Sterile Neutrino Dark Matter

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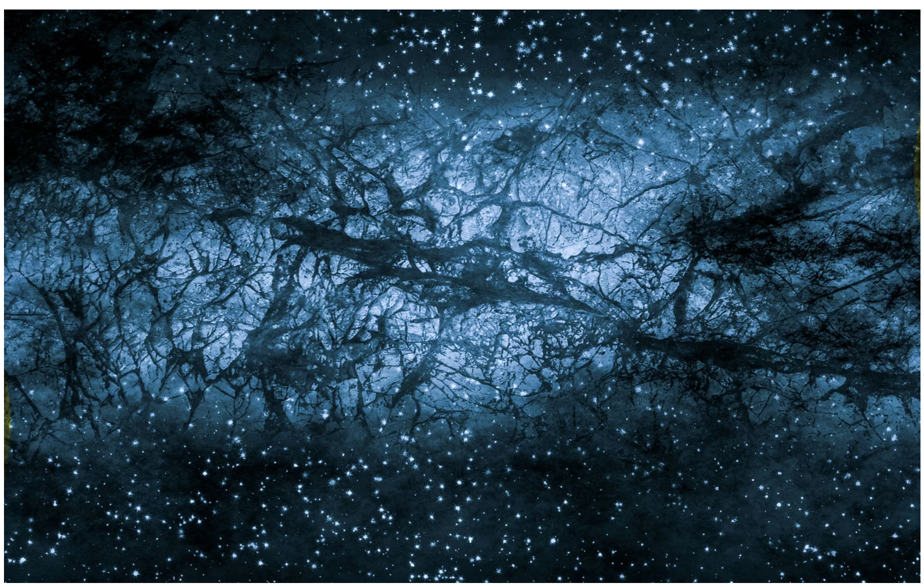

Contents

1 Introduction

Sterile Neutrinos have become a serious option for dark matter. In this thesis, the main goal is to get an understanding of what a sterile neutrino is and what its properties are. For this, a large part will be information that is necessary to understand sterile neutrinos, coming from an undergraduate physics background. Furthermore, this thesis will be explaining why these hypothetical particles could be dark matter and how they would fit within established theories. Due to the inherent properties of the sterile neutrino, it does not fit in the Standard Model of Elementary particles. This means that this thesis will also go into theories beyond the Standard Model. A short but important part about experiments with sterile neutrinos will be added as well. In conclusion, after reading this thesis, it will be clear to the reader why the sterile neutrino is such an interesting dark matter candidate. This thesis will be a collection of known information and the idea is to give a good and clear overview of all the significant theory extensions necessary to introduce sterile neutrino dark matter, to eventually get a good idea of the interesting options for follow-up studies on these particles.
Those already familiar with the basic physics of phenomenology and quantum mechanics, can go directly to the sterile neutrino section. Chapters 2 and 3 will help with the understanding of the more advanced parts further on. The equations in this thesis use the natural units ($\hbar$ = c = 1).

2 Dark Matter

Dark matter is a hypothetical form of matter that consists of yet undiscovered very weakly interacting particles. Its existence is necessary for many observed physical phenomena. Looking, for example, at rotating galaxies, the visible mass[1] is not enough to account for the observed rotation speed curve of individual stars and gas (figure 1)[10]. Observations of the motion of galaxy clusters and gravitational lensing[2] measurements also show the necessity of dark matter. Furthermore, observations of the Cosmic Microwave Background

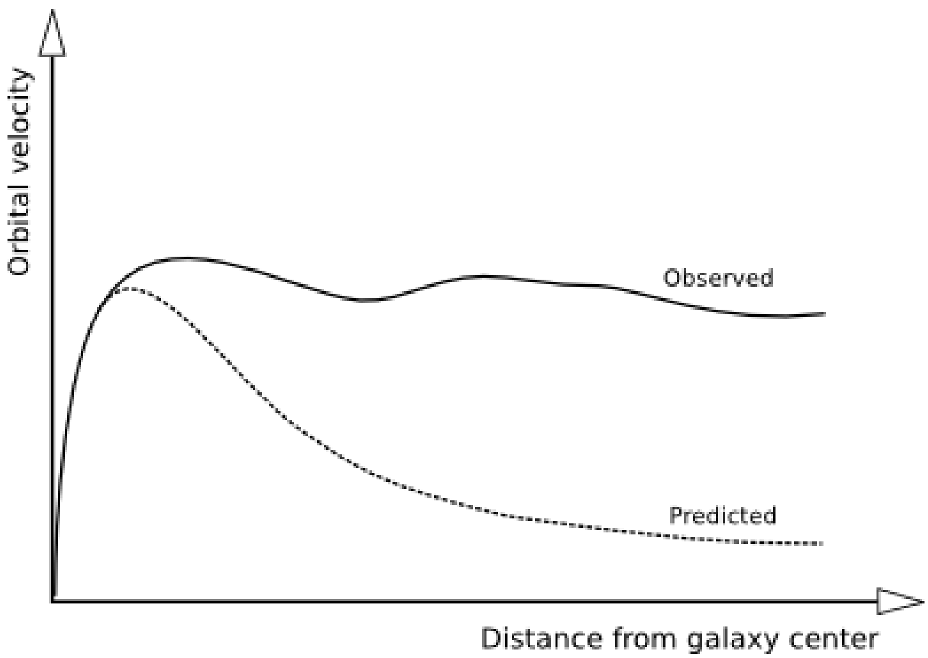

Figure 1: *The rotation speed of the stars and gas of a galaxy is expected to decrease when the distance from the centre of that galaxy is increased. This rotation speed curve was predicted with the amount of visible mass that was measured in other experiments. The observed curve shows that the rotation speed does not decrease with growing distance from the centre.* Taken from 1) in [43]

(CMB) combined with calculations to achieve the observed rotation curves, show that at least 85% of the matter has to be some kind of dark mass. Other methods to understand dark matter, like modifying gravity, adding old and nearly invisible stars or other heavy but unseen galactic objects, have not yet provided an explanation of the observations. This initiated the 'hunt' for dark matter particles and gave birth to many different theories (figure 2).

[1] mass that emits a certain frequency of light within the light spectrum

[2] The measurement of bending light around a massive galactic object like a galaxy

Another line of evidence is the structure formation of the universe. The currently observed structure formation of galaxies and clusters would not have been possible in the given amount of time since the big bang with the ordinary matter as the only ingredient. This is because this visible matter is affected by radiation. This radiation slows down the condensation process of matter, making it unable to form the observed arrangement of matter of the universe within the time from the Big Bang until now. Dark matter is not affected by this radiation and can provide a solution to this problem. Simulations of dark matter combined with normal matter reproduce the universe that is observed today, making physicists believe that dark matter has to be out there.

The Standard Model neutrino could be a dark matter particle since it is almost invisible and flies around in great numbers. Unfortunately, after the observation of the background of relic neutrinos[3], it was found that these neutrinos are way too light to satisfy the observed dark matter density Ω_{DM}.

So why not theorize a neutrino that fits the bill? This is where the sterile neutrino comes into play. To understand what kind of dark matter the sterile neutrino is, two defining parameters, *free streaming length* and *phase space density*, are introduced. Both will return in the sterile neutrino dark matter section.

The free streaming length and phase space density are parameters to understand the structure of the dark matter. The free streaming length could be seen as a parameter that explains when particles will interact with each other. The value tells us how dark matter may be formed and how it moves through space. The phase space density is a parameter explaining how many particles can exist in a certain portion of space. It helps to understand how dark matter is formed.

The interaction between particles is all mathematically written down in terms of probabilities. The total interaction rate is estimated with the following parameters, the *cross-section* (σ) defining the effective target area inside which particles have to meet to interact with each other and the flux of incoming particles. The probability of particles scattering can be influenced by increasing or decreasing the flux. For example, the flux of particles grows, when the averaged velocity of the involved particles increases. Another way to influence the flux is by changing a the spatial particle density. A higher density causes a larger flux. The density itself can be influenced by thermodynamic parameters like temperature and pressure. The velocity is also important because relativistic particles follow different energy equations than non-relativistic ones.

[3]Neutrinos that were left behind with too little density and energy to change into other particles by means of collision processes, because of the expansion (or cooling down) of the universe.

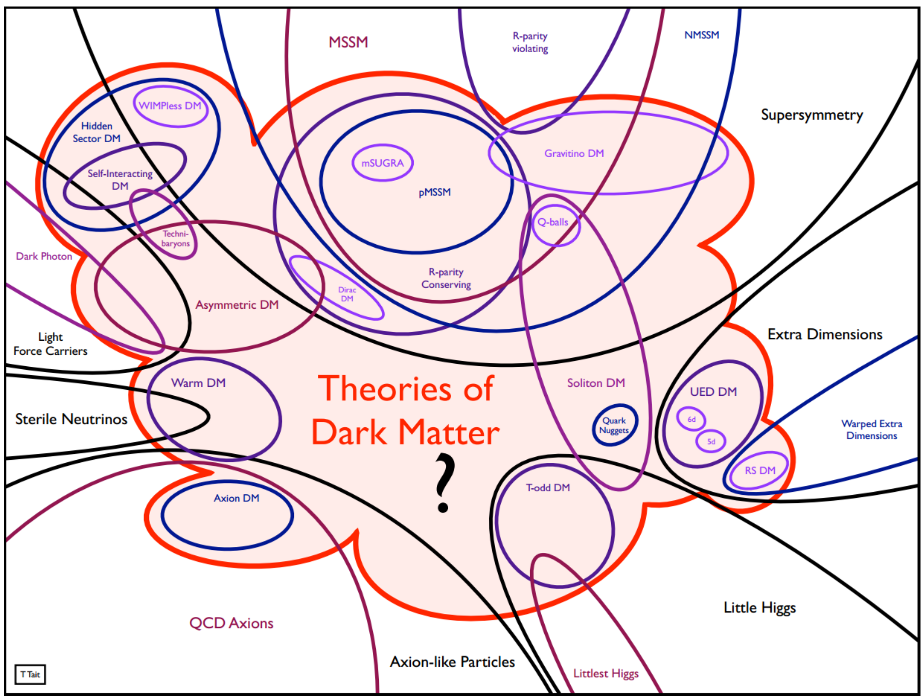

Figure 2: *A collection of all the different proposed and theorized dark matter theories. Sterile neutrinos are on the centre left within the warm dark matter portion (warm dark matter will also be elaborated on in the sterile neutrino physics section).* Taken from 2) in [43]

3 Neutrino Physics

3.1 Standard Model and Fundamental Forces

The standard model is a collection of building blocks of the universe. It has had a lot of success in predicting experiments, which is why it is so respected as a theory [3]. The model consists of all the known elementary particles (figure 3), which are categorized in fermions and bosons. In the standard model,

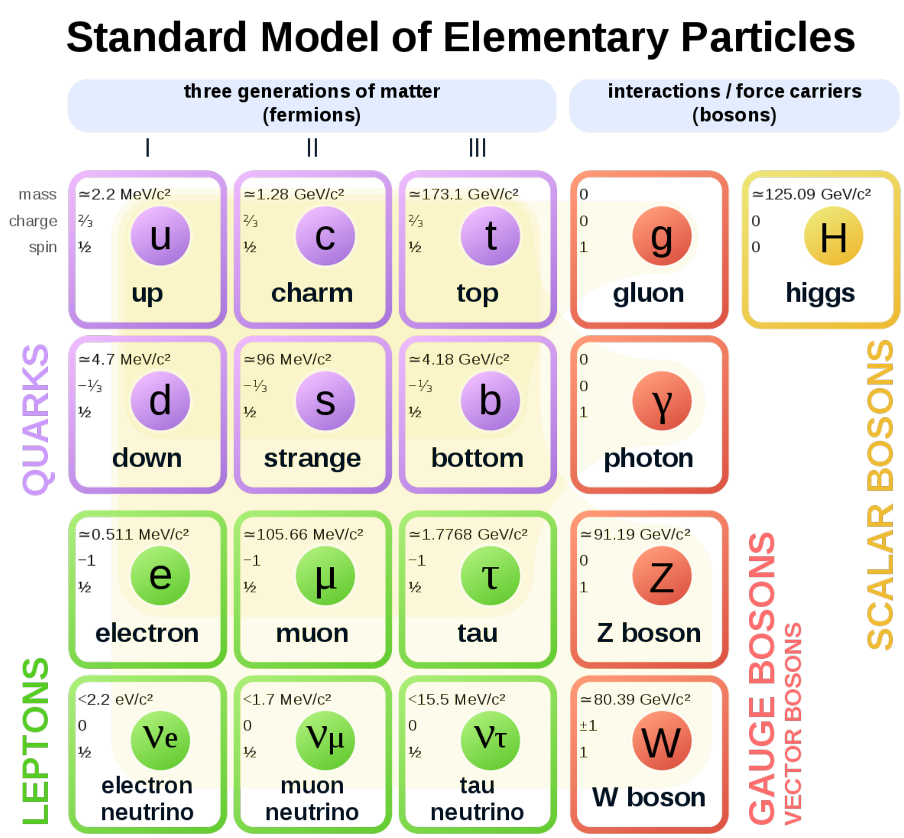

Figure 3: *This figure shows the particle content of the standard model. The quarks and leptons both consist of three generations and they make up the fermions (spin= $\frac{1}{2}$). The gauge bosons (spin=1) and Higgs boson (spin=0) are the bosons. Not included in this table are the oppositely charged anti-fermions of the SM.* Taken from 3) in [43]

there are four fundamental forces (figure 4) and three of those forces have their own quantum field theory (more on this in section 3.4). Each quantum theory has its own mediators called the (gauge) bosons. For the strong and electromagnetic force these mediators are respectively the gluons and the photon and for the weak force, these are the $W^{\pm}$ and Z^0 boson. Neutrinos only possess

