– SQUARK-GLUINO PRODUCTION –

Next-to-Leading Order Contributions to the Scattering Cross-Section in the Threshold Region

Bachelor Thesis

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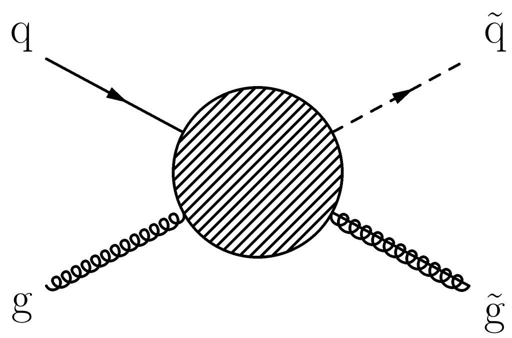

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Chapter 1

Introduction

In the last decades there has been a lot of attention for supersymmetry (SUSY). This theory is an extension to the Standard Model (SM) and comprises a lot of additional particles. However, there is no glimpse of these predicted particles yet. At the moment, the search for supersymmetric particles is one of the main goals for the current particle accelerators. Inasmuch as SUSY has not been found yet, there are two possible scenarios: either SUSY simply does not exist or we are just not able to produce supersymmetric particles because their masses are too large. An important quantity for collision events, which can be measured by experimentalists and calculated by theorists, is the (scattering) cross-section. As long as SUSY is not found, detailed knowledge about the cross-section can be used to raise the lower bounds on the masses of the supersymmetric particles. If SUSY is found, this knowledge can be exploited to determine the masses of these particles more accurately.

In this thesis the production process for squark-gluino pairs is discussed. If SUSY exists, it is expected that these supersymmetric particles are produced abundantly in hadron colliders. Theorists have long been working on the cross-section for this process, including more quantum corrections to improve the prediction for the cross-section. Since squarks and gluinos must have large masses, they will have virtually no kinetic energy upon creation. In this thesis an analytical expression will be derived for the cross-section of squark-gluino pairs in the limit of production at the kinematic threshold.

First an introduction on SUSY will be given. Then the notion of cross-section will be elucidated and in particular the cross-section for a process with two incoming and two outgoing particles will be addressed. In the subsequent chapter, we will discuss why we are after this specific expression for the cross-section. Subsequently, we will consider the process of squark-gluino pair production, followed by a discussion of the various quantum corrections that will be taken into account. After this, the calculation of the cross-section will be performed. Several problems and their solutions will be presented. Finally, a numerical check of the obtained analytical expression will be performed and the result will be discussed.

1.1 Conventions

Throughout this thesis the use of natural units is implied, so $\hbar = c = 1$. Furthermore, indices of four-vectors are omitted and vector quantities are indicated in boldface.

Chapter 2

Introduction to Supersymmetry

2.1 The Standard Model

The SM was developed in the early and mid 20th century by several physicists. In the 70s, the current wording of the SM was completed, because the existence of quarks was confirmed by experiments. As far as we know, there exist just four fundamental forces in nature: the strong and weak nuclear force, the electromagnetic force and gravity. Only the first three are described within the SM. The SM describes the properties of the elementary particles, including the way they interact. The collection of elementary particles consists both of fermions (particles with half-integer spin) and bosons (particles with integer spin). Figure (2.1) shows all the particles of the SM.

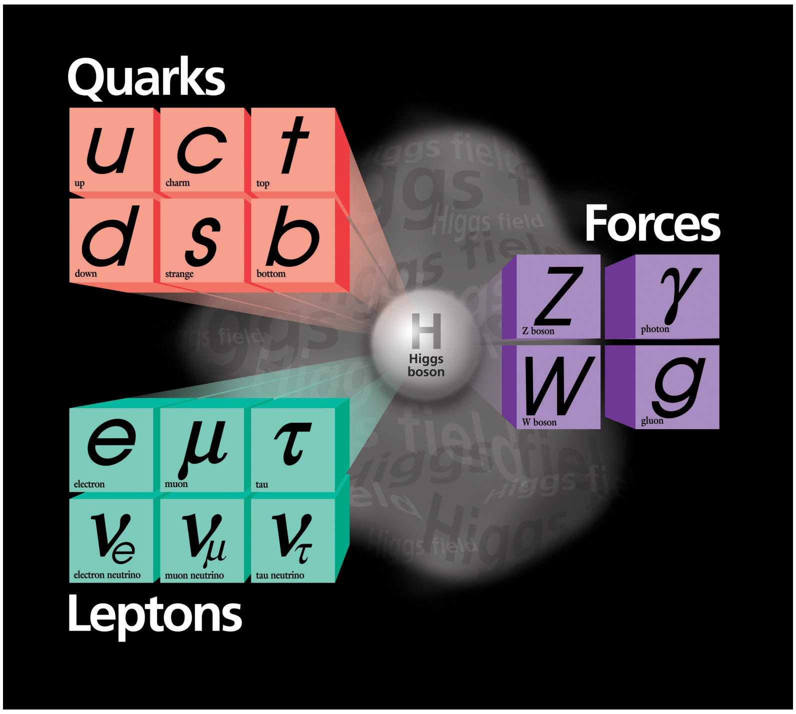


Figure 2.1: The elementary particles in the Standard Model.

Most of the particles in the SM are fermions. For every fermion (all of them are spin-$\frac{1}{2}$ particles) there exists a corresponding antifermion. All fermions are sensitive to the weak interaction (and gravity). The fermions can be divided into two groups by looking at the strong nuclear interaction: six quarks (up, down, charm, strange, top, bottom) and six leptons. The property of quarks that defines them as such, is the possession of color charge. This enables quarks to participate in strong interactions. The quarks do not roam freely in nature, on the contrary, they are only encountered within composite, color-neutral particles, like protons and neutrons. The other fermions do not possess color charge and are called leptons (electron, electron neutrino, muon, muon neutrino, tau, tau neutrino). The electron, muon and tau carry electric charge, whereby they are able to take part in electromagnetic processes. The neutrinos (the neutral leptons), on the other hand, do not carry electric charge and hence they are only influenced by the weak nuclear force. Therefore, these very light particles are hardly affected by matter and extremely difficult to detect. Both quarks and leptons can be divided into pairs which appear in three generations. The first generation includes the lightest pairs of particles and the third generation the heaviest ones. Apart from the neutrinos, particles from the second and third generation have very short life times and are solely observed in high-energy environments.

Another type of particles in the SM is constituted by the gauge bosons. These particles are the mediators of the strong, weak and electromagnetic forces. The electromagnetic interaction between electrically charged particles is transferred by (the massless) photons. The massive $W^{\pm}$ and Z bosons are the force carriers of the weak interaction. The $W^{\pm}$ boson exclusively interacts with left-handed particles and right-handed antiparticles, while the neutral Z boson couples to both handednesses[1]. Particles possessing color charge are able to interact via the strong nuclear force, which is mediated by the massless gluons. There are eight gluons and they all carry a net color charge enabling them to interact among themselves as well. One very important particle in the SM, whose existence still has to be borne out by experiments, is the Higgs boson. This massive, scalar particle is related to the mechanism that explains why particles have mass.

The force of gravity is a stranger in our midst, for the relative strength of this force is much smaller than the other forces. As yet, no one has succeeded in establishing a good quantum theory of gravity in a consistent fashion[2]. To this day, gravity is explained by Einstein's theory of general relativity and has no place (yet) in a theory of elementary particles. So, could the SM be the final story?
The SM is extremely succesful in describing phenomena at the highest attainable energies. Yet, most people agree that the SM is merely an effective description of a more complete theory. This is because various things are not well (or even not at all) explained within the SM. Probably the most important example of this is the complete absence of gravity in this theory. Several new theories try to explain the deficiencies of the SM. One of them is the extension of the SM by SUSY. In the coming decades, particle accelerators probably will answer the question whether SUSY and other theories will bring us a step closer to a Theory of Everything!

2.2 Motivation for Supersymmetry

SUSY is a symmetry that relates bosons to fermions and vice versa. Accordingly, every particle in the SM has a so-called "superpartner", which has yet to be discovered. The Minimal Supersymmetric Standard Model (MSSM) is the minimal extension of the SM that realizes SUSY. But why have we actually developed this theory? Here follows a list of the main problems of the SM which SUSY claims to solve:

- **Hierarchy Problem:**

Initially, the MSSM was proposed in 1981 to solve the hierarchy problem. This problem amounts to the fact that within the SM the mass of the Higgs boson inevitably collects very large quantum corrections. As a result, the Higgs boson gets too heavy. However, in addition to the particles in the SM, in the MSSM also the corresponding superpartners contribute to quantum corrections. Inasmuch as these corrections have opposite signs for fermions and bosons, SUSY ensures that the undesirable contributions exactly cancel out.

- **Grand Unification:**

In the 60s, the physicists Glashow, Weinberg and Salam succeeded in combining the weak and elec-

[1] However, with a different strength.
[2] Perhaps because it simply does not exist!

tromagnetic force into the so-called “electroweak interaction”. Both forces turn out to be different manifestations of the electroweak interaction. The relative weakness of the weak force compared to the electromagnetic force is just a consequence of the enormous masses of the $W^{\pm}$ en Z bosons. The logical next step is, of course, to try to combine the strong interaction with the electroweak interaction, which is called a “Grand Unified Theory” (GUT).
The couplings, which set the strength for the various forces, are not constant but depend on the energy. Could there be an energy for which all the couplings take the same value? This could mean that all forces are, in fact, different manifestations of a single underlying interaction. At approximately 10^{16} GeV (the GUT scale), the couplings come very close together, but unfortunately, there is no intersection. SUSY, however, possibly ensures these couplings to be equal at the GUT scale [1].

- **Dark Matter:**

The visible matter in our universe does not have enough mass to explain the speed of moving galaxies within the framework of the general theory of relativity. Presumably, 22% of the energy in our universe is constituted by an invisible form of matter, which is called, quite suggestively, “dark matter”.
Apparently, this form of matter is neither affected by strong, nor electromagnetic interactions, so dark matter might be made up of so-called “Weakly Interacting Massive Particles” (WIMPs). These particles can solely interact through the weak force and gravity. In the MSSM the lightest supersymmetric particle (LSP) cannot decay. Moreover, in most scenarios (see also the next section) this particle is both color and charge neutral, making it a WIMP and an attractive candidate for dark matter.

2.3 The Minimal Supersymmetric Standard Model

In the MSSM, every particle has a superpartner whose spin differs by half a unit. All particles have the same internal quantum numbers (apart from spin) as their supersymmetric partners. The names of the spin-0 superpartners start with an “s”, referring to “scalar” [2]. On the other hand, the names of the fermionic superpartners end in “ino”. The supersymmetric particles in the MSSM can be divided into five groups: squarks, sleptons, gluinos, charginos and neutralinos. Table (2.1) gives an overview of the particles in the SM and their corresponding superpartners.

