BTZ and LST

– a picture of the bulk –

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Revisiting some DVV classics . . .

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BPS Spectrum of the Five-Brane and Black Hole Entropy

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Abstract

We propose a formulation of 11-dimensional M-theory in terms of five-branes with closed strings on their world-volume. We use this description to construct the complete spectrum of BPS states in compactifications to six and five dimensions. We compute the degeneracy for fixed charge and find it to be in accordance with U-duality (which in our formulation is manifest in six dimensions) and the statistical entropy formula of the corresponding black hole. We also briefly comment on the compactification to four dimensions.

Fivebranes & Strings

- The BPS states of n_5 fivebranes wrapped on $\mathcal{M}\times\mathbb{S}^1_y$ ($\mathcal{M}=\mathbb{T}^4$ or K3) include those of n_1 strings wrapping $\mathbb{S}^1_y$ and wiggling along the remaining compactification (Strominger-Vafa '96, **Dijkgraaf-Verlinde² '96,'97**)
- In a "weak coupling" cusp of the moduli space, these BPS states match those of a symmetric product orbifold $(\mathcal{M}^N)/S_N$ describing a Fock space of free "*little strings*", where $N=n_1 n_5$
- These *little strings* that account for the entropy are *fractionated* strings – the fivebranes break ordinary strings into n_5 pieces.
- In **D1**-**D5** at weak coupling these fractional strings are instantons in the **D5** SYM theory
- In M-theory they come from **M2**-branes (the lift of IIA **F1**'s) breaking on the **M5**'s (the lift of IIA **NS5**'s)

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Elliptic Genera of Symmetric Products and Second Quantized Strings

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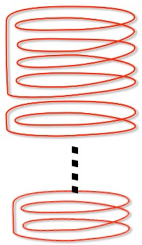
In this note we prove an identity that equates the elliptic genus partition function of a supersymmetric sigma model on the N-fold symmetric product M^N/S_N of a manifold M to the partition function of a second quantized string theory on the space $M \times S^1$. The generating function of these elliptic genera is shown to be (almost) an automorphic form for $O(3, 2, \mathbf{Z})$. In the context of D-brane dynamics, this result gives a precise computation of the free energy of a gas of D-strings inside a higher-dimensional brane.

BPS states of the symmetric product

- It's useful to focus on BPS states, as these are robust across moduli space and might allow us to connect the weakly coupled brane picture of the symmetric product to the supergravity regime

- The general **½-BPS** ground state $|\{N_k^I\}\rangle$ is labelled by a decorated conjugacy class of the symmetric group – a tensor product of N_k cyclic twists sewing together k copies of the CFT on $\mathcal{M}=\mathbb{T}^4$ or K3; each cycle can be in any ½-BPS ground state of $\mathcal{M}$ labeled by I, and $\sum_k kN_k^I = N = n_1 n_5$



- For **¼-BPS** states, apply your favorite collection of fractionated, left-moving oscillator excitations to each cycle

$$\big|\{n_{\ell,a}\},\{m_{j,\beta}\},I\big\rangle_{k\cdot cycle} = \prod_{\ell,a;j,\beta} (\alpha^a_{-\ell/k})^{n_{\ell,a}} (\psi^\beta_{-j/k})^{m_{j,\beta}} \big|I\big\rangle_k$$

- Symmetrize over identically excited cycles

Whither the *little string* ?

- Where is all this stringy physics in the bulk dynamics, in the gravity regime of the moduli space?

- Does it all collapse into the black hole singularity of the effective gravity theory, hidden behind a horizon?

- If so, why does the gravity description seem to retain structures reminiscent of an underlying brane bound state picture of the black hole?

- Are there instead stringy effects that persist out to the horizon scale? The fuzzball paradigm says yes . . .

- If so, do such effects resolve the pesky and persistent information puzzles associated to black holes?

