

Quantum Cryptography

Christian Schaffner

Research Center for Quantum Software

Institute for Logic, Language and Computation (ILLC)
University of Amsterdam



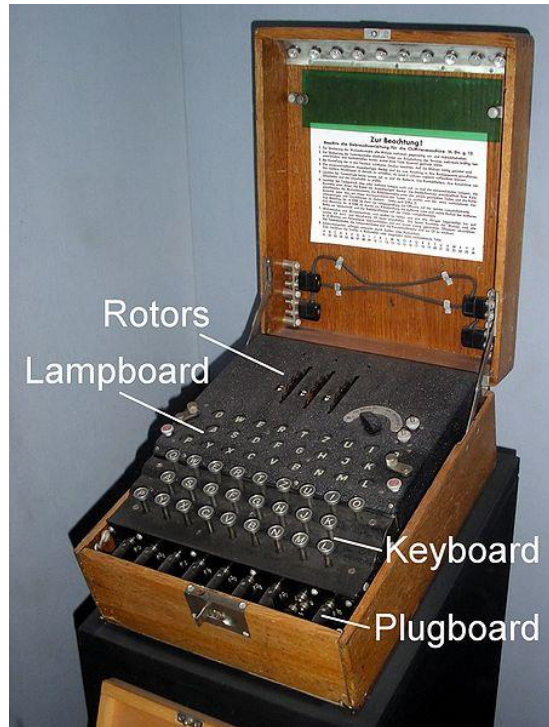
Logic, Language and Computation
Monday, 24 September 2018

Classical Cryptography

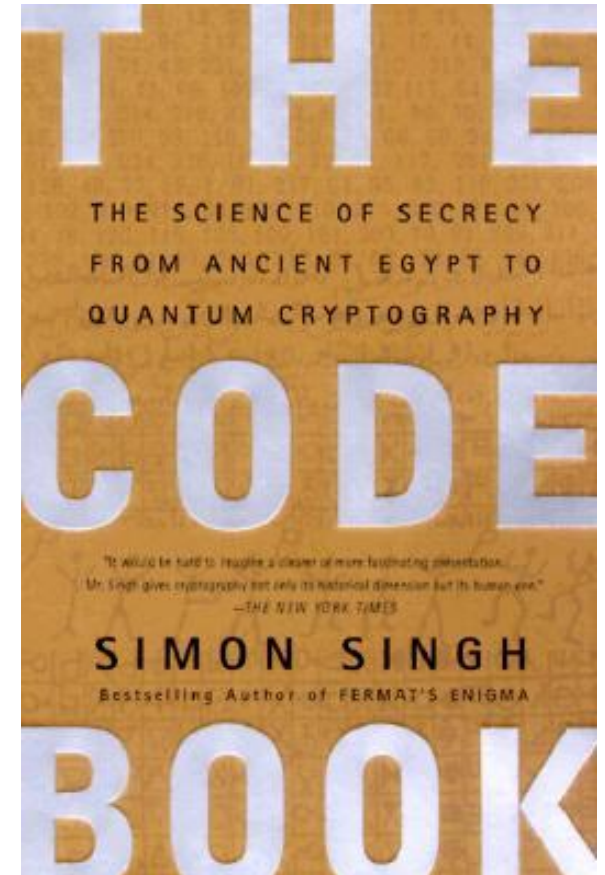
- 3000 years of fascinating history
- Until 1970: **private communication** was the only goal



Scytale



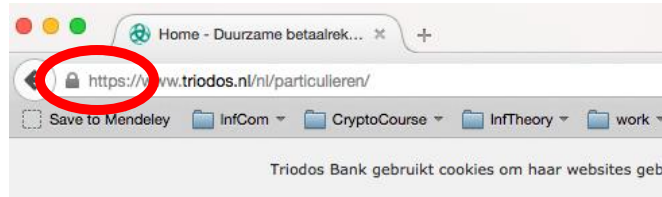
Enigma



Modern Cryptography

- is everywhere!
- is concerned with all settings where people do not trust each other

Edward Snowden



What will you Learn from this Talk?

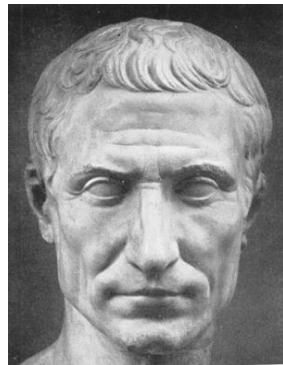
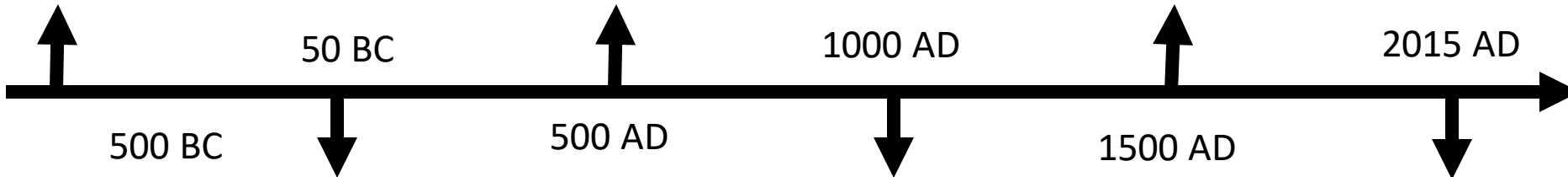
- Classical Cryptography (& Politics)
- Quantum Mechanics
- Crypto Threat of Quantum Computing
- Quantum Cryptography
- Quantum Future

Ancient Cryptography

Scytale



Blaise de Vigenère



Caesar Cipher (ROT4)
(variant still [in use](#))

Additional Hints ([Decrypt](#))

baqre jngre / nna xrggvat

Additional Hints ([Encrypt](#))

onder water / aan ketting



Decryption Key

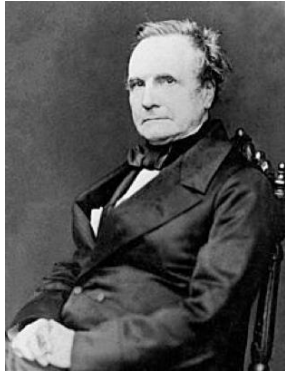
A|B|C|D|E|F|G|H|I|J|K|L|M

N|O|P|Q|R|S|T|U|V|W|X|Y|Z

(letter above equals below, and vice versa)

Ancient Cryptography

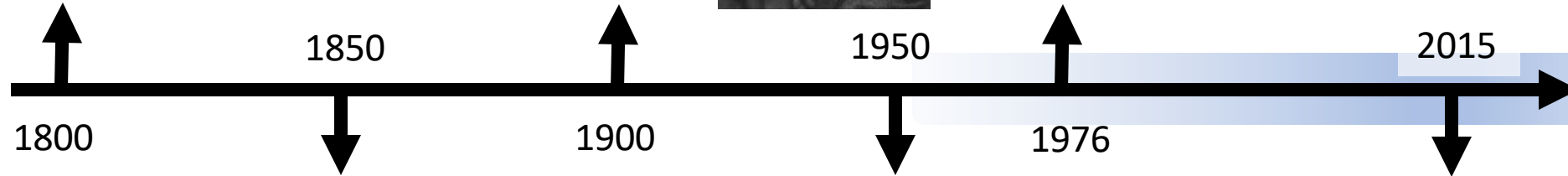
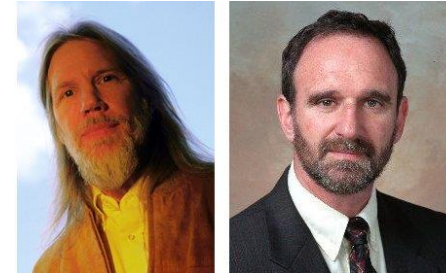
Charles Babbage



Claude Shannon



Diffie / Hellman



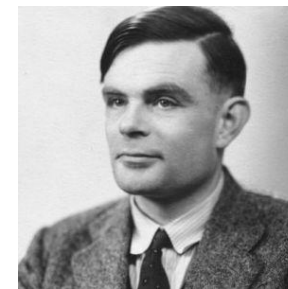
“a cryptographic system should be secure even if everything but the key is known to the adversary”



 Auguste Kerckhoffs



Enigma

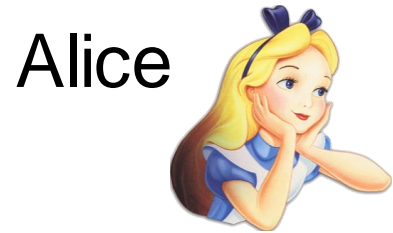


Alan Turing
([The Imitation Game](#))

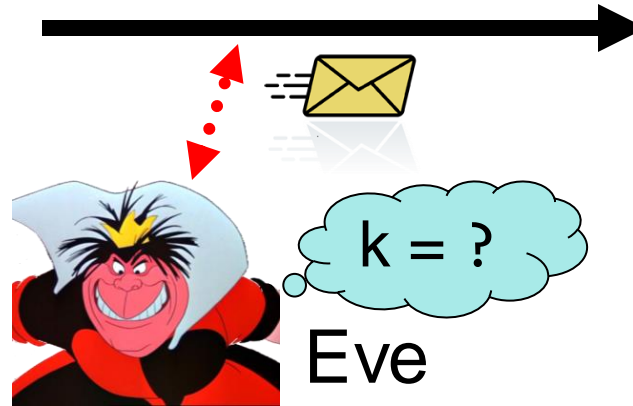


Secure Encryption

$m = \text{'doe you'}$



$k = 0101\ 1011$



Bob



$k = 0101\ 1011$

- Goal: Eve **does not learn** the message
- Setting: Alice and Bob share a secret key k

eXclusive OR (XOR) Function (exclusive disjunction)

x	y	$x \oplus y$
0	0	0
1	0	1
0	1	1
1	1	0

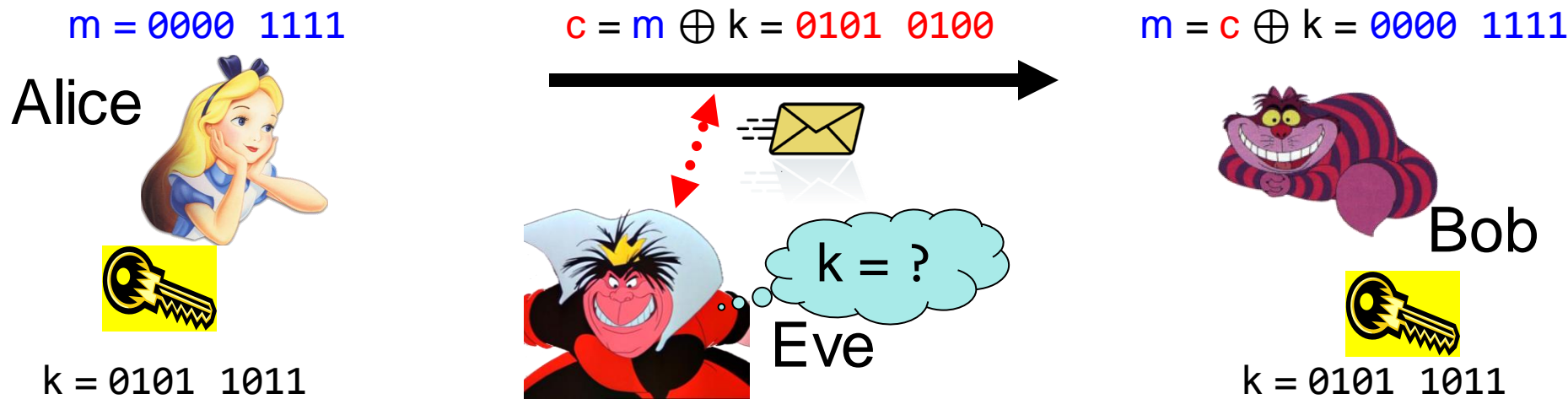
- Some properties:

- $\forall x : x \oplus 0 = x$

- $\forall x : x \oplus x = 0$

$$\forall x, y : x \oplus y \oplus y = x$$

One-Time Pad Encryption



- Goal: Eve **does not learn** the message
- Setting: Alice and Bob share a key k
- Recipe:

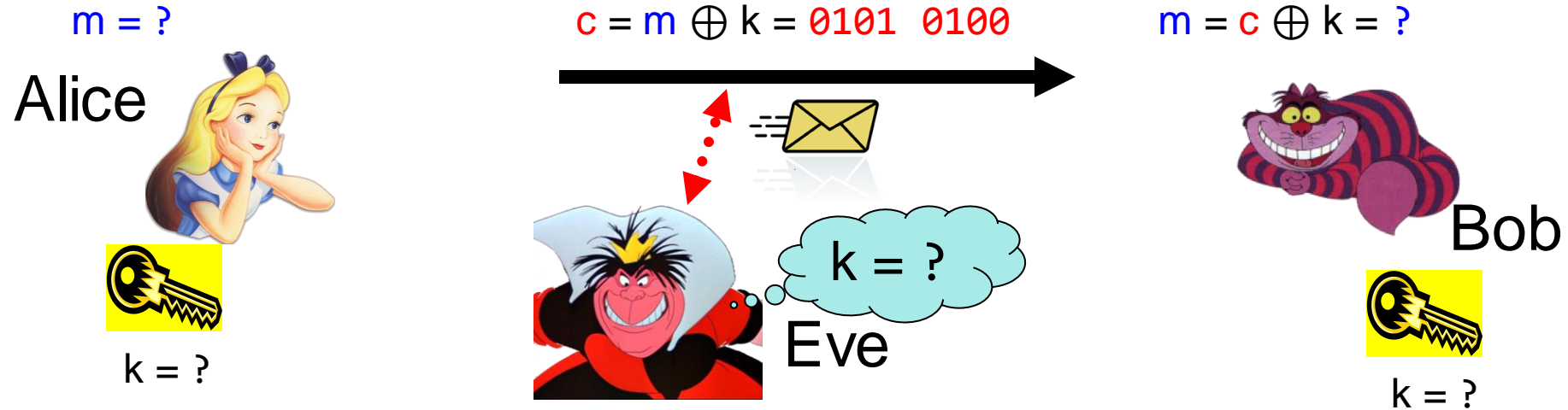
$$\begin{aligned}
 m &= 0000 \ 1111 \\
 k &= 0101 \ 1011 \\
 c &= m \oplus k = 0101 \ 0100
 \end{aligned}$$

$$\begin{aligned}
 c &= 0101 \ 0100 \\
 k &= 0101 \ 1011 \\
 c \oplus k &= 0000 \ 1111 \\
 c \oplus k &= m \oplus k \oplus k = m \oplus 0 = m
 \end{aligned}$$

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

- Is it secure?

