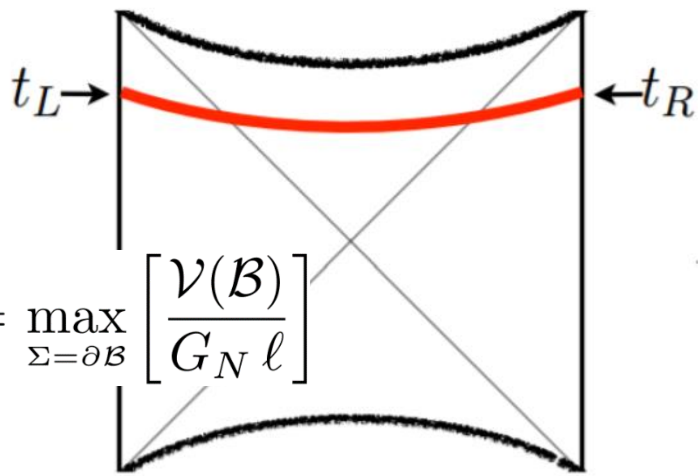

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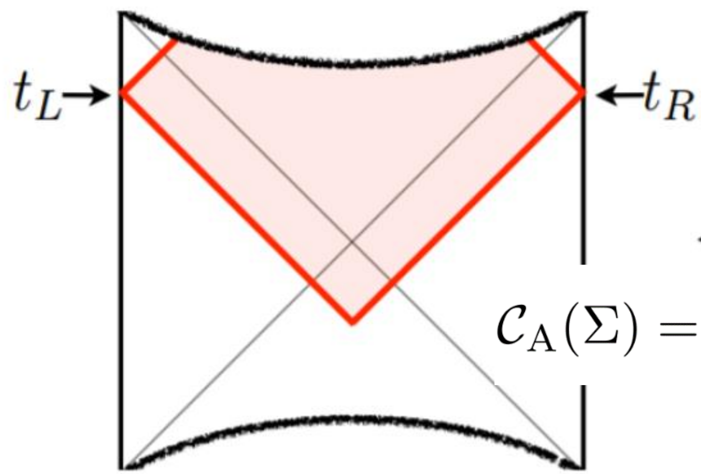

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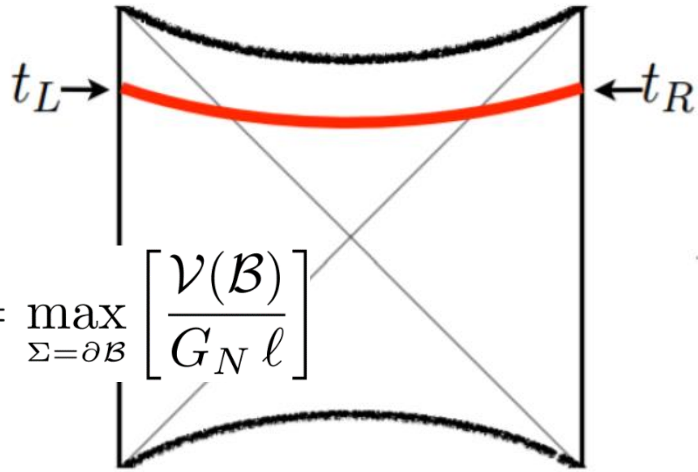

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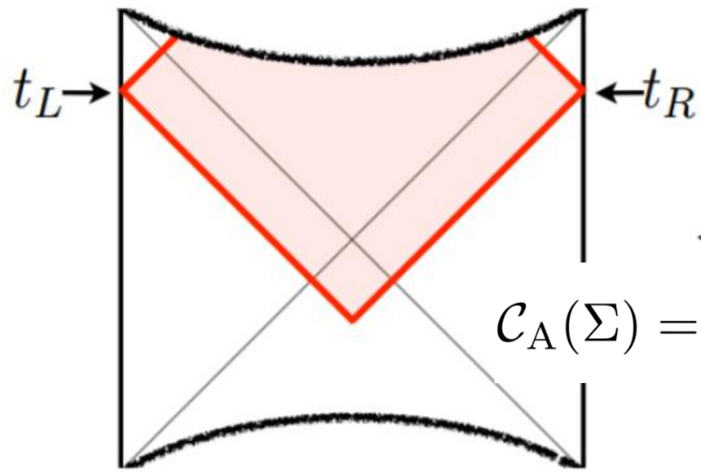

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- both of these gravitational “observables” probe the black hole interior (at arbitrarily late times on boundary)

Holographic Complexity: A Tale of Two or more Dualities

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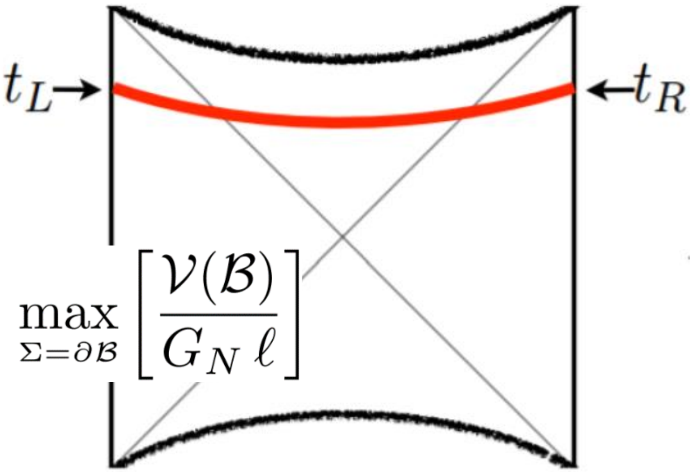

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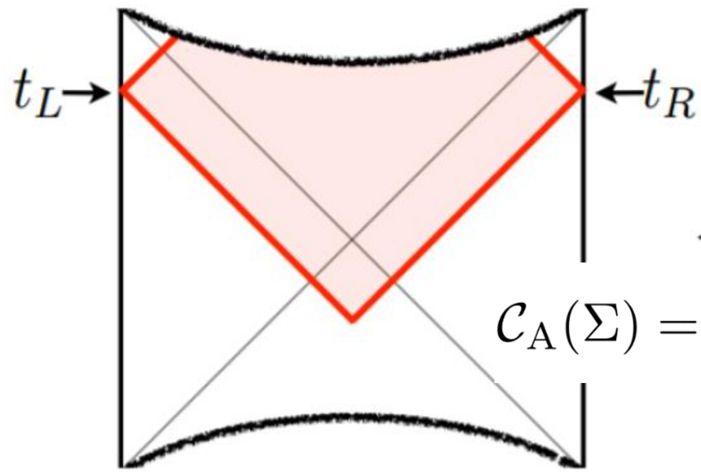

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- complexity=volume**2.0**: evaluate spacetime volume of WDW patch

$$\mathcal{C}'_{\rm V}(\Sigma) = \frac{V_{\rm WDW}}{G_N\,\ell^2}$$

(Couch, Fischler & Nguyen)

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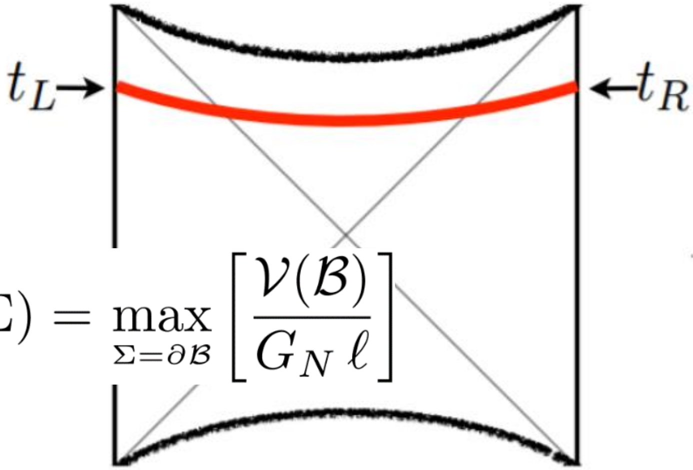

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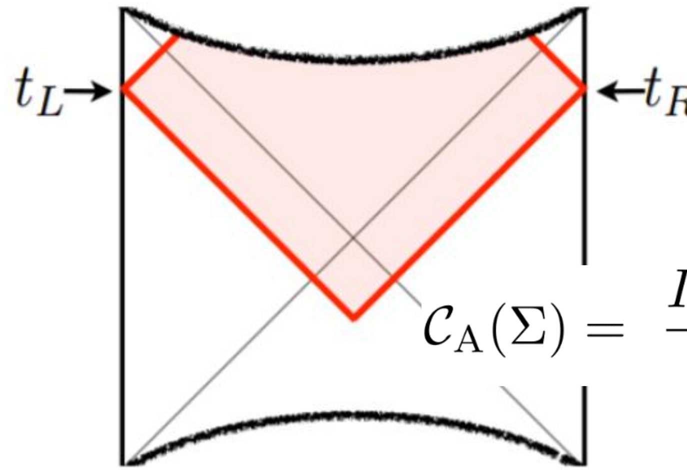

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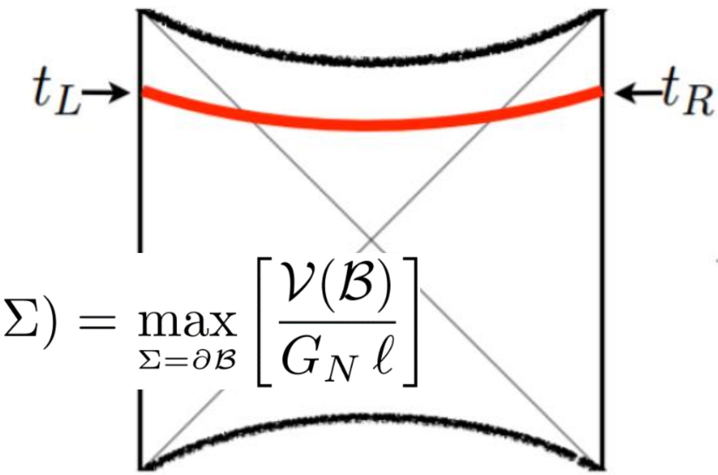

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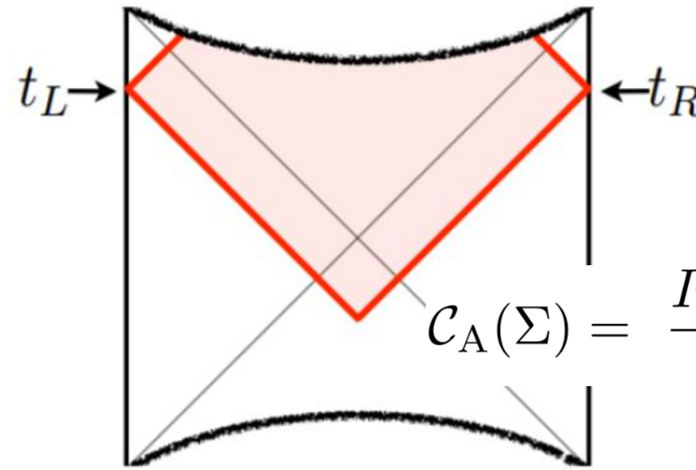

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- connection of complexity=volume to AdS/MERA
- linear growth (at late times) (d = boundary dimension)

$$\left.\frac{d\mathcal{C}_{\rm V}}{dt}\right|_{t\to\infty} = \frac{8\pi}{d-1}\,M \quad \text{(planar)}$$

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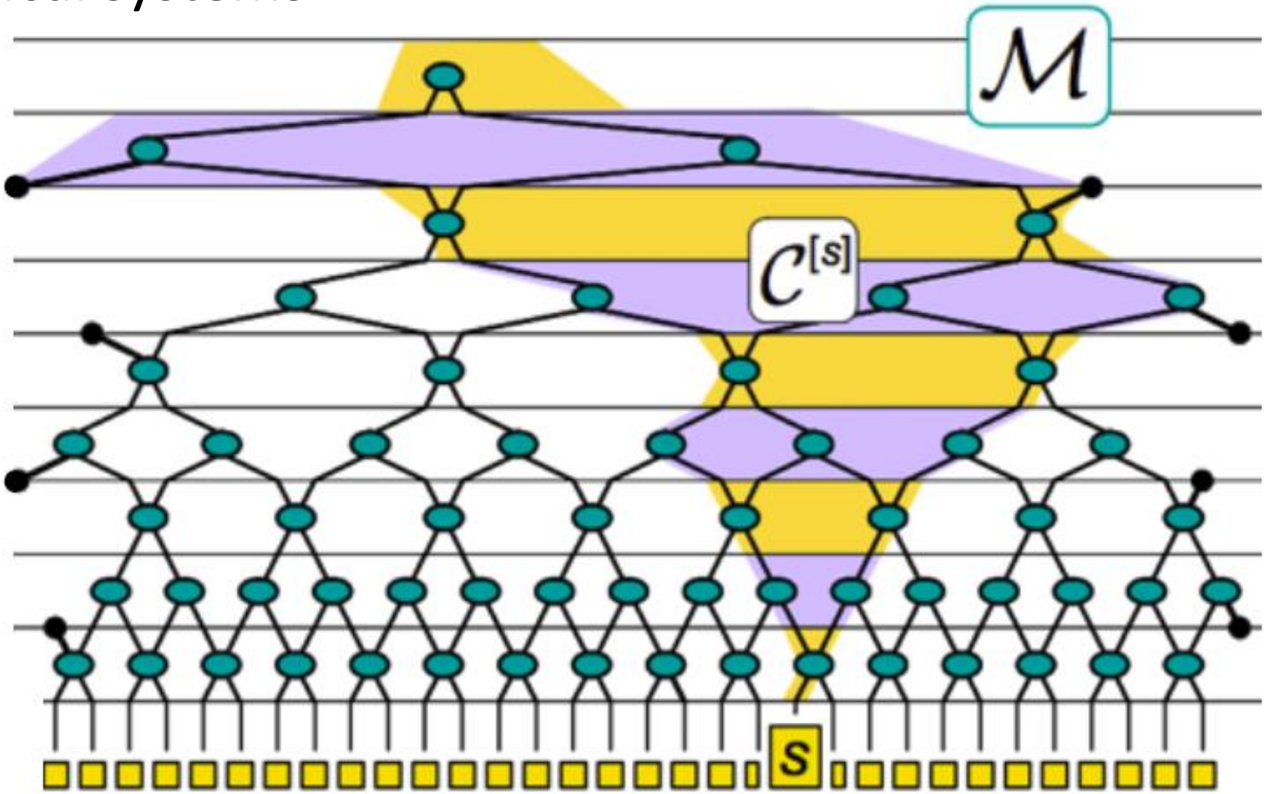
- “switchback” effect (information scrambling)

AdS/MERA:

- MERA (Multi-scale Entanglement Renormalization Ansatz) provides efficient tensor network representation of ground-state wave-function in d=2 critical systems (Vidal; Vidal & Evenbly)



