Radboud university

Department of high energy physics

Bachelor thesis

Dark matter neutrinos

Author:
Tom de Wilt

Supervisor:
Prof. Dr. Wim Beenakker

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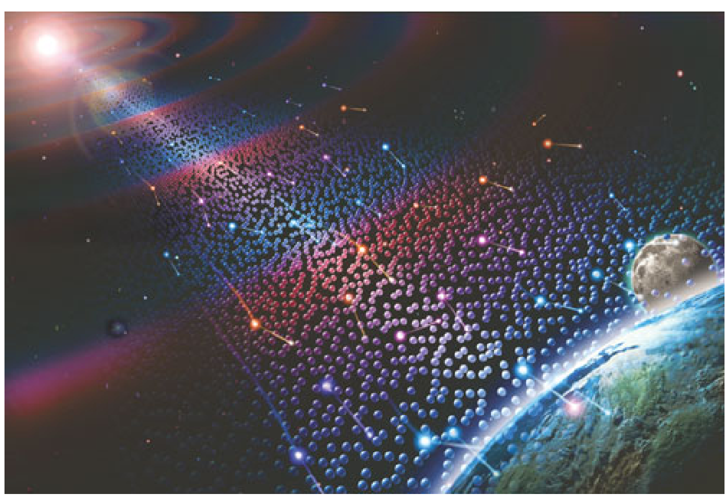

Acknowledgements

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1 Introduction

Physics is known as a field of interest where observational quantities can be described by fundamental theories. As soon as our laws start failing to explain new observations (provided that these observations are accurate and trustworthy) we know that changes have to be made to our theories in order to be able to explain these new observations. This has been a guiding principle in the progress that the physics community has made and is still making.

Dark matter is primarily postulated to explain the velocity curves for the orbital motion of stars and gas in galaxies [1]. Throughout the twentieth century, claims have been made that stars and galaxies move faster than what Newtonian mechanics would suggest based on visible matter. As technology improved and measurements became more precise, the claims on the observations became more solid [2]. If Newtonian mechanics is assumed to be correct, it would follow that stars and gas that are orbiting around the center of a galaxy should have decreasing velocities with radial distance from it at larger radii, when most of the galaxy's luminous mass is inside the orbit. However, this is not found. Rather the orbital speed shows no tendency of decreasing over large radial distances.

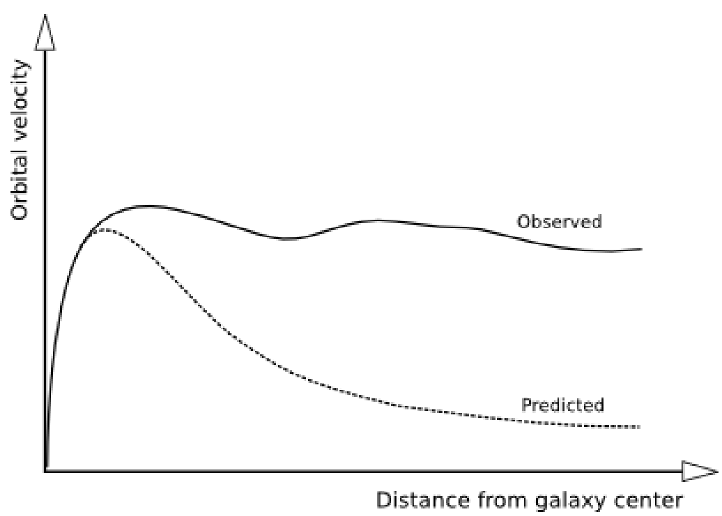

Figure 1: Example of the rotational curves.
Source: *http://en.citizendium.org/wiki/Galaxy_rotation_curve*

Weak gravitational lensing is another motivation for dark matter [3]. When looking at the shape and orientation of background sources behind a cluster of galaxies, one can determine the mass of the cluster and then compare this with the visible mass. Gravitational lensing experiments show higher masses in galaxies than we see at the moment and this gives rise to the need of dark matter. These kind of experiments give us a good base to start with, because we can describe the amount of dark matter in galaxies or clusters of galaxies and start modelling this.

Dark matter is also needed to explain the properties of the small fluctuations in the Cosmic Microwave Background [4]. Peaks in the power spectrum of the CMB represent the amount of matter in the universe at that time. Having only baryonic matter does not explain the power spectrum. The observed location and amplitude of the peaks in the power spectrum of the CMB can only be explained by including dark matter.

Lastly, there is the argument that the simulations of the process of structure formation of the universe that leads us to include dark matter in order to produce the observed structure of the universe [5].

Several theories try to explain these problems in physics. First, there is the adjustment of our laws of gravity, such as "modified Newtonian dynamics". Second, there is non-particle dark matter, such as "massive astrophysical compact halo objects". And lastly, there is particle dark matter. The first two approaches have problems explaining all four problems and are not backed up by observations. In this thesis the theory of particle dark matter is considered. The four problems mentioned earlier give us a solid ground to model a dark matter mass distribution in order to explain the observations.

When looking at this particle dark matter, we can quite quickly set constraints on the properties that such a particle should have. It should definitely be massive, since it interacts through gravity as we have seen in the above discussion. It should not interact through the strong nuclear force or the electromagnetic force. If it would have done so, other experiments should have already found them. Particularly we would have found dark matter if it interacted through the electromagnetic force, since it would not be "dark". In general it is also assumed that the particle does interact through the weak force. This does not necessarily have to be true, there are dark matter models that only interact through gravity. In this thesis we want to study dark matter that is bound inside a star. That would not be a possibility if the dark matter particle would only interact through gravity, because then a dark matter particle that accelerates towards a star will simply pass through the star without being captured. So for this reason, weakly interacting dark matter models are used in this thesis. The last property that dark matter should have is that it is a stable particle. Dark matter is still around today, it should have played a role in the formation of the galaxies and it should have been here ever since the beginning of the universe.

The most logical thing to do is look in our standard model to find a particle with such properties. There is a candidate in our standard model in terms of the stated properties: the neutrino. It has mass, it is weakly interacting and it is stable. Nonetheless, this particle is not a good candidate for dark matter, because the mass that it has is very tiny. The small rest mass of neutrinos (estimated to bin in the range of $\sim$ eV) means that neutrinos are relativistic during most of the history of the Universe. This would make clustering and the structure formation impossible to do. Thus we are looking for a new particle. It is commonly called a Weakly Interacting Massive Particle (WIMP).

Several experiments are trying to find dark matter, so far unsuccessfully in finding hard evidence, but there are tantalising experimental hints that give good reasons to believe in it, such as the Galactic center excess [6]. Creation and scattering of WIMPs are two direct detection methods. Creation tries to produce two WIMPs by colliding standard model particles. If you can determine a difference in the total energy of the incoming and outgoing particles in the interaction, it would most likely represent a weakly interacting particle that escapes detection and could potentially be a WIMP. These kind of experiments are for example done at the LHC [7]. Scattering experiments try to get dark matter to interact with ordinary particles from our standard model, such as nuclei. If liquid xenon is used as detection material, the nuclear recoils caused by the WIMPs produce photons and electrons. LUX does this for example [8]. But as stated, we are dealing with weakly interacting particles. Therefore indirect detection through dark matter annihilations should also be a useful way of getting a lot more constraints on the particle's identity. A general annihilation process of two WIMPs is shown in figure 2.

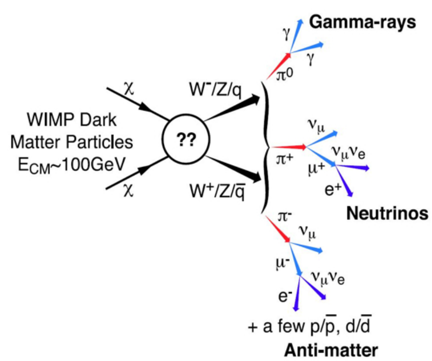

Figure 2: Schematic annihilation of two WIMPs.
Source: *https://fermi.gsfc.nasa.gov/science/eteu/dm/*

Figure 2 shows WIMP annihilation to known particles from our standard model of particle physics. When we look at an astronomical scale there are a couple of particles that would be able to reach our detectors on Earth or near-Earth satellites e.g. gamma-rays, neutrinos and matter/anti-matter such as protons/antiprotons. Annihilation becomes relevant when we look at dark matter rich regions. Dark matter rich regions would be high-mass systems with a relatively large mass density, such as the Galactic center and the Sun. Based on physical models of such a massive system that predict the particle flux, we can try to find an excess in one of these particle fluxes and see if it fits the postulated dark matter distribution.