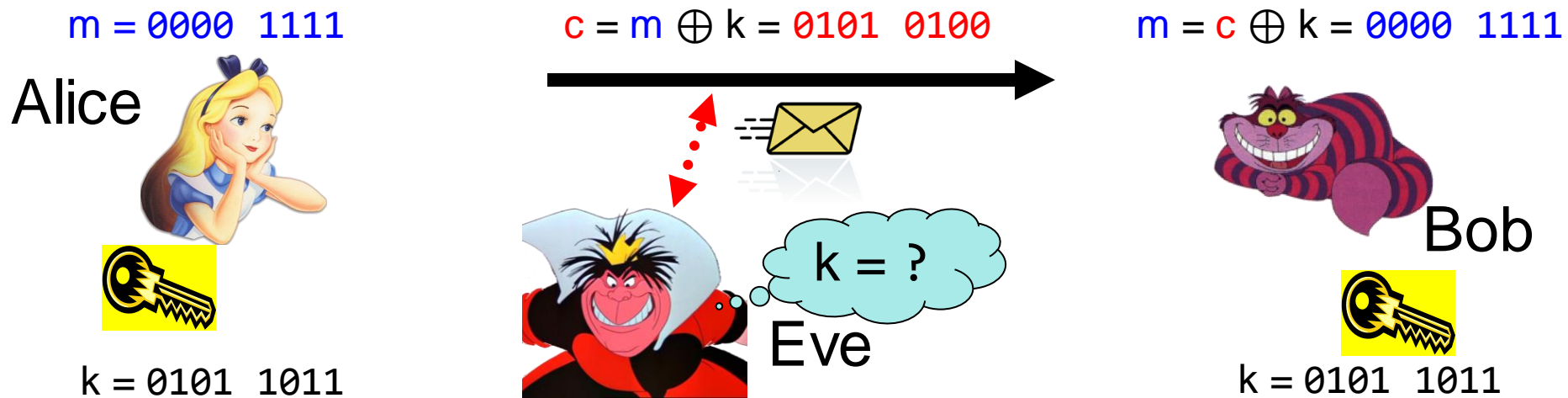
Perfect Security



- Given that
 - is it possible that
 - Yes, if $c = 0101 \ 0100,$
 $m = 0000 \ 0000 ?$
 $k = 0101 \ 0100.$
 - is it possible that $m = 1111 \ 1111 ?$
Yes, if $k = 1010 \ 1011.$
 - it is possible that $m = 0101 \ 0101 ?$
Yes, if $k = 0000 \ 0001$
- In fact, every m is possible.
- Hence, the one-time pad is **perfectly secure!**

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

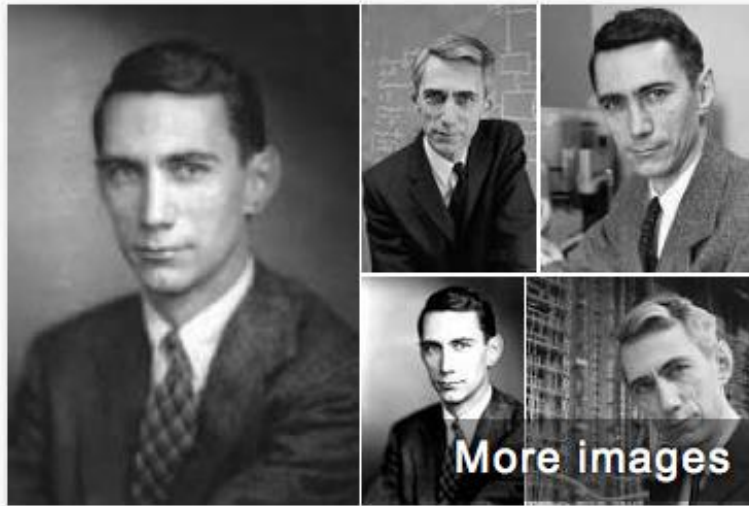
Problems With One-Time Pad



- The key has to be **as long as** the message.
- The key can only be **used once**.

Information Theory

- 6 EC MoL course, given in 2nd block: Nov/Dec 2018
- mandatory for Logic & Computation track
- first lecture: Tuesday, 30 October 2017, 9:00
- <http://homepages.cwi.nl/~schaffne/courses/inftheory/2018/>



Claude Shannon

Mathematician

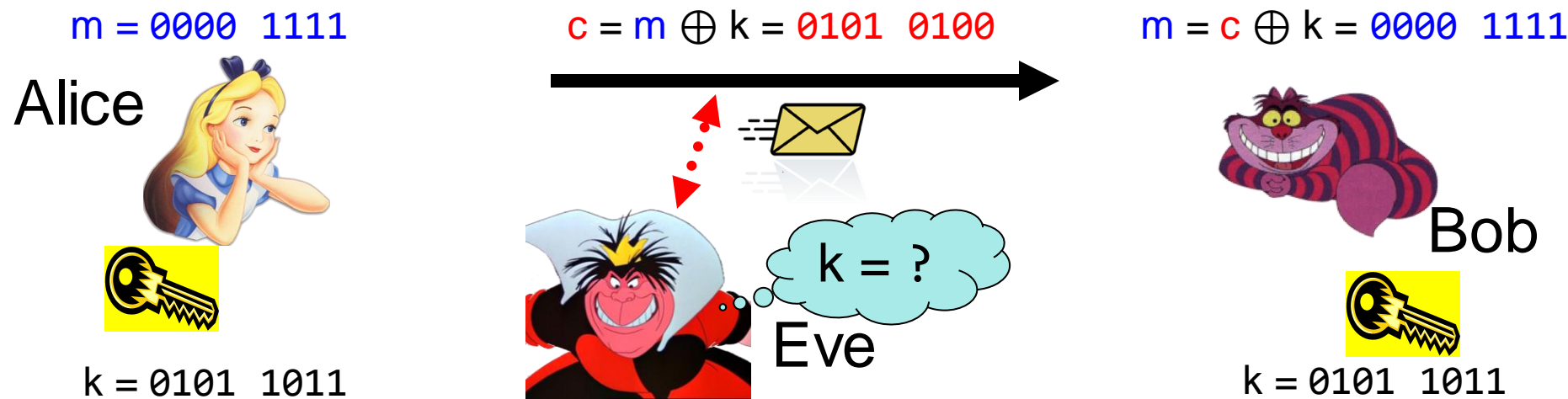
Claude Elwood Shannon was an American mathematician, electronic engineer, and cryptographer known as "the father of information theory". Shannon is famous for having founded information theory with a landmark paper that he published in 1948.

[Wikipedia](#)

Born: April 30, 1916, [Petoskey, Michigan, United States](#)

Died: February 24, 2001, [Medford, Massachusetts, United States](#)

Problems With One-Time Pad



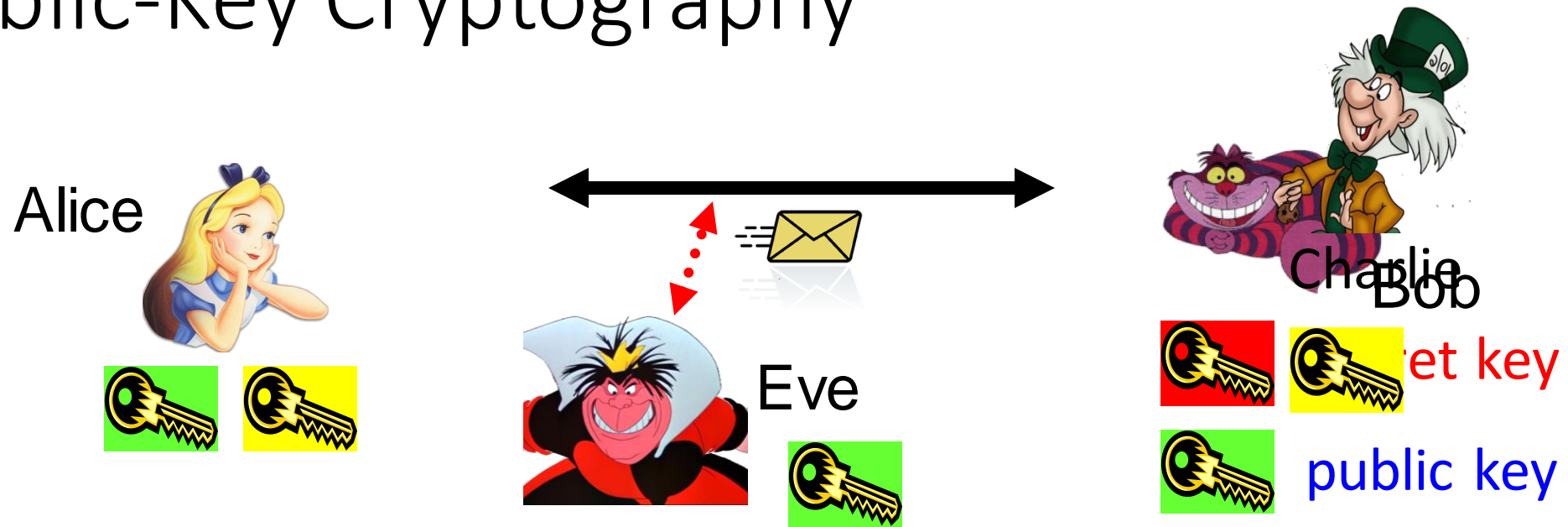
- The key has to be **as long as** the message.
- The key can only be **used once**.
- In practice, other encryption schemes (such as [AES](#)) are used which allow to encrypt long messages with short keys.
- One-time pad does not provide [authentication](#):
Eve can easily flip bits in the message

Symmetric-Key Cryptography



- Encryption insures **secrecy**:
Eve **does not learn** the message, e.g. [one-time pad](#)
- Authentication insures **integrity**:
Eve **cannot alter** the message
- General problem: players have to exchange a key to start with

Public-Key Cryptography



- Solves the key-exchange problem.
- Everyone can encrypt using the [public key](#).
- Only the holder of the **secret key** can decrypt.
- [Digital signatures](#): Only **secret-key** holder can sign, but everyone can verify signatures using the [public-key](#).

History of Public-Key Crypto



- Early 1970s: invented in the „classified world“ at the British Government Communications Head Quarters (GCHQ) by Ellis, Cocks, Williamson



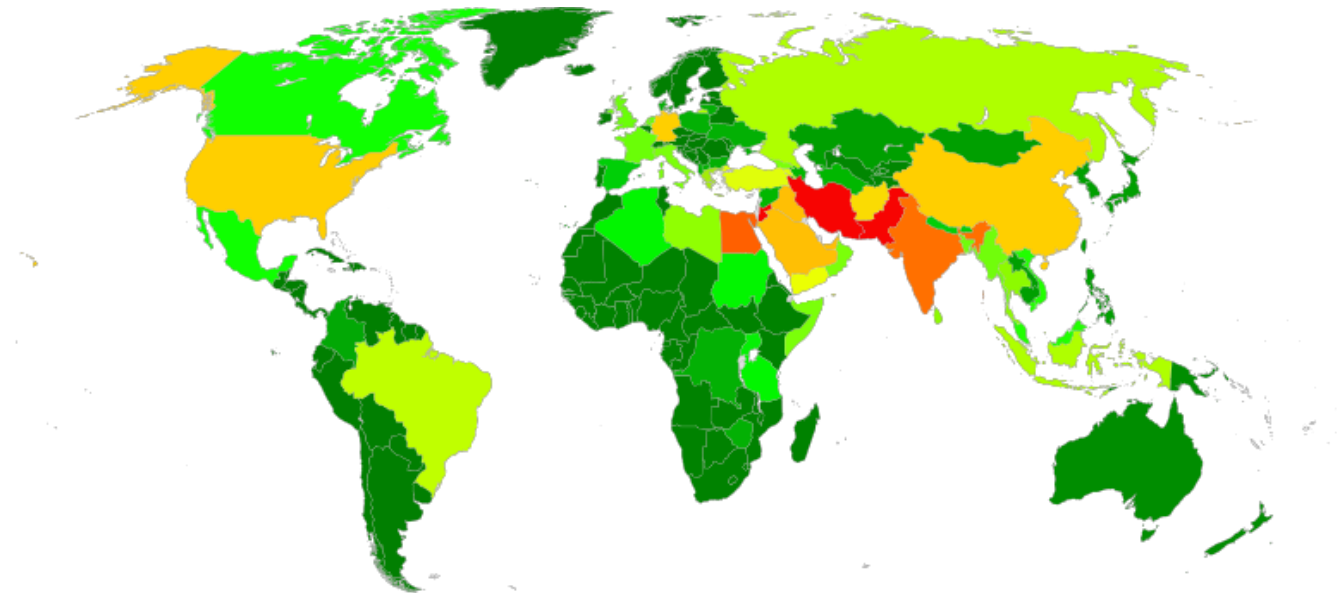
- Mid/late 1970s: invented in the „academic world“ by Merkle, Hellman, Diffie, and Rivest, Shamir, Adleman (RSA)



Politics of Cyberwar



- [Edward Snowden](#), former CIA and NSA employee, now [whistleblower](#) and on (temporary) asylum in Russia
- In 2013, he leaked many thousand top secret documents to various media, documenting
- [mass surveillance programs](#) by secret services from all over the world



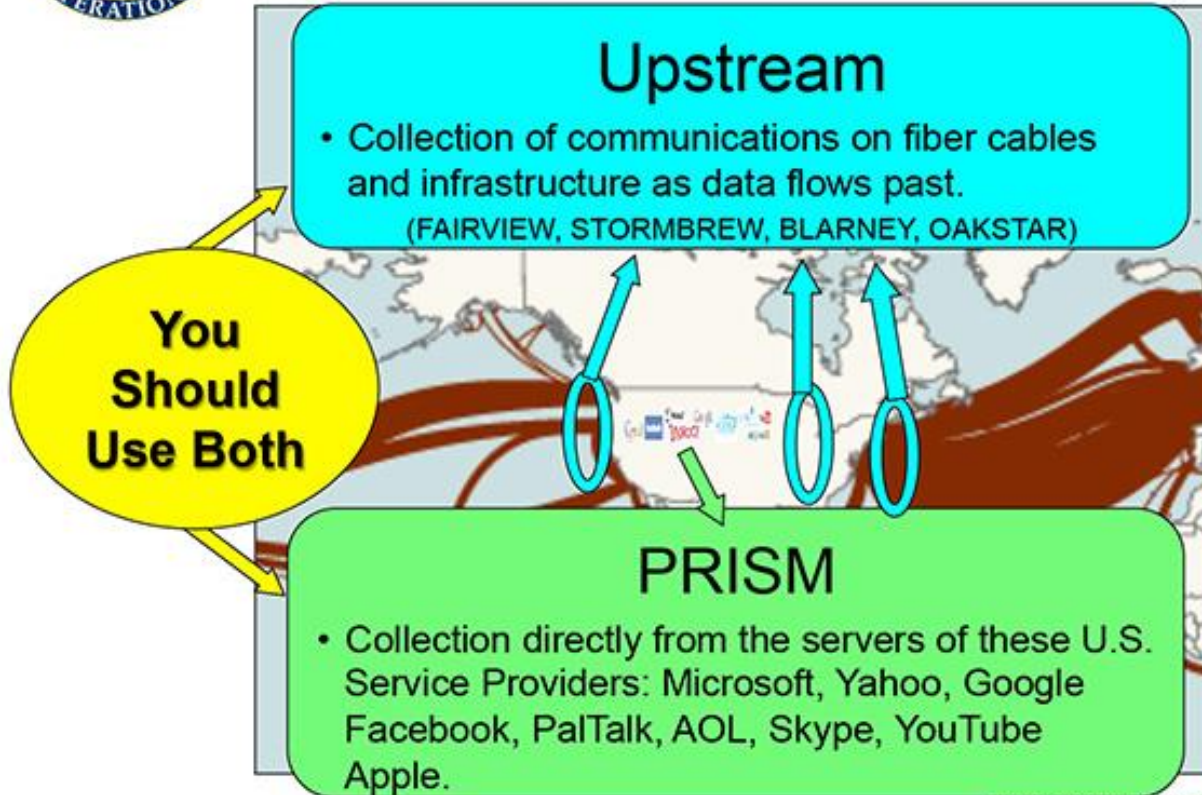
Politics of Cyberwar



TOP SECRET//SI//ORCON//NOFORN



(TS//SI//NF) FAA702 Operations *Two Types of Collection*



TOP SECRET//SI//ORCON//NOFORN

Politics of Cyberwar



- Methods:
 - **Break** cryptography
 - **Influence** industrial standards
 - **Pressure** manufacturers to make insecure devices
 - **Infiltrate** hardware and software (communication infrastructure, computers, smartphones etc.)