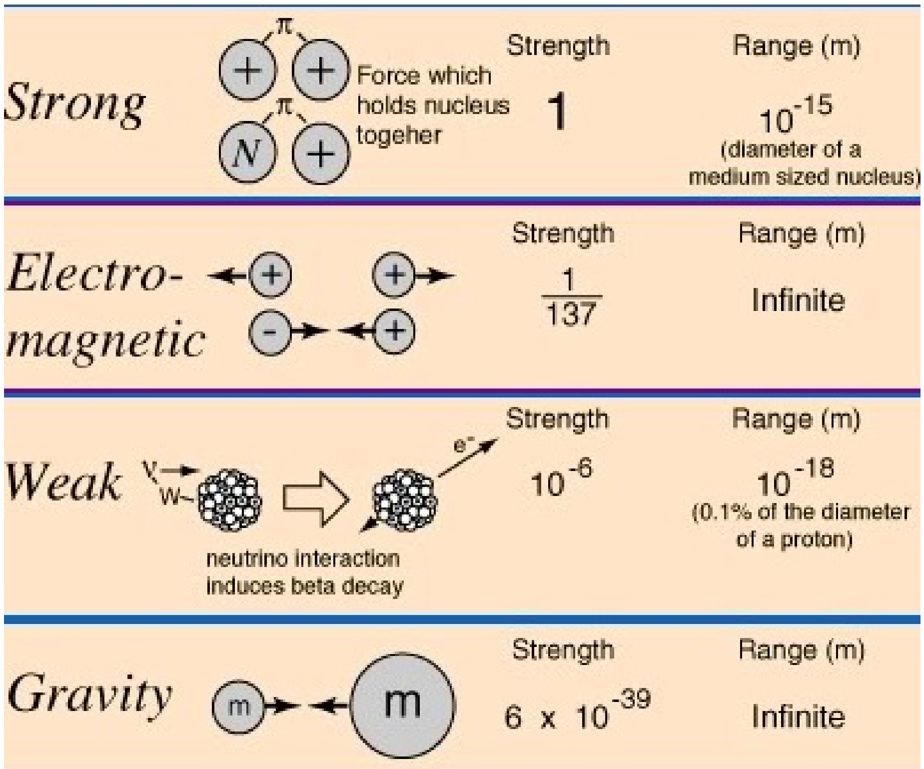

Figure 4: *The 4 fundamental forces in comparison to each other.* Edited from 4) in [43]

weak charge (except sterile neutrinos, they do not even have a weak charge), so they have no colour or electromagnetic charge. This means that they only 'feel' the weak force [15], and not surprisingly, the other forces leave the neutrino untouched. Thus, they are invisible to photons and gluons, both very visible forces because of their relatively strong interaction strength. Only being involved in these weak interactions is what makes them so hard to detect [42]. Gravity has actually no 'charge' analogy, but the gravity effects caused by *individual* neutrinos are simply too weak to observe, so they are rendered negligible. As a side note, gravity effects do become increasingly important when a great number of neutrinos are clustered together (as with dark matter). At this large scale, neutrinos would definitely 'feel' and exert gravity. Nevertheless, for individual interactions at small (standard model) scale, neutrinos only interact with the W and Z^0 boson.
The beta decay is one of the most important decays to observe neutrinos. These decays are the effect of the weak force working on nuclei. The three different

beta decays are:

β^--decay

$$_{Z}X \rightarrow {}_{Z+1}Y + e^- + \overline{v_e} \tag{1}$$

β^+-decay

$$_{Z}X \rightarrow {}_{Z-1}Y + e^+ + v_e \tag{2}$$

Electron capture (later referred to as K-capture)

$$_{Z}A + e^- \rightarrow {}_{Z-1}B + v_e \tag{3}$$

Here the subscript Z stands for the number of protons within the nucleus. All of the beta decays are mediated by the W boson since they involve a charge transfer. There is a more scientific name for these weak interactions with charge transfer, namely the *Charged Current* (CC) interactions, which is the basis of the nuclear β-decay. Neutrinos are not only involved in charged interactions, but also in neutral flavour conserving currents called the *Neutral Current* (NC) interactions, mediated by the Z boson. For most neutrino (and sterile neutrino) experiments the charged current interactions are utilised (see the sterile neutrino experiment section 5).

3.2 Symmetries and Conservation

In physics, fundamental symmetries are the building blocks of theories [2]. These symmetries are conservation properties of a physical system under specific transformations. Symmetries make calculations easier and they help theorize about extensions of existing theories (e.g. supersymmetry (SuSy) [28]). Nevertheless, non-conserved symmetries are often much more interesting, because new physics is usually the cause of violation of the symmetry or conservation law. The difference between conserved and non-conserved quantities can, for example, be found in experiments. A symmetry that seems to be exact at a fundamental level is the *CPT-symmetry*. Consisting of Charge conjugation (C), Parity (P) and Time reversal (T) symmetry.
Charge parity (C) is the charge conjugation of a particle or antiparticle, which means the transformation of a particle to its corresponding antiparticle (or the other way around), notation $C|\psi\rangle = |\overline{\psi}\rangle$. C-symmetry means that under charge conjugation, the physical laws of a system do not change.
Parity (P) is the transformation that flips the sign of spatial coordinates (also called inversion). This operator transforms a certain particle into its mirror image. When a mirror image of a particle does not behave in the same way as its normal version, this is called parity asymmetry (figure 6). Parity symmetry would mean that the mirror version does behave in the same way within the same physical system. *Time Reversal* (T) is the operator that reverses the direction of time in a system. The symmetry of time implies symmetry of physical laws under time reversal.

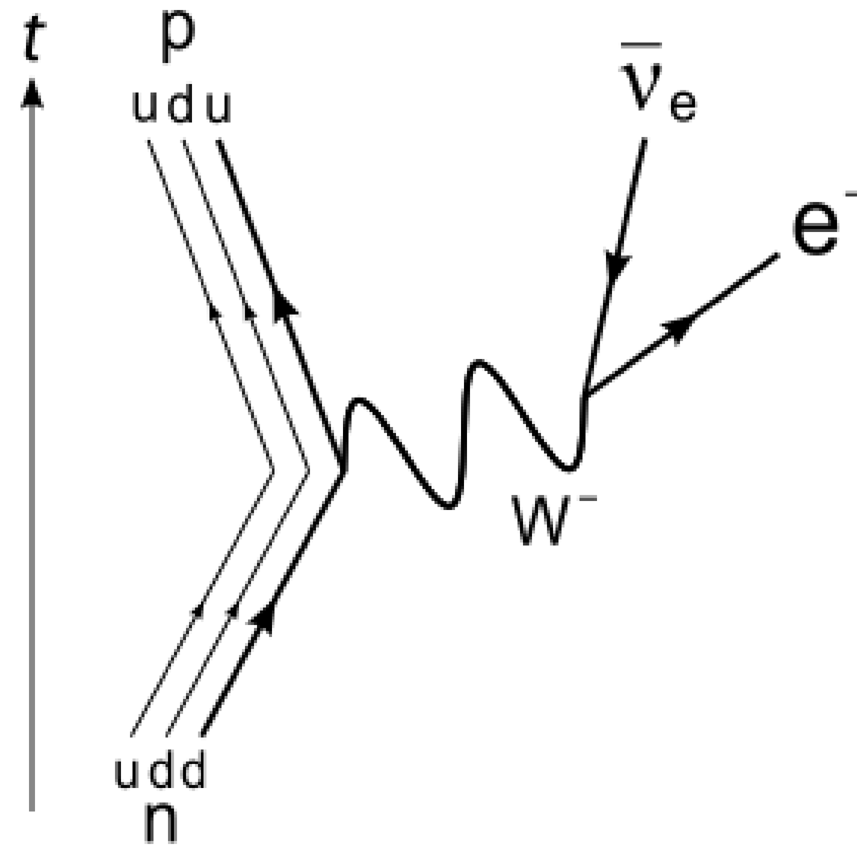

Figure 5: *The lowest order (tree-level) Feynman Diagram describing beta decay. This weak interaction decay is mediated by the W boson.* Taken from 5) in [43]

Both C-symmetry and P-symmetry are violated by the weak interactions. The combination of the two, which is called the CP-symmetry, is violated in the weak interactions of the quark sector and is expected to be violated in the leptonic sector as well.
So even though CPT is conserved, if it is reduced to CP it is not conserved. This CP violation, which can be experimentally determined through the existence of complex phases in the leptonic sector (section 3.7), could reveal new physics. This new physics is thought to explain why there is an imbalance between matter and antimatter in the universe [18].
An important and returning conservation law in the standard model is Lepton number (L) conservation. It is divided into the three generations $L = L_e + L_\mu + L_\tau$. So L consists of L_e, L_μ, L_τ.

A +1 is assigned to every particle of the corresponding lepton family (e.g. to L_μ for μ^- and ν_μ). A -1 is assigned to every antiparticle of its corresponding family (e.g. to L_e for e^+ and $\overline{\nu_e}$).
Not entirely surprisingly, this conservation law is violated when the Standard Model is extended with Majorana particles, which are particles that are the same as their anti-particle. Until the existence of these Majorana particles is experimentally proven, lepton number seems to be conserved in weak interactions. For more information on symmetries in the Standard Model, I refer to [2].

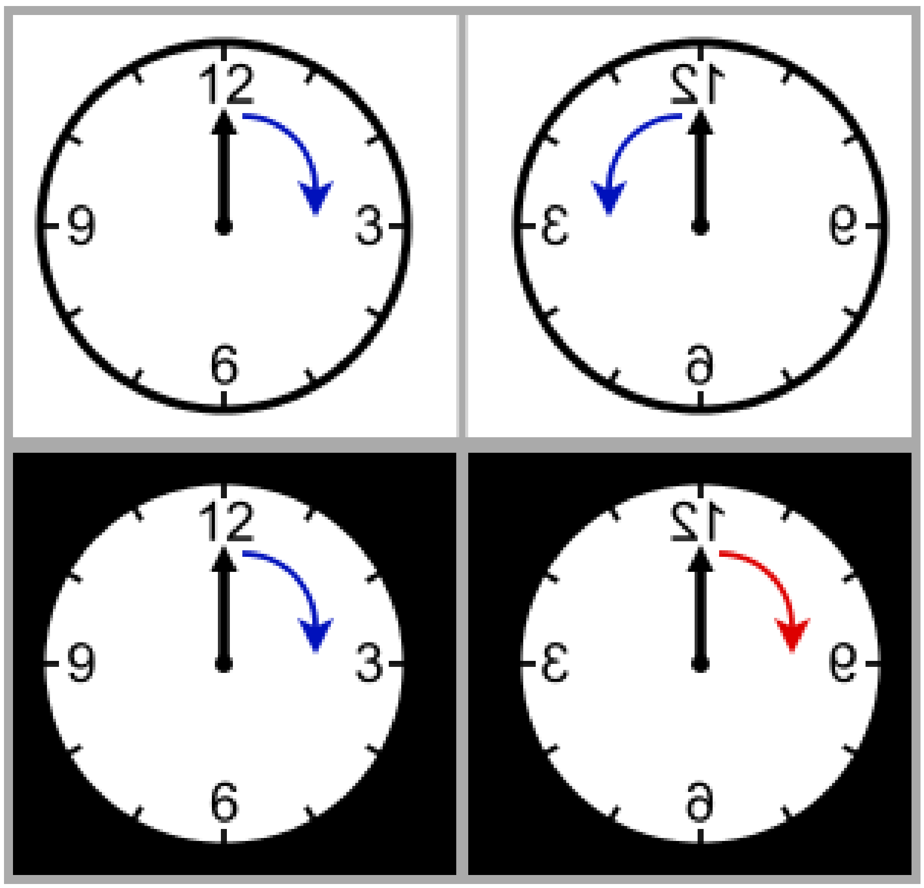

Figure 6: *The top clocks obey P symmetry, the bottom clocks do not.* Taken from 6) in [43]

3.3 Chirality

Chirality is a quantum mechanical inherent property of a spin-$\frac{1}{2}$ particle. To understand it a little better, a comparison to helicity can be made. Helicity describes the same property as chirality, but only when particles have no mass. In this mass-less limit chirality is the projection of a particle's spin on the direction of motion (figure 7). Parallel aligned spin and momentum results in a Right-Handed (RH) particle. When it is aligned antiparallel, this particle is called Left-Handed (LH).