Particle	**Symbol**	**Spin**	**Superpartner**	**Symbol**	**Spin**
quark	q	$\frac{1}{2}$	squark	$\tilde{\mathrm{q}}$	0
lepton	l	$\frac{1}{2}$	slepton	$\tilde{\mathrm{l}}$	0
$W^{\pm}$	$W^{\pm}$	1	wino	$\tilde{W}^{\pm}$	$\frac{1}{2}$
Z	Z	1	zino	$\tilde{Z}$	$\frac{1}{2}$
photon	γ	1	photino	$\tilde{\gamma}$	$\frac{1}{2}$
gluon	g	1	gluino	$\tilde{\mathrm{g}}$	$\frac{1}{2}$
Higgs	H	0	Higgsino	$\tilde{H}$	$\frac{1}{2}$

Table 2.1: Standard Model particles and their superpartners.

The gauginos mix with Higgsinos[3], the superpartners of the Higgs bosons. The resulting linear combinations are called charginos or neutralinos, belonging to electrically charged and neutral combinations respectively. There are four fermionic charginos and also four fermionic neutralinos.

[3]There are actually *four* different Higgsinos, two charged and two neutral ones. In SUSY there are five Higgs bosons.

A very important quantum number within the context of supersymmetric theories is R-parity. This multiplicative number is defined to be +1 for particles in the SM and −1 for their superpartners. If R-parity is conserved (which is true in the MSSM) every interaction vertex must contain an even number of supersymmetric particles. Consequently, supersymmetric particles can only be produced in pairs. Furthermore, the LSP must be absolutely stable.

Exact (unbroken) SUSY predicts particles and their superpartners to have equal masses. Obviously, this is not the case, since otherwise SUSY would already have been observed in particle accelerators! So if SUSY exists, the supersymmetric particles must be much *heavier* than the particles in the SM. We speak of SUSY as a "broken" symmetry of nature. It is not yet known exactly how the breaking of SUSY works at a fundamental level. There are several scenarios for this.

2.4 Particle Accelerators

Currently, finding supersymmetric particles is one of the main experimental objectives of high-energy physics. Two particle accelerators are actively looking for SUSY: the Tevatron and the Large Hadron Collider (LHC). Since both accelerators are hadron colliders[4], particles that are sensitive to the strong interaction are produced most abundantly. Hence, it is expected that squarks and gluinos are produced predominantly, because they possess color charge. The final states of the decay cascades of both squarks and gluinos contain LSPs (the lightest neutralino), which are stable in the MSSM because of R-parity conservation. So, with the production of squarks and gluinos, jets are detected *and* at the same time a "missing energy signal" will occur, for the LSP will leave the detector unseen.
The masses of the supersymmetric particles must exceed the masses of the SM particles considerably. At the same time, SUSY has to solve the hierarchy problem[5], which implies that the masses should not exceed $\mathcal{O}(1\ \text{TeV})$ [2]. Currently, the LHC operates at a center-of-mass (CM) energy of 7 TeV, but this will be increased to 14 TeV in the near future. As a result, the range of sensitivity to squark and gluino masses will be extended to 3 TeV [3]. Hence, if SUSY exists it seems unlikely that it will not be observed in the near future!

The most important quantity in scattering events that experimentalists can measure and theorists are able to calculate, is the scattering cross-section. The more precise the cross-sections are known for supersymmetric particle production processes, all the better the masses of these particles could be determined by comparing with the measured cross-sections. Due to the emergence of LSPs in the cascade decays and their surreptitious escape from the detectors, the masses of the squarks and gluinos cannot be deduced merely from experimental data [4]. Thus, theoretical predictions are of great importance! In the next chapter we will study the concept of cross-section in more detail.

[4]The Tevatron is a proton-antiproton collider and the LHC is a proton-proton collider.
[5]Recall that SUSY was initially invented to solve this very problem!

Chapter 3

From Matrix Element to Scattering Cross-Section

The process by which incoming particles turn into outgoing particles via an interaction, is called scattering. In high-energy physics one is interested in calculating the probabilities of the various outcomes in a scattering experiment. A convenient way to graphically display a scattering process is by making use of Feynman diagrams. These diagrams are drawn according to the Feynman rules, which depend on the considered interaction Lagrangian. Every diagram can be associated with a complex number, the so-called "matrix element" (or amplitude) $\mathcal{M}$. This number is a measure of the likelihood of the occurrence of a process. The total probability that a given initial state becomes a certain final state, can be obtained by adding up *all* possible Feynman diagrams. Since an infinite number of interactions are possible between given initial and final states, also an infinite number of Feynman diagrams correspond to a certain process. Clearly, transition probabilities cannot be calculated exactly. However, the diagrams do not contribute equally to the total transition amplitude. Most notably because every interaction contributes a factor to the matrix element $\mathcal{M}$ that is proportional to the interaction strength g, which is generally a number much smaller than one[1]. Basically, a Taylor expansion is possible where the interaction strength g serves as the expansion parameter. In situations where g is much smaller than one, it suffices to consider only the first orders of the expansion in order to arrive at an accurate answer. Once the total matrix element has been obtained, $|\mathcal{M}|^2$ can be used to calculate various physical quantities, e.g. decay rates and scattering cross-sections.

3.1 Scattering Cross-Section

A very useful tool in particle physics is the (scattering) cross-section. The concept of cross-section is most easily explained by considering an incident particle being scattered off a stationary target particle (in this section we will stick to this situation). The cross-section σ is then a measure for the probability that the particle will be scattered off the target, bearing the dimensions of area[2]. It depends on the properties of both the particle and the target, since these determine the type of interaction that will occur. Furthermore, the closer the incident particle gets to the target, the stronger the interaction usually will be. So the cross-section is generally not simply equal to the (projected) area of the target particle as seen by the incident particle. But it can be viewed as the effective hard sphere scattering area that the target particle represents.

In a similar way an infinitesimal effective area $\mathrm{d}\sigma$ can be assigned to incoming particles that are scattered into a solid angle $\mathrm{d}\Omega$. The proportionality factor between $\mathrm{d}\sigma$ and $\mathrm{d}\Omega$ is called the differential cross-section and it equals $\mathrm{d}\sigma/\mathrm{d}\Omega$. The number of incident particles per unit of time, per unit of area, is called the "luminosity" (flux) $\mathcal{L}$. This means that $\mathrm{d}N = \mathcal{L}\,\mathrm{d}\sigma$ represents the number of particles that passes through the area $\mathrm{d}\sigma$ per unit of time. From this it follows that the

[1]This is not the case for strong interactions at low energies!

[2]The most common unit is the barn (b), $1\,\mathrm{b} = 10^{-28}\,\mathrm{m}^2$.

differential cross-section can be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}N}{\mathcal{L}\,\mathrm{d}\Omega}\,. \tag{3.1}$$

In collision experiments one controls the luminosity $\mathcal{L}$. Furthermore, the detector can be viewed as consisting of small segments that subtend a solid angle $\mathrm{d}\Omega$ and count the number of particles $\mathrm{d}N$ that end up in this area. Thus, experimentalists are able to measure the differential cross-section for a particular scattering process. But how is the differential cross-section determined by theorists?

3.2 Cross-Section for Two-Body Scattering

The concept of cross-section is not merely applicable to a process where an incoming particle collides on a stationary target particle. It can be generalized to include processes where two moving particles collide with each other.

To calculate a cross-section, you need two things: the matrix element $\mathcal{M}$ and the available phase space. The matrix element contains all dynamical information about the process and can be calculated by evaluating all relevant Feynman diagrams. The factor for the phase space, however, contains only kinematic information, so it depends on the masses and the four-momenta of the initial and final state particles. This factor is larger for processes where the particles have more freedom, which implies that the process is more likely to occur. Fermi's Golden Rule states that a transition rate (probability of transition per unit of time) is given by the product of the phase space and the absolute value of the matrix element squared. Armed with this rule we are able to compute decay rates and scattering cross-sections.

Suppose particle 1 and 2 collide and produce the particles 3 to n:

$$1 + 2 \;\rightarrow\; 3 + 4 + \cdots + n\,.$$

The form of Fermi's Golden Rule appropriate to calculating scattering cross-sections is given by [5]

$$\begin{aligned}\sigma = {} & \frac{SK_{12}}{4\sqrt{\left(p_1\cdot p_2\right)^2 - \left(m_1 m_2\right)^2}} \int \left|\mathcal{M}\right|^2 (2\pi)^4\, \delta^{(4)}\left(p_1 + p_2 - \sum_{i=3}^{n} p_i\right) \\ & \times \prod_{j=3}^{n} 2\pi\,\delta\left(p_j^2 - m_j^2\right)\theta\left(p_j^0\right)\frac{\mathrm{d}^4 p_j}{\left(2\pi\right)^4}\,,\end{aligned} \tag{3.2}$$

where m_i and p_i, respectively, represent the mass and four-momentum of particle i. S is a statistical correction for the possible double counting of identical particles in the final state. Thus, $S \times (n-2)!$ equals the number of ways in which the $n-2$ particles in the final state can be ordered. The factor K_{12} incorporates the averaging over possible (indistinguishable) initial states. The denominator of the factor in front of the integral is proportional to the flux of the incident particles [6]. It contains the Minkowskian inner product $p_1 \cdot p_2$ of the initial state four-momenta p_1 and p_2.

The presence of the step function and the delta functions can be understood by the three kinematic restrictions on the phase space:

1. Each outgoing particle lies on its **mass shell**, that is to say $p_j^2 = m_j^2$. With this, you actually state that $E_j^2 = \mathbf{p}_j^2 + m_j^2$ should apply. This is effectuated by the delta function $\delta\left(p_j^2 - m_j^2\right)$.

2. Each outgoing particle has a **positive energy**, so $p_j^0 = E_j > 0$. The step function[3] $\theta\left(p_j^0\right)$ ensures that this always holds.
3. **Four-momentum is conserved**, hence the delta function $\delta^{(4)}\left(p_1 + p_2 - \sum_{i=3}^n p_i\right)$.