In this thesis I will focus on the WIMP annihilation to neutrinos and in particular from dark matter in the Sun. This is by itself a hard task, because we are trying to find The Monster of Loch Ness with ghosts. The Sun is one of the nearest dark matter rich regions and we know a lot about the sun compared to other dark matter rich regions. I will talk about the dark matter physics in the sun and different neutrino detectors and their use in the search for dark matter. I use and verify DarkSUSY in order to simulate the dark matter distribution in the Sun and find expected dark-matter-induced-neutrino rates on Earth for a given annihilation model of the WIMPs.

2 Theory

2.1 Dark matter

There are theoretical models that contain dark matter candidates. Supersymmetric models are most popular in this context. In this thesis and the programs used, the dark matter model implemented is the Minimal Supersymmetric Standard Model (MSSM). Supersymmetry is a theory where every particle from our standard model has a supersymmetric partner. This supersymmetric partner is a heavier particle and its spin differs by 1/2, i.e. each fermion from our standard model has a heavy bosonic supersymmetric partner and each boson from our standard model has a heavy fermionic supersymmetric partner. One of the best candidates in this model for dark matter is the lightest neutralino. This is a linear combination of supersymmetric partners of the photon, the Z boson and neutral scalar Higgs particles [9]. This particle is heavy, neutral, stable and weakly interacting and thus this would be a good candidate for the WIMP. In this model the neutralino, denoted by $\widetilde{\chi}$, is its own antiparticle, because it has no charge. So from this we can get an annihilation process as shown in figure 2. Possible annihilation channels used in this thesis are $\widetilde{\chi}\widetilde{\chi} \longrightarrow W^+W^-$ and $\widetilde{\chi}\widetilde{\chi} \longrightarrow t\bar{t}$, where W^+ and W^- are the standard model W gauge bosons and t and $\bar{t}$ are the top-and antitop quarks. The reason why I used these models is because the W gauge boson model has not been excluded by other research and fits the tantalising Galactic center excess [10]. The top quark model is used as a reference to look at a different kind of muon neutrino energy spectrum. The W^+, W^-, t and $\bar{t}$ will decay further through different kinds of channels and from this we expect to have neutrinos as decay products, e.g. through $W^- \longrightarrow e^-\bar{\nu}_e$ (which could of course happen for all leptons). Masses of the neutralinos are then appropriately set for the annihilation process to be kinematically possible.

2.2 Capturing and annihilating neutralinos in the Sun

Since we are trying to look at signs of dark matter in the Sun, it seems relevant how the dark matter is actually captured and how it is annihilated. WIMPs have two possible interactions, gravity and weak force interactions. Gravity will cause the WIMP to be attracted towards the Sun. Since these WIMPs are heavy, they are non-relativistic, i.e. their speed is low with respect to the speed of light. Our solar system orbits the center of our galaxy, moving through a cloud of dark matter (at average) at rest with respect to the Galactic center. We may assume that dark matter particles have an isotropic velocity distribution (thermal motion) while the stars and gas inside the Galaxy moves on circular orbits around the Galactic center in the plane of the Galaxy. The average speed of the solar system orbiting around the Galactic center is $250 kms^{-1}$. The weak force will allow some WIMPs to be captured inside the Sun. Having a scattering interaction with matter in the Sun transfers momentum and in this sense the velocity of the WIMP can drop below the escape velocity of the Sun, i.e. $v_{WIMP} < v_{esc}$. There are a couple of cross-sections that are important for the capture of WIMPs in the Sun: the spin (in)dependent proton/neutron scattering cross-sections σ^p_{SD}, σ^p_{SI}, σ^n_{SD} and σ^n_{SI}. The Sun has a lot more protons than neutrons, therefore the neutron capture rate is negligible compared to the proton capture rate. Furthermore, σ^p_{SI} is by default $\sim 10^5$ times smaller than σ^p_{SD}, so spin-independent scattering may be neglected when considering the total capturing inside the Sun. Leading for the Sun is σ^p_{SD}.

DarkSUSY distinguishes spin independent and spin dependent capture rates [11, 12]. It uses a reasonably good approximation for the spin dependent capture rate which is relatively easy to compute:

$$\frac{C_{\odot,SD}}{(1.3*10^{23}s^{-1})(270kms^{-1}/\overline{v})} = \left(\frac{\rho_\chi}{0.3GeVcm^{-3}}\right)\left(\frac{100GeV}{m_\chi}\right)\left(\frac{\sigma^{p\chi}_{SD}}{10^{-40}cm^2}\right) \tag{1}$$

Here ρ_χ is the local dark matter mass density, $\overline{v}$ is the dark matter velocity dispersion and m_χ is the neutralino mass. The spin independent capture rate can also contribute significantly in less proton dominated massive systems, such as planets, but is slightly more complicated. It is given by:

$$\begin{aligned}\frac{C_{\odot,SI}}{(4.8*10^{22}s^{-1})(270kms^{-1}/\overline{v})} = \left(\frac{\rho_\chi}{0.3GeVcm^{-3}}\right)\left(\frac{100GeV}{m_\chi}\right)* \\ \sum_A \left(\frac{\sigma^{A\chi}_{SI}}{10^{-40}cm^2}\right) F_A(m_\chi) f_A \phi_A S(m_\chi/m_A)/m_A\end{aligned} \tag{2}$$

Here A indicates an element. F_A is the form factor. f_A is the mass fraction of that element. ϕ_A describes the distribution of element A in the system. The mass of the element is denoted by m_A. S is a kinematic suppression factor

[12, 13]. DarkSUSY uses models for the composition of Earth and the Sun to calculate these capture rates. For Earth the capture rate is dominated by spin independent interactions. This needs a more detailed analysis. Gould's approximation can then be used [14]. In this thesis I will focus on the Sun.

So the number of WIMPs in the Sun will increase due to capture and the number of WIMPs in the Sun will decrease due to annihilation. Other factors such as evaporation can be ignored compared to these two processes [15]. So then we can write a differential equation for the number of WIMPs in the Sun:

$$\frac{dN}{dt} = C_{\odot} - C_A N^2 \tag{3}$$

Keep in mind that C_A is not an annihilation rate, but more like an annihilation efficiency, since the annihilation rate depends on the number of WIMPs in the Sun. The annihilation rate is given by the relation:

$$\Gamma_A = \frac{1}{2} C_A N^2 \tag{4}$$

The factor $\frac{1}{2}$ follows from the fact that an annihilation needs two WIMPs to take place. The differential equation has an exact solution for N(t):

$$N(t) = \sqrt{\frac{C_{\odot}}{C_A}} tanh\Big(\sqrt{C_{\odot} C_A} t\Big) \tag{5}$$

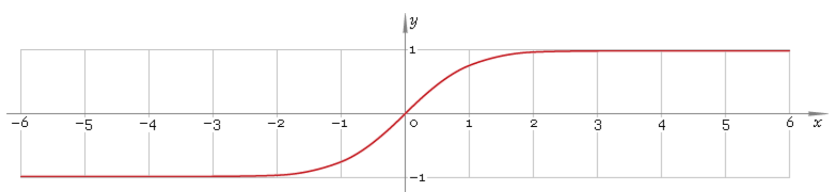

Figure 3: y=tanh(x)

In figure 3 can be seen that N(t) saturates towards a constant as t gets large, or to be more specific when:

$$t > \frac{1}{\sqrt{C_{\odot} C_A}} = \tau \tag{6}$$

We might as well call that the saturation time τ.