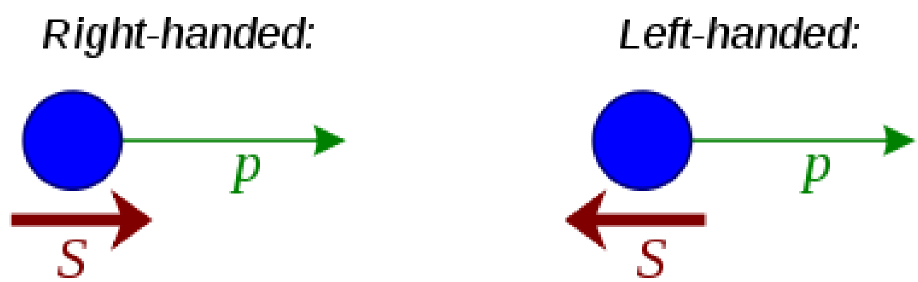

Figure 7: *P stands for the particles momentum and S for the spin.* Taken from 7) in [43]

This comparison, unfortunately, does not work when a particle has mass. The difference between chirality and helicity can actually be observed. Let's assume that a particle has mass, then the helicity of this particle can be flipped with a change of reference frame. A particle with mass can by definition not go with the speed of light (it would need infinite energy to reach this). If the observer goes faster than the particle, the momentum of the particle flips and with that its helicity. This is fundamentally different from chirality, which can only be changed with charge conjugation.
Therefore, when a particle has mass, helicity is not the same as chirality. Then what is chirality? Chirality indicates how the phase of the fermion's spin vector changes under a spatial rotation. The effect on the phase is opposite for right and left chiral fermions. Thus this property ensures that right and left chiral particles are inherently different.
From now on, LH and RH refer to respective left and right chiral (or handed) particles. The symmetries obeyed within a theory determine if RH en LH are treated differently and thus if a theory is chiral or non-chiral. For example, theories obeying P-symmetry, like electromagnetism, are non-chiral theories. In contrast, chiral theories such as the weak force, violate this P-symmetry, and treat left and right-handed particles completely differently.
For the weak force, the quantum number weak charge is introduced to explain this apparent preference. right-handed particles do not have this weak charge and therefore they cannot interact with the weak force. With this knowledge of

chirality, the beta-decay 1, done explicitly with the Wu Experiment [5], shows something fundamental about neutrinos. The particles (e^-, v_e) in these interactions are left-handed and the anti-particles (e^+, $\overline{v_e}$) are right-handed. This yields an interesting conclusion: In the universe, all the 'visible' neutrinos are left-handed. This also immediately means that if right-handed neutrinos existed, they would be practically invisible (or sterile).

3.4 Quantum Field Theory of the Standard Model

To understand the symmetries within the standard model, knowledge is necessary of the basics of Quantum Field Theory. Without going into the complex details, the Quantum Field Theory has an understandable nature to it. The idea is that every fermion, boson and fundamental force has its own field, and the interactions between these fields result in visible effects. The fields describe all the possible quantum states for a given system. The particles interacting with a force field create excitations in that force field [23].
Interactions between different particles are described with Lagrangians and can be visualized with *Feynman Diagrams*. The Feynman Diagrams are used to get a simplified and correct overview of a certain interaction between particles (in figure 5 a tree diagram[4] of a β-decay is given). The *Lagrangian Density* consists of all the involved fields and derivatives of fields and is the mathematical representation of the Feynman diagrams. It also contains the coupling constants of the fields involved in the interaction. The Lagrangian Density (or the Lagrangian) itself has to obey certain symmetries
The standard model is a *gauge theory*. This means that the Lagrangian of the system is invariant under certain gauge transformations (generalised phase transformations). In other words, it obeys gauge symmetry. Each gauge group is a Lie group with its own group generators. In the standard model, the generators are linked to the force mediating gauge bosons and each of the bosons is described by a corresponding quantum field. The standard model consists of the following gauge groups:

- $SU(3)_c$ representing the strong force with 8 group generators linked to the 8 force-mediating gluons
- $SU(2)$ representing the weak force with the weak isospin generators (T_1, T_2 and T_3) linked to force carriers W1, W2 and W3
- $U(1)_Y$ which is the hypercharge gauge group with the hypercharge generator Y, linked to the force mediating B-boson

The direct product $SU(2)_L \times U(1)_Y$ is the electroweak gauge-group. The subindex L stands for the fact that this group treats left-handed particles different from right-handed.
The fermion fields are important for the sterile neutrinos, because these fields

[4]A tree diagram is the most basic Feynman Diagram of a certain interaction with no loop interactions or extra virtual particles.

provide an idea of how they would look mathematically. In the standard model there are three left-handed lepton doublets L_α.

$$L_\alpha = \begin{pmatrix} \nu_{\alpha L} \\ \alpha_L \end{pmatrix} \quad (4)$$

With $\alpha = e, \mu, \tau$ and quantum numbers $(2, -\frac{1}{2})$, where the 2 stands for the doublet (allows two isospin states $T_3 = \pm\frac{1}{2}$) and the $-\frac{1}{2}$ is the weak charge (Y_W) according to the definition $Q = T_3 + Y_W$ for the electromagnetic charge. The right-handed components are singlets (only allows $T_3 = 0$ states) under $SU(2)_L \times U(1)_Y$. If the right-handed neutrinos exist, they are bound to be isospin singlet states and therefore not subject to weak interactions.
$SU(2)_L \times U(1)_Y$ symmetry prevents the existence of fermionic mass terms, because these terms would violate the symmetry (as will be explained in section 3.6). This means that there is another mechanism necessary to generate the mass of the particles. In the standard model, all the mass terms are generated by the Higgs Mechanism through symmetry breaking[5]. These mass terms can be represented by a Yukawa interaction with a Yukawa coupling constant y. A Yukawa interaction term would then look as follows

$$\mathcal{L}_Y = y\bar{\psi}\phi\psi \quad (5)$$

The ϕ corresponding to the Higgs field with quantum numbers $(2, \frac{1}{2})$. So the Higgs field is also represented by a doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad (6)$$

The Lagrangian of the interaction with the Higgs field would show us why neutrinos were thought to be massless. The first order mass terms would need right-handed neutrinos to exist, because the Higgs field works with Dirac mass terms (see equation 5). The fact that neutrinos do have mass is immediate proof for the existence of physics beyond the standard model.

3.5 Thermodynamics of particle production

Particles produced at the beginning of the universe are all subject to the laws of thermodynamics. Dark matter seen today has to submit to these laws. This gives us constraints for dark matter production and for the phase space density that is allowed.
Interactions between particles can either be in or out of equilibrium. Suppose there is a process $A \rightarrow B$ possible at a certain energy level, which refers to their averaged velocity and the thermodynamic parameters shaping the surroundings of the particles like the density. When this interaction is in *thermal equilibrium*, B changes into A as much as the other way around. Suppose that B

[5]For a better understanding of this mechanism read reference [4] and [7]

changes into A via a collision of particles. To change into A with a reasonable probability, the particles of B needs a certain amount of energy. If the energy of B drops below this amount due to, for example, the expansion (cooling down) of the universe, the reverse process from B back into A will not happen as frequently as earlier or not at all. This reaction is then *out of equilibrium*. The density of particles B that is left in the early universe after such an energy drop is called the *relic density*. The temperature at which the reverse process does not happen anymore is called the *decoupling temperature* T_{dec}. Suppose the process involves a very small interaction strength, in that case the energy level (defined by the earlier described parameters) for any interaction to happen has to be much higher, because the increased flux of the colliding particles has to compensate for the smaller cross-section. Hence, the decoupling from other particles happens earlier in the expansion of the universe. As particles decouple they release entropy. This cools down the out-of-equilibrium particles by diluting and red-shifting them, called *the chilling effect*[16].

Freeze out and Freeze in
The method of production of particles in the early universe heavily depends on whether they were ever in thermal equilibrium or not. There are two different possibilities in the early universe for relic particles.

- At first the particles are in thermal equilibrium, but they move out of it by external factors like the expansion of the universe. This process leaves behind a relic density of particles that has no efficient reactions left to disappear other than decays into other particles. This process is called a *freeze out*.
- The particles are never in thermal equilibrium. They are produced via a process that creates them and they can only disappear via decays. This process is called a *freeze in*.

Thermal and Non-Thermal
Particles can be produced thermally or non-thermally. Thermal distributions depend at least partially on temperature (T) whereas non-thermal ones do not. As an example, a thermal distribution of the momentum of particles is given by equation 7, also called the Fermi-Dirac distribution.

$$f(p,T) = \frac{1}{e^{\sqrt{p^2+m^2}/k_B T} + 1} \tag{7}$$

In this equation p is the absolute value of the momentum of the particles (for example dark matter), m the mass and T the temperature. The only restrictions on non-thermal momentum spectra are those parameters that make the spectra physical [17]. The momentum distribution of dark matter also does not always depend on T. The production of warm dark matter, such as sterile neutrinos (as will be shown later on), is also mostly non-thermal, so it inherits a non-thermal distribution as well.

3.6 Neutrino Properties

3.6.1 Basic properties

The standard model has room for three different flavours of light left-handed neutrinos. This has experimentally been proved with the decay of the Z^0 boson. The total decay width[6] was compared to the visible decay width. With the difference between these values, the best fit for the amount of (almost invisible) light neutrino flavours was approximately three.
The existing neutrinos are very light compared to their lepton flavour brothers (e.g. the electron is at least a factor 10^5 heavier than the neutrinos). Unfortunately, within the standard model, there is no explanation for this lightness. This is called the *mass problem*. When neutrinos were first introduced into the standard model, they were even thought to be massless. That neutrinos, in fact, did have mass has experimentally been proved with the discovery of neutrino oscillations. This comes from the simple fact that the massless neutrino flavours would have a well-defined mass eigenstate[7] that would evolve independently from each other. This would directly imply no mixture between these flavour states, refuting the discovery of oscillations. Thus, these oscillations allow flavour (or interaction) eigenstates of a neutrino to be a linear combination of multiple mass states, see sec 3.6.4.
The lightest neutrino mass state itself has not been found yet. Neutrino masses are so small, that our instruments are not precise enough to determine an exact value. To resolve this problem, experimenters try to exclude. The sum of the masses has an upper boundary. Data from the Cosmic Microwave Background (CMB) showed that the sum of masses is at least lower than this bound. To try to pin down the mass eigenstates further, oscillation experiments are used. These experiments are only sensitive to squared mass differences. The following values for the squared mass differences have been found.

$$\Delta m^2_{21} = \Delta m^2_{sol} = 7.4x10^{-5}eV^2 \tag{8}$$

$$\mid \Delta m^2_{31} \mid \approx \mid \Delta m^2_{32} \mid = \Delta m^2_{atm} = 2.5x10^{-3}eV^2 \tag{9}$$

The hierarchy of the mass eigenstates, given in figure 8, has until now not been found. It could be normal, following the hierarchy of the charged leptons, or inverted. Depending on whether mass eigenstate ν_3 is the heaviest or the lightest mass state [8].