NS5-F1 ½-BPS ground states

- The general **½-BPS** string condensate on a stack of **NS5**'s has N_k^I excitations of winding number k w/choice of 8B+8F polarizations I (for $\mathbb{T}^4$);

$$\alpha, \dot\beta, A, B = \pm$$

$$|\Psi\rangle = \prod_{\substack{k,\ell,p,q \\ pol's\ I}} \overbrace{\Big(\underbrace{|\alpha\dot\beta\rangle}_{\perp\ \text{scalars}}{}_k\Big)^{N_k^{\alpha\dot\beta}} \Big(\underbrace{|AB\rangle}_{\text{internal}}{}_\ell\Big)^{N_\ell^{AB}}}^{\text{bosons}} \overbrace{\Big(|\alpha B\rangle_p\Big)^{N_p^{\alpha B}} \Big(|A\dot\beta\rangle_q\Big)^{N_q^{A\dot\beta}}}^{\text{fermions}}$$

$$\textstyle\sum_{k,I} k N_k^I = N = n_1 n_5$$

internal (NS5 gauge multiplet)

mode number = ***little string*** winding

- Easily understood via T-duality to **NS5-P** where this is the Fock space of fractionated BPS momentum modes on a single NS5 wrapping $\mathbb{S}^1_{\tilde{y}}$ n_5 times.
- For K3, there are no fermionic modes, and 16 extra internal bosonic modes
- ***Each of these states corresponds to a particular horizonless geometry***

Lunin-Mathur '01, Taylor '05, Kanitscheider-Skenderis-Taylor '07

NS5-F1 supertube geometry

- The ½-BPS **NS5-F1** supertube geometry

$$ds^2 = -Z_1^{-1}\big(du+\omega\big)\big(dv+\beta\big) + Z_5\, d\mathsf{x}_\perp\cdot d\mathsf{x}_\perp + ds^2_{\mathbb{T}^4} \qquad e^{2\Phi} = g_s^2\,\frac{Z_5}{Z_1}$$

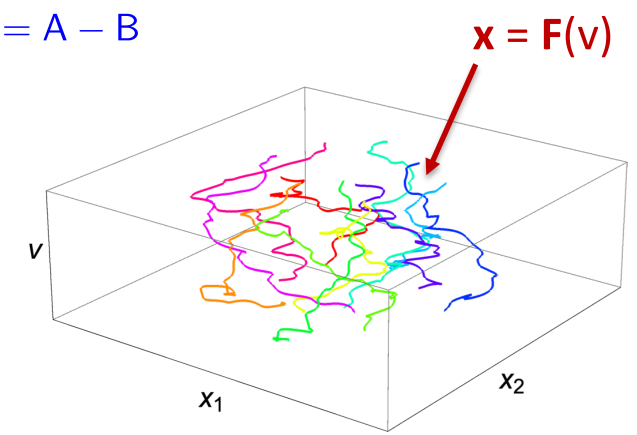
$$B = \tfrac{1}{2} Z_1^{-1}(du+\omega)\wedge(dv+\beta) + b_{ij}\, dx^i\wedge dx^j\,. \qquad db = *_\perp dZ_5$$

is specified by a set of harmonic functions and one-forms sourced by functions $\mathsf{F}^I(v)$ along a contour (Lunin-Mathur '01)

$$Z_5 = \frac{n_5}{2\pi L}\int_0^{2\pi L}\frac{dv}{|\mathbf{x}-\mathsf{F}(v)|^2} \qquad \beta = \mathsf{A}+\mathsf{B}\ , \quad \omega = \mathsf{A}-\mathsf{B}$$

$$\mathsf{A}_i = \frac{n_5}{2\pi L}\int_0^{2\pi L}\frac{dv\,\dot{F}_i(v)}{|\mathbf{x}-\mathsf{F}(v)|^2}\ , \qquad d\mathsf{B} = *_\perp d\mathsf{A}$$

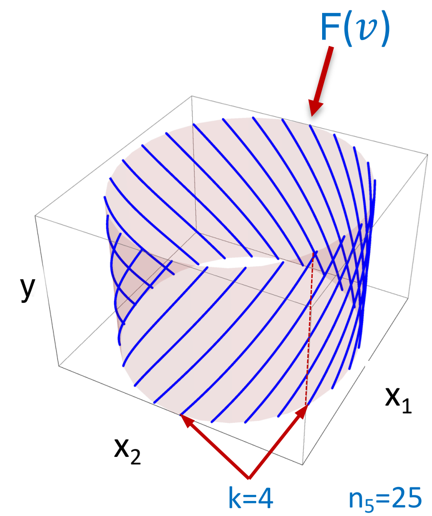
$$Z_1 = 1+\frac{n_5}{2\pi L}\int_0^{2\pi L}\frac{dv\,\dot{\mathsf{F}}_I\cdot\dot{\mathsf{F}}_I}{|\mathbf{x}-\mathsf{F}(v)|^2}$$