The four-dimensional volume element $\mathrm{d}^4 p_j$ can be split up into a temporal and a spatial part:

$$\mathrm{d}^4 p_j = \mathrm{d}p_j^0 \, \mathrm{d}^3 \mathbf{p}_j \, . \tag{3.3}$$

Using the identity $\delta\left(x^2 - a^2\right) = \frac{1}{2a}\left[\delta\left(x-a\right) + \delta\left(x+a\right)\right]$, for $a > 0$, we obtain

$$\begin{aligned} \theta\left(p_j^0\right)\delta\left(p_j^2 - m_j^2\right) &= \theta\left(p_j^0\right)\delta\left[\left(p_j^0\right)^2 - \mathbf{p}_j^2 - m_j^2\right] \\ &= \frac{1}{2\sqrt{\mathbf{p}_j^2 + m_j^2}}\,\delta\left(p_j^0 - \sqrt{\mathbf{p}_j^2 + m_j^2}\right) \, , \end{aligned} \tag{3.4}$$

where the step function $\theta\left(p_j^0\right)$ has killed the delta function $\delta\left(p_j^0 + \sqrt{\mathbf{p}_j^2 + m_j^2}\right)$.

Now, the integrals over p_j^0 in equation (3.2) can be performed and then the expression simplifies to

$$\sigma = \frac{S K_{12}}{4\sqrt{\left(p_1 \cdot p_2\right)^2 - \left(m_1 m_2\right)^2}} \int |\mathcal{M}|^2 \left(2\pi\right)^4 \delta^{(4)}\left(p_1 + p_2 - \sum_{i=3}^n p_i\right) \prod_{j=3}^n \frac{1}{2\sqrt{\mathbf{p}_j^2 + m_j^2}} \frac{\mathrm{d}^3\mathbf{p}_j}{\left(2\pi\right)^3} \, , \tag{3.5}$$

with $p_j^0 \to \sqrt{\mathbf{p}_j^2 + m_j^2}$ whenever p_j^0 appears in $\mathcal{M}$ or in delta functions.

Let us consider now the two-body scattering process

$$1 + 2 \to 3 + 4 \, .$$

It is most convenient to work in the CM frame. Consequently, the incident particles have opposite momenta with equal magnitudes. Thus $\mathbf{p}_2 = -\mathbf{p}_1$, so

$$\sqrt{\left(p_1 \cdot p_2\right)^2 - \left(m_1 m_2\right)^2} = \left(E_1 + E_2\right)|\mathbf{p}_1| \, . \tag{3.6}$$

Equation (3.5) can now be written as follows:

$$\sigma = \left(\frac{1}{8\pi}\right)^2 \frac{S K_{12}}{\left(E_1 + E_2\right)|\mathbf{p}_1|} \int |\mathcal{M}|^2 \frac{\delta^{(4)}\left(p_1 + p_2 - p_3 - p_4\right)}{\sqrt{\mathbf{p}_3^2 + m_3^2}\sqrt{\mathbf{p}_4^2 + m_4^2}} \, \mathrm{d}^3\mathbf{p}_3 \, \mathrm{d}^3\mathbf{p}_4 \, . \tag{3.7}$$

The delta function can be rewritten as

$$\delta^{(4)}\left(p_1 + p_2 - p_3 - p_4\right) = \delta\left(E_1 + E_2 - p_3^0 - p_4^0\right)\delta^{(3)}\left(\mathbf{p}_3 + \mathbf{p}_4\right) \, , \tag{3.8}$$

If we now use $p_j^0 \to \sqrt{\mathbf{p}_j^2 + m_j^2}$, the integral over $\mathbf{p}_4$ can be carried out:

$$\sigma = \left(\frac{1}{8\pi}\right)^2 \frac{S K_{12}}{\left(E_1 + E_2\right)|\mathbf{p}_1|} \int |\mathcal{M}|^2 \frac{\delta\left(E_1 + E_2 - \sqrt{\mathbf{p}_3^2 + m_3^2} - \sqrt{\mathbf{p}_3^2 + m_4^2}\right)}{\sqrt{\mathbf{p}_3^2 + m_3^2}\sqrt{\mathbf{p}_3^2 + m_4^2}} \, \mathrm{d}^3\mathbf{p}_3 \, . \tag{3.9}$$

[3]The step function $\theta\left(x\right)$ has the following properties: $\theta\left(x\right) = 0$ if $x < 0$ and $\theta\left(x\right) = 1$ if $x > 0$.

In general, $|\mathcal{M}|^2$ depends on all four-momenta. However, in the CM frame $\mathbf{p}_2 = -\mathbf{p}_1$ and $\mathbf{p}_4 = -\mathbf{p}_3$, so $|\mathcal{M}|^2$ is only a function of $\mathbf{p}_1$ and $\mathbf{p}_3$. We can simplify the expression further by realizing that $|\mathcal{M}|^2$ is a *scalar* quantity and therefore cannot depend on a vector. There are three ways in which one can obtain a scalar from $\mathbf{p}_1$ and $\mathbf{p}_3$: $\mathbf{p}_1 \cdot \mathbf{p}_1$, $\mathbf{p}_3 \cdot \mathbf{p}_3$ and $\mathbf{p}_1 \cdot \mathbf{p}_3 = |\mathbf{p}_1|\,|\mathbf{p}_3| \cos\theta$. Hence $|\mathcal{M}|^2$ may depend on both the magnitude r and the direction θ of $\mathbf{p}_3$.[4] For the sake of convenience, we switch to spherical coordinates in momentum space, so $\mathrm{d}^3\mathbf{p}_3 \to r^2 \sin\theta \,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi = r^2\,\mathrm{d}r\,\mathrm{d}\Omega$, with $r \equiv |\mathbf{p}_3|$. There are no delta functions containing θ, so the integration over θ cannot be performed trivially. However, the integral over r *can* be carried out, since there is still a delta function of r left over. Without knowing $|\mathcal{M}|^2$ we can only calculate the differential cross-section $\mathrm{d}\sigma/\mathrm{d}\Omega$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{SK_{12}}{(E_1+E_2)\,|\mathbf{p}_1|} \int_0^\infty |\mathcal{M}|^2 \,\frac{\delta\left(E_1+E_2-\sqrt{r^2+m_3^2}-\sqrt{r^2+m_4^2}\right)}{\sqrt{r^2+m_3^2}\sqrt{r^2+m_4^2}}\, r^2\,\mathrm{d}r\,. \tag{3.10}$$

The argument of the delta function can be simplified by the definition

$$u \equiv \sqrt{r^2+m_3^2}+\sqrt{r^2+m_4^2}\,, \tag{3.11}$$

so

$$\frac{\mathrm{d}u}{\mathrm{d}r} = \frac{ur}{\sqrt{r^2+m_3^2}\sqrt{r^2+m_4^2}}\,. \tag{3.12}$$

Equation (3.10) now becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{SK_{12}}{(E_1+E_2)\,|\mathbf{p}_1|} \int_{m_3+m_4}^\infty |\mathcal{M}|^2 \,\frac{\delta\left(E_1+E_2-u\right) r}{u}\,\mathrm{d}u\,. \tag{3.13}$$

The integral can now be performed (which sends $u \to E_1+E_2$). Thus, we conclude

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = SK_{12}\left[\frac{|\mathcal{M}|}{8\pi\,(E_1+E_2)}\right]^2 \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}\,, \tag{3.14}$$

where $|\mathbf{p}_f|$ and $|\mathbf{p}_i|$ denote the magnitudes of either outgoing and incoming momentum respectively.

The total cross-section σ can be determined by integrating the differential cross-section over the solid angle Ω. To determine the differential cross-section one must first compute $|\mathcal{M}|^2$. However, *every* Feynman diagram (with the right initial and final states) contributes to the total matrix element. Hence, the more diagrams are taken into account the more accurately σ is determined. The diagrams that contribute most in the perturbative expansion are the leading order (LO) ones. For two-to-two body processes these diagrams are usually proportional to g^2, whereas at one order higher −at next-to-leading order (NLO)− the diagrams are proportional to g^4. In chapter 5 equation (3.14) will be used to calculate the cross-section for the production of squark-gluino pairs.

[4]The dependence on $|\mathbf{p}_1|$ is not relevant here, for this is just a constant as far as the integration is concerned.

Chapter 4

Threshold Resummation

The LO and NLO contributions to the cross-sections for the production of squarks and gluinos are already known [4]. However, for various reasons it is not sufficient to consider only these contributions. It is necessary to include higher order corrections as well. The most important arguments for this are:

- The uncertainties on the theoretical predictions of the LO cross-sections are about as large as the cross-sections themselves. By adding higher order corrections, these uncertainties are much smaller, but still considerable [4][7].
- The NLO contributions turn out to be of the same order of magnitude as the LO contributions. Therefore it might be relevant to include higher orders as well [4][7].

If SUSY exists, the particles are expected to have large masses. Thus in case they are formed, the CM energy will be close to the kinematic threshold for the production of these particles. Therefore, a significant part of the NLO corrections can be attributed to the threshold region.
In the threshold region the NLO corrections are dominated by the contributions due to the emission of soft gluons[1] by both the initial and final state particles. These corrections give rise to powers of $L \equiv \log\left(8\beta^2\right)$ in the cross-section [4]. We will come to the exact definition of β in the next chapter and it will be shown that at threshold $\beta = 0$. Therefore these logarithms diverge in the threshold limit, making them very important! Consequently, we would like to take as many of them into account as possible. Miraculously, the soft-gluon corrections can be taken into account to *all* orders in perturbation theory by means of threshold resummation [7]! Threshold resummation is a way to organize the perturbation series. The resummed cross-section schematically takes the form:

$$\sigma = \sigma^{\mathrm{thr}} e^{L P_1(\alpha_\mathrm{s} L)} e^{P_2(\alpha_\mathrm{s} L)} e^{\alpha_\mathrm{s} P_3(\alpha_\mathrm{s} L)} \dots , \tag{4.1}$$

where the functions indicated with P represent polynomials in $\alpha_\mathrm{s} L$. The strong coupling constant α_s is proportional to the square of the strong interaction strength g_s.
The first exponential function is called the leading logarithmic (LL) contribution, the first two exponential functions together are called the next-to-leading logarithmic (NLL) contribution and when also the third exponential function is included, we speak of the next-to-next-to-leading logarithmic (NNLL) contribution.

Up to NLL accuracy it suffices to calculate the LO cross-section at threshold: $\sigma^{\mathrm{thr}} = \sigma^{\mathrm{thr}}_{\mathrm{LO}} \propto \beta$. But when we also want to include the NNLL contribution we have to know the NLO cross-section at threshold $\sigma^{\mathrm{thr}}_{\mathrm{NLO}}$ up to first order in β [8]. The calculation of the NLO cross-section away from threshold has been carried out already [4]. However, we do not have an analytical expression for the threshold region yet. In this thesis an analytical expression is obtained for the NLO cross-section at threshold for the production of squark-gluino pairs at hadron colliders. This is an important ingredient in the calculation of the NNLL corrections for the squark-gluino production process.