3.6.2 Dirac or Majorana and mass terms

For neutrinos that have nonzero mass, it is necessary to know which type of fermionic particle we are working with. There are two types of fermionic particles, a Dirac particle and a Majorana particle. A Majorana particle is its own

[6]The decay width Γ of a particle consists of all the different possible particles it can decay into. A fractional decay width for one possibility is called a branching ratio (BR). Notation of the neutrino BR of the Z boson: $\frac{\Gamma(Z\rightarrow\nu\bar{\nu})}{\Gamma_Z}$

[7]Eigenstates of the Hamiltonian

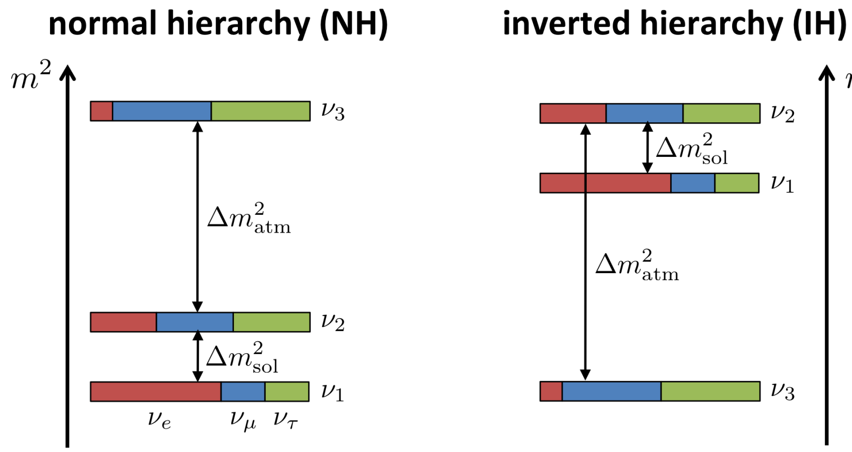

Figure 8: *Normal or inverted hierarchy.* Taken from 8) in [43]

antiparticle (such as a photon is its own antiparticle), in contrary to a Dirac particle where this is not the case. Which type of fermionic particle a neutrino is, is still up for debate. It could be that both types exist in nature or just one of them. There is no experimental proof yet for any preference for either of the two. The Majorana neutrino could experimentally be proven with the observation of neutrinoless double beta-decay [11]. Choosing for the Dirac neutrino seems the obvious choice since all the other fermions are also Dirac particles, but the neutrinos having no electrostatic charge, makes this choice less obvious. This means that we have to discuss both.
Firstly, let's look at how the chiral projection operator works. The fermion field ψ gets a chiral direction with the projection operator P_R and P_L:

$$\psi = \psi_L + \psi_R \tag{10}$$

Where $P_L\psi = \psi_L$ and $P_R\psi = \psi_R$.
ψ_L (same as $\overline{\psi_L}^{\rm C}$)[8] creates a RH antiparticle and destroys a LH particle, whereas ψ_R (same as $\overline{\psi_R}^{\rm C}$) creates a LH antiparticle and destroys a RH particle. For the adjoint fields $\overline{\psi_R}$ and $\overline{\psi_L}$, the statements are correct if the words 'antiparticle' and 'particle' are interchanged.
Dirac fermions consist of two Weyl fermions, one Weyl fermion is the left or right-handed part of the same particle. The mass of these Dirac particles is generated by the Higgs mechanism, with mass terms shown below.

$$-m_D(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L) \tag{11}$$

[8]This superscript C means that this ${\psi_L}^{\rm C}$ is the charge conjugated version of ψ_L

Looking at this, for Dirac neutrinos to have mass, the right-handed neutrino has to exist. This said, it does not add new additional eigenstates, but only new spin states.

Majorana fermions consist of a single Weyl fermion. In addition, for the Majorana field the following condition holds.

$$\psi = \psi^{C} \tag{12}$$

The Majorana mass terms can, like the fermion field ψ, be split into two terms: the right-handed mass terms

$$-\frac{1}{2}\overline{\psi_R}^{C} m_R \psi_R + h.c. \tag{13}$$

and the left-handed mass terms

$$-\frac{1}{2}\overline{\psi_L}^{C} m_M \psi_L + h.c. \tag{14}$$

These terms violate lepton number L by 2 units. In Majorana's approach, these numbers make no sense, since there is no difference between particle and antiparticle.[9] The right-handed terms are invariant under $SU(2)_L \times U(1)_Y$ and the M_R is allowed because ψ_R and $\overline{\psi_R}^{C}$ are gauge singlets.

The left-handed terms do violate $SU(2)_L \times U(1)_Y$, since the weak hypercharge carried by left-handed neutrinos is not cancelled to zero, but doubled to two units in this term. In other words, this term is not gauge invariant. Therefore the SM needs to be extended, for example with a double Higgs term (for further explanation of this read [10]). Another option is the extension with a higher dimensional term with a new energy scale Λ. This will be elaborated on in section 4.3. The combination of Dirac and Majorana mass terms is the basis of new beyond the standard model physics, namely the basis of the seesaw mechanism. It can be written in a Lagrangian as follows

$$\mathcal{L}^{\text{D+M}} = -m_D \bar{\nu}\nu - \frac{1}{2} m_L (\overline{\nu_L}^{C} \nu_L + \overline{\nu_L} \nu_L{}^{C}) - \frac{1}{2} m_R (\overline{\nu_R}^{C} \nu_R + \overline{\nu_R} \nu_R{}^{C}) \tag{15}$$

Where the ν stands for the neutrino field, instead of the above used fermion field ψ. To find definite masses this equation can be written in a matrix form (see section 4.3).

3.6.3 Neutrino Oscillation and Mixing angles

Pontecorvo proposed neutrino oscillations in 1958, along with the suggestion that neutrinos had small masses. This idea is visualized in figure 9 and can be mathematically written down as follows.

$$\nu_{eL} = \cos\theta\, \nu_{1L} + \sin\theta\, \nu_{2L} \tag{16}$$

The left-handed electron neutrino is composed of two mass eigenstates v_1 and

[9] Within these mass terms, the violation of the lepton number might not make sense. In interactions with charged leptons, Majorana neutrinos will definitely violate this number.

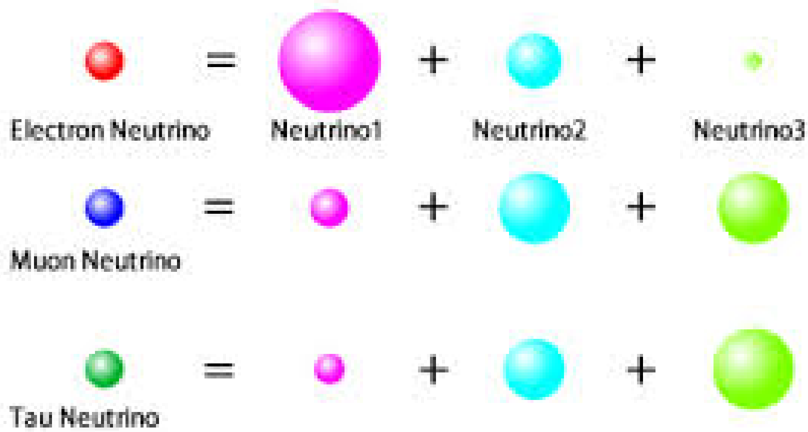

Figure 9: *The visualization of neutrino oscillations. Each neutrino flavour (electron, muon and tau) can be seen as a superposition of neutrino mass eigenstates (neutrino 1, 2 and 3).* Taken from 9) in [43]

v_2, with their mixing ratio defined by the mixing angle θ. The mixing angles are defined as the ratio between different eigenstates, thus what part of the neutrino flavour consists of mass eigenstate 1 and what part of eigenstate 2 and so on.
In contrary to the other leptons, the neutrino flavours (e,μ,τ) have no well-defined mass eigenstates. The 3 flavour eigenstates v_α can be written in terms of the 3 different mass eigenstates v_i as follows

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i} v_i \text{ with } \alpha = (e, \mu, \tau) \tag{17}$$

In this equation $U_{\alpha i}$ is an element of the mixing matrix U_ν, which is an analogy of the quark mixing CKM matrix. This CKM matrix is unitary, while for U_ν this is still open for debate. This is called the *flavour puzzle* [6]. The mixing matrix U_ν shows what the ratio of mass eigenstates is for each flavour eigenstate. The U_ν can be written in terms of two real rotation matrices V_{ij} representing the mixing matrix between state i and j and one matrix with a complex phase component. The complex phase is defined through a diagonal matrix with the Dirac phase δ. Finally, if neutrinos are Majorana neutrinos a final phase matrix with Majorana phases α_1, α_2 has to be added:

$$U_\nu = V_{23}A_\delta V_{13}A_\delta V_{12}B_{\alpha_1,\alpha_2} =$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix} \tag{18}$$

In this mixing matrix $s_{ij} = sin\theta_{ij}$, $c_{ij} = cos\theta_{ij}$ with θ_{12}, θ_{13} and θ_{23} being the 3 mixing angles, δ is the Dirac phase and α_1, α_2 the Majorana phases. More on phases in sec. 3.7.

3.6.4 Oscillation probabilities

The probability of oscillation is important for the neutrino experiments. (This will return explicitly in section 5). For simplicity let's assume there are only two neutrino flavours. To get the probability of 2D oscillation in vacuum the following approximations are used (for more details see [12])

- The flavour of an active neutrino can be determined using the Charged Current (CC)
- Equation 17 and a simple 2D version of matrix 18
- The plane-wave approach regarding the state of the neutrino after time T with energy E

The resulting transition (or disappearance[10]) probability depends on the mixing angle θ between flavour state α and β, the squared mass difference (Δm^2)) between the states, the baseline[11] length L and energy of the particle E:

$$P_{v_\alpha \to v_\beta}(L) = sin^2(2\theta)\; sin^2(\frac{\Delta m^2 L}{4E}) \tag{19}$$

The survival (or appearance[12]) probability can be defined with the transition probability:

$$P_{v_\alpha \to v_\alpha}(L) = 1 - sin^2(2\theta)\; sin^2(\frac{\Delta m^2 L}{4E}) \tag{20}$$

When dealing with more than two flavours the survival probability is defined by 1 - all the transition probabilities a state can evolve to.
In these calculations, the neutrino oscillations are considered to be in a vacuum. The situation changes drastically when neutrinos move through matter. The

[10] Amount of oscillations from flavour state $v_\alpha \to \nu_\beta$ over a distance L with energy E. Experimenters look directly for the appearance of certain other states.

[11] Distance from the neutrino source.

[12] Amount of oscillations from state $\nu_\alpha \to \nu_\alpha$ over a distance L with energy E. In other words, does the neutrino in flavour state α also stay in α.

oscillation probability for neutrinos in matter is namely very different from that in vacuum. The math of these oscillations goes beyond the scope of this thesis. For that subject, I refer to [12].

3.7 CP violation

CP violation is one of the *Sakharov conditions* for the early universe. According to Sakharov there are three conditions necessary for the baryogenesis[13] of the universe. These three are 1) Baryon number[14] violation, 2) CP (and C) violation, 3) Interactions are out of thermal equilibrium.

3.7.1 CP violating phases

CP violation has been proved in the quark sector, but the amount of violation was not enough to explain the matter-antimatter imbalance. Therefore, physicists are still looking for other sources of CP violation. In the lepton sector, with what is known today, only the neutrino mixing could be a viable source. To find how many parameters could violate CP, the total amount of parameters of the mixing matrix U has to be analysed.
In general, there are n^2 free parameters in a n x n unitary matrix. In this matrix there are $\frac{n(n-1)}{2}$ independent real rotations and thus mixing angles. The rest (total - mixing angles) of the parameters are phases. For the CP violation, the physical phases, the ones that cannot be absorbed[15], are the interesting ones. How many can be absorbed depends on which type of fermion is being mixed. For Dirac fermions $\frac{(n-2)(n-1)}{2}$ physical phases remain[16]. For Majorana fermions $\frac{n(n-1)}{2}$ phases remain[17]. For the mathematics of these values, I refer to [13].

3.7.2 CP violation and the matter and antimatter problem

The fact that there remain physical phases in a mixing matrix is enough to explain CP violation. CP symmetry means that it does not matter for an interaction rate if it is dealing with particles or antiparticles. CP violation implies thus by definition an imbalance between the production of those two. Since C involves the complex conjugation of the fields phases play a key role.
The phases remain in certain interaction rates and this makes the rates different when the complex conjugate of the phases is taken. Thus, there is a difference

[13]Hypothetical processes that produce an imbalance between matter (baryons) and antimatter (anti-baryons)

[14]The baryon number is analogous to the lepton number. Particles consisting of quarks and/or antiquarks are assigned a baryon number with $B = \frac{1}{3}(n_q - n_{\overline{q}})$. Where $n_q/n_{\overline{q}}$ is the number of quarks/antiquarks in the particle.