- **Why** mass surveillance?
 - Other than to combat terrorism, these surveillance programs have been employed to **assess the foreign policy** and economic stability of other countries, and to **gather "commercial secrets"**.



Why worry?



- „I have nothing to hide“ is a very naive reaction.
- Think about what your smartphone knows about you.
- Think about what your smartphone does not know about you.

TS//SI//REL to USA, FVEY
(S//REL) iPhone Location Services



(U) Who knew in 1984...

TS//SI//REL to USA, FVEY

TS//SI//REL to USA, FVEY
(S//REL) iPhone Location Services



(U) ...that this would be big brother...

TS//SI//REL to USA, FVEY

TS//SI//REL to USA, FVEY
(S//REL) iPhone Location Services



(U) ...and the zombies would be paying customers?

TS//SI//REL to USA, FVEY

TECHNOLOGY

- 2014: **Facebook to Pay \$19 Billion for WhatsApp**
price of \$42 per user for WhatsApp.
- 2016: **MICROSOFT BUYS LINKEDIN FOR \$26.2 BILLION**

Why worry?



- „I have nothing to hide“ is a [very naive reaction](#).
- Everyone's personal privacy is at stake!
- [George Orwell](#)'s surveillance state from his book [1984](#) is coming true...
- *"They (the NSA) can use the system to go back in time and scrutinize every decision you've ever made, every friend you've ever discussed something with, and attack you on that basis to sort of derive suspicion from an innocent life and paint anyone in the context of a wrongdoer." – Edward Snowden*

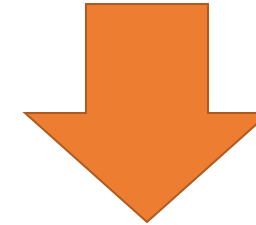


Dutch Example: Dragnet Law

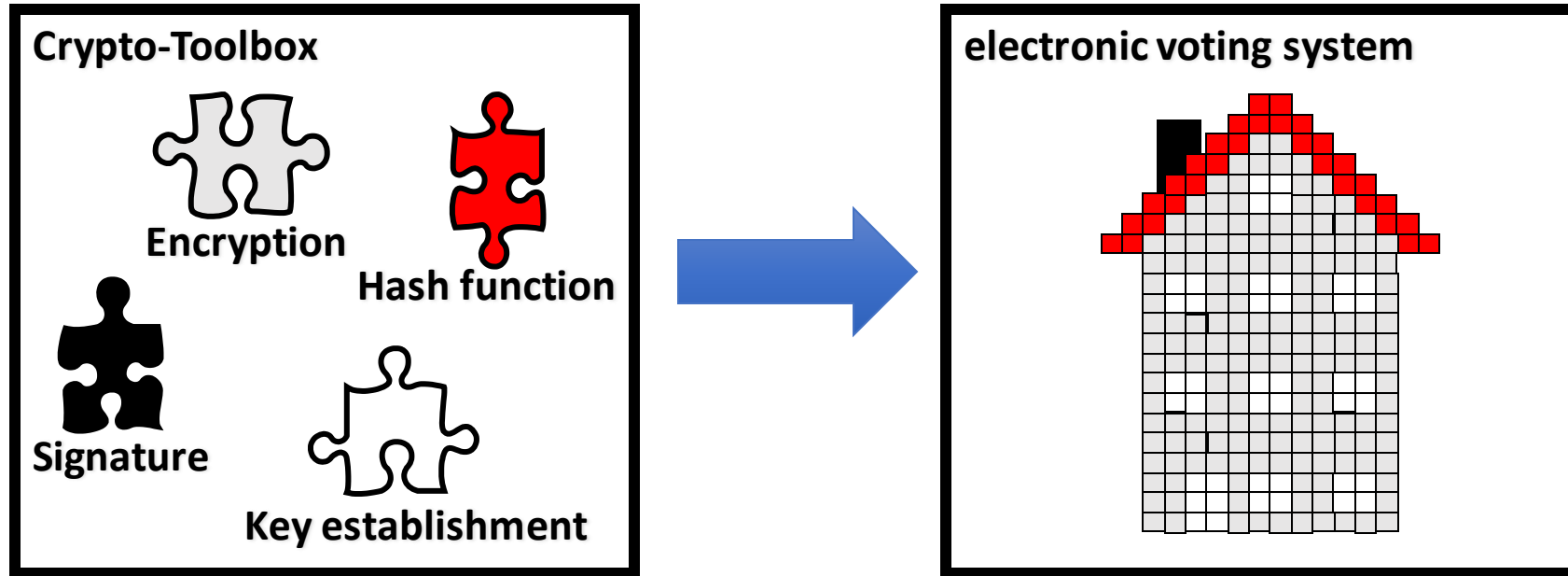
- Law for the intelligence and security services (Wet inlichten en veiligheidsdiensten, Wiv)
- Accepted by parliament in 2017
- Referendum initiated by (former) ILLC students
- 21 March 2018: (6.7 mio votes, participation 51%)
46.5% pro, 49.4% against, 4% empty
- Cosmetic adjustments were done by parliament
- Law is now in effect, allowing Dutch secret services to tap communication infrastructures of “bystanders”, store it for 3 years, share data with foreign services etc.
- <https://www.bitsoffreedom.nl/dossiers/sleepnet/>
<https://geensleep.net/>



Algemene Inlichtingen- en
Veiligheidsdienst
Ministerie van Binnenlandse Zaken en
Koninkrijksrelaties



Cryptography and Logic



- Cryptographic protocols for advanced tasks are built from basic building blocks
- Problem: security proofs become **very hard to verify**
- Solution:
 - Logic provides formal methods to specify tools and task
 - Use automatic proof checkers to verify security

What will you Learn from this Talk?

✓ Classical Cryptography (& Politics)