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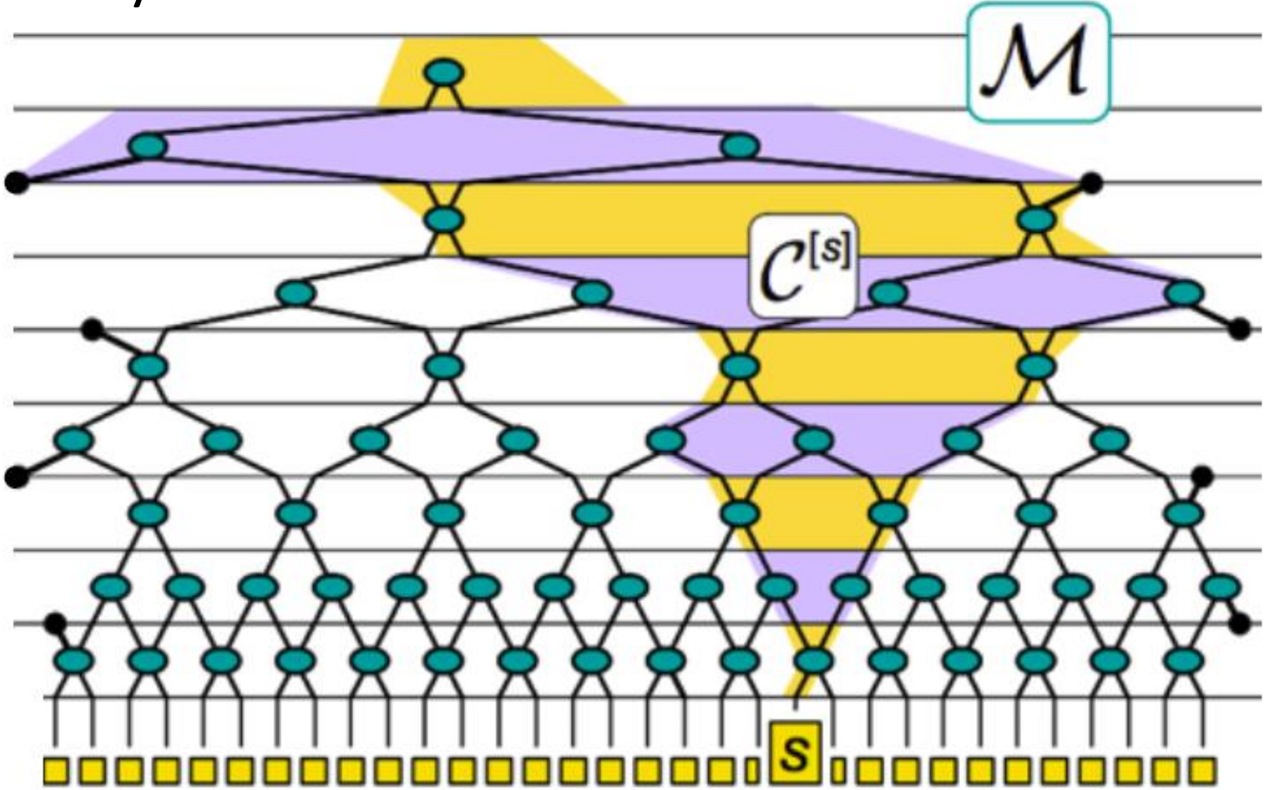

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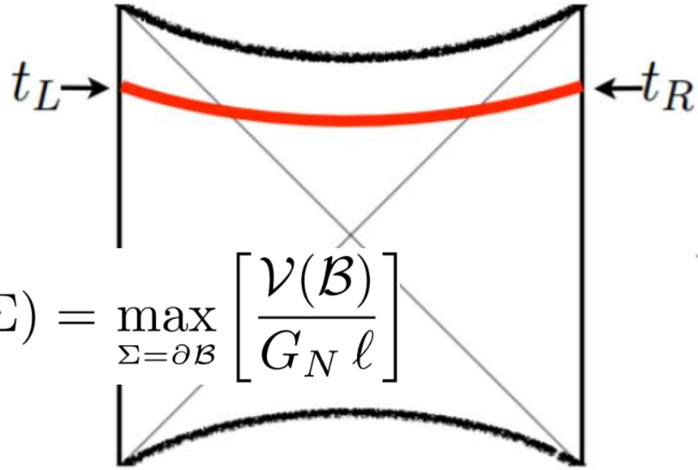

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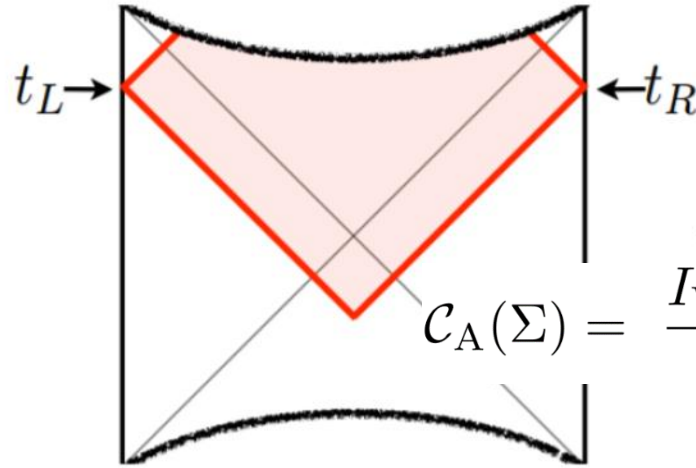

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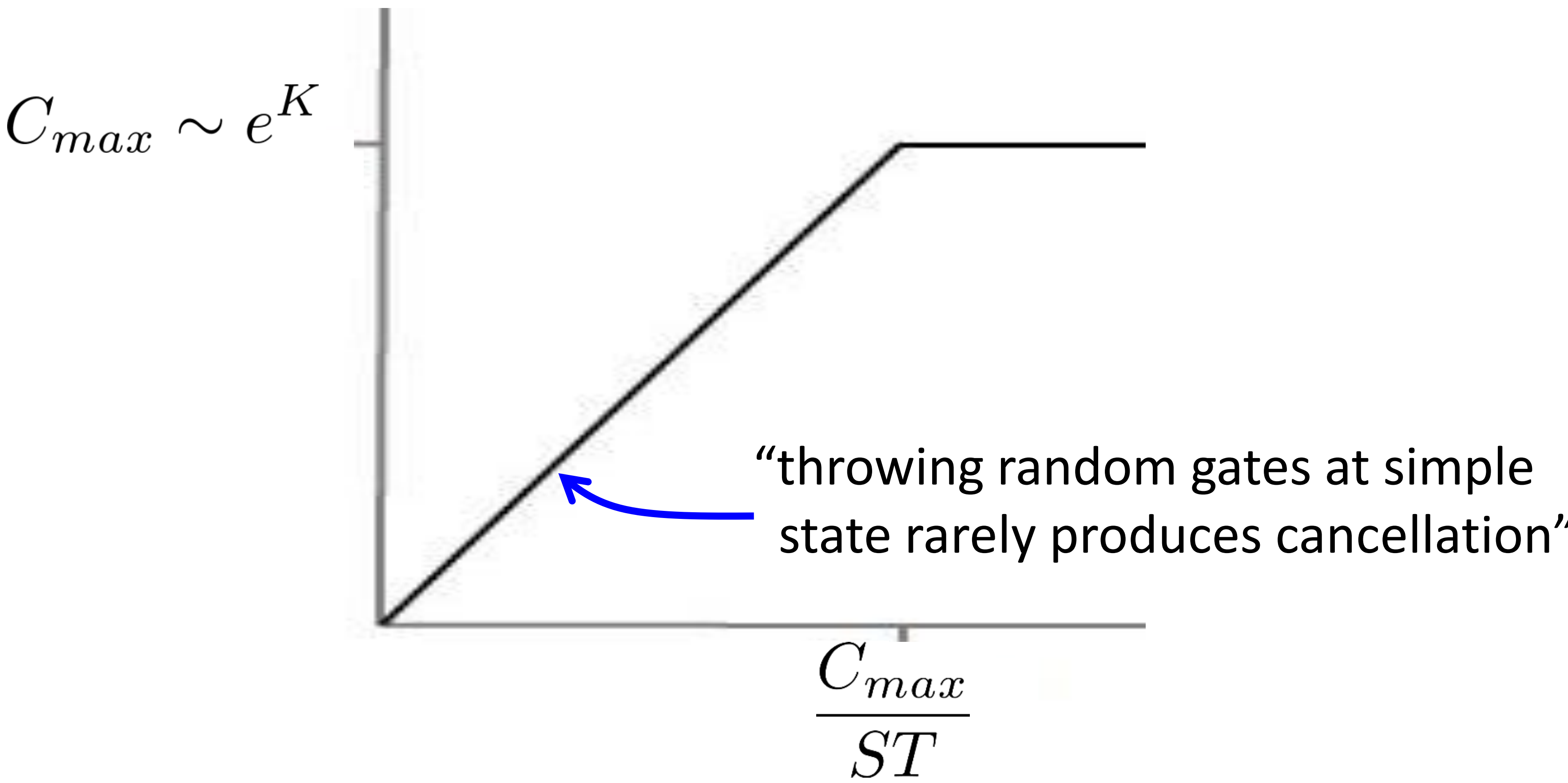
- rate proportional # degrees of freedom K → thermal entropy S; make up dimensions with temperature T (complexity extensive)



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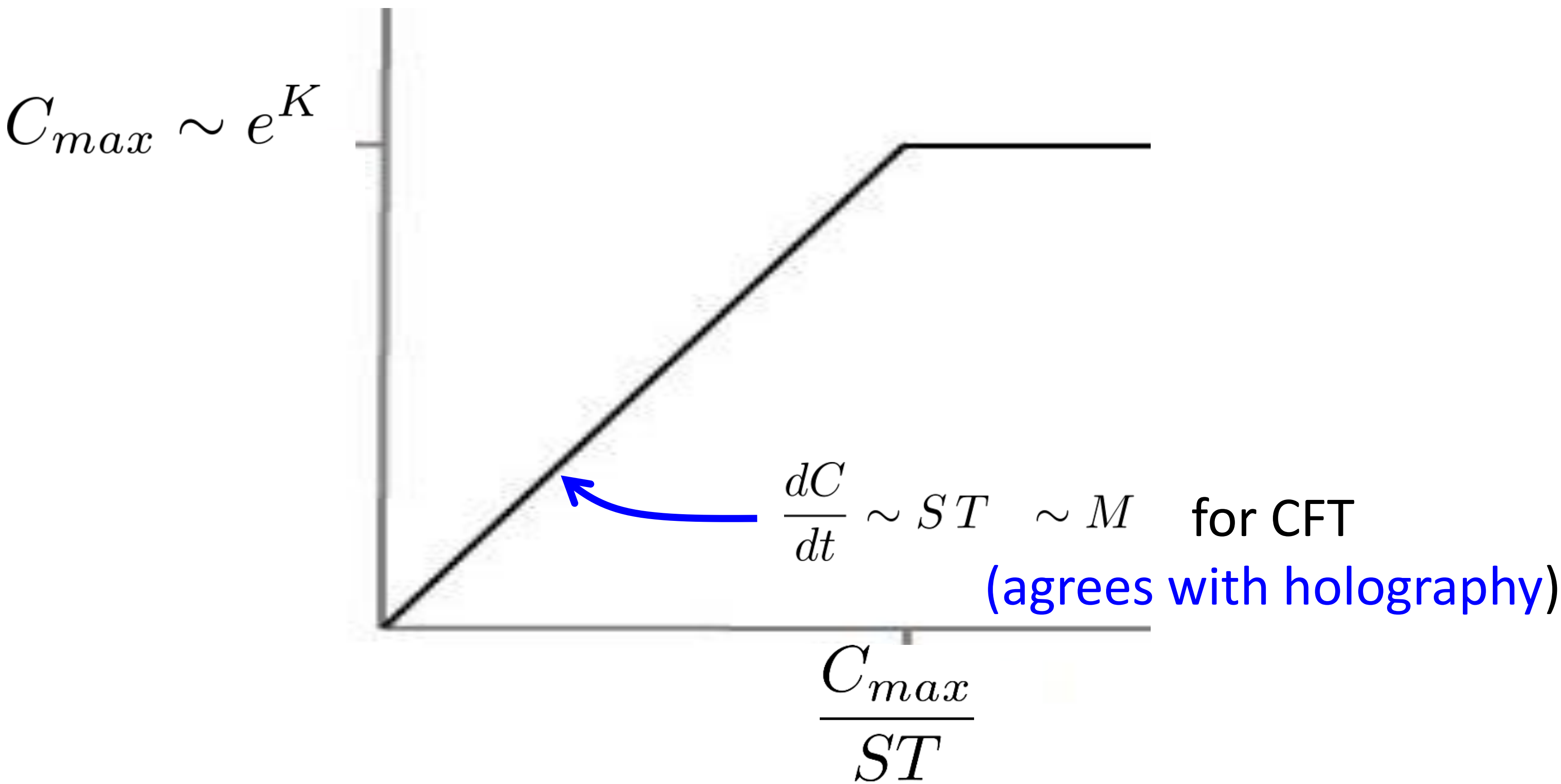

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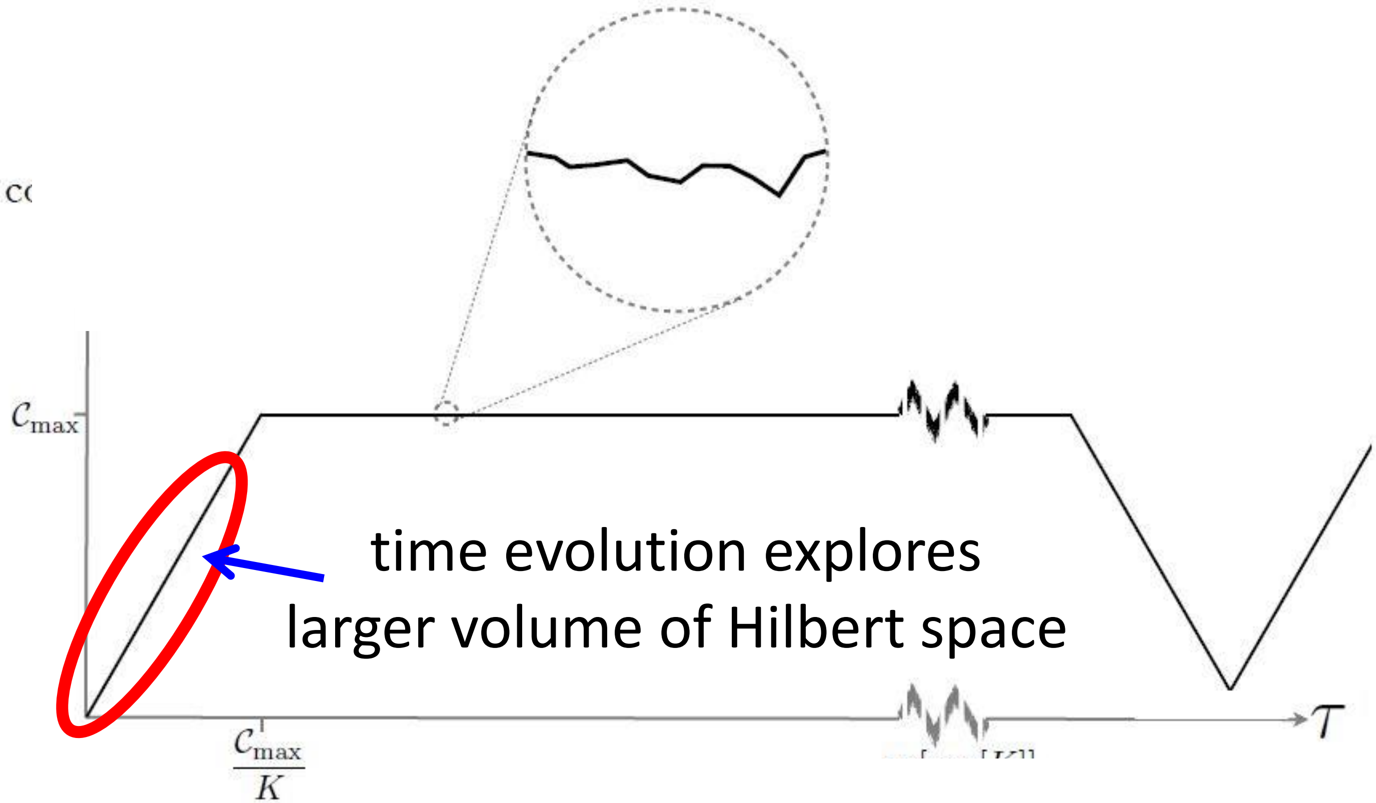

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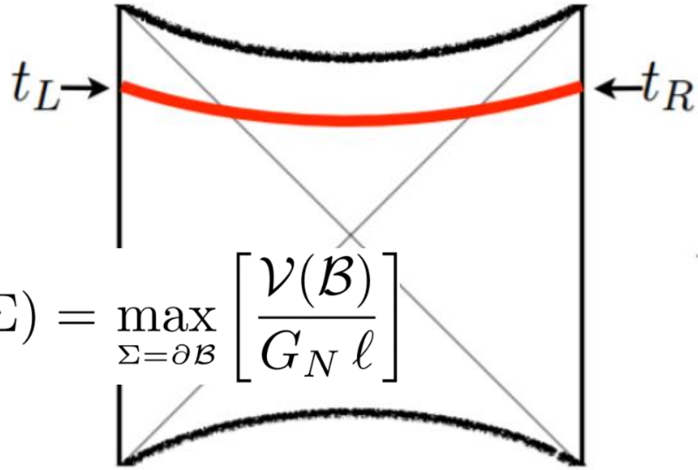

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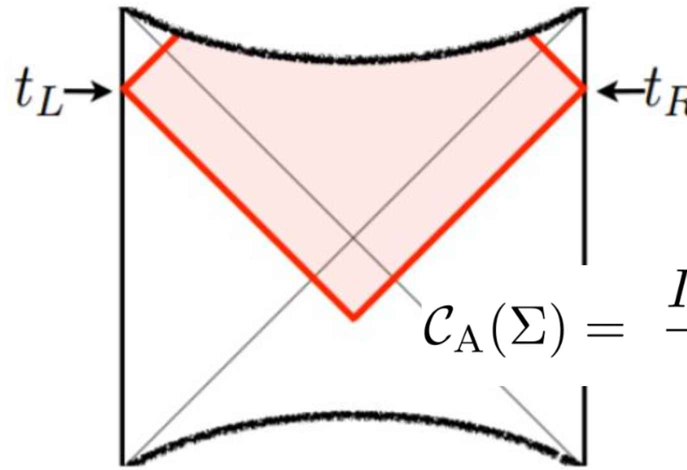

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Stanford, Susskind, . . .