When t gets bigger than τ, an equilibrium in the number of WIMPs is about to be reached in the Sun and the capture rate and annihilation rate should be equal, except for the fact that for each annihilation two WIMPs need to be captured. Usually τ is less than a few billion years, which does not exceed our solar system's age and therefore it should be a valid assumption that our Sun has reached WIMP saturation. So, in general for every two WIMPs that are captured, two also annihilate. Therefore it is not actually necessary to know the annihilation cross section. The smaller the annihilation cross section would get, the bigger τ would get. But in equilibrium we can relate the annihilation rate to the capture rate as $\Gamma_A = \frac{1}{2} C_\odot$

The saturation time τ depends strongly on the parameters of the system, such as the size of the system, the local density of WIMPs the system orbits in, the capture cross-sections and the annihilation cross-sections. Several experiments constrain these parameters. These constraints strongly depend on the dark matter model that is used and therefore these models yield very different parameter values. However, τ can be modelled and be taken into account. Especially when looking at smaller bodies, such as Earth, this has to be done, because smaller bodies tend to have a much larger value for τ. But in most studies the equilibrium assumption is used for large systems.

2.3 Neutrino physics

Now that we have established the groundwork for dark matter to cluster and annihilate, it's time to figure out how we can measure anything. As far as this thesis goes, the neutrinos coming out of WIMP annihilations are interesting. Other products, such as photons and charged particles are not suited, because the Sun is totally opaque to them.

As seen in figure 2, all kinds of neutrinos are products of the annihilation: ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$, ν_τ and $\bar{\nu}_\tau$. Since they are coming from the Sun, neutrino oscillations could be of importance. Neutrinos are weakly interacting and very light. This is a good and a bad thing. Once created, neutrinos do not interact inside the Sun. Therefore they move in straight lines and their origin can be located once the direction of flight is determined in a detector. Of course this also happens when a neutrino goes through our detector. So our neutrino count will be low compared to the total number of neutrinos traversing it.

As said neutrinos have very little interactions. There are two interactions: charged current interactions and neutral current interactions. Charged current interactions are for example $\bar{\nu}_e + p^+ \longrightarrow n + e^+$ and $\nu_\mu + n \longrightarrow p^+ + \mu^-$. Both of these use the exchange of W^+/W^- as shown in figure 4. This process of course goes for every neutrino flavor. The charged lepton can then radiate Cerenkov light. The other kind of interaction is when a neutrino scatters with another particle and transfers momentum to it using the exchange of a Z^0 particle. This interaction is irrelevant for Cerenkov light detectors and therefore irrelevant for this thesis.

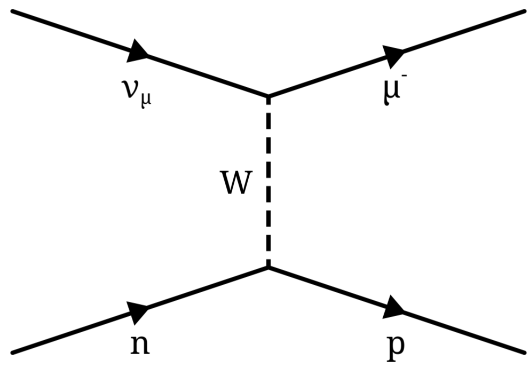

Figure 4: Charged current interaction where μ can be detected by Cerenkov radiation. One down quark of the neutron changes to a up quark to go out as a proton. **Source:** *https://physics.stackexchange.com/questions/145358/neutrino-neutron-interaction-feynman-diagram-w-boson-direction*

With charged current interactions we need a certain minimum energy of the neutrino, because the lepton rest mass is higher than the mass of the neutrino. This is a prerequisite that is fulfilled in our set-up, since we are dealing with neutrinos from for example W^+ and W^- decays. Such neutrinos will most likely have energies in the order of GeVs. Charged current interactions also leave information behind of the flavor of the neutrino, as the measured lepton will always have the same flavor as its partner neutrino.

Neutrinos can be detected by the use of Cerenkov light through charged current interactions. Cerenkov radiation is emitted when a charged particle in a medium has a velocity greater than the phase velocity of light in that medium. This cannot happen in vacuum because then the particle would exceed the speed of light in vacuum which is forbidden [16]. For example, in water the speed of the propagation of light is $0.75c$ and particles can have greater speeds than this in water. The resulting effect is comparable with breaking the sound barrier: a kind of "shock wave" is created. The physical properties are of course totally different. When a charged particle travels through a medium it disturbs the electrodynamic potential locally. The medium fails to respond fast enough and this leaves a cone of radiation light.

The energy of the neutrinos can be estimated. For detecting purposes, detectors mainly detect muons (and therefore muon neutrinos) because their tracks are the clearest/sharpest. The estimation of the energy is based on the amount of Cerenkov light output by a muon track in the vicinity of the detector.

2.4 Neutrino sources

If we measure a neutrino we will not be able to tell by itself what the origin of the neutrino would be. This is a huge problem, because we measure so few neutrinos compared to the incoming flux. There are three major neutrino sources to take into account when looking for dark matter neutrinos: solar neutrinos, atmospheric neutrinos and background neutrinos. Background neutrinos are by itself not a big problem, because we can measure the background by looking in other directions than towards the Sun.

There are several kinds of neutrino detectors on which I will get back to later on, but an important feature of the detectors is that big neutrino detectors are sensitive to high energy neutrinos (with energies $\geq$ 10 GeV).

Solar neutrinos are mostly produced by fusion processes. These are neutrinos on MeV scales. These are not the neutrinos that we would measure with our detectors (or at least a negligible amount), so they do not have to be taken into account.

Cosmic rays cause atmospheric neutrinos. These are very much in the same energy range as dark matter neutrinos would be.

Dark matter neutrinos will primarily be in the energy range from $\sim$ 1 GeV up to the neutralino mass (which is unknown, but the Gev/TeV scale is most common). Atmospheric neutrinos are ranging from $\sim$ 1GeV up to several TeV energies. They are the very reason why we cannot find an excess yet in this class of energetic neutrinos. Furthermore, atmospheric neutrinos are very unpredictable, because a lot of them are being created in showers. All showers are different and there are numerous ways for the creation of a neutrino or a lepton partner of the neutrino in such a shower.

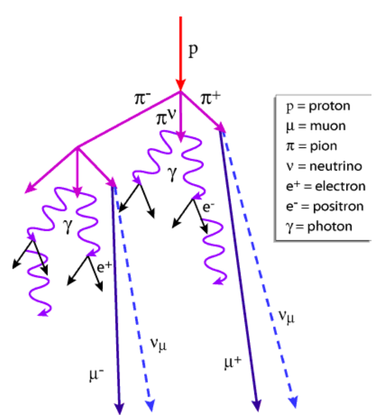

Figure 5: Cosmic ray air shower producing multiple neutrinos. **Source:***https://www.hawc-observatory.org/science/cosmicrays.php*

In figure 5 can be seen that in a cosmic-ray induced air shower multiple (charged) leptons and neutrinos are created. These can both be detected in our neutrino detector through Cerenkov radiation. As mentioned Cerenkov radiation is pointing, i.e. we know where the detected particle came from. Charged particles, in particular muons, have the possibility to arrive at our detectors, even though our detectors are placed somewhere deep in a sea/under the ground. But if we use Earth as a filter, then charged particles from the other side of Earth will not arrive at our detector. This helps us to get rid of downward going charged particles from cosmic ray air showers, but this does not get rid of the atmospheric neutrinos. In fact, this remains a problem. Atmospheric neutrinos and dark matter neutrinos are by itself similar, since the energies are in the same region. But if we would for example look at the energy spectrum of these neutrinos we will not expect them to look exactly the same. So in that sense they could in principle be distinguished.

2.5 Neutrino telescopes

There are a lot of neutrino telescopes. A rough rule is that the bigger the telescope, the more sensitive it is to high-energy neutrinos. There tend to be more low-energy neutrinos than high-energy neutrinos. A small telescope measures muons with all energies as long as it travels through the detector. Therefore the number of measured high-energy neutrinos is fairly small. Big detectors are not sensitive to low-energy neutrinos and will measure high-energy neutrinos. Therefore we need a bigger detector to increase the expected amount of high-energy neutrinos measured. High-energy neutrinos emit more Cerenkov radiation and therefore can be detected when the particle is further away from an optical module than low-energy neutrinos. The density of optical modules is in big detectors (often) a lot lower than in small detectors due to financial reasons. So what we need in order to find dark matter neutrinos is a large detector with a medium optical module density to see all neutrinos above a certain cut-off energy. Super-Kamiokande (which measures neutrinos of energy 1 GeV or higher) for example is too small to detect dark matter neutrinos. I will discuss the leading detectors for dark matter searches. Most large detectors do have a typical energy threshold of the order of tens of GeV. So they are mainly sensitive to neutralinos of 100 GeV or more.