■ Introduction to Quantum Mechanics

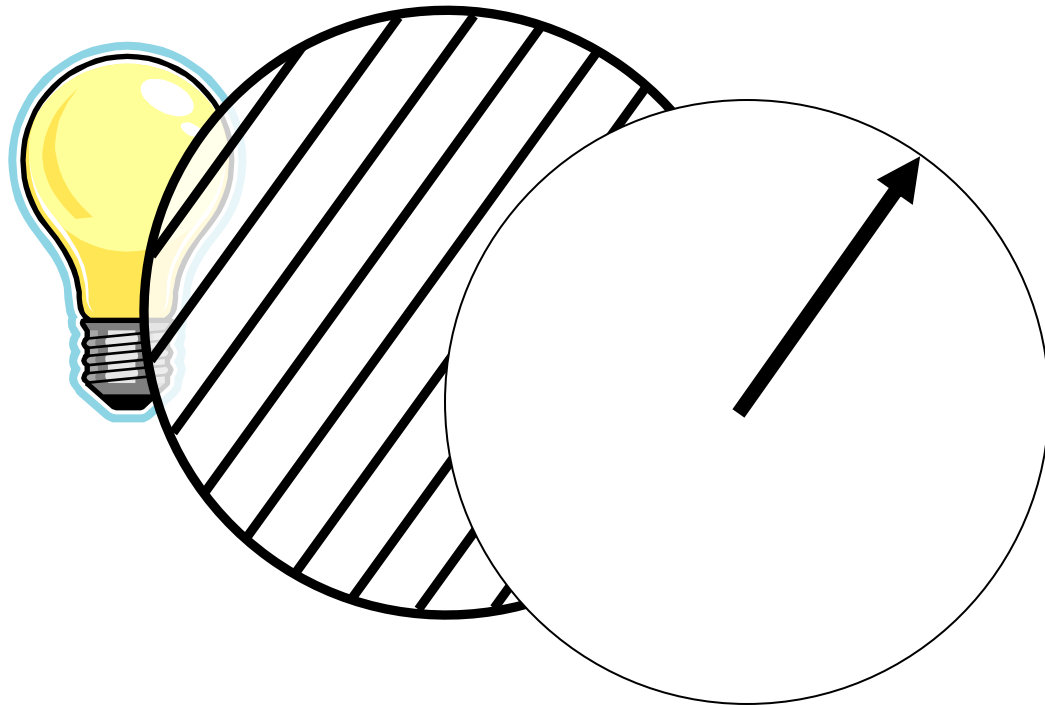
■ Crypto Threat of Quantum Computing

■ Quantum Cryptography

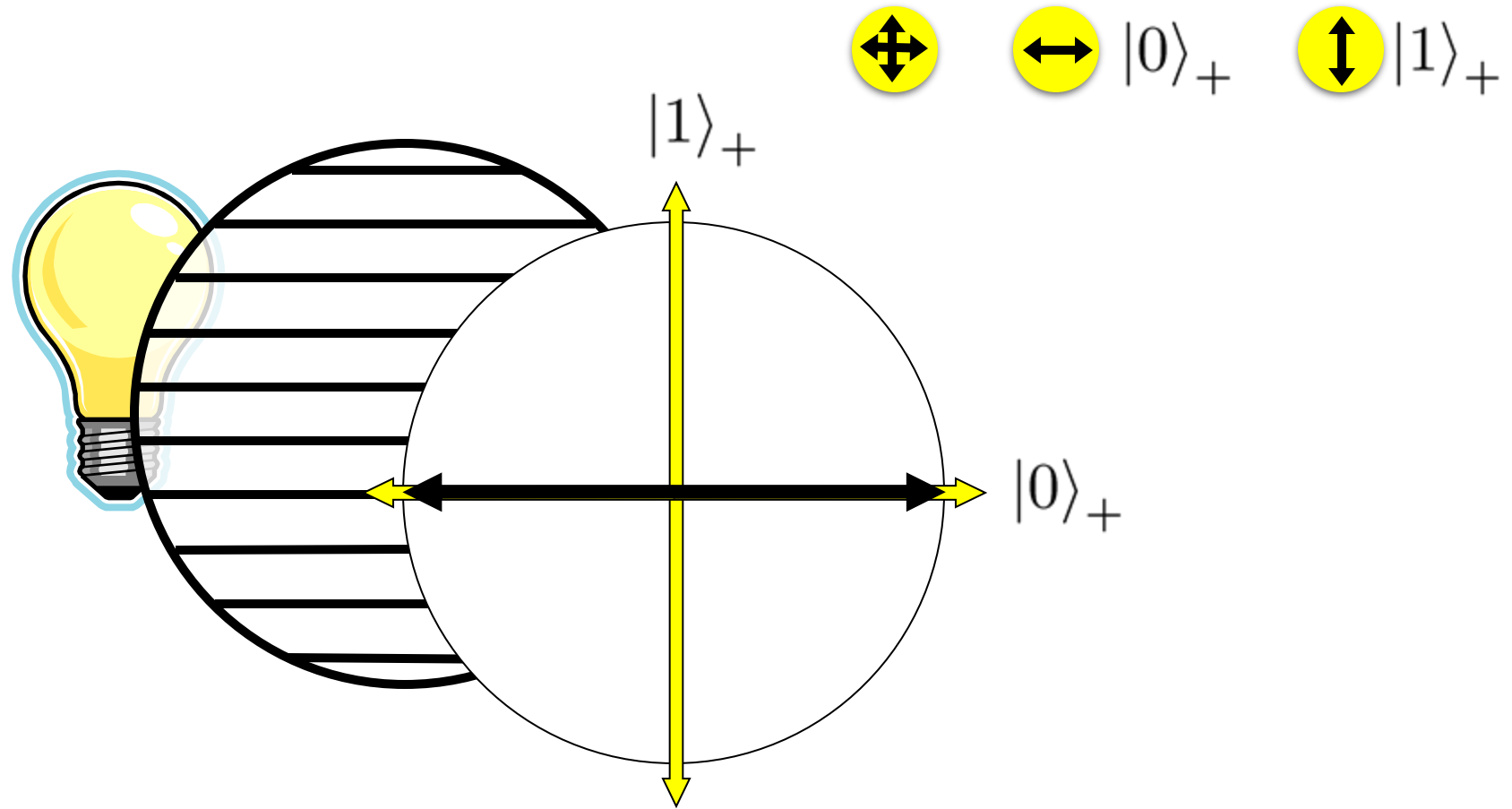
■ Quantum Future

Quantum Bit: Polarization of a Photon

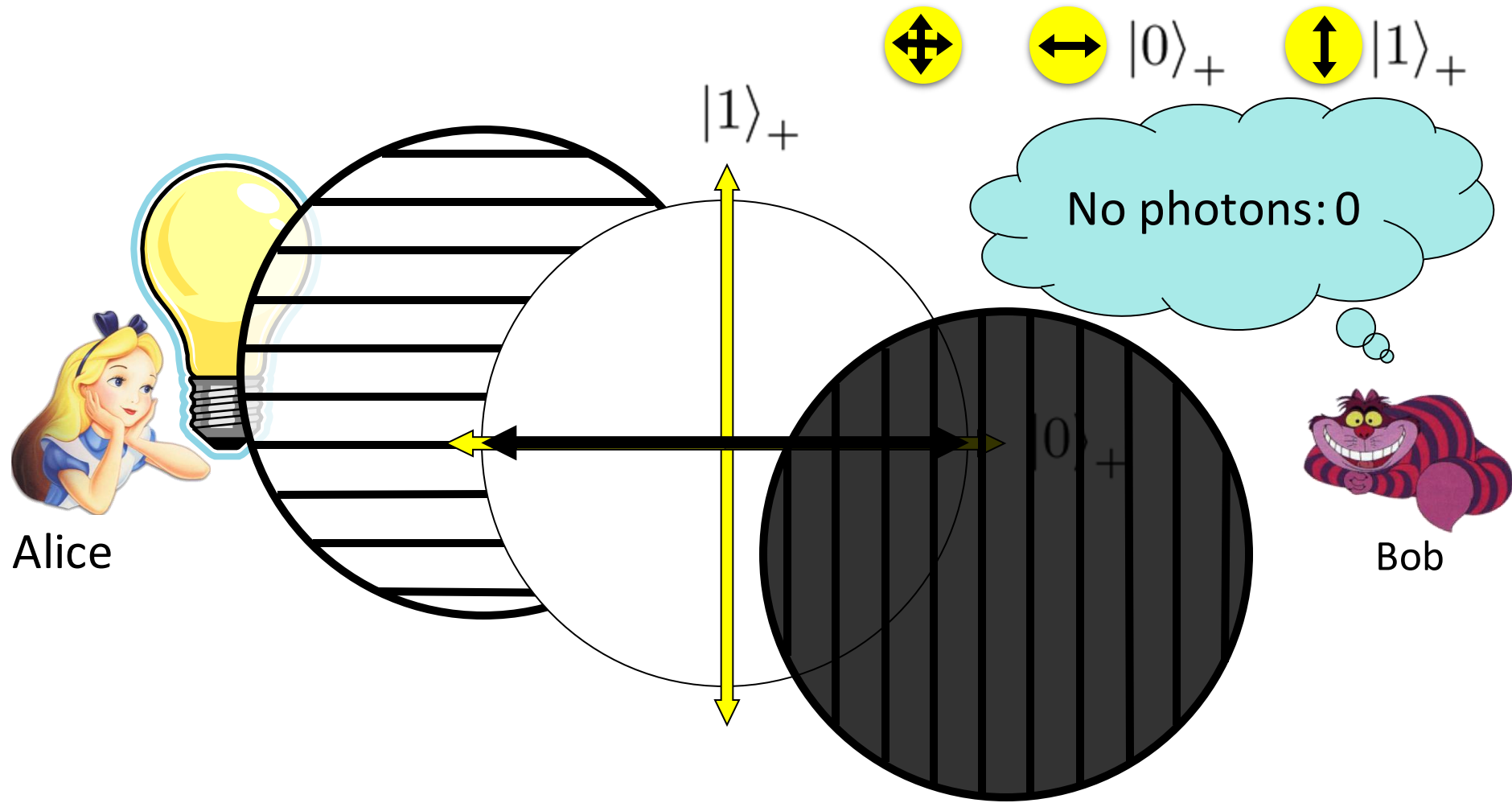
qubit as unit vector in \mathbb{C}^2



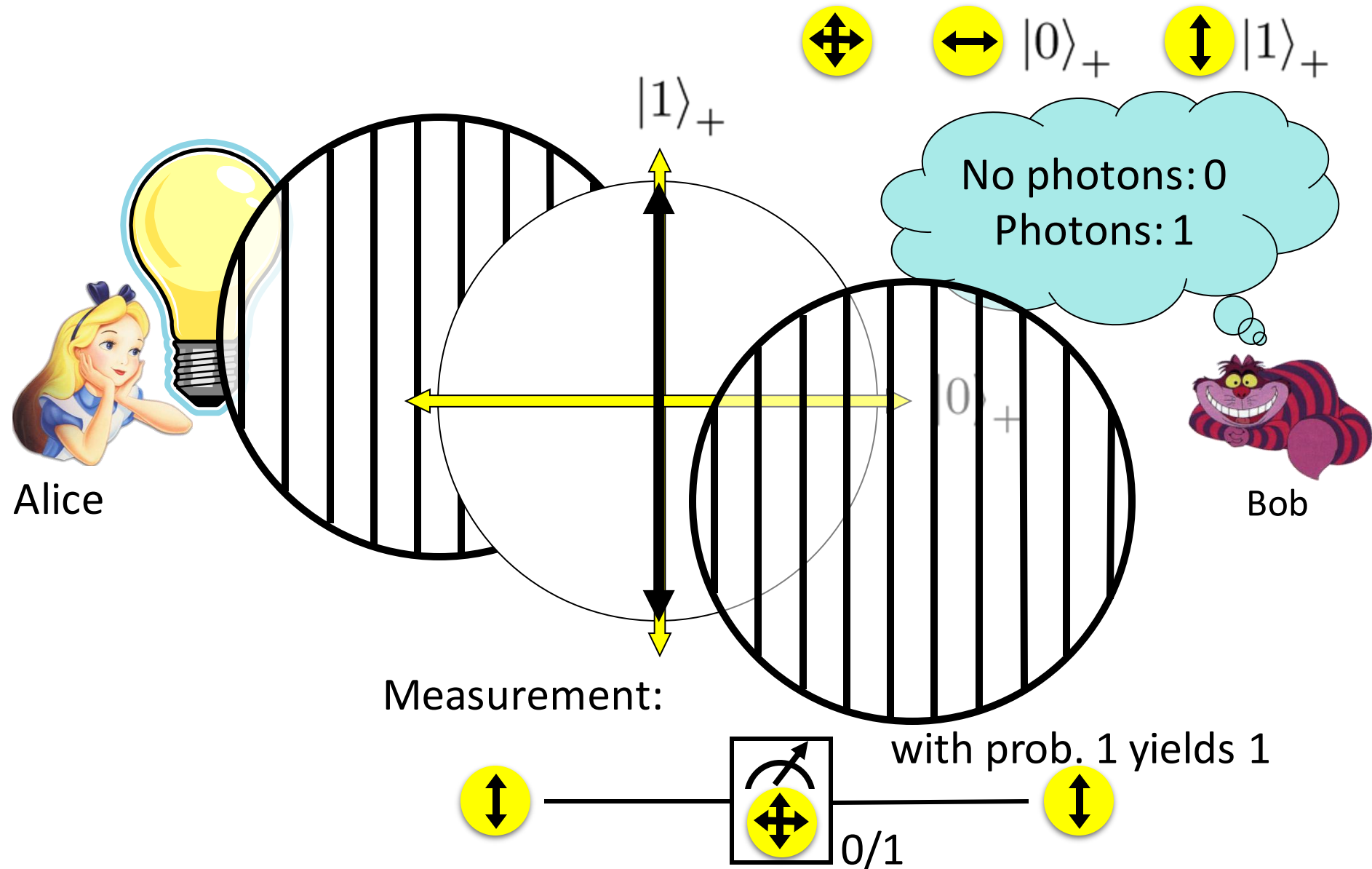
Qubit: Rectilinear/Computational Basis



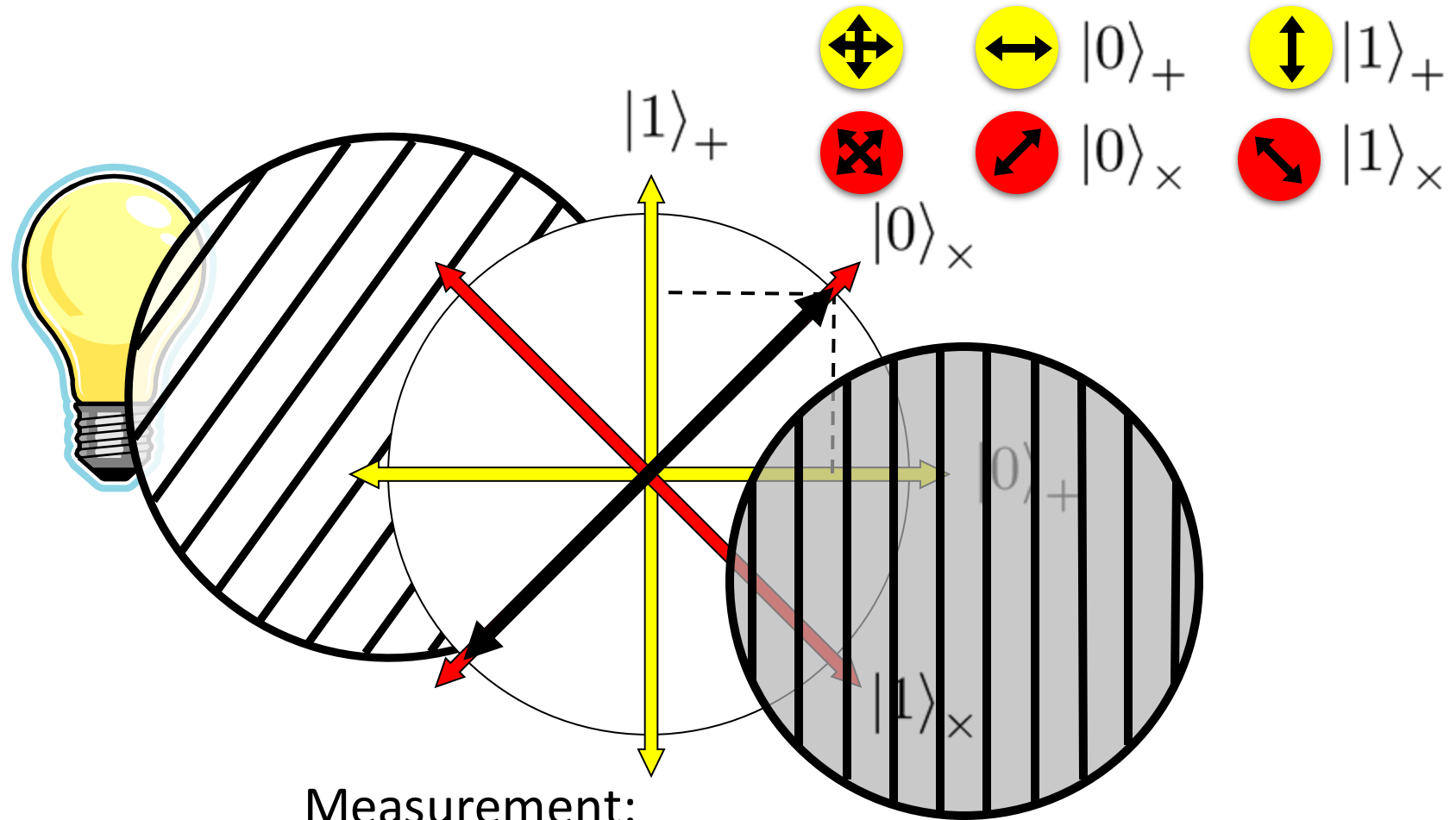
Detecting a Qubit



Measuring a Qubit



Diagonal/Hadamard Basis

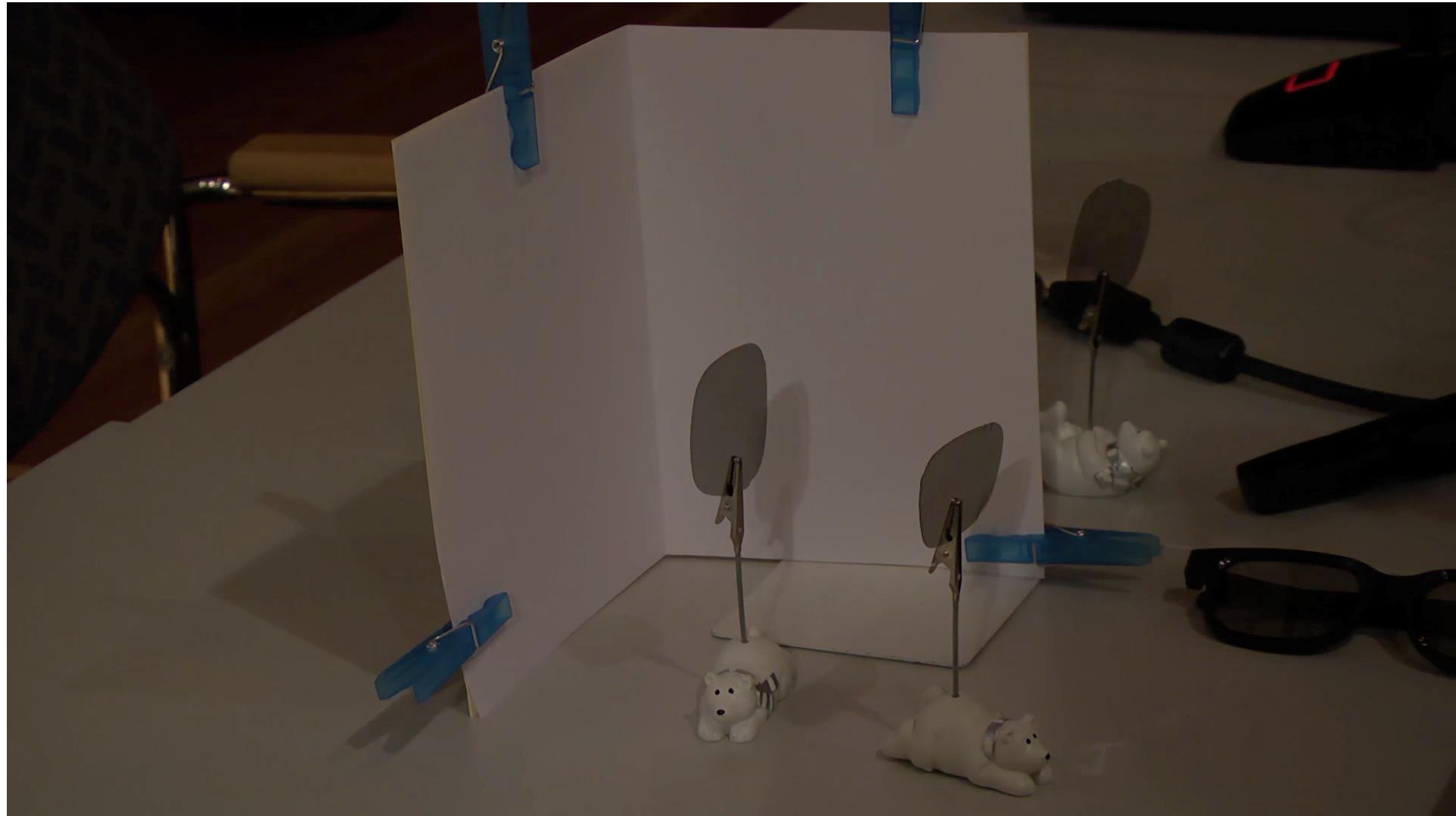


Measurement:

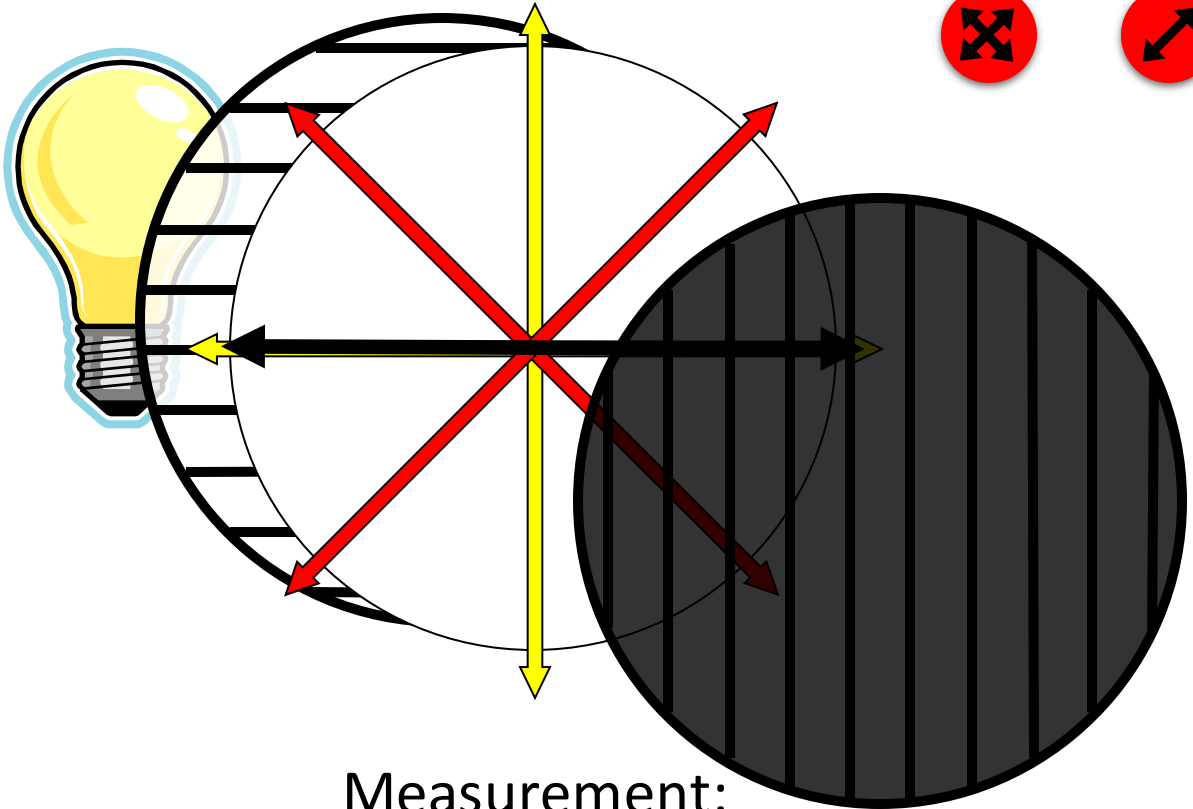
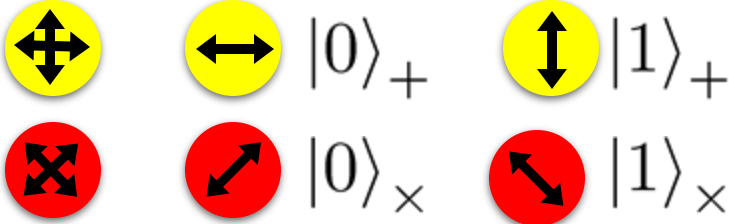
$$\frac{\begin{array}{c} \text{yellow circle with } \longleftrightarrow \\ + \\ \text{yellow circle with } \updownarrow \end{array}}{\sqrt{2}} = \begin{array}{c} \text{red circle with } \nearrow \\ \text{red circle with } \searrow \end{array} \text{---} \boxed{\begin{array}{c} \text{yellow circle with } \nearrow \\ \text{yellow circle with } \oplus \end{array}} \text{---} \begin{array}{c} \text{yellow circle with } \longleftrightarrow \\ \text{yellow circle with } \updownarrow \end{array}$$

with prob. ½ yields 0 yellow circle with \longleftrightarrow
 with prob. ½ yields 1 yellow circle with \updownarrow

Video



Measuring Collapses the State

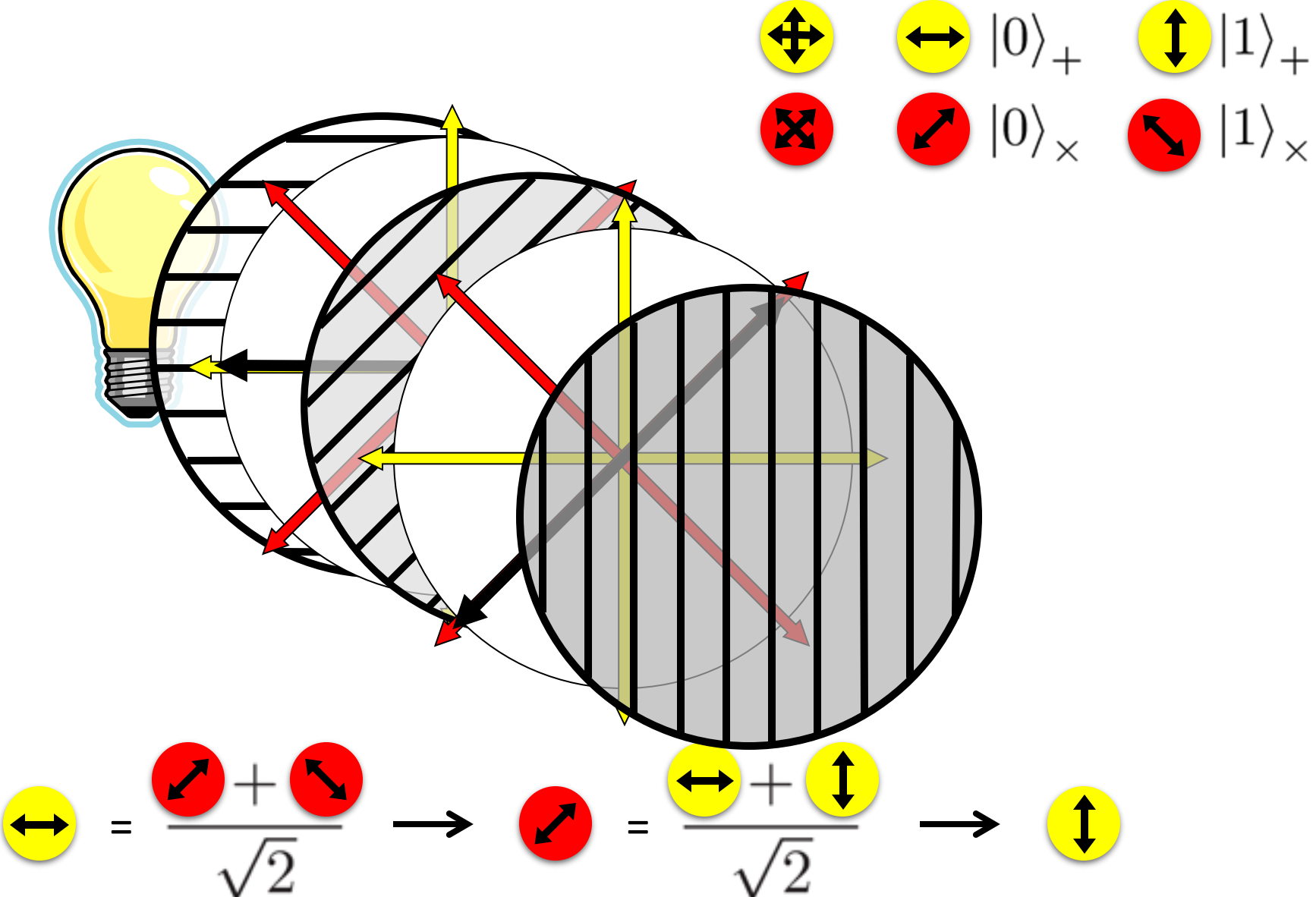


Measurement:

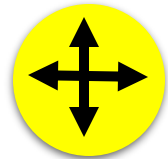
$$\frac{|0\rangle_+ + |1\rangle_+}{\sqrt{2}} = |0\rangle_x \text{ --- } \boxed{\text{Measurement}} \text{ --- } \begin{matrix} |0\rangle_+ \\ |1\rangle_+ \end{matrix}$$

with prob. 1/2 yields 0 $|0\rangle_+$
 with prob. 1/2 yields 1 $|1\rangle_+$

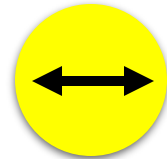
Measuring Collapses the State



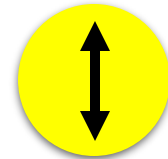
Quantum Mechanics



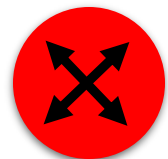
+ basis



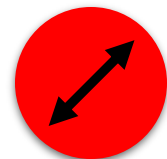
$|0\rangle_+$



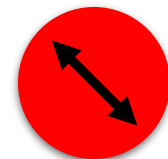
$|1\rangle_+$



x basis



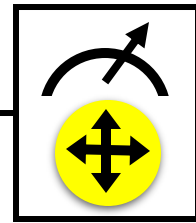
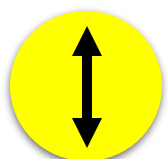
$|0\rangle_x$



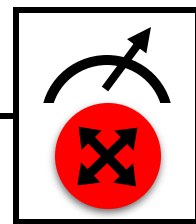
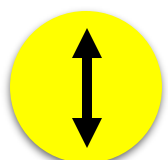
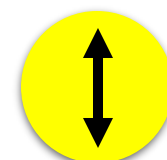
$|1\rangle_x$

Measurements:

with prob. 1 yields 1



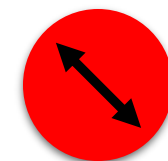
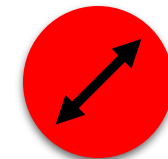
0/1



0/1

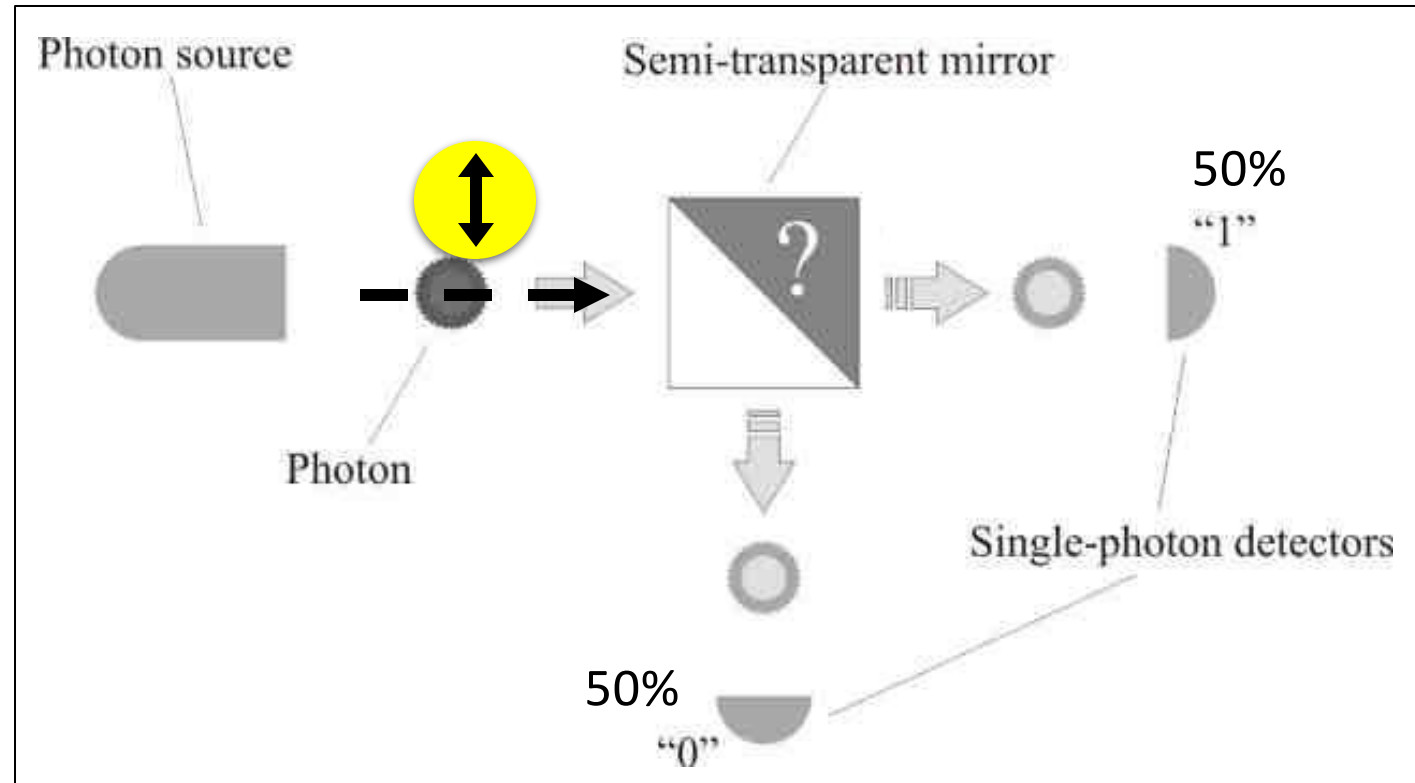
with prob. $\frac{1}{2}$ yields 0

with prob. $\frac{1}{2}$ yields 1



Demonstration of Quantum Technology

- generation of random numbers



(diagram from idQuantique white paper)