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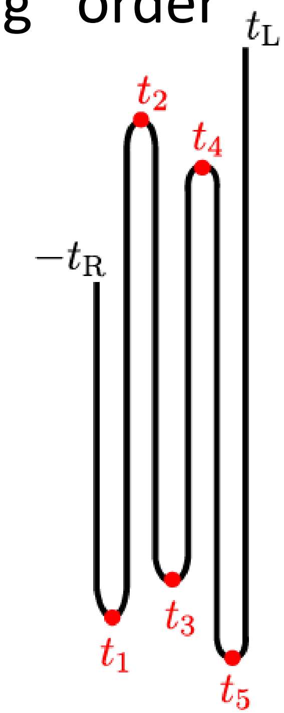
where $t_1, t_2, \dots, t_n$ are in an alternating “zig-zag” order



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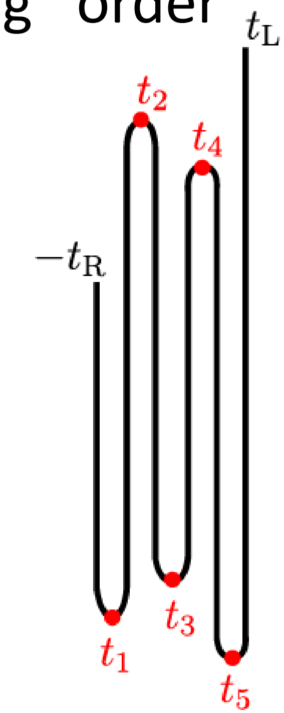
- complexity $\propto |t_R - t_1| + |t_2 - t_1| + \cdots$

$$+|t_L - t_n| - 2\,n\,t_*$$



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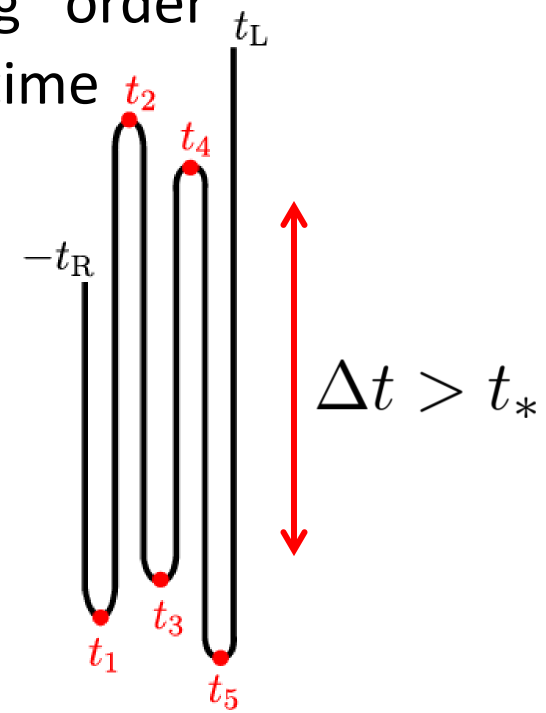
total “folded” time

# of folds

scrambling time:

$$t_* = \frac{1}{2\pi T}\,\log(N^2)$$



Stanford, Susskind, . . .

"Switchback effect":

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total "folded" time

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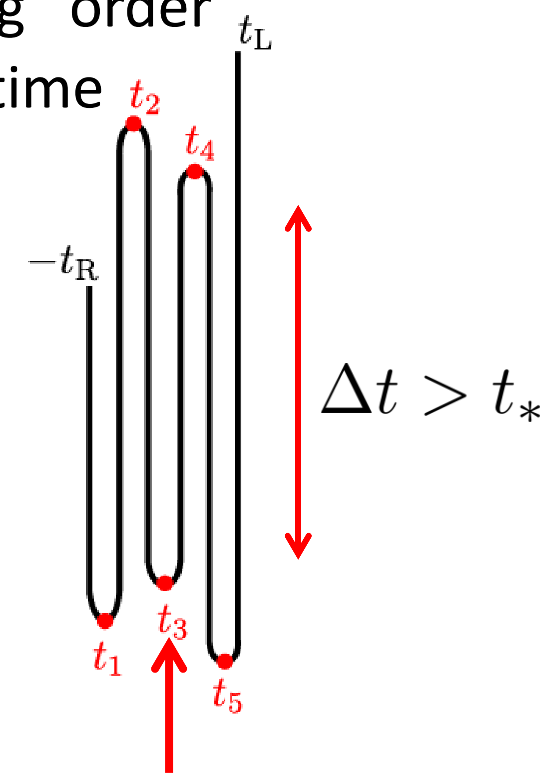

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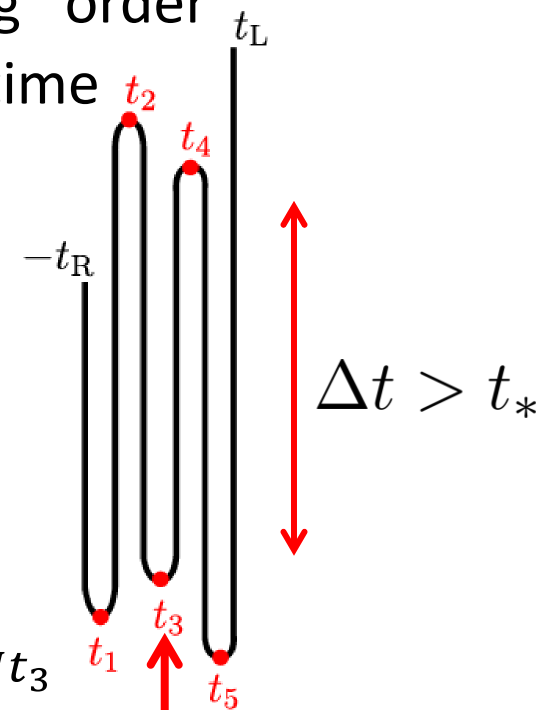

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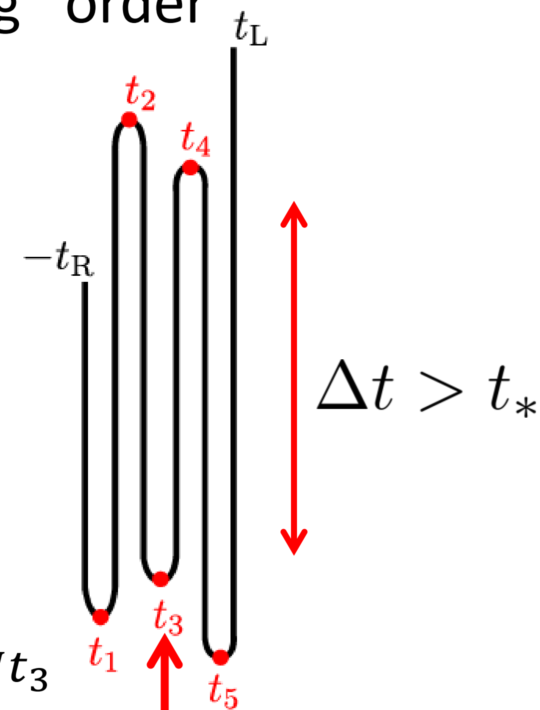

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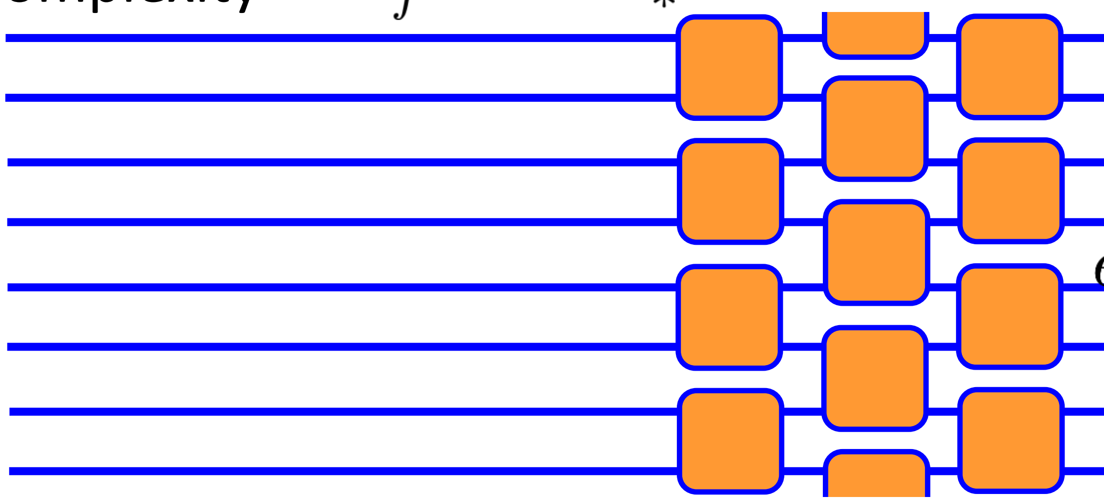

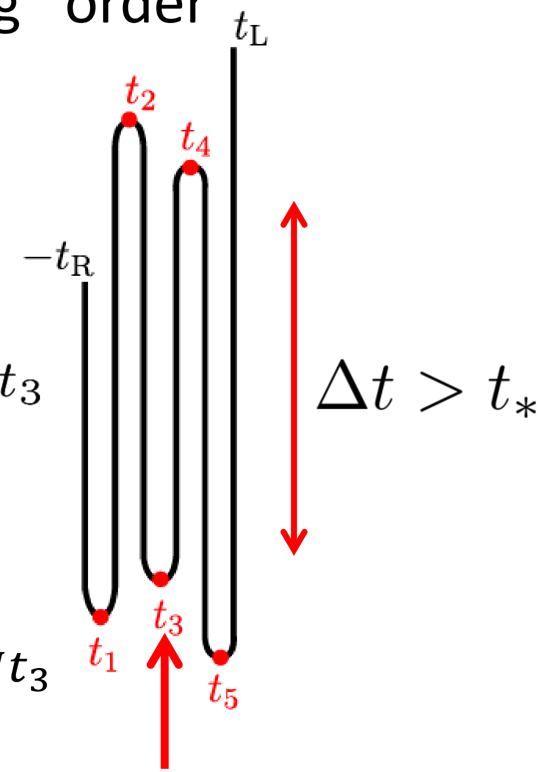

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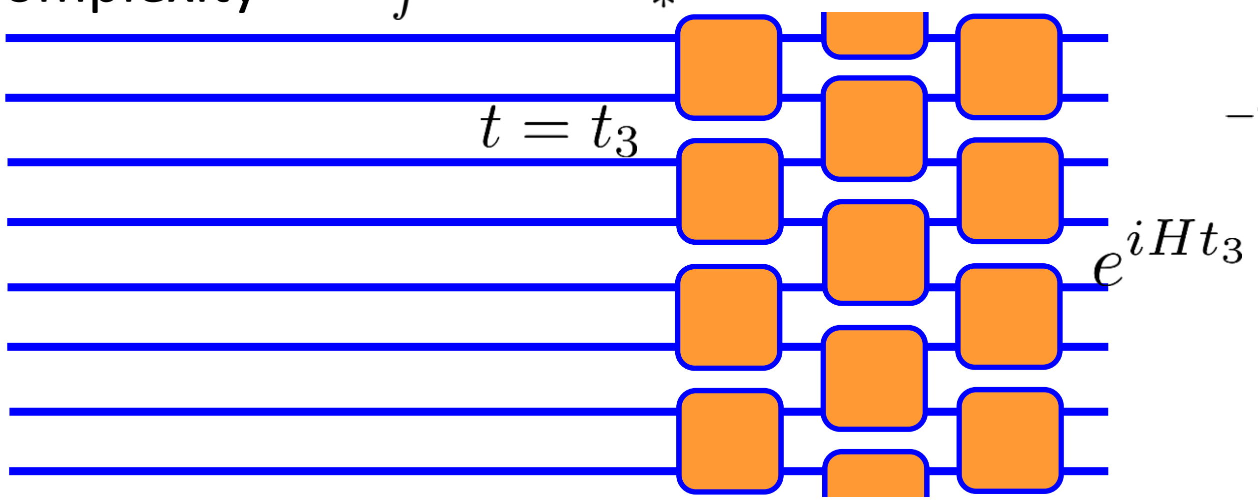

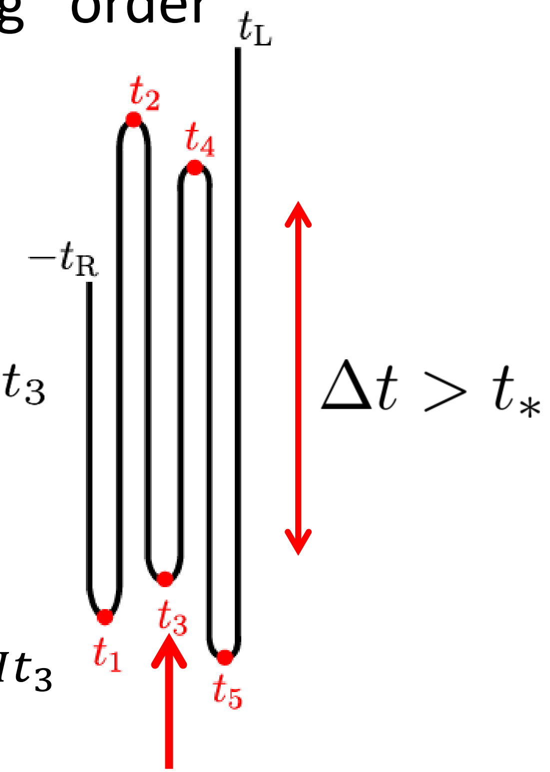

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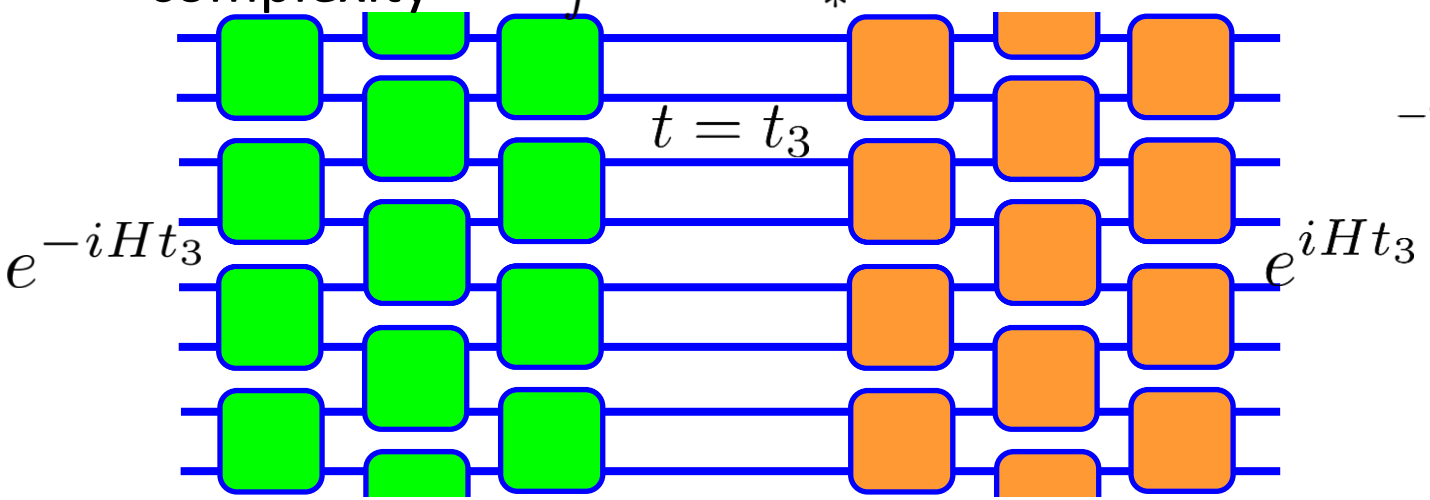