2.5.1 IceCube

We are looking for high-energy neutrinos and only few telescopes measure a significant amount of high-energy neutrinos. IceCube is the leading detector at the moment in this field of research [17]. It is located at the south pole and is an "ice cube" of one cubic kilometer. The detector has been designed to measure high energy neutrinos. It is a successor of the telescope AMANDA (Antarctic Muon And Neutrino Detector Array) which was at the same location but it was a smaller design. The detector basically measures neutrinos in energies from $\sim$ 30GeV up to the TeV scale.

As we want to use Earth as a filter against atmospheric muons, the south pole limits the detector in the regions where it can look. The Sun will be down during a whole season, therefore the dark matter research will be seasonal. In this research area IceCube's field of view will change on a seasonal basis.

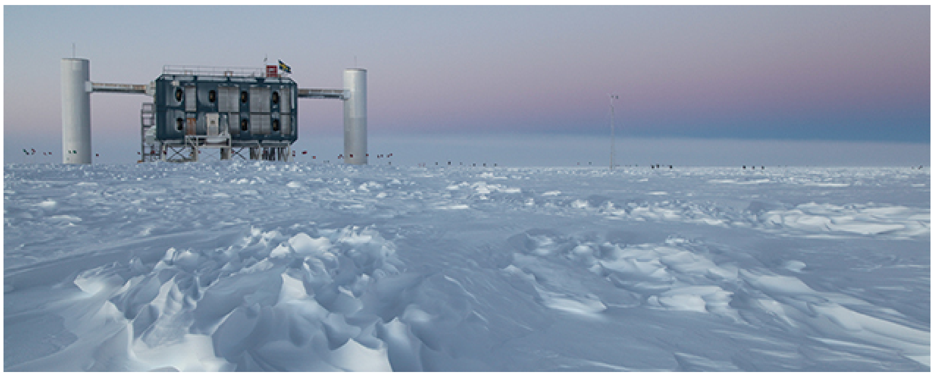

Figure 6: The IceCube laboratory at the south pole, Antarctica. **Source:**_http://icecube.wisc.edu/about/overview_

IceCube is set-up in ice between about 1450 and 2450 meter below the surface of the ice sheet. This, as you might be able to imagine, is a dark place. No daylight gets anywhere near this place. The detector is frozen in optically clear ice. This is a good thing for our detector, because our detector needs to be really sensitive to small amounts of photons from the emitted Cerenkov radiation. In a square kilometer there are 86 cables (strings) set-up from 1450 meters to 2450 meters below the surface. On each of these cables 60 Digital Optical Modules (DOMs) are mounted. The DOMs contain highly sensitive light detectors such as photomultiplier tubes and from these light detectors data is transmitted towards the surface of the south pole.

IceCube is composed of three distinct components. The part described above is for ultra high energy neutrinos. Then we have IceTop which is used for particle showers and is set-up at the top surface of the south pole with 162 icetanks to have a partial veto against the down-going background of muons created by cosmic-ray interactions in the atmosphere above IceCube. Both components are not really sensitive for finding dark matter neutrinos. DeepCore was built to lower the energy threshold to 10 GeV [18]. Its location in IceCube is around 2100 meter below the surface. At this level the ice is exceptionally clear. Also, because we have DOMs above DeepCore, the downward-going muons produced in cosmic-ray air showers can be filtered out of the data. DeepCore has a DOM density that is about 5 times higher than the rest of IceCube. Also it has enhanced photomultiplier tubes with 35% higher efficiency compared to the rest. All this makes that DeepCore increases the sensitivity to neutrinos from dark matter annihilations and atmospheric neutrinos.

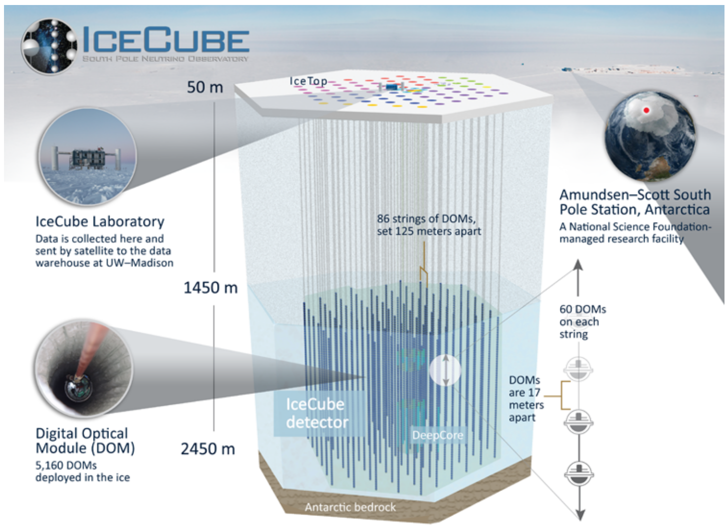

Figure 7: The IceCube detector.
Source:*http://icecube.wisc.edu/science/icecube/detector*

2.5.2 ANTARES

ANTARES (Astronomy with a Neutrino Telescope and Abyss environmental RESearch project) is a neutrino telescope located in the Mediterranean sea that is also used for the measurements of high energy neutrinos [19]. Since it is on the orbiting part of Earth, its field of view will change on a daily basis in contrast to IceCube.

ANTARES is smaller than IceCube. Its size is 200x300x350 meter. So its volume is about 50 times smaller, which makes the number of measured neutrinos also basically this much smaller. ANTARES is set-up at a depth of 2150-2500 meters. 12 lines are attached to the bottom of the sea and held straight by a buoy at the top. Of course, due to this set-up the lines will vibrate a bit. The position is therefore constantly monitored by an acoustic calibration system. The lines are separated horizontally by 65 meters. ANTARES does not use DOMs, it uses so called storeys which are comprised of 3 optical modules.

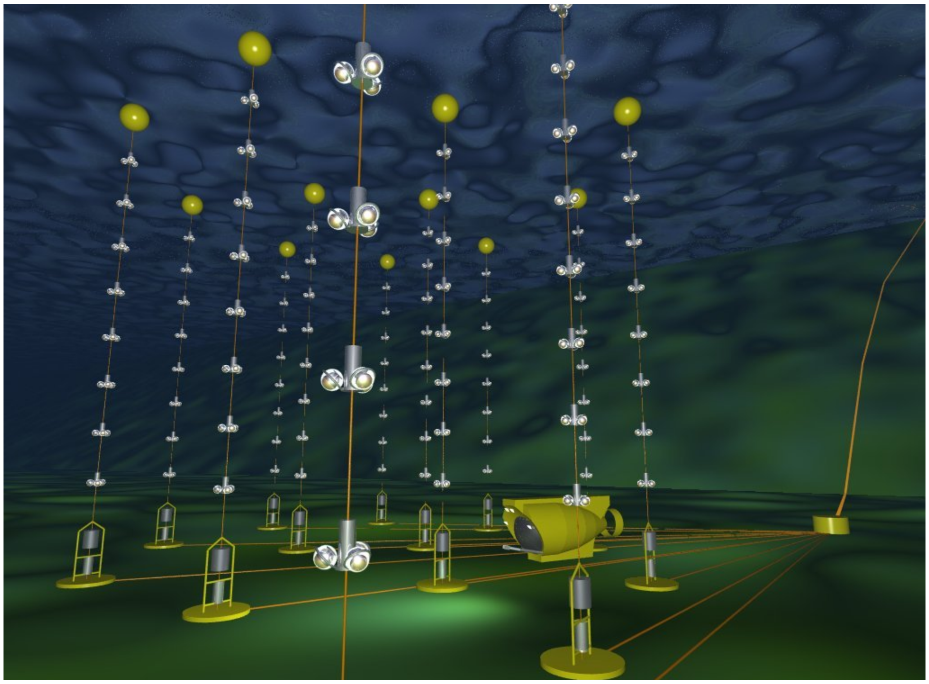

Figure 8: The ANTARES detector.
Source:$https://en.wikipedia.org/wiki/ANTARES\_(telescope)$