[1] Low-energetic gluons.

Once these corrections are completed, they can be combined with NNLL results for other processes involving squarks and gluinos. However, the latter goes beyond the scope of this thesis.

By looking at similar hadronic processes [8] it is expected that higher order corrections to the cross-sections are positive and large. There are now two possible scenarios. One scenario is that squarks and gluinos are not found. In that case the significant increase of the cross-sections raises the lower bounds (exclusion limits) on the masses of the squarks and gluinos accordingly. The more exciting option is that squarks and gluinos are found. Then it is essential to know the cross-sections very accurately. It might not be possible to deduce the squark and gluino masses merely through reconstruction of the original squark and gluino states (due to the undetected LSPs). When experimental data is compared to theoretical predictions for the cross-sections, the masses can be determined more accurately.

The next chapter is about the squark-gluino production process. In chapter 6 we will consider the various NLO contributions to this process. In the subsequent chapter the calculation of the NLO cross-section in the threshold region will be discussed. Finally, the obtained analytical expression will be checked numerically.

Chapter 5

The Squark-Gluino Production Process

If SUSY exists it is expected that the supersymmetric particles which possess color charge are produced most copiously in hadron colliders. Therefore, it is particularly useful to study the processes within the supersymmetric QCD (SUSY-QCD) sector of the MSSM. The particle content of SUSY-QCD comprises quarks, squarks, gluons and gluinos. In the MSSM R-parity is conserved, which implies that supersymmetric particles can only be produced in pairs. The decay cascades of both squarks and gluinos contain jets and LSPs. The LSPs leave the detector unseen and thus momentum conservation *seems* to be violated. This is typical for SUSY events and therefore this is something that is searched for in particle accelerators. Squarks and gluinos can be produced in various ways. In this thesis we investigate in particular the production of squark-gluino pairs.

5.1 Squark-Gluino Final States

At LO, the final state consisting of a squark ($\tilde{\mathrm{q}}$) and a gluino ($\tilde{\mathrm{g}}$) can only be obtained by the collission of a quark (q) and a gluon (g):

$$\mathrm{q}_i\,(g_1) + \mathrm{g}\,(g_2) \;\rightarrow\; \tilde{\mathrm{q}}_i\,(p_1) + \tilde{\mathrm{g}}\,(p_2)\ .$$

Between the brackets, the corresponding four-momenta are indicated and the index i denotes the flavor of the quarks and squarks. It is also possible that in this process the quarks and squarks are replaced by their antiparticles.

Figure (5.1) depicts the three LO Feynman diagrams corresponding to the production of squark-gluino pairs. The meaning of the different types of lines can be found in the appendix.

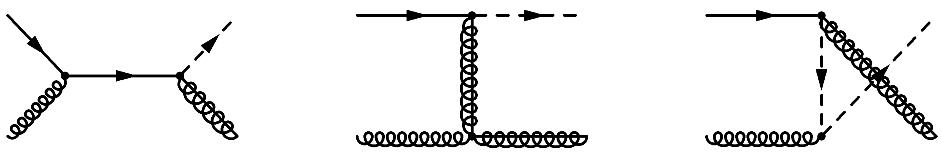


Figure 5.1: The leading order Feynman diagrams for squark-gluino final states.

The gluino mass is denoted by $m_{\tilde{\mathrm{g}}}$ and it is assumed that all squark flavors have the same mass $m_{\tilde{\mathrm{q}}}$. The masses of all *light* quarks[1] are neglected and the presence of the top squark in the final

[1] All quarks except for the top quark.

state is excluded, since a top quark is not found within a proton and therefore cannot occur in the initial state. Furthermore, all incoming and outgoing particles are assumed to be on their respective mass shells, which means that $g_1^2 = g_2^2 = 0$, $p_1^2 = m_{\tilde{\mathrm{q}}}^2$ and $p_2^2 = m_{\tilde{\mathrm{g}}}^2$.

The process is regarded in the CM frame, which implies that $\mathbf{g}_2 = -\mathbf{g}_1$. Thus, the incoming four-momenta are given by

$$g_1 = \begin{pmatrix} E_{\mathrm{q}} \\ \mathbf{g}_1 \end{pmatrix} = \begin{pmatrix} |\mathbf{g}_1| \\ \mathbf{g}_1 \end{pmatrix} \qquad \text{and} \qquad g_2 = \begin{pmatrix} E_{\mathrm{g}} \\ \mathbf{g}_2 \end{pmatrix} = \begin{pmatrix} |\mathbf{g}_1| \\ -\mathbf{g}_1 \end{pmatrix} . \tag{5.1}$$

From momentum conservation it follows that $\mathbf{p}_2 = -\mathbf{p}_1$, so the four-momenta of the outgoing particles are given by

$$p_1 = \begin{pmatrix} E_{\tilde{\mathrm{q}}} \\ \mathbf{p}_1 \end{pmatrix} = \begin{pmatrix} \sqrt{\mathbf{p}_1^2 + m_{\tilde{\mathrm{q}}}^2} \\ \mathbf{p}_1 \end{pmatrix} \qquad \text{and} \qquad p_2 = \begin{pmatrix} E_{\tilde{\mathrm{g}}} \\ \mathbf{p}_2 \end{pmatrix} = \begin{pmatrix} \sqrt{\mathbf{p}_1^2 + m_{\tilde{\mathrm{g}}}^2} \\ -\mathbf{p}_1 \end{pmatrix} . \tag{5.2}$$

As will be shown in the next section, to parametrize $|\mathbf{p}_1|$ the dimensionless threshold parameter

$$\beta \equiv \sqrt{1 - \frac{(m_{\tilde{\mathrm{q}}} + m_{\tilde{\mathrm{g}}})^2}{E_{\mathrm{CM}}^2}} \tag{5.3}$$

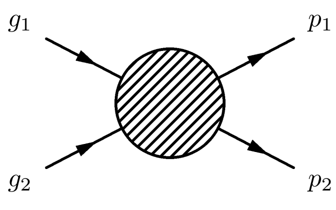
can be used. When the outgoing particles are produced at threshold, i.e. when $\mathbf{p}_1 = \mathbf{0}$, then

$$E_{\mathrm{CM}} = E_{\tilde{\mathrm{q}}} + E_{\tilde{\mathrm{g}}} = m_{\tilde{\mathrm{q}}} + m_{\tilde{\mathrm{g}}} . \tag{5.4}$$

Consequently, we see that $\beta \to 0$ in the threshold limit as it should.

5.2 Mandelstam Variables

In high-energy physics one frequently makes use of the so-called “Mandelstam variables”. These Lorentz invariant, scalar quantities contain information about the kinematics of the considered process. For two-body scattering processes there are three Mandelstam variables, indicated by s, t and u. Let us denote the incoming four-momenta by g_1 and g_2 and the outgoing four-momenta by p_1 and p_2. We consider the generic two-body scattering as shown below:



The Mandelstam variables are defined as follows:

$$\begin{aligned} s &= (g_1 + g_2)^2 = (p_1 + p_2)^2 \\ t &= (g_1 - p_1)^2 = (g_2 - p_2)^2 \\ u &= (g_1 - p_2)^2 = (g_2 - p_1)^2 . \end{aligned} \tag{5.5}$$

Consequently, by applying conservation of four-momentum one finds that these variables are interrelated by

$$s + t + u = g_1^2 + g_2^2 + p_1^2 + p_2^2 = \sum_{i=1}^{4} m_i^2\,. \tag{5.6}$$

The Mandelstam variable s is also known as the square of the invariant mass (the CM energy). This is easily understood by looking at the CM frame, because then $g_1 + g_2 = (E_{\mathrm{CM}}, 0, 0, 0)$. It now follows immediately that $E_{\mathrm{CM}} = \sqrt{s}$.

In this section, the Mandelstam variables for the squark-gluino production process are expressed in the threshold parameter β for later use. The expression for $s\,(\beta)$ follows in a trivial fashion from equation (5.3):

$$s\,(\beta) = \frac{\left(m_{\tilde{\mathrm{q}}} + m_{\tilde{\mathrm{g}}}\right)^2}{1 - \beta^2}\,. \tag{5.7}$$

To determine $t\,(\beta)$ and $u\,(\beta)$ we first must write the incoming and outgoing four-momenta as a function of β:

$$\left(g_1 + g_2\right)^2 = \left(E_{\mathrm{q}} + E_{\mathrm{g}}\right)^2 = \left(|\mathbf{g}_1| + |\mathbf{g}_2|\right)^2 = 4\,\mathbf{g}_1^2\,. \tag{5.8}$$

At the same time $\left(g_1 + g_2\right)^2 = s$, by definition. Thus by combining equation (5.7) and (5.8), we find for the magnitude of the incoming momentum:

$$|\mathbf{g}_1| = \frac{m_{\tilde{\mathrm{q}}} + m_{\tilde{\mathrm{g}}}}{2\sqrt{1 - \beta^2}}\,. \tag{5.9}$$

We also have

$$\begin{aligned}\left(p_1 + p_2\right)^2 &= \left(E_{\tilde{\mathrm{q}}} + E_{\tilde{\mathrm{g}}}\right)^2 = \left(\sqrt{\mathbf{p}_1^2 + m_{\tilde{\mathrm{q}}}^2} + \sqrt{\mathbf{p}_2^2 + m_{\tilde{\mathrm{g}}}^2}\right)^2 \\ &= m_{\tilde{\mathrm{q}}}^2 + m_{\tilde{\mathrm{g}}}^2 + 2\,\mathbf{p}_1^2 + 2\sqrt{\left(\mathbf{p}_1^2 + m_{\tilde{\mathrm{q}}}^2\right)\left(\mathbf{p}_1^2 + m_{\tilde{\mathrm{g}}}^2\right)}\,.\end{aligned} \tag{5.10}$$

But $\left(p_1 + p_2\right)^2 = s$ as well. When equation (5.7) and (5.10) are combined one obtains the following expression for the magnitude of the outgoing momentum:

$$|\mathbf{p}_1| = \beta\sqrt{\frac{4m_{\tilde{\mathrm{q}}}m_{\tilde{\mathrm{g}}} + \beta^2\left(m_{\tilde{\mathrm{q}}} - m_{\tilde{\mathrm{g}}}\right)^2}{4\left(1 - \beta^2\right)}}\,. \tag{5.11}$$