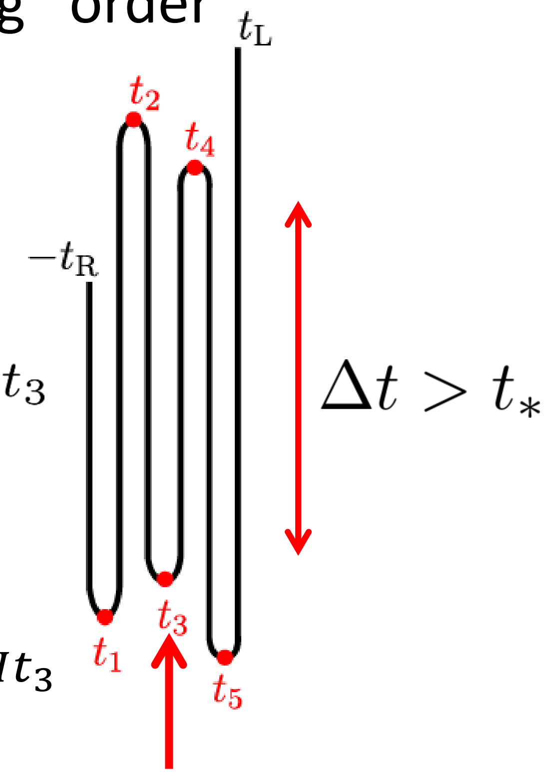

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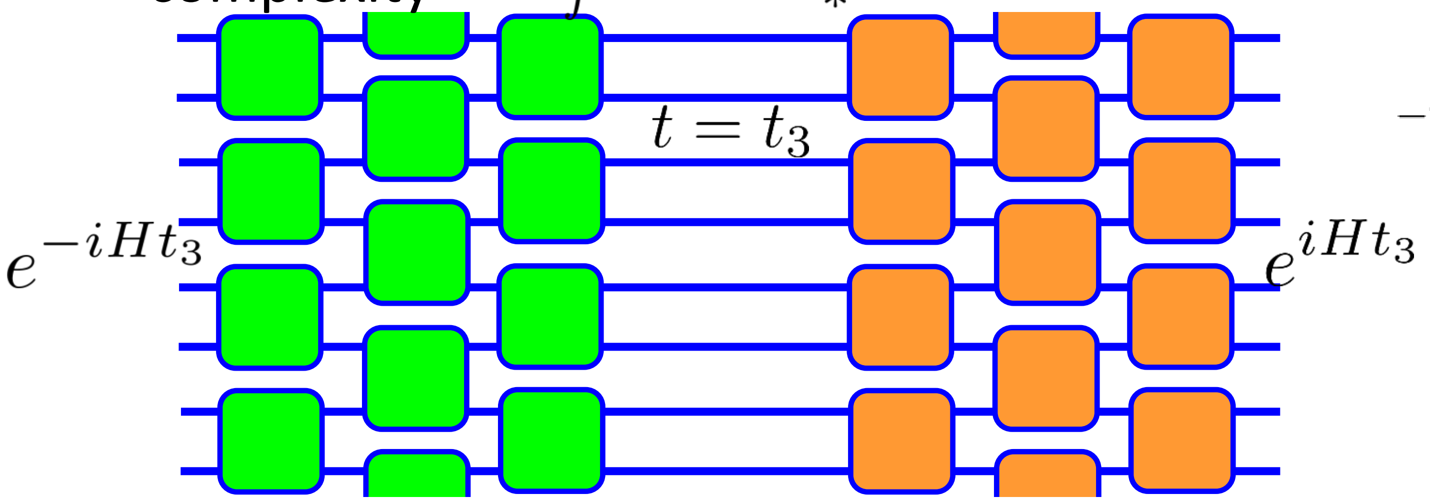

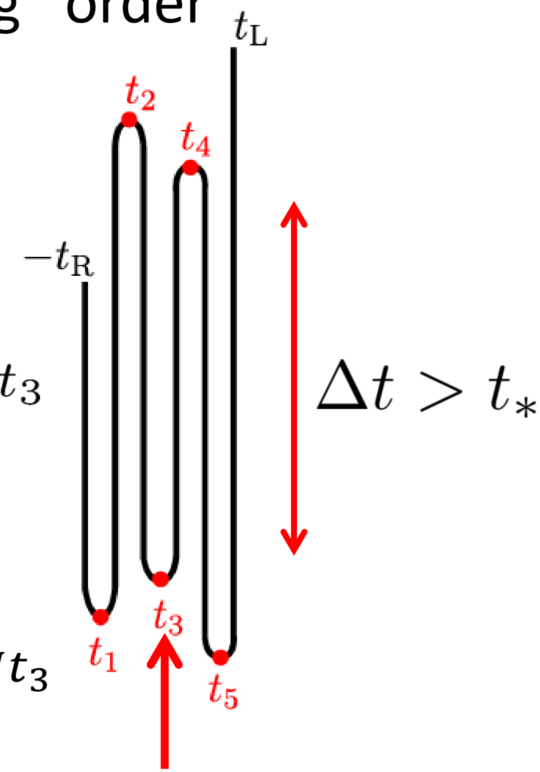

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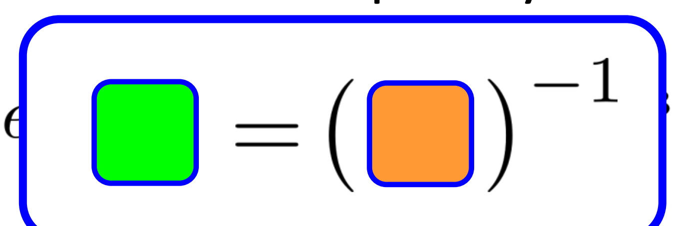

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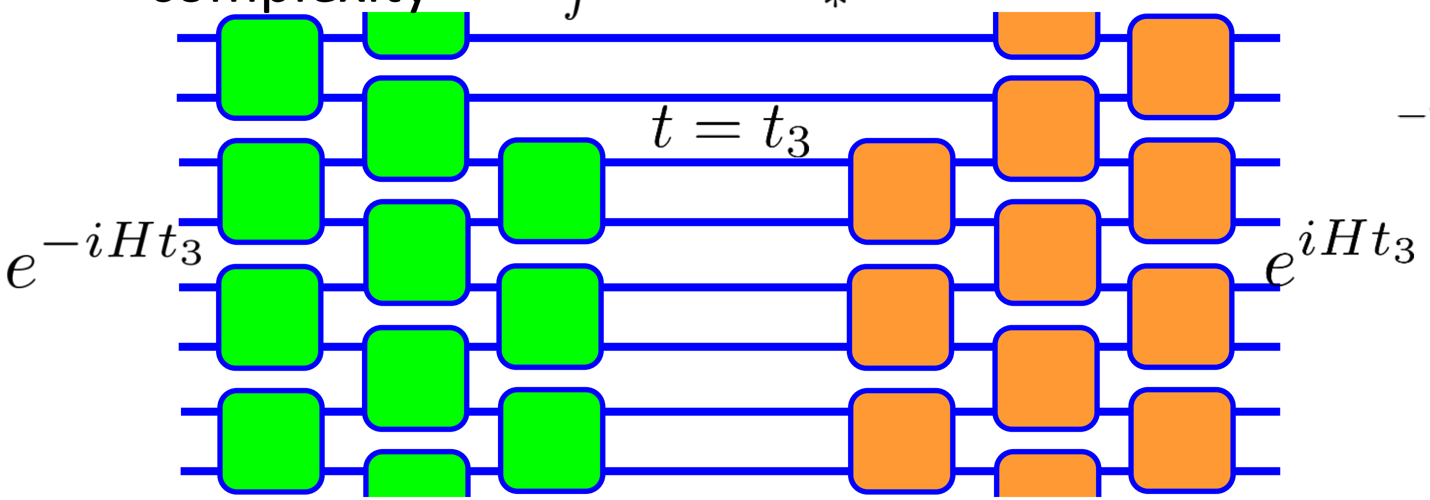

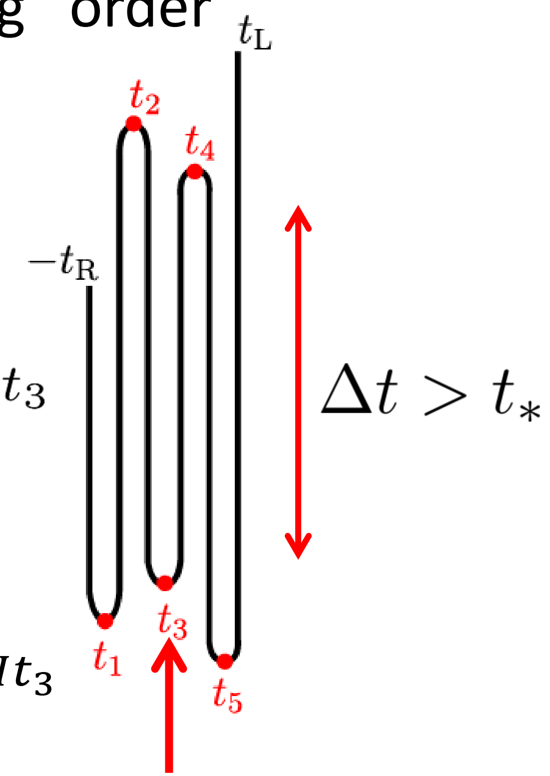

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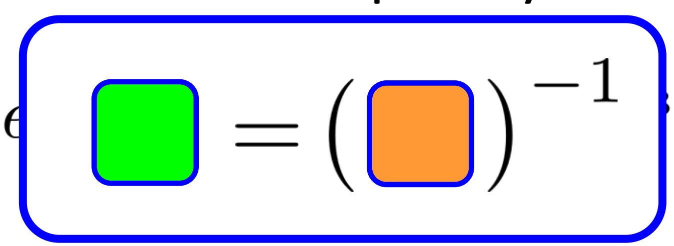

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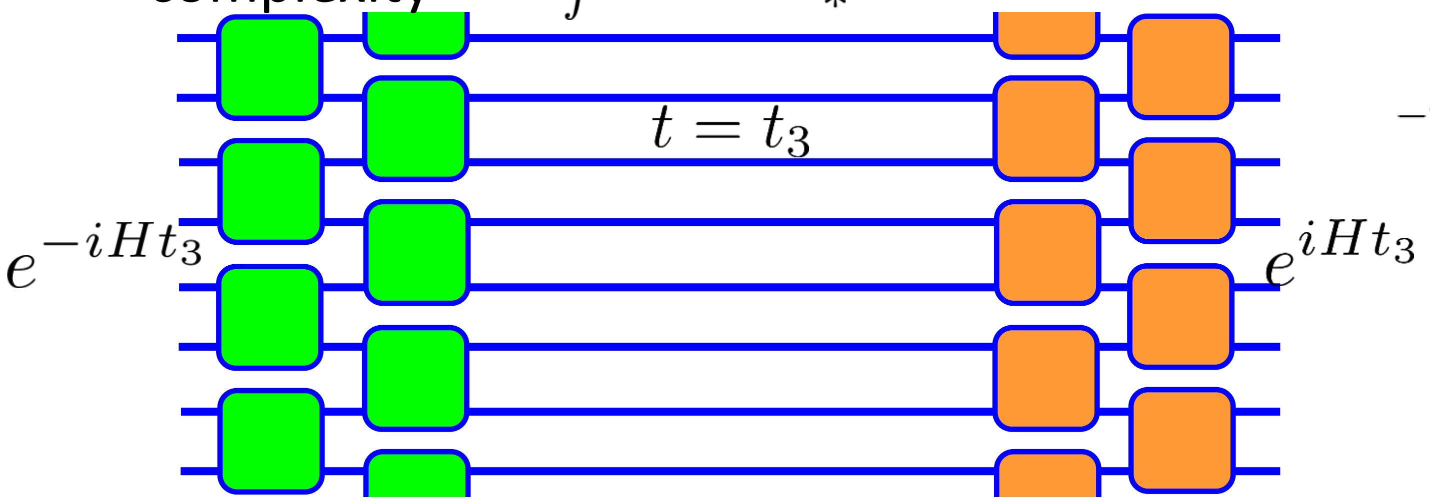

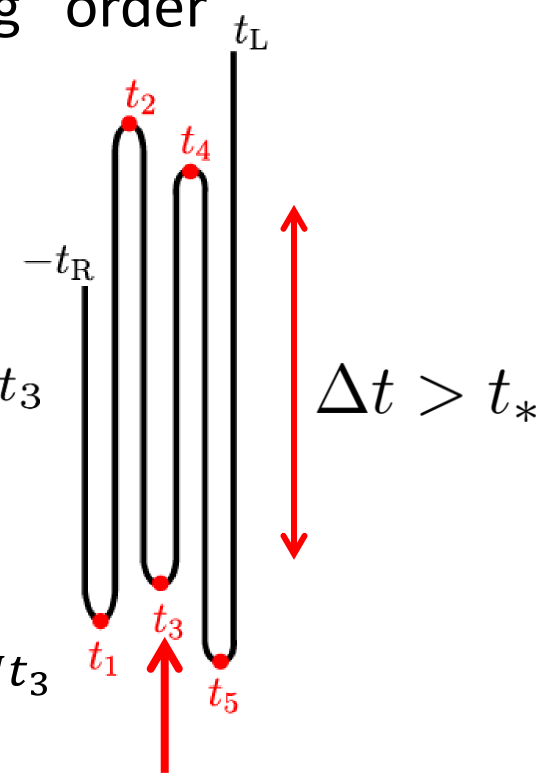

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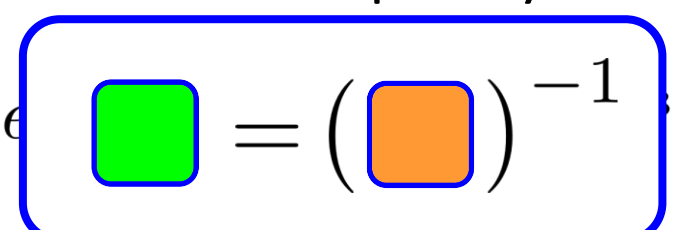

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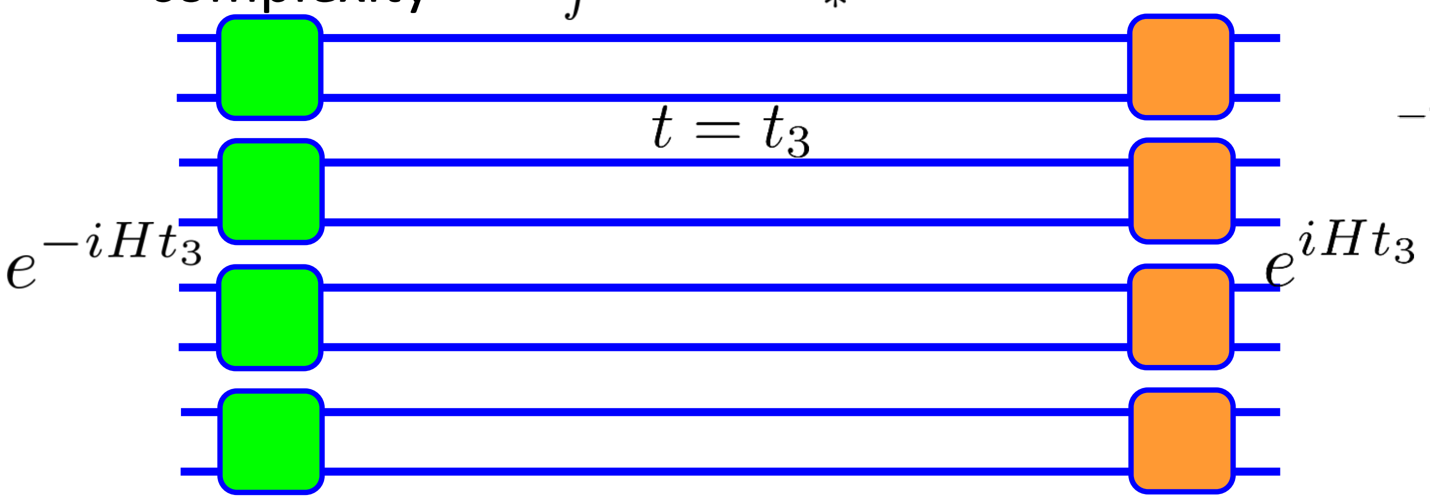