Let's have a look at the important differences with IceCube. The fact that ANTARES has been built in water and uses this as a medium for Cerenkov radiation has pros and cons. Due to the flow of the water more measurements must be done in order to be sure of the track of a muon, because we also need to know exactly what the position of the line/storey is. On the other hand, when an optical module is broken, this would be a way easier fix than in IceCube. ANTARES does however also have to deal with luminescent animals. The reason why ANTARES is a lot less suited in the search for dark matter is its size. The detector volume is quite small. In order to find a significant excess we need a lot of data. Detecting 11 neutrinos where you would expect 10 neutrinos could easily be a coincidence. Detecting 1100 neutrinos where you would expect 1000 neutrinos can barely be a coincidence. Using Poisson statistics, a noise of $\sqrt{N}$ is expected. For 11 and 1100 detected neutrinos this would mean $\sim$ 3,32 resp. $\sim$ 33,2 neutrinos. For 1100 neutrinos this would be a significant excess, but for 11 neutrinos this does not hold. The estimation of ANTARES is that it would find 3000 upward going neutrinos per year. In comparison, IceCube detects about 100.000 atmospheric neutrinos per year. Therefore IceCube outperforms ANTARES by a lot in terms of finding dark matter neutrinos.

2.5.3 KM3NeT

KM3NeT is a project under development [20]. It is to ANTARES as what IceCube is to AMANDA. Its base design is ready, but not definite. Therefore it could be really interesting to simulate how certain DOM set-ups would increase the efficiency for dark matter searches. KM3NeT stands for cubic kilometer Neutrino Telescope. Similar to ANTARES, KM3NeT will be in the Mediterranean sea. KM3NeT claims that they will be complementary to IceCube in its field of view and furthermore that it would exceed IceCube substantially in sensitivity.

KM3NeT will be composed of two parts: ARCA (Astroparticle Research with Cosmics in the Abyss) and ORCA (Oscillations Research with Cosmics in the Abyss). I will not say a lot about ARCA since its focus will lie on astrophysical neutrinos of energies higher than 100 GeV. ORCA will be of more importance [21]. This will be located also near the French coast in the Mediterranean sea like ANTARES. It will be focused on resolving the neutrino mass hierarchy problem and it will be sensitive to neutrinos from 1 to 100 GeV. Therefore this detector would also be qualified for dark matter searches. The density of DOMs could really influence the energy resolution and therefore be relevant for dark matter searches.

So far, 3 strings in total are deployed for ORCA. ORCA will consist of a single block with 115 vertical detection units with 18 DOMs on each with 31 photomultiplier tubes per DOM. Its size is quite small (height of 150m and radius of 110m), but this makes the density of DOMs relatively high and lowers the energy threshold to 1 GeV, which is significantly lower than IceCube.

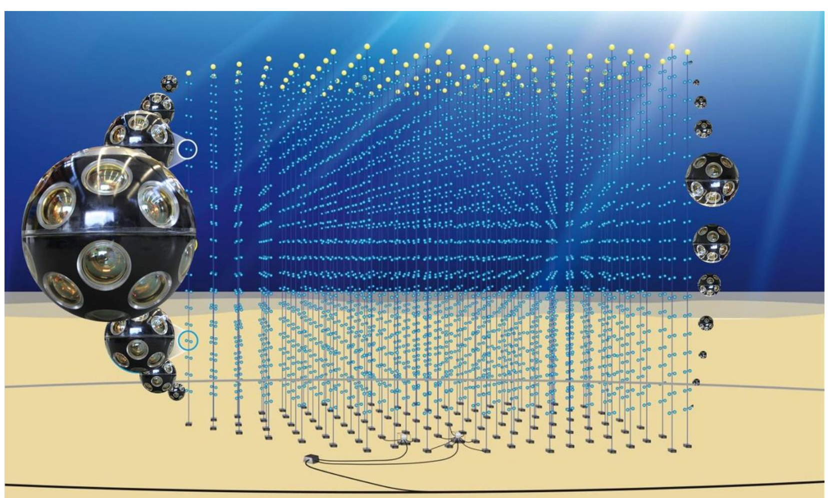

Figure 9: The KM3NeT detector.
Source:$https://en.wikipedia.org/wiki/KM3NeT$

3 Research

DarkSUSY

DarkSUSY is a publicly available Fortran 95 simulation programme for neutralino dark matter calculations [9, 22, 23, 24]. Fortran 95 has its benefits because it is quick with hard calculations. In terms of using this for a bachelor thesis, this is hard in my opinion, especially if it is your first time in a different programming language and working with someone else's programme. It takes some time to get used to things and the programme consists of a lot of individual components. But a lot of it is quite clear, the comments cover the most essential topics to get you started.

DarkSUSY can calculate a lot for a given dark matter model, such as the neutralino density in the Universe nowadays. Most importantly is the claim that it can compute the indirect detection signal of neutrinos from neutralino annihilations from the center of Earth or from the Sun. Little is known about what a realistic supersymmetric model would look like in its detail. Therefore DarkSUSY uses the MSSM with some additional simplifying assumptions. The MSSM is a good model to test ideas about detection and it has features which are expected to be universal for any supersymmetric dark matter model.

Implementations of DarkSUSY

As said before, neutrino telescopes usually work with a minimum energy detection range. DarkSUSY gives a function where this lower boundary can be set. It gives as output the muon neutrino flux produced by dark matter annihilations in units $yr^{-1}km^{-2}$. Furthermore DarkSUSY claims to have implemented the annihilation rate from formula 4 with the number of particles from formula 5, resulting in [9]:

$$\Gamma_A = \frac{C_\odot}{2} tanh^2\left(\frac{t}{\tau}\right) \tag{7}$$

So DarkSUSY computes τ. The Sun's and Earth's composition is also included in order to calculate the capture rate. The cross-sections for interactions between neutralinos and matter in the Sun are hard to calculate, because they are heavily dependent on the neutralino mass and form factors. Therefore these are parameters that can be tweaked to set limits on the expected neutrino flux.

My research

My research was quite general and it was to start highlighting the importance of neutrino research in relation to dark matter within the dark matter research group at the Radboud University.

My main task was to figure out whether DarkSUSY is a valid tool for predicting detection signals of neutrinos from dark matter annihilation in the Sun.

Let me start by saying that outside of the main file, DarkSUSY has a lot of small components and therefore has a lot of "black boxes". Making adjustments is therefore not an easy thing to do. The first thing to do for me is to check the validity of DarkSUSY. The behaviour of the parameters $< \sigma v >$ (velocity-weighted annihilation cross-section) and σ^p_{SD} is examined in the context of formula 7. The validity check is continued by checking the results for ANTARES [25]. In this context I will try to reproduce the muon neutrino flux limit in figure 10, starting from the σ^p_{SD} limit in figure 11. If this can be done by DarkSUSY, DarkSUSY can also be used for the production of a cross-section limit plot for any other neutrino telescope. Lastly, the energy spectrum of the neutrinos should be really interesting to see, because neutrino telescopes work above a certain energy threshold. Also, detectors are trying to improve on classifying the energy of a neutrino.

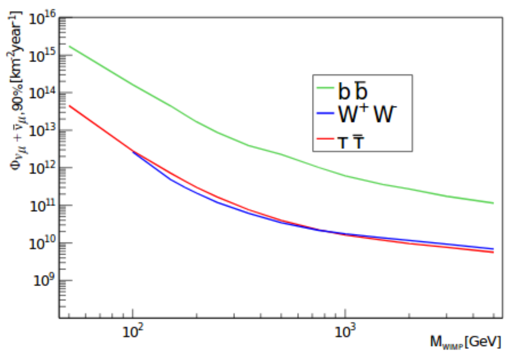

Figure 10: ANTARES limits on m_χ vs the muon neutrino flux [25].