[15]Some phases can be absorbed (or can be made to vanish) through a redefinition of the fields they are multiplied with

[16]In the 3 by 3 case this is the Dirac phase δ.

[17]In the 3 by 3 case these are the Dirac phase δ and the Majorana phases α_1 and α_2

for the same interaction between matter and antimatter. In other words, the associated reaction is out of balance.

4 Sterile Neutrino Physics

4.1 Overview

Chirality has been defined in order to understand what sterile means in terms of the standard model. It has become clear that the handedness of a particle has an impact on the way it interacts with gauge bosons (See figure 10). According to the theory already covered, it follows that the existence of sterile neutrinos implies the introduction of right-handed neutrinos. Within the $SU(2)_L$ they could be introduced as singlets in contrast to their lighter left-handed brothers, who are represented by doublets together with the charged leptons. The existing mixing matrix gets extended with a new dimension for each new mass state corresponding to a sterile flavour (see section 4.4.1). If there are additional sterile flavours, the following questions would be invoked. How many sterile flavours are out there? How strongly do they couple to their active counterparts and how does the mixing between the active and sterile flavours work? Additional questions arise when this proposed sterile neutrino also has to fit within the currently measured bounds for Dark Matter and experimental data like the CMB. What is the mass-range of these Dark Matter neutrinos and how does this range influence the active-sterile mixing angles? How can these neutrinos be produced in the early universe and what causes them to still exist at this point in time? Overall, what mechanisms make the sterile neutrino a plausible dark matter candidate? When answering these questions it follows that the sterile neutrinos are predicted by many theoretical extensions of the standard model. A few of those will be discussed in the next sections.

4.2 Possible Sterile Neutrinos

The sterile neutrino is definitely featured in some new beyond the standard model (BSM) physics. Not only to explain existing neutrino anomalies[18], but also for other unsolved problems within particle physics. There are many types of sterile neutrinos. However, for dark matter only a few are interesting. First, let's sum up a few possibilities and filter out the useful ones.

Sterile neutrinos can either be light or heavy. Both options are solutions to different problems. It is possible that they co-exist, but for now, only the existence of the heavy ones will be reviewed. First, let's review the two most important options.

Singlets

The sterile neutrino is introduced as a right-handed singlet fermion. It has the possibility to oscillate to its left-handed doublet counterparts, due to a tiny left-handed admixture. The addition of sterile neutrino flavours also implies the existence of extra definite sterile mass states. So a certain neutrino flavour does not only consist of the three known mass eigenstates, but it also consists of a

[18]Anomalies are yet unsolved problems. Most of the time the anomalies cannot be understood with known physics.

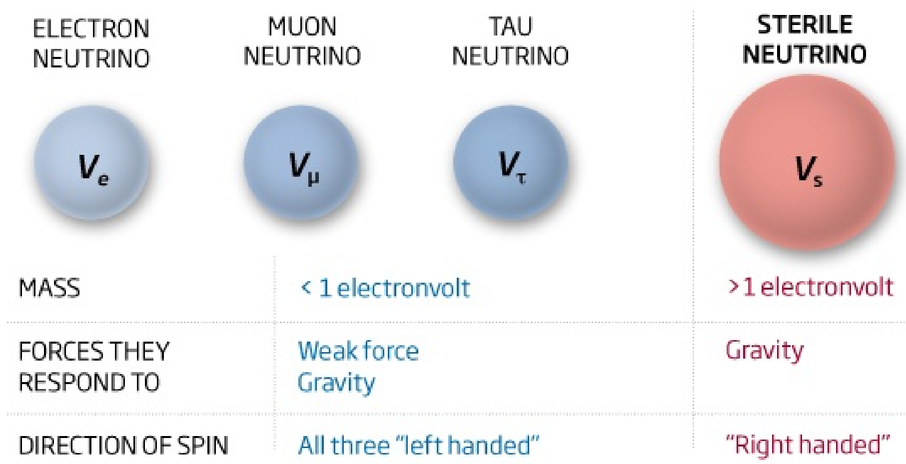

Figure 10: *The visualization of the sterile neutrino flavour as the fourth neutrino flavour.* Edited from [42]

very small portion of right-handed sterile neutrino via the new neutrino mixing mechanism. The mass generation mechanism of such particles is described in the seesaw mechanism.
Doublets
In this model, the sterile neutrino is in fact not fully sterile[19], but only partially[20]. The standard model is extended with a right-handed weak force [21]. The gauge group would be $SU(2)_R$ and it would imply the existence of hypothetical particles like the exotic right-handed W boson and an extra neutral Z' boson.

4.3 Seesaw Mechanism

The seesaw mechanism is an extension of the standard model regarding the size of mass terms and providing an option to combine both Dirac and Majorana masses into one model. It comes in different forms. Each form is useful for different purposes and for explaining different physics. All of these models are explanations of solutions to the *mass problem*. For dark matter purposes, the *seesaw type I* will be reviewed. First, let's look at the mathematics of a 2-dimensional seesaw model.

[19]This particle does not interact with any existing gauge bosons (also the not yet discovered ones).
[20]This particle does not interact with the SM gauge bosons.
[21]This can, for example, be done with an extension to a weak left-right symmetric model, further referred to in [6] and [41].

4.3.1 Seesaw 2-dimensional mathematics

The seesaw mechanism was proposed to understand why the neutrino masses are so much smaller than all the other SM masses. The small masses can be explained with extraordinarily small Yukawa couplings, but why would this coupling be only so tiny for the neutrinos? In order to evade these small couplings, the seesaw mechanism gives a possible explanation.
In this two-dimensional example, the idea of the seesaw mechanism is to introduce 2 neutrino fields; a heavy and a light field. The superposition of the two fields results in the left-handed neutrinos observed and the theorized right-handed ones.
The underlying math of the seesaw mechanism is a rotation of the mass matrix. Let's first consider a generic matrix A

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 100 \end{pmatrix} \tag{21}$$

$$R = \begin{pmatrix} cos\,\theta & sin\,\theta \\ -sin\,\theta & cos\,\theta \end{pmatrix} \tag{22}$$

The matrix components can be rotated by a slight angle θ. Let's assume the rotation angle $\theta = 1°$. After rotation of the basis vector with this rotation angle, the matrix A is transformed into B:

$$R\ A\ R^{\mathrm{T}} = B = \begin{pmatrix} 0.034 & 1.744 \\ -1.744 & 99.96 \end{pmatrix} \tag{23}$$

This rotated matrix B is approximately the same, but not exactly. The idea is that this is the phenomenon that is observed in nature. Most of the mass is in the lower right corner representing the heavy right-handed neutrinos. The almost zero value in the upper left corner represents the mass of the light left-handed neutrinos. The physical analogy to the rotation of the matrix is the mixing angle between the active and sterile neutrinos. [34]

4.3.2 Seesaw as a mass generation Mechanism

Now let's look at the actual seesaw mechanism for neutrino mass generation. The mass matrix M has the following form

$$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \tag{24}$$

Using this, the mass term can be written in terms of the vector V of the two neutrino fields

$$V = \begin{pmatrix} \nu_L \\ {\nu_R}^{\mathrm{C}} \end{pmatrix} \tag{25}$$

Now equation (15) can be reduced to the following matrix form

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2}\overline{V}M(V)^{\mathrm{C}} + h.c. \tag{26}$$

To find the mass eigenstates of this two-dimensional mass matrix, the same rotation principle with rotation matrix R is used to find the diagonalized matrix M_λ. The eigenvalues $m_{1,2}$ are found by reviewing this as an eigenvalue problem [12].

$$M_\lambda = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = R^{\mathrm{T}}\, M\, R \tag{27}$$

This gives the eigenvalues

$$m_{2,1} = \frac{1}{2}(m_R + m_L) \pm \frac{1}{2}\sqrt{(m_L - m_R)^2 + (2m_D)^2} \tag{28}$$

and it also constrains θ

$$tan\ (2\theta) = \frac{2m_D}{m_R - m_L} \tag{29}$$

The eigenvalues $m_{1,2}$ can either be positive or negative. The actual mass eigenvalues of course cannot be negative. $m_{1,2}$ are multiplied with a squared phase $\alpha_{1,2}$ that can either be 1 or -1. In case of the negative eigenvalue, this squared phase is also negative, making the mass eigenstate positive and real. This phase α is the Majorana CP violating phase already seen earlier.
This example can lead to the seesaw parameters given in seesaw type I under the following assumptions. [13]

- m_D are Dirac masses generated with the Higgs mechanism
- In matrix M, m_L can be chosen 0, because the v_L mass term breaks gauge invariance
- $m_R >> m_D$, where m_R is around the scale of Λ.

This leads to the following mass eigenvalue equations.

$$m_1 \approx m_D \frac{m_D}{m_R}$$

$$m_2 \approx m_R \tag{30}$$

$$\theta \approx \frac{m_D}{m_R}$$

Eventually, these parameters lead to the neutrino flavour eigenstate ν_α having both a part of mass eigenstate 1 and of 2:

$$\nu_\alpha = c_1\nu_1 + c_2\nu_2 \tag{31}$$

Analysing the seesaw parameters can lead to the following conclusions. First, one of the eigenvalues becomes very small, because of $m_R >> m_D$, explaining what is seen in nature. The other eigenvalue is around the scale of new physics $m_2 = m_R \sim \Lambda$. Moreover, the mixing parameter θ, defining the oscillation

probability between the fields, is very small ($\frac{m_D}{m_R} << 1$). This means that the almost sterile neutrino state is supplemented with a tiny active admixture.
Even though this model only represent one generation of neutrinos, it helps us understand the consequences for a larger model with more neutrino fields, because physically the conclusions do not change when more families or flavours are added.

4.3.3 Seesaw type I

In the seesaw type I model, the Standard Model is extended with n right-handed neutrino flavours N_i. In this thesis N_i is used to denote the corresponding 'sterile' mass eigenstate, the flavour eigenstate notation is v_{si}.[22] In comparison to the simple 2-field model, not one but n right-handed fields are added to the 3 existing left-handed ones. This means that m_R is replaced by the Majorana mass matrix M_m, which does not have one but n eigenvalues. The masses of the left-handed neutrinos can still be written in terms of the Dirac masses and the left-handed Majorana masses. Only the math changes a little, since the multiplication is now with matrices instead of scalar values. For example, the Lagrangian becomes a multiplication of different matrices (for the actual mathematical representation, I refer to [13]).
The n eigenvalues of the Majorana mass matrix M_m represent the masses of the individual sterile neutrino mass eigenstates. These eigenstates are found at the scale Λ. This high scale generates the possibility for new physics. At this scale, for example lepton number violation could be allowed, possibly explaining why Majorana states could exist. A more exotic idea is the extension of the standard model with a new gauge group $SU(2)_R$, the right-handed weak force. As usual, m_D is given by the corresponding Yukawa couplings and the Higgs vacuum expectation value (vev). The mixing matrix θ_{as} gives insight into the active and sterile mixing component. Every components of this matrix shows what portion of a particular flavour is a definite mass eigenstate m_i.
The Higgs generated mass matrix m_D term can be written in terms of the active-sterile mixing matrix θ_{as} and the mass matrix M_m. This relation implies that for every left-handed neutrino a right-handed neutrino with a non-zero mixing component is necessary to explain their lightness [6]. Therefore, if this lightness is only generated by the seesaw mechanism, then by definition there have to be at least two right-handed neutrinos (one for every light neutrino with nonzero mass).

[22] It should be noted that the sterile flavour eigenstates ν_{si} are approximately the same as their corresponding mass eigenstate N_i. When looking at, for example, decays of sterile neutrinos, the mass eigenstates are the ones involved and not the flavour eigenstates. The flavour eigenstates are as shown in section 4.4.1 a superposition of left and right-handed mass states, but these flavour states are not involved in any interaction (they are sterile). A right-handed mass eigenstate, which consists of a superposition of flavour eigenstates, is supplemented with tiny admixtures of left-handed flavours. Therefore it does have a small probability to decay to left handed states or even to join left-handed weak interactions.