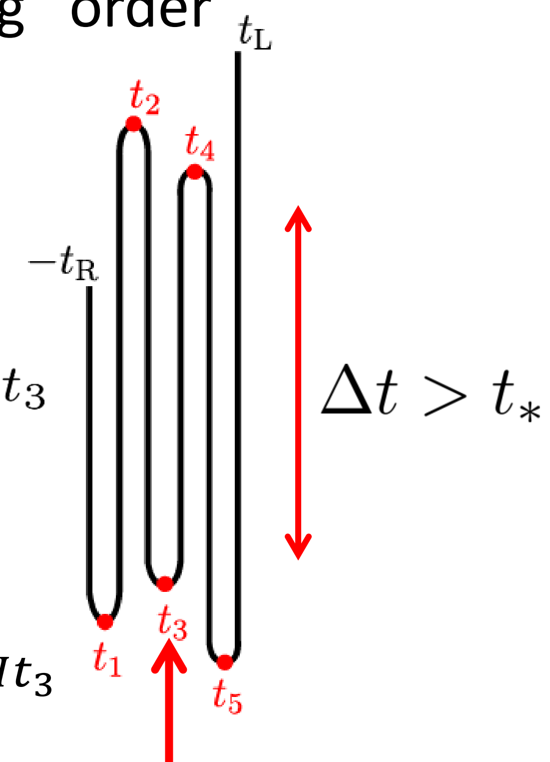

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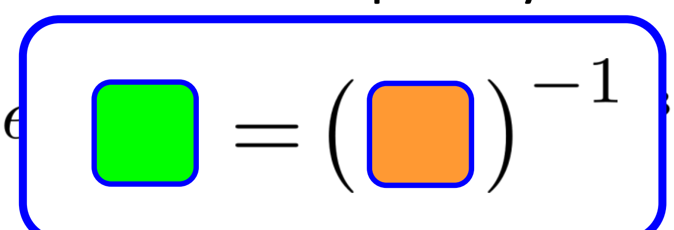

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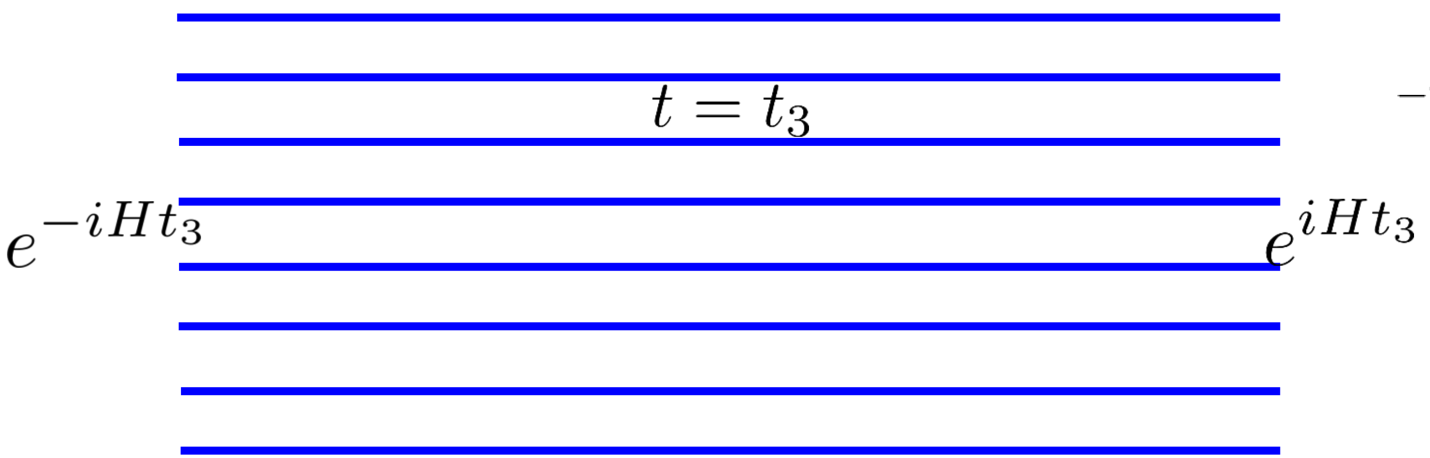

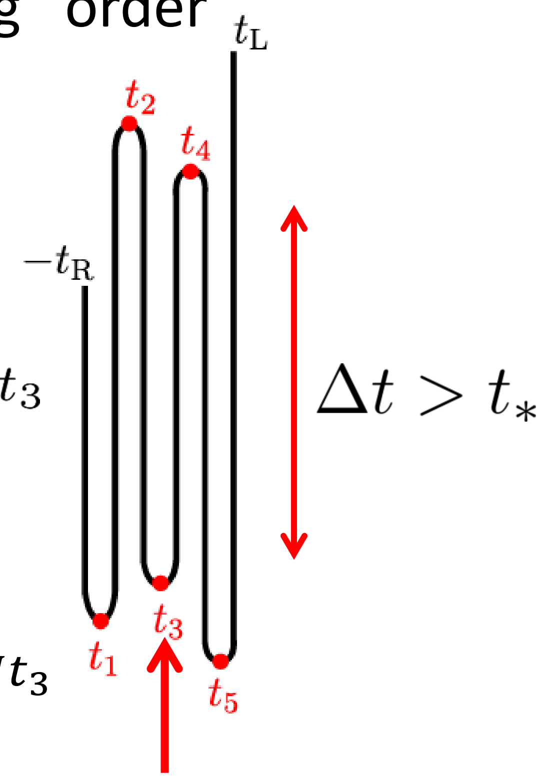

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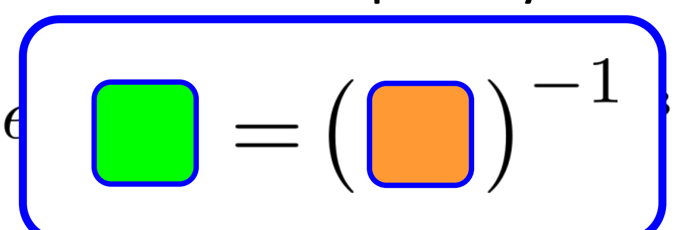

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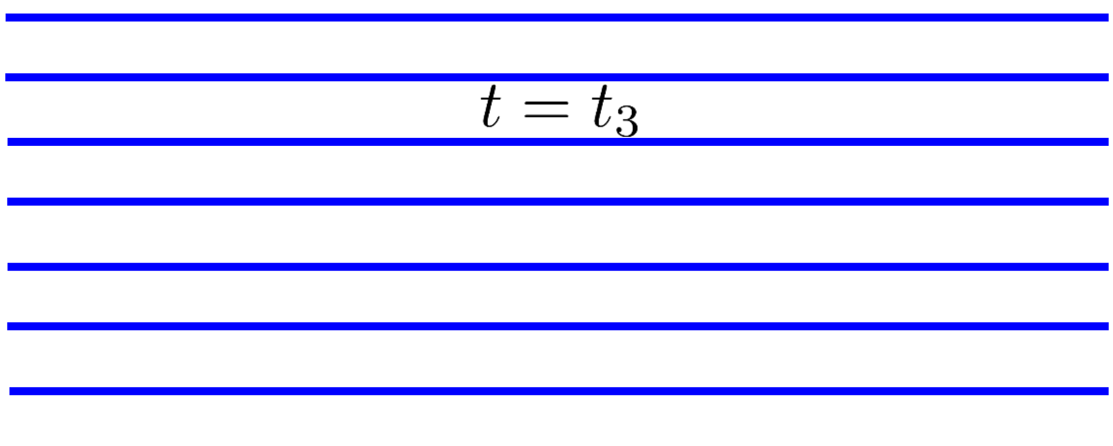

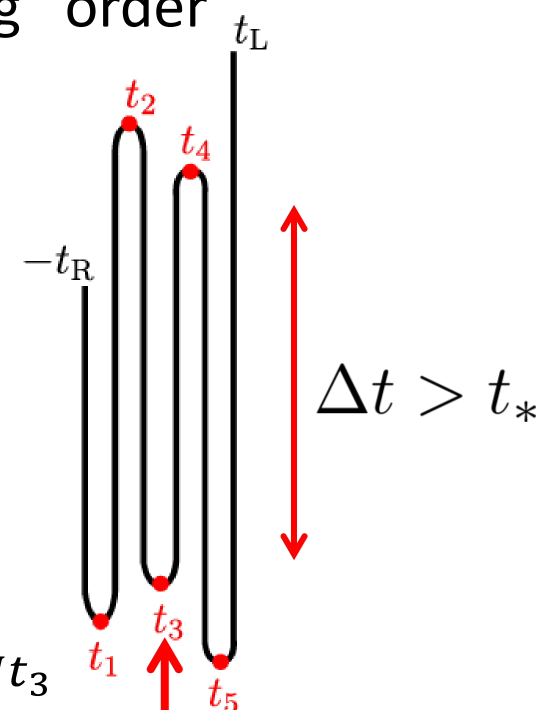

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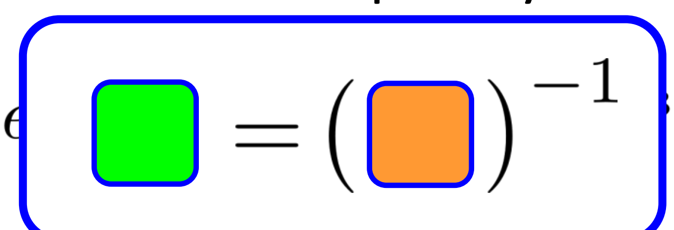

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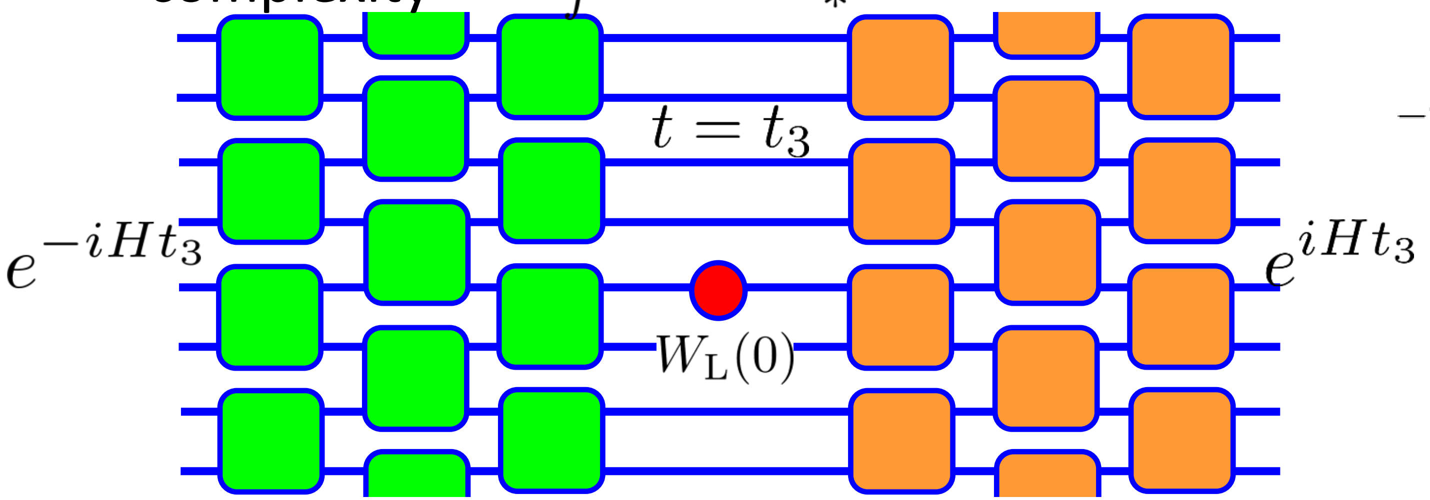

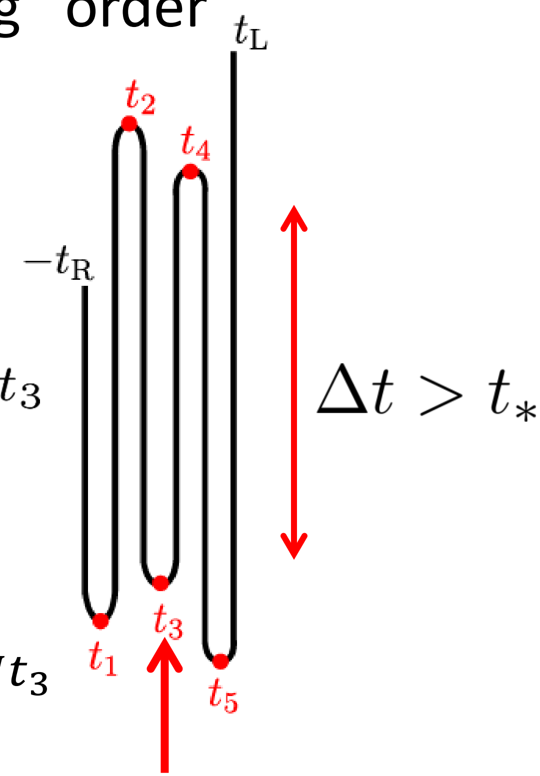

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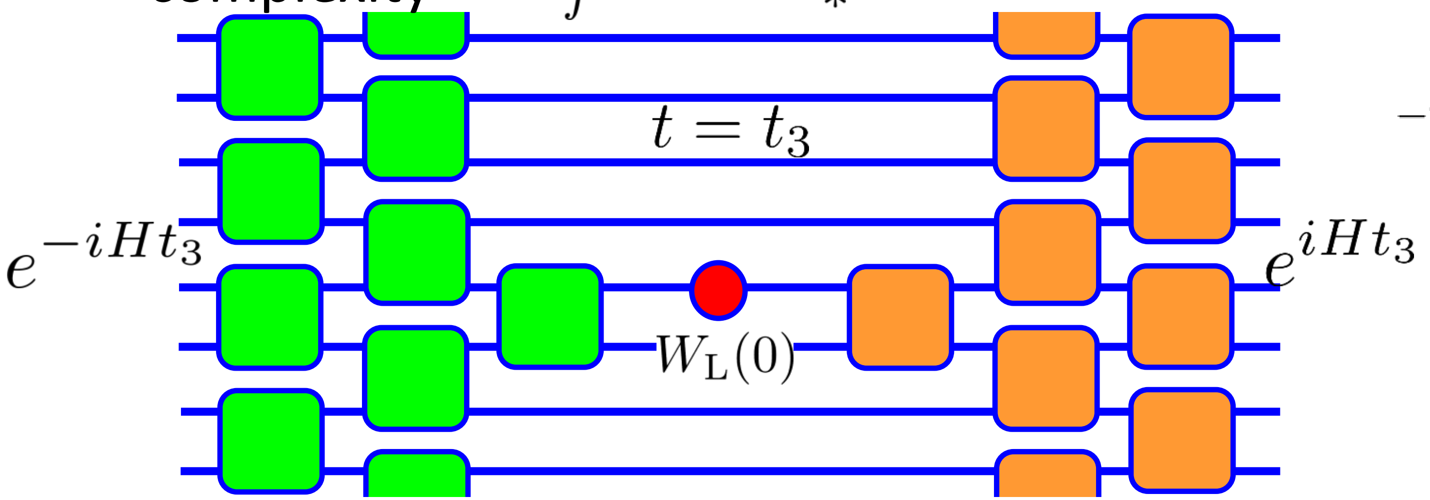