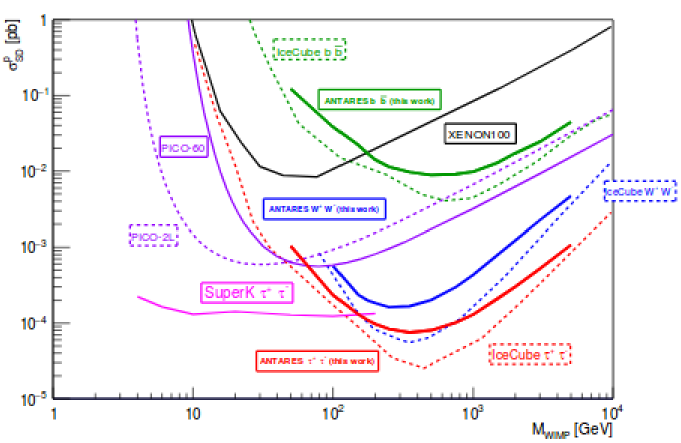

Figure 11: ANTARES limits on m_χ vs σ^p_{SD} [25].

4 Results

When testing DarkSUSY, I got NaN (not a number) values for the neutrino flux. To get rid of this problem I had to change a file in DarkSUSY and the how-to is shown in appendix A. After configuring DarkSUSY again, I got rid of the problem. If nothing is said about what model is used, then the W^+W^- model is used. DarkSUSY does not give me error bars, because it performs straight up calculations given a dark matter model. But since most parts are qualitative results to understand what is going on, error bars are less relevant.

4.1 Equilibrium check: cross-section dependence

As described in the theory, there should be an equilibrium time for a given dark matter rich region depending on the capture rate and the annihilation efficiency. DarkSUSY claims that they have implemented this in their programme. The way to check this is to see how muon neutrino fluxes change whilst changing relevant parameters, $< \sigma v >$ and the spin (in)dependent proton/neutron cross-sections. For convenience, all spin (in)dependent proton/neutron cross-sections are set to 0, except for σ_{SD}^p. This is so we can just look at the influence of one parameter. The term σ_{SD}^p is chosen because it is the dominant term in the capture rate and the effect should be the easiest to observe.

The parameters to be adjusted can be found in the root files dssigmav.f and dsddneunuc.f. Unfortunately DarkSUSY is not easy to use in this sense, because just changing these values does not change the main file programme. So DarkSUSY has to be configured when changing root files in order to change the main file. I did not find an easier way than this and I put in the values manually which is inefficient and time consuming.

First I looked at the behaviour of $< \sigma v >$ by changing it from very small up to very large values with fixed σ_{SD}^p (and the other cross-sections taken to be zero). The standard value for $< \sigma v >$ in this W^+W^- model is $7.69 * 10^{-27} cm^3 s^{-1}$. In figure 12 is a logarithmic plot shown of $< \sigma v >$ versus the muon neutrino flux.

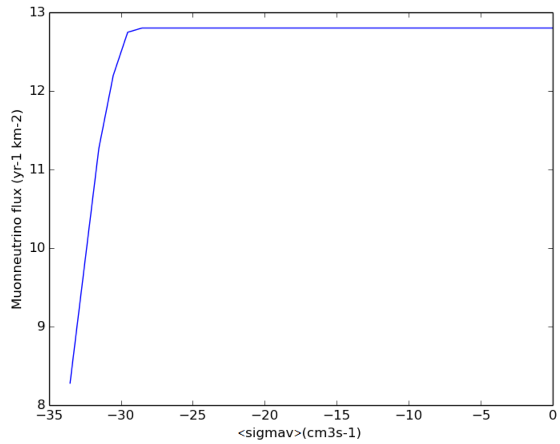

Figure 12: Logarithmic equilibrium check #1 for the annihilation rate $< \sigma v >$ versus the neutrino flux with fixed capture cross-sections $\sigma^p_{SD} = 6.0 * 10^{-4} pb$.

This plot shows us some nice expected features. For increasing $< \sigma v >$ there seems to be a critical value where the muon neutrino flux from the Sun does not increase anymore. This can be explained with formula 6. As C_A increases, the equilibrium time decreases. So around the equilibrium point we expect τ to be the same as the age of our solar system. This gives us reason to believe the claim of DarkSUSY that the equilibrium time is implemented. To further investigate this claim, σ^p_{SD} was set as a parameter and it was observed what the equilibrium tip-over point (ETOP) is. The ETOP is the $< \sigma v >$ value where the generated output is 95% of the saturation limit. I calculated the ETOP for several σ^p_{SD}. For doubling σ^p_{SD} the ETOP is halved.

If we again take a look at formula 6 then we observe the following: for increasing $C_\odot$ by a factor, C_A has to be lowered by the same factor in order to maintain the same τ. In other words $C_\odot * C_A$ needs to stay constant in order to maintain the same τ, which my calculations confirm. Thus the claim that DarkSUSY has formula 7 implemented seems confirmed.

4.2 Reproducing limit plots

To understand the limit plots that ANTARES made (figure 11), I tried to reproduce such a m_χ vs muon neutrino flux plot out of a σ^p_{SD} vs m_χ plot with DarkSUSY. The quantities m_χ and σ^p_{SD} can be adjusted in the root files dssigmav.f and dsddneunuc.f, such that they are on a point of the blue solid line in figure 11. Doing this for several points gives figure 13.

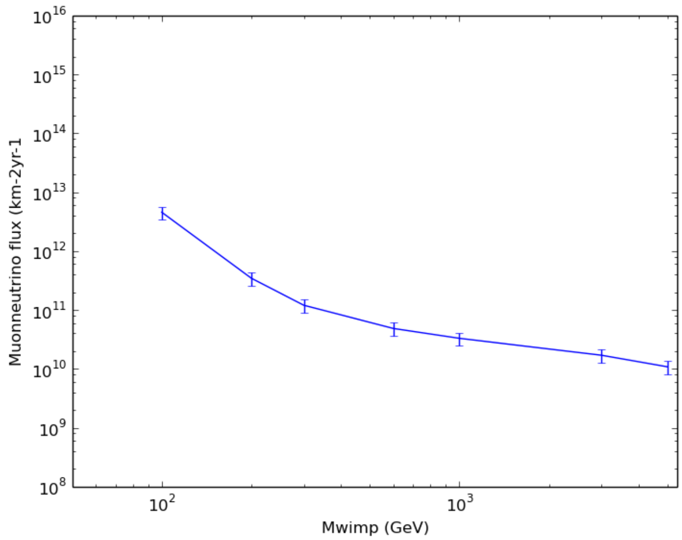

Figure 13: Reproduced limit plot for the W^+W^- dark matter model using figure 11.

In this plot error bars are based on the visual errors made by extracting the parameters by eye from figure 11. This plot reproduces within error bars the W^+W^- curve in figure 10. Generally speaking, this shows that DarkSUSY is a representative dark matter computation programme for neutrino calculations and that these plots can be made for any neutrino telescope as long as you have either of the limits.

4.3 Energy spectrum

DarkSUSY has two functions for neutrino fluxes: an integrated flux above an energy threshold (see appendix B) and an energy differential function for the flux per GeV (see appendix C). This seems a proper function to get the energy spectrum, but it only gave fluxes of value 0. Therefore I used the energy threshold function and I iterated over the energies in order to get the spectrum. The function gives an energy cut-off from below, so we can iterate from above to get a spectrum (this is shown in appendix D). I did this while keeping in mind that the function is not necessarily built to make a spectrum, but only to give an integrated value. I could not find information about the validity of trying this, but the uncertainties will most likely increase from this. The energy spectrum for the W^+W^- model is shown in figure 14 and the one for $t\bar{t}$ is shown in figure 15. Note that I manually set the parameters σ^p_{SD} equal in both models and made sure that $< \sigma v >$ is such that there is an equilibrium in the number of WIMPs in the Sun. This is done because default set-up of the relevant parameters, which my specific models used, are diverse. In this way we get similar integrated neutrino fluxes. Now both models are quantitatively comparable to one another. Most importantly, the shape of the energy spectra is relevant.