- no **quantum computation**, only **quantum communication** required

Quantum (Optics) Games

- Single-player puzzle game: [Laser Maze](#), beam bending logic game

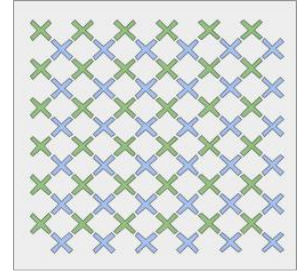
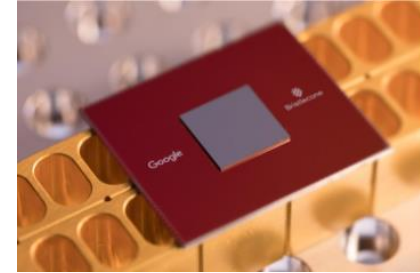
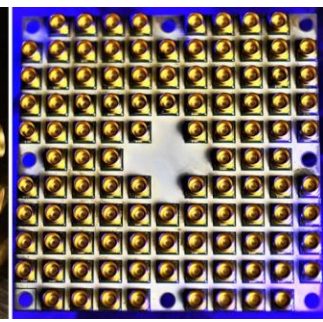
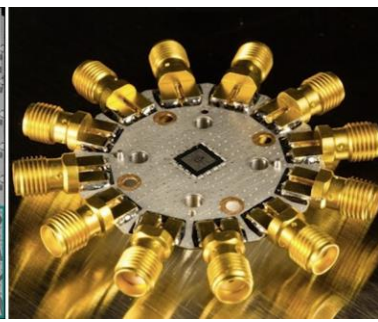
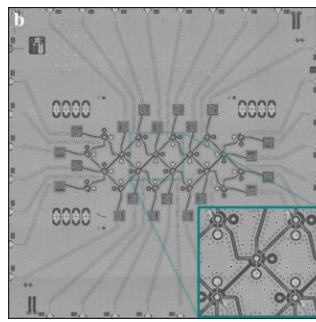
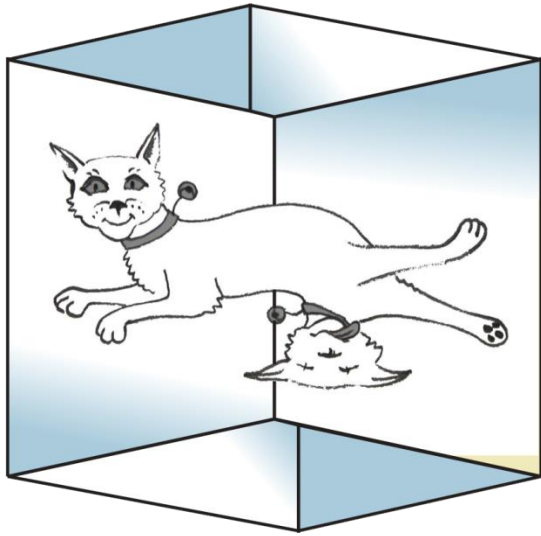


- Virtual game: <http://quantumgame.io/>



This Device complies with
21 CFR part 1040.10 and 1040.11

0



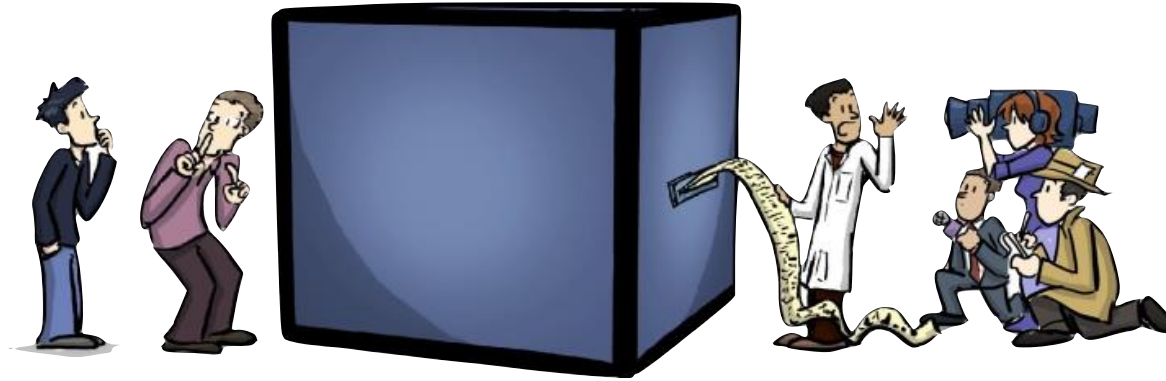
Wonderland of Quantum Mechanics



What will you Learn from this Talk?

- ✓ Classical Cryptography (& Politics)
- ✓ Introduction to Quantum Mechanics
- Crypto Threat of Quantum Computing
- Quantum Cryptography
- Quantum Future

Quantum Computing



A Quantum
COMPUTER

- Classical bit: 0 or 1
- Quantum bit: can be in **superposition of 0 and 1**
- Yields a (probably) more **powerful computational model**

$$\frac{\text{↔} + \text{↕}}{\sqrt{2}} = \text{↗} \quad \text{0}$$

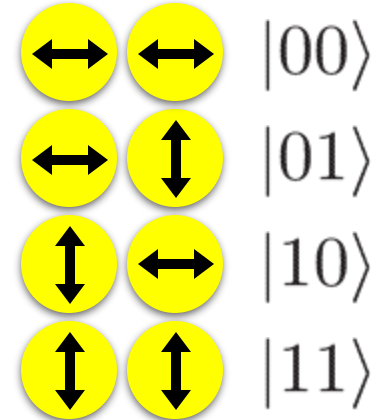
Many Qubits

- 1 qubit lives in a 2-dimensional space, can be in a superposition of 2 states
- 2 qubits live in a 4-dimensional space, can be in a superposition of 4 states

$$\frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2}$$

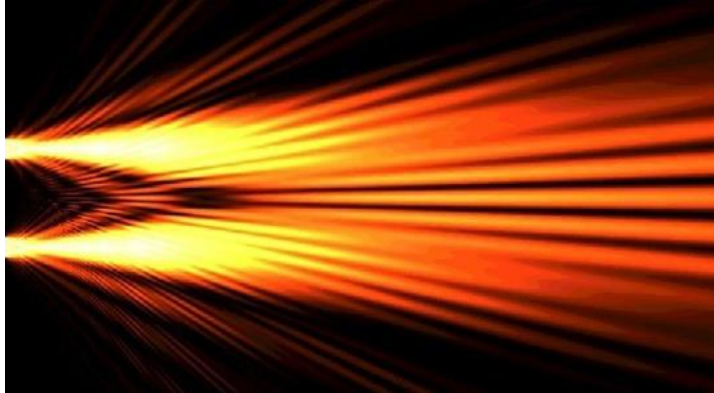
- 3 qubits can be in superposition of 8 states
- n qubits can be in superposition of 2^n states
- So, with 63 qubits, one can do $2^{63} = 9223372036854775808$ calculations simultaneously!
- Problem: **Measuring** this huge superposition **collapses** everything and yields only one random outcome

$$\frac{\left(\longleftrightarrow\right) + \left(\updownarrow\right)}{\sqrt{2}} = \left(\nearrow\right)$$

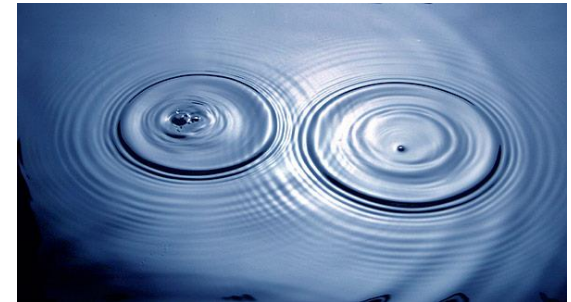


Quantum Computing

- With n qubits, one can do 2^n calculations simultaneously
- Problem: **Measuring** this huge superposition will **collapse** the state and only give one random outcome
- Solution: Use **quantum interference** to measure the computation you are interested in!



$$\frac{\left(\begin{array}{c} \leftarrow \rightarrow \end{array} \right) - \left(\begin{array}{c} \uparrow \downarrow \end{array} \right)}{\sqrt{2}} = \left(\begin{array}{c} \nearrow \searrow \end{array} \right)$$



- Seems to work for specific problems only
- Requires clever design of quantum algorithms and quantum software!

Quantum Algorithms: Factoring

- [Shor '94] Polynomial-time quantum algorithm for factoring integer numbers

- $15 = 3 * 5$

- $27 =$

- $31 =$

- $57 =$

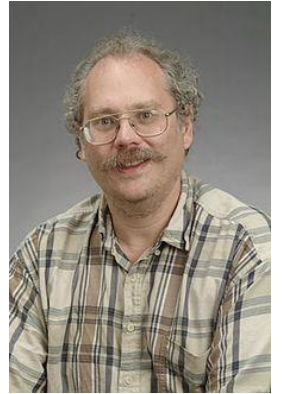
- $91 =$

- $173 =$

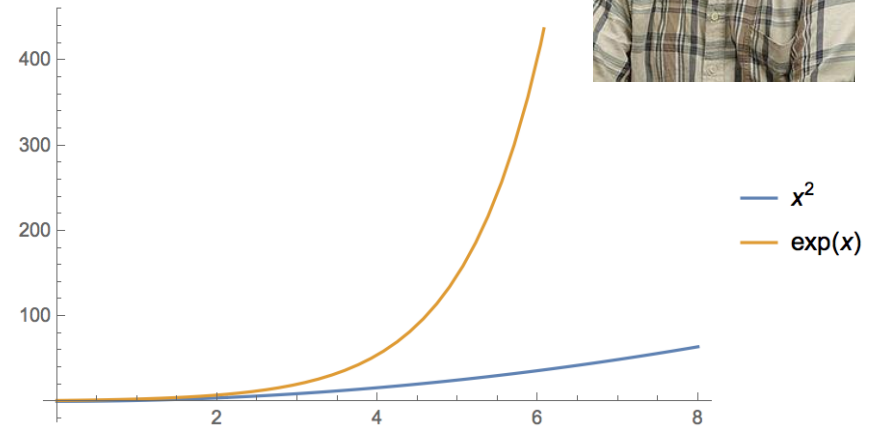
- [RSA-100](#) =
15226050279225333605356183781326374297180681149613806886579084945801
22963258952897654000350692006139 =



Quantum Computer breaks Public-Key Crypto



- [Shor '94] Polynomial-time quantum algorithm for factoring integer numbers
- Classical Computer : **Exponential time**
- Quantum Computer : **Poly-time: n^2**
- For a 600-digit number (RSA-2048)
 - **Classical: age of universe**
 - **Quantum: few minutes**



- Consequence: Large enough quantum computers **break all** currently used public-key cryptosystems!!!

Can We Build Quantum Computers?

- Possible to build in theory, no fundamental theoretical obstacles have been found yet.
- Enormous technical challenge (control vs decoherence)



- 5 March 2018:

Google moves toward quantum supremacy with 72-qubit computer

IBM and Intel recently debuted similarly sized chips

Conventional Quantum-Safe Cryptography

- **Wanted:** new assumptions to replace factoring and discrete logarithms in order to build conventional public-key cryptography



<https://csrc.nist.gov/Projects/Post-Quantum-Cryptography>

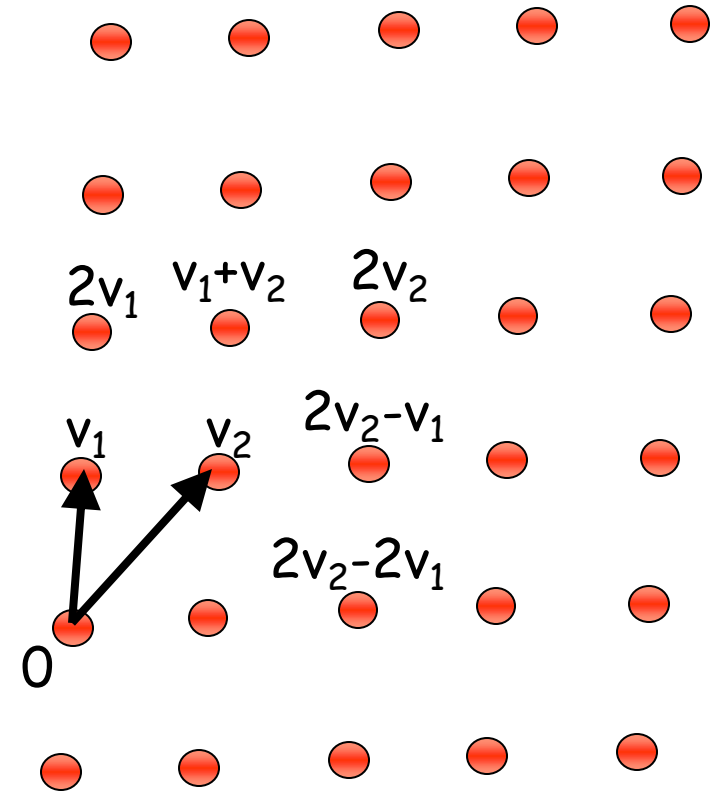
- NIST “competition”: 82 submissions (23 signature, 59 encryption schemes)
- Several submissions have already been broken and withdrawn
- April 2018: First-round workshop in Florida
- Expected: 3-5 years of crypto-analysis
- New standards, world-wide adoption

QuSoft

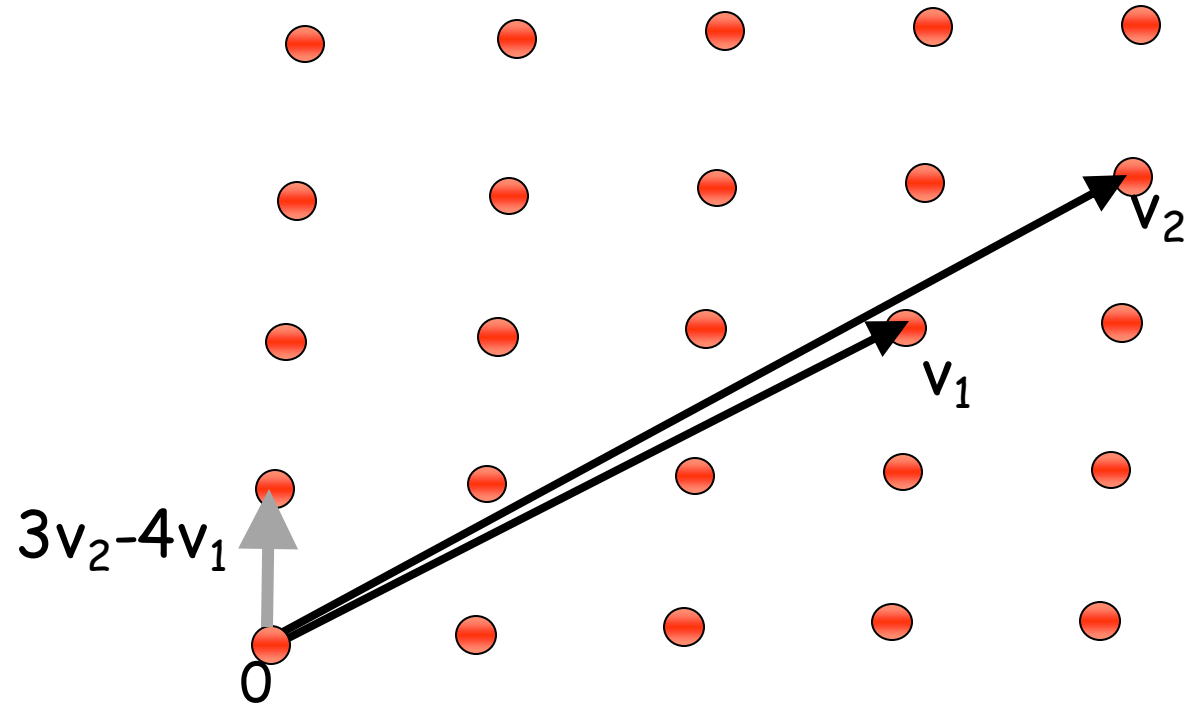
CWI

Example: Lattice-Based Cryptography

- For any vectors v_1, \dots, v_n in \mathbb{R}^n , the **lattice** spanned by v_1, \dots, v_n is the set of points $L = \{a_1v_1 + \dots + a_nv_n \mid a_i \text{ integers}\}$
- **Shortest Vector Problem (SVP)**: given a lattice L , find a shortest (nonzero) vector



Example: Lattice-Based Cryptography



- **Shortest Vector Problem (SVP):** given a lattice, find a shortest (nonzero) vector
- **no efficient (classical or quantum) algorithms known**
- public-key encryption schemes can be built on the computational hardness of SVP

What will you Learn from this Talk?