4.3.4 Seesaw Scale

The scale of Λ could be chosen anywhere. For the purpose of explaining the lightness of the known neutrino masses, it is typically chosen high above the Electro Weak (EW) scale[23]. So $\Lambda >> v$, where v is the vacuum expectation value of the Higgs mechanism after symmetry breaking. The mass eigenstates of the new particles introduced in the seesaw mechanism, including the sterile neutrinos, are at the order of Λ. This immediately would explain why they have not been observed earlier[24]. As with the Fermi 4 point interaction [25], at this scale the new physics introduces particles with such high masses that their interactions cannot be observed at the EW scale where the standard model works. They would only interact virtually with a very low probability.
A more interesting scale for observable sterile neutrinos could be chosen. Namely the Low Scale Seesaw. For sterile neutrino dark matter, the mass value of the mass eigenstates can be chosen under the EW scale. In that case, the heavy neutrino that can be the dark matter candidate, would have a mass eigenvalue around the keV range. A good example of the low scale seesaw in action is the νMSM (see section 4.4.1). In that model, a lower scale is chosen to avoid problems with the lifetime of the dark matter. The sterile neutrinos are produced via oscillation with active neutrino flavours (see section 4.5) and for this the mixing parameter has to be at least above certain values, accomplished with this lower scale Λ. The magnitude of the specific mixing parameter is bound, because the sterile neutrino has to be produced at the beginning of the universe and still has to exist today. Besides the sterile neutrino production, there are more advantages to a lower scale. Namely the particle becomes observable in several other physical phenomena, but more importantly, the sterile neutrino could explain more than just the dark matter. It could also explain for example Pulsar kicks and could help with understanding the formation of early stars.[16]
The lifetime bound has important implications when the high scale ($\Lambda >> v$) is chosen. For this high-scale seesaw, the lightest active neutrino eigenstate has to be practically massless. Otherwise, the mixing parameter θ has to be greater than its upper lifetime bound to be able to generate the $m_{lightest} > 10^{-3} eV$. Moreover, this implies that the mass of the introduced sterile neutrino is much smaller than Λ or that it is too short-lived to be dark matter[6]. More about this balance between mixing and lifetime is discussed in the production section 4.5.

[23] Beside the fact that this would also create the possibility of other new physics at this scale

[24] With the technology of today, the amount of energy necessary to create these particles by far exceeds the amount that experimental devices are able to create

[25] The weak decay looks like an interaction between four particles without a mediating W boson, because the energy of the interaction is by far not enough to create a real W boson. So this interaction is mediated by a really short-lived and almost not detectable virtual W boson.

4.4 Dark matter Sterile Neutrinos

In the first generation of direct detection experiments, looking for the Weakly Interacting Massive Particles (WIMPs), the possibility of finding WIMPs was heavily reduced. The Feebly Interacting Massive Particle (FIMP) such as the sterile neutrino evades these constraints[26]. It seems to be a natural dark matter candidate, because of this feeble interaction strength, although interacting so weakly does restrain the production of the right-handed states. Somewhere in the history of the universe, there should have been some particle(s) that produced it. Because of its feeble interaction strength the production only works efficiently at very high, early universe temperatures. The moment at which a particle is introduced, has all kinds of implications for its lifetime, free-streaming length, structure formation et cetera. For example, the possibility that it mixes with its active neutrino counterparts is heavily constrained by these cosmological parameters, in fact, this production is almost ruled out by them (more about this in section 4.5).
With the initial production of FIMPs comes the question why they still exist today, but luckily their feeble interaction strength helps out there. They were never in thermal equilibrium, even not in the early universe. This has some interesting implications for their relic density, which are explained in section 4.4.2
So the dark matter version of the sterile neutrino has quite specific characteristics. The properties of these particles not only determine how they can behave as dark matter, but also how they could be detected or even how they could be ruled out. First, I will elaborate on a theoretical extension of the standard model that proposes a few properties, the neutrino minimal standard model (νMSM).

4.4.1 Neutrino Minimal Standard Model

The neutrino minimal standard model (νMSM) was proposed with the Ockham's razor[27] principle. This model gives a natural solution to a number of the earlier stated (neutrino) particle physics anomalies with a small number of newly introduced parameters. The idea is based on a low scale seesaw type I. It hypothesizes the introduction of three right-handed singlets (N_i), that all have mass eigenstates below the electroweak scale. The lightest eigenstate N_1 has a mass in the keV range. The other two, $N2$ and N_3, have approximately the same mass somewhere between 100 MeV and the electroweak scale. First, these two heavier states explain the lightness of the left-handed states via the seesaw mechanism. Furthermore, via leptogenesis[28], the heavier sterile neutrinos lead to CP violation. This could possibly provide an answer for the matter-

[26]The introduction of FIMP dark matter does not exclude the existence of WIMP dark matter, they could perfectly exist alongside each other to explain the observed dark matter density.

[27]When to choose which hypothesis corresponds the best with the universe, choose the one that explains the most phenomena while introducing the least new parameters

[28]Hypothetical processes and interactions at the beginning of the universe that create an asymmetry between leptons and anti-leptons.

antimatter imbalance. Solutions for other unexplained phenomena are given in [20].
In the νMSM, the mixing matrix becomes a 6 x 6 matrix with all the mass and flavour eigenstates having their own mixing ratio c_{ij} in relation to each other. A single flavour eigenstate, following equation 17, would look as follows:

$$v_e = c_{11}v_1 + c_{12}v_2 + c_{13}v_3 + c_{14}N_1 + c_{15}N_2 + c_{16}N_3 \tag{32}$$

The red part is the sterile admixture part. The oscillation components for the sterile states are small, which means that the probability to find them in such a state is small as well. These components, as stated earlier, are also coupled to their production. The idea is that these three neutrinos react so weakly that their production interaction was never in thermal equilibrium (more on that in section 4.4.2). This means that there is an upper bound on the number of sterile neutrinos that could have been produced.
The addition of three right-handed states seems to be very natural, since every kind of fermion in the standard model also has three generations. However, it creates some new constraints. For example, three right-handed states restrain the mass of the lightest mass eigenstate to be of the order of 10^{-5} eV, due to their constrained mixing angle value.
The sterile neutrinos solve many problems, but the introduction of the particles also creates new ones. The main production method of the particles, at least when they were introduced, is the *Dodelson Widrow mechanism*. The mass-mixing angle ratio set by this production mechanism, unfortunately, is excluded by the Lyman-α bounds and x-ray searches. That said, this exclusion can be evaded with a new approach to this DW mechanism, as will be shown in section (4.5).

4.4.2 Dark Matter properties

The most interesting property of the sterile neutrino is its mass, since dark matter candidates have to be heavy in order to satisfy Ω_{DM}. It can be chosen in the eV [26], keV [6], MeV [41] and GeV-TeV [41] range. The mass-range choice heavily influences the boundaries for the active-sterile mixing angles. Equation 30 shows us how the mass size influences these mixing angles. The heavier the mass of the sterile neutrino the shorter its lifetime. Therefore, the MeV and GeV-TeV range neutrinos are too short-lived to be dark matter and thus are ruled out to be dark matter. Moreover, to constitute 100% of the dark matter the right-handed states have to have a mass higher than 0.4 keV [41]. The mass-range constraints in combination with x-ray constraints gives regions of restricted but also allowed parameter choices. These regions can be seen in figure 12. Taking the constraints into account, it seems that the best choice for the dark matter sterile neutrino mass is around a few keV (This keV range mass was also proposed in the νMSM and various other references [6], [36], [41]).
With a mass in the keV range, the sterile neutrino will be light[29], sufficiently

[29] lighter than the EW scale

stable and probably detectable (see section 5.4). To be sufficiently stable, the lifetime of the produced N has to be at least of the order of the age of the Universe. That way it is ensured that the N still exists in abundance today. This constrains the lifetime of the decays of the sterile states (shown in section 4.4.3). As has been explained, these decays are linked to active-sterile mixing angles (see section 4.3).

In the proposed form, the heavy neutrinos can be labelled as some kind of warm dark matter, although the 'warm' label is mainly historical. It has been found that the distribution of dark matter could indeed be thermal, but it could also be non-thermal, disregarding the immediate need for a thermal representation. A sterile neutrino dark matter particle could be produced non thermally [6], which then, in turn, creates a non-thermal distribution.

Due to the chilling effect, the 'warm' sterile neutrino dark matter can also be chilled, or even become cool[30] dark matter, all with a different free streaming length and phase space density. Combined with their momentum distribution, these parameters describe the structure formation of the dark matter sterile neutrinos.

Another important property of dark matter sterile neutrinos is their *relic number density*. In combination with their mass, this density shows if the particles can account for the observed dark matter density. The sterile neutrinos were never in thermal equilibrium[31], so a first conclusion is that the relic density of active neutrinos is much larger than that of sterile neutrinos ($\rho_\nu >> \rho_N$). Furthermore, the relic density left by these sterile neutrinos could satisfy the observed dark matter density bound, without exceeding standard model lepton properties such as the bound on leptonic warm dark matter, the *Tremaine-Gunn Bound*.

4.4.3 Sterile Neutrino Decays

It is obvious that any possible decay of the dark matter neutrino has a very long lifetime, since it still has to be out there today. It is important to understand which decays could actually happen. On the one hand, a decay is an indirect way of detecting the particles, by observing product particles with certain energies. On the other hand, the decay width can give another insight into how long the lifetime of these sterile neutrinos is.

Sterile neutrinos have two different decay possibilities via the left handed admixture of the right-handed mass eigenstates. Without extending the standard model too much[32], both decays have a decay width that depends for the most part on the particle's mass and the active-sterile mixing angle. Suppose there

[30] Both cool and chilled dark matter have a structure that corresponds more with cold dark matter than with warm

[31] This depends on the chosen production model. For the ones described in this thesis, this statement about thermal equilibrium is true.

[32] More outlandish decays in higher order loops may also be possible, but have not been researched yet.

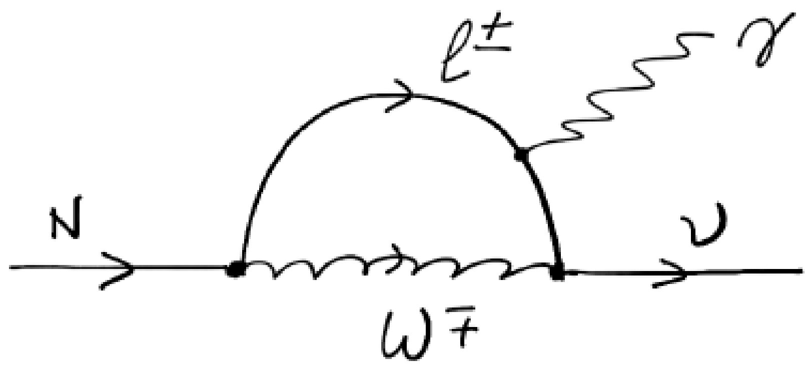

Figure 11: *A Feynman diagram of the $N \to \gamma\nu$ decay. Here N is the right-handed mass eigenstate, and with the help of its tiny left handed admixture it has a possibility to decay to a left handed flavour eigenstate. In this decay the l, standing for the involved charged lepton, determines which flavour eigenstate is released (following lepton number conservation). Another possibility where the photon is emitted by the W boson is not shown here.* Taken from [6]

is a right-handed state N. The decay channel to left-handed neutrinos is then given by: $N \to \nu_\alpha \nu_\beta \overline{\nu}_\beta$. Because N is a Majorana particle, the decay can have all combinations of different flavours and charge conjugations [6].
The other possible decay would be a loop decay to a standard model neutrino and a photon (for the Feynman diagram see figure 11). This decay, notation $N \to \nu + \gamma$, has an interesting observation possibility if the sterile neutrino is indeed the main source of dark matter. The photon in this interaction acquires half of the decay energy coming from the sterile neutrino mass ($E_\gamma \propto \frac{1}{2} M_N$). These photons can then be found by observing a certain emission line within the 1-100 keV range [30].
This line, also called *the 3.5 keV line*, has possibly already been observed by two independent groups [37]. It can be seen as a signal coming from the decay of a dark matter particle with a mass of a few keV. Other interpretations such as an atomic transition could also be the cause of the observed emission line [38]. Unfortunately, the signal had a significance of 3-4 σ, so statistically speaking it could also be just a fluctuation, rendering all theories of what it might be wrong. Despite this, it could definitely be an exciting observation, especially when linking it to the gamma decay of sterile neutrinos.