Now the Mandelstam variables $t\,(\beta)$ and $u\,(\beta)$ can be determined:

$$\begin{aligned} t &\equiv \left(g_1 - p_1\right)^2 = g_1^2 + p_1^2 - 2g_1 \cdot p_1 \\ &= m_{\tilde{\mathrm{q}}}^2 - 2\left(E_{\mathrm{q}}E_{\tilde{\mathrm{q}}} - \mathbf{g}_1 \cdot \mathbf{p}_1\right) \\ &= m_{\tilde{\mathrm{q}}}^2 - 2\,|\mathbf{g}_1|\left(\sqrt{\mathbf{p}_1^2 + m_{\tilde{\mathrm{q}}}^2} - |\mathbf{p}_1|\cos\theta\right)\,,\end{aligned} \tag{5.12}$$

with θ the angle between $\mathbf{g}_1$ en $\mathbf{p}_1$. Using the expressions (5.9) and (5.11) one finds after some tedious but straightforward algebra:

$$t(\beta) = \frac{2m_{\tilde{q}}m_{\tilde{g}} - \beta\left(m_{\tilde{q}} + m_{\tilde{g}}\right)\cos\theta\sqrt{4m_{\tilde{q}}m_{\tilde{g}} + \beta^2\left(m_{\tilde{q}} - m_{\tilde{g}}\right)^2} + \beta^2\left(m_{\tilde{q}}^2 + m_{\tilde{g}}^2\right)}{2\left(\beta^2 - 1\right)} \,. \tag{5.13}$$

In a completely analogous way we arrive at the following expression for $u(\beta)$:

$$u(\beta) = \frac{2m_{\tilde{q}}m_{\tilde{g}} + \beta\left(m_{\tilde{q}} + m_{\tilde{g}}\right)\cos\theta\sqrt{4m_{\tilde{q}}m_{\tilde{g}} + \beta^2\left(m_{\tilde{q}} - m_{\tilde{g}}\right)^2} + \beta^2\left(m_{\tilde{q}}^2 + m_{\tilde{g}}^2\right)}{2\left(\beta^2 - 1\right)} \,. \tag{5.14}$$

In the threshold limit, β goes to zero, hence we can expand the obtained expressions for the Mandelstam variables around $\beta = 0$. The first order expansions in β are as follows:

$$\begin{aligned} s &= \left(m_{\tilde{q}} + m_{\tilde{g}}\right)^2 \\ t &= -\, m_{\tilde{q}}m_{\tilde{g}} + \beta\sqrt{m_{\tilde{q}}m_{\tilde{g}}}\left(m_{\tilde{q}} + m_{\tilde{g}}\right)\cos\theta \\ u &= -\, m_{\tilde{q}}m_{\tilde{g}} - \beta\sqrt{m_{\tilde{q}}m_{\tilde{g}}}\left(m_{\tilde{q}} + m_{\tilde{g}}\right)\cos\theta \,. \end{aligned} \tag{5.15}$$

Note that at zeroth order in β there is no distinction between t and u.

5.3 The Cross-Section

In chapter 3 we studied the cross-section for a two-body process. Equation (3.14) gives the differential cross-section. In the case of squark-gluino production the final state contains two different particles, so the statistical factor S equals one. Furthermore, the incoming and outgoing momenta can be replaced by $\mathbf{g}_1$ and $\mathbf{p}_1$ respectively and the energies E_{q} and E_{g} both equal $|\mathbf{g}_1|$. Thus we get the following expression for the differential cross-section for squark-gluino final states:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = K_{\mathrm{qg}}\frac{|\mathcal{M}|^2\,|\mathbf{p}_1|}{256\,\pi^2\,|\mathbf{g}_1|^3} \,. \tag{5.16}$$

The factor K_{qg} effectuates the averaging over the colors and spins of the quarks and gluons in the initial state. Both quarks and gluons have two different spin polarizations. Furthermore, quarks appear in three colors and gluons in eight. This gives $2 \times 2 \times 3 \times 8 = 96$ possibilities in the initial state. Thus K_{qg} is given by:

$$K_{\mathrm{qg}} = \frac{1}{96} \,. \tag{5.17}$$

Using equations (5.9) and (5.11) the differential cross-section can be expressed in β. At first order in β it equals:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\beta\,|\mathcal{M}|^2\sqrt{m_{\tilde{q}}m_{\tilde{g}}}}{3072\,\pi^2\left(m_{\tilde{q}} + m_{\tilde{g}}\right)^3} \,. \tag{5.18}$$

Consequently, the total cross-section σ is given by:

$$\begin{aligned}\sigma =& \frac{\beta\sqrt{m_{\tilde{\mathrm{q}}} m_{\tilde{\mathrm{g}}}}}{3072\,\pi^2\left(m_{\tilde{\mathrm{q}}}+m_{\tilde{\mathrm{g}}}\right)^3}\int |\mathcal{M}|^2\;\mathrm{d}\Omega \\ =& \frac{\beta\sqrt{m_{\tilde{\mathrm{q}}} m_{\tilde{\mathrm{g}}}}}{1536\,\pi\left(m_{\tilde{\mathrm{q}}}+m_{\tilde{\mathrm{g}}}\right)^3}\int_{-1}^{1} |\mathcal{M}|^2\;\mathrm{d}\cos\theta\,,\end{aligned} \tag{5.19}$$

where in the last step the integral over ϕ could be performed because the process exhibits azimuthal symmetry (i.e. $\mathcal{M}$ does not depend on ϕ).

Chapter 6

Next-to-Leading Order Corrections

The aim of this thesis is to calculate the NLO cross-section to first order in β for squark-gluino production. The reason for this was discussed in chapter 4. The NLO corrections consist of real and virtual contributions. Real contributions change the particle content of the final state by the addition of one or more extra particles. Virtual contributions do not change the final state, but involve the exchange of extra internal (virtual) particles that cannot be detected directly and need not be on their mass shells. Diagrammatically this corresponds to the addition of internal lines in the LO diagrams, resulting in the occurrence of closed loops. These diagrams are referred to as "loop diagrams".

The real corrections for squark-gluino production comprise gluon radiation (the addition of one extra gluon in the LO diagrams) and the emission of "massless" quarks [9]. In this thesis the virtual corrections will be dealt with. The virtual part covers self-energy corrections, vertex corrections and box diagrams. In the next section these corrections will be explained in more detail. We are interested in the virtual contributions to the NLO cross-section without the so-called "Coulomb term". The latter originates from the exchange of gluons between the slowly moving particles in the final state[1] and these corrections give rise to a singular factor $\propto 1/\beta$ in the matrix element [9].

6.1 Scalar Integrals

The virtual corrections at NLO comprise various one-loop diagrams, which give rise to scalar integrals[2]. In the loops the complete SUSY-QCD spectrum is used, thus all quarks, squarks, gluons and gluinos[3].

Some examples of different one-loop diagrams are now given and the nomenclature of the corresponding integrals will be elucidated. For most diagrams the notation of the associated integral is not uniquely determined at all. In the next chapter this freedom is exploited to reduce the number of integrals. For each line in the Feynman diagrams the corresponding four-momentum is shown, with the arrow denoting its direction. The meaning of the various lines can be found in the appendix.

[1]One could regard this as a bound state effect.

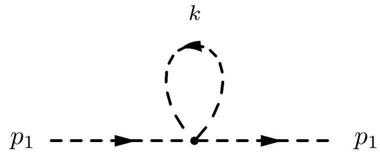
[2]Actually the one-loop diagrams give rise to tensor integrals, but with special techniques these integrals have been simplified to scalar integrals.

[3]Recall that the top squark is excluded from the final state. It *is* included inside the loops for consistency. In addition, it is assumed that the top squark has the same mass as the other squarks.

- **Self-Energy Corrections:**

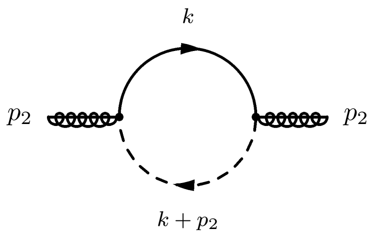
The first type:

$A_0\,(m_{\tilde{\mathrm{q}}})$

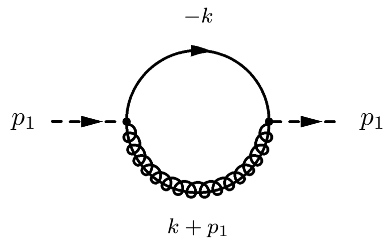


The second type:

$B_0\,(p_2, 0, m_{\tilde{\mathrm{q}}})$

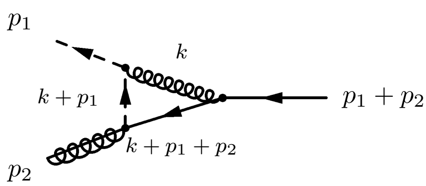


$B_0\,(p_1, 0, m_{\tilde{\mathrm{g}}})$

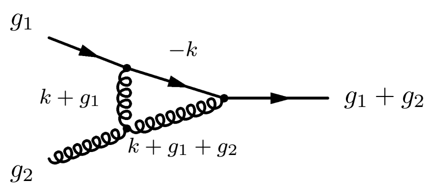


- **Vertex Corrections:**

$C_0\,(p_1, p_2, 0, m_{\tilde{\mathrm{q}}}, 0)$

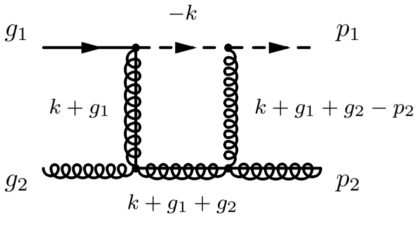


$C_0\,(g_1, g_2, 0, 0, 0)$

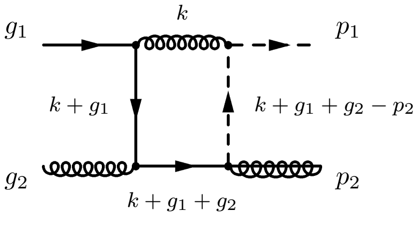


- **Box Diagrams:**

$D_0\,(g_1, g_2, -p_2, m_{\tilde{\mathrm{q}}}, m_{\tilde{\mathrm{g}}}, m_{\tilde{\mathrm{g}}}, 0)$



$D_0\,(g_1, g_2, -p_2, 0, 0, 0, m_{\tilde{\mathrm{q}}})$



The scalar integrals appear in five types (A_0, B_0, B_0', C_0 and D_0) and can depend on all four-momenta and masses. Each internal line contributes a factor $\propto 1/\left(p^2 - m^2\right)$ to the matrix element, where p represents the four-momentum and m the mass of the corresponding particle. As we can see from the diagrams above, there is an unknown four-momentum k inside the loops and because of the superposition principle we have to integrate over k. The integrals are defined as follows:

$$
\begin{aligned}
A_0\left(m_1\right) &\propto \int \frac{1}{D_1}\,\mathrm{d}^n k \\
B_0\left(p_1, m_1, m_2\right) &\propto \int \frac{1}{D_1 D_2}\,\mathrm{d}^n k \\
C_0\left(p_1, p_2, m_1, m_2, m_3\right) &\propto \int \frac{1}{D_1 D_2 D_3}\,\mathrm{d}^n k \\
D_0\left(p_1, p_2, p_3, m_1, m_2, m_3, m_4\right) &\propto \int \frac{1}{D_1 D_2 D_3 D_4}\,\mathrm{d}^n k\,,
\end{aligned}
\tag{6.1}
$$

with

$$
\begin{aligned}
D_1 &= k^2 - m_1^2 \\
D_2 &= \left(k + p_1\right)^2 - m_2^2 \\
D_3 &= \left(k + p_1 + p_2\right)^2 - m_3^2 \\
D_4 &= \left(k + p_1 + p_2 + p_3\right)^2 - m_4^2\,.
\end{aligned}
\tag{6.2}
$$

The B_0' integral is somewhat different and related to the B_0 integral:

$$
B_0'\left(p, m_1, m_2\right) = \frac{\mathrm{d}B_0\left(p, m_1, m_2\right)}{\mathrm{d}p^2}\,. \tag{6.3}
$$

The integrals diverge in 4 dimensions and that is why they are performed in $n = 4 + \epsilon$ dimensions, with $\epsilon \to 0$ at the end of the calculation. We will come back to this in chapter 8.