- ✓ Classical Cryptography (& Politics)
- ✓ Introduction to Quantum Mechanics
- ✓ Crypto Threat of Quantum Computing

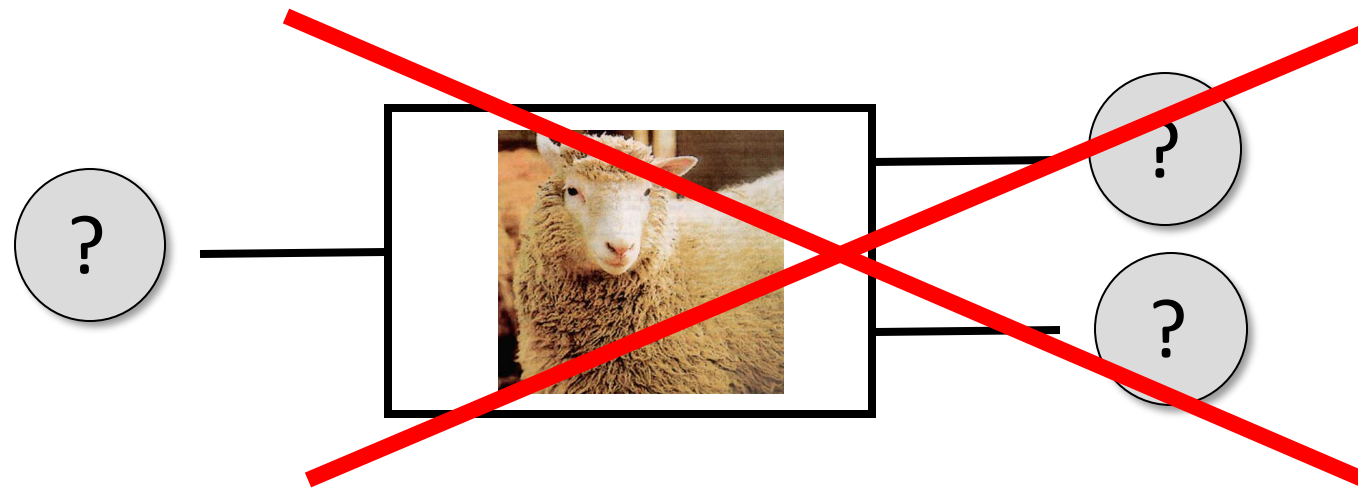
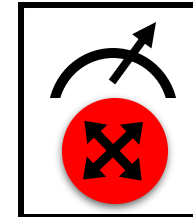
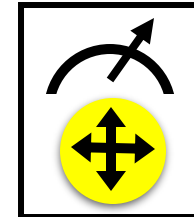
- Quantum Cryptography

- Quantum Future

No-Cloning Theorem

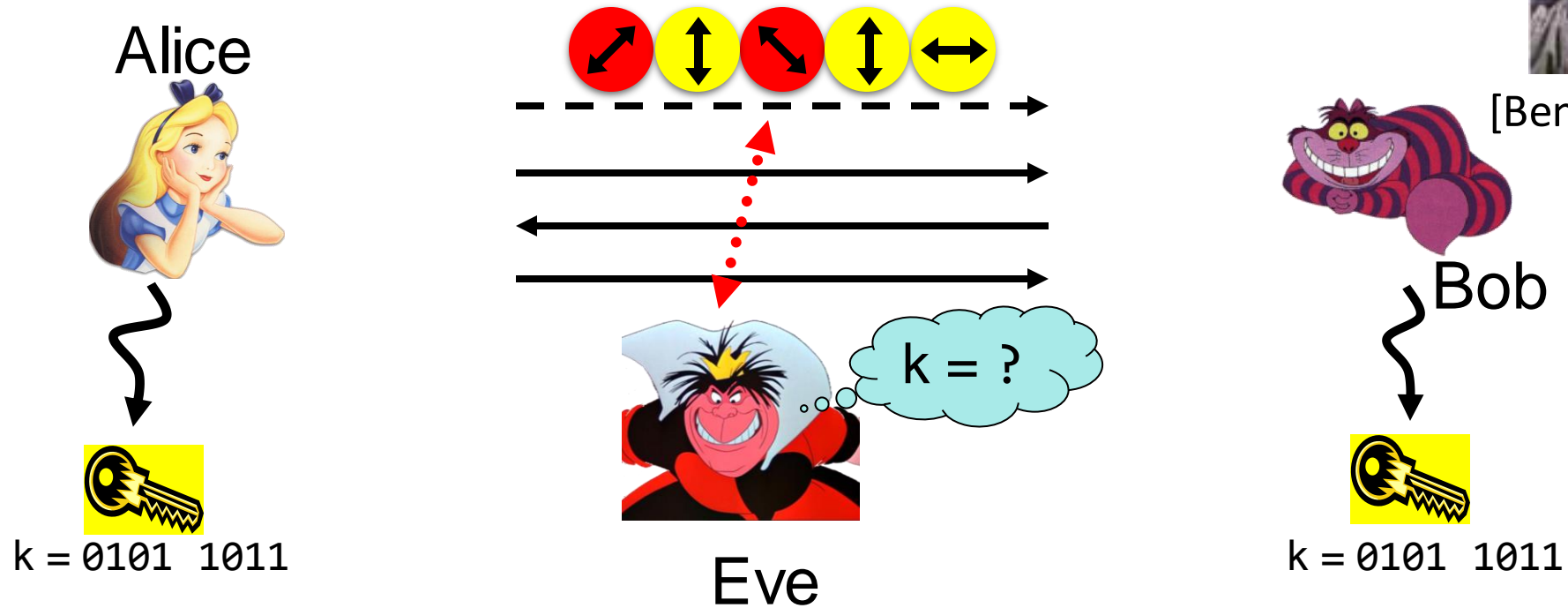


Quantum operations: U



Proof: copying is a **non-linear operation**

Quantum Key Distribution (QKD)

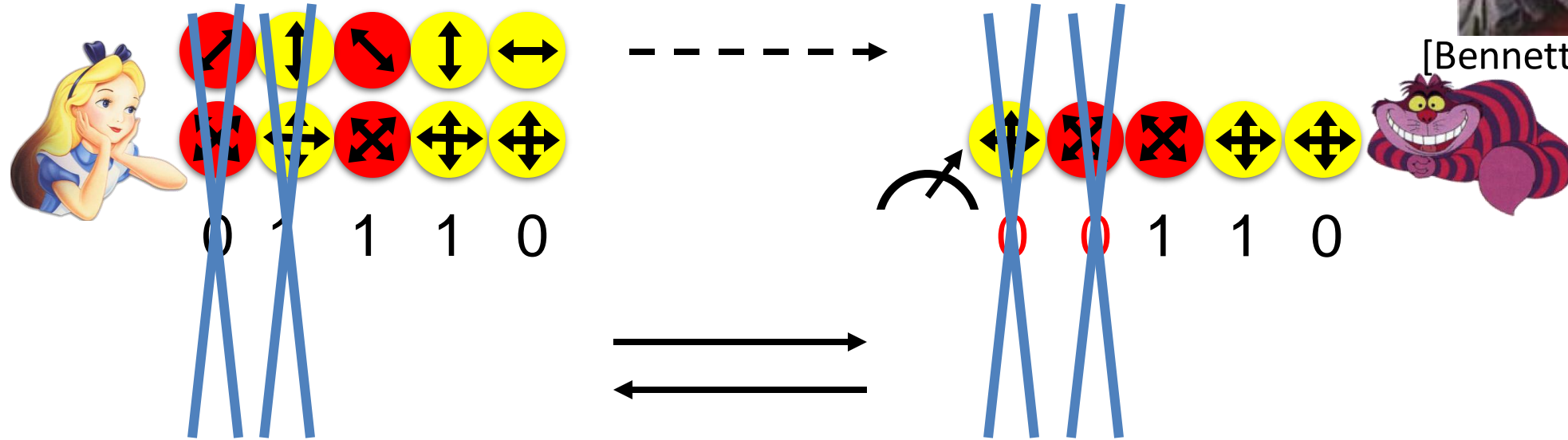



- Offers a **quantum solution** to the key-exchange problem
- Puts the players into the starting position to use symmetric-key cryptography (encryption, authentication etc.).


Quantum Key Distribution (QKD)



[Bennett Brassard 84]



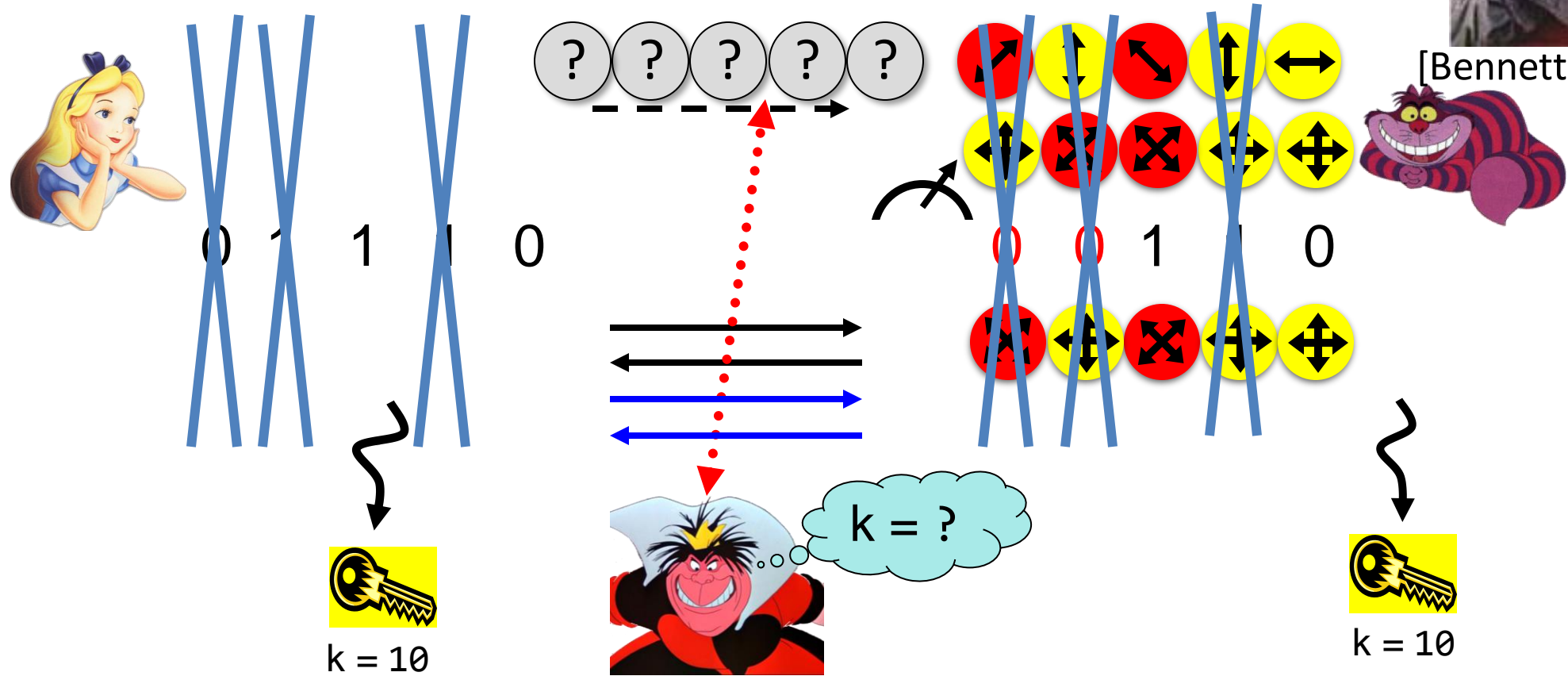

k = 110


k = 110

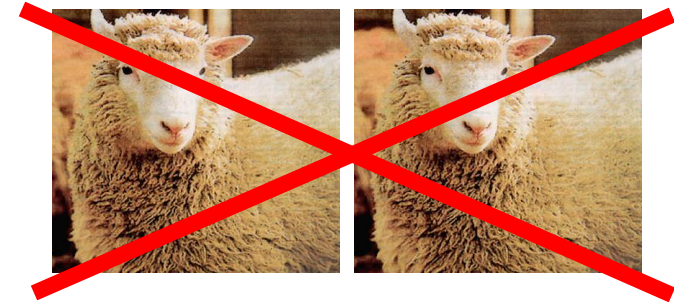
Quantum Key Distribution (QKD)



[Bennett Brassard 84]