4.5 Sterile Neutrino Production

The production of dark matter sterile neutrinos can be accomplished in many interesting ways. In this thesis, the emphasis will lie on those models that are most likely to be the actual method of production for the particles. The selection of these models is based on, for example, taking into account the data constraints from x-ray searches or the CMB.
I have found that the production can be split up into three different main sources:

- Oscillation production (freeze in)[6] [19] [21] [36]
- New gauge interaction production (freeze out)[6] [41]
- Decay production [6] [16] [19] [20] [29]

The keV mass-range for the sterile neutrino is a particularly good choice for oscillation production. In principle, the other method allows the mass scale to be much larger than the EW scale [6].

4.5.1 Production via Oscillations

The addition of n right-handed neutrinos creates a framework for active-sterile mixing (see section 4.3). The mixing induces a very natural way of the production of sterile neutrinos, because it is an inherent property of both active and sterile neutrinos.
Oscillation production can be split into two methods: *non-resonant* and *resonant*. For both of them, the main idea is that any reaction that produces an active neutrino also has a small probability to produce a sterile neutrino. This is due to the sterile admixture, where at least some of the active-sterile mixing ratios (c_{ij}) have to be non-zero. This means that the sterile neutrinos actually do participate in the known weak left-handed interactions, but with only a small probability. To show this mathematically, for simplicity only one generation with 2 neutrinos is considered (without changing the physics, this can be extended to more neutrino fields).
In this model there is one left-handed active neutrino state ν_l and one right-handed sterile flavour state ν_r (=v_{s1}) with mass eigenstates ν_1 and N_{s1}. The mixing angle θ determines their mixing ratio.

$$\nu_l = \cos\theta\, \nu_1 + \sin\theta\, N_{s1} \tag{33}$$

$$\nu_r = -\sin\theta\, \nu_1 + \cos\theta\, N_{s1} \tag{34}$$

Looking at these equations it is easy to see that when θ is small, $m_1 \approx 0$ and $m_{N1} \approx M_{DM}$, where M_{DM} is at least in the keV range.

Non-Resonant Production (Dodelson-Widrow mechanism)
The production of the sterile neutrinos via oscillations took place through scattering of particles in the primordial plasma[33]. This is called the Dodelson-Widrow (DW) mechanism. The primordial plasma has consequences for the probability of production depending on the mixing angle between active and sterile states and on the cross-section of the interacting particles.
The most important consequence of the reactions happening in the plasma is that the oscillation process can't be assumed to be occuring in a vacuum. In other words, oscillation matter effects have to be taken into account. In matter, the mixing angle is estimated with the following equation:

$$\sin^2 2\theta_m = \frac{(\frac{\Delta m^2}{2p}\sin 2\theta)^2}{(\frac{\Delta m^2}{2p}\sin 2\theta)^2 + (\frac{\Delta m^2}{2p}\cos 2\theta - V_{Dens} - V_{Temp})^2} \tag{35}$$

The mass difference $\Delta m^2 = m_2^2 - m_1^2$, where m_1 and m_2 are the mass eigenvalues of the mass eigenstates in vacuum. θ_m is the matter mixing angle and θ is the vacuum mixing angle value. This equation also depends on the mass m, the momentum of the particle p and the matter potentials V_{Temp} and V_{Dens} [6]. For non-resonant production, the lepton asymmetry l_a in the particle density is assumed to be of the order of the baryon asymmetry $\eta_B \approx 10^{-10}$. This renders the V_{Dens} insignificant for this production, because it explicitly depends on the value l_a. [34]. The production rate of sterile neutrinos depends on a temperature factor T^5 (Coming from the cross-section of this interaction combined with the flux) and on $\sin^2 2\theta_m$. This means that the production is suppressed at low temperatures. V_{Temp} suppresses the interaction at higher temperatures. The production is at its peak at. This suppression by V_{Temp} causes the sterile neutrinos that are produced never to be in thermal equilibrium (called freeze-in production). At the peak of the production, the momenta of the created neutrinos are relatively high. This results in a momentum distribution corresponding to warm dark matter.
For non-zero mixing angles, this production method leads to a sufficient amount of dark matter via the DW mechanism. The resulting warm dark matter structure should be relatively easy to detect, also due to the fact that its requirements for the mass and the mixing angle are quite strictly set by the DW mechanism. With these parameters, a maximum of 15% of the DM density could be made up from these sterile neutrinos [41]. To constitute more, the parameters are only able to become a combination of mass and mixing angle (see figure 12) that has been strongly disfavoured by cosmological data (e.g. the Lyman-α bound). Therefore, this scenario seems to be excluded as a possible production method.
Yet, this scenario has been looked into because these bounds can be evaded if in the primordial plasma a larger value of lepton asymmetry l_a is assumed.

[33] The particle soup in the early universe.
[34] l_a can be estimated with the difference in neutrino and antineutrino number densities divided by the photon density.

Resonant Production
Resonant production in comparison to non-resonant means that the production peaks only at certain parameter values and that the production is relatively low in other circumstances. The non-resonant production can have certain resonances due to the Mikheyev-Smirnov-Wolfenstein (MSW) effect. The mathematics of this MSW mechanism (given in for example [41]) go beyond the scope of this thesis, therefore only the basics and results of these resonances are discussed.
The main idea is that lepton asymmetry causes the matter potential V_{Dens} to be non-zero, which in turn allows active neutrinos to have mass level crossings with the sterile neutrinos. It also has an effect on the matter mixing angle θ_m as was shown in the non-resonant section. The production still creates sterile neutrinos in the same way, but it gives a completely different energy distribution It leaves us with a more distorted spectrum with an overall lower averaged momentum for the non-thermal distribution [31], hence with a cooler dark matter structure.

With the larger value of lepton asymmetry, resonant production can cause production for the right keV mass-range with a much lower mixing angle. Also, this sterile neutrino could now definitely constitute 100 per cent of the total DM density Ω_{DM}.

4.5.2 New gauge interaction production

For this production method, the sterile neutrino is assumed to have a new gauge charge. This new gauge group then interacts with the particle, immediately implying that the neutrino is not fully sterile after all. The production would come from interactions in the early universe with other particles equipped with this new gauge charge. The most interesting and not unusual extension would be that of the right-handed weak force. For example, the right-handed neutrino flavour eigenstates would then come from interactions with right-handed particles mediated by a new right-handed W-boson W_R. The introduction of this W_R can be for example done with the extension of the weak force to a left-right symmetric model [29].
The introduced gauge group acts only in the early universe circumstances with reasonable probability and therefore at this point in time (when the universe has (almost) completely cooled down), there are no efficient interactions left because they have decoupled from the standard model during the expansion of the universe.
At a certain point, the production of right-handed neutrinos via the interactions would have been subject to the freeze out mechanism, explaining its relic density and its apparent 'sterileness'. With temperatures smaller than the rest mass energy of the new W_R boson, the interactions with other right-handed particles to create or annihilate the sterile neutrinos is kept in thermal equilibrium via a mechanism analogous to the *Fermi four-point interaction*.

To show its compatibility with the dark matter density, the energy density contribution of the produced right-handed state can be shown by calculating the corresponding relic abundance (actual computation shown in [6]).
The estimated interaction rate Γ_N depends mainly on the cross-section of the particles. The temperature of freeze out T_F can be calculated with the assumption that Γ_N is equal to the Hubble expansion rate (H). The relic abundance can then be calculated using this T_F. Eventually, this leads to a large energy density contribution. Too large for the observed dark matter density Ω_{DM}. This inconsistency can be explained in various ways, the most interesting being a model where the energy density of N_1 is diluted after the freeze out of the N_1 particles. The foundation of this dilution is extra entropy coming from for example the decay of one of the heavier right-handed states (e.g. N_2). This decay then creates the observed baryon asymmetry via leptogenesis.
Therefore, the advantage of this model is that this dilution of the right-handed neutrinos cools the momentum distribution, making it easier to comply with the structure formation bounds. Moreover, this model, while producing a dark matter candidate, can also create leptogenesis, as explicitly shown in [33].

4.5.3 Decay Production

The decay production method arises naturally with an extension of the νMSM [20]. As said, the production of sterile neutrinos comes with an introduction of a singlet scalar S. The decay $S \rightarrow NN$ then produces the sterile neutrinos[35]. The decay into the sterile neutrino is driven by the Yukawa Coupling. There are a few different possibilities for the chosen mass of the scalar (shown in figure 12), but for every model, the mass of the scalar is way above the EW scale. The mass of the dark matter sterile neutrino (around a few keV) can be calculated with the expectation value of the singlet scalar combined with the corresponding Yukawa coupling y:

$$M_{N_1} = y\langle S \rangle = 10\,keV \tag{36}$$

In this equation $y = 10^{-8}$ and $\langle S \rangle = 1TeV$.

The advantage of this production mechanism is that it is not constrained by the same data that excludes the non-resonant oscillation production. Even better, the sterile neutrinos can, with an extension of this mechanism, additionally be produced via oscillation production. Using the oscillation production as an additional source, it can fill up the gaps in the observed momentum distribution. This makes it possible to fulfil the dark matter bounds while simultaneously agree with the cosmological data of the history of the universe [17].

[35] Other ways of production from this singlet scalar e.g. freeze in or freeze out are reviewed in [20].

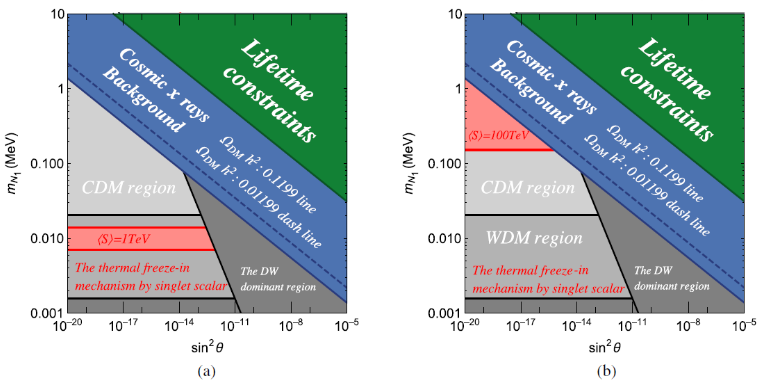

Figure 12: *A plot showing the relation between the mixing angle in terms of* $\sin^2\theta$ *and the mass of the lightest right-handed state* N_1*. a) Shows the lifetime constraints and possible parameter combinations of cold and warm dark matter. Furthermore, it shows the allowed region for the singlet scalar decay production method with* $\langle S \rangle = 1$ *TeV b) shows the same plot as a, but with the allowed region for* $\langle S \rangle = 100$ *TeV.* Taken from [20]

5 Experiments

5.1 Theory of sterile neutrino experiments

Experiments with sterile neutrinos are not very different from other dark matter searches, for example, searches for WIMPs. The best approaches for laboratory searches for dark matter are direct detection and the evaluation of the kinematics of radioactive decays and these will be reviewed. Other indirect methods are considered in [6], [24], [25] and [26].
For simplicity, only one sterile eigenstate N_{s1} is considered, with a certain mass M and mixing angles. In addition, this sterile neutrino is supposed to satisfy the entire observed dark matter density. This sterile neutrino has a mass generated with the seesaw mechanism.

5.2 Direct Sterile Neutrino Detection

The direct detection of dark matter particles can be split into two different methods. First, the sterile neutrino could be captured by a radioactive nucleus inducing β-decay [39].

$$_{Z}X + \nu \rightarrow {}_{Z+1}Y^{+} + e^{-} \tag{37}$$

This interaction can be done with, for example, the stable element 163-dysprosium. The sterile neutrinos would induce a detectable electron signal of $K_e = M_{N_1} + E_0$, where K_e is the kinetic energy of the electron, and E_0 is the endpoint energy of the β-decay and N_1 is the mass eigenstate of the lightest sterile neutrino. The only problem is the heavy suppression of this decay due to the tiny active-sterile mixing parameter.
The second method of direct detection is the scattering of sterile neutrinos [40]. When this neutrino accounts for all the dark matter clustered in halos around galaxies, the density of this sterile neutrino is a factor 10^3 larger than the current observed cosmic ν background for active neutrinos. Therefore, a search for capturing and scattering events with DM sterile neutrinos seems to be a well-motivated experiment, even though they are strongly suppressed by the mixing between active and sterile components. For further comments on these experiments, I refer to [6] and [41].

5.3 Sterile neutrino detection with β-decays

With a β-decay releasing enough energy, the production of the sterile neutrino mass eigenstates is definitely possible (if they exist of course). The decay only has to have an endpoint energy E_0 that is at least equal to the keV range mass of the eigenstate M_{N_1}. This is necessary to be able to produce the sterile admixture component with a high enough probability.
In this thesis, three detection methods with β-decays will be reviewed.