- ***The Fourier mode amplitudes a_k^I of $\mathsf{F}^I(v)$ are coherent state parameters specifying $\langle N_k^I\rangle$***

The Return of the String

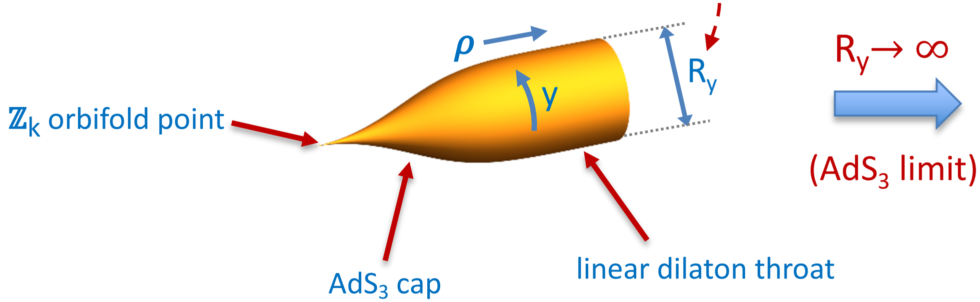
- The worldsheet **string** dynamics in the **fivebrane** throat is *exactly solvable* when the n_5 fivebranes are evenly distributed along a circle in a ⊥ plane.
  (Giveon-Kutasov '99, EJM-Massai '17)

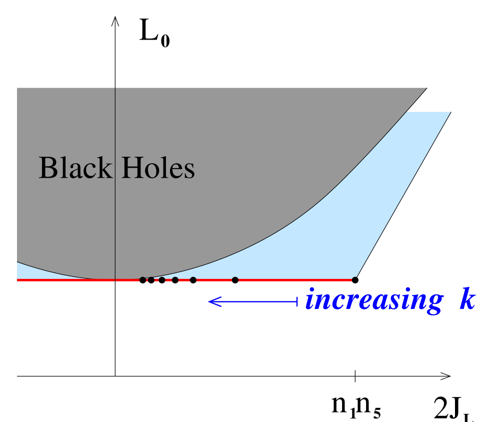


- The worldsheet theory is a gauged WZW model on

$$\frac{\mathcal{G}}{\mathcal{H}} = \frac{\left(\mathbb{R}_t \times \mathbb{S}^1_y \times \mathbb{T}^4\right)_{||} \times \left(SL(2,\mathbb{R}) \times SU(2)\right)_{\perp}}{U(1)_L \times U(1)_R}$$

where the gauge orbits under $\mathcal{H}$ are null isometries of $\mathcal{G}$





String propagation in a black hole geometry

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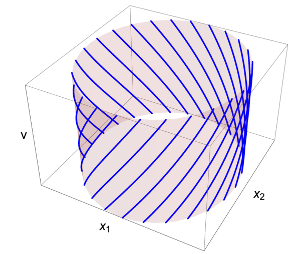
We consider string theory in the background of the two-dimensional black hole as described by the SL(2, ℝ)/U(1) coset theory recently introduced by Witten. We study the spectrum of this conformal field theory, and give explicit representations for the tachyon vertex-operators in terms of SL(2, ℝ) matrix elements. This is used to compute the scattering of strings off the black hole and to show that the string propagator exhibits Hawking radiation. We further discuss the role of winding states and the appearance of bound states in the euclidean solution. We find that target-space duality in the lorentzian theory interchanges the black hole horizon with the space-time singularity. We conclude with a comparison with the non-critical $c = 1$ string and its formulation as a gauged SL(2, ℝ) WZW model.

supertube source perturbations



- This circular supertube corresponds to the fivebrane state with only a single transverse scalar mode populated macroscopically

$$|\Psi\rangle = \Big(\underbrace{|{+}{+}\rangle}_{\perp\text{ scalar}}{}_k\Big)^{N/k}$$