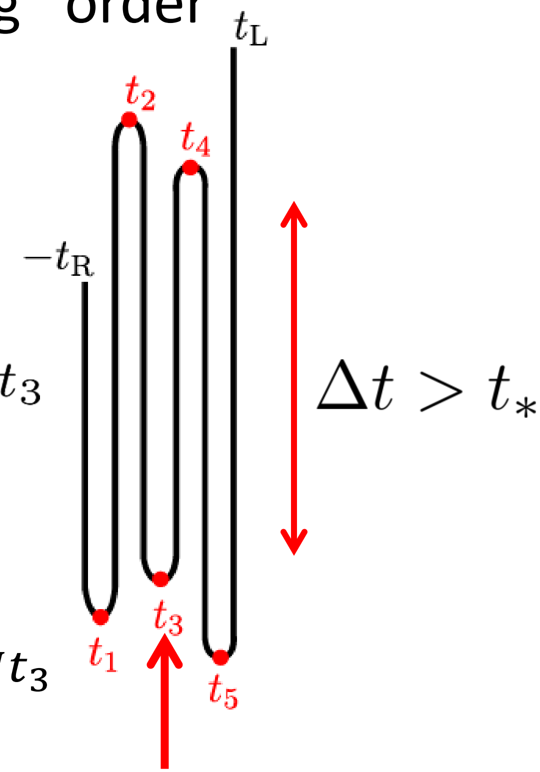

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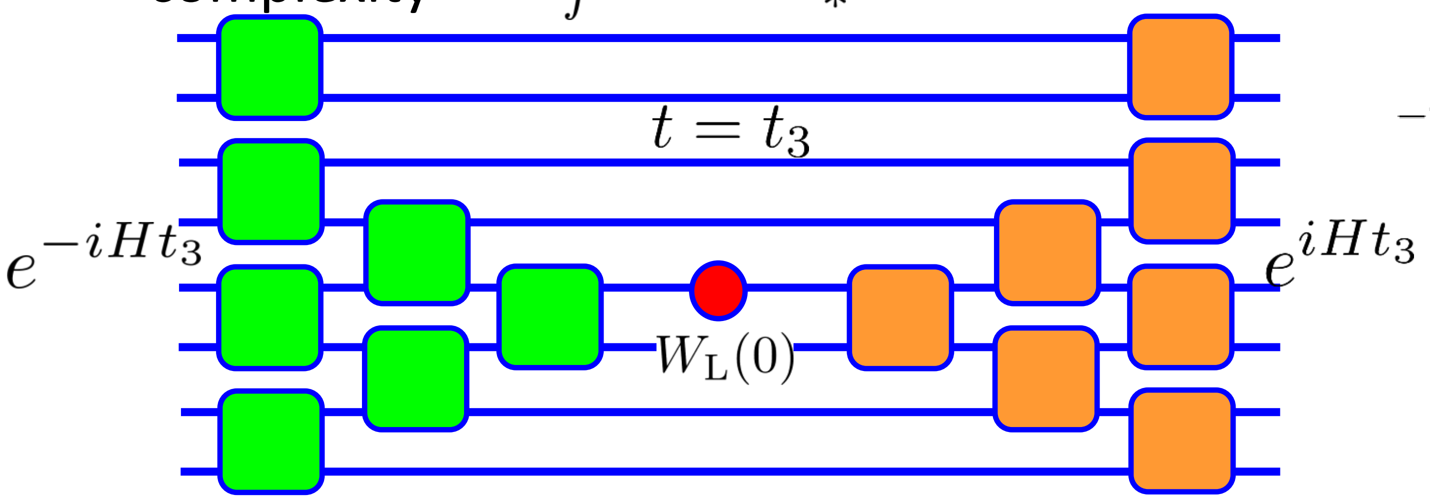

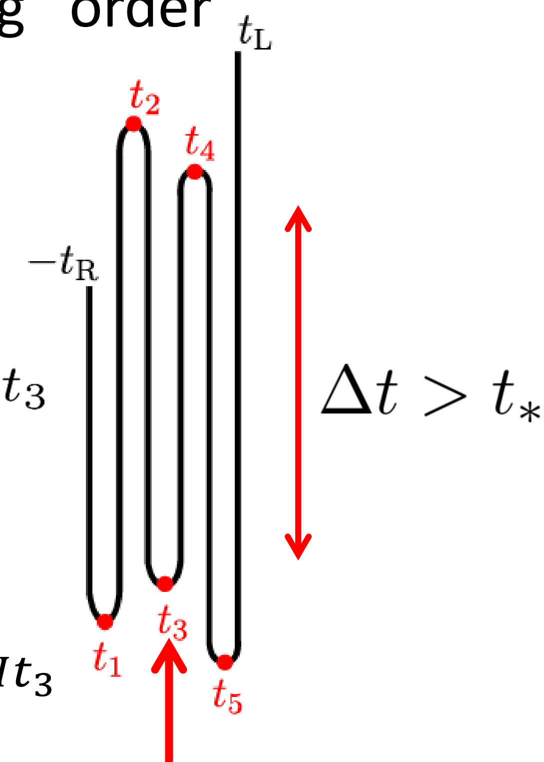

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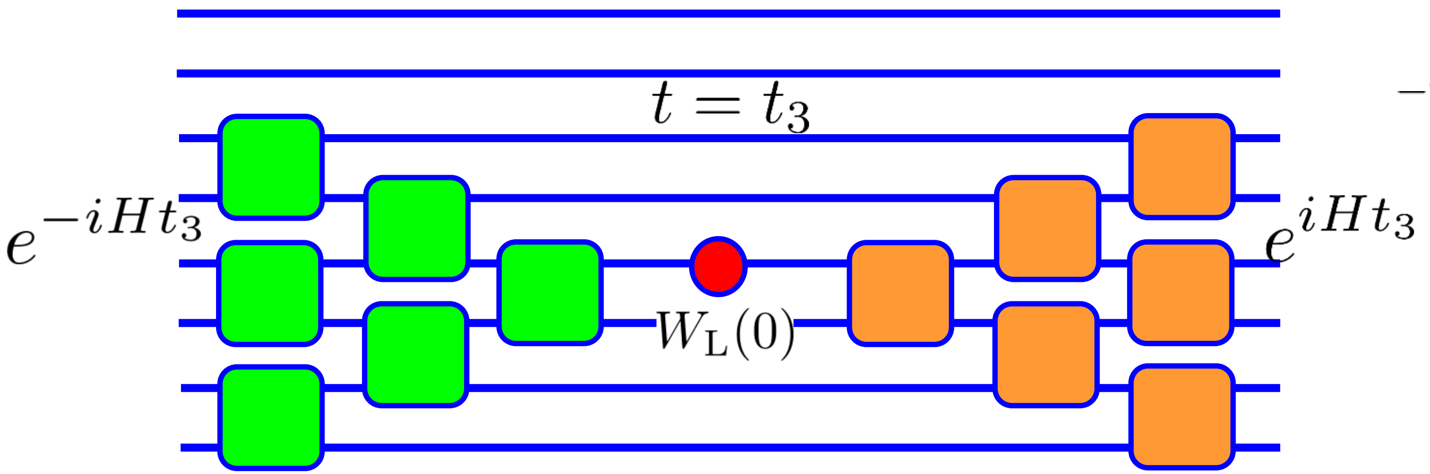

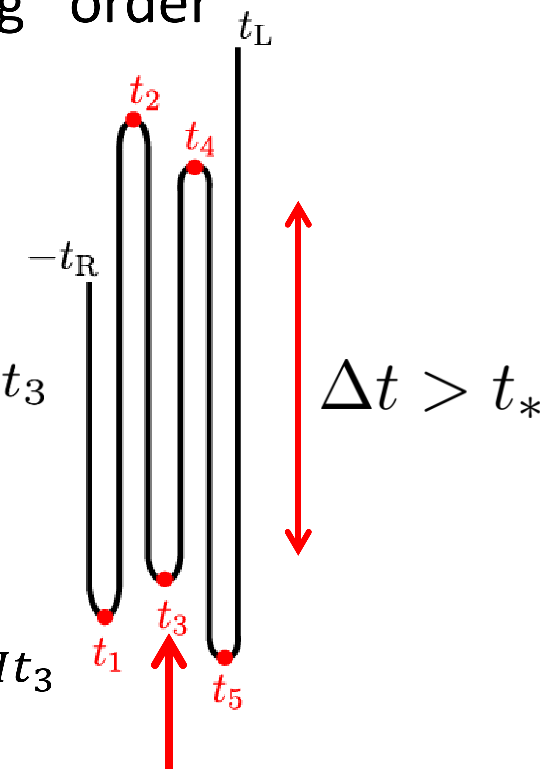

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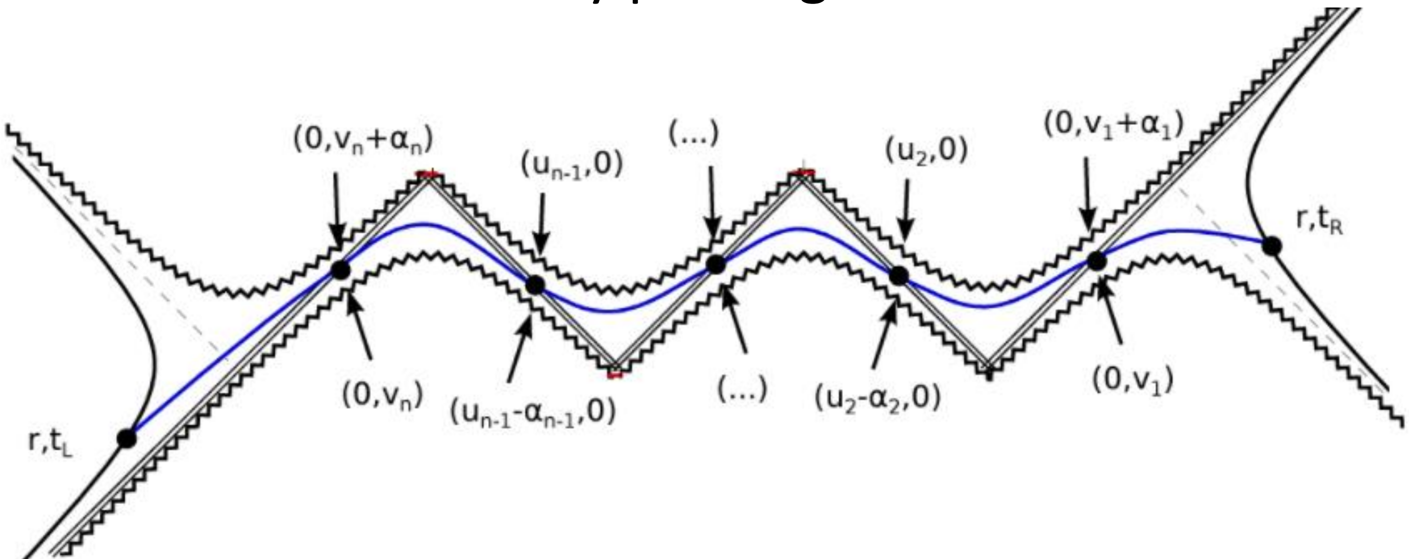
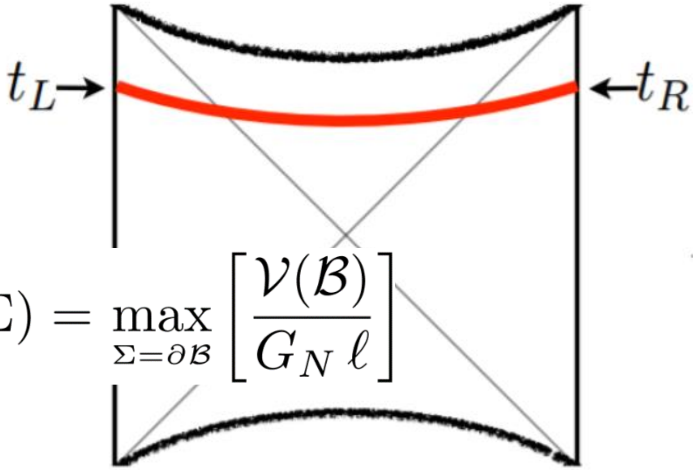
- “switchback effect” is $-2n\,t_*$ due to partial cancellation of forward and backward evolutions along the time folds
- reproduce this behaviour by probing black hole with shock waves



Holographic Complexity:

Complexity = Volume



$$\mathcal{C}_{\rm V}(\Sigma) = \max_{\Sigma=\partial\mathcal{B}}\left[\frac{\mathcal{V}(\mathcal{B})}{G_N\,\ell}\right]$$

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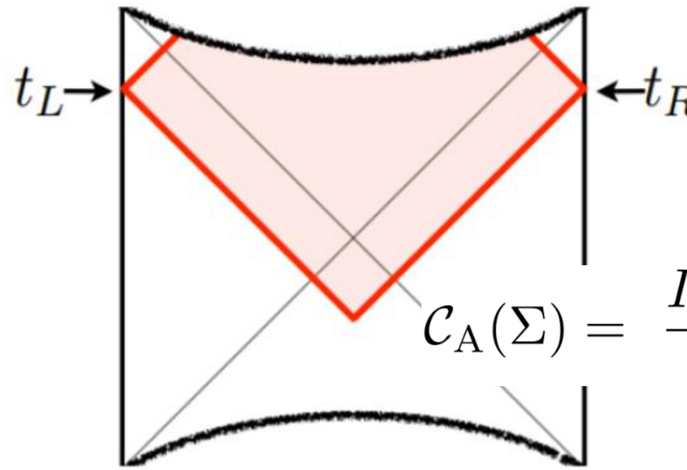

$$\mathcal{C}_{\rm A}(\Sigma) = \frac{I_{\rm WDW}}{\pi\,\hbar}$$

WHY COMPLEXITY??

- connection of complexity=volume to AdS/MERA ✓
- linear growth (at late times) (d = boundary dimension)

$$\left.\frac{d\mathcal{C}_{\rm V}}{dt}\right|_{t\to\infty} = \frac{8\pi}{d-1}\,M \quad \text{(planar)} \qquad \left.\frac{d\mathcal{C}_{\rm A}}{dt}\right|_{t\to\infty} = \frac{2M}{\pi}$$ ✓

- "switchback effect" (probe black holes with shock waves) ✓

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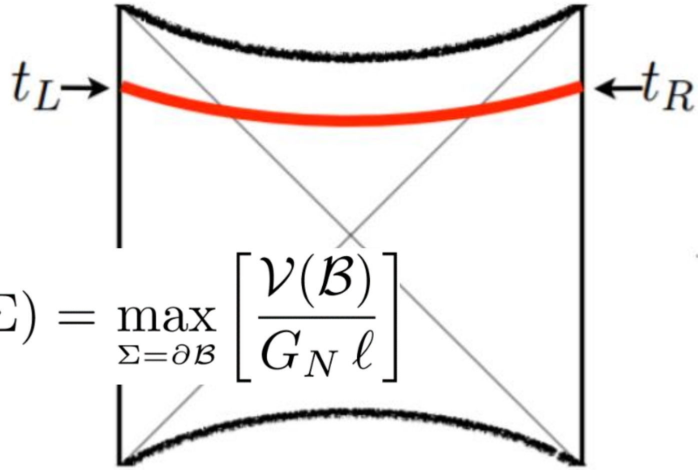

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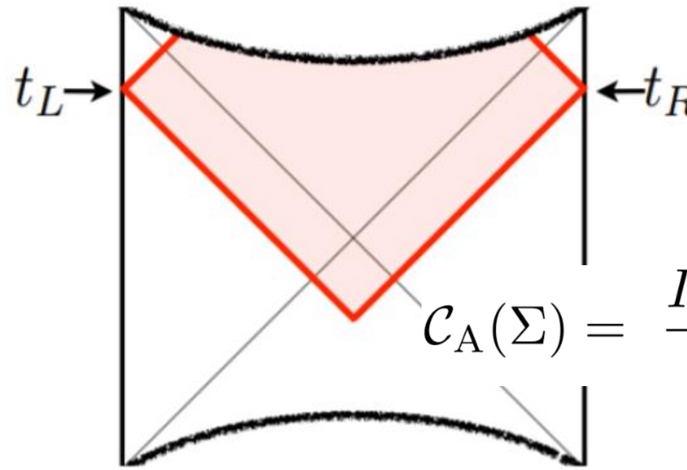
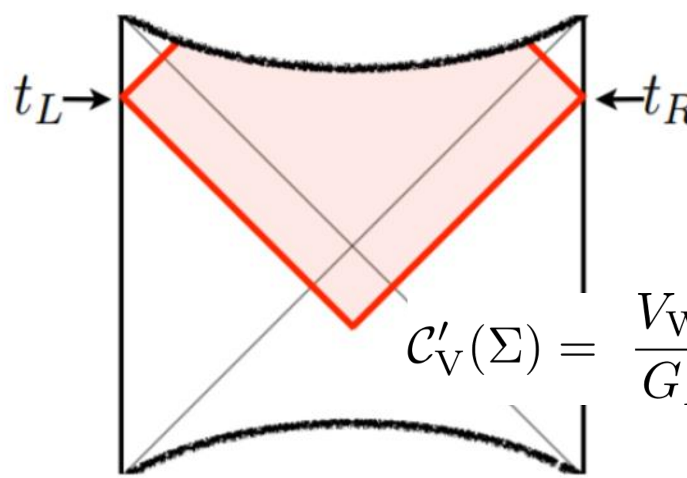


$$\mathcal{C}_\mathrm{A}(\Sigma) = \frac{I_\mathrm{WDW}}{\pi\,\hbar}$$

WHY Volume
or Action
or Spacetime Volume???

