# **LECTURE 2**

(direct) CP violation neutral meson mixing measuring an oscillation frequency

### **CP** violation

• what is it?

#### CP-symmetry violation (CP):

$$P(A \to B) \neq P(\bar{A} \to \bar{B})$$

decay width:  $\Gamma$  [E] cross-section:  $\sigma$  [L<sup>2</sup>]

- why is it interesting?
  - big picture: matter versus anti-matter
  - small picture: precision tests of SM (electroweak symmetry breaking)

$$A_f = \mathcal{A}(B^0 \to f) = \langle f | H_{\text{weak}} | B^0 \rangle$$

$$= \sum_{\text{diagrams}} \text{"CKM matrix elements"} \times \langle f | \mathcal{O} | B^0 \rangle$$

$$\bar{A}_{\bar{f}} = \mathcal{A}(\bar{B}^0 \to \bar{f}) = \langle \bar{f} | H_{\text{weak}} | \bar{B}^0 \rangle$$

 $= \sum_{\text{diagrams}} \text{"CKM matrix elements"} \times \left\langle \bar{f} \right| \mathcal{O} \left| \bar{B}^0 \right\rangle$ 



• from amplitudes to counting events

$$\Gamma(i \to f) = \int |\mathcal{A}(i \to f)|^2 \, \mathrm{d(phase space)}$$

(silly representation of Fermi's Golden Rule)

- decay width proportional to amplitude-squared
- consequence: no CP asymmetry if there is only a single 'diagram'

## Interfering amplitudes

• CP violation is consequence of (at least two) interfering amplitudes:



• need both "CP-violating" and "CP-conserving" phases

$$\mathcal{A}_f \equiv \frac{N(\overline{B} \to \overline{f}) - N(B \to f)}{N(\overline{B} \to \overline{f}) + N(B \to f)} = \frac{2\sin\Delta\phi_w \sin\Delta\phi_s}{|A_1/A_2| + |A_2/A_1| + 2\cos\Delta\phi_w \cos\Delta\phi_s}$$

• sizable only if interfering amplitudes are of similar magnitude

## weak vs strong phases

- <u>weak (CP-odd) phase</u>: changes sign with CP transformation
  - only phases from Higgs-Yukawa couplings
  - called 'weak' because only affects (charged) weak interaction vertices
- <u>strong (CP-even) phase</u>: does not change sign with CP transformation
  - phases come from 'time-evolution'
    - simplest example:  $\Phi(t) = e^{-iEt}\Phi(0)$
    - same for particle and anti-particle
  - called `strong' because in SM strong interaction generates non-trivial CP-even phases

CP violation is a true quantum interference effect ۲  $A_1$  $A_1$ *K*<sup>-</sup>  $K^+$ CP  $B^0$  $B^0$ **``** <u>\_\_\_\_</u> 0 •  $\pi$  $\pi^{\tau}$  $A_2$  $A_2$ 

• as we shall see, there are different sources for the interference

# Example: $B^0 \rightarrow K^+ \pi^-$

"CKM-suppressed tree-diagram"





## Example: $B^0 \rightarrow K^+ \pi^-$



"penguin-diagram"



- penguin amplitude: 'loop-suppressed'
- tree amplitude: 'CKM suppressed'

similar sized amplitudes with different weak phases!

• SM computation difficult because of 'hadronic stuff': A ~ -0.1

#### Direct CP-violation in B->Kpi

$$B^{0} \rightarrow K^{+}\pi^{-}$$
 decays anti- $B^{0} \rightarrow K^{-}\pi^{+}$  decays









### Neutral meson mixing

• SM has small set of 'stable' neutral mesons:

name	quark content	$\mathrm{mass/MeV}$	lifetime/ps
$K^0$	$d\overline{s}$	498	180
$D^0$	$c ar{u}$	1865	0.41
$B^0$	$d\overline{b}$	5280	1.5
$B_s^0$	$s \overline{b}$	5367	1.5