- ½-BPS string vertex operators $\mathcal{V}^{\alpha\dot\beta}_{j,w_y}$ (NS·NS) and $\mathcal{S}^{AB}_{j,w_y}$ (R·R) implement transitions

$$\mathcal{V}^{\alpha\dot\beta}_{j,w_y}\ :\ \big(|{+}{+}\rangle_k\big)^{2j+1}\ \longrightarrow\ |\alpha\dot\beta\rangle_{(2j+1)k+w_y n_5}$$

$$\mathcal{S}^{AB}_{j,w_y}\ :\ \big(|{+}{+}\rangle_k\big)^{2j+1}\ \longrightarrow\ |AB\rangle_{(2j+1)k+w_y n_5}$$

← 16 extra of these for K3

- These perturbations take us toward arbitrary ½-BPS backgrounds

bosons (⊥ scalars, internal); fermions (absent for K3)

$$|\Psi\rangle = \prod_{\substack{k,\ell,p,q\\ pol's\ I}} \Big(\underbrace{|\alpha\dot\beta\rangle}_{\perp\text{ scalars}}{}_k\Big)^{N_k^{\alpha\dot\beta}} \Big(\underbrace{|AB\rangle}_{\text{internal}}{}_\ell\Big)^{N_\ell^{AB}} \Big(|\alpha B\rangle_p\Big)^{N_p^{\alpha B}} \Big(|A\dot\beta\rangle_q\Big)^{N_q^{A\dot\beta}}$$

½-BPS worldsheet perturbations

- These ½-BPS vertex operators are supergraviton-like, *e.g.*

$$\mathcal{V}^{++}_{j,w_y} = \big(J\bar{J}\,\Phi^{sl}_{j+1}\big)_{j;j,j}\,\Phi^{su}_{j;j,j}\; e^{iw_yR_y(t+\tilde{y})} \qquad + \ldots$$

Clebsch — winding on $\mathbb{S}^1_y$

$$\mathcal{V}^{--}_{j,w_y} = \Phi^{sl}_{j+1}\,\big(J\bar{J}\,\Phi^{su}_{j}\big)_{j+1}\; e^{iw_yR_y(t+\tilde{y})} \qquad + \ldots$$

$$\mathcal{S}^{AB}_{j,w_y} = \big(S^A\bar{S}^B\,\Phi^{sl}_{j+\frac{1}{2}}\,\Phi^{su}_{j-\frac{1}{2}}\big)_{j|j}\; e^{iw_yR_y(t+\tilde{y})} \qquad + \ldots$$

BPS condition selects highest weight sates in SL(2,R) and SU(2)

Kutasov-Larsen-Leigh '98

subject to the Virasoro and null gauge constraints.

EJM-Massai-Turton '18, '20
Bufalini-Iguri-Kovensky-Turton '22

- There is a 1-1 map of ½-BPS worldsheet vertex operators and ½-BPS deformations. The generic state is obtained by condensing these operators (exponentiating them into the worldsheet action). One sees that indeed the LM backgrounds are coherent string condensates bound to the **NS5**'s.

¼-BPS (& Stringy) perturbations

- There are **¼-BPS** generalizations; some are in sugra and some are stringy, *e.g.*

½-BPS $\mathcal{V}^{++}_{j,w_y} = \left(J\bar{J}\,\Phi^{sl}_{j+1}\right)_{j;j,j}\,\Phi^{su}_{j;j,j}\,e^{iw_y R_y(t+\tilde{y})}$ $+\dots$

¼-BPS *sugra* $\mathcal{V}^{++}_{j,n,m} = \left(J\bar{J}\Phi^{sl}_{j+1}\right)_{j;j+n,j}\,\Phi^{su}_{j;j-m,j}\,\exp\left[i\frac{n_y}{R_y}(t+y)\right]$ $+\dots$

not highest wt ⇒ not BPS on left — carrying momentum on $\mathbb{S}^1_y$

¼-BPS *Stringy* $\mathcal{P}(L\ osc.)\left(\bar{J}\Phi^{sl}_{j+1}\right)_{j,j+n,j}\,\Phi^{su}_{j,j-m,j}\,\exp\left[iw_yR_y(t+\tilde{y})+i\frac{n_y}{R_y}(t+y)\right]$ $+\dots$

L stringy excitations — not L highest wt ⇒ not BPS on L — carrying momentum **and** winding on $\mathbb{S}^1_y$ ⇒ $n+m = kn_y = \frac{k}{w_y}N_L$ (level matching)