Chapter 7

Calculation

The virtual contribution to the NLO cross-section has to be calculated up to first order in β. Ultimately, the squark and gluino mass should be the only unknowns in this expression. The calculation of the full NLO cross-section has already been carried out, but there is no analytical expression for the threshold region yet. The latter will be obtained in this thesis.
All computations are carried out with the symbolic computer program FORM [10] and the computational program Mathematica [11].

In order to obtain the cross-section up to first order in β, the matrix element squared has to be calculated at zeroth order in β, according to equation (5.19). The expression for $|\mathcal{M}|^2$ is written in terms of scalar integrals and prefactors containing invariants (among these invariants are also the Mandelstam variables). We want to compute $|\mathcal{M}|^2$ at zeroth order in β, so why not just plug in the threshold values for the Mandelstam variables and expand all the integrals up to zeroth order in β? There are several reasons why this would not work.
First this chapter will explain how the expansion of the prefactors works. Next we will consider how to cope with the imaginary parts of the expression. Subsequently, a number of tricks will be described that can be applied to the integrals. The Coulomb integrals will be dealt with separately in the subsequent section. Finally, it is described how we can further simplify the expression for the cross-section.

7.1 Expansion of the Mandelstam Variables

The expression for $|\mathcal{M}|^2$ contains several singular factors (like $\propto 1/\beta$). Therefore, it is clear that we better not put β equal to zero at this stage. The only true $1/\beta$ dependence in $|\mathcal{M}|^2$ should come from the Coulomb term and enters through the scalar integrals. Initially, when the integrals are written as A_0, B_0, B_0', C_0 or D_0, there should not be any true β divergences. This means that every β in a denominator should be accompanied by a β in the numerator and thus can be divided out.

Putting the integrals aside for the moment, it turns out that the largest divergences are factors that are proportional to $1/\beta^2$. These divergences originate from the factor

$$\frac{1}{s^2 + m_{\tilde{g}}^4 + m_{\tilde{q}}^4 - 2sm_{\tilde{g}}^2 - 2sm_{\tilde{q}}^2 - 2m_{\tilde{g}}^2 m_{\tilde{q}}^2}\,.$$

The largest divergence that will enter through the integrals is a $1/\beta$ pole, which comes from the Coulomb integrals. Consequently, all factors need to be expanded up to third order in β in order to obtain $|\mathcal{M}|^2$ at zeroth order in β.

After inserting the third order expansions of the Mandelstam variables (and their reciprocals), it turns out that there are still $1/\beta$ singularities left over. This implies that a zeroth order expansion

of the integrals is not sufficient. We should rather expand them all up to first order in β, which is much more work. Might there be a way out of this inconvenience?
It appears that the divergent factors only combine with integrals that do not depend on the scattering angle θ. Thus all the $1/\beta$ terms could only have a θ dependence in the factors and not in the integrals. Ultimately, we are interested in the cross-section and not in the square of the matrix element. From equation (5.19), it follows that

$$\sigma \propto \int_{-1}^{1} |\mathcal{M}|^2 \,\mathrm{d}\cos\theta\,. \tag{7.1}$$

Consequently, we see that all terms in $|\mathcal{M}|^2$ that are proportional to an odd power of $\cos\theta$ will vanish when we turn to the cross-section. It turns out that all $1/\beta$ terms are proportional to $\cos\theta$. As a result, it is unnecessary to perform first order expansions of the integrals. Fortunately, zeroth order in β suffices!

7.2 Imaginary Parts

Logarithms and dilogarithms are abundant in the integrals. A dilogarithm is defined as follows:

$$\mathrm{Li}_2(z) \equiv -\int_0^z \frac{\log(1-u)}{u}\,\mathrm{d}u, \qquad \forall\, z \in \mathbb{C} \setminus [1,\infty)\,. \tag{7.2}$$

Sometimes the argument of a logarithm is negative or the argument of a dilogaritm is greater than one. This yields an imaginary part in the outcome. A single function of such a form is not a problem, since we are ultimately only interested in the real part as $|\mathcal{M}|^2$ is real. However, when two of these functions are multiplied with each other, the imaginary parts combine into an additional real part. That is why the imaginary parts should be treated with care!
A dilogarithm with an argument greater than one can always be turned into a dilogarithm with an argument smaller than one. The imaginary part is then shifted to an additional logarithm. A logarithm of $z = a + bi$ with $a, b \in \mathbb{R}$, $a < 0$ and $b \to 0$ can be written as

$$\log(z) \equiv \log|z| + i\arg(z) = \log|z| \pm i\pi\,, \tag{7.3}$$

where the plus sign is used if $b \downarrow 0$ and the minus sign if $b \uparrow 0$.
Thus it is important to know whether the imaginary part of the argument of a (di)logarithm goes to zero from either the positive or the negative direction. Consequently, when performing the integral expansions the signs of the imaginary parts need to be determined.

An example of a dilogarithm that appears in one of the integrals:

$$\mathrm{Li}_2\left(1 + \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}\frac{m_{\tilde{q}}^2 - m_{\tilde{g}}^2 - i\delta}{t - m_{\tilde{g}}^2 + i\delta}\right),$$

where $\delta \downarrow 0$. After separating the argument of the dilogarithm into a real and an imaginary part, we obtain:

$$\mathrm{Li}_2\left(1 + \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}\frac{m_{\tilde{q}}^2 - m_{\tilde{g}}^2 - i\delta}{t - m_{\tilde{g}}^2 + i\delta}\right) = \mathrm{Li}_2\left[\frac{m_{\tilde{q}}^3 + m_{\tilde{g}}^3}{m_{\tilde{q}}^3 + m_{\tilde{q}}^2 m_{\tilde{g}}} + \frac{m_{\tilde{g}}^2\left(m_{\tilde{q}}m_{\tilde{g}} - m_{\tilde{q}}^2 + 2m_{\tilde{g}}^2\right)}{m_{\tilde{q}}^2\left(m_{\tilde{q}}m_{\tilde{g}} + m_{\tilde{g}}^2\right)^2}\,i\delta\right]. \tag{7.4}$$

We used that $t = -m_{\tilde{q}} m_{\tilde{g}}$ at threshold, which follows from equation (5.15). If the real part is smaller than one, we can directly put δ equal to zero. However, since $m_{\tilde{q}}$ and $m_{\tilde{g}}$ are unknown, we cannot tell whether the real part is greater or smaller than one. If the real part is greater than one, then

$$m_{\tilde{q}}^3 + m_{\tilde{g}}^3 > m_{\tilde{q}}^3 + m_{\tilde{q}}^2 m_{\tilde{g}} \quad \Rightarrow \quad m_{\tilde{g}}^2 > m_{\tilde{q}}^2 . \tag{7.5}$$

In this case the following applies to the imaginary part:

$$m_{\tilde{q}} m_{\tilde{g}} - m_{\tilde{q}}^2 + 2m_{\tilde{g}}^2 > m_{\tilde{q}} m_{\tilde{g}} + m_{\tilde{g}}^2 > 0 . \tag{7.6}$$

Thus we conclude:

$$\lim_{\delta \to 0} \mathrm{Li}_2 \left(1 + \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2} \frac{m_{\tilde{q}}^2 - m_{\tilde{g}}^2 - i\delta}{t - m_{\tilde{g}}^2 + i\delta} \right) = \lim_{\delta \to 0} \mathrm{Li}_2 \left(\frac{m_{\tilde{q}}^3 + m_{\tilde{g}}^3}{m_{\tilde{q}}^3 + m_{\tilde{q}}^2 m_{\tilde{g}}} + i\delta \right) , \tag{7.7}$$

where we used that only the *sign* of the imaginary part matters, since δ goes to zero anyway.

7.3 Tricks with Integrals

There are a lot of integrals in the expression. Fortunately, there are several methods by which the number of different integrals can be reduced. For instance, some integrals can be expressed in simpler ones by means of a partial fraction decomposition. Also through integral transformations one can show that some integrals are in fact identical. But before addressing these tricks, we first make use of a significant overall simplification.

Might for example the integrals $C_0(-g_1, p_1, 0, 0, m_{\tilde{g}})$ and $C_0(-g_2, p_1, 0, 0, m_{\tilde{g}})$ be equal? First we should realize that the integrals are scalar quantities and can therefore solely depend on scalars and not on four-vectors. The only way to obtain a scalar quantity from a four-vector, is by means of an inner product. This means that $C_0(-g_1, p_1, 0, 0, m_{\tilde{g}})$ can only depend on $g_1^2 = 0$, $p_1^2 = m_{\tilde{q}}^2$ and t, while $C_0(-g_2, p_1, 0, 0, m_{\tilde{g}})$ depends only on $g_2^2 = 0$, $p_1^2 = m_{\tilde{q}}^2$ and u. At threshold $t = u$ applies, which follows from equation (5.15). As a result, these integrals must be identical! More generally we can set $g_1 = g_2$ in most integrals.