- meson and anti-meson state 'degenerate' if only strong+EM
  - weak interaction 'mixes' the meson and anti-meson states
  - removes degeneracy
  - leads to interesting mixing and CPV phenomenology

## Mixing diagram

• leading order diagrams for neutral meson mixing:





- sensitive to CP-violating phases
- sensitive to virtual quark masses

# Mixing formalism

• consider a two-component state

$$|\psi(t)\rangle = a(t) |B^{0}\rangle + b(t) |\bar{B}^{0}\rangle$$
  
eigenstates of  $H_{\text{strong}} + H_{\text{em}}$ 

• time-evolution follows from Schrodinger equation

$$i\hbar \frac{\mathrm{d}\psi}{\mathrm{d}t} = H\psi$$

what is H?

# Mixing formalism

• without weak interaction, solution is, in "rest frame" (E=m)



# Mixing formalism

• without weak interaction, solution is, in "rest frame" (E=m)



- with weak interaction
  - states mix
  - states 'decay'  $\rightarrow$  described by 'open system Hamiltonian'

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

open system  $\rightarrow$  not Hermitian H

reflects transitions into Hilbert space of final states

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = M - \frac{i}{2}\Gamma$$
$$M \equiv \frac{1}{2}(H + H^{\dagger})$$
$$\Gamma \equiv i(H - H^{\dagger})$$
Hermitian matrices

$$P(t) = |e^{-i(M-i\Gamma/2)t}|^2 = e^{-\Gamma t}$$

"exponential decay

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix}$$

- CPT invariance:  $M_{11}=M_{22}$  and  $\Gamma_{11}=\Gamma_{22}$
- $\rightarrow$  6 real parameters:



#### Interpretation of off-diagonal elements



#### interference!

#### diagonalize: mass eigen values

compute eigenvalues of H (details: your homework)

$$\mu_{\rm H,L} = M_{11} - \frac{i}{2}\Gamma_{11} \pm \sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} \qquad \text{H: "heavy"} L: "light"}$$

mass and width of the mass eigenstates are

$$m_L \equiv \operatorname{Re}(\mu_L) \qquad \Gamma_L \equiv -2\operatorname{Im}(\mu_L)$$
$$m_H \equiv \operatorname{Re}(\mu_H) \qquad \Gamma_H \equiv -2\operatorname{Im}(\mu_H)$$

• mass and width usually expressed in terms of average and difference

$$m \equiv (m_H + m_L)/2 = M_{11}$$

$$\Gamma \equiv (\Gamma_H + \Gamma_L)/2 = \Gamma_{11}$$

$$\Delta m \equiv m_H - m_L = 2 \operatorname{Re} \left( \sqrt{(M_{12} - \frac{i}{2}\Gamma_{12}) (M_{12}^* - \frac{i}{2}\Gamma_{12})} \right)$$

$$\Delta \Gamma = \Gamma_H - \Gamma_L = -4 \operatorname{Im} \left( \sqrt{(M_{12} - \frac{i}{2}\Gamma_{12}) (M_{12}^* - \frac{i}{2}\Gamma_{12})} \right)$$

• mass and width usually expressed in terms of average and difference

$$m = M_{11}$$

$$\Gamma = \Gamma_{11}$$

$$\Delta m = 2 \operatorname{Re} \left( \sqrt{\left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)} \right)$$

$$\Delta \Gamma = -4 \operatorname{Im} \left( \sqrt{\left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)} \right)$$
also hard to compute, but this is where the interesting physics is!