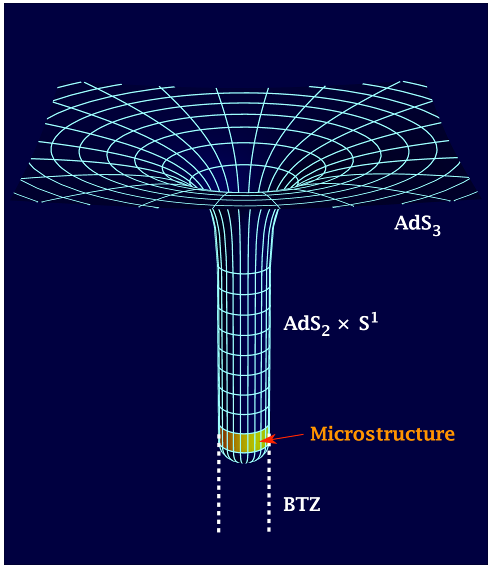
oscillator excitations required by world sheet constraints

- ***There are many more (perturbative) stringy deformations than supergravity deformations, yet still falling short of the BTZ entropy.***

Nonlinear ¼-BPS deformations

- *superstrata* are the *exact* supergravity solutions obtained by condensing the ¼-BPS *supergravity* vertex ops into the WS action

- All the known *superstratum* modes correspond to ¼-BPS supergravity vertex ops in the worldsheet construction. But there are a few more, suggesting generalizations



- *The finest modings are absent* – c.o.m. momentum excitations have moding $\propto 1/k$, string oscillators contribute momentum $\propto 1/w_y$, while the cycle length created by the vertex operator is $(2j+1)k+w_y n_5$

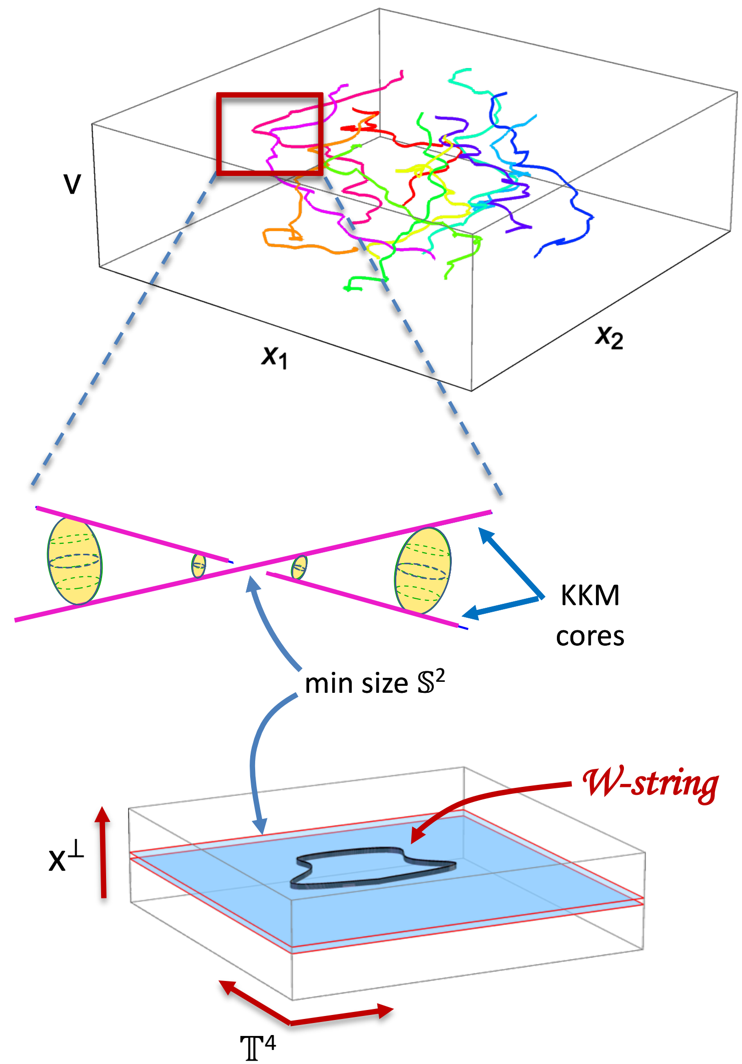
- Consistent with the idea that the finest modings are *little string* excitations rather than **F1** excitations.