7.3.1 Partial Fraction Decomposition

At threshold, sometimes a partial fraction decomposition can be used to express an integral in several simpler integrals. This method is easiest elucidated with an example. Let us consider the following four D_0 integrals:

$$\begin{array}{lcl} D_0(p_2, -g_2, p_1, m_{\tilde{g}}, 0, 0, m_{\tilde{q}}) , & & D_0(p_2, -g_2, p_1, m_{\tilde{q}}, 0, 0, m_{\tilde{g}}) , \\ D_0(p_2, -g_2, p_1, 0, m_{\tilde{g}}, m_{\tilde{g}}, 0) & \text{and} & D_0(p_2, -g_2, p_1, 0, m_{\tilde{q}}, m_{\tilde{q}}, 0) . \end{array} \tag{7.8}$$

All these integrals depend on the same four-momenta, but the mass dependences differ. We can generically write these four integrals as:

$$D_0(p_2, -g_2, p_1, m_1, m_2, m_3, m_4) \propto \int \frac{1}{D_1 D_2 D_3 D_4} \mathrm{d}^n k , \tag{7.9}$$

where m_i with $i \in \{1,2,3,4\}$ equals 0, $m_{\tilde{\mathrm{q}}}$ or $m_{\tilde{\mathrm{g}}}$ and

$$\begin{aligned} D_1 &= k^2 - m_1^2 \\ D_2 &= \left[k + m_{\tilde{\mathrm{g}}} p_1 / m_{\tilde{\mathrm{q}}}\right]^2 - m_2^2 \\ D_3 &= \left[k + m_{\tilde{\mathrm{g}}} p_1 / m_{\tilde{\mathrm{q}}} - g_2\right]^2 - m_3^2 \\ D_4 &= \left[k + \left(1 + m_{\tilde{\mathrm{g}}}/m_{\tilde{\mathrm{q}}}\right) p_1 - g_2\right]^2 - m_4^2\,, \end{aligned} \tag{7.10}$$

where we used that at threshold

$$p_2 = \frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}}\, p_1\,. \tag{7.11}$$

We can now use a partial fraction decomposition to decompose these D_0 integrals into four C_0 integrals:

$$\begin{aligned} \frac{1}{D_1 D_2 D_3 D_4} &= \frac{A}{D_1 D_2 D_3} + \frac{B}{D_1 D_2 D_4} + \frac{C}{D_1 D_3 D_4} + \frac{D}{D_2 D_3 D_4} \\ &= \frac{A D_4 + B D_3 + C D_2 + D D_1}{D_1 D_2 D_3 D_4}\,, \end{aligned} \tag{7.12}$$

where the coefficients A, B, C and D need to be solved. We obtain:

$$\begin{aligned} 1 =& A\left\{\left[k + \left(1 + m_{\tilde{\mathrm{g}}}/m_{\tilde{\mathrm{q}}}\right) p_1 - g_2\right]^2 - m_4^2\right\} + B\left\{\left[k + m_{\tilde{\mathrm{g}}} p_1/m_{\tilde{\mathrm{q}}} - g_2\right]^2 - m_3^2\right\} \\ & + C\left\{\left[k + m_{\tilde{\mathrm{g}}} p_1/m_{\tilde{\mathrm{q}}}\right]^2 - m_2^2\right\} + D\left\{k^2 - m_1^2\right\} \\ =& k^2\left[A+B+C+D\right] + 2k\cdot p_1\left[A + (A+B+C)\, m_{\tilde{\mathrm{g}}}/m_{\tilde{\mathrm{q}}}\right] - 2k\cdot g_2\left[A+B\right] \\ & + p_1^2\left[A\left(1 + 2m_{\tilde{\mathrm{g}}}/m_{\tilde{\mathrm{q}}}\right) + (A+B+C)\, m_{\tilde{\mathrm{g}}}^2/m_{\tilde{\mathrm{q}}}^2\right] + g_2^2\left[A+B\right] \\ & - 2g_2\cdot p_1\left[A + (A+B)\, m_{\tilde{\mathrm{g}}}/m_{\tilde{\mathrm{q}}}\right] - A m_4^2 - B m_3^2 - C m_2^2 - D m_1^2\,, \end{aligned} \tag{7.13}$$

where in the last step the expression was cast in a more convenient form. The terms that contain k have to be equal to zero, for k is the integration variable. Consequently A, B, C and D can be solved from the following system of four equations:

$$\begin{aligned} 0 =& A + B + C + D \\ 0 =& 2\left[A + (A+B+C)\, m_{\tilde{\mathrm{g}}}/m_{\tilde{\mathrm{q}}}\right] \\ 0 =& -2\,(A+B) \\ 1 =& A m_{\tilde{\mathrm{q}}}\left(m_{\tilde{\mathrm{q}}} + 2m_{\tilde{\mathrm{g}}}\right) + (A+B+C)\, m_{\tilde{\mathrm{g}}}^2 - \left(m_{\tilde{\mathrm{q}}} + m_{\tilde{\mathrm{g}}}\right)\left[A m_{\tilde{\mathrm{q}}} + (A+B)\, m_{\tilde{\mathrm{g}}}\right] \\ & - A m_4^2 - B m_3^2 - C m_2^2 - D m_1^2\,. \end{aligned} \tag{7.14}$$

We used that $p_1^2 = m_{\tilde{\mathrm{q}}}^2$, $g_2^2 = 0$ and $2g_2\cdot p_1 = g_2^2 + p_1^2 - (g_2 - p_1)^2 = m_{\tilde{\mathrm{q}}}^2 - u = m_{\tilde{\mathrm{q}}}\left(m_{\tilde{\mathrm{q}}} + m_{\tilde{\mathrm{g}}}\right)$. Now we obtain for A, B, C and D:

$$B = -\,A = \frac{m_{\tilde{g}}}{m_1^2 m_{\tilde{q}} - m_2^2 m_{\tilde{q}} - m_3^2 m_{\tilde{g}} + m_4^2 m_{\tilde{g}}}$$

$$C = -\,D = \frac{m_{\tilde{q}}}{m_1^2 m_{\tilde{q}} - m_2^2 m_{\tilde{q}} - m_3^2 m_{\tilde{g}} + m_4^2 m_{\tilde{g}}} \,. \tag{7.15}$$

Now we have succeeded in decomposing the D_0 integrals into easier C_0 integrals. Of course, a partial fraction decomposition is only useful if the C_0 integrals already occur in the expression. Otherwise one has to expand *four* C_0 integrals instead of one D_0 integral, which is definitely more time-consuming. In the next subsection we show how to use integral transformations to bring the obtained C_0 integrals into forms that are already present in the expression.
Besides, a partial fraction decomposition is not merely applicable to some D_0 integrals. Also C_0 integrals that contain both p_1 and p_2 can be decomposed (into three B_0 integrals). Coulomb integrals cannot be decomposed, since the coefficients become singular.

7.3.2 Integral Transformations

It has already been noted in chapter 6 that the nomenclature for the integrals is not unique. One can use integral transformations to show that several integrals are in fact equal. In order to clarify this, let us consider the four C_0 integrals from the previous subsection, which are defined by equation (7.12). None of these integrals seem to occur in the expression, but by redefining the integration variables, we obtain integrals that *are* already present. An example:

$$\int \frac{\mathrm{d}^n k}{D_1 D_2 D_4} = \int \frac{\mathrm{d}^n k}{[k^2 - m_1^2][(k+p_2)^2 - m_2^2][(k+p_2-g_2+p_1)^2 - m_4^2]} \,. \tag{7.16}$$

By applying conservation of four-momentum we obtain:

$$\int \frac{\mathrm{d}^n k}{D_1 D_2 D_4} = \int \frac{\mathrm{d}^n k}{[k^2 - m_1^2][(k+p_2)^2 - m_2^2][(k+g_1)^2 - m_4^2)]} \tag{7.17}$$

and by sending $k \to k - g_1$ we conclude:

$$\int \frac{\mathrm{d}^n k}{D_1 D_2 D_4} = \int \frac{\mathrm{d}^n k}{[k^2 - m_4^2][(k-g_1)^2 - m_1^2][(k-g_1+p_2)^2 - m_2^2]}$$

$$\propto C_0\,(-g_1, p_2, m_4, m_1, m_2) \,. \tag{7.18}$$

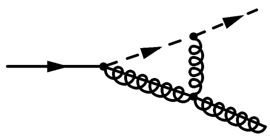
The latter integral already occurs in the expression. After performing a partial fraction decomposition and integral transformations the D_0 integral (7.9) can be written as follows:

$$\begin{aligned} D_0\,(p_2, -g_2, p_1, m_1, m_2, m_3, m_4) = {} & A \times C_0\,(-g_2, p_2, m_3, m_2, m_1) + \\ & B \times C_0\,(-g_1, p_2, m_4, m_1, m_2) + \\ & C \times C_0\,(-g_1, p_1, m_1, m_4, m_3) + \\ & D \times C_0\,(-g_2, p_1, m_2, m_3, m_4) \,, \end{aligned} \tag{7.19}$$

with A, B, C and D given by equation (7.15).

7.4 Coulomb Integrals

All Coulomb integrals are proportional to $1/\beta$. Hence, we are not allowed to simply insert all the threshold values for the Mandelstam variables, as we do for the other integrals. As an example, let us have a look at the following vertex correction:



The squark and the gluino exchange a gluon. Consequently this diagram gives rise to a Coulomb integral, namely [12]:

$$C_0\left(p_1, p_2, m_{\tilde{\mathrm{q}}}, 0, m_{\tilde{\mathrm{g}}}\right) = \frac{x_\mathrm{s}}{m_{\tilde{\mathrm{q}}} m_{\tilde{\mathrm{g}}}\left(1 - x_\mathrm{s}^2\right)} \left\{ \log\left(x_\mathrm{s}\right)\left[\frac{2}{\epsilon} + \log\left(\frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}}\right) - \frac{1}{2}\log\left(x_\mathrm{s}\right)\right] + \frac{1}{2}\log\left(\frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}}\right) \right.$$
$$\left. -\mathrm{Li}_2\left(1 - x_\mathrm{s}^2\right) + \mathrm{Li}_2\left(1 - \frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}} x_\mathrm{s}\right) + \mathrm{Li}_2\left(1 - \frac{m_{\tilde{\mathrm{q}}}}{m_{\tilde{\mathrm{g}}}} x_\mathrm{s}\right) \right\}, \tag{7.20}$$

where

$$x_\mathrm{s} = \frac{\beta_\mathrm{s} - 1}{\beta_\mathrm{s} + 1} \qquad \text{and} \qquad \beta_\mathrm{s} = \frac{m_{\tilde{\mathrm{q}}} + m_{\tilde{\mathrm{g}}}}{2\sqrt{m_{\tilde{\mathrm{q}}} m_{\tilde{\mathrm{g}}}}}\,\beta\,. \tag{7.21}$$

In this expression only x_s depends on β. The $1/\beta$ proportionality comes from the factor $1/\left(1 - x_\mathrm{s}^2\right)$. Thus, in order to get the proper zeroth order expression for this integral, we should determine $1/\left(1 - x_\mathrm{s}^2\right)$ to zeroth order in β, while the remaining β dependent parts should be expanded up to first order in β.