#### mass eigenstates

define 'light' and 'heavy' mass states

$$|B_L\rangle = p |B^0\rangle + q |\overline{B}^0\rangle$$
$$|B_H\rangle = p |B^0\rangle - q |\overline{B}^0\rangle$$

these are eigenstates of H if

۲

$$p = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}.$$

normalization:

$$|p|^2 + |q|^2 = 1$$

- common phase of p and q can be absorbed in states
- the physical parameters are phase and magnitude of p/q

### CP violation in mixing

• suppose  $\arg(\Gamma_{12}) = \arg(M_{12}) \equiv \phi_M$ 

$$\frac{p}{q} = \sqrt{\frac{M_{12} - i\Gamma_{12}/2}{M_{12}^* - i\Gamma_{12}^*/2}} \longrightarrow e^{i\phi_M}$$

- we shall later see that  $|q / p| \neq 1$  leads to "<u>CP violation in mixing</u>"
- in case of no CPV in mixing, we have

$$\Delta m \stackrel{CP}{=} 2|M_{12}|$$

$$\Delta \Gamma \stackrel{CP}{=} |\Gamma_{12}|$$

### **CP** eigenstates

flavour eigenstates 

$$\left| K^{0} \right\rangle = \left| d\bar{s} \right\rangle$$
$$\left| \bar{K}^{0} \right\rangle = \left| \bar{d}s \right\rangle$$

$$|K_{\text{even}}\rangle = \frac{1}{\sqrt{2}} \left( |K^0\rangle + |\bar{K}^0\rangle \right)$$
$$|K_{\text{odd}}\rangle = \frac{1}{\sqrt{2}} \left( |K^0\rangle - |\bar{K}^0\rangle \right)$$

**CP** eigenstates ۲

if we can produce an 'even' (or 'odd) state, then without CP violation, • it will remain even (or odd)

### epsilon

• alternative parametrization, traditional for Kaon system

$$\frac{p}{q} = \frac{1+\epsilon}{1-\epsilon}$$
 CPV in mixing:  $\operatorname{Re}(\epsilon) \neq 0$ 

• mass eigenstates expressed in terms of the CP-even and CP-odd states

$$|K_{\text{light}}\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} \left[ |K_{\text{even}}\rangle + \epsilon |K_{\text{odd}}\rangle \right]$$
$$|K_{\text{heavy}}\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} \left[ |K_{\text{odd}}\rangle + \epsilon |K_{\text{even}}\rangle \right]$$

in practice, epsilon turns out to be small

### summary of definition of states

• flavour eigenstates: can be produced in hadronization process

$$\left| K^{0} 
ight
angle = \left| d ar{s} 
ight
angle$$
  
 $\left| ar{K}^{0} 
ight
angle = \left| ar{d} s 
ight
angle$ 

• CP eigenstates: states with definite CP

$$\begin{aligned} |K_{\text{even}}\rangle &= \frac{1}{\sqrt{2}} \left( \left| K^{0} \right\rangle + \left| \bar{K}^{0} \right\rangle \right) \\ |K_{\text{odd}}\rangle &= \frac{1}{\sqrt{2}} \left( \left| K^{0} \right\rangle - \left| \bar{K}^{0} \right\rangle \right) \end{aligned}$$

mass eigenstates: eigenstates of 'open' Hamiltonian

$$\begin{aligned} |K_{\text{light}}\rangle &= p |K^{0}\rangle + q \left| \bar{K}^{0} \right\rangle &= \frac{1}{\sqrt{1 + |\epsilon|^{2}}} \left[ |K_{\text{even}}\rangle + \epsilon |K_{\text{odd}}\rangle \right] \\ |K_{\text{heavy}}\rangle &= p \left| \bar{K}^{0} \right\rangle - q |K^{0}\rangle &= \frac{1}{\sqrt{1 + |\epsilon|^{2}}} \left[ |K_{\text{odd}}\rangle + \epsilon |K_{\text{even}}\rangle \right] \end{aligned}$$

## K-short and K-long

• without CPV in mixing, mass eigenstates correspond to "CP eigenstates"

$$|K_{\text{light}}\rangle = \sqrt{\frac{1}{2}} \left( |K^0\rangle + |\bar{K}^0\rangle \right) \equiv |K_{\text{even}}\rangle$$
CP-even  
$$|K_{\text{heavy}}\rangle = \sqrt{\frac{1}{2}} \left( |K^0\rangle - |\bar{K}^0\rangle \right) \equiv |K_{\text{odd}}\rangle$$
CP-odd