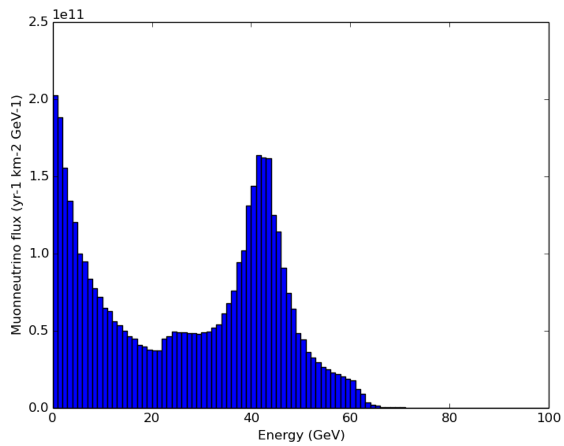

Figure 14: Energy spectrum of the ν_μ-flux from the Sun produced by the W^+W^- dark matter annihilation model for constant σ_{SD}^p and $< \sigma v >$ set such that an equilibrium in the number of WIMPs has been reached in the Sun.

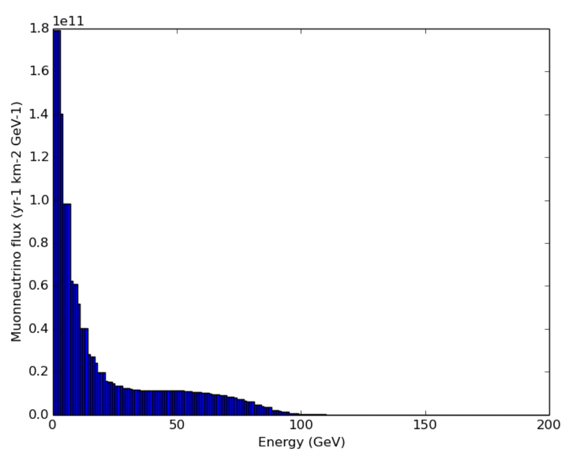

Figure 15: Energy spectrum of the ν_μ-flux from the Sun produced by the $t\bar{t}$ dark matter annihilation model for constant σ_{SD}^p and $< \sigma v >$ set such that an equilibrium in the number of WIMPs has been reached in the Sun.

These plots need some discussion. Let's first discuss the energy spectrum from the W^+W^- model. The first thing to note is that this graph is obviously composed of two individual parts. It peaks at low energies and has a long tail for higher energies up to energies of $\sim$ 85GeV. Note that high energy neutrinos ($\sim$ 80GeV) cannot be seen in the figure due to their small fluxes compared to the rest of the figure. The other part looks more like a Gaussian on top of the first part, peaking at $\sim$ 42GeV. There is some physical sense in this energy spectrum when we look at how a W^- (or W^+) decays into a neutrino: $W^- \longrightarrow \mu^- + \bar{\nu_\mu}$. The W^- particle has an energy that is almost equal to the mass of the neutralino, which is taken to be $\sim$ 85GeV. So, the W^- and W^+ are produced with a small amount of kinetic energy. The neutrino can get any energy from its own mass up to the neutralino energy minus the muon rest mass, because the mass of the neutrino and its lepton partner are negligible compared to the mass of the mass of the W. However, in the considered W^- (or W^+) decay the distribution peaks at equal muon and neutrino energies, each carrying half the W^- (or W^+) energy. The peak is broadened by the width and velocity of the W bosons. This could well explain the Gaussian. The first part can be explained by longer W^+W^- decay chains. We of course have a lot of decay channels as shown in figure 2. The more intermediate particles you have before getting a neutrino, the lower the energy will be of this neutrino. This could explain the low-energy peaking behaviour with a long tail.

Now onto the $t\bar{t}$-induced energy spectrum. The neutralino mass is a lot higher in this model i.e. $\sim$ 175GeV. Also here the graph does not show this, but neutrinos are found up to energies of 175GeV. The $t\bar{t}$-decay is dominated by b-jets. Therefore the peak is mostly in the low-energy neutrino region. In these decays energetic W^+ and W^- are created as well. They give rise to a smeared energy distribution with a shallow bump at $\sim m_\chi/3$.

Let's take a look at the neutrino fluxes of both models compared to one another. The integrated flux of the W^+W^- model is $\sim$ twice as big as the integrated flux of the $t\bar{t}$ model. This may seem remarkable at first sight, because we expect more neutrinos to come out of a $t\bar{t}$ annihilation due to b-jets and the decay into W bosons. Also σ^p_{SD} is set equal and $< \sigma v >$ is set such that equilibrium has taken place. The answer can be found in equation 1. The local density ρ_χ is in both models the same. But m_χ differs by $\sim$ a factor 2. So the capture rate for the $t\bar{t}$ model is damped $\sim$ twice as much as for the W^+W^- model.

5 Concluding remarks

The dark matter neutrino search field is quite an active field with several big projects going on. As time passes the detectors accumulate an increasing amount of data, becoming (in effect) more sensitive to dark matter neutrinos or will set more stringent limits on several dark matter cross-sections. As far as KM3NeT concerns, the most important questions to answer are: on what time scale can it be competitive with IceCube in terms of dark matter searches, can it be complementary to IceCube, what changes can be made to the detector to improve the sensitivity to dark matter neutrinos and is it worth the money to actually do so. If limits can be simulated on the muon neutrino flux, my research has shown that DarkSUSY can be used to convert these to cross-section limits.

Good follow-up research topics are then in my opinion simulation studies of the incoming muon neutrino flux within different kind of detector set-ups, to make a rough estimate of how this data grows over time to see on what time basis it can outperform other detectors and how the sensitivity changes for different kind of set-ups. Probably the energy resolution will play one of the largest roles in dark matter neutrino searches, because dark matter has a distinct energy spectrum that is most likely different from the atmospheric neutrino spectrum.

The last thing to note is that dark matter does not necessarily have to be captured inside a star (with a lot of emitted radiation). It can be captured by planets. It might be useful to look at Earth or another planet, although this may be a hard thing to do. Moreover, the equilibrium time τ_{planet} will usually be a lot bigger the age (~ 5 Gyr) of the solar system. Therefore equilibrium between capture and annihilation rate has not occurred yet. This makes the estimates much harder, since $< \sigma v >$ becomes an active parameter again. Within DarkSUSY the computations for this can be done, because the annihilation rate is not simply assumed to be in equilibrium with the capture rate. The second problem with planets is that the number of annihilations per unit time is much smaller, because the body is much less dense and heavy than the Sun is. Therefore the neutrino flux coming from planets is a lot smaller as well and sensitivity could become a really important issue, because there will still be a large contribution from background sources. On the upside, we can look at a planet without a lot of noise. It does not emit a lot of neutrinos or cosmic rays by itself. Jupiter or Earth are then the first two planets of interest to look at, because they are either the heaviest or the closest. DarkSUSY has already implemented equations for the muon neutrino flux emanating from Earth.

References

[1] H.C. van de Hulst, E. Raimond, and H. van Woerden. "Rotation and density distribution of the Andromeda nebula derived from observations of the 21-cm line". In: *Bulletin of the Astronomical Institutes of the Netherlands* 14 (Nov. 1957). URL: http://adsabs.harvard.edu/abs/1957BAN....14....1V.

[2] V. C. Rubin, W.K. Ford, and N. Jr. Thonnard. "Rotational properties of 21 SC galaxies with a large range of luminosities and radii, from NGC 4605 /R = 4kpc/ to UGC 2885 /R = 122 kpc/". In: *Astrophysical Journal* 238 (June 1980), pp. 471–487. DOI: 10.1086/158003. URL: http://adsabs.harvard.edu/abs/1980ApJ...238..471R.

[3] Johan Richard Richard Massey Thomas Kitching. "The dark matter of gravitational lensing". In: *Rep. Prog. Phys.* 73 (Jan. 2010), p. 086901. DOI: 10.1088/0034-4885/73/8/086901. URL: arXiv:1001.1739.

[4] Martin Bucher et al. "Physics of the cosmic microwave background anisotropy". In: *Int. J. Mod. Phys. D* 24 (Jan. 2015), p. 1530004. DOI: 10.1142/S0218271815300049. URL: arXiv:1501.04288.

[5] J. Diemand and B. Moore. "The structure and evolution of cold dark matter halos". In: *Advanced Science Letters* 4 (June 2009), p. 497. DOI: 10.1166/asl.2011.1211. URL: arXiv:0906.4340.