The breakdown of supergravity

- Smooth supergravity solutions are associated to an "abelianized" regime of fivebrane dynamics, where the fivebrane source is well-separated from itself in its transverse space as it wiggles around.
- High redshifts are associated to more compact sources, with smaller radius of gyration. The "Higgsing" of non-abelian fivebrane dynamics goes away as we approach the black hole threshold
- We can see the restoration of non-abelian dynamics develop locally where the wiggly fivebrane profile develops a self-intersection . . .
- The partial restoration of non-abelian structure is manifested by the appearance of light "W-strings" of *little string theory* which are expected to be the entropic degrees of freedom in the black hole phase.
- ***Note that we are seeing all of this in the bulk gravity description of the AdS_3/CFT_2 regime*** *(because NS5's are solitonic and have large back-reaction)*

supertube singularities

- The geometry has a **KKM** core located along the contour $\mathbf{x} = F(v)$ where a fibered circle smoothly degenerates.
- Local **KKM** pairs form a minimal $\mathbb{S}^2$ that shrinks away if/where they collide
- Singularities arise when the profile $F(v)$ self-intersects; this describes *intersecting* **NS5**-*branes* in the effective geometry
- The singularity is resolved by D3-branes wrapping the vanishing $\mathbb{S}^2$. These are the *little W-strings* of nonabelian fivebrane dynamics; string perturbation theory breaks down . . .



Solvable models of *little strings* ?

- The BTZ entropy is accounted for by little string oscillations along $\mathbb{T}^4$ rather than **F1**'s wiggling in all 10 dimensions.

- One can corroborate this thesis by *eliminating* the $\mathbb{T}^4$. This eliminates little string entropy and makes a bulk *noncritical* string theory on $AdS_3 x \mathbb{S}_b^3$

- The dual spacetime CFT is once again a deformation of a symmetric product $(\mathcal{M}^N)/S_N$ (by a $\mathbb{Z}_2$ twist interaction that implements string interactions), but now $\mathcal{M} = \mathbb{R}_\phi x \mathbb{S}_b^3$ describes the configuration space of **F1**'s *transverse* to the fivebranes rather than the space of little strings *internal* to the fivebranes as one has in the critical dimension (think of these as different phases for **F1**'s)

(Balthazar-Giveon-Kutasov-EJM '21)

- There are many intriguing parallels with another non-critical string duality – that of 2d string theory and matrix QM – including **(1)** the presence of the radial direction in the configuration space on both sides of the duality; **(2)** asymptotically free dynamics; and **(3)** *absence of BH's in the spectrum*.

TOPOLOGICAL STRINGS IN $d < 1$

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LOOP EQUATIONS AND VIRASORO CONSTRAINTS IN NON-PERTURBATIVE TWO-DIMENSIONAL QUANTUM GRAVITY

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We give a derivation of the loop equation for two-dimensional gravity from the KdV equations and the string equation of the one-matrix model. We find that the loop equation is equivalent to an infinite set of linear constraints on the square root of the partition function satisfying the Virasoro algebra. We give an interpretation of these equations in topological gravity and discuss their extension to multi-matrix models. For the multi-critical models the loop equation naturally singles out the operators corresponding to the primary fields of the minimal models.

Summary

- AdS_3 string theory is a limit of *little string theory* holography

- Mode fractionation is a key to understanding BTZ black hole entropy. Ordinary **F1** strings are missing a factor of n_5 in their fractionation of momentum modes; *little strings* are appropriately fractionated

- We see this in the BPS spectrum of strings in an exactly solvable worldsheet construction of supertubes and superstrata (microstate geometries) in the critical dimension. *Little strings* appear as light excitations at the threshold of black hole formation.

- Further evidence comes from "noncritical LST" $AdS_3 \times \mathbb{S}_b^3$ models. These lack a large *little string* phase space but the dual CFT is again a symmetric product. The asymptotic density of states consists of highly excited **F1** strings – the additional fractionation is absent, and correspondingly there are no black holes in the spectrum. Are there integrable structures as in 2d strings?

Gefeliciteerd Erik & Herman!