Complexity = Volume2.0



$$\mathcal{C}'_\mathrm{V}(\Sigma) = \frac{V_\mathrm{WDW}}{G_N\,\ell^2}$$

Holographic Complexity:

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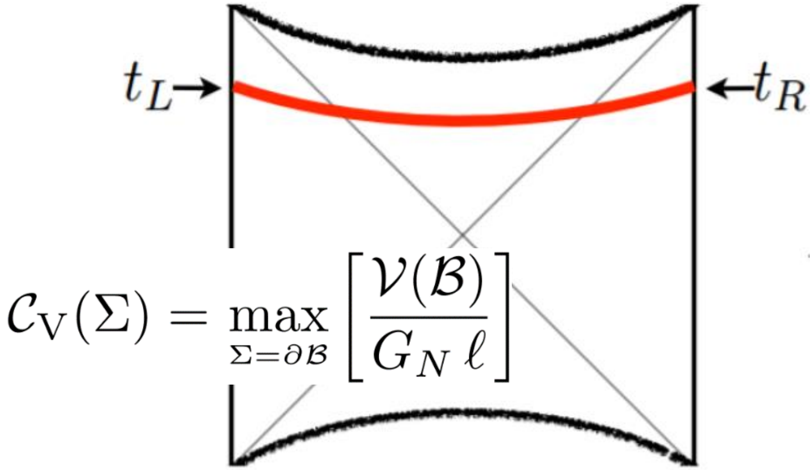

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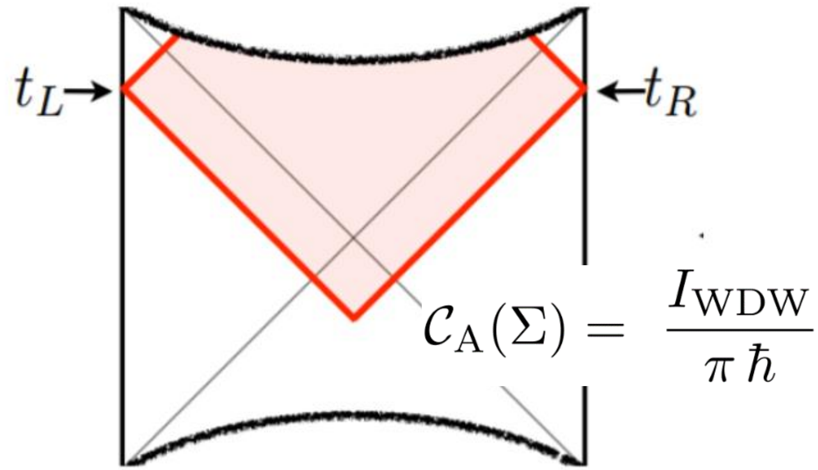

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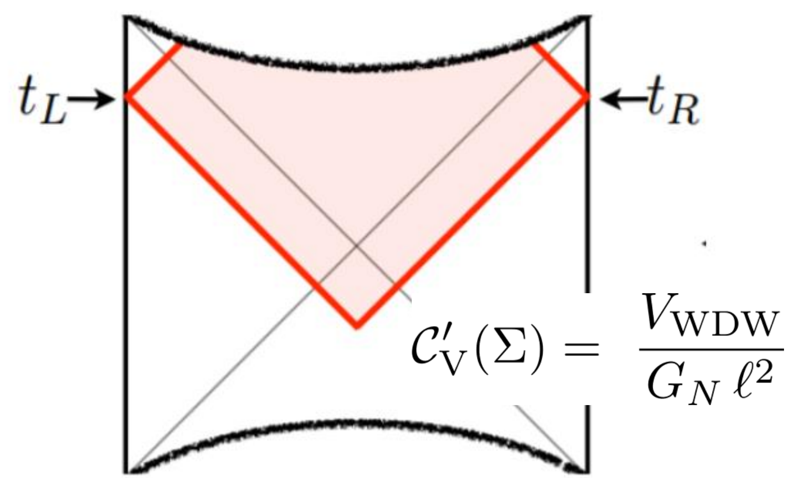


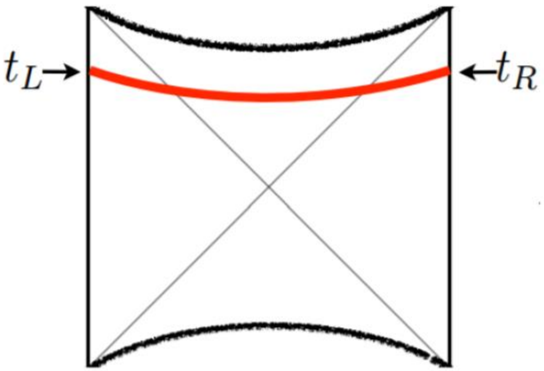
Ambiguities in defining complexity?

Belin, RCM, Ruan, Sarosi & Speranza (2111.02429)

Complexity=Volume Revisited:

- complexity=volume: evaluate proper volume of extremal codim-one surface connecting Cauchy surfaces in boundary theory (cf holo EE)

(Stanford & Susskind)



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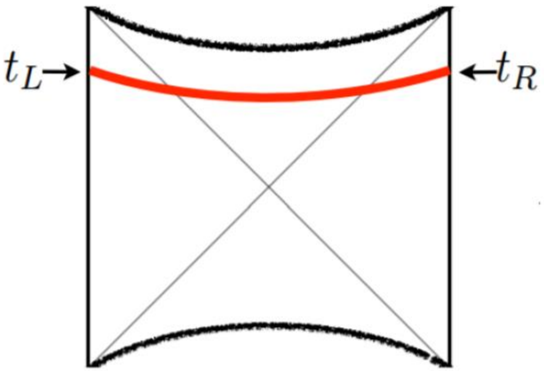

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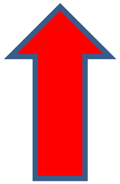

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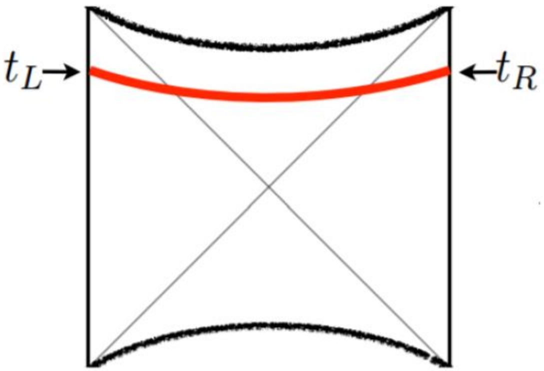

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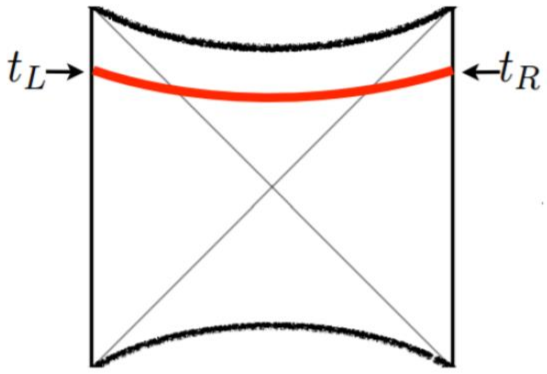

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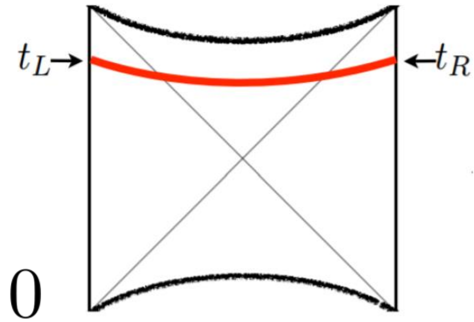
- yields "nice" diffeomorphism invariant observable

Belin, RCM, Ruan, Sarosi & Speranza (2111.02429)

Generalize two step procedure:



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$$\delta_X \left(\int_\Sigma d^d\sigma \sqrt{h}\; F_2(g_{\mu\nu}; X^\mu) \right) = 0$$

- F_2 is *scalar* function of bkgd metric $g_{\mu\nu}$ and embedding $X^\mu(\sigma)$

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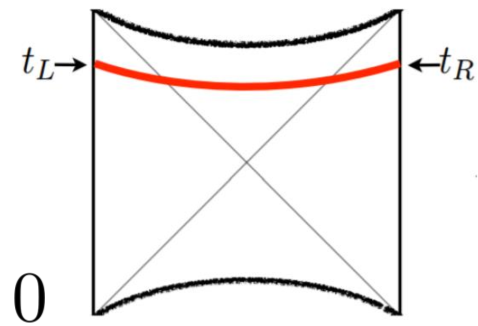

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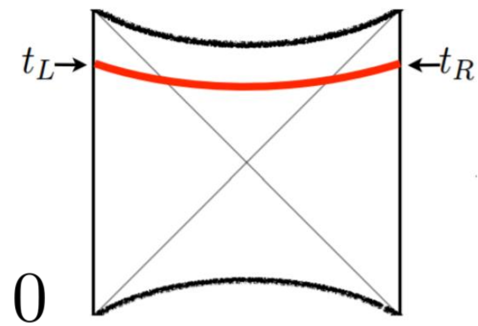

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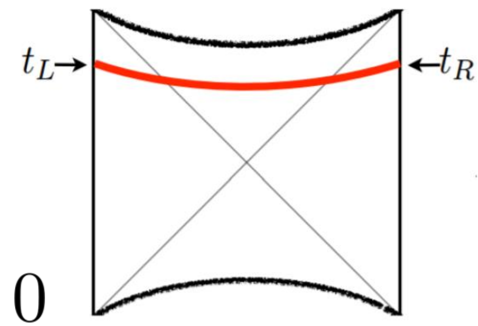

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So what?

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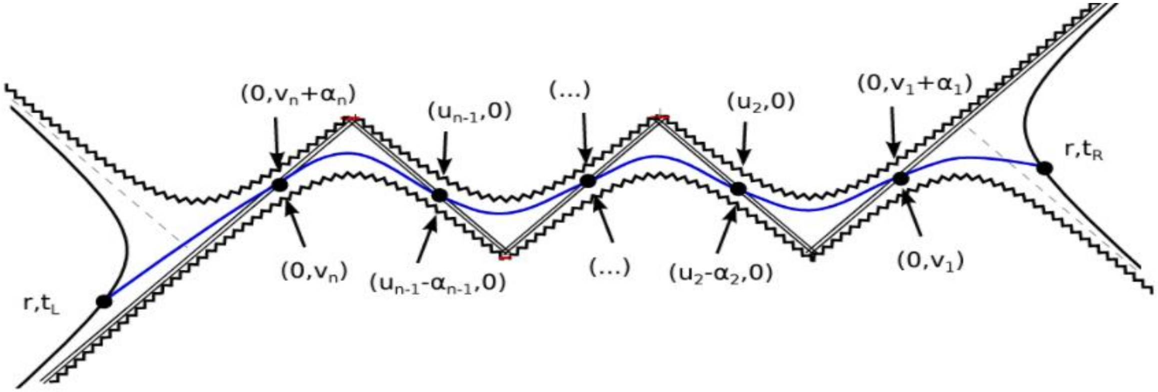
1) Observables grow linearly with time at late times:

$$\lim_{\tau \to \infty} O_{F_1, \Sigma_{F_2}}(\tau) \sim P_\infty \, \tau$$

where in large *T* limit, the constant P_∞ ∝ mass

2) Observables exhibit "switchback effect", ie, universal time delay in response to shock waves falling into the dual black hole



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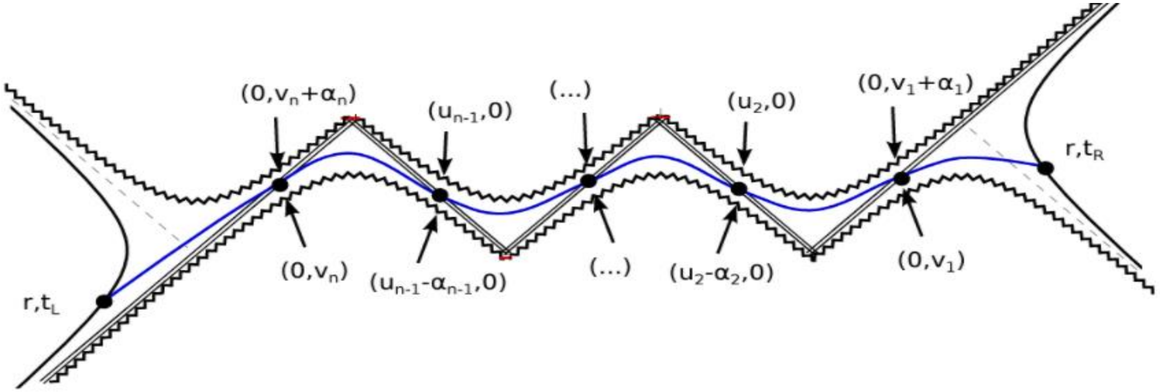


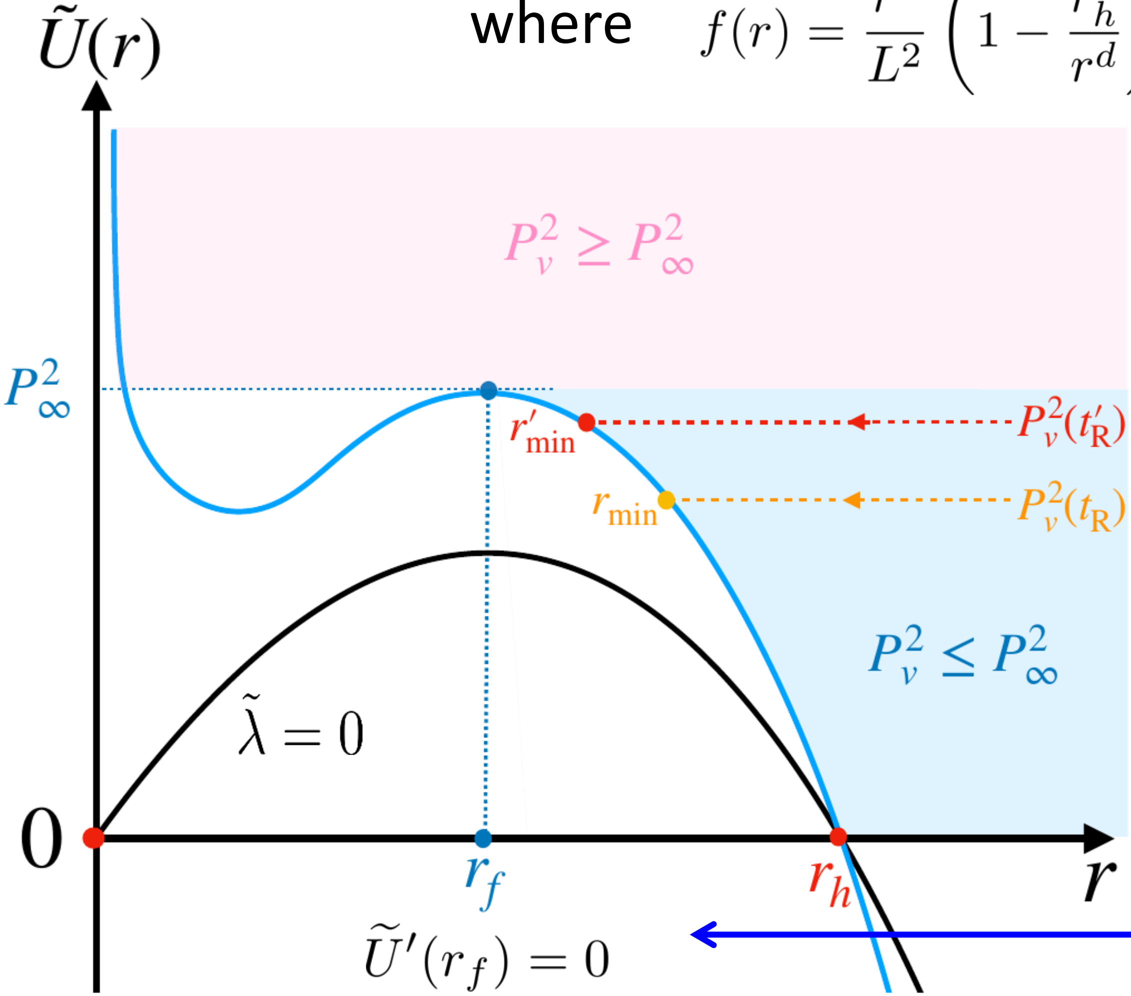
- when F_1 and F_2 constructed with bkgd curvtures, analysis similar to extremal volume . . . . .