7.5 Further Simplifications

After performing the expansions we obtain an expression for $|\mathcal{M}|^2$ at zeroth order in β. By using equation (5.19) we find the required expression for the cross-section. However, the obtained expression is quite cumbersome[1]. Therefore it is desirable to make further simplifications.

Inasmuch as FORM is not able to simplify fractions with numerators consisting of more than one term, we have to make several definitions to ensure that a numerator always consists of only one term. An example of such a definition is the average mass m_av:

$$m_\mathrm{av} \equiv \frac{m_{\tilde{\mathrm{q}}} + m_{\tilde{\mathrm{g}}}}{2}\,. \tag{7.22}$$

The following identities give the relation between m_av, $m_{\tilde{\mathrm{q}}}$ and $m_{\tilde{\mathrm{g}}}$ in a slightly different way:

$$\frac{m_{\tilde{\mathrm{g}}}}{m_\mathrm{av}} = 2 - \frac{m_{\tilde{\mathrm{q}}}}{m_\mathrm{av}} \qquad \text{and} \qquad \frac{m_{\tilde{\mathrm{q}}}}{m_\mathrm{av}} = 2 - \frac{m_{\tilde{\mathrm{g}}}}{m_\mathrm{av}}\,. \tag{7.23}$$

The expression can be simplified further by inserting these identities.

[1]Think of an expression of a several hundred pages long!

FORM cannot, for example, simplify (di)logarithms and square roots. Thus it is convenient to insert identities which involve such functions. For instance one can make use of

$$\mathrm{Li}_2\left(1+\frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}}-i\delta\right)=-\mathrm{Li}_2\left(-\frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}}\right)-\log\left(-\frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}}+i\delta\right)\log\left(1+\frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}}\right)+\frac{\pi^2}{6} \tag{7.24}$$

and

$$\log\left(-\frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}}+i\delta\right)=\log\left(\frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}}\right)+i\pi\,. \tag{7.25}$$

To ensure that more logarithms can combine, it is also fruitful to normalize all of them to the same variable. For example the arguments of all logarithms can be expressed in terms of m_{av}, so for instance:

$$\log\left(\frac{m_{\tilde{\mathrm{g}}}}{m_{\tilde{\mathrm{q}}}}\right)=\log\left(\frac{m_{\mathrm{av}}}{m_{\tilde{\mathrm{q}}}}\right)-\log\left(\frac{m_{\mathrm{av}}}{m_{\tilde{\mathrm{g}}}}\right)\,. \tag{7.26}$$

Of course we want to check whether we have arrived at the correct analytical expression for the cross-section in the threshold region. It is possible to perform a numerical check. This will be discussed in the next chapter.

Chapter 8

Numerical Results

At the very end, the analytical expression obtained can be checked with the Fortran-program PROSPINO [13]. Before this numerical check is carried out, we first check whether there are some singularities left over in the expression. Recall that the integrals have been performed in $n = 4 + \epsilon$ dimensions, where $\epsilon \to 0$. As a result, several ϵ-poles show up. However, these singularities should be fake. In order to make sure that all ϵ-poles cancel out exactly, a lot of additional simplifications are implemented. Ultimately, all ϵ-poles drop out. The Coulomb term is already known from [4] and is in agreement with the one we have found.

8.1 The Numerical Check

The so-called "virtual-soft scaling function" $f^{\mathrm{V+S}}$ is defined as follows [4]:

$$f^{\mathrm{V+S}} \equiv \frac{m_{\mathrm{av}}^2}{4\pi\alpha_{\mathrm{s}}^3}\,\sigma\,, \tag{8.1}$$

where σ represents in this case the virtual part (V) of the NLO cross-section in the threshold region without the Coulomb term. Also the contributions from (real) radiation of soft-gluons (S) are included, since these gluons cannot be detected separately. After β has been divided out, the dimensionless scaling function $f^{\mathrm{V+S}}$ can be checked numerically. Since we have an expression for the threshold region, also PROSPINO must perform the limit $\beta \to 0$. Just taking $\beta = 0$ is not possible, but $\beta > 0$ can always be set smaller.
The only unknowns in our expression for the cross-section are the squark and gluino masses. In addition, there is a third mass dependence, since the expression also contains the mass of the top quark (m_{t}). However, this mass is known and is set to 175 GeV.
Since the ordering of the masses determines whether the outcome of a (di)logarithm has an imaginary part, the final answer must be checked for all possible orderings of $m_{\tilde{\mathrm{q}}}$, $m_{\tilde{\mathrm{g}}}$ and m_{t}. By inserting specific values for the masses $m_{\tilde{\mathrm{q}}}$ and $m_{\tilde{\mathrm{g}}}$ in the expression, we get a numerical answer. If the answer provided by PROSPINO matches for all mass orderings, it means that the analytical expression is correct. From figure (8.1) it can be inferred that our expression is correct for at least one ordering of the masses.

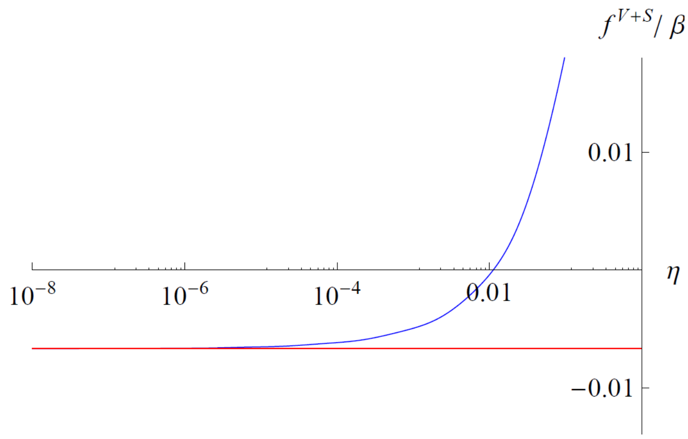


Figure 8.1: On the vertical axis $f^{\mathrm{V+S}}$ is shown after β has been divided out. The red line represents the value resulting from our expression, while the blue line is generated by PROSPINO. The threshold limit $\eta \equiv \beta^2/\left(1-\beta^2\right) \to 0$ is shown on the horizontal axis. For this graph $m_{\tilde{\mathrm{q}}} = 450$ GeV and $m_{\tilde{\mathrm{g}}} = 500$ GeV are used. Thus the ordering $m_{\mathrm{t}} < m_{\tilde{\mathrm{q}}} < m_{\tilde{\mathrm{g}}}$ is checked.

The results from PROSPINO converge to our results for *all* mass orderings. Thus we conclude that we have obtained a correct analytical expression.

8.2 The Result

To compare the LO and NLO cross-sections quantitatively, we can study the so-called K-factor. In the threshold region this factor is defined as [4]:

$$K^{\mathrm{thr}} \equiv \frac{\sigma_{\mathrm{NLO}}^{\mathrm{thr}}}{\sigma_{\mathrm{LO}}^{\mathrm{thr}}}\,. \tag{8.2}$$

For the computation of K^{thr}, also the real contributions must be included. Figure (8.2) shows the behaviour of K^{thr} when the squark mass is varied.

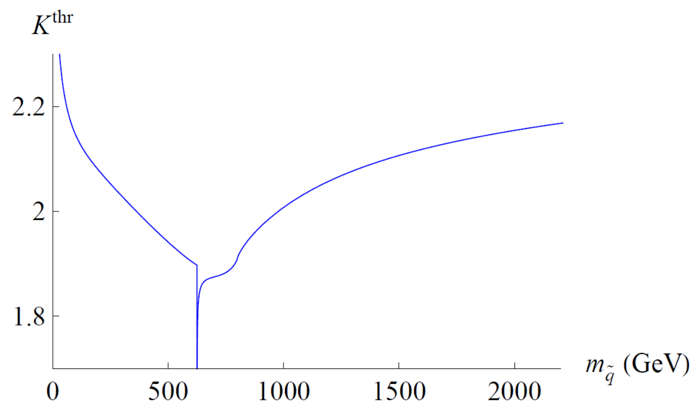


Figure 8.2: The K-factor at threshold versus the squark mass. The gluino mass is set to 800 GeV.

The peak at $m_{\tilde{\mathrm{q}}} = 625$ GeV in figure (8.2) arises from the fact that the unknown lifetime of the gluino has been set to infinity. This peak disappears if a finite lifetime is taken into account. Obviously, the NLO contributions to the cross-section are significant. For some squark masses they are even more important than the LO contributions!

Chapter 9

Conclusion

In this thesis we have calculated the virtual part of the NLO cross-section for the production of squark-gluino pairs at the kinematic threshold. We have obtained the correct analytical expression up to first order in the threshold variable β. We conclude that at threshold the NLO contributions are of the same order of magnitude as the LO contributions.

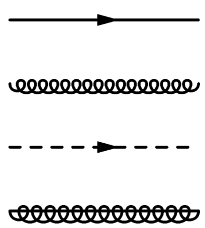
The NLO cross-section at threshold is an important ingredient in the calculation of the NNLL corrections. A next step is to complete these corrections. This result then has to be combined with the NNLL results for other processes involving squarks and gluinos.

If squarks and gluinos are not found, the NNLL corrections can be used to raise the lower limits of the squark and gluino masses. If they are found, the improved knowledge about the cross-section can be used to improve the determination of the squark and gluino masses, by comparing the theoretical predictions to experimental data.

Appendix A

Diagrammatic Notation

Every particle has its own type of line in a Feynman diagram. The conventions for the SUSY-QCD particles are as follows:



quark (q)

gluon (g)

squark ($\tilde{q}$)

gluino ($\tilde{g}$)

Particles flow along the arrow, whereas the corresponding antiparticles flow against it. Since the gluon and gluino are their own antiparticle, there is no arrow shown.

Abbreviations

CM	Center of Mass
GUT	Grand Unified Theory
LHC	Large Hadron Collider
LL	Leading Logarithmic
LO	Leading Order
LSP	Lightest Supersymmetric Particle
MSSM	Minimal Supersymmetric Standard Model
NLL	Next-to-Leading Logarithmic
NLO	Next-to-Leading Order
NNLL	Next-to-Next-to-Leading Logarithmic
SM	Standard Model
SUSY	Supersymmetry
SUSY-QCD	Supersymmetric Quantum Chromodynamics
WIMP	Weakly Interacting Massive Particle

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