• as a result, the  $K^0$  decays are predominantly

$$K_{\text{light}} \rightarrow \pi \pi$$
  
 $K_{\text{heavy}} \rightarrow \pi \pi \pi$ 

much less phase space: much longer lifetime!  $\Gamma_{\text{light}} \gg \Gamma_{\text{heavy}}$  "K-short"

"K-long"



- production happens through flavour eigenstates (strong interaction)
- detection (usually) also happens through flavour eigenstates (EM interaction)
- in between: propagation of mass eigenstates



• time-evolution of *mass eigenstates* follows from SE:

$$|B_{\rm L}(t)\rangle = e^{-i\,\mu_{\rm L}\,t} |B_{\rm L}\rangle \qquad |B_{\rm H}(t)\rangle = e^{-i\,\mu_{\rm H}\,t} |B_{\rm H}\rangle$$
eigenvalue of 'open' Hamiltonian

• remember: mu is complex, so this includes the decay:

$$\mu_L \equiv m_L - \frac{i}{2}\Gamma_L$$
oscillation
exponential decay

• time-evolution of *mass eigenstates* follows from SE:

$$|B_{\rm L}(t)\rangle = e^{-i\,\mu_{\rm L}\,t}\,|B_{\rm L}\rangle \qquad |B_{\rm H}(t)\rangle = e^{-i\,\mu_{\rm H}\,t}\,|B_{\rm H}\rangle$$
  
eigenvalue of 'open' Hamiltonian

• now consider time-evolution of *flavour eigenstates* 

$$|B_{\rm L}\rangle = p |B^0\rangle + q |\bar{B}^0\rangle$$

$$|B_{\rm H}\rangle = p |B^0\rangle - q |\bar{B}^0\rangle$$

$$|B_{\rm H}\rangle = \frac{1}{p} (|B_{\rm L}\rangle + |B_{\rm H}\rangle)$$

$$|\bar{B}^0\rangle = \frac{1}{q} (|B_{\rm L}\rangle - |B_{\rm H}\rangle)$$

• after doing the math, find that for state produced as  $B^0$ 

$$\left|B^{0}(t)\right\rangle = g_{+}(t)\left|B^{0}\right\rangle + \left(\frac{q}{p}g_{-}(t)\left|\bar{B}^{0}\right\rangle\right)$$

$$\left|\bar{B}^{0}(t)\right\rangle = g_{+}(t) \left|\bar{B}^{0}\right\rangle + \left(\frac{p}{q}g_{-}(t) \left|B^{0}\right\rangle\right.$$

$$B^{0} \xrightarrow{q} g_{-}(t)$$

$$B^{0} \xrightarrow{g_{+}(t)} B^{0}$$

#### with

$$g_{\pm}(t) = \frac{1}{2} \left( e^{-i\,\mu_{\rm L}\,t} \pm e^{-i\,\mu_{\rm H}\,t} \right)$$

# Mixing 'probability'

• mixing probability:

$$\Gamma(B^{0} \to B^{0}) \propto \left| \left\langle B^{0} \middle| B^{0}(t) \right\rangle \right|^{2} = |g_{+}(t)|^{2}$$
  

$$\Gamma(\bar{B}^{0} \to \bar{B}^{0}) \propto \left| \left\langle B^{0} \middle| B^{0}(t) \right\rangle \right|^{2} = |g_{+}(t)|^{2}$$
  

$$\Gamma(B^{0} \to \bar{B}^{0}) \propto \left| \left\langle B^{0} \middle| B^{0}(t) \right\rangle \right|^{2} = |g_{-}(t)|^{2} \left| \left| \frac{q}{p} \right|^{2} \right|$$
  

$$\Gamma(\bar{B}^{0} \to B^{0}) \propto \left| \left\langle B^{0} \middle| B^{0}(t) \right\rangle \right|^{2} = |g_{-}(t)|^{2} \left| \frac{p}{q} \right|^{2}$$

CP violation in mixing:  $\Gamma(B^0 \to \bar{B}^0) \neq \Gamma(\bar{B}^0 \to B^0)$ 

• what is  $|g_{\pm}(t)|^2$  ?