Simple Example: $F_1 = F_2 = 1 + \lambda L^4 C_{abcd}C^{abcd}$

- profile determined by classical mechanics problem

$$\dot{r}^2 + \widetilde{U}(r) = P_v^2 \quad \text{with} \quad \widetilde{U}(r) = -f(r)a^2(r)\left(\frac{r}{L}\right)^{2(d-1)},$$

where $f(r) = \frac{r^2}{L^2}\left(1 - \frac{r_h^d}{r^d}\right)$ and $a(r) = 1 + \tilde{\lambda}\left(\frac{r_h}{r}\right)^{2d}$



- turning point:

$$P_v^2 = \widetilde{U}(r_{min})$$

$$P_\infty^2 = U(r_f) = \xi\left(\frac{r_h}{L}\right)^{2d}$$

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surfaces reach critical radius in BH interior

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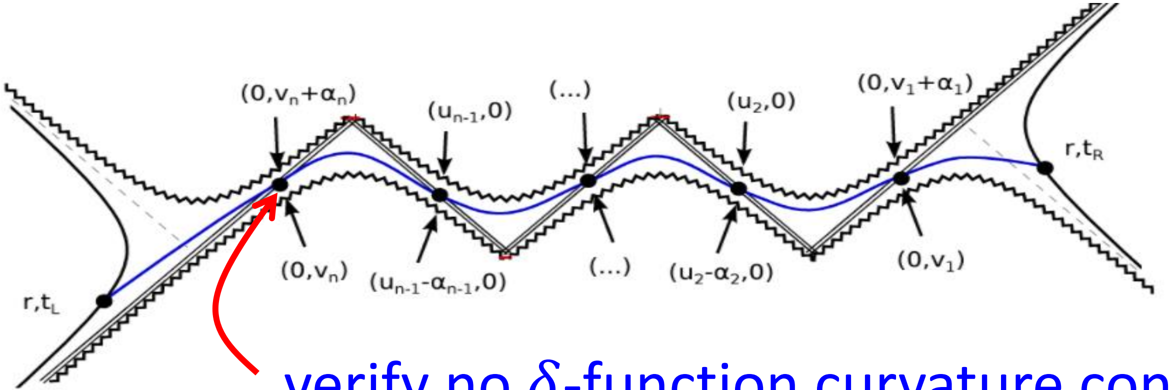

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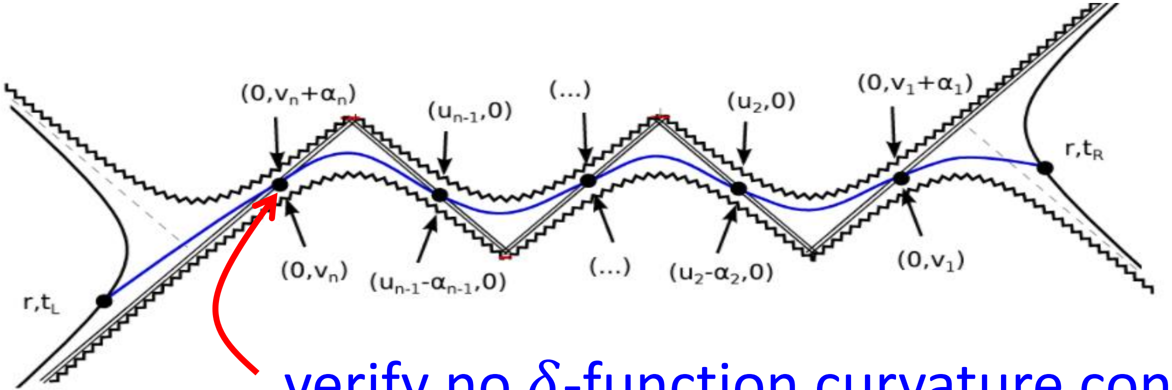

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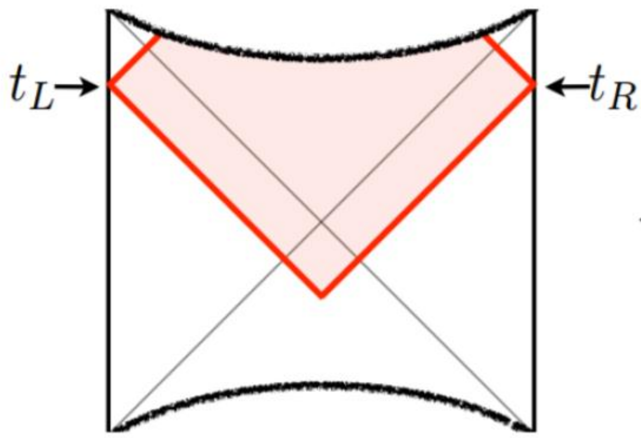
- **universality displayed observables suggests that all of them are equally viable candidates for holographic complexity!!**

What about extensions of CA and CV2.0?

- complexity=action: evaluate gravitational action for Wheeler-DeWitt patch = domain of dependence of bulk time slice connecting boundary Cauchy slices in CFT (Brown, Roberts, Swingle, Susskind & Zhao)
- complexity=volume**2.0**: evaluate spacetime volume of WDW patch (Couch, Fischler & Nguyen)



$$\mathcal{C}_{\rm A}(\Sigma) = \frac{I_{\rm WDW}}{\pi\,\hbar}$$

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- no extremization procedure implemented; surfaces bounding volume (ie, codimension-zero region) are light sheets

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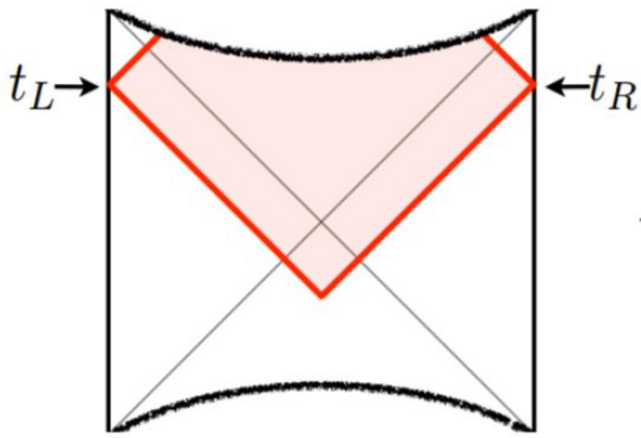

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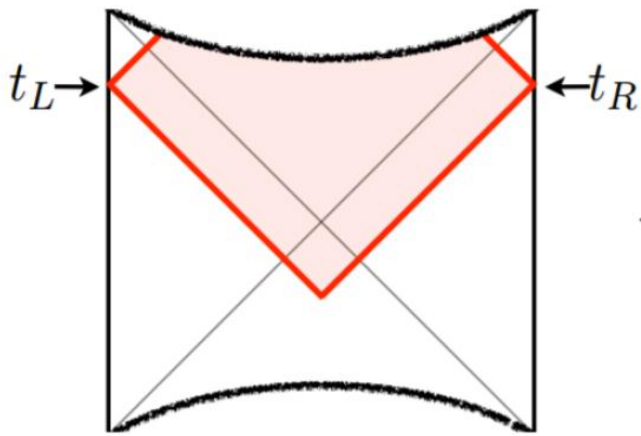

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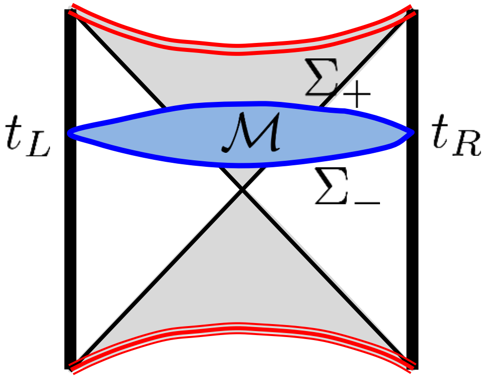
2) Evaluate geometric feature of codim.-0 region (& bounding surfaces)

- yields "nice" diffeomorphism invariant observable

Generalized procedure for codim.-0 observables:



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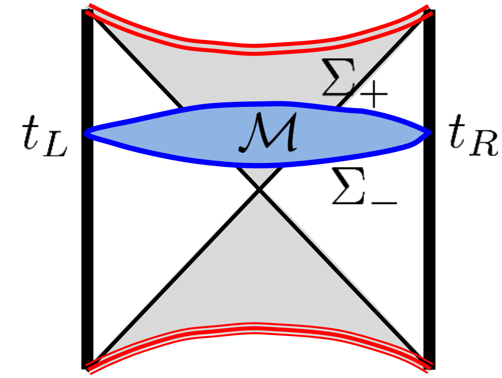

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2) Evaluate geometric feature of corresponding region:

$$O(\Sigma_{CFT})=\frac{1}{G_N\,L}\int_{\Sigma_+} d^d\sigma\,\sqrt{h}\;F_1(g_{\mu\nu};X^\mu_+)
+\frac{1}{G_N\,L}\int_{\Sigma_-} d^d\sigma\,\sqrt{h}\;F_2(g_{\mu\nu};X^\mu_-)+\frac{1}{G_N\,L^2}\int_{\mathcal{M}} d^{d+1}\sigma\,\sqrt{-g}\;F_3(g_{\mu\nu})$$

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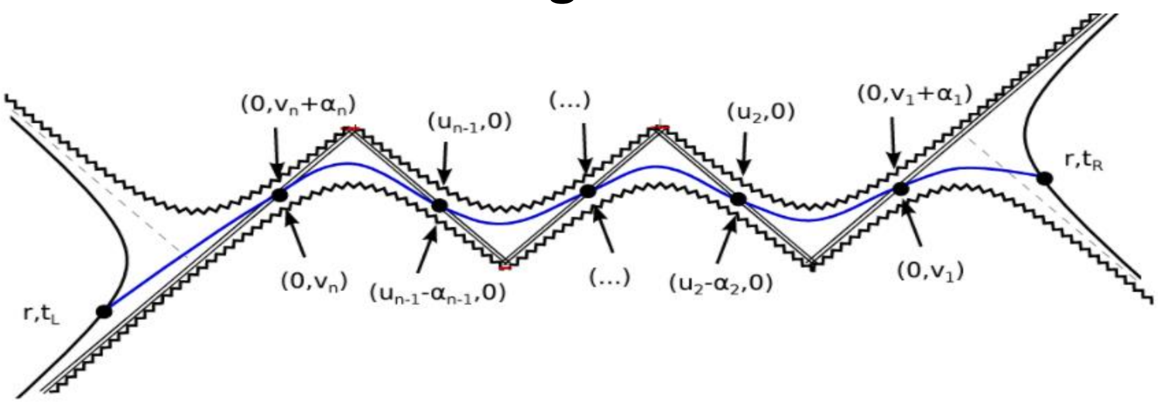

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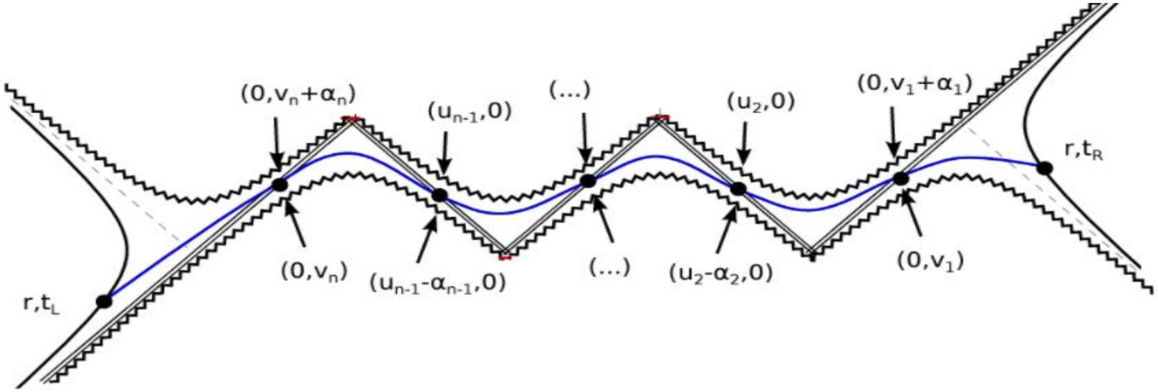

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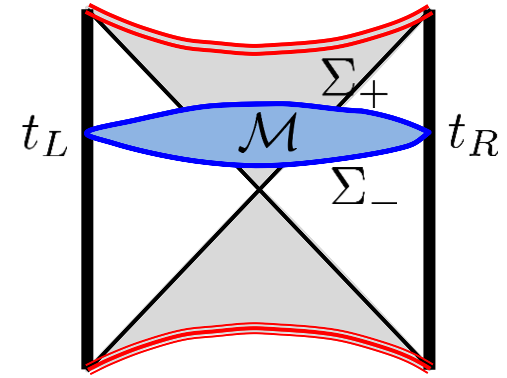

- extremize the functional

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- evaluating the volumes of the bounding surfaces $\Sigma_\pm$ weighted by coefficients $\alpha_\pm$, as well as of volume of codim.-0 region $\mathcal{M}$
- extremal equations yields CMC surfaces (eg, see Witten)

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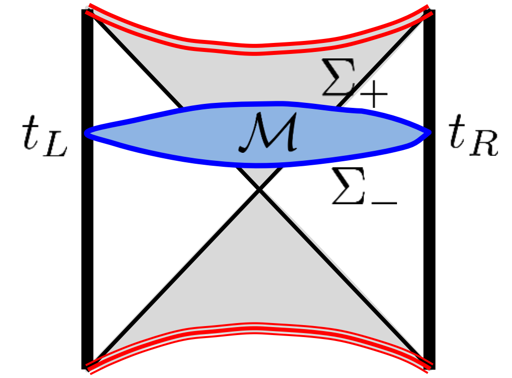

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- in limit $\alpha_\pm \to 0$, these surfaces become the future/past light sheets

⟶ $\mathcal{M}$ becomes WDW patch!

- evaluate volume (same functional) ⟶ CV2.0
- evaluate action (including bdy terms) ⟶ CA

Conclusions/Questions/Outlook:

- simple example but "classical mechanics" analysis readily extends to $F_1(g_{\mu\nu}, \mathcal{R}_{\mu\nu\rho\sigma}, \nabla_\mu)$ and to observables where $F_1 \neq F_2$
- couplings for curvature invariants should not be too large
- similar behaviour appears to hold for functionals including dependence on extrinsic curvature
- **infinite class of holographic observables equally viable candidates for gravitational dual of complexity!!**
- can freedom in constructing gravitational observables be related to freedom in constructing complexity model in boundary QFT
- is there something that singles out maximal volume?
- what is role of extremal solutions which are not global maxima and probe very near to singularity?
- further investigation of codimension-zero observables
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- is there so…

Lots to explore!

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BRIAN SIBLEY