[6] The Fermi-LAT Collaboration. "The Fermi Galactic Center GeV Excess and Implications for Dark Matter". In: *The Astrophysical Journal* 840 (Apr. 2017), p. 43. DOI: 10.3847/1538-4357/aa6cab. URL: arXiv:1704.03910.

[7] Patrick J. Fox et al. "Missing Energy Signatures of Dark Matter at the LHC". In: *Phys. Rev. D* 85 (Sept. 2011), p. 056011. DOI: 10.1103/PhysRevD.85.056011. URL: arXiv:1109.4398.

[8] The LUX collaboration. "First results from the LUX dark matter experiment at the Sanford Underground Research Facility". In: *Phys. Rev. Lett.* 112 (Oct. 2013), p. 091303. DOI: 10.1103/PhysRevLett.112.091303. URL: arXiv:1310.8214.

[9] P. Gondolo et al. "DarkSUSY: Computing Supersymmetric Dark Matter Properties Numerically". In: *JCAP* 0407 (June 2004), p. 008. DOI: 10.1088/1475-7516/2004/07/008. URL: arXiv:astro-ph/0406204.

[10] Abraham Achterberg et al. "A description of the Galactic Center excess in the Minimal Supersymmetric Standard Model". In: *JCAP* 1508 (Feb. 2015), p. 006. DOI: 10.1088/1475-7516/2015/08/006. URL: arXiv:1502.05703.

[11] P. Gondolo et al. *Manual and long description of routines*. DarkSUSY. http://www.darksusy, Feb. 2013.

[12] G. Jungman, M. Kamionkowski, and K. Griest. "Supersymmetric Dark Matter". In: *Phys.Rept.* 267 (Mar. 1995), pp. 195–377. DOI: 10.1016/0370-1573(95)00058-5. URL: arXiv:hep-ph/9506380.

[13] M. Kamionkowski. In: *Phys. Rev.* D44 (1991), p. 3021.

[14] Andrew Gould. "Resonant enhancements in weakly interacting massive particle capture by the earth". In: *The Astrophysical Journal* 321 (Sept. 1987), pp. 571–585.

[15] K. Griest and D. Seckel. "Cosmic asymmetry, neutrinos and the sun". In: *Nuclear Physics B* 296 (1987), pp. 1034–1036. DOI: 10.1016/0550-3213(88)90409-9. URL: arXiv:0903.2986.

[16] J. V. Jelley. "Cerenkov radiation and its application". In: *Br. J. Appl. Phys.* 6 (1955), p. 227. URL: http://iopscience.iop.org/article/10.1088/0508-3443/6/7/301/pdf.

[17] The IceCube Collaboration. *IceCube Preliminary Design Document.* IceCube. Oct. 2001. URL: https://icecube.wisc.edu/icecube/static/reports/IceCubeDesignDoc.pdf.

[18] The IceCube Collaboration. "The Design and Performance of IceCube DeepCore". In: *Astroparticle Physics* 35 (Sept. 2011), pp. 615–624. DOI: 10.1016/j.astropartphys.2012.01.004. URL: arXiv:1109.6096.

[19] The ANTARES Collaboration. *Technical Design Report of the ANTARES 0.1 km2 project.* ANTARES. July 2001. URL: http://antares.in2p3.fr/Publications/TDR/v1r0/Chap1_Introduction.pdf.

[20] The KM3NeT Collaboration. *KM3NeT: Technical Design Report for a Deep-Sea Research Infrastructure in the Mediterranean Sea Incorporating a Very Large Volume Neutrino Telescope.* KM3NeT. 2009. URL: http://inspirehep.net/record/1366114/files/TDRKM3NeT.pdf.

[21] ANTARES and KM3NeT Collaborations. *Highlights from ANTARES, and prospects for KM3NeT.* ANTARES and KM3NeT. 2016. URL: https://pos.sissa.it/236/024/pdf.

[22] Sven Heinemeyer et al. Feynhiggs. Mar. 2003. URL: http://www.feynhiggs.de/.

[23] P. Bechtle et al. HiggsBounds. Feb. 2009. URL: http://higgsbounds.hepforge.org/.

[24] the Max-Planck-Institute for Physics. *SLHALib - a library for SUSY Les Houches Accord I/O.* Aug. 2004. URL: http://www.feynarts.de/slha/.

[25] The ANTARES Collaboration. "Limits on Dark Matter Annihilation in the Sun using the ANTARES Neutrino Telescope". In: *Physics Letters B,* 759 (Aug. 2016), pp. 69–74. DOI: 10.1016/j.physletb.2016.05.019. URL: arXiv:1603.02228.

Appendices

A DarkSUSY correction

Edit the file dsfromslha in src/slha darksusy
and

```
c...JE FIX: Note: the following lines will mean that we instead of
c...using running Yukawas from DarkSUSY will use whatever Yukawas that
c...are specified in the SLHA file. In principle it is more consistent
c...to use the Yukawas from the file, but they might not be at the
c...scale we want them.
c rruiz do g1=1,3 since 0 and 1 are no given
      do g1=1,3
        call dssetfromslha(Ye_Yf(g1,g1),yukawa(kl(g1)),0,'')
        call dssetfromslha(Yu_Yf(g1,g1),yukawa(kqu(g1)),0,'')
        call dssetfromslha(Yd_Yf(g1,g1),yukawa(kqd(g1)),0,'')
      enddo
```

replace by

```
      do g1=3,3
        call dssetfromslha(Ye_Yf(g1,g1),yukawa(kl(g1)),0,'')
        call dssetfromslha(Yu_Yf(g1,g1),yukawa(kqu(g1)),0,'')
        call dssetfromslha(Yd_Yf(g1,g1),yukawa(kqd(g1)),0,'')
      enddo
```

The reason is that in the slha file we don't have Yukawas for
second and third generations. So the original leads to 0 entrances
which are dangerous.

B Main file code

```
 write(*,*) 'Calculating rates in neutrino telescopes'
 eth=1.0d0       ! energy threshold (of neutrino/muon), GeV
 thmax=30.0d0    ! the maximum half-aperture angle, degrees
 rtype=1         ! 1=neutrino flux
                 ! 2=neutrino to muon conversion rate
                 ! 3=muon flux
 ptype=3         ! 1=particles only
                 ! 2=anti-particles only
                 ! 3=sum of particle and anti-particle rates
 call dsntrates(eth,thmax,rtype,ptype,rateea,ratesu,istat)

 write(*,*) '  Flux from the Earth = ',rateea,
&  ' km^-2 yr^-1'
 write(*,*) '  Flux from the Sun =   ',ratesu,
&  ' km^-2 yr^-1'
```

C Differential rates code

```
energy=10.0d0  ! energy (of neutrino/muon), GeV
theta=30.0d0   ! angle from center of Earth/Sun, degrees
rtype=3        ! 1=neutrino flux
               ! 2=neutrino to muon conversion rate
               ! 3=muon flux
ptype=3        ! 1=particles only
               ! 2=anti-particles only
               ! 3=sum of particle and anti-particle rates
call dsntdiffrates(energy,theta,rtype,ptype,rateea,ratesu,istat)
```

D Energy spectrum code

```
write(*,*) 'Calculating rates in neutrino telescopes'
do i = 0, 199
  eth=200.0d0-i*1          ! energy threshold (of neutrino/muon), GeV
  thmax=30.0d0             ! the maximum half-aperture angle, degrees
  rtype=1                  ! 1=neutrino flux
                           ! 2=neutrino to muon conversion rate
                           ! 3=muon flux
  ptype=3                  ! 1=particles only
                           ! 2=anti-particles only
                           ! 3=sum of particle and anti-particle rates
  call dsntrates(eth,thmax,rtype,ptype,rateea,ratesu,istat)

  eth2=200.0d0-(i+1)*1     ! energy threshold (of neutrino/muon), GeV
  thmax2=30.0d0            ! the maximum half-aperture angle, degrees
  rtype2=1                 ! 1=neutrino flux
                           ! 2=neutrino to muon conversion rate
                           ! 3=muon flux
  ptype2=3                 ! 1=particles only
                           ! 2=anti-particles only
                           ! 3=sum of particle and anti-particle rates
  call dsntrates(eth2,thmax2,rtype2,ptype2,rateea2,ratesu2,istat)

  write(*,*) ratesu2-ratesu
enddo
```