# Mixing probability

• after some algebra find

$$|g_{\pm}(t)|^{2} = \frac{e^{-\Gamma t}}{2} \left[ \cosh \frac{\Delta \Gamma t}{2} \pm \cos \Delta m t \right]$$
exponential decay
exponential decay
exponential decay

• in the  $B_d^0$  system,  $\Delta\Gamma \ll \Gamma$ , giving for the 'mixing asymmetry'

$$A_{\rm mix}(t) \equiv \frac{N({\rm flavour didn't change}) - N({\rm flavour changed})}{N({\rm flavour didn't change}) + N({\rm flavour changed})} = \frac{1}{2} \left(1 + \cos(\Delta m t)\right)$$

to characterize time-behaviour define  $x = \Delta m / \Gamma$ 









## Measuring the oscillation frequency



- we can identify flavour at decay by looking at final state
- but how do we know the flavour at production?

## **Flavour tagging**



# **Flavour tagging**

two different inputs:

- 1. "opposite-side flavour tagging"
  - measure the flavour of the other b-quark in the event
  - at production, it has the opposite charge
- 2. "same-side flavour tagging"
  - measure correlation with other particles during hadronization phase
  - only available at LHC (not at B-factories)

no perfect method: all information combined in MV classifier

#### For the exercise

• in the exercise we will measure  $\Delta m_d$  on a sample of  $B^0 \rightarrow \psi K^{*0}$  events



## The 'Feynman diagram'



- charge of the Kaon is related to the *flavour of the B at decay*
- in the ntuple, B flavour at decay is represented by the variable *pid*

### mistag rate

- flavour tag determines *B flavour at production*, but it makes errors:
  - oscillations of tag side B
  - picked wrong OS kaon/lepton
  - picked wrong SS pion
- output of flavour tagger in ntuple is
  - **q**: flavour of B
  - eta: estimate of mistag rate

Prob(right tag) =  $(1 - \eta)$ Prob(tag wrong) =  $\eta$ 

## Dilution from mistag rate

- tagging errors lead to a 'dilution' of observed asymmetry
- perfect tagging:

 $A_{\rm mix}(t) \equiv \frac{N({\rm flavour didn't change}) - N({\rm flavour changed})}{N({\rm flavour didn't change}) + N({\rm flavour changed})} = \frac{1}{2} \left(1 + \cos(\Delta m t)\right)$ 

• tagging with mistag rate  $\eta$ :

$$A_{\text{mix}}^{\text{obs}}(t) = (1 - 2\eta) \frac{1}{2} (1 + \cos(\Delta m t))$$
  
"dilution factor:"  $D^{\text{tag}} = (1 - 2\eta)$ 

### Asymmetry dilution due to resolution

- finite decay time resolution also leads to a 'dilution'
- effective dilution factor from gaussian resolution  $\sigma_t$ :

$$D^{\text{reso}} = \exp\left[-\left(\Delta m \,\sigma_t\right)^2\right]$$

- dilution from resolution is
  - tiny effect for measurement of  $B_d^0$  oscillations
  - big effect for measurement of  $B_s^0$  oscillations



$$N(t) = \frac{N_0}{2} e^{-t/\tau} [1 + \cos(\Delta m t)]$$



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$$N(t) = \frac{N_0}{2} e^{-t/\tau} \left[1 + \cos(\Delta m t)\right]$$

**x** D<sup>reso</sup>



$$N(t) = \frac{N_0}{2} e^{-t/\tau} \left[1 + \cos(\Delta m t)\right]$$

#### x D<sup>reso</sup>

#### + non-oscillation backgrounds



$$N(t) = \frac{N_0}{2} e^{-t/\tau} \left[1 + \cos(\Delta m t)\right]$$

#### x D<sup>reso</sup>

#### + non-oscillation backgrounds

+ acceptance effects due to cuts removing prompt backgrounds



• see README.md file at

https://github.com/wouterhuls/FlavourPhysicsBND2023/

• now: exercises 5-7