- β-decay spectroscopy
- Kinematic analysis of the β-decay spectroscopy

5.3.1 β-decay spectroscopy

The β-decay spectroscopy method depends on the detection of a deformation of a β-decay spectrum [41]. As shown in equation 1, these decays involve the flavour eigenstate electron neutrinos. With knowledge of the neutrino oscillations, it can be established that this electron neutrino consists of three different mass eigenstates. These eigenstates cause the spectrum of a β-decay to be a superposition of the three mass eigenstates' own spectra. All these eigenstates have very small mass differences. With today's technology, these splittings are too small to detect, but this situation changes completely when the electron neutrino would consist of a fourth (or even fifth or sixth) mass eigenstate. This would cause a detectable distortion in the measured spectrum of the β-decay. The mass term will no longer be one approximate term of the light mass eigenstates $m(v_i)$, but it will consist of two terms: one term still representing the light neutrino mass eigenstates together, and the other additional term, representing the heavy keV range mass of the right-handed state M_{N_1}. This means that the differential spectrum can be written as ([6]):

$$\frac{d\Gamma}{dE} = \sin^2\theta_{as}\frac{d\Gamma}{dE}(M_{N_1}) + \cos^2\theta_{as}\frac{d\Gamma}{dE}(m(v_e)) \tag{38}$$

Here θ_{as} describes the mixing between the active and sterile states. It determines how distorted the spectrum will be. Though, the effect will undeniably be small, see figure 13.

5.3.2 Kinematic analysis of the β-decay spectroscopy

This method uses a kinematic reconstruction of either a β-decay or an electron capture to detect sterile neutrino states. The reconstruction would provide a mass spectrum, on which a sterile neutrino mass eigenstate would be visible if it is involved in any of the analysed events [6]. The sterile neutrino would leave a footprint by missing out. The reconstructed mass spectrum would usually only consist of the three known light neutrino mass eigenstates, but if sterile neutrinos exist, the spectrum will have small detectable bumps (see figure 14), created by the extra sterile mass eigenstates.

A reconstruction can be done through the following procedure. First, the momenta of all the particles involved in the event are measured as precisely as possible. For a β-decay this means the momentum of the daughter ion and of the emitted electron[36] [41]. Then the missing energy represents the mass squared of the sterile mass eigenstate and can be calculated with the following equation:

$$M_{N_1}^2 = (Q - E_d - E_e)^2 - (\mathbf{p} + \mathbf{k})^2 \tag{39}$$

Here E_e is the kinetic energy of the electron and E_d is the kinetic energy of the daughter ion, Q is the total energy of the interaction, $\mathbf{p}$ is the momentum of the

[36] Assuming the parent ion is at rest at the time of the decay. This can be done by, for example, cooling the mother nucleus down $\sim 1mK$

daughter ion and **k** is the momentum of the electron.
To find out if there is an actual sterile neutrino mass eigenstate, the trick is to analyse many events and find bumps in the resulting reconstructed mass spectrum (see figure 14).

5.4 Sterile neutrino experiments and the Hunter experiment

Searching for papers that review experiments with dark matter sterile neutrinos requires a bit of caution. Many of the papers that talk about sterile neutrinos, are actually talking about *light* sterile neutrinos. Neutrinos with that property have already been ruled out as a dark matter candidate.
An actual dark matter search is the Hunter experiment. It is using both the mass spectrum and the beta-spectrum analysis of K-capture events, to find keV range sterile neutrinos. As explained, if this keV mass neutrino exists, the beta spectrum will show a tiny difference, because of the sterile admixture part (as shown in figure 13). Experiments to find the hypothesised particle were already done in [45], unfortunately in this experiment actual mass eigenstates of a keV sterile neutrino couldn't be distinguished from background sources. The technical challenge is to overcome the noise of the two most significant background sources. The first one, unobserved photons that take part in the decay and add an extra momentum parameter to equation 39. The second one is scattering of source atoms with the daughter ions.[41] When these backgrounds could be controlled better, the sterile neutrino keV mass eigenstate could be detectable in, for example, a beta-decay mass spectrum reconstruction experiment with for example a cold ($3mK$) ^{3}H-source. The expected results of such an experiment need a technical review that exceeds the scope of this thesis, for this I refer to [41]. However, together with all the earlier described beta-decay experiments, the expectation is that within a foreseeable time the sterile neutrino, with mass and mixing angle within the allowed parameter region (figure 12), is definitely detectable. In most experiments, overcoming the background is the technical challenge. This means that improving sensitivity and resolution of the setup, will boost the likelihood of detection of sterile neutrinos if they exist.

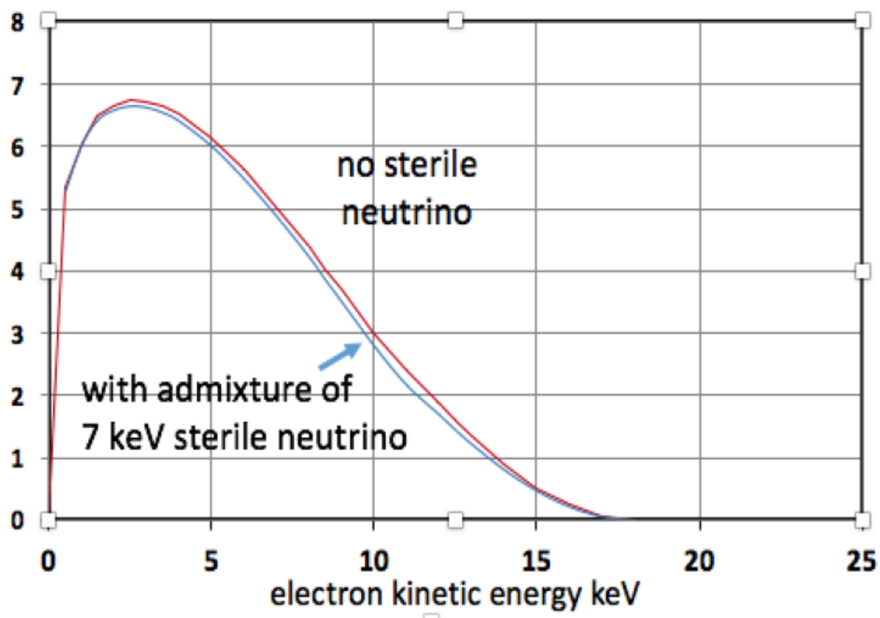

Figure 13: *The spectrum of a beta decay. The distorted spectrum induced by sterile flavours is the blue spectrum.* Taken from [27]

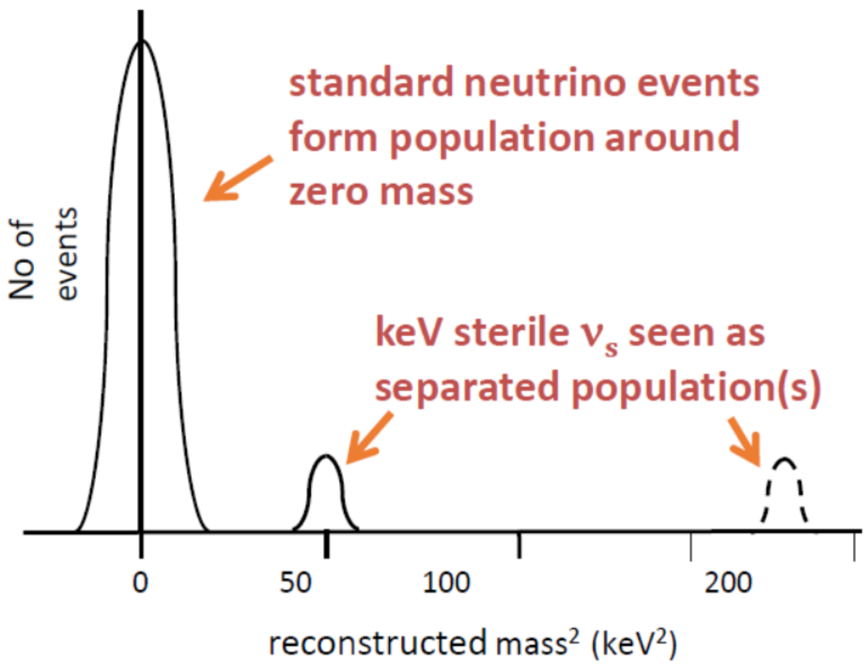

Figure 14: *The expected reconstructed mass spectrum of the experiment if there exist sterile neutrino states. The first smaller bump is the hypothesized dark matter candidate with a mass around* $(7keV)$. Taken from [27]

6 Conclusion

The sterile neutrino seems to be an adequate dark matter candidate. Being a neutrino provides it with properties that are expected of particles that fit the current dark matter models. For the standard model neutrino, their mass turned out to be too small for it to be a dark matter candidate. This isn't a problem for the sterile neutrino because its mass isn't restricted to such a tiny size. That said, its mass size can't be chosen entirely freely. For example, the seesaw model showed that the mass profoundly determines the mixing angle. This mixing parameter is the most constraining factor in the choice of the mass size, because it determines a large part of both the neutrino oscillations and the sterile neutrino decay modes. Where the latter puts the most profound bounds on the mixing angle size. As was shown, if the mixing angle is too big, too many sterile neutrinos would have already decayed. In comparison, if it is too small, there would be too many sterile neutrinos left, rendering them unable to form the observed dark matter structure. The mass and mixing angle together are constrained by the cosmological data. The trick is to hit the sweet spot for both the size of the mass and of the mixing angle. For the mass, the best value lies around a few keV. Then the mixing angle has a small possible region, taking into account the lifetime considerations and cosmological data seen in figure 12. In this region, the sterile neutrino can be a stable right-handed fermion. Moreover, in the neutrino minimal standard model, this choice of parameters makes the sterile neutrino able to constitute 100% of the observed dark matter density Ω_{DM}. This νMSM has an additional beneficial side effect because it introduces other heavier sterile neutrinos that explain other not yet understood neutrino properties (e.g. lightness problem) and particle physics anomalies (e.g. pulsar-kicks and baryogenesis).

Observations of the universe do not only constrain the mass and mixing angle, but also the structure formation of the sterile neutrinos. The dark matter sterile neutrino is probably not warm dark matter (because this kind of dark matter is strongly disfavoured by, for example, the Lyman-α bounds), but some form of cool dark matter. However, in this thesis it was also shown that these bounds could be evaded, either by assuming different circumstances in the early universe, which would then result in for example resonant production, or by choosing a totally different production model. At the moment, the models shown in this thesis are the most promising ones for this research field. To find which model corresponds best with nature two future approaches are possible. One is by trying to identify sterile neutrinos via detection experiments. The other is by excluding models by trying to improve cosmological data with for example x-ray searches. The first path is obviously interesting since it will finally give theorists a hint about the properties of dark matter and maybe even solve the dark matter problem. Furthermore, from the detection of sterile neutrinos, by either direct detection (e.g. missing energy via kinematic reconstruction) or indirect detection (e.g. 3.5 keV γ emission line), one could deduce the best corresponding production model. The second path, excluding a single production model or certain combinations of parameter values (e.g. a region in

the mass-mixing angle parameter space), can be interesting because it provides exclusions and thus models that don't need further research. Following these statements, the νMSM model is an interesting subject for the second path of future research because it provides us with a production method, namely the oscillation production, that could be completely ruled out.
Personally, I would like to dive deeper into the thermodynamics of particle creation and annihilation. Then more specifically, try to understand the actual mathematics of some of the sterile neutrino production models. For example, for a master thesis I would like to expand my knowledge on the oscillation production and look for optimisations and actual experiments to eventually prove or rule out this model. I think that oscillation production is the most promising model, because it follows naturally from the inherent properties of neutrinos. It also makes use of other already existing physical processes, such as a freeze out of relic particles. Furthermore, to make it fit the cosmological bounds, for example, more lepton asymmetry in the primordial plasma is a possible solution. More lepton asymmetry is not only necessary for this production model of sterile neutrinos, but also for leptogenesis that in turn can lead to baryogenesis. As was seen, baryogenesis could explain the observed baryon asymmetry.
A more speculative thesis, for example to hypothesize new dark matter models or to try to find solutions for the technical challenges currently holding back sterile neutrino observations, is also something I would fancy. Overall, while doing this thesis, I did not only raise awareness for dark matter sterile neutrinos, I also learned that researching and understanding the field of theoretical high energy physics is something I really enjoy.

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