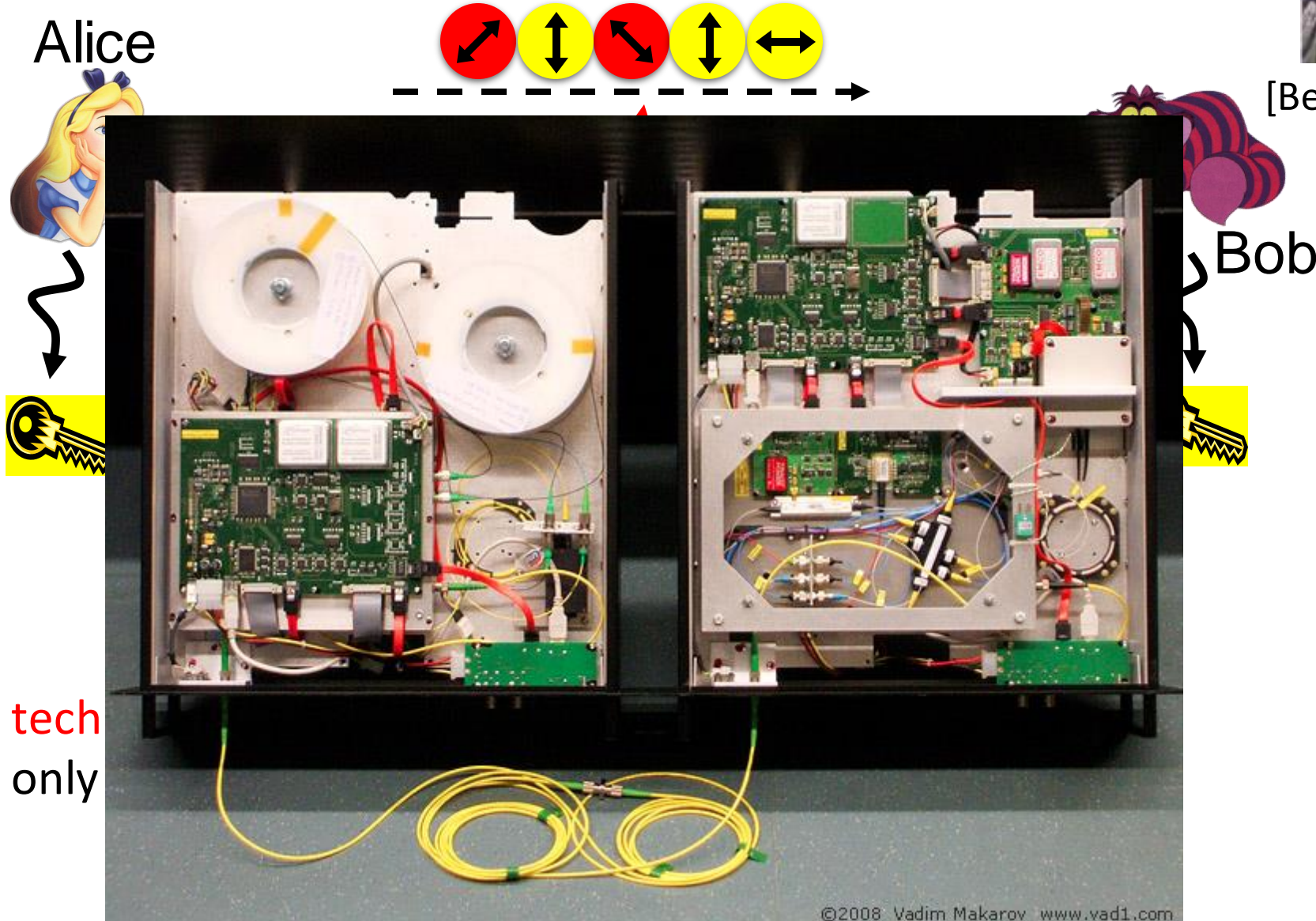
- Quantum states are unknown to Eve, she **cannot copy them**.
- Honest players can **test** whether Eve interfered.



Quantum Key Distribution (QKD)



[Bennett Brassard 84]

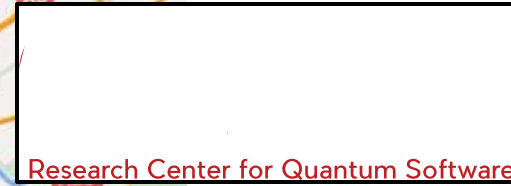
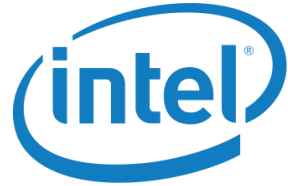


What will you Learn from this Talk?

- ✓ Classical Cryptography (& Politics)
- ✓ Introduction to Quantum Mechanics
- ✓ Crypto Threat of Quantum Computing
- ✓ Quantum Cryptography

■ Quantum Future

Quantum Research in NL



QuTech: 135 Mio € , 50 Mio \$ Intel

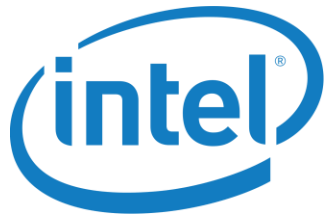
Nov 2017: [Quantum Software Consortium](#), NWO 18.8 Mio € for 10 years

Quantum Research in EU



Starting 2018: 1 (or 2?) Bio. € flagship program on Q technologies

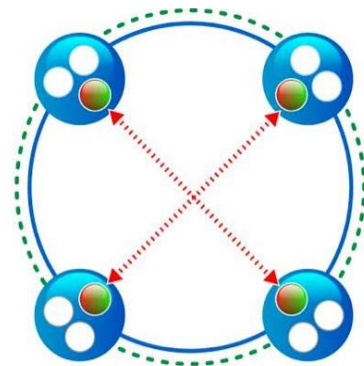
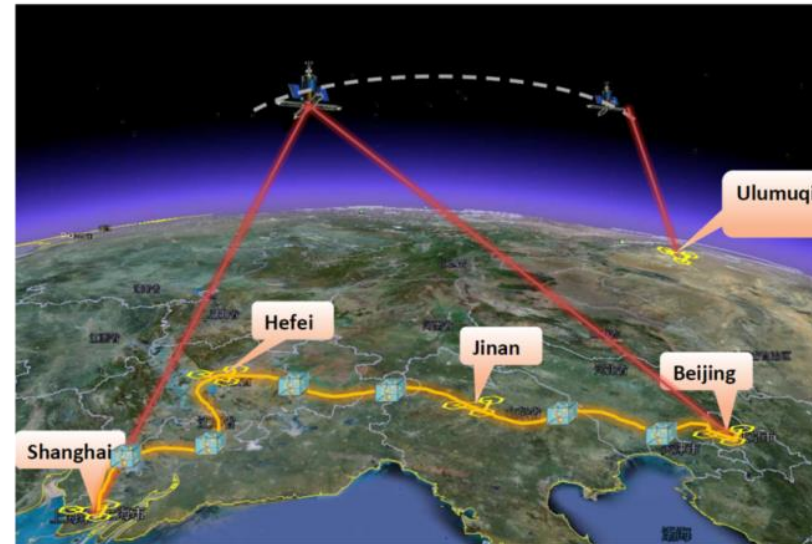
Quantum Research Worldwide



Waterloo, Singapore, Santa Barbara, China, ...

Quantum Networks

- 2000km QKD backbone network between Beijing and Shanghai
- first QKD satellite launched in 2016 from China
- Quantum entanglement allows to generate secure keys (like QKD)



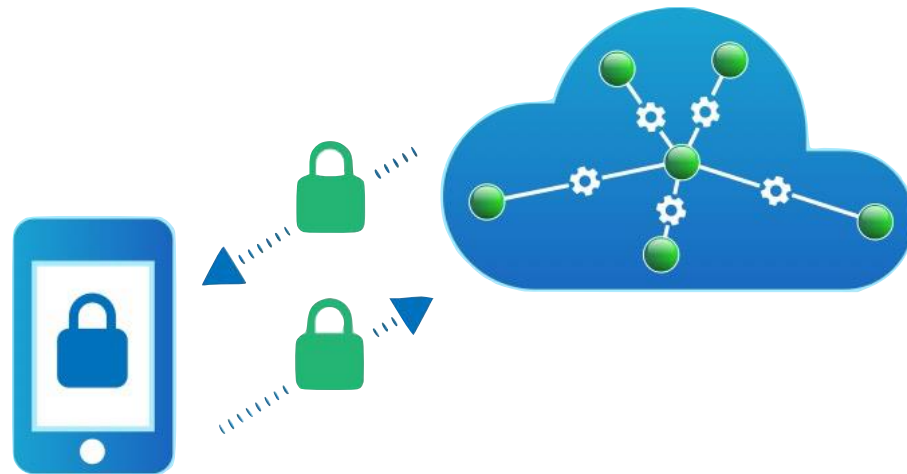
EXAMPLE QUANTUM NETWORK

- Network node
- Unused Qubit memory
- Used Qubit memory
- Physical quantum communication link
- - - Physical classical communication link
- ⋯ Virtual link via entanglement



Secure Computing in Quantum Cloud

- Distributed quantum computing
- Recent result: quantum homomorphic encryption allows for secure delegated quantum computation



Y. Dulek, C. Schaffner, and F. Speelman, arXiv:1603.09717
Quantum homomorphic encryption for polynomial-sized circuits, in CRYPTO 2016, QIP 2017

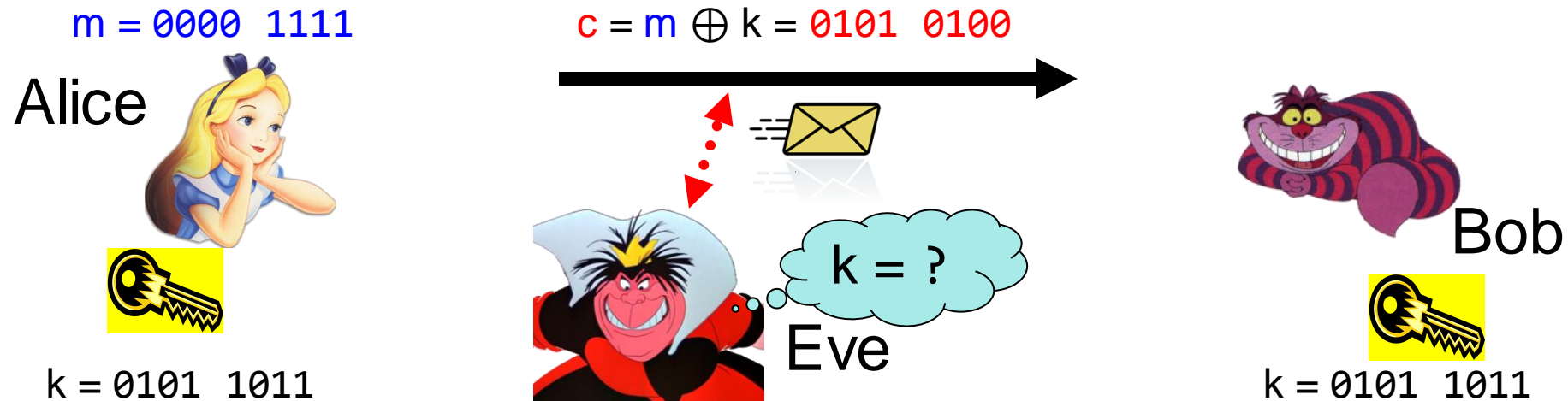
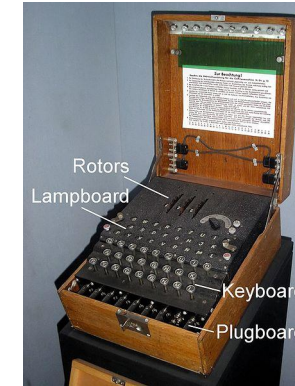


<https://quantumexperience.ng.bluemix.net>

What Have You Learned from this Talk?

✓ Classical Cryptography

- Long [history](#)
- [One-time pad](#)



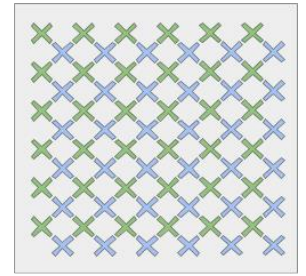
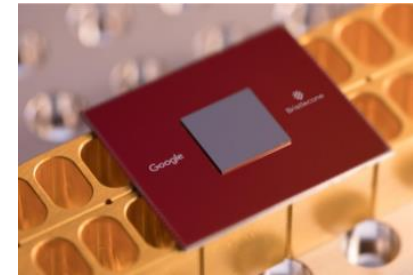
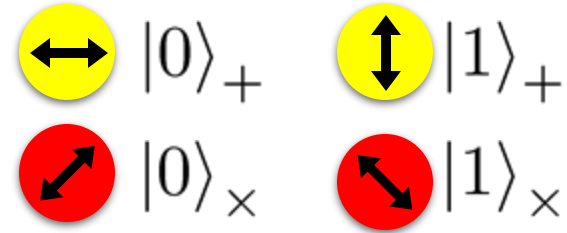
- [Public-key cryptography](#)
- Links to [Logic](#) and [Politics](#)



What Have You Learned from this Talk?

✓ Quantum Mechanics

- [Qubits](#)
- [Quantum Computer](#)
- [Shor's Algorithm](#)
- [No-cloning](#)



Answers to the Quantum Threat

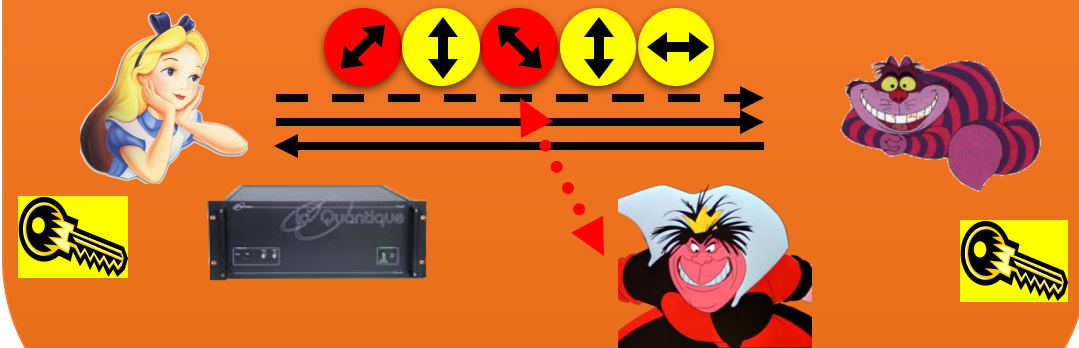
Conventional Post-Quantum Cryptography

- Can be deployed without quantum technologies
- Believed to be secure against quantum attacks of the future



Quantum Cryptography

- Requires some quantum technology (but no large-scale quantum computer)
- Typically no computational assumptions



Thank you for your attention!

Questions



<http://www.qusoft.org/education/>